Supplementary Information

Gait-phase-dependent neural dynamics are altered in treated Parkinson's disease

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Table S1: Per-subject epoch counts after artefact rejection confirm that preprocessing retained the majority of data in both cohorts and conditions. For each subject, the number of epochs retained (n_{kept}) out of the originally segmented epochs (n_{orig}) is shown for the resting baseline and the walking condition, with the percentage retained in parentheses. Rest epochs are fixed-length 1.1 s windows; walking epochs are linearly time-warped gait cycles. Joint EEG–LFP rejection used AutoReject (see Methods) on EEG and a 99.5^{th}-percentile single-channel threshold on STN-LFPs.

PID	Cohort	Rest (baseline)	Walking (pre8walk)
101	HC	141 / 141 (100.0%)	75 / 77 (97.4%)
102	HC	119 / 119 (100.0%)	102 / 116 (87.9%)
103	HC	106 / 122 (86.9%)	140 / 151 (92.7%)
104	HC	118 / 120 (98.3%)	110 / 130 (84.6%)
105	HC	124 / 124 (100.0%)	99 / 118 (83.9%)
106	HC	120 / 120 (100.0%)	75 / 82 (91.5%)
107	HC	123 / 123 (100.0%)	84 / 86 (97.7%)
108	HC	124 / 124 (100.0%)	84 / 85 (98.8%)
109	HC	107 / 115 (93.0%)	74 / 77 (96.1%)
110	HC	96 / 111 (86.5%)	82 / 95 (86.3%)
201	PD	106 / 109 (97.2%)	113 / 119 (95.0%)
202	PD	97 / 109 (89.0%)	36 / 37 (97.3%)
203	PD	108 / 109 (99.1%)	113 / 141 (80.1%)
204	PD	105 / 109 (96.3%)	121 / 142 (85.2%)
205	PD	115 / 116 (99.1%)	96 / 101 (95.0%)
206	PD	129 / 131 (98.5%)	59 / 60 (98.3%)
207	PD	138 / 155 (89.0%)	101 / 102 (99.0%)
208	PD	14 / 15 (93.3%)	100 / 105 (95.2%)
209	PD	141 / 142 (99.3%)	96 / 97 (99.0%)
210	PD	114 / 128 (89.1%)	62 / 64 (96.9%)

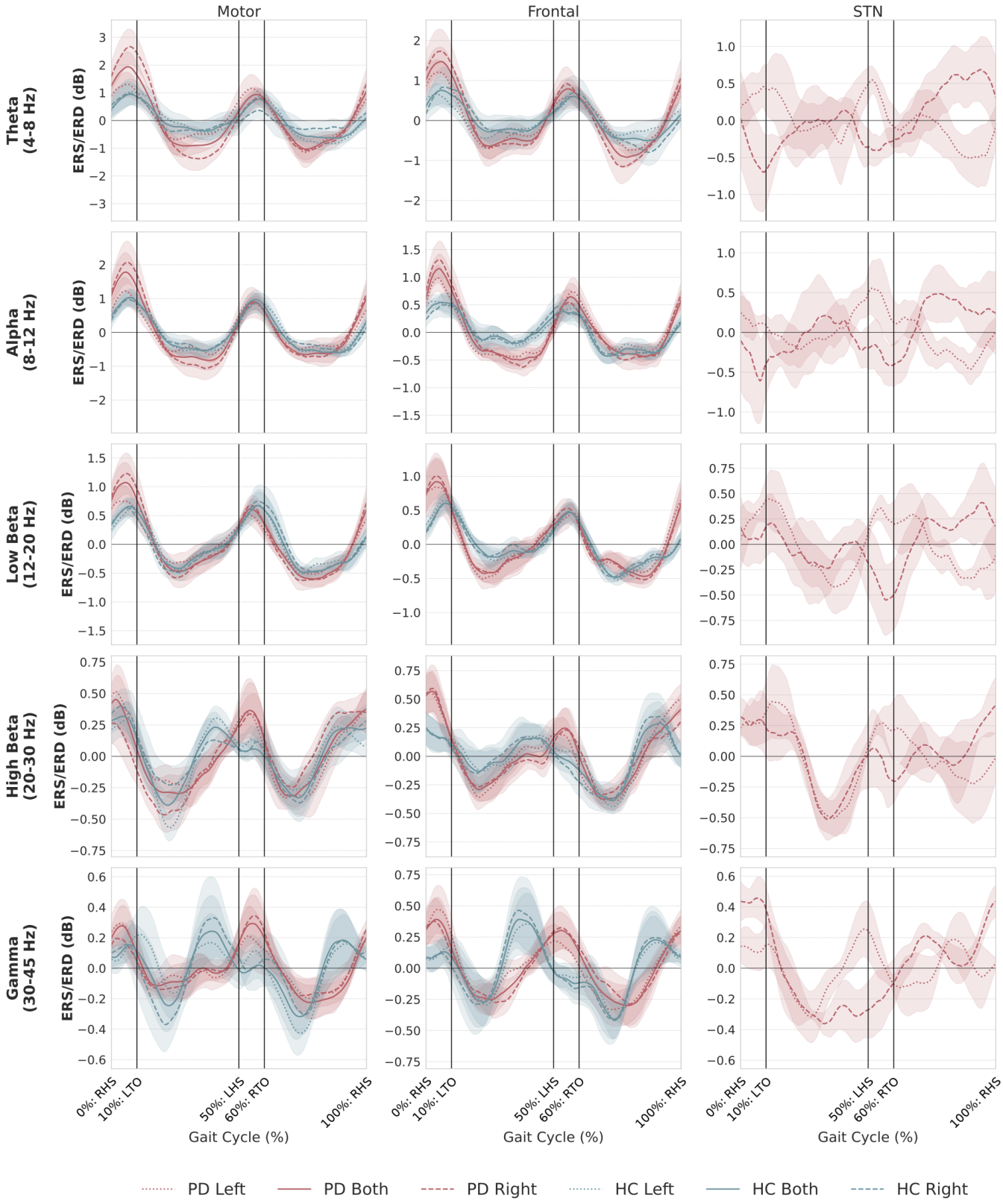
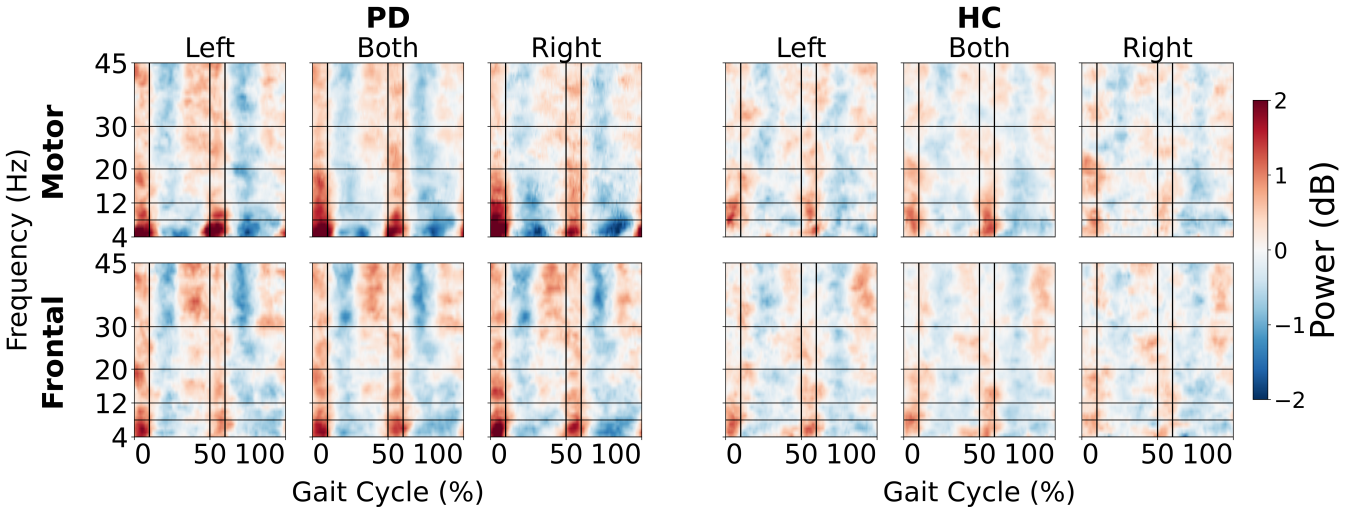
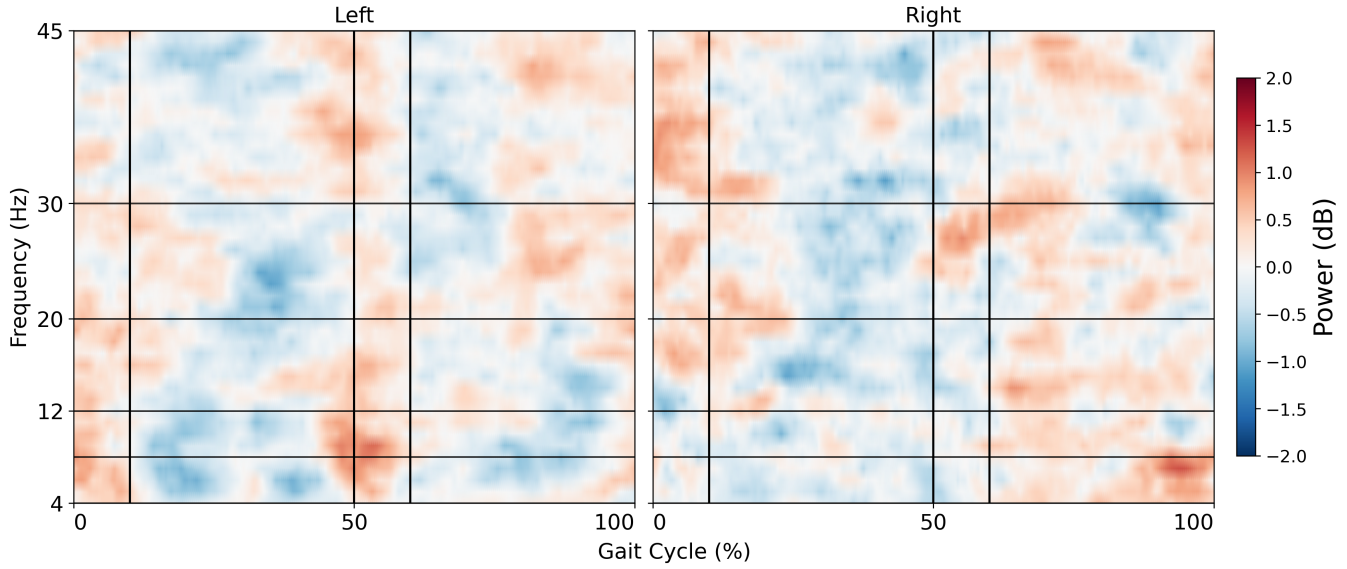


Figure S1: Average-gait-cycle-normalised gERD/ERS profiles emphasise phase-localised cohort and hemisphere differences after removing slow walking-state offsets. Mean gait-event-related de-/synchronisation (gERD/ERS) extracted from the average-gait-cycle-normalised gERSPs in Fig. S2a and Fig. S2b, broken down by frequency band (theta, alpha, low-beta, high-beta, gamma) across the gait cycle. Cortical traces compare PD ($N = 10$) and HC ($N = 10$); subcortical traces compare right STN ($N = 6$) and left STN ($N = 4$) in the PD cohort.

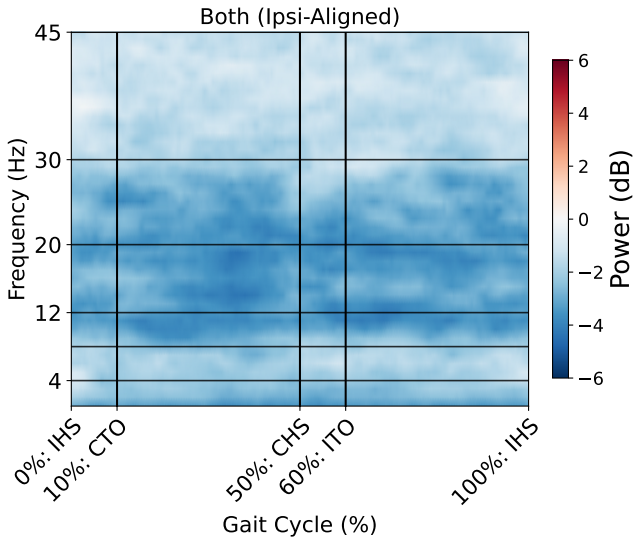


(a) Within-cycle cortical gERSPs (average-gait-cycle normalisation) isolate phase-dependent modulation by removing the slow walking-state DC offset. Group-median cortical gERSPs across one normalised gait cycle; PD ($N = 10$) on the left, HC ($N = 10$) on the right. Values are centred per (channel, frequency) on the within-cycle mean.

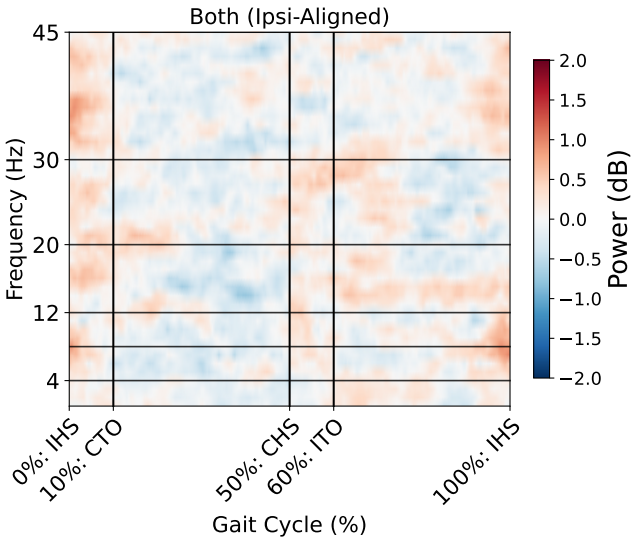


(b) Within-cycle STN gERSPs show clearer phase-dependent hemispheric asymmetry under average-gait-cycle normalisation. Group-median STN-LFP gERSPs in the PD cohort; right STN for $N = 6$ participants and left STN for $N = 4$ participants (one hemisphere recorded per subject).

Figure S2: Average-gait-cycle-normalised cortical and subcortical gERSPs. Cortical **(a)** and subcortical **(b)** gERSPs computed with a Morlet wavelet transform (target FWHM = 200 ms) and then centred per (channel, frequency) on the within-cycle mean, so that the displayed values reflect phase-dependent modulation rather than the absolute walking-state offset (decibels, dB).



(a) Baseline-normalised gERSPs (relative to seated rest).



(b) Average-gait-cycle normalised gERSPs (relative to mean walking power).

Figure S3: Ipsi/contra-aligned STN-LFP gait dynamics highlight that residual hemispheric modulation in treated PD localises around the loading phases of either foot rather than to a single contralateral swing. To visualise STN dynamics relative to the specific stepping limb, left and right STN recordings were re-aligned so that 0% of the gait cycle always corresponds to the heel-strike ipsilateral to the recording site (IHS), and 50% to the heel-strike of the contralateral foot (CHS); operationally, this is implemented by relabelling Left-STN gait phases by 50% (see Methods). **(a)** gERSPs normalised to the seated resting baseline (absolute change in power during walking relative to rest). **(b)** gERSPs normalised to the average power across the gait cycle (within-cycle phase modulation isolated by removing the sustained walking-state offset). Colour scales are in dB. Key gait events: IHS (0%/100%), contralateral toe-off (CTO, ∼10%), CHS (50%), and ipsilateral toe-off (ITO, ∼60%).

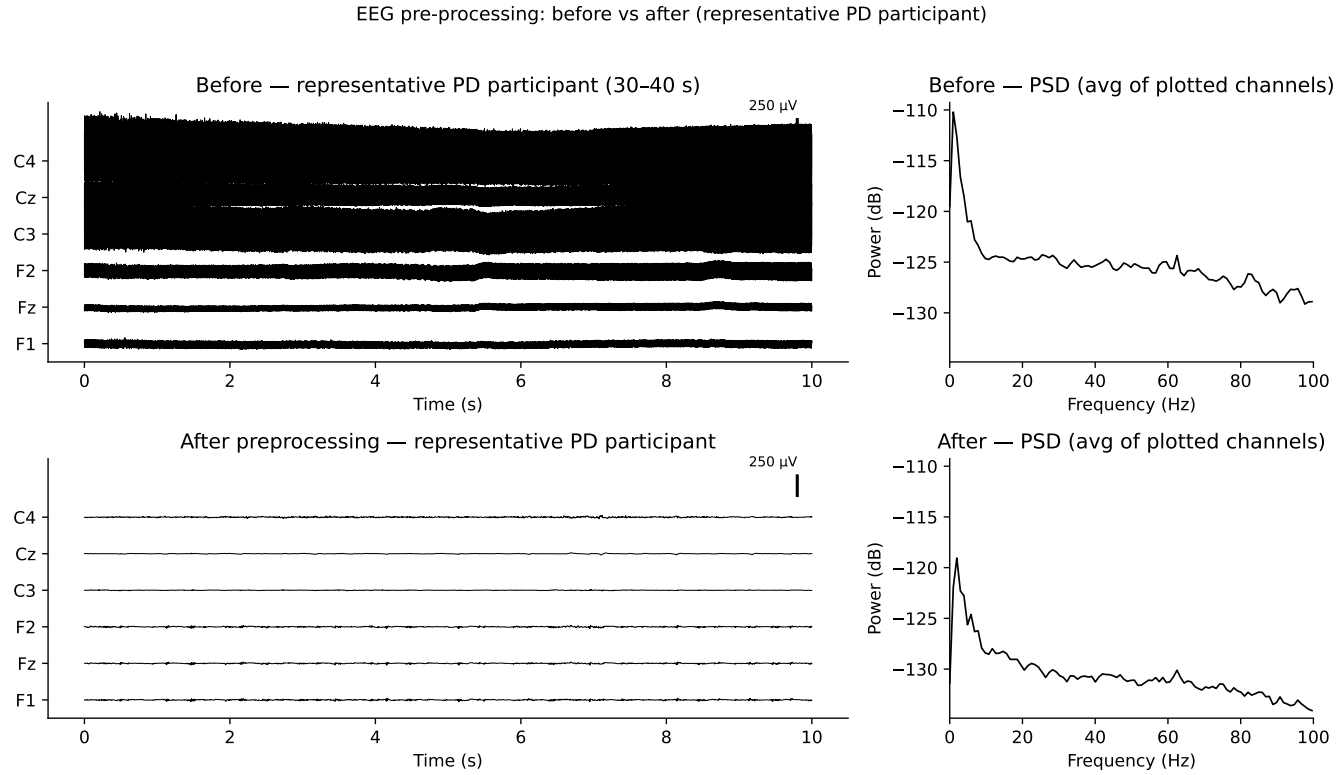


Figure S4: EEG preprocessing markedly attenuates motion and muscle artefacts. Representative before-and-after example of the EEG preprocessing pipeline in one PD participant during overground walking (a 10 s window over six representative fronto-central channels). **Top**, raw signal before preprocessing: multichannel time series (left) and average power spectral density (PSD, right), showing large-amplitude, broadband motion and muscle contamination. **Bottom**, the same data after the full pipeline (zero-phase 4–100 Hz band-pass filtering, common-average referencing, Infomax independent component analysis with ICLabel-based rejection, and AutoReject; see Methods), with artefacts strongly attenuated across channels and frequencies. Only channels common to both stages are shown; outer-ring electrodes removed during preprocessing are omitted.

Table S2: Aperiodic spectral parameters (FOOOF exponent and offset) do not differ between cohorts or between rest and walking in any ROI. Comparisons of aperiodic exponent and offset extracted by FOOOF on the power spectral densities (PSDs) across all ROIs (STN, prefrontal cortex, motor cortex) and conditions. Diff is the mean difference for t-test rows and the median difference for Mann-Whitney rows; positive values indicate value at Rest > value during walking (within-cohort) or PD > HC (between-cohort).

ROI	Comparison	Parameter	Test Used	Diff	p_{raw}	p_{FDR}
STN	Rest vs. Walk (PD)	Exponent	Paired T-Test	-0.098	0.514	0.514
		Offset	Paired T-Test	-0.297	0.168	0.337
Prefrontal Cortex	Rest vs. Walk (PD)	Exponent	Paired T-Test	-0.076	0.484	0.494
		Offset	Paired T-Test	-0.100	0.424	0.494
	Rest vs. Walk (HC)	Exponent	Paired T-Test	0.073	0.494	0.494
		Offset	Paired T-Test	-0.177	0.286	0.494
	PD vs. HC (Rest)	Exponent	Welch T-Test	-0.216	0.266	0.819
		Offset	Welch T-Test	-0.040	0.895	0.895
	PD vs. HC (Walk)	Exponent	Welch T-Test	-0.068	0.614	0.819
		Offset	Welch T-Test	-0.116	0.572	0.819
Motor Cortex	Rest vs. Walk (PD)	Exponent	Paired T-Test	0.116	0.375	0.654
		Offset	Paired T-Test	0.064	0.654	0.654
	Rest vs. Walk (HC)	Exponent	Paired T-Test	0.162	0.266	0.654
		Offset	Paired T-Test	-0.137	0.575	0.654
	PD vs. HC (Rest)	Exponent	Mann-Whitney	-0.036	0.623	0.791
		Offset	Mann-Whitney	0.350	0.791	0.791
	PD vs. HC (Walk)	Exponent	Welch T-Test	-0.230	0.170	0.339
		Offset	Welch T-Test	-0.390	0.127	0.339

Note: $^{\circ}$ indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S3: STN-LFP periodic spectral parameters do not differ between rest and walking in treated PD. Statistical comparison of FOOOF-derived periodic parameters (alpha and beta sub-bands: centre frequency and power) in the PD cohort, rest vs. walking. p-values are false discovery rate (FDR)-corrected per parameter family (Benjamini–Hochberg). Missing peaks were imputed with 0 in the power.

Parameter	Comparison	Test Used	Statistic	Median Diff	p_{raw}	p_{FDR}
Periodic Components						
Alpha CF	Rest vs. Walk	Paired T-Test	-0.40	-1.061	0.702	0.702
Alpha Power	Rest vs. Walk	Paired T-Test	0.84	0.059	0.422	0.702
Low Beta CF	Rest vs. Walk	Paired T-Test	-1.20	-1.085	0.270	0.702
Low Beta Power	Rest vs. Walk	Paired T-Test	1.10	0.057	0.301	0.702
High Beta CF	Rest vs. Walk	Paired T-Test	-0.52	-1.671	0.653	0.702
High Beta Power	Rest vs. Walk	Wilcoxon	1.00	0.200	0.063°	0.500

Note: $^{\circ}$ indicates $0.05 < p \leq 0.1$.

Table S4: Post-hoc decomposition of cortical baseline-normalised Cohort-by-Region and Cohort-by-GaitPhase interactions: PD-vs-HC differences in high-beta and gamma are concentrated in the prefrontal cortex but extend to motor cortex in gamma. Post-hoc HC-vs-PD comparisons for the significant interactions of the LMM in Table 3 of the main text, by region and by gait-phase. p-values are FDR-corrected (Benjamini–Hochberg) within each emmeans family.

Frequency Band	Interaction Level	df	t-ratio	p-value
Theta	*Cohort × Region*			
	Prefrontal Cortex	22.2	1.651	0.1127
	Motor Cortex	22.2	1.339	0.1941
	Cohort × GaitPhase			
	FDS	22.3	1.401	0.1750
	IDS	22.3	1.121	0.2744
	LLS	22.3	1.764	0.0913°
	RLS	22.3	1.692	0.1046
Alpha	*Cohort × Region*			
	Prefrontal Cortex	22.3	0.679	0.5039
	Motor Cortex	22.3	0.799	0.4325
	Cohort × GaitPhase			
	FDS	22.3	0.613	0.5461
	IDS	22.3	0.539	0.5954
	LLS	22.3	0.935	0.3597
	RLS	22.3	0.869	0.3943
Low-Beta	*Cohort × Region*			
	Prefrontal Cortex	22.3	1.050	0.3051
	Motor Cortex	22.3	0.650	0.5226
	Cohort × GaitPhase			
	FDS	22.4	0.805	0.4291
	IDS	22.4	0.545	0.5909
	LLS	22.4	1.067	0.2972
	RLS	22.4	0.977	0.3390
High-Beta	*Cohort × Region*			
	Prefrontal Cortex	22.3	2.593	0.0165*
	Motor Cortex	22.3	1.975	0.0608°
	Cohort × GaitPhase			
	FDS	22.4	2.288	0.0319*
	IDS	22.4	2.064	0.0508°
	LLS	22.4	2.352	0.0279*
	RLS	22.4	2.416	0.0242*
Gamma	*Cohort × Region*			
	Prefrontal Cortex	22.3	3.477	0.0021**
	Motor Cortex	22.3	2.124	0.0450*
	Cohort × GaitPhase			
	FDS	22.4	2.551	0.0181*
	IDS	22.4	2.722	0.0123*
	LLS	22.4	2.904	0.0081**
	RLS	22.4	3.012	0.0063**

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S5: Cortical average-gait-cycle-normalised LMM: Cohort-by-GaitPhase interactions remain significant in every band, but the Cohort-by-Region interaction prominent under baseline normalisation drops out under average-gait-cycle normalisation. Type III ANOVA results from the LMM on cortical average-gait-cycle-normalised gERD/ERS, with gERSPs in Supplementary Fig. S2a and bar profiles in Supplementary Fig. S1. Model: `gERD/ERS ~ Cohort * Region + Cohort * GaitPhase + (1|SubjectID) + (1|GaitCycle:SubjectID)`.

Variable	**Theta**	**Alpha**	**Low-Beta**	**High-Beta**	**Gamma**
Cohort	df = 1, 19.6 F = 3.83 p = 0.065°	df = 1, 20.9 F = 4.04 p = 0.058°	df = 1, 1837 F = 5.71 p = 0.017*	df = 1, 20.7 F = 0.94 p = 0.344	df = 1, 19.2 F = 2.25 p = 0.150
Region	df = 1, 12859 F = 1.64 p = 0.201	df = 1, 12859 F = 0.04 p = 0.842	df = 1, 12859 F = 3.98 p = 0.046*	df = 1, 12859 F = 11.25 p < 0.001***	df = 1, 12859 F = 16.87 p < 0.001***
GaitPhase	df = 3, 12859 F = 519.54 p < 0.001***	df = 3, 12859 F = 240.38 p < 0.001***	df = 3, 12859 F = 155.05 p < 0.001***	df = 3, 12859 F = 58.58 p < 0.001***	df = 3, 12859 F = 59.98 p < 0.001***
Cohort × Region	df = 1, 12859 F = 1.67 p = 0.196	df = 1, 12859 F = 0.18 p = 0.674	df = 1, 12859 F = 0.05 p = 0.817	df = 1, 12859 F = 1.07 p = 0.300	df = 1, 12859 F = 1.76 p = 0.185
Cohort × GaitPhase	df = 3, 12859 F = 65.47 p < 0.001***	df = 3, 12859 F = 11.49 p < 0.001***	df = 3, 12859 F = 12.76 p < 0.001***	df = 3, 12859 F = 4.48 p = 0.004**	df = 3, 12859 F = 10.61 p < 0.001***

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S6: Cortical average-gait-cycle-normalised Cohort-by-GaitPhase post-hocs: significant HC-vs-PD differences cluster in the double-support phases (FDS, IDS) across multiple bands. Post-hoc HC-vs-PD comparisons for the significant interactions of the LMM in Table S5; Cohort-by-Region was not significant in any band, so those rows are listed as not performed. p-values are FDR-corrected (Benjamini–Hochberg) within each emmeans family.

Frequency Band	Interaction Level	df	t-ratio	p-value
Theta	*Cohort* × *Region*	Not performed (ANOVA $p > 0.05$)		
	Cohort × *GaitPhase*			
	FDS	33.1	-3.043	0.0046**
	IDS	33.1	-6.435	<0.0001***
	LLS	33.1	1.771	0.0858°
	RLS	33.1	0.930	0.3592
Alpha	*Cohort* × *Region*	Not performed (ANOVA $p > 0.05$)		
	Cohort × *GaitPhase*			
	FDS	59.9	-3.100	0.0030**
	IDS	59.9	-3.973	0.0002***
	LLS	59.9	1.002	0.3205
	RLS	59.9	0.056	0.9554
Low-Beta	*Cohort* × *Region*	Not performed (ANOVA $p > 0.05$)		
	Cohort × *GaitPhase*			
	FDS	66.5	-1.982	0.0516°
	IDS	66.5	-5.614	<0.0001***
	LLS	66.5	1.017	0.3128
	RLS	66.5	-0.174	0.8624
High-Beta	*Cohort* × *Region*	Not performed (ANOVA $p > 0.05$)		
	Cohort × *GaitPhase*			
	FDS	51.4	-0.790	0.4330
	IDS	51.4	-2.679	0.0099**
	LLS	51.4	-0.106	0.9163
	RLS	51.4	0.557	0.5801
Gamma	*Cohort* × *Region*	Not performed (ANOVA $p > 0.05$)		
	Cohort × *GaitPhase*			
	FDS	44.3	-3.598	0.0008***
	IDS	44.3	-1.951	0.0574°
	LLS	44.3	-0.139	0.8900
	RLS	44.3	0.858	0.3956

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S7: STN baseline-normalised Hemisphere-by-GaitPhase post-hocs: no single gait-phase shows a significant left-vs-right contrast in any band. Post-hoc Left-vs-Right comparisons by gait-phase for the LMM in Table 4 of the main text; tests were performed only for bands with a significant omnibus interaction. p-values are FDR-corrected (Benjamini–Hochberg) within each emmeans family.

Frequency Band	Interaction Level (Left - Right)	df	t-ratio	p-value
Theta	FDS	8.11	-1.424	0.1919
	IDS	8.11	-1.410	0.1956
	LLS	8.11	-1.539	0.1619
	RLS	8.11	-1.853	0.1006
Alpha	Not performed (ANOVA $p > 0.05$)			
Low-Beta	Not performed (ANOVA $p > 0.05$)			
High-Beta	Not performed (ANOVA $p > 0.05$)			
Gamma	FDS	8.57	0.328	0.7511
	IDS	8.57	-0.776	0.4589
	LLS	8.57	-0.080	0.9384
	RLS	8.57	-0.343	0.7399

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S8: STN average-gait-cycle-normalised LMM: Hemisphere-by-GaitPhase interactions in theta and gamma persist under average-gait-cycle normalisation. Type III ANOVA results from the LMM on STN-LFP average-gait-cycle-normalised gERD/ERS shown in Supplementary Fig. S2b. Model: `gERD/ERS ~ Hemisphere * GaitPhase + (1|SubjectID)`.

Variable	Theta	Alpha	Low-Beta	High-Beta	Gamma
Hemisphere	df = 1, 3640 F = 2.46 p = 0.117	df = 1, 3640 F = 2.15 p = 0.142	df = 1, 3640 F = 1.15 p = 0.285	df = 1, 3640 F = 0.07 p = 0.794	df = 1, 3640 F = 0.26 p = 0.609
GaitPhase	df = 3, 3640 F = 0.49 p = 0.691	df = 3, 3640 F = 0.90 p = 0.443	df = 3, 3640 F = 3.35 p = 0.018*	df = 3, 3640 F = 4.08 p = 0.007**	df = 3, 3640 F = 3.84 p = 0.009**
Hemisphere × GaitPhase	df = 3, 3640 F = 4.21 p = 0.006**	df = 3, 3640 F = 1.72 p = 0.161	df = 3, 3640 F = 0.49 p = 0.692	df = 3, 3640 F = 0.42 p = 0.737	df = 3, 3640 F = 4.25 p = 0.005**

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S9: STN average-gait-cycle-normalised Hemisphere-by-GaitPhase post-hocs: theta and gamma show significant left-vs-right contrasts concentrated in the double-support phases (IDS and FDS for both bands; RLS additionally for theta). Post-hoc Left-vs-Right comparisons by gait-phase for the LMM in Table S8; tests were performed only for bands with a significant omnibus interaction. p-values are FDR-corrected (Benjamini–Hochberg) within each emmeans family.

Frequency Band	Interaction Level (Left - Right)	df	t-rat
Theta	FDS	111	2.08
	IDS	111	2.25
	LLS	111	0.95
	RLS	111	-2.16
Alpha	Not performed (ANOVA $p > 0.05$)		
Low-Beta	Not performed (ANOVA $p > 0.05$)		
High-Beta	Not performed (ANOVA $p > 0.05$)		
Gamma	FDS	111	2.20
	IDS	111	-2.67
	LLS	111	0.34
	RLS	111	-0.89

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S10: Ipsi/contra-aligned STN LMM (baseline-normalised): theta and gamma Hemisphere-by-GaitPhase interactions are preserved after re-anchoring 0% to the ipsilateral heel-strike. Type III ANOVA results from the LMM on baseline-normalised STN-LFP gERD/ERS after relabelling left-STN gait phases to align both hemispheres to their ipsilateral heel-strike (see Supplementary Fig. S3). Model: `gERD/ERS ~ Hemisphere * GaitPhase + (1|SubjectID)`.

Variable	Theta	Alpha	Low-Beta	High-Beta	Gamma
Hemisphere	df = 1, 8 F = 2.44 p = 0.157	df = 1, 8 F = 6.47 p = 0.035*	df = 1, 8 F = 1.22 p = 0.301	df = 1, 8 F = 0.19 p = 0.671	df = 1, 8 F = 0.05 p = 0.830
GaitPhase	df = 3, 3632 F = 1.87 p = 0.133	df = 3, 3632 F = 1.04 p = 0.374	df = 3, 3632 F = 2.51 p = 0.057°	df = 3, 3632 F = 1.85 p = 0.136	df = 3, 3632 F = 5.13 p = 0.002**
Hemisphere × GaitPhase	df = 3, 3632 F = 3.28 p = 0.020*	df = 3, 3632 F = 1.07 p = 0.359	df = 3, 3632 F = 0.97 p = 0.408	df = 3, 3632 F = 1.56 p = 0.197	df = 3, 3632 F = 3.26 p = 0.021*

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S11: Ipsi/contra-aligned STN post-hocs (baseline-normalised): no individual gait-phase reaches a significant Left-vs-Right contrast in any band. Post-hoc Left-vs-Right comparisons by gait-phase for the LMM in Table S10; tests were performed only for bands with a significant or marginal omnibus interaction. p-values are FDR-corrected (Benjamini–Hochberg) within each emmeans family.

Frequency Band	Interaction Level (Left - Right)	df	t-rat
Theta	FDS	8.11	-1.3
	IDS	8.11	-1.4
	CLS	8.11	-1.6
	ILS	8.11	-1.7
Alpha	Not performed (ANOVA $p > 0.1$)		
Low-Beta	Not performed (ANOVA $p > 0.1$)		
High-Beta	Not performed (ANOVA $p > 0.1$)		
Gamma	FDS	8.57	0.1
	IDS	8.57	-0.6
	CLS	8.57	0.0
	ILS	8.57	-0.4

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S12: Ipsi/contra-aligned STN LMM (average-gait-cycle-normalised): theta and gamma Hemisphere-by-GaitPhase interactions are preserved. Type III ANOVA results from the LMM on average-gait-cycle-normalised STN-LFP gERD/ERS after ipsi/contra alignment (see Supplementary Fig. S3). Model: `gERD/ERS ~ Hemisphere * GaitPhase + (1|SubjectID)`.

Variable	Theta	Alpha	Low-Beta	High-Beta	Gamma
Hemisphere	df = 1, 3640 F = 2.46 p = 0.117	df = 1, 3640 F = 2.15 p = 0.142	df = 1, 3640 F = 1.15 p = 0.285	df = 1, 3640 F = 0.07 p = 0.794	df = 1, 3640 F = 0.26 p = 0.609
GaitPhase	df = 3, 3640 F = 1.71 p = 0.162	df = 3, 3640 F = 1.30 p = 0.274	df = 3, 3640 F = 2.72 p = 0.043*	df = 3, 3640 F = 3.09 p = 0.026*	df = 3, 3640 F = 4.75 p = 0.003**
Hemisphere × GaitPhase	df = 3, 3640 F = 2.98 p = 0.030*	df = 3, 3640 F = 1.32 p = 0.267	df = 3, 3640 F = 1.12 p = 0.341	df = 3, 3640 F = 1.41 p = 0.239	df = 3, 3640 F = 3.34 p = 0.019*

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S13: Ipsi/contra-aligned STN post-hocs (average-gait-cycle-normalised): theta and gamma yield significant Left-vs-Right contrasts in the double-support phases (theta after the contralateral heel-strike, FDS; gamma after the ipsilateral heel-strike, IDS). Post-hoc Left-vs-Right comparisons by gait-phase for the LMM in Table S12; tests were performed only for bands with a significant omnibus interaction. p-values are FDR-corrected (Benjamini–Hochberg) within each emmeans family.

Frequency Band	Interaction Level (Left - Right)	df	t-rat
Theta	FDS	111	2.60
	IDS	111	1.73
	CLS	111	0.04
	ILS	111	-1.25
Alpha	Not performed (ANOVA $p > 0.1$)		
Low-Beta	Not performed (ANOVA $p > 0.1$)		
High-Beta	Not performed (ANOVA $p > 0.1$)		
Gamma	FDS	111	1.77
	IDS	111	-2.24
	CLS	111	0.70
	ILS	111	-1.25

Note: ° indicates $0.05 < p \leq 0.1$; * indicates $p \leq 0.05$; ** indicates $p \leq 0.01$; *** indicates $p \leq 0.001$.

Table S14: STN linear mixed-model structure comparison (gERD/ERS): the Hemisphere×GaitPhase model is hypothesis-driven, with information-criterion support concentrated in theta and gamma. Each PD subject contributes one STN hemisphere, so Hemisphere is a between-subjects factor with `(1|Subject)` supplying its error term. Models were fit by maximum likelihood, so AIC and BIC are comparable across both fixed and random terms; Δ values are relative to the best model within each band. The **FINAL** row is the model reported in the manuscript. Anatomical and ipsi/contra alignments give identical fits (factor relabelling only), so the comparison is shown once. *additive*: Hemisphere + GaitPhase; *FINAL*: Hemisphere × GaitPhase; *interaction+cycle*: FINAL plus a gait-cycle random intercept (lowers AIC in theta/gamma but yields singular fits, hence not adopted). k is the number of model parameters.

Norm.	Band	Model	k	AIC	ΔAIC	BIC	ΔBIC
baseline	Theta	additive	7	20,052.0	33.7	20,095.4	8.9
		FINAL	10	20,037.6	19.4	20,099.7	13.1
		interaction+cycle	11	20,018.3	0.0	20,086.5	0.0
	Alpha	additive	7	20,828.2	1.1	20,871.6	0.0
		FINAL	10	20,827.1	0.0	20,889.1	17.5
		interaction+cycle	11	20,829.1	2.0	20,897.3	25.7
	Low beta	additive	7	20,965.5	0.0	21,008.9	0.0
		FINAL	10	20,967.9	2.3	21,029.9	20.9
		interaction+cycle	11	20,969.9	4.3	21,038.1	29.1
	High beta	additive	7	19,099.5	0.0	19,142.9	0.0
		FINAL	10	19,103.8	4.3	19,165.8	22.9
		interaction+cycle	11	19,105.8	6.3	19,174.0	31.1
	Gamma	additive	7	16,506.5	18.5	16,549.9	0.0
		FINAL	10	16,501.9	13.9	16,563.9	14.0
		interaction+cycle	11	16,488.0	0.0	16,556.2	6.3
avg-gait	Theta	additive	7	19,914.1	32.1	19,957.5	7.3
		FINAL	10	19,901.4	19.4	19,963.4	13.2
		interaction+cycle	11	19,882.0	0.0	19,950.2	0.0
	Alpha	additive	7	20,726.7	1.4	20,770.1	0.0
		FINAL	10	20,725.3	0.0	20,787.3	17.2
		interaction+cycle	11	20,727.3	2.0	20,795.5	25.4
	Low beta	additive	7	20,895.6	0.0	20,939.1	0.0
		FINAL	10	20,897.1	1.4	20,959.1	20.1
		interaction+cycle	11	20,899.1	3.4	20,967.3	28.3
	High beta	additive	7	18,989.0	0.0	19,032.4	0.0
		FINAL	10	18,993.7	4.7	19,055.7	23.3
		interaction+cycle	11	18,995.7	6.7	19,063.9	31.5
	Gamma	additive	7	16,461.0	16.9	16,504.4	0.0
		FINAL	10	16,457.0	12.9	16,519.0	14.6
		interaction+cycle	11	16,444.1	0.0	16,512.3	7.9