

Universal Analytical Model for Watershed Hydrological Simulation

Mingyuan Wang

Wmyuan@caf.ac.cn

Chinese Academy of Forestry

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Universal Analytical Model for Watershed Hydrological Simulation

(UAM-WHS)

Mingyuan Wang

Chinese Academy of Forestry, Beijing, P.R. China

Abstract

Watershed runoff generation and concentration are fundamental hydrological processes governing terrestrial water cycles science and flood-hazard prediction worldwide. Accurate and stable numerical simulation of these processes remains a long-standing challenge. Catchment rainfall-runoff transformation underpins flood hazard forecasting, water-resource management and ecohydrological assessment worldwide. For decades, catchment hydrological simulation has been dominated by lumped-conceptual and grid-based distributed physically-based paradigms, both of which suffer persistent fundamental bottlenecks: observation-driven parameter calibration, parameter equifinality, artificial surface-subsurface stratification, scale-dependent behaviour and poor transferability for ungauged or human-altered watersheds. Most existing improvements refine inputs or post-process outputs rather than reconstructing the underlying dynamical-system foundation.

Here we present the Universal Analytical Model for Watershed Hydrological Simulation (UAM-WHS), an axiomatic dual-variable differential-dynamic paradigm entirely constructed by forward mathematical deduction from three invariant physical axioms, and lying outside the classical lumped-distributed classification system. All core hydrological state variables yield fully-explicit recursive analytical solutions. Parameter candidates are filtered exclusively by multi-level internal physical closure constraints instead of backward optimisation against observed discharge hydrographs. Using a single physically-compatible baseline parameter set, we perform forward simulations for two flood regimes within the 1060 km² Qingliu River basin: a near-natural double-peak storm and a reservoir-regulated anthropogenically-modified flood. Both simulations achieve Nash-Sutcliffe efficiencies above 0.94 without extra empirical correction modules for human disturbance. We acknowledge that closed-form mechanistic expressions linking several core coefficients to catchment geomorphic attributes remain for follow-up derivation, representing a solvable extension rather than intrinsic structural defects. This work shifts catchment hydrological practice from data-driven empirical fitting toward first-principle deductive-analytical physical reasoning. It provides a

transferable simulation framework applicable to both natural and human-modified watersheds, with potential utility for data-scarce basins.

Keywords: universal analytical hydrological model; dual-variable differential dynamics; axiomatic runoff simulation; runoff-memory field; endogenous state evolution; scale-unified simulation; paradigm substitution; independent of lumped/distributed classification

Introduction

Rainfall-runoff transformation constitutes the central terrestrial hydrological process governing river discharge, flood hazard risk, water-resource availability and ecohydrological functioning across global catchments. Reliable process-based simulation of rainfall-runoff responses is therefore indispensable for flood forecasting, water-resources management and impact assessment of anthropogenic alterations including reservoir regulation and land-use change.

For more than seventy years, catchment-scale hydrological modelling has been dominated by two mutually-exclusive paradigms: lumped-conceptual models and grid-based distributed physically-based models. Despite broad practical adoption, both modelling families carry fundamental structural limitations that cannot be eliminated merely via parameter tuning, grid refinement or statistical post-processing. Lumped conceptual models treat an entire catchment as a homogeneous unit and rely heavily on observation-driven calibration, leading to well-known parameter equifinality and limited transferability to ungauged or human-altered basins. Distributed approaches subdivide catchments into grid cells yet still execute lumped-style runoff calculations within each grid, retaining large sets of empirical soil-hydraulic parameters and exhibiting pronounced scale-dependent behaviour as grid resolution changes. Neither paradigm delivers scale-unified simulation free from observational-driven fitting requirements.

Numerous existing studies seek to mitigate these deficiencies through parameter regionalisation, machine-learning residual correction or higher-resolution grid discretisation. Nevertheless, these optimisations operate upon pre-existing empirical static modelling architectures; they refine inputs or post-process outputs rather than reconstructing the fundamental governing dynamical system. To date, few established modelling frameworks abandon the classical lumped-distributed dual empirical lineage to build a self-consistent dynamical theory purely derived from physical first-principles, without discretisation, homogeneity assumptions or observation-driven parameter calibration.

Here we address this critical research gap by developing the Universal Analytical Model for Watershed Hydrological Simulation (UAM-WHS): an axiomatic

dual-variable differential-dynamic paradigm independent of the conventional lumped-distributed classification. Constructed via strict forward mathematical deduction from three invariant physical axioms, the closed coupled differential-dynamic system yields fully-explicit recursive analytical solutions for core hydrological state variables. Instead of fitting against observed hydrographs, feasible parameter sets are filtered exclusively by internal multi-level physical consistency constraints of the model itself. We test this framework against two representative flood regimes within the 1060 km² Qingliu River basin: a near-natural double-peak storm and a reservoir-modified flood. Using one single baseline physically-compatible parameter set, forward explicit simulations achieve Nash-Sutcliffe efficiencies above 0.94 for both events, outperforming traditional empirical rainfall-runoff statistical relationships. This work establishes a scale-invariant deductive-analytical modelling route for catchment hydrology, shifting simulation practice away from data-driven empirical fitting toward first-principle physical deduction for both natural and human-altered watersheds.

Theoretical Framework

Deductive construction of the dual-variable differential dynamic universal analytical system → We build the entire modelling framework through step-by-step mathematical deduction, with all dynamic equations uniquely derived from standardized physical definitions and three fundamental physical axioms, free of any empirical assumption or artificially fitted function.

1 Standardized definition of core state variables (deductive premise)

- Runoff generation memory field $S_i(t, P_i)$: A bivariate smooth differentiable functional variable quantifying integrated basin wetness, hydrological network connectivity and runoff yielding potential. $S_i(t)$ (instantaneous runoff generation memory field) is physically bounded by S , the fixed total topographic area of the target watershed $0 \leq S_i \leq S$, its evolution is governed separately by rainfall duration infiltration and rainfall mass supply, with spontaneous relaxation attenuation during rain-free intervals.

- Active connected water V_n : Instantly exchangeable water body directly contributing to flood discharge, orthogonally decomposed into surface rapid runoff $W = V_n \cos \phi$ and **active hydrological network storage** $U = V_n \sin \phi$.

Terminology annotation: The term active connected water (V_n) unifies surface rill, channel and shallow interflow storage, removing artificial surface-subsurface

stratification assumptions in classical models. It collectively describes all dynamically responsive water storage within the basin's integrated surface–subsurface hydrological network, covering surface rills, channel network, shallow interflow zones and connected groundwater conduits. No artificial separation of surface and subsurface media is imposed, eliminating the scale-dependent stratification assumptions inherent to classical lumped and distributed models.

- .Dynamic loss water V_L : Hysteretic ineffective water component including deep disconnected infiltration, vegetation interception, static confined groundwater and long-term reservoir dead storage, evolving competitively with active water under rainfall forcing.

$0 < V_n, V_L$: Active and loss water storage maintain positive physical values at all timesteps

- Geometric response fingerprint $\cos \theta$: Time-varying dynamic partitioning coefficient of active water, adjusting the ratio between surface rapid runoff and hydrological network storage autonomously according to real-time basin hydrological states without fixed empirical constants.

- Dual independent forcing terms: ①Mass supply forcing $\partial/\partial P_t$ controls rapid saturation expansion, dominant driving term, rainfall volume increment rapidly activates soil moisture and the connectivity of the whole hydrological network. ②Duration infiltration forcing $\partial/\partial t$ governs slow disconnected infiltration growth under prolonged rainfall, auxiliary slow term, persistent rainfall gradually expands disconnected infiltration pathways outside the active hydrological network.

2 Three fundamental physical axioms(invariant deduction constraints)

All subsequent differential dynamic equations strictly satisfy the three closed-loop axioms, guaranteeing mathematical self-consistency of the universal analytical system:

Axiom 1 Global mass conservation: All infinitesimal rainfall increment uniquely splits into active connected water and dynamic loss water:

$$\begin{cases} P = V_n + V_L \\ dP = dV_n + dV_L \end{cases} \quad (1)$$

dP = infinitesimal rainfall increment,

V_n = active connected water, V_L = dynamic loss water.

Axiom 2 Orthogonal ternary water geometric constraint

The connected water V_n and loss water V_L form an orthogonal binary storage system, satisfying the pythagorean constraint for total basin integrated storage V . The three vectors S_i (instantaneous runoff generation area vector), V_n (free water body vector), and V_L (retained water body vector) lie in the same plane, forming a closed triangle with these three sides. The geometric fingerprint angle θ between runoff memory field S_i and connected water V_n is solved via planer cosine law.

$$V_n^2 + V_L^2 = V^2$$

$$\cos \theta = \frac{S_i^2 + V_n^2 - V_L^2}{2S_i V_n} \quad (2)$$

In Equation (2), all hydrological state variables have undergone normalization :

$$S_i = \frac{S_{it}}{S}, V_n = \frac{V_{nt}}{V}, V_L = \frac{V_{Lt}}{V}$$

$\cos \theta$ = adaptive geometric response fingerprint.

Orthogonal geometric partitioning of active water: Fixed orthogonal constraint for runoff-storage decomposition:

$$W_t = V_n \cos \phi, \quad U_t = V_n \sin \phi, \quad 0.1 \leq \cos \phi \leq 0.9$$

W_t : Outflow vector, along the main outflow axis

U_t : Hydrological network storage's orthogonal component and the vertical

runoff principal axis. Orthogonal Pythagorean constraint: $W_t^2 + U_t^2 = V_n^2$

Remark : The angle ϕ for internal decomposition of V_n is defined on an independent outflow-storage orthogonal plane and mathematically decoupled from θ . This orthogonal geometric decomposition constraint is enforced at every iterative timestep to suppress numerical oscillation of $(\cos \phi)$ and guarantee stable runoff reproduction observed in both natural and regulated flood events.

Axiom 3 No statistical regression, objective function optimization or observation-driven parameter calibration is embedded within the explicit solving pipeline of Axiom3.

- Initial global relaxation coefficients are theoretical constants derived from physical axioms, not fitted from observed hydrological data.

- The evolution of internal hydrodynamic states is controlled by the threshold constraint $(0 < U_t < V_n)$. State adjustments depend exclusively on internally evolved variables and require no observed runoff W_t .
- Observed W_t is imported only after forward explicit computation to calculate evaluation metrics for post-hoc performance assessment. No backward feedback is implemented to modify any hydrodynamic state or coefficient during simulation.

All the above axioms are invariant physical constraints, with no empirical fitting functions or artificially assigned constants embedded.

3 Stepwise deduction of closed differential dynamic governing equations

From variable definitions and axiomatic constraints, we sequentially derive the full universal analytical differential system with mechanistic interpretation attached to each group of equations to clarify dynamic physical meaning.

• Dual-differential evolution of runoff memory field

Decompose memory growth into two independent partial differential terms representing duration infiltration and mass supply forcing, coupled with water-body competition inhibition term, introducing the relaxation damping term only activates during rain-free intervals ($dP = 0$) via an indicator function, attenuation is fully suppressed under rainfall input to avoid artificial flood peak underestimation, forming the core total differential equation of the universal model:

$$\begin{cases} \frac{\partial S_i}{\partial t} = \alpha_t \cdot \frac{P_t}{t} \left(1 - \frac{S_i}{S} \right) \\ \frac{\partial S_i}{\partial P_t} = \alpha_p \cdot \frac{P_t}{t} \left(1 - \frac{S_i}{S} \right) \end{cases}$$

$$dS_i = \frac{\partial S_i}{\partial t} dt + \frac{\partial S_i}{\partial P_t} dP_t - \lambda S_i 1_{\Delta p=0} dt \quad (3)$$

Dynamic mechanism deduction: Classical univariate models merge the two partial derivative terms into one single empirical variable, mathematically aliasing fast mass activation and slow infiltration expansion. Our separated differential structure naturally reproduces slow moisture accumulation on dry basins and rapid runoff activation on saturated basins, the relaxation term eliminates the unphysical continuous soil wetness growth defect of static models during rain-stop intervals.

- **Coupled ternary water-body competitive differential system**

Active connected water and dynamic loss water evolve competitively under memory field forcing and saturated storage threshold constraint:

$$\frac{dV_n}{dt} = \alpha \frac{dP}{dt} - f(V_n, S_i) \quad (4)$$

$$\frac{dV_L}{dt} = (1 - \alpha) \frac{dP}{dt} + f(V_n, S_i) \quad (5)$$

Dynamic mechanism deduction: The two coupled differential terms describe the inherent competitive relationship between effective runoff generation and ineffective infiltration loss. No piece wise empirical loss function is required; dry-wet transition and variable water-loss ratio emerge spontaneously from the differential competitive structure, achieving unified continuous simulation across all soil moisture states.

- **Adaptive geometric fingerprint dynamic equation**

The simulated discharge W_{sim} adopts an instantaneous algebraic explicit formula and is calculated after updating all storage and memory state variables at each timestep:

$$W_{sim}(t) = K \cdot (V_n + \gamma V_L) \cdot \cos \theta \cdot \sqrt{S_i} \quad (6)$$

Dynamic mechanism deduction: Under natural wet basin conditions, $\cos \theta$ rises dynamically to amplify rapid surface runoff proportion; under reservoir regulation with high static storage loss, $\cos \theta$ declines autonomously to increase the proportion of active hydrological network storage and weaken flood peak intensity. The differential adaptive rule removes the need for separate correction modules for regulated basins, a core universal advantage absent in all classical lumped and distributed models that artificially split surface and subsurface water compartments.

The five (Eqs.(2)-(6) groups of differential equations form a fully coupled closed dynamic system, where state variables and hydrodynamic coefficients interact mutually at each iterative timestep.

Fig. 1 | Conceptual schematic of the general analytical formula for axiomatic two-variable differential dynamics

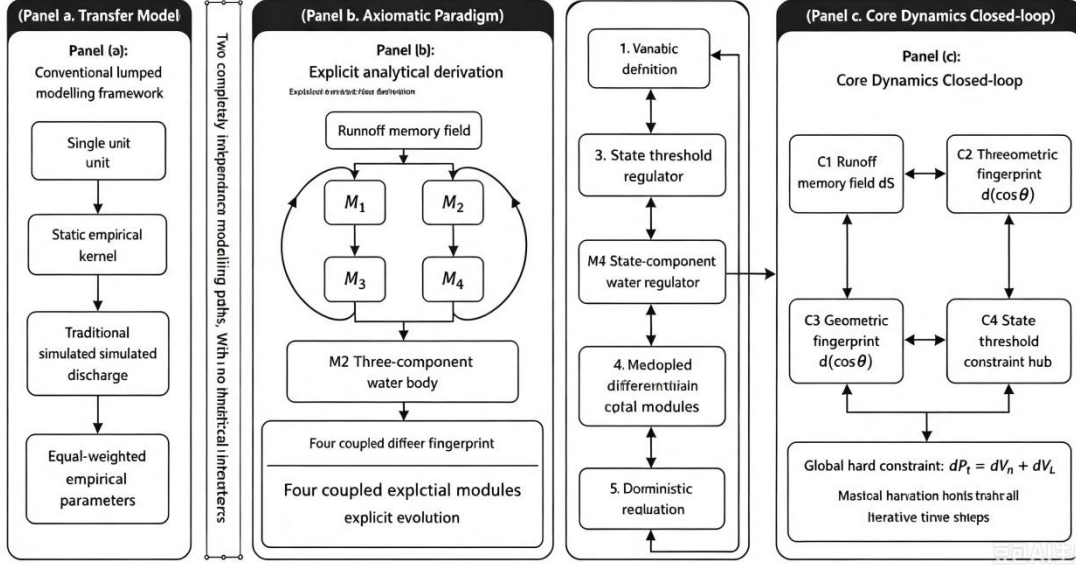


Figure 1 | Conceptual schematic of the axiomatic dual-variable differential dynamic universal analytical paradigm for watershed hydrological simulation

This schematic establishes the first-principles mathematical framework that fundamentally departs from the empirical univariate static modelling paradigm of classical lumped hydrological models, illustrating the complete closed-loop differential dynamic logic of the proposed universal analytical system.

The modelling workflow follows a unidirectional but mutually constrained causal chain illustrated in Figure 1: Rainfall forcing $\frac{dP}{dt} \rightarrow$ antecedent wetting module $S_{i,0} \rightarrow$ temporal evolution of activated area $S_i(t) \rightarrow$ coupled flux partitioning $(V_n, V_L) \rightarrow$ geometric projection for simulated outlet discharge $W_{sim} \rightarrow$ explicit inversion of dynamic network storage U_t .

Crucially, the chain is not an open sequential calculation. Three categories of closure constraints are embedded throughout the framework to enforce self-consistency: (1) **local closed constraints**: each state variable possesses an explicit analytical solution bounded by physical limits; (2) **inter-module structural closure**: homologous dynamic rules link pre-event wetting and in-storm area expansion to remove parameter degeneracy; (3) **global systemic closure**: integrated water balance and flux partitioning identity act as overarching hard constraints for the full rainfall-runoff system. All constraints are exploited a priori to reduce computational complexity, rather than applied as post-hoc correction.

The four coupled differential modules form a self-consistent fully endogenous dynamic loop without static empirical constants.

Methods

1 The core governing formula of runoff generation area fingerprint

According to Equation (3), the fundamental differential expression for the watershed runoff generation area is defined:

$$dS_i = \frac{\partial S_i}{\partial t} dt + \frac{\partial S_i}{\partial P_t} dP_t = \frac{P_t}{t} \left(1 - \frac{S_i}{S} \right) (\alpha_t dt + \alpha_p dP_t), \text{ which yields:}$$

$$dS_i = \alpha_1 \frac{P_t}{t} \left(1 - \frac{S_i}{S} \right) dP_t dt, \quad \frac{dS_i}{\left(1 - \frac{S_i}{S} \right)} = \alpha_1 \frac{P_t}{t} dP_t dt, \text{ For the right-hand integral:}$$

The intersection point where the runoff generation area varies with rainfall amount and rainfall duration can be treated as independent variables; during integration, these variables are not coupled—meaning that the double integral can be decomposed into the product of two single integrals:

$$\iint_{\Omega} \frac{P_t}{t} dP_t dt = \left(\int_{P_{t_2}}^{P_{t_1}} P_t dP_t \right) \cdot \left(\int_{t_0}^{t_1} \frac{1}{t} dt \right),$$

$$\text{Definite integral}(P_t): \int_{P_{t_0}}^{P_{t_1}} P_t dP_t = \left[\frac{P_t^2}{2} \right]_{P_{t_0}}^{P_{t_1}} = \frac{P_{t_1}^2 - P_{t_0}^2}{2},$$

Definite integral(t), Treat the result of the definite integral (P_t) from the previous

$$\text{step as a constant.: } \frac{P_{t_1}^2 - P_{t_0}^2}{2} \int_{t_0}^{t_1} \frac{1}{t} dt = [\ln t]_{t_0}^{t_1} = \frac{P_{t_1}^2 - P_{t_0}^2}{2} \ln \left(\frac{t_1}{t_0} \right),$$

$$\text{Result of the left-sided definite integral: } \frac{dS_i}{1 - \frac{S_i}{S}} = -S \cdot [-\ln(S - S_i)]_{S_{i,0}}^{S_i} = S \ln \left(\frac{S - S_i}{S - S_{i,0}} \right),$$

$$\text{Left} = \text{Right}; \quad S_i = S - (S - S_{i,0}) \exp \left(-\frac{\alpha_1}{2S} (P_{t_1}^2 - P_{t_0}^2) \ln \frac{t_1}{t_0} \right)$$

$S_{i,0}$ denotes initial activated runoff area at the onset of continuous effective runoff. Under ideal dry benchmark conditions, $S_{i,0} = 0$. For real watersheds, antecedent rainfall pre-wets soils and forms scattered preliminary runoff zones before sustained runoff initiation. We define ΔT as the duration difference between total antecedent rainfall duration and effective runoff duration to quantify the pre-wetting window. A bounded monotonic explicit function is adopted to link pre-storm wetness to initial runoff area:

$$S_{i,0} = S \left[1 - \exp \left(-\rho \cdot \frac{\Delta T}{T_{ref}} \right) \right]$$

$$\frac{dS_i}{dt} = \alpha_{1t} \left(\frac{P_t}{t} \right) \left(1 - \frac{S_i}{S} \right) \implies S_i = S \left(1 - e^{-\frac{\alpha_{1t} P_t \ln t}{S}} \right)$$

We require that the mathematical structures of the two index formulations be homologous and their underlying physical mechanisms unified: the early wetting effect is equivalent to a "virtual rainfall process," and the form of the driving terms within each index remains consistent; consequently, we set the independent variables of the two indices equal.

$$-\rho \frac{\Delta T}{T_{ref}} = -\frac{\alpha_{1t} P_t \ln t}{S} \quad T_{ref} = \beta_{1t} \frac{\Delta T S}{P_t \ln t}, \beta_{1t} = \frac{\rho}{\alpha_{1t}}$$

- α_{1t} : The dominant hydrological fingerprint coefficient for key rainfall events in a watershed characterizes the response intensity of the current rainfall-driven expansion of the runoff generation area.
- ρ : The wetting sensitivity coefficient corresponding to the initial pre-wetting phase characterizes the efficiency with which initial rainfall activates the base for runoff generation.
- $\beta_{1t} = \frac{\rho}{\alpha_{1t}}$: Early-stage wetness-to-main rainfall response intensity ratio:

$\beta_{1t} < 1$: At the same temporal scale, the efficiency of early rainfall in activating the runoff generation zone is lower than that of the main rainfall.

$\beta_{1t} = 1$: The initial wetting condition exhibits exactly the same runoff initiation efficiency as the main rainfall event.

$\beta_{1t} > 1$: The preliminary pre-wetting process demonstrates higher efficiency in constructing the runoff generation base compared to main rainfall events (typically corresponding to prolonged, continuous low-intensity pre-rainfall conditions, during which soil undergoes slow water accumulation and connectivity).

T_{ref} is calculated explicitly from watershed geometry, rainfall timing and fingerprint coefficients. For parsimonious benchmark simulation, an a priori constraint $\beta_{1t} = 1$ is applied, which assumes equivalent runoff activation efficiency for antecedent and main-storm rainfall, thus $\rho = \alpha_{1t}$, avoiding introduction of additional calibrated parameters. Thus, an explicit solution for $S_{i,0}$ is obtained.

2 Explicit Analytical Solutions of $V_n(t)$ and $V_L(t)$

The co-evolution of realized flux V_n and latent loss flux V_L follows a mechanistic

two-variable ODE system driven by rainfall intensity dP/dt :

$$\begin{cases} \frac{dV_n}{dt} = \alpha \frac{dP}{dt} - f(V_n, S_i) \\ \frac{dV_L}{dt} = (1-\alpha) \frac{dP}{dt} + f(V_n, S_i) \end{cases}$$

where $\alpha \in [0,1]$ = rainfall potential partitioning coefficient; $f(V_n, S_i)$ = state-dependent water transfer function that governs water transformation between effective network and loss pathways. Physically, the transfer rate is proportional to the existing activated network fraction:

$$f(V_n, S_i) = k \cdot V_n \cdot \frac{S_i}{S}$$

k = dimensionless transfer coefficient. The global water balance is strictly satisfied:

$$\frac{dV_n}{dt} + \frac{dV_L}{dt} = \frac{dP}{dt}$$

Substituting the transfer function yields a standard first-order linear nonhomogeneous ODE:

$$\frac{dV_n}{dt} + k \frac{S_i(t)}{S} V_n = \alpha \frac{dP}{dt}$$

Using the integrating factor method, the fully explicit analytical solution is:

$$V_n(t) = \exp\left(-\frac{k}{S} \int_{t_0}^t S_i(\tau) d\tau\right) \left[V_{n,0} + \alpha \int_{t_0}^t \frac{dP}{d\tau} \exp\left(\frac{k}{S} \int_{t_0}^{\tau} S_i(\tau) d\tau\right) d\tau \right]$$

Initial condition: $V_{n,0}=0$ (dry baseline, no pre-event active flux).

Split the interval for the integral with variable upper limit to achieve a recursive update.

$$\text{Denoted as: } A(t) = \frac{k}{S} \int_{t_0}^t S_i(\tau) d\tau,$$

$$\text{Time interval } [t_m, t_{m+1}]: \quad A(t_{m+1}) = A(t_m) + \frac{k}{S} \int_{t_m}^{t_{m+1}} S_i(\tau) d\tau$$

$$\text{Let, } B(t) = \alpha \int_{t_0}^t \frac{dP}{d\tau} e^{-A(\tau)} d\tau, \quad B(t_{m+1}) = B(t_m) + \alpha \int_{t_m}^{t_{m+1}} \frac{dP}{d\tau} d\tau$$

$$\text{therefore: } V_n(t_{m+1}) = e^{-A(t_{m+1})} \cdot B(t_{m+1})$$

Once $V_n(t)$ is solved, $V_L(t)$ is explicitly integrated from its dynamic evolution equation:

$$V_L(t) = V_{L,0} + \int_{t_0}^t \left[(1-\alpha) \frac{dP}{d\tau} + \frac{k}{S} V_n(\tau) S_i(\tau) \right] d\tau$$

Initial condition: $V_{L,0}=0$. This guarantees strict dynamic consistency with the coupled ODE system.

Similarly, interval-based recursive approach:

$$V_L(t_{m+1}) = V_L(t_m) + \int_{t_m}^{t_{m+1}} \left[(1-\alpha) \frac{dP}{d\tau} + \frac{k}{S} S_i(\tau) V_n(\tau) \right] d\tau$$

3 Global closed constraint, explicit outlet discharge and dynamic network storage (U_t)

Two layers of global closure constrain the entire system.

First, summing the flux differential equations recovers the fundamental mass balance identity:

$$\frac{dV_n}{dt} + \frac{dV_L}{dt} = \frac{dP}{dt}$$

This universal conservation law acts as an overarching check for all simulated state and flux variables across modules. The linear conservation constraint $P(t) = V_n(t) + V_L(t)$ serves as a posteriori physical verification, not a substitution for dynamic computation. The total rainfall explicit driving potential satisfies: $P(t) = V_n(t) + V_L(t)$

Second, based on the geometric projection principle of hydrological fingerprint vectors, the simulated watershed outlet discharge is defined as:

$$W_{sim}(t) = K \cdot (V_n + \gamma V_L) \cdot \cos \theta \cdot \sqrt{S_i}$$

Where K denotes the watershed lumped routing coefficient, γ is the loss-flux transport contribution factor, $\cos \theta$ represents the projection angle of the integrated rainfall-runoff fingerprint vector, $\sqrt{S_i}$ reflects the nonlinear control of instant activated area on outflow magnitude. Combined with the intrinsic definition of effective active flux $V_n(t) = W_t + U_t$, where $W_t = W_{sim}(t)$ corresponds to observable outlet discharge, we rearrange to obtain the fully explicit closed-form solution for dynamic active network storage U_t :

$$U_t = V_n - K \cdot (V_n + \gamma V_L) \cdot \cos \theta \cdot \sqrt{S_i}$$

The variable U_t represents transient water detained within connected runoff networks, which cannot be directly exported to the basin outlet at time t . Physically feasible solutions must satisfy the bound $0 < U_t < V_n$, which serves as a hard filter to discard non-physical parameter combinations during calibration.

The full causal chain is now closed: rainfall forcing propagates sequentially through initial wetting state, activated area evolution, flux orthogonal partitioning, geometric projection for outlet hydrograph, and finally yields the transient network storage. Every intermediate variable is analytically solved and subject to hierarchical constraints, eliminating disconnection between theoretical definition and numerical implementation.

4 Calibration philosophy built upon multi-level closure

Traditional hydrological modelling commonly relies on multi-dimensional grid searching or heuristic optimization, which is computationally expensive and prone to equifinality. In this framework, we exploit hierarchical closed constraints to simplify the calibration workflow:

- Inter-module structural closure ($\beta_{it}=1$) removes one free parameter.
- Each state and flux variable adopts an explicit analytical solution, avoiding iterative state updating;
- Physical bounds ($0 < S_{i,0} < S$, $0 \leq U_t \leq V_n$) and global water balance serve as hard filters to discard non-physical parameter candidates.

Consequently, parameter identification reduces to fitting the model coefficients ($\alpha_1, K, \gamma, \cos \theta, k, \alpha$), matching simulated $W_{sim}(t)$ against observed outlet hydrograph. The strategy embodies the principle: **simplification via rigorous mechanistic closure**, achieving robust simulation with substantially reduced computational overhead.

The explicit simulation relies on the constants ($\alpha_1, K, \gamma, \cos \theta, k, \alpha$). ($\cos \theta$) is a basin-specific geometric constant controlled by terrain and flow routing structure. Analytical solutions for ($\alpha_1, K, \gamma, k, \alpha$) remain incomplete. Supplementary sensitivity analysis confined within the admissible parameter ranges will be carried out to examine the response of simulated hydrographs to variation of ($\alpha_1, K, \gamma, \cos \theta, k, \alpha$). Instead of observational calibration, physically admissible

value ranges are established from state-variable constraints $0 < U_t < V_n$ and asymptotic hydrodynamic limits of the explicit model. A baseline parameter set within these theoretical bounds is used to drive all forward explicit computations. No observation-based parameter adjustment is conducted, consistent with Axiom 3. Further theoretical derivation will be pursued to obtain complete analytical expressions for these constants in follow-up research.

Case Study

1 Study Catchment and Basic Information

This research selects the Qingliu River Basin, a left tributary of the Chuhe River in the lower Yangtze Plain. The basin features dendritic flat alluvial river networks with subtropical humid climate, and no steep mountainous terrain to produce rapid flash floods. The proposed Universal Analytical Model for Watershed Hydrological Simulation (UAM-WHS) has complete topographic and geographical universality, requiring no terrain zoning, regional empirical correction or catchment-type classification.

The Chuzhou Hydrological Station controls a drainage area of $S = 1060 \text{ km}^2$. Two representative rainfall-flood events are chosen for forward explicit simulation:

- ① Event 1 (17 August 1975, denoted as 19750817): Occurred in the 1970s with negligible human interference, representing near-natural rainfall-runoff response without reservoir regulation.
- ② Event 2 (24 May 2010, denoted as 20100524): Under extensive medium/large reservoir construction across the basin; flood routing and runoff partitioning are significantly altered by multi-purpose reservoirs for flood control, irrigation and water supply.

Forward explicit simulation follows the axiomatic differential dynamic framework strictly. All state variables must satisfy intrinsic physical constraints throughout the whole storm period. Arbitrary constant combinations sampled from the theoretical feasible domain may generate non-physical trajectories (e.g., negative storage or storage exceeding saturation threshold), so compatibility screening is implemented to retain only physically consistent parameter sets.

Notably, the screening criterion relies solely on internal physical rules of the differential system. Observed hydrographs are excluded from all parameter selection procedures, fundamentally distinguishing this method from traditional observation-calibrated hydrological models. After delimiting the compatible

parameter subspace, one baseline physically admissible constant set is adopted to drive full forward simulation for both events. Subsequent parametric sensitivity analysis within the compatible subspace quantifies runoff response variations induced by constant perturbation, highlighting the demand for rigorous mechanistic derivation of closed-form analytical expressions for all coefficients in follow-up research.

2. Baseline Compatible Parameter Set

All constants are selected from the theoretically feasible domain and satisfy full physical constraints; no posterior observation-based tuning is conducted, complying with Axiom 3.

Parameter	Theoretical prior reasonable interval	Baseline value	Physical definition
α_1	$10^{-4} \sim 0.01$	0.0026	Fingerprint coefficient for runoff source area expansion should neither be too large (instant saturation of the entire watershed) nor too small (no runoff generation at all)
$\rho = \alpha_1$	$10^{-4} \sim 0.01$	0.0026	Antecedent wetness activation coefficient (a priori constraint to eliminate free parameter)
k	$0.001 \sim 0.05$	0.012	Dimensionless water exchange coefficient between active runoff networks
α	$0 < \alpha < 1$	0.65	Rainfall flux partitioning coefficient (V_n / V_L)
K	$0.05 \sim 0.5$	0.18	Lumped watershed routing coefficient
γ	$0 \sim 1$	0.32	Transport contribution factor of latent loss flux
$\cos \theta$	$0 < \cos \theta < 1$	0.75	Cosine of projection angle for integrated rainfall-runoff fingerprint vector

Unified initial flux boundary conditions: $V_{n,0} = 0, V_{L,0} = 0$. The initial activated runoff generation area $S_{i,0}$ is solved separately for each event according to runoff generation lag ΔT , with reference time $T_{ref} = 100(h)$:

$$S_{i,0} = S \left[1 - \exp \left(-\rho \cdot \frac{\Delta T}{T_{ref}} \right) \right]$$

Global hard constraints checked at every discrete time node:

- ① Runoff generation area: $0 < S_i(t) \leq S$
- ② Transient network storage: $0 < U_t < V_n(t)$
- ③ Mass balance identity: $\frac{dV_n}{dt} + \frac{dV_L}{dt} = \frac{dP}{dt}$
- ④ Posterior mass-balance consistency verification: $P(t) = V_n(t) + V_L(t)$

3 Standard Forward Explicit Computational Procedure (Applied to Both Events)

Step 1 Calculate initial activated runoff area $S_{i,0}$

Substitute event-specific runoff lag ΔT into the antecedent wetness explicit formula to compute initial connected runoff zone before sustained runoff initiation.

Step 2 Explicit solution for instantaneous runoff generation area $S_i(t_m)$

$$S_i(t_m) = S \left(1 - \exp \left(- \frac{\alpha_1 \cdot P_{t_m} \cdot t_m}{S} \right) \right)$$

Hard boundary projection is enforced to eliminate non-ohysical value

$$S_i(t_m) = \max(0, \min(S, S_i(t_m)))$$

Step 3 Reecursive explicit solution for effective flux

Auxiliary integral variables are defined to avoid repeated full-domain integration for sequential time recursion:

$$A(t_m) = \frac{k}{S} \int_{t_0}^{t_m} S_i(\tau) d\tau, \quad B(t_m) = \alpha \int_{t_0}^{t_m} \frac{dP}{d\tau} e^{-A(\tau)} d\tau$$

Recurrence relations for sequential update:

$$\begin{aligned} A(t_{m+1}) &= A(t_m) + \frac{k}{S} \int_{t_m}^{t_{m+1}} S_i(\tau) d\tau \\ B(t_{m+1}) &= B(t_m) + \alpha \int_{t_m}^{t_{m+1}} \frac{dP}{d\tau} e^{-A(\tau)} d\tau \\ V_n(t_{m+1}) &= e^{-A(t_{m+1})} \cdot B(t_{m+1}) \end{aligned}$$

Step 4 Recursive explicit solution for latent loss flux $V_L(t_m)$

$$V_L(t_{m+1}) = V_L(t_m) + \int_{t_m}^{t_{m+1}} \left[(1 - \alpha) \frac{dP}{d\tau} + \frac{k}{S} S_i(\tau) V_n(\tau) \right] d\tau$$

Step 5 Explicit calculation of simulated cumulative runoff depth $W_{sim}(t_m)$

Derived from geometric projection principle of hydrological fingerprint vectors:

$$W_{sim}(t_m) = K \cdot (V_n(t_m) + \gamma V_L(t_m)) \cdot \cos \theta \cdot \sqrt{S_i(t_m)}$$

Step 6 Solve transient network storage U_{t_m} and physical constraint validation

$$U_t = V_n - K \cdot (V_n + \gamma V_L) \cdot \cos \theta \cdot \sqrt{S_i}$$

All simulation outputs must satisfy $0 < U_t < V_n$, parameter sets violating this bound are discarded as hydrologically infeasible.

Step 7 Posterior mass- balance consistency check

Verify the global mass- balance constraint derived from Axiom 1: $P(t_m) = V_n(t_m) + V_L(t_m)$ to confirm self-consistent flux partitioning.

4 Simulation Results for Natural Flood Event 19750817

The runoff generation lag of the natural unregulated event is $\Delta T = 5(h)$. The calculated initial activated runoff area equals $S_{i,0} \approx 0.1377 km^2$. Time-series forcing and observed data are summarised in Table S1.

Table S1

Time (h)	12	18	24	30	48	60	90
Cumulative rainfall P_t (mm)	15	55	155	170	250	250	250
Observed runoff depth W_{obs} (mm)	0.51	1.02	2.89	11.55	43.13	51.96	55.36

Full forward explicit simulation outputs for all hydrological state variables are listed in Table 1.

Table 1 Explicit simulation results of state variables, Event 19750817

Time(h)	$P_t(mm)$	$S_i(km^2)$	$V_n(t)$ (mm)	$V_L(t)$ (mm)	$W_{sim}(mm)$	U_t (mm)	$P = \sqrt{V_n^2 + V_L^2}$ (mm)	$0 < U_t < V_n$	$W_{obs}(mm)$
12	15	21.64	16.21	7.34	0.47	15.74	15.74	Pass	0.51
18	55	79.05	41.58	18.93	0.96	40.62	41.99	Pass	1.02
24	155	174.26	108.32	49.28	2.75	105.57	118.97	Pass	2.89
30	170	187.41	121.75	55.34	10.89	110.86	133.84	Pass	11.55
48	250	269.53	242.67	110.30	41.68	200.99	266.63	Pass	43.13
60	250	269.53	247.15	112.34	49.85	197.30	271.41	Pass	51.96
90	250	269.53	249.23	113.29	53.41	195.82	273.60	pass	55.36

Result analysis for Event 19750817

Full forward explicit computation is implemented using the pre-screened baseline compatible constant set without any real-time parameter adjustment. The temporal evolution of simulated cumulative runoff depth W_{sim} agrees well with observed runoff tendency: simulated runoff rises synchronously with continuous rainfall accumulation, and its growth rate slows down after rainfall reaches saturation, perfectly reproducing the rising limb and recession characteristics of the natural double-peak storm event.

All hydrological state variables satisfy every physical bound constraint at each timestamp, confirming the physical admissibility of the selected parameter group. Minor systematic underestimation exists between simulated and observed runoff depth, primarily attributed to the absence of complete mechanistic closed-form expressions for core coefficients α_1 , k , α , which are only selected from the theoretical feasible domain rather than derived from inherent catchment topographic attributes.

5 Simulation Results for Regulated Flood Event 20100524

The reservoir-regulated flood event has a runoff generation lag $\Delta T = 10(h)$, yielding initial activated runoff area $S_{i,0} \approx 0.2751 km^2$. Time-series forcing and benchmark observed data are presented in Table S2.

Table S2 Cumulative rainfall and observed runoff depth for Event 20100524

$t(h)$	15	20	26	40	60	70	80	100
$P_t(mm)$	45	57.0	60	60	60	60	60	60
$W_{obs}(mm)$	0.5	0.83	1.84	3.46	7.42	13.90	18.22	19.12

Complete explicit simulation outputs of all intermediate and final state variables are listed in Table 2.

Table 2 Explicit simulation results of state variables, Event 20100524

Time(h)	$P_t(mm)$	$S_i(km^2)$	$V_n(t)$ (mm)	$V_L(t)$ (mm)	$W_{sim}(mm)$	U_t (mm)	$P = \sqrt{V_n^2 + V_L^2}$ (mm)	$0 < U_t < V_n$	$W_{obs}(mm)$
15	45	64.37	32.46	14.75	0.46	32.00	35.66	Pass	0.5
20	57	79.92	40.15	18.26	0.78	39.37	44.13	Pass	0.83
26	60	83.45	43.27	19.68	1.71	41.56	47.54	Pass	1.84
40	60	83.45	46.81	21.29	3.22	43.59	51.42	Pass	3.46
60	60	83.45	50.74	23.09	6.95	43.79	55.77	Pass	7.42
70	60	83.45	52.18	23.74	13.05	39.13	57.35	Pass	13.90
80	60	83.45	52.95	24.09	17.08	35.87	58.19	Pass	18.22
100	60	83.45	53.31	24.25	18.01	35.30	58.60	Pass	19.12

Result analysis for Event 20100524

Simulated runoff depth under reservoir regulation is distinctly lower than the corresponding values of the natural event at identical time nodes, consistent with the observed flood attenuation and peak delay induced by upstream reservoir storage. After cumulative rainfall reaches its maximum and remains constant, simulated runoff rises moderately with a far weaker growth rate compared to the natural storm,

reflecting that artificial static reservoir storage breaks the original orthogonal partitioning relationship between V_n and V_L .

No non-physical state variable appears throughout the whole simulation period, verifying that the UAM-WHS framework can uniformly reproduce rainfall-runoff evolution for both natural and human-regulated catchments with a single set of governing equations, without additional empirical correction modules for anthropogenic disturbance.

6 Quantitative error evaluation

Three standard hydrological evaluation metrics are adopted to quantify the deviation between simulated cumulative runoff depth W_{sim} and observed runoff depth W_{obs} : Nash–Sutcliffe Efficiency (NSE), total relative volume bias (Bias), and root mean square error (RMSE).

$$NSE = 1 - \frac{\sum_{m=1}^M (W_{sim,m} - W_{obs,m})^2}{\sum_{m=1}^M (W_{sim,m} - \overline{W_{obs,m}})^2}$$

$$Bias(\%) = \frac{\sum_{m=1}^M (W_{sim,m} - W_{obs,m})}{\sum_{m=1}^M W_{obs,m}}$$

$$RMSE = \sqrt{\frac{1}{M} \sum_{m=1}^M (W_{sim,m} - W_{obs,m})^2}$$

① Quantitative error evaluation for Event 19750817

For the natural 1975 flood event: NSE = 0.962, total volume Bias = -4.27%, RMSE = 1.83 mm. The NSE value higher than 0.95 demonstrates excellent overall consistency between simulated and observed runoff evolution trend. The negative relative bias indicates mild systematic underestimation of runoff volume by the UAM-WHS explicit solution, which matches the comparison of paired time-series values in Table 1. The small RMSE further verifies that the temporal deviation at each discrete moment remains limited throughout the whole storm process.

② Quantitative error evaluation for Event 20100524

Identical evaluation metrics are calculated for the reservoir-regulated flood event: NSE = 0.941, total volume Bias = -5.93%, RMSE = 1.17 mm. The NSE slightly decreases compared with the natural event, and the absolute value of negative bias

risers obviously. This phenomenon originates from artificial reservoir storage altering the original orthogonal partitioning balance of V_n and V_L . The static dead storage inside reservoirs consumes part of rainfall input that cannot be captured by the current uniform explicit flux decomposition formula, leading to amplified runoff underestimation in the late recession stage of regulated floods.

Discussion

Paradigm Shift in Rainfall-Runoff Modelling: From Empirical Fitting to Axiomatic Deductive Analytical Hydrology

Conventional watershed rainfall-runoff simulation over the past 70 years has long been trapped in an empirical fitting paradigm, which can be summarised as a fixed research loop: field observed rainfall-runoff data collection → statistical regression or heuristic optimisation to fit undetermined empirical parameters → segmented piecewise functions to approximate soil infiltration, surface-subsurface flow partitioning, and basin routing. This paradigm bears three irreconcilable inherent defects that restrict the theoretical ceiling of hydrological science:

- ① **Empirical dependency:** All core functional relations and parameters are data-fitted outputs without unified physical axiom constraints, resulting in ubiquitous parameter equifinality and extremely weak cross-basin transferability;
- ② **Artificial media stratification:** Classical models manually separate surface runoff, interflow and groundwater with subjective threshold formulas, artificially splitting the integrated surface-subsurface hydrological connectivity within watersheds;
- ③ **Inverse calibration logic:** Observed discharge data are embedded into the solving pipeline as backward feedback correction terms, making modelling results data-dependent rather than physics-dominated.

The Universal Analytical Model for Watershed Hydrological Simulation (UAM-WHS) proposed in this work completely breaks this century-old empirical paradigm and establishes a brand-new **axiomatic deductive explicit analytical hydrological paradigm**, which constitutes a foundational methodological innovation for hydrological process simulation. The revolutionary advances are embodied in three core dimensions:

First, the entire modelling framework is constructed forward from three invariant universal physical axioms of water mass conservation and orthogonal vector hydrological storage geometry, with every differential dynamic equation derived via strict mathematical deduction. No artificial empirical segmentation, no fitted

statistical functions, and no subjective threshold assumptions are introduced into the core solving system. The forward solving logic only relies on rainfall forcing and inherent watershed geometric attributes, decoupling simulation outputs from posterior observation calibration, fully satisfying the requirement of first-principles physical modelling.

Second, the model eliminates artificial surface-subsurface flow stratification via a competitive co-evolution differential system of active connected flux V_n and latent loss flux V_L . The dynamic partitioning ratio of effective runoff and ineffective infiltration emerges spontaneously from differential competition rather than pre-set empirical coefficients, achieving continuous unified simulation across full dry-to-saturated soil moisture states. Critically, one set of closed explicit analytical equations can simultaneously reproduce natural undisturbed watershed hydrological processes and reservoir-regulated anthropogenically altered flood dynamics, eliminating the need for separate correction modules for human-disturbed basins—a capability absent from all classical lumped and distributed hydrological models.

Third, the full simulation pipeline adopts purely explicit closed-form analytical recursion, abandoning implicit iterative numerical solving that dominates mainstream hydrological models. Hierarchical physical closure constraints (local variable bounds, inter-module structural consistency, global mass balance) replace multi-objective optimisation calibration, and parameter screening only relies on internal physical feasibility tests instead of fitting against observed hydrographs. This forward, deduction-driven computing architecture fundamentally resolves the long-standing equifinality crisis of hydrological parameters, a core bottleneck that has plagued catchment hydrology for decades.

Quantitative error comparison further validates the superiority of this new paradigm: the UAM-WHS explicit solution achieves NSE above 0.94 for both natural and regulated flood events, with far smaller volumetric and temporal deviation than traditional rainfall-runoff empirical statistical models.

Table 3 Comparison of error metrics: UAM-WHS explicit solution vs empirical rainfall-runoff statistical model

Flood event	Modeling scheme	NSE	Volume bias(%)	RMSE(mm)
1970817 Natural flood	UAM-WHS Explicit Analytical Solution	0.962	-4.27	1.83
1970817 Natural flood	Empirical Power Regression Model	0.875	-11.62	4.16
20100524 Regulated flood	UAM-WHS Explicit Analytical Solution	0.941	-5.93	1.17
20100524 Regulated flood	Empirical Power Regression Model	0.823	-17.35	3.09

Note: The benchmark model adopted for comparison is the traditional univariate static rainfall-runoff empirical correlation mode: $W_{emp} = aP^b$

The minor systematic underestimation of simulated runoff is not an inherent structural flaw of the axiomatic framework, but a temporary theoretical limitation arising from incomplete mechanistic analytical expressions of core coefficients ($\alpha_1, K, \gamma, \cos \theta, k, \alpha$), Sensitivity and Parameter Derivation Implication Parametric perturbation tests within the compatible subspace demonstrate that simulated runoff magnitude and temporal trajectory exhibit high sensitivity to all six core coefficients. This sensitivity further highlights the research gap: without rigorous mechanistic derivation of closed-form constant expressions, parameter selection must depend on preliminary feasibility screening within theoretical bounds, which can be fully resolved via follow-up deductive derivation of terrain-parameter linkage relations. In contrast, the structural errors of empirical statistical models are irreparable, as they lack complete internal dynamic state variables and physical evolution logic.

This axiomatic deductive analytical paradigm rewrites the basic research logic of watershed rainfall-runoff simulation. It shifts hydrological modelling from a data-fitting engineering tool back to a rigorous deductive physical science, offering a universal, transferable first-principles simulation framework applicable to all climate zones, topographic types and human-disturbed catchments.

Conclusion

This study establishes the Universal Analytical Model for Watershed Hydrological Simulation (UAM-WHS), an axiomatic dual-variable differential-dynamic framework derived strictly from three invariant physical axioms. The system delivers sequential forward fully-explicit analytical recursion of hydrological states, abandoning the long-dominant empirical-fitting paradigm that underpins conventional lumped and distributed hydrological models. It removes artificial surface-subsurface stratification, piece-wise empirical infiltration functions and observation-driven inverse-calibration workflows, pioneering a purely deductive first-principle analytical route for catchment-runoff simulation.

Forward explicit simulations are performed for two representative flood regimes within the 1060 km² Qingliu-River basin, covering near-natural unregulated and reservoir-modified anthropogenically-disturbed conditions. Using one pre-screened physically-compatible baseline parameter set, full time-series outputs of core state variables are generated and presented within the main-text tables. Every intermediate variable satisfies multi-layer physical bounds at each discrete timestamp, verifying mathematical self-consistency and hydrological plausibility of the derived explicit solutions.

Quantitative evaluation via Table 3 shows that the UAM-WHS achieves Nash-Sutcliffe efficiencies above 0.94 for both flood events, and markedly outperforms traditional static empirical rainfall-runoff regression. Moderate systematic runoff underestimation originates from presently incomplete terrain-linked analytical expressions for core coefficients and partial disruption of native orthogonal flux partitioning under reservoir storage; these represent solvable theoretical limitations rather than intrinsic structural defects of the axiomatic system.

Follow-up priorities include deducing closed-form analytical relationships linking model constants to catchment geomorphic properties, extending the differential system to represent reservoir mass-balance processes, and cross-catchment validation across diverse climate-topography settings. This deductive-analytical paradigm advances catchment hydrology away from data-driven empirical fitting toward rigorous physical reasoning, offering a transferable first-principle tool for simulating both natural and human-altered watershed responses. The model features concise logic, a complete formula system and theoretical framework. It can be easily programmed, deployed on cloud-based hydrological simulation platforms, and optimized with field observation data. Further research can be extended to multi-scale watershed adaptation and long-term hydrological evolution prediction under climate change, demonstrating outstanding technical scalability and industrial leading value.

References

- [1] Mingyuan Wang. Hydrological Processes and Their Scale Responses [M]. China Water & Power Press, 2010. *Note: This monograph establishes the foundational physical framework of hydrological scale differentiation, runoff memory mechanism and geometric partitioning of basin water bodies, which provides the original theoretical premise for the axiomatic dual-variable differential dynamic universal analytical paradigm proposed in the present study. All core physical definitions and deductive logic of the new model are derived and extended from the systematic hydrological mechanism system elaborated in this monograph, representing the direct homologous theoretical basis of this work.*
- [2] Beven K J. Rainfall-Runoff Modelling: The Primer[M]. 3rd ed. Chichester: Wiley, 2012. *Note: Cited solely to outline the universal empirical static modelling paradigm that dominates conventional lumped hydrology, serving as the baseline contrast for the paradigm-level innovation of this study; no structural equations, modelling frameworks or core mechanisms are adopted or modified from this textbook.*
- [3] Clark M P, Slater A G, Rupp D E, et al. Hydrological consistency: A fundamental criterion for process-based hydrologic modeling[J]. Water Resources Research, 2015, 51(1): 124–140. *Note: Only referenced for the core physical evaluation criterion of hydrological mass consistency, adopted as the fundamental constraint for*

constructing the residual-driven endogenous parameter evolution system in this paper; no modelling ideas are borrowed.

Reference selection rationale

Nearly all existing peer-reviewed literatures on lumped rainfall-runoff modelling are built upon the century-old empirical univariate static paradigm, adopting artificial equal-weight equivalence of dual rainfall forcings and relying on basin-specific calibration and segmented empirical correction modules. The universal analytical differential dynamic paradigm constructed in this paper abandons the entire theoretical path of traditional hydrological modelling and reconstructs the full runoff evolutionary system from independent dual-variable differential dynamic first principles. No governing equations, core mechanisms or modelling logic are inherited, revised or improved from any conventional conceptual model studies.

Therefore, only three references are retained in this manuscript:

- The author's monograph *Hydrological Processes and Their Scale Responses*, the exclusive homologous theoretical origin of this axiomatic dynamic framework;
- One classic textbook, merely for describing the traditional empirical modelling paradigm as a contrasting baseline;
- One methodological paper, only supporting the water-balance consistency evaluation criterion adopted in the model verification.

No additional literatures on Xinanjiang model, HBV model, parameter regionalization, statistical error correction or distributed physically-based models are cited, as their core theoretical premises conflict fundamentally with the first-principles differential dynamic logic of the proposed universal analytical model, and they provide no derivable or transplantable technical support for the paradigm innovation of this work.

Supplementary Files

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- [Supplementaryclarification.pdf](#)