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Nikolaos D. Tantaroudas

`nikolaos.tantaroudas@iccs.gr`

Institute of Communications and Computer Systems (ICCS)

Ilias Karachalios

University of Thessaly

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Rapid Worst-Case Gust Identification for Very Flexible Aircraft Using Reduced-Order Models

Nikolaos D. Tantaroudas^{1*} and Ilias Karachalios²

^{1*}Institute of Communications and Computer Systems (ICCS), 9 Iroon
Politechniou Street, Zografou, Athens, 15773, Greece.

²University of Thessaly, Larissa, Greece.

*Corresponding author(s). E-mail(s): nikolaos.tantaroudas@iccs.gr;

Abstract

The identification of worst-case gust loads is critical to the airworthiness certification of very flexible aircraft, yet nonlinear full-order simulations make exhaustive parametric searches impractical. A reduced-order model (ROM) based methodology is presented for rapid identification of worst-case gust conditions, achieving speedups of up to 11,000 times per gust case. The approach exploits nonlinear model order reduction through Taylor series expansion and eigenvector projection of the coupled fluid-structure-flight dynamic system. Validation covers a three-degree-of-freedom aerofoil, the restrained half-model of a Global Hawk-like UAV, and a free-flying very flexible flying-wing. For the two configurations that are reduced, the linear ROM screens the gust parameter space accurately enough to locate the critical gust to within one step of the sweep, while the load at that gust is confirmed by a single full-order run.

Keywords: worst-case gust, flexible aircraft, reduced-order model, gust loads, certification, nonlinear aeroelasticity

1 Introduction

The airworthiness certification standards CS-25 and FAR-25 mandate the identification of worst-case gust loads across a comprehensive range of flight conditions, gust gradient distances, and gust intensities. For conventional aircraft possessing moderate wing flexibility, this identification process is routinely performed using linear analysis tools. However, the current generation of high-altitude long-endurance (HALE) and solar-powered aircraft platforms is characterised by very high-aspect-ratio

wings that experience substantial structural deformations during flight operations. These large deformations invalidate the fundamental principle of linear superposition, thereby mandating the employment of fully nonlinear coupled aeroelastic-flight dynamic simulations for reliable load prediction [1–3].

The computational burden of nonlinear simulations presents a formidable obstacle to the certification process. A single time-domain simulation of the coupled system typically requires several hours of wall-clock time. Consequently, a worst-case gust search involving $O(10^2)$ to $O(10^3)$ parameter combinations would demand weeks to months of continuous computation. The investigation into the NASA Helios mishap [2] brought into sharp focus the critical importance of understanding gust response for very flexible aircraft, as the interaction between turbulence-induced large deformations and the resulting changes in aerodynamic and inertial characteristics contributed directly to the catastrophic structural failure.

Model order reduction (MOR) provides a systematic approach to overcoming this computational bottleneck. A variety of MOR techniques have been explored for aeroelastic applications, including proper orthogonal decomposition (POD) [4], harmonic balance methods [5], and system identification approaches [6]. The nonlinear model order reduction (NMOR) technique employed in the present work [7–9] projects the coupled system onto a compact eigenvector basis derived from the Jacobian matrix eigenspectrum, preserving nonlinear terms through a Taylor series expansion. A particularly valuable property for worst-case gust searches is that, once the ROM has been constructed, it is excitation-independent and can be applied to arbitrary gust profiles without regeneration.

Since 2015, considerable progress has been achieved in reduced-order modelling techniques for flexible aircraft dynamics. Wang et al. [10] proposed a normal-mode-based model reduction approach for geometrically nonlinear slender structures, subsequently generalised to a comprehensive nonlinear modal aeroservoelastic analysis framework [11]. Artola et al. [12] demonstrated aeroelastic control and state estimation using a minimal nonlinear modal description. Goizueta et al. [13] introduced adaptive sampling techniques for interpolation of parametric ROMs. Riso and Cesnik [14] conducted a systematic assessment of the accuracy of low-order modelling approaches for aeroelastic predictions of very flexible wings. In the context of worst-case gust identification, Balatti et al. [15] investigated the influence of folding wingtips on worst-case gust loads. A comprehensive treatment of coupled flight mechanics, aeroelasticity, and control has been provided in the monograph by Palacios and Cesnik [16]. Most recently, Gao et al. [17] developed a gust load alleviation framework for flexible flying wings using linear parameter-varying modelling, Wang et al. [18] presented an H_∞ control law for gust alleviation with experimental validation, and Zheng et al. [19] proposed a gust response alleviation methodology based on passive wingtip bending.

The NMOR methodology was originally developed for gust loads analysis in [7] and for nonlinear control applications in [8]. Subsequent extensions encompassed free-flying flexible aircraft with coupled flight dynamics [20, 21], experimental validation against wind tunnel data [22–24], and a comprehensive treatment in the book chapter [9]. The influence of aerodynamic modelling fidelity on the framework performance was evaluated in [25]. A detailed derivation of the NMOR formulation for coupled

aeroelastic-flight dynamic systems is presented in the companion paper [26]. The complete coupled aeroelastic-flight dynamic framework upon which the present work is built, including the geometrically-exact beam formulation, unsteady strip aerodynamics, and quaternion-based flight dynamics, is documented in [27]. A model reference adaptive control (MRAC) framework that achieves real-time gain adjustment for nonlinear aeroelastic systems is presented in [28].

Two recent journal publications by the authors form the immediate basis of the present work. In [29] a parametric investigation quantified how nonlinear flexibility modifies the trim, flutter and gust response of high-aspect-ratio wings and established the deformation regimes in which linear analysis ceases to be adequate; in [30] the same excitation-independent ROMs served as design plants for an H_∞ gust load alleviation controller whose robustness was assessed on the full nonlinear model. The present paper builds on both, since the flexibility-dependent trends of [29] motivate the linear-versus-nonlinear ROM validity boundary examined here, and the ROM construction validated for control design in [30] is exploited for the exhaustive gust sweeps that worst-case identification requires.

1.1 Original Contributions of This Paper

The present paper is one of a set of companion studies built on the same nonlinear ROM framework, and its scope is delimited as follows. The NMOR formulation itself, the eigenvector-basis selection, the convergence of the reduced model with the number of retained modes, and the validation of the ROM against full-order solutions for a three-degree-of-freedom aerofoil, a HALE aircraft and the very flexible flying-wing were reported in the companion paper [26], which also quoted, in aggregate form, the computational speedups obtained in the gust sweeps. The parametric influence of structural flexibility on trim, flutter and gust response was the subject of [29], and the use of the ROM as a design plant for H_∞ gust load alleviation of the Global Hawk-like UAV and of the flying-wing was presented in [30]. None of these studies addressed how the critical gust is actually found.

New to the present paper are: (i) the formulation of worst-case gust identification as a search over the gust length and its four-stage ROM-based implementation (Sec. 4), building on the strategy outlined in [9], with the gust models of Sec. 2; (ii) the sweeps themselves, not previously published in a peer-reviewed journal: 1,000 cases for the aerofoil (Figs 3–4), the 80-case search for the restrained UAV half-model (Figs 5–8) and 38 cases for the free-flying flying-wing (Figs 9–11), which show that the peak load is a non-monotonic function of gust duration; (iii) the validity domains of the linear and nonlinear ROMs as a practical selection rule and the two-stage screening-and-confirmation workflow (Sec. 6); (iv) the flexible-versus-rigid trajectory comparison at the worst-case gust (Fig. 12); and (v) the interpretation of the worst-case gust lengths through the lowest coupled aeroelastic mode of each configuration (Sec. 6.1). All results reported here were recomputed with the current implementation of the framework, whose reduction procedure is that of [26]; the model sizes and the aggregate speedups of Tabs 1–2 are therefore this paper’s own measurements. They were timed on a single core of one machine, with an implementation different from the one used for [26] and for the authors’ preprint [31], and the full-order cost per case differs from that setting

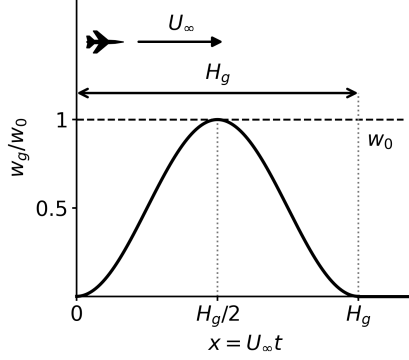


Fig. 1 The “1-minus-cosine” discrete gust of Eq. (1): gust length $H_g = 2H$, peak velocity w_0 , penetrated at U_∞

by about an order of magnitude; the wall-clock times and the speedup ratios quoted in those works are therefore not comparable with the ones reported here, and only the ratios measured within this paper should be read as its result. The paper also extends that preprint.

2 Gust Modelling

2.1 Discrete Gust: 1-Minus-Cosine Profile

The standard discrete gust profile mandated by certification regulations CS-25.341(a) and FAR-25.341(a) is the “1-minus-cosine” gust, illustrated in Fig. 1:

$$w_g(x) = \frac{w_0}{2} \left(1 - \cos \left(\frac{2\pi x}{H_g} \right) \right), \quad 0 \leq x \leq H_g \quad (1)$$

where w_0 denotes the peak gust velocity and $x = U_\infty t$ is the distance penetrated into the gust at the aircraft forward speed U_∞ . The parameter H_g is here the *gust length*, the distance travelled through the complete gust, so that Eq. (1) contains one full cosine cycle; it is twice the gradient distance H of CS-25.341(a), which measures the distance to the peak, $H_g = 2H$. The gust length is the quantity swept in Secs 5, and it is quoted below either directly, in metres or in semichords, or through the equivalent gust duration $t_g = H_g/U_\infty$. For time-domain simulation, the spatial profile is converted to a temporal profile using the frozen-turbulence (Taylor) hypothesis:

$$w_g(t) = \frac{w_0}{2} \left(1 - \cos \left(\frac{2\pi U_\infty (t - t_0)}{H_g} \right) \right), \quad t_0 \leq t \leq t_0 + \frac{H_g}{U_\infty} \quad (2)$$

2.2 Continuous Turbulence: Von Kármán Spectrum

For continuous turbulence analysis as mandated by CS-25.341(b), the Von Kármán power spectral density (PSD) is employed:

$$\Phi_{ww}(\Omega) = \sigma_w^2 \frac{L_w}{\pi} \frac{1 + \frac{8}{3}(1.339 L_w \Omega)^2}{\left(1 + (1.339 L_w \Omega)^2\right)^{11/6}} \quad (3)$$

where σ_w is the turbulence intensity, L_w is the turbulence scale length (typically 2,500 ft at high altitude), and $\Omega = 2\pi f/U_\infty$ is the spatial frequency [32]. The searches reported in Sec. 5 are all discrete-gust searches, since it is the discrete gust that carries a length parameter to be swept. The continuous-turbulence model is stated here because the reduced models used for the search are excitation-independent (Sec. 3.2): the same reduced model integrates a Von Kármán realisation at the same online cost, so the screening stage extends to CS-25.341(b) without reconstruction.

2.3 Gust Penetration Effects

For aircraft with long wing spans, the gust penetration phenomenon introduces spanwise phase variations in the gust loading. Each spanwise strip i at position y_i experiences the gust with a time delay:

$$\Delta t_i = \frac{y_i \sin \Lambda}{U_\infty} \quad (4)$$

where Λ is the wing sweep angle. For the straight-wing configurations examined here ($\Lambda = 0$) the penetration delay vanishes, and the gust would reduce to a spatially uniform encounter.

A spatially uniform encounter is, however, degenerate for a free-flying vehicle. On a uniform free-free wing a spanwise-uniform load is balanced by the inertial relief of the accelerating aircraft at every spanwise station, so that the exact elastic bending is identically zero: the aircraft translates and pitches, and the wing does not bend. A spanwise-varying encounter is therefore a prerequisite for an elastic gust response of the free-flying configuration, and it is the encounter to which the flying-wing of Sec. 5.3 is subjected. The restrained configurations are not affected, since the reaction at the root replaces the inertial relief and a uniform gust bends them.

The gust field applied to the free-flying flying-wing of Sec. 5.3 is accordingly written in the separable form $w_g(y, t) = g(y) h(t)$, in which $h(t)$ is the “1-minus-cosine” history of Eq. (2) and $g(y)$ is a symmetric spanwise gust cell of width L_y centred on the centreline,

$$g(y) = \frac{1}{2} \left(1 - \cos \left(\frac{2\pi (y + L_y/2)}{L_y} \right) \right), \quad |y| \leq \frac{L_y}{2} \quad (5)$$

and zero outside it, with L_y taken as the full span. The cell is unity on the centreline and vanishes at both tips, and being symmetric it excites no rolling response, so the

plunge and pitch response of the vehicle is unchanged in character. The aerofoil of Sec. 5.1 is two-dimensional and has no spanwise dimension, and the UAV of Sec. 5.2 is restrained at the root; both retain the uniform encounter of Eq. (2).

3 Aeroelastic Model and Reduction

3.1 Coupled System Formulation

The coupled aeroelastic-flight dynamic system is formulated in first-order state-space form following [9]:

$$\frac{d\mathbf{w}}{dt} = \mathbf{R}(\mathbf{w}, \mathbf{u}_c, \mathbf{u}_d) \quad (6)$$

with the state vector $\mathbf{w} = \{\mathbf{w}_f, \mathbf{w}_s, \mathbf{w}_r\}^T$ organised into aerodynamic ($\mathbf{w}_f$), structural ($\mathbf{w}_s$), and rigid-body ($\mathbf{w}_r$) partitions. The rigid-body partition is present only for free-flying configurations; for restrained models, in which the rigid-body degrees of freedom are constrained, it is absent. The aerodynamic model employs unsteady strip theory with Wagner and Küssner indicial functions [33–35]. The structural model is based on geometrically-exact nonlinear beam elements [36, 37]. The flight dynamics employ quaternion-based attitude propagation [20].

3.2 Nonlinear Model Order Reduction

The NMOR procedure follows the Taylor-expansion and eigenvector-projection approach documented in [7, 8, 20]. The nonlinear residual is expanded about a steady reference state $\mathbf{w}_0$, retaining terms up to the second order:

$$\mathbf{R}(\mathbf{w}) \approx \mathbf{A}\Delta\mathbf{w} + \frac{1}{2}\mathcal{B}(\Delta\mathbf{w}, \Delta\mathbf{w}) + \mathbf{B}_g\Delta\mathbf{u}_d \quad (7)$$

where $\mathbf{A}$ is the Jacobian evaluated at $\mathbf{w}_0$, $\mathcal{B}$ is the symmetric bilinear operator encoding second-order nonlinear interactions, and $\mathbf{B}_g$ is the gust input matrix. For the restrained models the reference state is the nonlinear static aeroelastic equilibrium at the flight condition, about which the wing is already deformed; for the free-flying flying-wing it is the undeformed vehicle at the reference airspeed, which is the state from which the gust encounters of Sec. 5.3 are started, so that the expansion point and the initial condition of every simulation coincide. The system is projected onto m biorthonormal eigenvectors of the Jacobian:

$$\Delta\mathbf{w} \approx \Phi\mathbf{z} + \bar{\Phi}\bar{\mathbf{z}}, \quad \Psi^H\Phi = \mathbf{I}_m \quad (8)$$

yielding the reduced-order equations:

$$\dot{z}_k = \lambda_k z_k + \sum_{i,j} D_{kij} z_i z_j + \psi_k^H \mathbf{B}_g \Delta\mathbf{u}_d, \quad k = 1, \dots, m \quad (9)$$

The m retained eigenvectors are selected by their *participation* in the response of interest, the modal contribution $|\psi_k^H \mathbf{B}_g| |\phi_{k,o}| / |\lambda_k|$ of mode k to a chosen output

degree of freedom o under the gust input, ranked in decreasing order [7, 26]. For the restrained models a single output, the wing tip, is sufficient, because every mode that matters to the loads is visible there. For the free-flying vehicle it is not: the slow flight-dynamic modes that carry the pitch and the vertical velocity have essentially no wing-tip component, so a ranking on the tip alone excludes them and fills the basis instead with members of the degenerate aerodynamic-lag cluster, whatever the basis size. The ranking used for the flying-wing of Sec. 5.3 is therefore taken over a *set* of outputs, the wing-tip deformation, the rigid-body pitch angle and the vertical velocity, each normalised by its own maximum participation and the modes ranked on the largest normalised value over the set. On a single output the criterion reduces identically to the conventional one.

The second-order interaction coefficients D_{kij} are computed through matrix-free finite differences [8]. A key property is the ROM's *excitation-independence*, since the coefficients are computed once offline, and the ROM can be excited by any gust profile without regeneration. The Jacobian, the basis and the second-order coefficients depend on the reference state and not on the gust, a property verified directly for the spanwise gust field of Sec. 2.3, whose Jacobian is numerically identical to that of the uniform encounter.

4 Worst-Case Gust Search Methodology

The worst-case gust search is formulated as:

$$H_g^* = \arg \max_{H_g \in [H_{\min}, H_{\max}]} J(\mathbf{w}(t; H_g, w_0(H_g))) \quad (10)$$

where J is the structural response metric. The ROM-based search is implemented as a four-stage procedure: (i) offline ROM construction; (ii) parametric sweep of gust lengths using the ROM; (iii) identification of the worst-case H_g^* ; (iv) validation through a single full-order simulation at the identified worst case, following the general strategy outlined in [9]. The gust encounter is illustrated in Fig. 1.

Two levels of ROM fidelity are compared, the *linear ROM*, retaining only the Jacobian term, and the *nonlinear ROM*, retaining the second-order terms of the Taylor expansion, which carry the geometric stiffening of the large-deformation regime. Their relative accuracy on the present test cases is quantified in Secs 5.2–5.3 and turned into a selection rule in Sec. 6.2.

5 Results

Three test cases of increasing complexity are considered. The aerofoil and the UAV half-model are restrained models without rigid-body states, so that the search addresses the elastic gust response alone; the flying-wing is a free-flying model with rigid-body and quaternion states, so that the gust also perturbs the flight path.

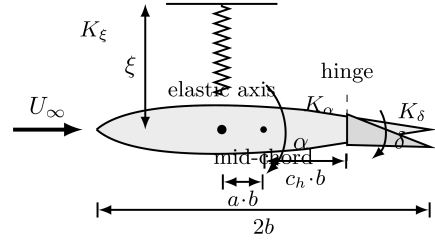


Fig. 2 Three-DOF aerofoil: pitch (α), plunge (ξ) and trailing-edge flap (δ) degrees of freedom

5.1 Three-Degree-of-Freedom Aerofoil

The first test case is a two-dimensional aerofoil with three DOF (pitch α , plunge ξ , trailing-edge flap δ) and 14 full-order states (8 aerodynamic + 6 structural) [7, 24]; the four-mode ROM of this configuration was constructed and validated in the companion paper [26], and the sweep reported here is full-order throughout. The aerofoil configuration is depicted in Fig. 2. The structural model incorporates cubic hardening nonlinearities with $K_{\alpha 3} = 3$ and $K_{\xi 3} = 1.0$.

The worst-case gust search encompasses 1,000 gust lengths spanning $H_g \in [0.1, 100]$ semichords at $U^* = 4.5$, below the linear flutter speed $U_L^* = 6.37$ of this parameter set, with a peak gust velocity of 14 % of the freestream velocity; the section model used here reproduces the flutter speeds of the reference configurations of [7, 24] (6.285, 4.663 and 6.37) to within 0.1 %. Fig. 3 shows the envelope of the pitch-angle response over the sweep for the nonlinear (NFOM) and the linearised (FOM) model, and Fig. 4 selected time histories. The nonlinear worst case is found at $H_g = 54$ semichords, with a pitch excursion of $+4.7^\circ / -8.7^\circ$; the linear model peaks at $H_g \approx 34$ semichords, close to the period of the pitch mode; its own worst load is a quarter lower (-6.5°), and at the nonlinear worst gust it under-predicts the negative excursion by a third (-5.7° against -8.7°). The plunge hardening thus shifts the worst gust to longer lengths and amplifies the pitch excursion, so that the linear model identifies both the wrong gust length and the wrong load.

5.2 Global Hawk-Like UAV

The second test case is the half-model of a Global Hawk-like UAV, a 17.75 m semi-wing discretised with 20 geometrically-exact beam elements, together with fuselage and tail elements [9, 21]; the configuration is shown in Fig. 5. The rigid-body degrees of freedom are constrained, so that no flight-dynamic states are carried and the search addresses the elastic gust response alone. The full-order model comprises $n = 540$ states (162 structural degrees of freedom and eight aerodynamic lag states per node), reduced to $m = 8$ modes, three flap-bending aeroelastic modes and five real aerodynamic-lag modes, with no rigid-body modes in the basis. The lowest flap-bending modes of the beam lie at 0.33, 1.61 and 3.58 Hz; it is the third of these that the modal correlation of the beam with the detailed finite-element model of the aircraft identifies as the first wing-bending mode (3.58 Hz) [9, 21].

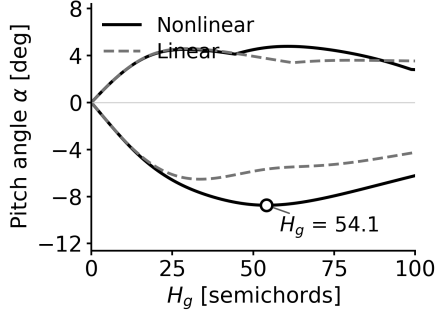


Fig. 3 Aerofoil worst-case gust search envelope: extreme pitch angles vs. gust length, nonlinear (NFOM) and linear (FOM)

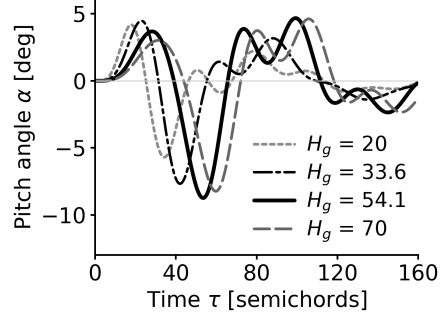


Fig. 4 Aerofoil pitch-angle histories for selected gust lengths; worst case $H_g = 54.1$ semichords (black)

Flight conditions: $U_\infty = 59$ m/s, $\rho = 0.0789$ kg/m³, reference incidence $\alpha_\infty = 4^\circ$, gust intensity 14 % of the freestream speed. A sweep over gust wavelengths from 0 to 776 MAC in steps of 9.7 MAC (80 calculations) is performed with the linear (FOM) and nonlinear (NFOM) full-order models. Fig. 6 shows the envelope of the largest and smallest wing-tip displacements over the sweep and Fig. 7 the responses at the worst case. The peak upward displacement has a single maximum at a gust duration of 3.4 s (NFOM) to 4.3 s (FOM), the downward rebound is largest for shorter gusts of about 2.4 s, and for long gusts both models converge to the quasi-static increment. About the static deflections of 2.7 m (NFOM) and 2.9 m (FOM) the nonlinear model reaches 7.7 m and the linear one 9.0 m, i.e. dynamic increments of 5.0 m (28 % of the semi-span) and 6.1 m, with the linear model lying above the nonlinear one throughout the sweep and its worst case 1 s longer.

The reduced-order models are constructed about the nonlinear static equilibrium with the 8-mode basis of Sec. 3.2, selected by participation in the wing-tip response, namely the first three flap-bending aeroelastic modes (0.32, 1.62 and 3.59 Hz) and five real aerodynamic-lag modes. Fig. 8 compares, for the worst-case gust, the full-order and reduced-order responses, where the linear ROM follows the linear FOM to within 6.3 % of the peak, and the nonlinear ROM with second-order terms reproduces the peak of the nonlinear FOM to within 9.3 %, on the conservative side, so that it gives the right indication of the worst case. The computational performance is summarised in Tab. 1, where the online integration of one gust case takes 0.1 s with the linear ROM and 2 s with the nonlinear ROM against 10–13 s and 148–191 s for the full-order models, i.e. speedups of 73–113 $\times$ and 74–91 $\times$; the second-order coefficients cost about 60 s once, offline, which is recovered after the first nonlinear case of the sweep.

5.3 Very Flexible Flying-Wing

The third test case is a 32 m span very flexible flying-wing [20]: chord 1 m, distributed mass 10 kg/m, bending stiffness $EI_2 = 2.5 \times 10^4$ Nm², torsional stiffness $GJ = 1.25 \times 10^4$ Nm², discretised with 80 geometrically-exact beam elements; with eight

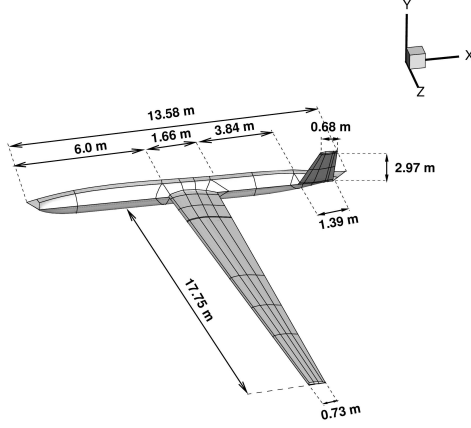


Fig. 5 Global Hawk-like UAV half-model (17.75 m semi-wing, 13.58 m fuselage, V-tail; as in [30]), rigid-body DOF constrained

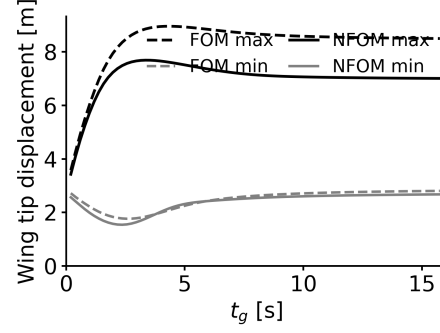


Fig. 6 UAV worst-case gust search envelope: extreme wing-tip displacements vs. gust duration (FOM dashed, NFOM solid)

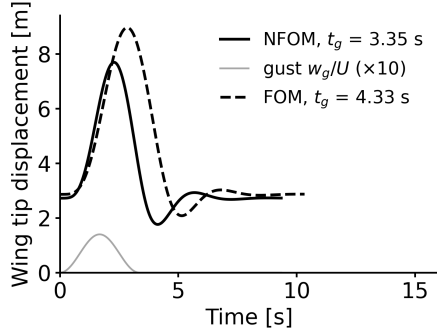


Fig. 7 UAV responses at the worst case of Fig. 6: NFOM ($t_g = 3.35$ s), FOM ($t_g = 4.33$ s), gust profile

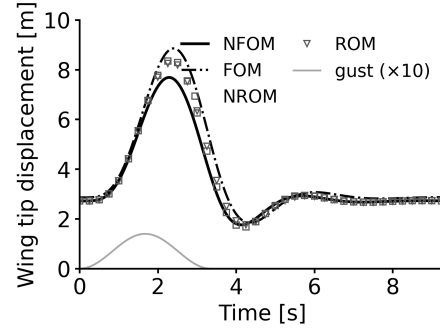


Fig. 8 UAV worst-case gust ($t_g = 3.35$ s): full-order (NFOM, FOM) vs. 8-mode reduced-order (NROM, ROM) responses

aerodynamic lag states per strip and the 12 rigid-body and 4 quaternion states the full-order model has $n = 1,616$ states, reduced to $m = 9$ modes. The aircraft is free-flying, so that the gust perturbs both the elastic deformation and the flight path of the tailless, neutrally stable vehicle. Operating conditions: altitude 20,000 m, $U_\infty = 25$ m/s, $\rho = 0.0889$ kg/m³, gravity neglected. The sweep comprises 38 gust durations from 0.5 s to 20 s (gust lengths of 12.5 m to 500 m) for a “1-minus-cosine” gust of normalised vertical velocity 0.25, that is a peak gust velocity of 6.25 m/s, distributed over the span by the symmetric gust cell of Eq. (5) and applied to the aircraft initially at rest in the undeformed position; each case is integrated for 20 s with the nonlinear full-order model (NFOM) and with the 9-mode linear (ROM) and nonlinear (NROM) reduced models built once about the undeformed state, the nine modes selected by

Table 1 Computational performance for the UAV worst-case gust search (online wall-clock time, single core)

Model	States	Time per case	Speedup
Full-order linear (FOM)	540	10–13 s	1×
Linear ROM	8	0.1 s	73–113×
Full-order nonlinear (NFOM)	540	148–191 s	1×
Nonlinear ROM	8	2 s	74–91×

the multi-output participation ranking of Sec. 3.2 over the wing-tip deformation, the pitch angle and the vertical velocity. The spanwise variation is not a refinement but a requirement of the free-flying configuration, for the reason given in Sec. 2.3: a uniform gust on this uniform free-free wing is balanced by inertial relief and produces no elastic bending at all. The elastic response of this configuration is mesh-convergent under the spanwise encounter: the governing tip excursion changes by 0.015 % between 20, 40 and 80 elements, so the deflections reported below are mesh-converged values.

Fig. 9 shows the envelope of the wing-tip deformation over the sweep and Fig. 10 that of the rigid-body pitch angle and of the peak vertical velocity. The gust cell loads the inboard wing while the tips carry the inertia of the accelerating aircraft, so the tip deforms downwards relative to the vehicle: the governing excursion is the downward one, largest at $t_g = 7.0$ s (-4.76 m, 29.7 % of the semi-span), and the worst-case gust is taken there ($H_g = 175$ m). The upward rebound is largest at $t_g = 5.0$ s ($+1.56$ m). Both branches of the envelope are non-monotonic: shorter gusts do not load the wing for long enough for the bending mode to develop ($+0.26/ - 0.73$ m at 0.5 s), and for gusts longer than the worst case the response decreases again, the rebound to -4.19 m and the upward peak to 0.08 m at 20 s. The flight-path response explains that decrease, since the pitch angle grows monotonically with gust duration, from 0.11° at 0.5 s to 0.59° at the worst case and 1.35° at 20 s, and the vertical velocity of the aircraft with it, from 0.80 to 3.27 m/s. The light, neutrally stable aircraft pitches up and follows the gust, so that the incidence seen by the wing falls and with it the elastic load, and it is the flight-dynamic response that bounds the worst case at an intermediate gust length.

The reduced models are run for the same 38 gusts (markers in Figs 9–10). The linear ROM locates the governing worst case at $t_g = 7.0$ s, exactly the full-order value, and the largest upward rebound at 4.8 s against the full-order 5.0 s, one step of the sweep, and reproduces the full-order envelope to within 3.0 % of the governing load on the upward branch and 1.5 % on the downward one, and the peak vertical velocity to within 7.3 % of its largest full-order value over the sweep. The rigid-body attitude, by contrast, is not reduced by a basis of this kind, and Fig. 10 therefore reports it from the full-order model alone. The attitude of a free-flying vehicle is a cyclic coordinate: its column of the coupled Jacobian is one of twelve exact zero eigenvalues, and ten of those, the attitude among them, are defective, so that no bi-orthogonal eigenvector pair exists for them and no modal basis can retain them at any size. A reduced attitude reconstructed from that state decays back towards zero once the gust has passed instead of holding, and recovering it as the quadrature of the

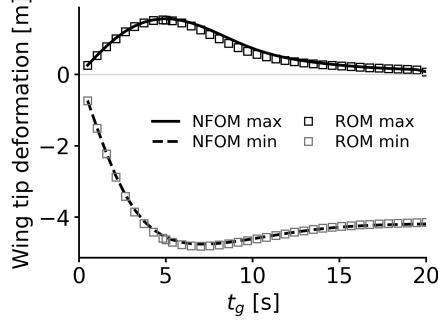


Fig. 9 Flying-wing worst-case gust search: extreme wing-tip deformations vs. gust duration (NFOM lines, ROM markers)

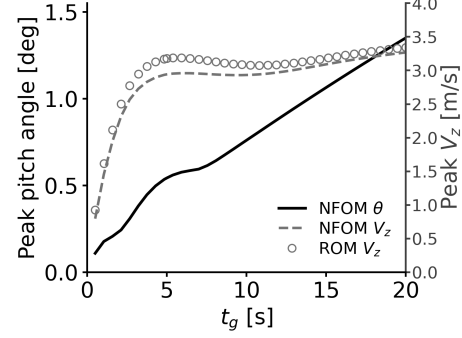


Fig. 10 Flying-wing worst-case gust search: peak rigid-body pitch angle and peak vertical velocity vs. gust duration (full-order; ROM markers on V_z)

reduced pitch rate is worse still, the rate itself being wrong; both were verified at 9, 12, 15, 20 and 30 modes. Predicting the attitude of a free-flying vehicle with this class of reduced model would require the defective block to be carried explicitly, as a Jordan chain or as an appended kinematic mode. The load-carrying channels, which are what the worst-case search needs, are unaffected. Retaining the second-order Taylor terms changes the reduced wing-tip envelope by less than 0.02 % over the whole sweep. It does not improve the flight-dynamic channels, which the truncation and not the expansion order limits: the second-order vertical velocity moves away from the full-order value rather than towards it (13 % against the linear model's 7.3 %), and the second-order attitude, being a reconstruction of the same defective state, comes out with the wrong sign. The second-order model is therefore reported here for the loads alone. The residual does have strong quadratic curvature about this expansion point, but its image is almost orthogonal to the retained subspace, since the quadratic self-interaction of a bending mode produces axial forces, and the axial and shear degrees of freedom are four orders of magnitude stiffer than the bending ones, so that 99.998 % of the quadratic term projects onto quasi-static modes above 5 kHz that no modal basis of this kind retains. The result is unchanged at 20 and 40 modes and on the nine modes of [26] (0.0006 %), and the accuracy of the reduced model is set by the truncation of the basis, not by the order of the expansion, and recovering the quadratic term would require a quadratic-manifold enrichment rather than a higher-order expansion. Fig. 11 compares the wing-tip histories at the worst case, with the NFOM (solid), the ROM (dashed) and the NROM (squares). The reduced models follow the full-order history through the gust and the first rebound, reproducing the governing downward excursion to 1.2 % (-4.82 against -4.76 m) while under-predicting the smaller upward rebound by 7.9 %, and reproduce the decay of the residual oscillation. Tab. 2 summarises the cost on a single core, where one 20 s full-order case takes 32.9 min, the median of the 38 runs, and the sweep 20.8 h, whereas the linear ROM integrates a case in 0.17 s, $11,000\times$ faster, after 27.8 min of offline construction; the complete search then takes

Table 2 Computational cost of the flying-wing worst-case gust search (single-core wall-clock time). The full-order figure is the median of the 38 runs; individual cases ranged from 27.2 to 51.8 min on a shared machine and their measured sum was 23.3 h, so the 20.8 h quoted here, and the speedups derived from it, are the conservative values

Model	States	Offline	Per case (online)	Total, 38 cases
Full-order nonlinear (NFOM)	1,616	–	32.9 min	20.8 h
Linear ROM	9	27.8 min	0.17 s	28 min
Nonlinear ROM	9	34.8 min	3.20 s	37 min

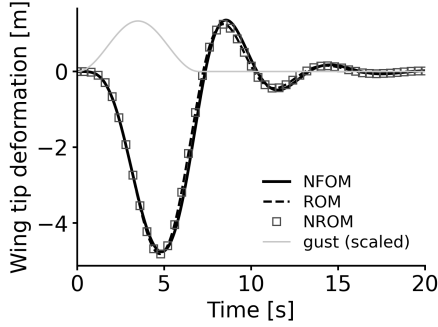


Fig. 11 Wing-tip deformation of the flying-wing at the worst-case gust: NFOM, 9-mode linear ROM and nonlinear ROM (NROM)

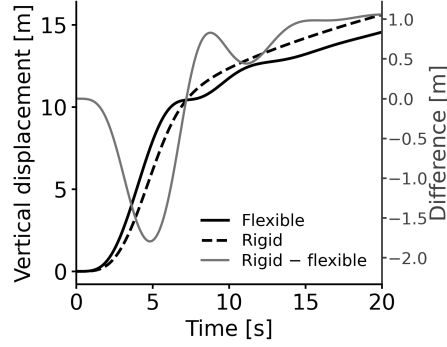


Fig. 12 Vertical displacement of the flying-wing at the worst-case gust: flexible vs. rigid (stiffness $\times 1,000$) and their difference

28 min, a factor of 45 (34 with the second-order coefficients, 34.8 min offline and 3.20 s per case). The offline cost is dominated by the finite-difference Jacobian, which is independent of the gust, so the same reduced model serves every gust shape and duration of the search.

Fig. 12 compares the vertical displacement of the flexible flying-wing with that of the rigid aircraft, the same free-flying model with the stiffness multiplied by 1,000, at the worst-case gust. The two trajectories cross. While the gust is on the aircraft the flexible one climbs faster, by as much as 1.79 m at 4.8 s (grey curve, right-hand scale), because the inboard wing bends up into the gust and carries a larger load than the rigid planform; once the gust has passed the wing springs back, the flexible aircraft loses that advantage and the rigid one overtakes it, ending 1.06 m higher after 20 s (15.6 against 14.6 m) with a pitch angle of 0.85° against 0.59° . Flexibility therefore does not simply reduce the gust response of the free-flying vehicle, it redistributes it in time, and the sign of the difference depends on when in the encounter the comparison is made. This is a stronger statement of the observation in [26] that the flight response of the flexible wing differs from that of the rigid wing, and it must be accounted for when worst-case gust loads of very flexible configurations are assessed in free flight.

6 Discussion

6.1 Gust Tuning and the Structural Modes

The worst-case gust lengths identified by the sweeps are those tuned to the lowest coupled aeroelastic mode of each configuration. A “1-minus-cosine” gust of duration t_g is a single pulse whose energy lies at frequencies below about $1/t_g$; a lightly damped mode of period T_1 responds most strongly when the pulse lasts about one period, because the gust then loads the structure in phase with its own motion and releases it as it rebounds. Shorter gusts do not load the mode for long enough, and longer gusts approach the quasi-static limit in which the response follows the gust profile and the dynamic overshoot disappears, which is the origin of the plateau in Fig. 6; for the free-flying wing (Fig. 9) the long-gust limit is a monotonic decrease instead, because the aircraft pitches up and follows the gust (Sec. 5.3). For the UAV, the in-vacuo flap-bending modes of the beam lie at 0.33, 1.61 and 3.58 Hz and, at the flight condition of Sec. 5.2, the lowest coupled aeroelastic mode has the eigenvalue $-0.98 \pm 2.03i$ rad/s, a frequency of 0.32 Hz, a period of 3.1 s and a damping ratio of 0.44, the aerodynamic damping of the bending mode being large at this dynamic pressure. The worst-case durations of 3.4 s (NFOM) and 4.3 s (FOM) in Fig. 6 are 1.1 to 1.4 periods of this mode, and the heavy damping is what makes the maximum of the envelope broad; the rebound is largest for a somewhat shorter gust (2.4 s), which unloads the wing while it springs back. For the flying-wing the first bending mode of the coupled system, computed here about the undeformed state, has the eigenvalue $-0.37 \pm 1.07i$ rad/s, a period of 5.9 s and a damping ratio of 0.33, and the worst case is found at 7.0 s, 1.2 periods; for the aerofoil the linear worst case (34 semichords) lies close to the period of the pitch mode (30 semichords), and the plunge hardening moves the nonlinear worst case to 54 semichords. The tuning is therefore governed by the lowest coupled mode, but the nonlinearity does not merely rescale the load but also moves the worst case, by 60 % for the aerofoil (34 to 54 semichords) and by one second for the UAV. A linear screening sweep therefore identifies the critical band rather than a single critical gust, and the full-order confirmation of Sec. 6.3 must cover a range of gust lengths around it.

6.2 Practical Guidelines for ROM Selection

The sweeps allow the choice of ROM fidelity to be stated from measurements. For screening, the linear ROM sufficed in both reduced configurations, since it located the worst case within one step of the sweep and followed the full-order envelope to within 3.0 % of the governing load for the flying-wing, whose tip reached 29.7 % of the semi-span (Sec. 5.3), and to within 6.3 % of the linear full-order peak for the UAV, whose tip reached 51 % of the semi-span in that linear model and 43 % in the nonlinear one (Sec. 5.2). Retaining the second-order terms changed the flying-wing tip envelope by less than 0.02 % and the UAV peak by 1.3 %, so that here the accuracy of the reduced model is limited by the truncation of the basis rather than by the order of the expansion, unlike the companion study [26]. The load at the identified worst case must therefore be confirmed by a full-order run, the second stage of the workflow of Sec. 6.3. The rule has one exception, established in Sec. 5.3: the rigid-body attitude of

a free-flying vehicle lies in a defective block of the coupled Jacobian and is not reduced by an eigenvector basis at any size, so a reduced model of this class is to be read as a predictor of loads and of the translational flight response, and not of the vehicle attitude.

6.3 Integration into Certification Workflows

The ROM-based methodology can be integrated into certification workflows through a two-stage process, consistent with the strategy proposed in [9]. First, the ROM sweeps the complete parameter space at negligible cost, identifying candidate worst-case conditions. Second, a limited number of full-order simulations confirm the ROM predictions, which for the flying-wing is not optional, because the reduced model under-predicts the upward rebound by up to 7.9 %. Per gust case the reduced models are 73–113 $\times$ (UAV, Tab. 1) and about 11,000 $\times$ (flying-wing) faster than the full-order models; for the complete 38-case flying-wing search, the offline construction of the reduced model included, the gain is 45 $\times$ (Tab. 2).

6.4 Limitations and Extensions

The present framework employs strip-theory aerodynamics, adequate for preliminary design of high-aspect-ratio wings. For detailed certification, higher-fidelity aerodynamic methods may be necessary [25, 38]. The NMOR framework is fully compatible with such upgrades [39]. Furthermore, the worst-case conditions identified by the present screening procedure constitute the natural design cases for the active gust load alleviation strategies developed by the authors in [28, 30], so that the same ROM serves both as the search engine for the critical gust and as the design plant for its alleviation.

7 Conclusions

A reduced-order model based methodology for the rapid identification of worst-case gust conditions has been presented and validated across three test cases spanning a wide range of complexity, from a three-degree-of-freedom aerofoil (14 states) through the restrained half-model of a Global Hawk-like UAV (540 states) to a free-flying very flexible flying-wing (1,616 states). The reduced models integrate a gust case two to three orders of magnitude faster than the full-order models (73–113 $\times$ for the UAV, about 11,000 $\times$ for the flying-wing); including the offline construction, the complete 38-case flying-wing search takes 28 minutes instead of 20.8 hours.

The linear ROM proved sufficient to screen the parameter space in both reduced configurations, up to tip deformations of 29.7 % of the semi-span for the flying-wing and 43 % for the UAV, locating the worst-case gust within one step of the sweep; the second-order terms changed the reduced tip envelope by less than 0.02 % for the flying-wing and the peak by 1.3 % for the UAV, so that the residual error of the reduced models, up to 7.9 % on the rebound, is a matter of basis truncation and must be removed by the full-order confirmation run rather than by a higher-order expansion. The same truncation places one limit that no basis size removes: the

attitude of the free-flying vehicle sits in a defective block of the coupled Jacobian and is not reproduced by an eigenvector basis, so that the reduced model predicts loads and the translational flight response but not the vehicle attitude. The worst-case gust has been shown to be a non-monotonic function of the gust length, with intermediate gust lengths producing the largest structural loads through resonant excitation of the dominant structural modes, the worst-case duration being close to the period of the lowest coupled aeroelastic mode (Sec. 6.1). This non-trivial dependence demonstrates that exhaustive parametric searches are necessary for reliable worst-case identification, and such searches become computationally feasible only through the ROM approach.

The methodology is directly applicable to certification workflows as a screening tool, in which the ROM identifies candidate worst-case conditions that are then confirmed by a few full-order simulations. Relative to the companion paper [26], which established the reduced model, the present paper contributes the search formulation, the worst-case sweeps for the three configurations, their interpretation through the lowest coupled aeroelastic mode, the ROM-selection rule and the certification workflow. Together with the authors' studies on nonlinear flexibility effects [29], H_∞ gust load alleviation [30] and model reference adaptive control [28], and the framework [27] and NMOR [26] companion papers, it completes a consistent ROM-based chain from flexibility characterisation, through worst-case gust identification, to active load alleviation; the framework is extensible to higher-fidelity aerodynamics without modification of the reduction procedure [9, 39].

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Declarations

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Ethics approval and consent to participate. Not applicable. This study is entirely computational and involved no human participants, human data or animals.

Consent for publication. Not applicable.

Data availability. The datasets generated and analysed during the current study are available from the corresponding author on reasonable request. All quantities on which the conclusions rest are reported in the figures and tables of this article.

Materials availability. Not applicable.

Code availability. The aeroelastic and reduced-order modelling framework used to generate the results, including the worst-case gust search driver, is available from the corresponding author on reasonable request.

Author contributions. N.D.T. conceived the study, developed the reduced-order and worst-case gust search methodology, implemented the simulation framework, performed all computations and analyses, produced the figures and wrote the manuscript. I.K. contributed to the interpretation of the results, to the formulation of the certification workflow, and to the critical revision of the manuscript. Both authors read and approved the final manuscript.

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