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Entanglement Transport Across Finite Registers: Schmidt-spectrum invariance, locality boundaries, and a falsification framework

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Abstract

Entanglement in a finite register is defined only after a bipartite tensor-product structure is supplied. For factorized isometric maps, every nonzero Schmidt weight is preserved; independent local Hamiltonians and local quantum channels likewise cannot generate entanglement from separable inputs. These standard facts yield a register-specific falsification criterion: a changed pure-state Schmidt spectrum disproves at least one of factorization, isometry, or closed-system completeness. The framework separates entanglement representation, transport, generation, no-signalling compatibility, Bell-inequality violation, and experimental realization, while identifying the additional operational assumptions required at each level.

1. Introduction

Entanglement is defined relative to a tensor-product decomposition, not by geometric resemblance. A centered diagram may display opposite cells or paired labels without identifying physical subsystems or a nonseparable state. This paper asks what follows after a bipartite Hilbert structure is explicitly supplied. [1, 2]

The main result concerns structural generation between adjacent finite registers. When the generation map factorizes into local isometries, it transports the complete Schmidt spectrum exactly. This yields a sharp diagnostic: any observed change of pure-state Schmidt data falsifies at least one of factorization, isometry, or closed two-register description.

2. Bipartite setting and exact tests

Let H_A and H_B be finite-dimensional complex Hilbert spaces and introduce H_AB = H_A tensor H_B as an additional state-space principle. A pure state is separable when it is a product |a> tensor |b>.

|psi > = sum_r sqrt(lambda_r)|a_r> tensor |b_r>, lambda_r > = 0, sum_r lambda_r = 1 (1)

Encoded Bell-sector theorem 2.1. If dim H_A > = 2 and dim H_B > = 2, H_AB contains a four-dimensional encoded two-qubit sector and hence the Bell state (|00>+|11>)/sqrt(2).

Proof

Choose two orthonormal vectors in each factor. Their four products span C^2 tensor C^2. The displayed vector has two Schmidt weights equal to 1/2 and is maximally entangled in that sector.

Relative to fixed product bases, write |psi > = sum_{i,j}C_ij|i>|j>.

Coefficient-rank theorem 2.2. The Schmidt rank of a pure bipartite state equals rank(C). Hence the state is separable exactly when rank(C) = 1.

Proof

A singular-value decomposition C = U Sigma V* becomes the Schmidt decomposition after absorbing U and V into local bases. Nonzero singular values are the square roots of the nonzero Schmidt weights.

Equal coefficients do not imply entanglement: (|00>+|01>+|10>+|11>)/2=|+> tensor |+> is separable, while (|00>+|11>)/sqrt(2) has coefficient rank two. The tensor structure, not the picture, decides the question. [1, 2]

3. Structural generation

Let V_A:H_A(n) to H_A(n + 1) and V_B:H_B(n) to H_B(n + 1) be isometries, and suppose the joint generation step is V_A tensor V_B.

Local-isometry preservation theorem 3.1. For every pure input, V_A tensor V_B preserves all nonzero Schmidt weights. Consequently it preserves Schmidt rank and every pure-state entanglement measure depending only on the Schmidt spectrum.

Proof

Apply V_A tensor V_B to the Schmidt decomposition (1). Isometries preserve orthonormality, so the resulting expansion is a Schmidt decomposition with the same coefficients.

Structural falsification corollary 3.2. A change in pure-state Schmidt spectrum across a claimed factorized-isometric generation step implies that at least one assumption is false: the map is nonfactorizing, nonisometric on the occupied subspace, or the modeled system has interacted with omitted degrees of freedom.

Proof

This is the contrapositive of Theorem 3.1 under the stated closed-system description.

A local coherence-to-event embedding is a special case of Theorem 3.1. Representation enlargement alone neither creates nor destroys pure-state entanglement.

4. Physical-time and channel boundaries

Keep physical time t distinct from structural label n. Consider H_AB = H_A tensor I + I tensor H_B + H_int. [1, 2]

Factorized-Hamiltonian no-go theorem 4.1. If H_int = 0, the unitary evolution factorizes as U_A(t) tensor U_B(t), preserves every pure-state Schmidt weight, and cannot entangle an initially separable pure state.

Proof

The two local Hamiltonian terms commute because they act on different factors. Their exponential factorizes, and local unitaries are isometries, so Theorem 3.1 applies.

A nonzero interaction is necessary for ordinary Hamiltonian creation of A|B entanglement from product inputs, but it is not sufficient for every state and time.

Local-channel no-go theorem 4.2. If rho_AB is separable and Phi_A,Phi_B are local completely positive trace-preserving maps, then (Phi_A tensor Phi_B)(rho_AB) is separable.

Proof

Write rho_AB=sum_k p_k rho_A^k tensor rho_B^k. The output is sum_k p_k Phi_A(rho_A^k) tensor Phi_B(rho_B^k), still a convex mixture of product states.

General local channels can reduce existing entanglement. The exact one-way statement is that they cannot create entanglement from a separable input without a joint operation or pre-existing entangled resource.

5. Conditional event statistics

Let {E_x^a} and {F_y^b} be complete local POVMs. Only for the compatibility benchmark, impose p(x,y|a,b) = Tr[rho_AB(E_x^a tensor F_y^b)].

No-signalling theorem 5.1. The local marginals are independent of the remote setting: sum_y p(x,y|a,b) = Tr[rho_AB(E_x^a tensor I)] and similarly for B.

Proof

Sum over the remote outcomes and use POVM completeness. Linearity of trace removes the remote setting.

No-signalling is therefore compatible with Bell-inequality violation; it does not turn correlations into controllable superluminal communication. [3–5]

6. Encoded CHSH benchmark

In the encoded Bell sector choose A_0=sigma_z, A_1=sigma_x, B_0=(sigma_z+sigma_x)/sqrt(2), and B_1=(sigma_z-sigma_x)/sqrt(2).

CHSH compatibility proposition 6.1. For |Phi+>=(|00>+|11>)/sqrt(2), the CHSH expression S = E(A_0,B_0) + E(A_0,B_1) + E(A_1,B_0)-E(A_1,B_1) equals 2 sqrt(2).

Proof

The four correlations are 1/sqrt(2), 1/sqrt(2), 1/sqrt(2), and − 1/sqrt(2), respectively. Substitution gives 4/sqrt(2) = 2 sqrt(2).

This reproduces standard two-qubit quantum mechanics. It does not derive the Bell state, settings, Born rule, independent choices, or spacelike laboratory separation. [3–7]

7. Operational levels and completion gates

Table 1
Algebraic entanglement, Bell nonlocality, and experimental demonstration are distinct claims.

Level	Required structure	Defensible conclusion
Internal-factor entanglement	declared H_A tensor H_B and nonseparable state	mathematical entanglement
Inter-register entanglement	operational subsystem identification and preparation	entanglement of modeled subsystems
Bell protocol	local settings, Born statistics, causal separation	Bell-nonlocal prediction
Experiment	implemented apparatus and controlled data	physical demonstration

Bell-completion boundary theorem 7.1. Finite-register geometry and a tensor-product label alone do not establish a physical Bell protocol. Such a claim additionally requires an operational factorization, preparation law, local observables, probability rule, causal localization, and a predeclared statistical test.

Proof

Each missing item enters a logically different part of p(x,y|a,b) and its physical interpretation. The tensor-product state space supplies none of them uniquely, so no one item can be inferred merely by relabeling geometric factors.

8. Novelty and literature boundary

Bell-sector existence, Schmidt decomposition, coefficient rank, local-unitary and local-channel restrictions, no-signalling from local completeness, and the CHSH value 2 sqrt(2) are standard. Current Bell and entanglement literature treats these as foundational tools, and modern experiments establish Bell violations independently of the finite-register construction. No novelty is claimed for those component results. [1–7]

The defensible contribution is their series-specific consolidation: factorized isometric register growth is shown to be entanglement-transparent; Schmidt-spectrum change becomes a falsification gate; and the paper separates mathematical factorization, transported entanglement, generated entanglement, Bell

nonlocality, and experiment. The source discussion of possible tests is retained only as a precommitment checklist, not as an empirical prediction.

9. Conclusion

Once a bipartite Hilbert structure is declared, finite registers can represent ordinary entanglement. Local factorized isometries transport the complete Schmidt spectrum, independent local Hamiltonians cannot create pure-state entanglement, and local channels preserve separability. A genuine generator must therefore cross the factorization or use an entangled resource. Conditional Born measurements give no-signalling and the standard CHSH benchmark, but the register geometry does not choose the physical factors, state, observables, probability law, causal arrangement, or experiment. Those remain explicit completion requirements.

Declarations

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References

1. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2010). https://doi.org/10.1017/CBO9780511976667
2. Horodecki, R., Horodecki, P., Horodecki, M., Horodecki, K.: Quantum entanglement. Rev. Mod. Phys. **81**, 865–942 (2009). https://doi.org/10.1103/RevModPhys.81.865
3. Bell, J.S.: On the Einstein Podolsky Rosen paradox. Phys. Phys. Fiz. **1**, 195–200 (1964). https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195
4. Clauser, J.F., Horne, M.A., Shimony, A., Holt, R.A.: Proposed experiment to test local hidden-variable theories. Phys. Rev. Lett. **23**, 880–884 (1969). https://doi.org/10.1103/PhysRevLett.23.880
5. Brunner, N., Cavalcanti, D., Pironio, S., Scarani, V., Wehner, S.: Bell nonlocality. Rev. Mod. Phys. **86**, 419–478 (2014). https://doi.org/10.1103/RevModPhys.86.419

6. Hensen, B., et al.: Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. Nature. **526**, 682–686 (2015). https://doi.org/10.1038/nature15759
7. Gühne, O., Haapasalo, E., Kraft, T., Pellonpää, J.-P., Uola, R.: Colloquium: Incompatible measurements in quantum information science. Rev. Mod. Phys. **95**, 011003 (2023). https://doi.org/10.1103/RevModPhys.95.011003