

Supplementary Information

Sliding-mode control of secondary depression dynamics: robustness to parameter uncertainty and external disturbances

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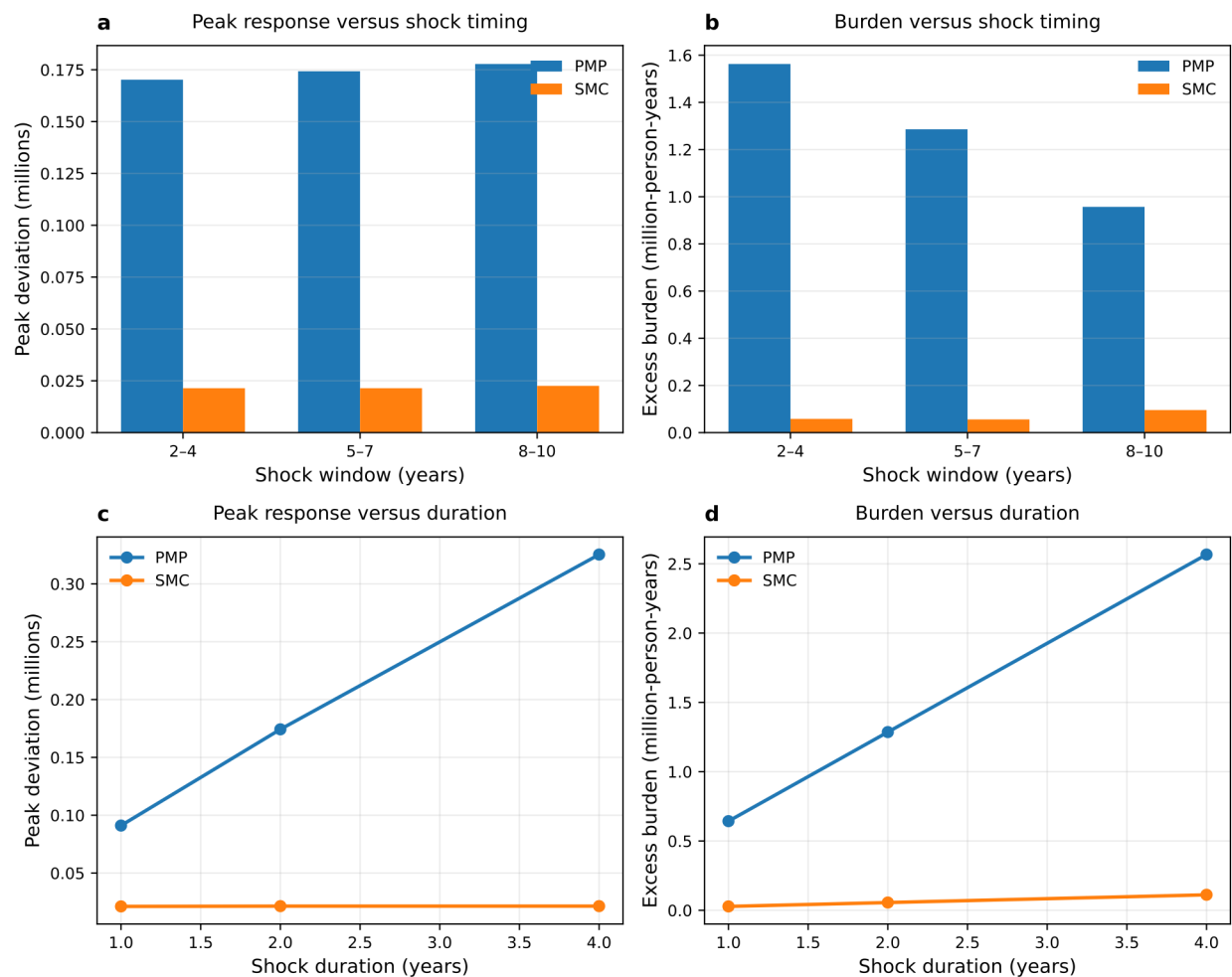
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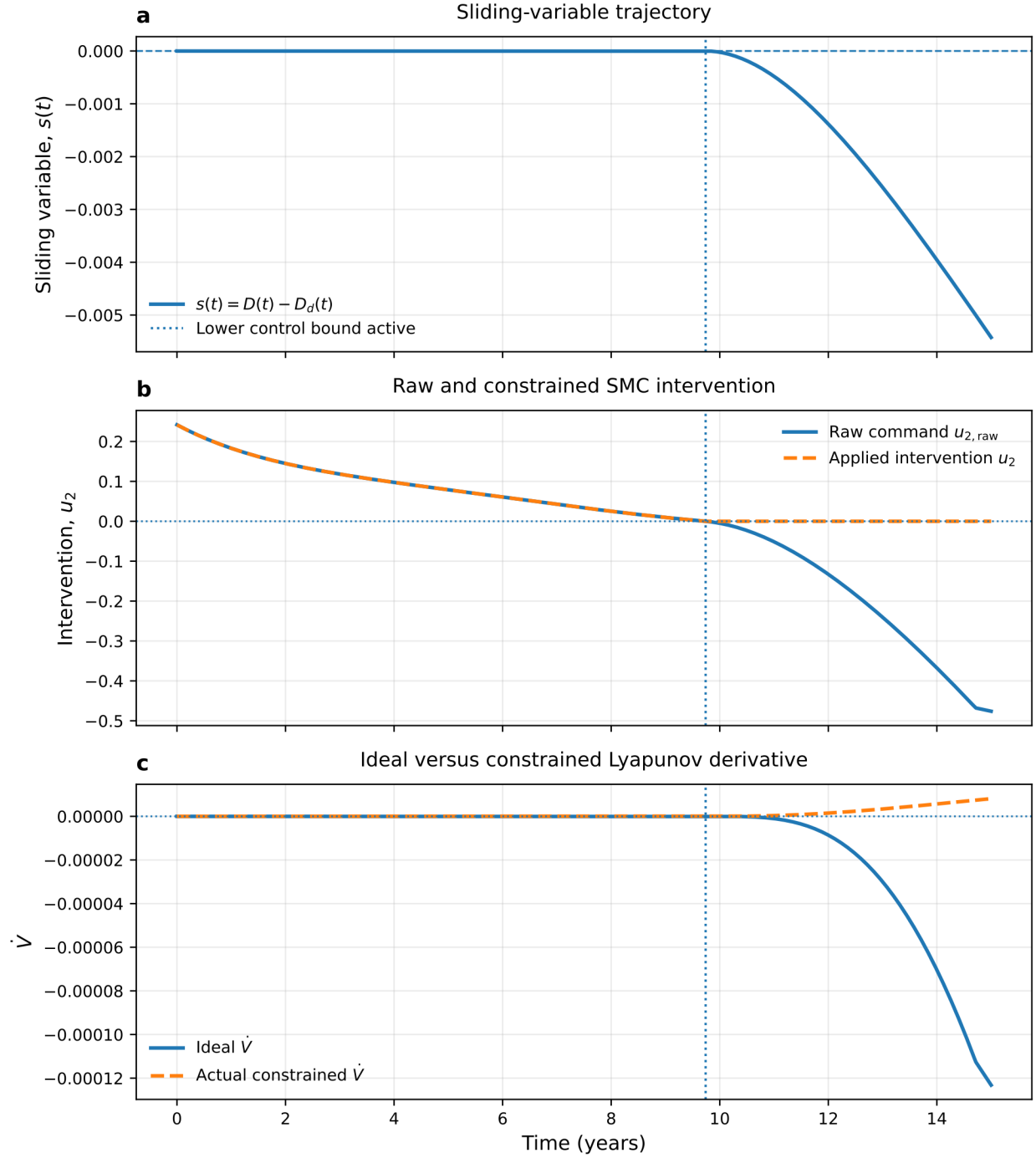
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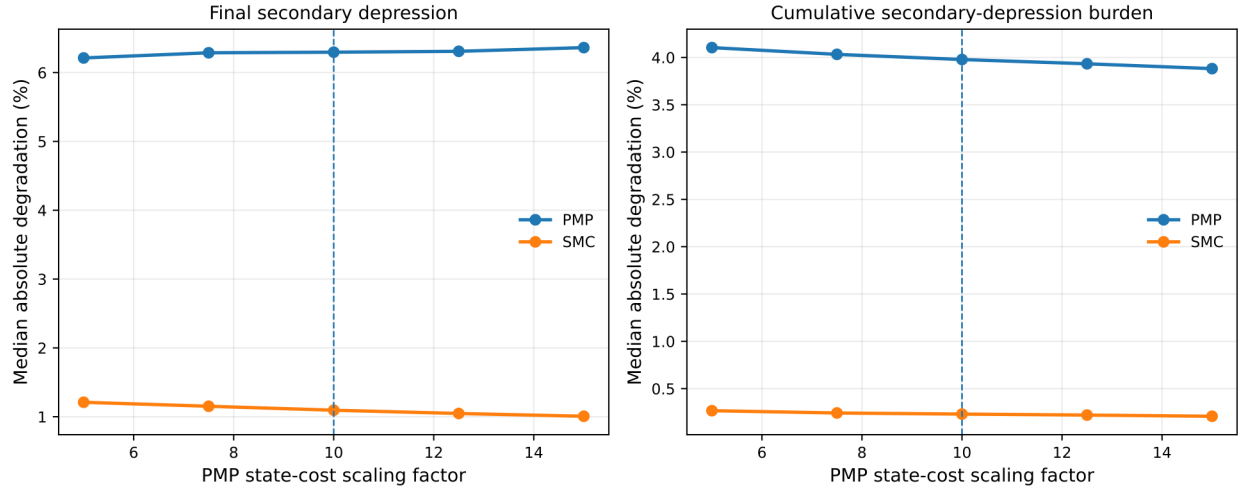
Supplementary Figure S1. Sensitivity of disturbance-response performance to shock timing and duration. Robustness of the nominal PMP and effort-matched SMC strategies was evaluated under changes in the timing and duration of the external disturbance. **(a)** Peak deviation in the

secondary-depression compartment (D) for disturbances applied during years 2–4, 5–7, and 8–10. **(b)** Corresponding excess cumulative secondary-depression burden for the three disturbance windows. **(c)** Peak deviation as a function of disturbance duration (1, 2, and 4 years). **(d)** Excess cumulative secondary-depression burden as a function of disturbance duration. Peak deviation represents the maximum absolute difference in D between the disturbed and corresponding nominal trajectory, whereas excess burden quantifies the time-integrated absolute deviation from the nominal D trajectory. Across the disturbance timings and durations examined, SMC consistently produced smaller peak deviations and excess burden than the nominal open-loop PMP strategy, demonstrating greater disturbance rejection within the evaluated scenarios.



Supplementary Figure S2. Sliding-variable, control-saturation, and Lyapunov behavior of the effort-matched SMC. (a) Sliding variable $s(t) = D(t) - D_d(t)$ for the nominal effort-matched SMC trajectory. The horizontal dashed line denotes the ideal sliding manifold $s = 0$, and the vertical dotted line indicates the first activation of the lower intervention bound. **(b)** Unconstrained SMC command $u_{2, \text{raw}}$ and applied intervention $u_2 = \text{clip}(u_{2, \text{raw}}, 0, 1)$. At approximately 9.74 years, the raw command becomes negative and the admissible lower bound

$u_2 = 0$ becomes active. (c) Lyapunov derivative computed from the ideal unsaturated reaching law, $\dot{V}_{\text{ideal}} = -Ks^2 - \eta s \text{sat}(s/\phi)$, compared with the derivative $\dot{V}_{\text{constrained}} = s\dot{s}$ evaluated along the constrained closed-loop trajectory. The ideal reaching condition is satisfied while the unconstrained command is admissible; once the lower control bound becomes active, the applied controller can no longer exactly impose the ideal reaching dynamics, and the constrained closed-loop Lyapunov derivative is not necessarily non-positive.



Supplementary Figure S3. Sensitivity of the robustness comparison to PMP reconstruction scaling. To assess whether the comparison depended on the state-cost scaling factor used in the calibrated PMP reconstruction, the PMP benchmark was reconstructed using scaling factors of 5, 7.5, 10, 12.5, and 15. At each scaling factor, the SMC reference endpoint was independently adjusted to match the nominal PMP control effort, after which both controllers were evaluated on the same 1,000 paired $\pm 20\%$ parameter-uncertainty realizations. Points show the median absolute percentage degradation relative to each controller's corresponding nominal solution for (a) final secondary depression and (b) cumulative secondary-depression burden. The dashed vertical line denotes the scaling factor of 10 used for the primary analysis. SMC retained substantially lower degradation across all PMP reconstructions.

Supplementary Table S1. Robustness across parameter-uncertainty levels.

Uncertainty	PMP final-D deg. (%)	SMC final-D deg. (%)	SMC win	PMP burden deg. (%)	SMC burden deg. (%)	SMC win
$\pm 5\%$	1.5879	0.2709	93.7%	1.0046	0.0567	97.9%
$\pm 10\%$	3.1856	0.5448	93.7%	2.0013	0.1133	97.9%
$\pm 20\%$	6.2964	1.0947	93.8%	3.9780	0.2312	97.8%
$\pm 30\%$	9.3680	1.6628	93.8%	5.9391	0.3475	98.2%

Supplementary Table S2. Disturbance-magnitude analysis.

Disturbance	Peak reduction with SMC	Extra-burden reduction	Final-residual reduction
0.0005	87.71%	95.64%	95.82%
0.0010	87.71%	95.64%	95.82%
0.0020	87.71%	95.64%	95.83%
0.0030	87.71%	95.65%	95.83%
0.0040	87.71%	95.65%	95.84%

Supplementary Table S3. Disturbance-timing analysis.

Shock window	Peak reduction	Burden reduction	Final-residual reduction
Years 2–4	87.41%	96.27%	94.56%
Years 5–7	87.71%	95.64%	95.83%
Years 8–10	87.31%	90.01%	91.10%

Supplementary Table S4. Disturbance-duration analysis.

Duration	Peak reduction	Burden reduction	Final-residual reduction
1 year	76.74%	95.64%	95.82%
2 years	87.71%	95.64%	95.83%
4 years	93.42%	95.64%	95.85%