

Impact of Fractal Dimension on Antenna Behavior: Sierpiński Carpet and Koch Snowflake Designs

Alireza Khalili Golmankhaneh

alirezakhalili2002@yahoo.co.in

Kyrgyz-Turkish Manas University

Diana T. Pham

The University of Texas at Arlington

Hamdullah Şevli

Van Yuzuncu Yil University

Rawid Banchuin

Siam University

Research Article

Keywords: Fractal antenna, Sierpiński Carpet, Koch Snowflake, Box-counting dimension, Multiband, Radiation pattern, Wireless communication

Posted Date: October 29th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10932477/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Impact of Fractal Dimension on Antenna Behavior: Sierpiński Carpet and Koch Snowflake Designs

Alireza Khalili Golmankhaneh^{1*}, Diana T. Pham²,
Hamdullah Şevli³, Rawid Banchuin⁴

^{1*}Department of Applied Mathematics and Informatics,
Kyrgyz-Turkish Manas University, Bishkek, Kyrgyzstan.

²Department of Biology, University of Texas at Arlington, TX,
Arlington, 76019, USA.

³Department of Computer Engineering, Faculty of Engineering, Van
Yuzuncu Yil University, Van, Turkey.

⁴Graduate School of Information Technology, Siam University,
Bangkok, Thailand.

*Corresponding author(s). E-mail(s):

alirezakhalili.golmankhaneh@manas.edu.kg (or)

alirezakhalili2002@yahoo.co.in;

Contributing authors: npham@mavs.uta.edu; hsevli@yyu.edu.tr;

rawid.ban@siam.edu;

Abstract

This paper presents a comparative study of two fractal antennas—the Sierpiński Carpet and Koch Snowflake—designed with identical dimensions and operating at 2.3 GHz. Although similar in size, their different fractal geometries lead to distinct electromagnetic behaviors. Key performance metrics such as input impedance, reflection coefficient, and radiation patterns are analyzed.

Keywords: Fractal antenna, Sierpiński Carpet, Koch Snowflake, Box-counting dimension, Multiband, Radiation pattern, Wireless communication

1 Introduction

Fractal geometry has emerged as a powerful tool in the analysis and design of complex structures that defy classical Euclidean dimensionality. Many natural and artificial systems exhibit forms that are better described using non-integer, or fractional, dimensions. The foundational work of mathematicians such as Georg Cantor, Helge von Koch, Gaston Julia, Pierre Fatou, and Sierpiński laid the groundwork for the study of such geometries. The term fractal, coined by Benoît Mandelbrot, formally characterizes these structures, which are defined by their self-similarity and fractal dimension [1–5].

Fractal objects possess several remarkable properties beyond their non-integer dimension. For instance, they may enclose a finite area while having an infinite perimeter, or occupy a finite volume while exhibiting infinite surface area. Most notably, their self-similarity allows them to replicate structural patterns across scales, a property that has profound implications in physics, engineering, and signal processing.

Related to fractals are space-filling curves—such as those introduced by Peano and Hilbert—which, although not fractals in the strict sense, fill a finite area with curves of infinite length. These mathematical constructs have opened new pathways for applications in electrical and electronic engineering [6–11].

Among the most prominent engineering applications of fractals are:

1. Fabrication of high-capacitance devices by maximizing conductor surface area within limited volumes;
2. Enhancement of integrated circuit (IC) performance through lateral capacitance in fractally partitioned layouts;
3. Development of compact, multiband antennas that utilize self-similar geometries for efficient impedance matching and frequency agility.

Fractal geometries provide unique advantages in antenna design, such as self-similarity, space-filling characteristics, and scalability. These properties address critical demands in modern wireless systems, including miniaturization, wide frequency coverage, and efficient multi-band performance [12–14]. Fractal antennas also reduce mutual coupling in array configurations, enhancing their effectiveness in compact systems [15–19].

Various fractal structures—such as the Sierpiński gasket, Koch curve, and Minkowski island—have been employed to design antennas that are lightweight, low-profile, and capable of supporting multiband or broadband operations without the need for additional matching networks [20–24].

Fractal antenna arrays, which use recursive or self-similar arrangements of radiating elements, further enhance design flexibility. These arrays enable efficient beamforming, reduced sidelobes, and compact multiband functionality by exploiting the geometrical advantages of fractals [19, 25–28].

For example, the Sierpiński gasket antenna has demonstrated multiband operation across five distinct frequency bands, a behavior attributed to its inherent self-similarity [29–32]. Similarly, Koch snowflake geometries have been employed in compact array designs that preserve bandwidth and directional performance [33].

In this paper, we present a comparative study of two widely known fractal antennas: the Sierpiński Carpet and the Koch Snowflake. Both designs are constructed

with identical physical dimensions and are tuned to operate at 2.3 GHz which is suitable for the renowned Wi-Fi application. We examine and contrast their electromagnetic properties—including input impedance, reflection coefficient (S_{11}), and radiation patterns—through numerical simulations. This analysis highlights how variations in fractal geometry, particularly the fractal dimension, can be leveraged to tailor antenna performance for specific wireless communication applications.

2 Fractal Geometry in Antennas

Fractal geometries such as the Koch curve, Sierpiński triangle, and Minkowski loop have been widely utilized in antenna design due to their unique properties. A key concept in evaluating these structures is the fractal dimension, typically calculated using Hausdorff or box-counting methods, which directly influences the electromagnetic behavior and radiation characteristics of the antenna [19].

3 Antenna Design Methodology

Fractal geometries were generated using iterative algorithms to develop two specific antenna types: the Koch snowflake antenna and the Sierpiński carpet antenna [29–32]. These designs are particularly effective for multiband and wideband applications due to their self-similar and space-filling properties [34–36]. The primary design parameters and geometry for each antenna are summarized below.

3.1 Koch Snowflake Antenna

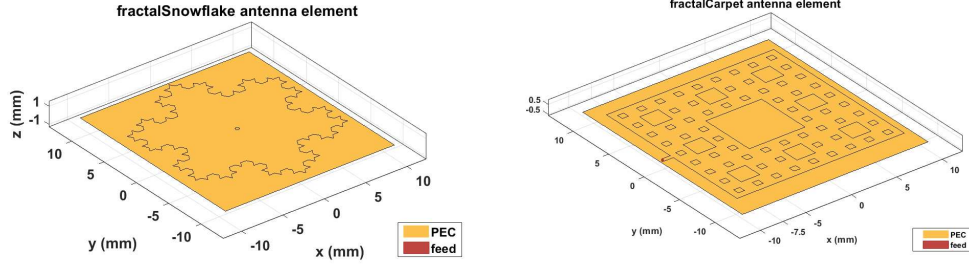
The Koch snowflake antenna is constructed using three recursive iterations. Its compact and symmetric layout enhances wideband performance, making it suitable for space-constrained environments.

- Iteration order: 3
- Length: 0.018 386 m, Height: 0.000 307 18 m
- Feed offset: [0, 0], Feed diameter: 0.000 421 95 m
- Fractal center offset: [0, 0]
- Ground plane: 0.020 424 m (length) $\times$ 0.022 473 m (width)
- Tilt: 0° about axis [1, 0, 0]

3.2 Sierpiński Carpet Antenna

The Sierpiński carpet antenna features a planar geometry with three levels of iteration. Its regular pattern supports multiband behavior and simplifies fabrication.

- Iteration order: 3
- Length and width: 0.018 386 m, Height: 0.000 307 18 m
- Stripline width: 0.000 421 95 m
- Feed offset: [−0.010 212, 0], Fractal center offset: [0, 0]
- Ground plane: 0.020 424 m (length) $\times$ 0.022 473 m (width)
- Tilt: 0° about axis [1, 0, 0]



(a) Geometry of the Koch snowflake antenna with three iterations. (b) Geometry of the Sierpiński carpet antenna with three iterations.

Fig. 1: Fractal antenna geometries: (a) Koch snowflake antenna and (b) Sierpiński carpet antenna.

As illustrated in Figure 1, the recursive nature of these fractal antennas facilitates frequency agility and compactness. Specifically, Figure 1a shows the Koch snowflake antenna, while Figure 1b depicts the Sierpiński carpet antenna. Both structures offer advantageous characteristics for wireless applications [19, 25–28].

3.3 Box-Counting Dimension of Fractal Antennas

To quantitatively assess the complexity of the antenna geometries, we used the box-counting method to estimate their fractal dimensions [37]. This technique overlays a binary image with grids of decreasing box size ε , counting the number of boxes $N(\varepsilon)$ that intersect the fractal pattern. The fractal dimension D is then approximated by the slope of the log-log plot of $N(\varepsilon)$ versus $1/\varepsilon$:

$$D = \lim_{\varepsilon \rightarrow 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)}.$$

This analysis was performed on two canonical fractal structures relevant to antenna design:

- Koch Snowflake: After converting the image to grayscale and binarizing it at a threshold of 128, the estimated box-counting dimension was:

$$D_{Koch} \approx 1.2619.$$

- Sierpiński Carpet: Using the same preprocessing steps, the box-counting dimension was found to be:

$$D_{Sierpiński} \approx 1.8928.$$

The box-counting dimensions of the Koch snowflake and the Sierpiński carpet are estimated using the log-log plots in Figure 2. In subplot (a), the linear trend confirms the self-similarity of the Koch snowflake, and the slope provides a numerical estimate

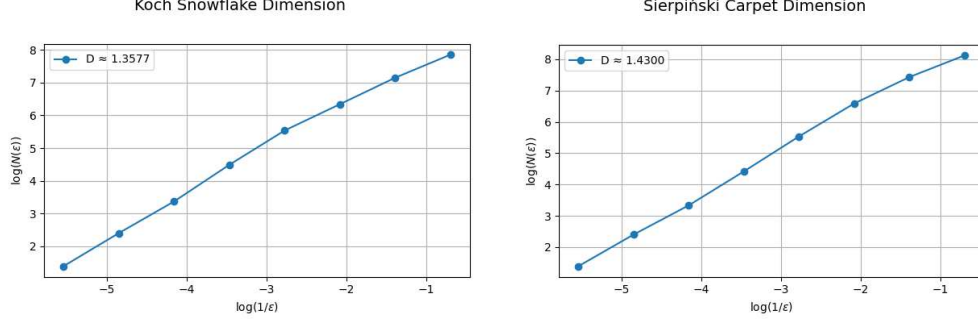


Fig. 2: (a) Log-log plot for box-counting dimension of the Koch snowflake. (b) Log-log plot for box-counting dimension of the Sierpiński carpet.

of its fractal dimension. Subplot (b) shows the Sierpiński carpet, whose higher slope reflects its more intricate and space-filling geometry.

4 Antenna Simulation Results

This section provides a comparative analysis of two fractal antenna geometries: the Koch Snowflake and the Sierpiński Carpet. Both designs share identical physical dimensions and are tuned to resonate at a center frequency of 2.3 GHz, which is suitable for Wi-Fi applications. Their performance is assessed based on input impedance, reflection coefficient (S_{11}), radiation patterns, and surface current distribution. Key differences and characteristics are illustrated and discussed with reference to the corresponding figures.

The simulations assume an ideal configuration in which air serves as the substrate layer between the antenna and the Perfect Electric Conductor (PEC) ground plane, while a layer of Teflon is inserted between the antenna and the copper substrate.

4.1 Koch Snowflake and Sierpiński Carpet with PEC Ground Plane and Air Substrate

This subsection presents a detailed comparison of two fractal antenna configurations—namely, the Koch Snowflake and the Sierpiński Carpet—both implemented with a Perfect Electric Conductor (PEC) ground plane and an air substrate, while a Teflon layer separates the antenna structure from the copper substrate. Performance metrics such as impedance, reflection coefficient, radiation characteristics, and surface current distribution are analyzed and discussed.

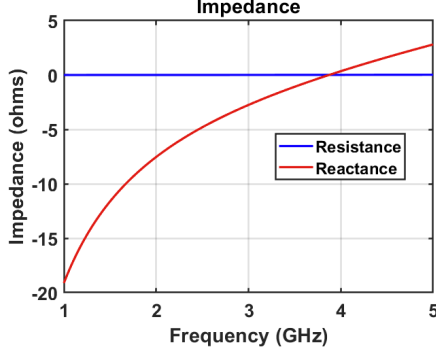
1. Koch Snowflake Antenna

The Koch Snowflake antenna is simulated over the frequency range 2–5 GHz. As shown in Figure 3a, the input impedance exhibits characteristic behavior around the center frequency of 2.3 GHz. The corresponding reflection coefficient (S_{11}) in

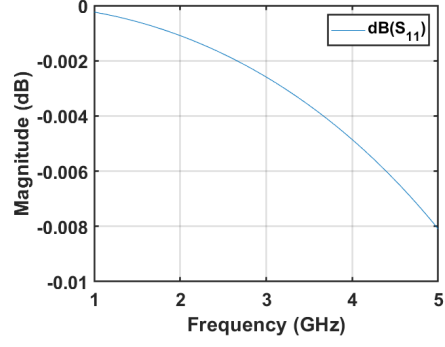
Figure 3b suggests the absence of strong resonance at that frequency, indicating poor impedance matching and higher return loss.

The far-field 3D radiation pattern at 2.3 GHz is presented in Figure 3c. A distinct directional main lobe is observed, which is advantageous for targeted communication. The azimuthal (XY-plane) and elevation (XZ-plane) cuts, shown in Figures 3d and 3e, respectively, further illustrate the angular dispersion and beam shape.

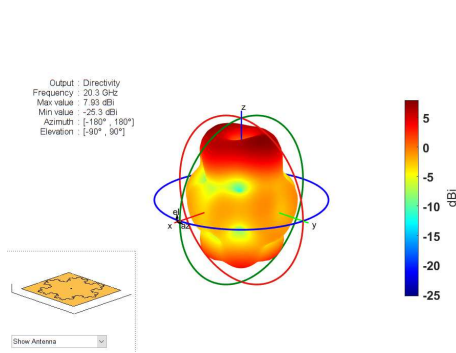
The surface current distribution at a higher frequency of 20.3 GHz is illustrated in Figure 3f. The current flow along the intricate fractal boundaries highlights the antenna's ability to support multi-frequency behavior.



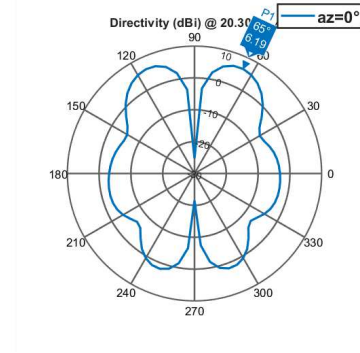
(a) Impedance response of the Koch Snowflake antenna.



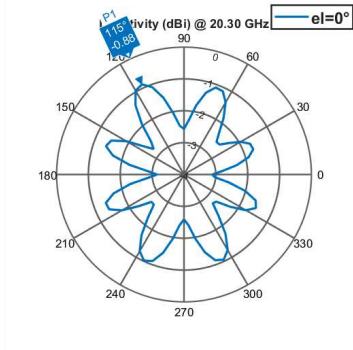
(b) Reflection coefficient (S_{11}) at 2.3 GHz.



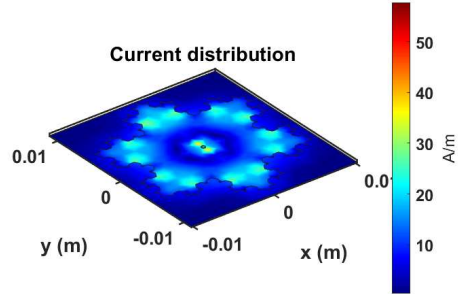
(c) Three-dimensional radiation pattern at 2.3 GHz.



(d) Azimuth radiation pattern in the XY -plane at 2.3 GHz.



(e) Elevation radiation pattern in the XZ -plane at 2.3 GHz.



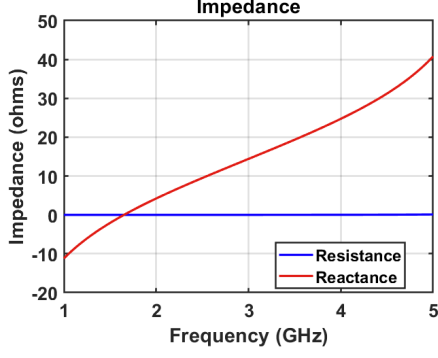
(f) Surface current distribution at 20.3 GHz.

Fig. 3: Simulation results for the Koch Snowflake antenna: (a) impedance response, (b) reflection coefficient S_{11} , (c) three-dimensional radiation pattern, (d) azimuth radiation pattern, (e) elevation radiation pattern, and (f) surface current distribution.

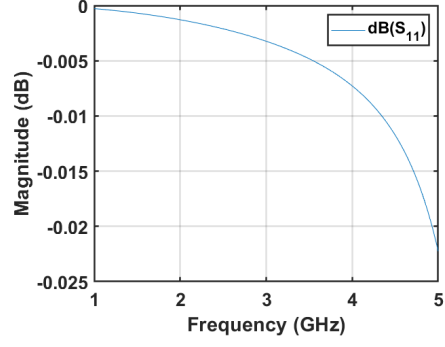
2. Sierpiński Carpet Antenna

The Sierpiński Carpet antenna is simulated over a broader frequency range of 1–5 GHz to capture its multi-band characteristics arising from its self-similar fractal geometry. As shown in Figure 4a, resonance peaks are evident, reflecting the inherent frequency-selective behavior. The corresponding S_{11} parameter in Figure 4b does not exhibit a clear dip at the target frequency of 2.3 GHz, suggesting that the antenna is not well matched at this frequency.

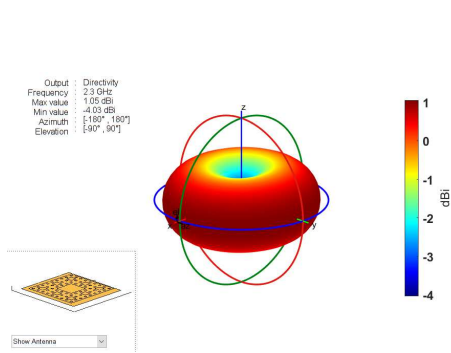
The 3D far-field radiation pattern at 2.3 GHz is displayed in Figure 4c, showing a relatively simple radiation structure comparable to that of the Koch antenna. The azimuth and elevation cuts are provided in Figures 4d and 4e, respectively. Finally, the surface current distribution at 20.3 GHz is depicted in Figure 4f.



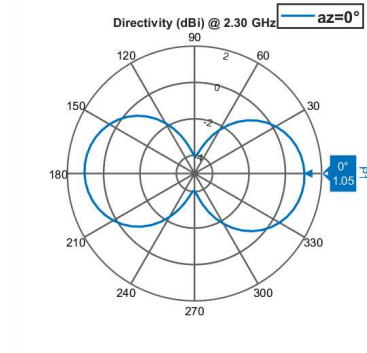
(a) Impedance response of the Sierpiński Carpet antenna.



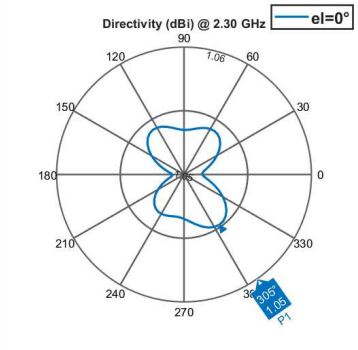
(b) Reflection coefficient (S_{11}) at 2.3 GHz.



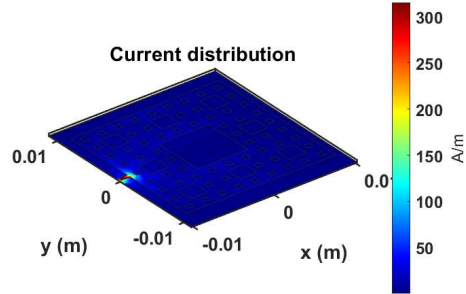
(c) Three-dimensional radiation pattern at 2.3 GHz.



(d) Azimuth radiation pattern in the XY -plane at 2.3 GHz.



(e) Elevation radiation pattern in the XZ -plane at 2.3 GHz.



(f) Surface current distribution at 20.3 GHz.

Fig. 4: Simulation results for the Sierpiński Carpet antenna: (a) impedance response, (b) reflection coefficient S_{11} , (c) three-dimensional radiation pattern, (d) azimuth radiation pattern, (e) elevation radiation pattern, and (f) surface current distribution.

5 Conclusion

In this study, a comparative analysis was carried out between two fractal antenna designs—the Sierpiński Carpet and the Koch Snowflake—both configured with identical physical dimensions and tuned to a resonant frequency of 2.3 GHz. Despite having the same size and operating frequency, differences in their box-counting fractal dimensions lead to distinct electromagnetic behaviors. Key performance metrics, including input impedance (Figures 3a and 4a), reflection coefficient (Figures 3b and 4b), and radiation patterns in both the azimuthal and elevation planes (Figures 3d, 3e, 4d, and 4e) were examined. The surface current distributions at higher frequencies (Figures 3f and 4f) further highlighted the influence of fractal geometry on current flow and resonance.

The results clearly demonstrate how variations in fractal geometry influence antenna characteristics such as bandwidth, radiation efficiency, and directivity. By modifying the fractal dimension while keeping other design parameters constant, antenna performance can be systematically tailored to meet specific requirements in modern wireless communication systems. All simulations were performed using MATLAB's Antenna Toolbox with a design frequency centered at 2.3 GHz.

Conflict of Interest: The authors declare that they have no conflicts of interest relevant to the content of this study.

Funding: No funding was received for this study.

Data Availability Statement: No data were generated or analyzed during the current study.

Author Contributions: A.K.G.: Investigation, Methodology, Software, Writing—Original Draft Preparation. D.T.P.: Investigation, Methodology, Software, Writing—Review and Editing. H.Ş. and R.B.: Investigation, Methodology, Writing—Review and Editing.

Acknowledgement: The authors have no acknowledgements to declare.

References

- [1] Mandelbrot, B.B.: The Fractal Geometry of Nature. WH freeman, New York (1982)
- [2] Barnsley, M.F.: Fractals Everywhere. Academic Press, New York (2014)
- [3] Falconer, K.: Fractal Geometry: Mathematical Foundations and Applications. John Wiley & Sons, New York (2004)
- [4] Lapidus, M.L., Radunović, G., Žubrinčić, D.: Fractal Zeta Functions and Fractal Drums. Springer, New York (2017)
- [5] Jorgensen, P.E.: Analysis and Probability: Wavelets, Signals, Fractals vol. 234. Springer, New York (2006)
- [6] Puente-Baliarda, C., Romeu, J., Pous, R., Cardama, A.: On the behavior of the

- sierpinski multiband fractal antenna. *IEEE Trans. Antennas Propag.* **46**(4), 517–524 (2002)
- [7] Song, C., Hall, P., Ghafouri-Shiraz, H., Wake, D.: Fractal stacked monopole with very wide bandwidth. *Electron. Lett.* **35**(12), 945–946 (1999)
 - [8] Walker, G., James, J.: Fractal volume antennas. *Electron. Lett.* **34**(16), 1536–1537 (1998)
 - [9] Song, C., Hall, P.S., Ghafouri-Shiraz, H.: Perturbed sierpinski multiband fractal antenna with improved feeding technique. *IEEE Trans. Antennas Propag.* **51**(5), 1011–1017 (2003)
 - [10] Tang, Wahid: Hexagonal fractal multiband antenna. *Antennas Wirel. Propag. Lett.* **3**, 111–112 (2004)
 - [11] Song, C., Hall, P.S., Ghafouri-Shiraz, H.: Multiband multiple ring monopole antennas. *IEEE Trans. Antennas Propag.* **51**(4), 722–729 (2003)
 - [12] Yakovlev, A.: Frontiers in electromagnetics. *IEEE Antennas Propag. Mag.* **43**(4), 96–97 (2001)
 - [13] Kalra, D.: Antenna miniaturization using fractals. PhD thesis, Citeseer (2007)
 - [14] Stutzman, W.L., Thiele, G.A.: *Antenna Theory and Design*. John Wiley & Sons, New York (2012)
 - [15] Dhaliwal, B.S.: *Fractal Antenna Design Using Bio-inspired Computing Algorithms*. Bentham Science Publishers, Sharjah (2023)
 - [16] Gianvittorio, J.P., Rahmat-Samii, Y.: Fractal antennas: A novel antenna miniaturization technique, and applications. *IEEE Antennas Propag. Mag.* **44**(1), 20–36 (2002)
 - [17] Werner, D.H., Petko, J.S., Spence, T.G.: Fractal antennas. *Antenna Engineering Handbook* **33** (2007)
 - [18] Brambila, F.: *Fractal Analysis: Applications in Physics, Engineering and Technology*. BoD–Books on Demand, Norderstedt (2017)
 - [19] Werner, D.H., Ganguly, S.: An overview of fractal antenna engineering research. *IEEE Antennas Propag. Mag.* **45**(1), 38–57 (2003)
 - [20] Balanis, C.A.: *Antenna Theory: Analysis and Design*. John wiley & sons, New York (2016)
 - [21] Malik, P.K., Padmanaban, S., Holm-Nielsen, J.B.: *Microstrip Antenna Design for Wireless Applications*. CRC Press, Florida (2021)

- [22] Cohen, N.: Fractal antenna applications in wireless telecommunications. In: Professional Program Proceedings. Electronic Industries Forum of New England, pp. 43–49 (1997). IEEE
- [23] Krzysztofik, W.J., Brambila, F.: Fractals in antennas and metamaterials applications. *Fractal Analysis: Applications in Physics, Engineering and Technology*, 953–978 (2017)
- [24] Awasthi, A.K., Sharma, A., Saurav, K.: *Practical Antenna: Design, Analysis, and Applications*. CRC Press, Florida (2024)
- [25] Pandey, A.: *Practical Microstrip and Printed Antenna Design*. Artech House, Massachusetts (2019)
- [26] Paun, M.-A., Nichita, M.-V., Paun, V.-A., Paun, V.-P.: Fifth-generation fractal antenna design based on the koch snowflake geometry. a fractal theory application. *Expert Syst.* **42**(1), 13242 (2025)
- [27] Malik, P., Lu, J., Madhav, B., Kalkhambkar, G., Amit, S.: *Smart Antennas: Latest Trends in Design and Application*. Springer, Cham (2021)
- [28] Biswas, B., Karmakar, A.: *Printed Antennas for Future Generation Wireless Communication and Healthcare*. CRC Press, Florida (2023)
- [29] Rahim, A., Malik, P.K.: Analysis and design of planar wide band antenna for wireless communication applications: Fractal antennas. In: *Design Methodologies and Tools for 5G Network Development and Application*, pp. 196–208. IGI Global, Pennsylvania (2021)
- [30] Triangle, S.: *Design of new microstrip multiband fractal antennas* (2017)
- [31] Malik, P.K.: *Planar Antennas: Design and Applications*. CRC Press, Florida (2021)
- [32] Puente Baliarda, C., Romeu Robert, J., Pous Andrés, R., Garcia, X., Benitez, F.: Fractal multiband antenna based on the sierpinski gasket. *Electron. Lett.* **32**(1), 1–2 (1996)
- [33] Saharsh, S., Viswasom, S., *et al.*: Design and analysis of koch snowflake fractal antenna array. In: *2020 Fourth International Conference on I-SMAC (IoT in Social, Mobile, Analytics and Cloud)(I-SMAC)*, pp. 194–197 (2020). IEEE
- [34] Waqas, M., Ahmed, Z., Ihsan, M.B.: Multiband sierpinski fractal antenna. In: *2009 IEEE 13th International Multitopic Conference*, pp. 1–6 (2009). IEEE
- [35] Gupta, M., Mathur, V.: Sierpinski fractal antenna for internet of things applications. *Materials Today: Proceedings* **4**(9), 10298–10303 (2017)

- [36] De Nicola, F., Puthiya Purayil, N.S., Spirito, D., Miscuglio, M., Tantussi, F., Tomadin, A., De Angelis, F., Polini, M., Krahne, R., Pellegrini, V.: Multiband plasmonic sierpinski carpet fractal antennas. *ACS Photonics* **5**(6), 2418–2425 (2018)
- [37] Golmankhaneh, A.K.: *Fractal Calculus and Its Applications*. World Scientific, Singapore (2022)