

Design of the flight controller

This document describes the development of the controllers enabling the simulated bird to follow a moving target. These controllers regulate the motion of the bird in all six degrees of freedom of its floating base (main body). They are used to control the dynamics of a bird flying in the wake of another bird. Figure 1 defines the coordinate system used in the design of the controller, and defines the spatial variables and angles of the bird. We define the x axis as the one aligned with the bird's body, y as the one pointing toward the right wing, and z as the vertical axis, pointing down. The corresponding rotation angles are noted ϕ for the roll/bank (around x), θ for the pitch (around y) and ψ for the yaw (around z). The forces along the three axes are noted F_x , F_y and F_z , and the moments about the three axes are noted L , M and N .

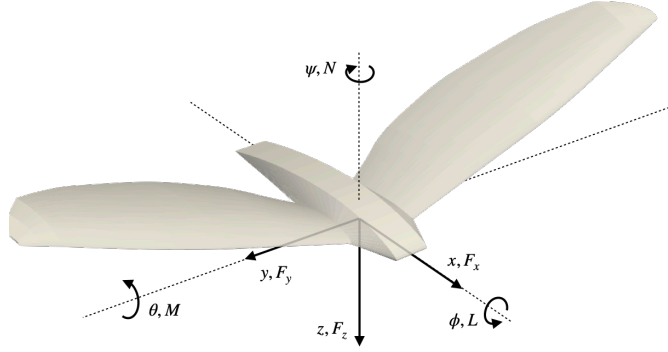


Figure 1: **Axes used in the design of the controller.** The x , y and z axes define the three translations of the bird, and the corresponding forces are F_x , F_y and F_z . The roll, pitch and yaw rotations are respectively noted ϕ , θ and ψ , and their corresponding moments are L , M and N .

1 Controller design

The main controller determines the reference velocity $\mathbf{u}_{ref}$ for the bird to reach its target. Figure 2 shows the block diagram representing this controller and illustrates the reference velocity resulting from a given configuration.

Although the C_x block of this diagram can be anything relating a velocity to the position and velocity of the target, we use the simple case of a reference velocity proportional to the distance to the target, to which the velocity of the target is added, as in

$$\mathbf{u}_{ref} = a_u \Delta \mathbf{x} + \mathbf{u}_t .$$

If the bird reaches the velocity $\mathbf{u}_{ref}$ instantly – or, more realistically, at a rate much faster than the one at which Δx varies –, the bird will reach the target, as

$$\dot{\Delta \mathbf{x}} = \mathbf{u}_t - \mathbf{u}_{ref} = -a_u \Delta \mathbf{x} ,$$

and

$$\Delta \mathbf{x}(t) = \Delta \mathbf{x}_0 \exp(-a_u t) .$$

Other controllers are dedicated to reaching the target velocity, as illustrated in Fig. 3, which describes the content of the $T_u(s)$ block in Fig. 2.

The velocity controller is split in two parts: one regulating the flight speed (the norm U of the velocity vector) and one regulating the flight direction h (the orientation of the velocity vector). The flight speed is modified by

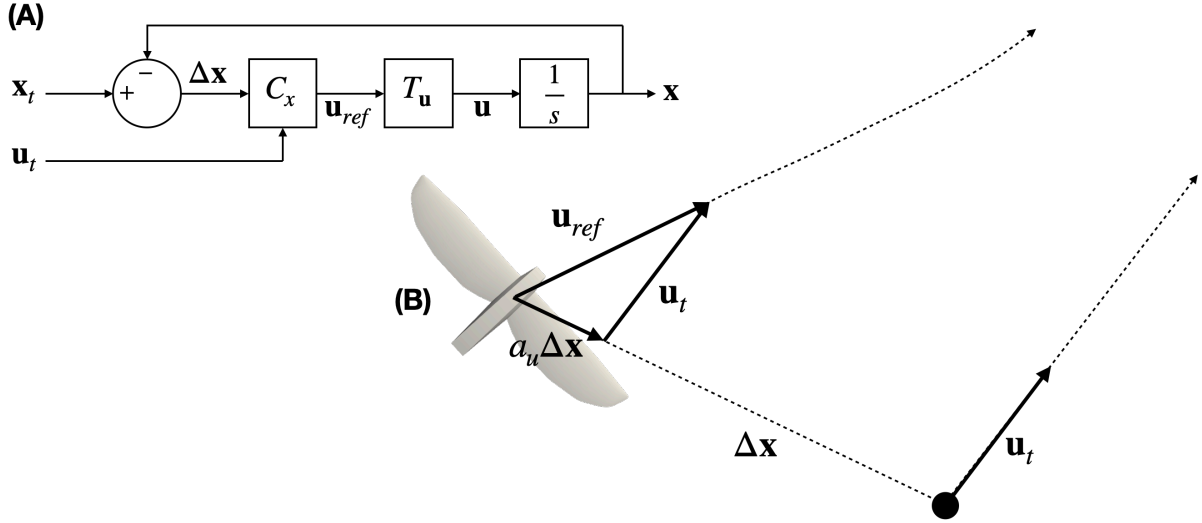


Figure 2: **Trajectory controller determining the velocity of the bird based on the relative position of the target and its velocity.** (A) Block diagram of the controller determining the velocity $\mathbf{u}$. The reference velocity $\mathbf{u}_{ref}$ is determined as a function of $\Delta \mathbf{x}$ and the velocity of the target $\mathbf{u}_t$. The actual velocity of the bird $\mathbf{u}$ results from another transfer function T_u , is integrated to get the position $\mathbf{x}$. (B) Illustration of the simple C_x used in this work, where the reference velocity $\mathbf{u}_{ref}$ is that of a target ($\mathbf{u}_t$) with an additional term proportional to the distance to the target $a_u \Delta \mathbf{x}$.

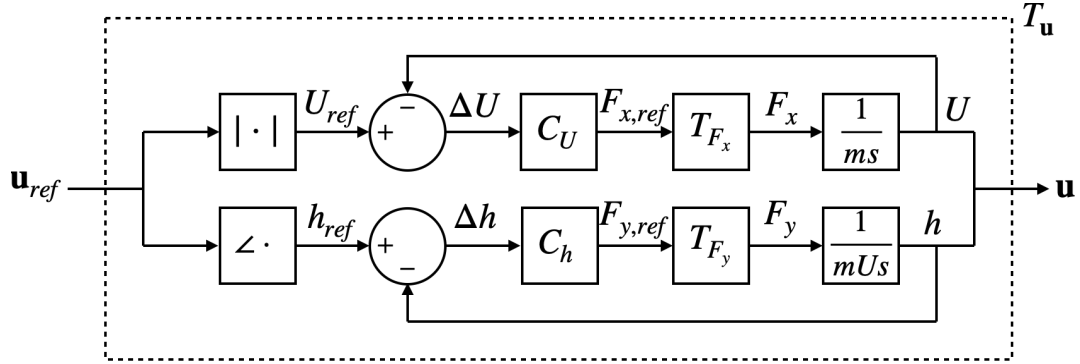


Figure 3: **Structure of the velocity controller.** The reference velocity $\mathbf{u}_{ref}$ is split into a magnitude U_{ref} and direction h_{ref} , which are controlled separately. Separate controllers C_U and C_h determine the forces along x and y respectively, which are obtained with the transfer functions T_{F_x} and T_{F_y} and integrated to get back to the velocity magnitude U and the heading h .

modifying the thrust, i.e. the force along the x axis (aligned with the bird body). The flight direction is modified by producing lateral forces, along the spanwise y axis.

Since any parasitic force would be integrated by the integrator on the right of Fig. 3, the controllers C_U and C_h also need to have an integral term. They are thus designed as PI controllers. The T_{F_x} block represents variations of wing kinematics leading to variations of thrust for the bird. The lateral force F_y is produced by changing the bank angle of the bird, as illustrated in Fig. 4.

Figure 4 shows that the bank angle ϕ induces a lateral force. This is due to the redirection of the lift vector, which remains normal to the spanwise axis of the bird. For small bank angles, the lateral component of the lift $L \sin \phi \simeq L\phi$ and if the lift supports the weight, $L = mg$. Thus, the lateral force is

$$F_y = mg \phi .$$

The bank angle is regulated by another controller C_ϕ , as represented in Fig. 5.

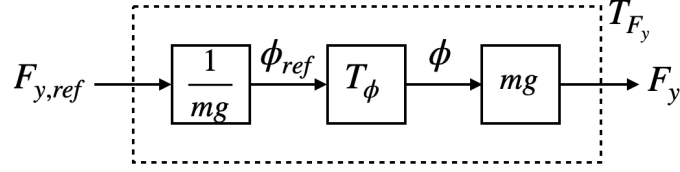


Figure 4: **Production of a lateral force.** A target lateral force $F_{y,ref}$ is obtained through a bank angle $\phi_{ref} = F_{y,ref}/(mg)$. The actual bank angle is obtained through a transfer function T_ϕ and is multiplied by the bird weight mg to get the actual lateral force F_y .

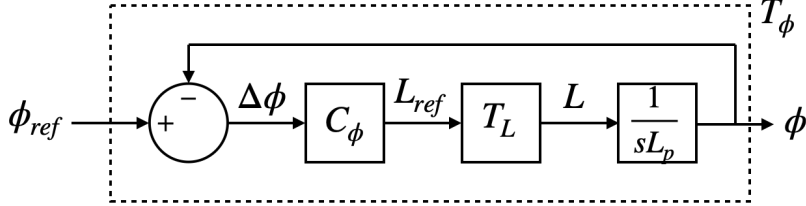


Figure 5: **Controller regulating the bank angle.** A controller C_ϕ takes as input the error in bank angle $\Delta\phi$ to determine a target rolling moment L_{ref} . The actual moment is obtained through a transfer function T_L and integrated (only once under the assumption of dominant roll damping) to get the bank angle ϕ .

In Fig. 5, the rolling moment is considered proportional to the rolling angular velocity. This assumes that the roll damping (i.e. the rolling moment opposing a rolling velocity) is dominant, and that the bird rapidly reaches an equilibrium angular velocity under a given rolling moment. The time constant of the roll damping dynamics being very small (about 1/20 of the flapping frequency), this assumption is realistic, and we can write $\dot{\phi} L_p = L$, where L_p is the roll damping coefficient.

The controller C_ϕ is chosen to be a proportional one. This may result in static errors with respect to the reference bank angle ϕ_{ref} , but since this angle is itself the output of a controller on the flight direction containing an integral term, it will adapt to reach the desired heading angle h . The T_L block represents the variations of the wing kinematics leading to a rolling moment for the bird.

The remaining degrees of freedom (namely the vertical translation and the pitching and yawing rotations) are regulated by separate controllers, ensuring a constant altitude and zero pitching and sideslip angles as represented in Fig. 6. The yaw controller on the third diagram of Fig. 6 notably includes the effect of changes in heading h to the sideslip angle β (between the longitudinal axis of the bird and its velocity vector), which has to be compensated by a rotation in yaw ψ to keep the bird aligned with its velocity (which corresponds to $\beta = 0$, β being the sideslip angle, between the body axis and the flight direction).

Finally, the pitch and sideslip angles (θ and β) of the bird have a large impact on the aerodynamic forces. To prevent these parasitic forces to hinder the control of the flight, their impact was quantified and is directly compensated in the computation of the reference forces. Additional terms are thus added to the outputs of the controllers. A term proportional to θ is added to the computation of the vertical and horizontal forces as well as the pitching moment ($F_{z,ref}$, $F_{s,ref}$ and M_{ref}), and a term proportional to β is added to the computation of the lateral force and the rolling and yawing moments ($F_{y,ref}$, L_{ref} and N_{ref}).

2 Production of aerodynamic forces

In the previous section, we did not describe the mechanisms through which the bird produces the references output by the controllers. This corresponds to the contents of blocks T_{F_x} , T_{F_z} , T_L , T_M and T_N (T_{F_y} is already covered in Fig. 4). To produce these forces, the bird adapts its wing motion, using flapping kinematics that produce a net force or moment. To find these kinematics, we separate the forces in two categories: F_x , F_z and M correspond to degrees of freedom in the longitudinal plane and are thus performed using set of kinematics that maintain symmetrical wing motions, while L and N require asymmetrical variations of the wing kinematics.

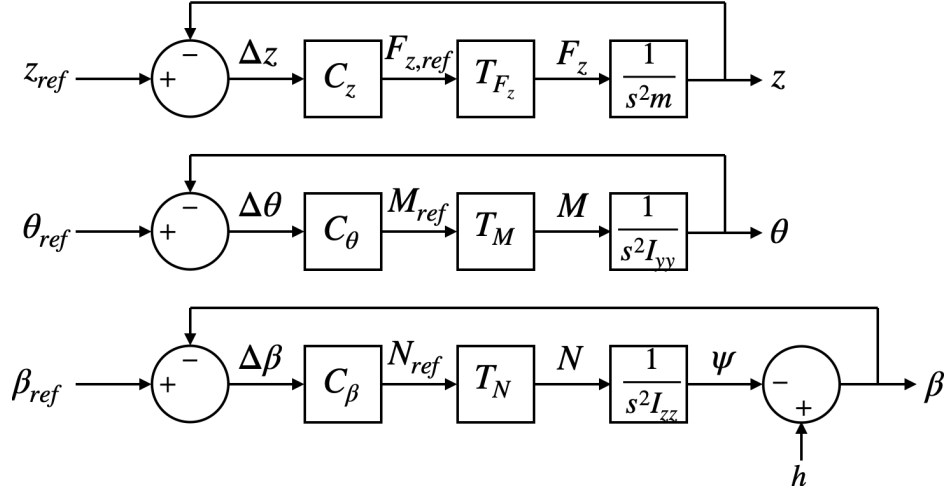


Figure 6: **Control schemes used for the vertical translation (z) and pitching (θ) and yawing (ψ) rotations.** The vertical controller C_z takes as input the error in vertical position Δz , and determines the reference vertical force $F_{z,ref}$. The pitch controller C_θ takes as input the error in pitch angle $\Delta\theta$ and determines the reference moment M_{ref} . The yaw controller C_β takes as input the error in sideslip angle $\Delta\beta$ and determines a reference yawing moment N_{ref} . The yaw angle ψ obtained after integrating the moment is subtracted to the heading angle h to get the sideslip angle β . Reference values are indicated for the pitch and sideslip angles (θ_{ref} and β_{ref} , respectively) to remain general, but the reference values will be zero in the present work.

Four kinematic parameters are selected for the longitudinal dynamics: $q_{0,s,y}$, $A_{s,y}$, $A_{s,x}$ and $q_{0,s,z}$. These parameters are all linked to the shoulder joint, and correspond respectively to its average rotation about the y axis (stroke-plane angle), the amplitude of its oscillations about the y axis, the amplitude of its oscillations about the x axis (flapping amplitude), and its mean angle in the z axis (sweep). To determine which values of these parameters will output the desired forces and moment, a linear regression has been performed on the forces generated by small variations of each of these angles. This results in a sensitivity matrix $\mathbf{S}_{lon}$ such that

$$\mathbf{F}_{lon} \simeq \mathbf{S}_{lon} \mathbf{q}_{lon} ,$$

where $\mathbf{F}_{lon} = [F_y, F_x, M]^T$ and $\mathbf{q}_{lon} = [q_{0,s,y}, A_{s,y}, A_{s,x}, q_{0,s,z}]^T$.

From this relation, we can identify linear combinations of the kinematic parameters that result in a force in a single direction (i.e. either pure vertical or forward force, or pure pitching moment). As there is one more parameter than force, an under-determination has to be solved first. In our case, we chose to minimize the norm of the vector $\mathbf{q}_{lon}$ for a given set of forces $\mathbf{F}_{lon,ref}$. This corresponds to

$$\mathbf{q}_{lon} = \mathbf{S}_{lon}^+ \mathbf{F}_{lon,ref} ,$$

where $\mathbf{S}_{lon}^+ = \mathbf{S}_{lon}^T (\mathbf{S}_{lon} \mathbf{S}_{lon}^T)^{-1}$ is the left pseudo inverse of $\mathbf{S}_{lon}$ and $\mathbf{S}_{lon} \mathbf{S}_{lon}^+ = \mathbf{I}$ is a 3x3 identity matrix.

For the lateral dynamics, the asymmetrical parameters are chosen as the mean angle and amplitude of oscillations of the twist of the distal wing, which result in rotations of the wrist along the y axis, and are noted $q_{0,w,y}$ and $A_{w,y}$. Similarly, the impact of these parameters on the dynamics are identified through a linear regression and we write

$$\mathbf{F}_{lat} \simeq \mathbf{S}_{lat} \mathbf{q}_{lat} ,$$

where $\mathbf{F}_{lat} = [L, N]^T$ and $\mathbf{q}_{lat} = [q_{0,w,y}, A_{w,y}]^T$. The kinematic parameters for given lateral moments are then determined by

$$\mathbf{q}_{lat} = \mathbf{S}_{lat}^{-1} \mathbf{F}_{lat,ref} .$$

As the forces are produced by variations in the flapping kinematics, they will not be achieved instantaneously, but rather with a delay similar to the flapping period. Therefore, the time scales of the controllers involving the production of these forces should be significantly larger than the flapping period. The only moment being produced significantly faster than the flapping period is the rolling moment. Indeed, the asymmetric changes of angle of attack near the tips result in almost instantaneous variations of the rolling moment.

3 Tuning of the controller gains

Following the design of the controllers, and the limiting time scale of the force production, we can now determine the gains of the controllers.

The only controller that can operate on a time scale faster than the flapping frequency is the bank angle regulator. As the rolling dynamics can be very fast, and the lateral force depends on the bank angle, we chose a time constant of $0.5T$ for the bank controller (where $T = 0.25$ s is the flapping period). As we decided to only use a proportional constant, the only control parameter is thus

$$C_\phi = k_{P,\phi} = \frac{L_p}{0.5T} ,$$

so that

$$\phi(s) = \frac{L}{sL_p} = \frac{C_\phi \Delta\phi}{sL_p} = \frac{\Delta\phi}{0.5Ts} ,$$

and

$$\phi(s) = \frac{\phi_{ref}}{1 + 0.5Ts} .$$

For both components of velocity control (i.e. the forward velocity and heading direction controllers), we synthesized the time constant to be $3T$. This is the result of a compromise between a quick convergence of the velocity vector to its desired value, and a time scale not too close to that of the flapping motion. As both of them are PI controllers, we need to determine the value of $k_{P,U}$, $k_{I,U}$, $k_{P,h}$ and $k_{I,h}$. For the velocity, we have

$$U(s) = \frac{C_U \Delta U}{ms} = \frac{sk_{P,U} \Delta U + k_{I,U} \Delta U}{s^2 m} ,$$

and thus

$$U(s) = \frac{s \frac{k_{P,U}}{m} + \frac{k_{I,U}}{m}}{s^2 + s \frac{k_{P,U}}{m} + \frac{k_{I,U}}{m}} .$$

In order to have critical damping, with a double pole at $1/(3T)$, the gains of the forward velocity controller will be $k_{P,U} = 2m/(3T)$ and $k_{I,U} = m/(3T)^2$. Following a similar development, we obtain the gains of the heading controller $k_{P,h} = 2mU/(3T)$ and $k_{I,h} = mU/(3T)^2$.

As stated in section above, the position controller needs to operate at a time scale slower than the velocity controller. We thus chose its unique pole (since it is a simple proportional controller) to be at $a_u = 1/(15T)$.

The three remaining controllers, illustrated in Fig. 6, all present a double integration of the control force to obtain the controlled variable (i.e. z , θ and β). Therefore, to stabilize the dynamics without static error, we use PID controllers. The gains of these controllers were chosen to operate on time scales of the same order as the velocity controllers, with an additional focus on suppressing oscillations through larger derivative terms. Table 1 summarizes the values of the gains.

Parameter	$k_{D,z}$	$k_{P,z}$	$k_{I,z}$	$k_{D,\theta}$	$k_{P,\theta}$	$k_{I,\theta}$	$k_{D,\beta}$	$k_{P,\beta}$	$k_{I,\beta}$
Value	$6 \frac{m}{4T}$	$3 \frac{m}{(4T)^2}$	$\frac{m}{(4T)^3}$	$4.5 \frac{I_{yy}}{2T}$	$3 \frac{I_{yy}}{(2T)^2}$	$\frac{I_{yy}}{(2T)^3}$	$-3 \frac{I_{zz}}{4T}$	$-3 \frac{I_{zz}}{(4T)^2}$	$-\frac{I_{zz}}{(4T)^3}$

Table 1: Gains of the vertical position, pitch and sideslip controllers.

In sum, actual numerical values of the gains can be obtained by plugging the values of morphological parameters and flight period T in the equations developed above. In the current implementation of the controllers, the design flight speed is 15 m s^{-1} , the bird mass is $m = 1.35 \text{ kg}$, the inertia of the bird about the z and y axes are considered constant and respectively estimated as $I_{zz} = 0.12366 \text{ kg m}^2$ and $I_{yy} = 0.117 \text{ kg m}^2$, and the roll damping coefficient is $L_p = 0.69939 \text{ kg m}^2 \text{ s}^{-1}$.