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How Much Additional Revenue Can a Government Raise by Increasing the Consumption Tax Rate?*

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Abstract

How much additional revenue can a government raise by increasing the consumption tax rate? We address this question using Laffer curves and the marginal cost of public funds (MCPF). We characterize analytically the global shape of the consumption-tax Laffer curve in a neoclassical model. The shape depends on the income effect on labor supply and on the use of tax revenue: KPR preferences rule out a hump, while additively separable and GHH preferences can generate one; rebating revenue makes humps less likely, whereas government absorption makes them more likely. Total tax revenue can peak before consumption-tax revenue or even decline from the outset, and the same cross-tax revenue effects determine the MCPF, which depends on marginal total revenue. At our quantitative parameter values under the rebate scheme, only GHH generates a hump; in that case, consumption-tax revenue can rise more than fivefold while total tax revenue rises by less than three percent.

Keywords: consumption tax; tax revenue; Laffer curve; tax interaction effect; marginal cost of public funds

JEL Classifications: H20, H21, H30, E62

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1 Introduction

The main objective of this paper is to ask how much additional revenue a government can raise by increasing the consumption tax rate. We address the question with two complementary objects: the Laffer curve for consumption tax, which gives the global revenue implications of the tax rate, and the marginal cost of public funds (MCPF), which gives the welfare cost of raising one more unit of public revenue at the margin. The first is the object we characterize analytically in a neoclassical model; the second follows from it. Wanniski (1978) attributes to Arthur Laffer the conjecture that tax revenue is hump-shaped in the tax rate: a higher rate raises revenue on the existing base but also erodes that base, and revenue falls once the erosion dominates.[1] For consumption taxes, Trabandt and Uhlig (2011, 2013) find numerically that the Laffer curve is monotonically increasing, in contrast to labor and capital income taxes, whose revenue curves have interior peaks. Whether this reflects a general property of the neoclassical model or a particular choice of preferences and fiscal rules has remained open. We answer that question, and then ask what the answer implies for the revenue a government can actually raise when other distortionary taxes are in place, and for the MCPF.

Our first and central contribution is a global characterization of the consumption tax revenue curve: the conditions under which it is hump-shaped rather than monotone, whether revenue is bounded, and the existence, uniqueness, and location of an interior peak. Two forces determine the shape. Preferences matter through the income effect on labor supply: King–Plosser–Rebelo (KPR) preferences rule out a hump for all parameter values, whereas additively separable and Greenwood–Hercowitz–Huffman (GHH) preferences can generate one. Monotonicity is therefore not a general property of the neoclassical model: it is guaranteed by the KPR specification that Trabandt and Uhlig (2011) employ, and it can fail under either alternative. The use of tax revenue matters as well: when the revenue is rebated lump sum, household income is protected and revenue can grow without bound, whereas when it is absorbed by government consumption, household resources fall, consumption is compressed further, and humps and bounded revenue become more likely; under GHH preferences with the revenue absorbed by government consumption, the curve is hump-shaped at every labor supply elasticity.

The consumption tax revenue curve, however, need not measure the government's fiscal capacity. With an existing labor income tax, and in the dynamic economy a capital income tax, a higher consumption tax also moves those tax bases, so the change in the government's budget is the change in total tax revenue, not in consumption tax revenue alone. We characterize the consumption tax revenue and total tax revenue curves side by side under a common fiscal rule. Whenever both have interior peaks, total tax revenue peaks strictly before consumption tax revenue; with sufficiently strong erosion of the existing bases, total

[1] Two usages of "Laffer curve" coexist: following Trabandt and Uhlig (2011), we use it for the tax revenue curve itself, whatever its shape; public finance textbooks such as Gruber (2015) reserve the term for the hump-shaped case.

tax revenue declines from the outset.

The distinction has a direct welfare counterpart. The MCPF for a marginal tax reform divides the household's welfare loss, measured in money terms, by the increase in the government's total tax revenue, so the relevant revenue margin is exactly the marginal total tax revenue just characterized. The consumption tax can then be on the rising branch of the consumption tax revenue curve while additional public funds are already arbitrarily expensive. One case is clean: in the static fund raising experiment in which the revenue is absorbed by government consumption, KPR preferences make labor supply, and with it the existing labor income tax base, invariant to the consumption tax rate, so the fiscal interaction vanishes and the MCPF equals one plus the consumption tax rate, the consumer price including the tax, for every parameter configuration. We confine the welfare interpretation to the static economy: comparisons across balanced growth paths hold no initial capital stock fixed, so dynamic results are reported as marginal revenue, not as welfare.

The classification of the consumption tax revenue curve carries over unchanged to the balanced growth path of a neoclassical growth economy in the tradition of Trabandt and Uhlig (2011), where total tax revenue follows the same logic with thresholds that also involve the capital income tax base. We evaluate both at parameter values based on their US calibration, with two parameters adapted to the present exercise, under the scheme in which the revenue is rebated to households. For that scheme the parameter restrictions required for a hump are demanding at conventional values, and the calibration shows what they imply: the consumption tax revenue curve is strictly increasing under KPR preferences and under additively separable preferences, and is hump-shaped only under GHH preferences; the total tax revenue curve behaves the same way at these parameters. Under a rebate scheme a hump is therefore possible but not typical, whereas absorption by government consumption makes one considerably easier to obtain. Under GHH preferences the two peaks are nevertheless far apart: consumption tax revenue peaks at a 100% tax rate and rises more than fivefold, whereas total tax revenue peaks at 22.4% and stays less than three percent above its baseline level, because almost all of the additional consumption tax revenue is offset by the erosion of the existing labor and capital income tax bases. At this calibration the fiscal interaction is present under all three specifications, so the total tax revenue results do not hinge on the contested parameter ranges.

Related literature. The theoretical literature on Laffer curves includes Ireland (1994) on capital income taxation in an AK model and Schmitt-Grohé and Uribe (1997) on labor income taxation in a neoclassical growth model. Trabandt and Uhlig (2011, 2013) compute Laffer curves for consumption, labor, and capital taxes for the US and the EU-14; Nutahara (2015) does so for Japan; Fève et al. (2018) study an economy with incomplete markets; Holter et al. (2019) study household heterogeneity and tax progressivity; Badel and Huggett (2017) develop a sufficient statistic approach to the peak of the labor

tax Laffer curve. Kobayashi (2014) examines the boundedness of consumption tax revenue under log utility. Relative to this literature, our contribution to positive analysis is an analytical characterization: exact conditions for hump shape and boundedness, jointly in preferences and in the use of tax revenue, for both consumption tax revenue and total tax revenue.[2] A hump-shaped consumption tax revenue curve can also arise from frictions that we deliberately exclude: Baydur and Yilmaz (2021) generate one from home production and the value added tax (VAT) treatment of financial institutions, Miravete et al. (2018) from market power, and Vasilev (2018) from tax evasion. Our results say what a frictionless model can and cannot deliver, and therefore when such frictions are needed. The results also bear on two applied literatures. Bi (2012), Leeper and Walker (2011), and Leeper (2013) study fiscal limits, the maximum debt burden sustainable without default or inflation, without giving the consumption tax a central role, although the maximum attainable revenue is exactly what boundedness of the revenue curve governs. Braun and Joines (2015), Hansen and Imrohoroglu (2016), and Kitao (2018) use the consumption tax as a financing instrument in fiscal reform analyses precisely because its revenue curve appears not to peak; our total tax revenue results qualify that reading substantially.[3]

On the public finance side, the marginal cost of public funds we use is the uncompensated concept of Mayshar (1990) and Slemrod and Yitzhaki (2001) for marginal reforms; see Dahlby (2008) for a monograph treatment and Johansson and Kriström (2025) for the commodity tax case. The erosion or expansion of an existing tax base by another tax is the tax interaction effect of Bovenberg and de Mooij (1994). That the MCPF can fall short of one because of income effects on the tax base goes back to Atkinson and Stern (1974) and Triest (1990); the connection between the MCPF and the Laffer curve is developed by Dahlby and Ferede (2018); and Jacobs (2018) shows that the MCPF equals one at an *optimal* tax system; our exercise is the complementary positive one, evaluating marginal reforms at observed tax rates, which need not be optimal. A particularly relevant comparison for this part of the paper is Bastani (2025), who provides a guide connecting the MCPF to the marginal value of public funds of Hendren and Sprung-Keyser (2020) and derives consumption tax expressions in a simple labor supply setting in which, in the consumption tax illustration, the existing labor income tax is normalized away; our exercise keeps that distortionary tax in place, so the response of the other tax base is part of the MCPF denominator. We claim no novelty for any of these concepts, formulas, or interaction effects taken separately. Combining them in one analytical framework is what the paper adds: the same globally characterized equilibrium responses that shape the Laffer curve for consumption tax also determine the response of the existing tax bases, the total tax revenue margin, and the resulting cost of funds, and this

[2] Trabandt (2006) finds numerically that the Laffer curve for consumption tax can be hump-shaped under GHH utility; our results provide the analytical conditions. Hiraga and Nutahara (2022) compare Laffer curves for consumption and labor taxes.

[3] The Laffer curve is a long run object; for the effects of consumption tax changes in the short run, see Baker et al. (2021) and Hino (2020).

permits statements in closed form about consumption tax revenue versus total tax revenue, the ordering of their peaks, and the erosion or expansion of existing bases. The KPR benchmark noted above is of this kind: the formula is the standard unit elasticity result; the model supplies the general equilibrium reason it applies exactly.

The remainder of the paper is organized as follows. Section 2 develops the main analytical results for the Laffer curve for consumption tax in a simple static economy, including the total tax revenue curve when a labor income tax is already in place. Section 3 studies the marginal cost of public funds and closes with a quantitative illustration of the marginal experiment. Section 4 extends the revenue curve analysis to the dynamic economy à la Trabandt and Uhlig (2011). Section 5 discusses the empirical plausibility of the hump conditions and reports quantitative results for both revenue curves and for the marginal revenue responses behind them. Section 6 concludes.

2 Laffer curve for consumption tax in a simple static economy

2.1 Model

A representative household consumes c, supplies labor n at wage w, receives a lump-sum transfer s from the government, and pays a consumption tax at rate τ^c. Its budget constraint is

$$(1+\tau^c)\,c \leq wn + s. \tag{1}$$

Firms are competitive and produce output y with the linear technology $y = n$, so $w = 1$ in equilibrium for every τ^c; there is no general equilibrium wage feedback, which is what makes the envelope arguments below clean. The government budget constraint is $s + g = T$, where g is government consumption and total tax revenue is $T = \tau^c c$ in this section; the resource constraint is $y = c + g$.

We consider three standard utility specifications. The first is the KPR form with constant labor supply elasticity,

$$U^{KPR} = \frac{1}{1-\eta}\left\{c^{1-\eta}\left[1-\kappa(1-\eta)n^{1+\lambda}\right]^{\eta} - 1\right\},$$

employed by, among others, Basu and Kimball (2005), Shimer (2009), Trabandt and Uhlig (2011), Kim and Katayama (2013), and Katayama and Kim (2018); see King and Rebelo (1999) for the balanced growth background; here η is relative risk aversion, $1/\lambda$ the (constant marginal utility of wealth) labor supply elasticity, and $\kappa > 0$ the preference weight on the labor term.[4] The second is additively separable (AS),

$$U^{AS} = \frac{c^{1-\eta}-1}{1-\eta} - \kappa n^{1+\lambda},$$

[4]We call $1/\lambda$ the labor supply elasticity although it is often interpreted as a Frisch elasticity; Section 5.1 discusses the calibration debate.

as in Gali (2008); the two coincide when $\eta = 1$. The third is the GHH form,

$$U^{GHH} = \frac{1}{1-\bar{\eta}}\Big\{\big(c - \kappa n^{1+\lambda}\big)^{1-\bar{\eta}} - 1\Big\},$$

which eliminates the income effect on labor supply and is employed by, among others, Jaimovich and Rebelo (2009), Gertler et al. (2012), and Korinek and Simsek (2016). We assume $\eta > 0$ and $\lambda \geq 0$ throughout, with $\lambda > 0$ and curvature $\bar{\eta} > 0$ for GHH utility. GHH utility is defined on allocations with $c - \kappa n^{1+\lambda} > 0$; this restriction never binds under the schemes considered below, but it matters for one of the alternative schemes of Section 2.4 that fix fiscal variables in levels.

Two benchmark fiscal rules, which we call *schemes*, specify the use of tax revenue:

Definition 1. *Under Scheme (1), all tax revenue is rebated as a lump-sum transfer: $s = T$, $g = 0$. Under Scheme (2), all tax revenue is used as government consumption: $g = T$, $s = 0$.*

Combining the production function, the resource constraint, and the government budget constraint, the two schemes pin down the equilibrium link between consumption and labor:

$$\text{Scheme (1):} \quad c = n, \qquad\qquad \text{Scheme (2):} \quad (1+\tau^c)\, c = n. \tag{2}$$

Alternative fiscal policy schemes are discussed in Section 2.4.

2.2 Consumption tax revenue curve for consumption tax

Consumption tax revenue is $R_c(\tau^c) = \tau^c\, c(\tau^c)$, where $c(\tau^c) > 0$ denotes equilibrium consumption; $R_c(0) = 0$. The object of this subsection is the relationship between the consumption tax rate τ^c and consumption tax revenue R_c. We ask (i) whether R_c is hump-shaped or monotone, (ii) whether it is bounded, and (iii) where its maximum lies when one exists. Because "hump-shaped" will carry necessary and sufficient conditions, we fix its meaning first.

Definition 2. *Let $f : [0,\infty) \to \mathbb{R}$ be continuously differentiable. We call f* strictly increasing *(*decreasing*) on an interval if $f' > 0$ ($f' < 0$) there. We call f* hump-shaped *if there exists $\tau^* > 0$ such that $f' > 0$ on $[0,\tau^*)$, $f'(\tau^*) = 0$, and $f' < 0$ on (τ^*,∞); then τ^* is the unique stationary point and the unique global maximizer of f on $[0,\infty)$. We call f* U-shaped *if $-f$ is hump-shaped.*

Remark 1. *Hump-shaped refers to global single-peakedness, not to global concavity: a hump-shaped revenue curve in the sense of Definition 2 need not satisfy $f'' < 0$ everywhere, and none of our results asserts concavity.*

Marginal revenue and the elasticity path. Let

$$\varepsilon_c(\tau^c) \;\equiv\; -\frac{d\ln c}{d\ln \tau^c} \tag{3}$$

denote the elasticity of equilibrium consumption with respect to the consumption tax rate. Throughout the paper ε_c is this *tax rate* elasticity; where the gross consumption price $1+\tau^c$ is more convenient, we work with the gross price elasticity $-\,d\ln c/d\ln(1+\tau^c)$ and convert using $\varepsilon_c = [\tau^c/(1+\tau^c)] \cdot (-\,d\ln c/d\ln(1+\tau^c))$; every statement about the shape of revenue is made in terms of ε_c.

Lemma 1. $R_c'(\tau^c) = c(\tau^c)\big[1-\varepsilon_c(\tau^c)\big]$. *Hence, at each tax rate, marginal revenue is positive, zero, or negative according as ε_c is below, equal to, or above one.*

Proof. $R_c' = c + \tau^c\, dc/d\tau^c = c\,[1 + (\tau^c/c)\, dc/d\tau^c] = c\,(1-\varepsilon_c)$, and $c>0$. □

Lemma 1 is exact but *pointwise*. Knowing that $\varepsilon_c = 1$ at some tax rate establishes only that marginal revenue vanishes there; it does not establish that the stationary point is unique, that it is a maximum, or that the curve is hump-shaped in the sense of Definition 2. A positive consumption function whose elasticity crosses one repeatedly generates a revenue curve with several interior maxima while satisfying Lemma 1 identically. Global statements require knowledge of the whole elasticity path:

Lemma 2. *Suppose ε_c is continuous on $[0,\infty)$ with $\varepsilon_c(0)=0$.*

(i) If $\varepsilon_c(\tau^c) < 1$ for all $\tau^c > 0$, then R_c is strictly increasing on $[0,\infty)$.

(ii) If there exists $\tau^* > 0$ such that $\varepsilon_c < 1$ on $(0,\tau^*)$, $\varepsilon_c(\tau^*) = 1$, and $\varepsilon_c > 1$ on (τ^*,∞), then R_c is hump-shaped with unique global maximizer τ^*.

(iii) In particular, if ε_c is strictly increasing with $\lim_{\tau^c\to\infty}\varepsilon_c(\tau^c) = L$, then case (i) applies when $L \le 1$ and case (ii) applies when $L > 1$, with τ^* the unique solution of $\varepsilon_c(\tau^c) = 1$.

Conditions (i)–(iii) are sufficient conditions; (iii) is not necessary for (ii). None of them restricts $\lim_{\tau^c\to\infty} R_c$, which must be established separately.

Proof. (i) By Lemma 1, $R_c' > 0$ wherever $\varepsilon_c < 1$; and $R_c'(0) = c(0) > 0$ since $\varepsilon_c(0) = 0$. (ii) By Lemma 1, $R_c' > 0$ on $(0,\tau^*)$, $R_c'(\tau^*) = 0$, and $R_c' < 0$ on (τ^*,∞), which is the definition of hump shape; uniqueness of the global maximizer follows since R_c is strictly increasing to the left of τ^* and strictly decreasing to its right. (iii) A continuous, strictly increasing ε_c with $\varepsilon_c(0) = 0$ and limit L satisfies $\varepsilon_c < 1$ everywhere if and only if $L \le 1$; if $L > 1$ it crosses one at a unique $\tau^* > 0$, with $\varepsilon_c < 1$ before and $\varepsilon_c > 1$ after. The final sentence holds because ε_c constrains only the derivative sign of R_c, not its level or asymptotic order. □

The last sentence matters: boundedness is a statement about the asymptotic order of $\tau^c\, c(\tau^c)$ and never follows from the local sign of marginal revenue alone. Throughout the paper, shape and boundedness are therefore proved by separate arguments.

Two closed forms for equilibrium consumption. Solving the household's intratemporal condition $-U_n/U_c = 1/(1+\tau^c)$, where subscripts on U denote partial derivatives, together with the equilibrium link (2) delivers equilibrium consumption $c(\tau^c)$ in closed form for each utility specification. What is classified below is not the utility functions themselves but this closed form response of equilibrium consumption to the consumption tax rate. In five of the six benchmark cases, consumption depends on the tax rate only through the gross consumption price $1+\tau^c$ and is a power of it,

$$c(\tau^c) = C\,(1+\tau^c)^{-\Omega},$$

with a constant $\Omega > 0$ that varies across cases; we call this the *power form*. The exceptional case, KPR utility under the rebate Scheme (1), instead delivers a power of a linear function of the tax rate itself,

$$c(\tau^c) = C\,(a+b\tau^c)^{-1/(1+\lambda)},$$

with constants $a, b > 0$; we call this the *KPR form*. The two labels are organizational, not a partition of function space: the power form is itself a power of a linear function of the tax rate, and the labels simply record which forms arise in equilibrium. Revenue is a product of a tax term and consumption, so the next two lemmas classify, once and for all, the global shape of products $C(\tau+m)\times$ (consumption form) with a location shift $m \geq 0$. The shift is also an organizational device. In this section $m = 0$, because consumption tax revenue is $\tau^c\,c$; in Section 2.3 the existing labor income tax makes total tax revenue proportional to $(\tau^c+\tau^n)\,c$, so $m = \tau^n$; and in the dynamic analysis of Section 4 the labor and capital tax revenue appears as the analogous shifts m_3 and m_4. In each lemma the derivative has the sign of an expression that is linear in the tax rate and can therefore change sign at most once; boundedness is a separate asymptotic question.

Lemma 3. *Let $G(\tau) = C(\tau+m)(1+\tau)^{-\Omega}$ on $\tau \geq 0$, with $C > 0$, $m \geq 0$, $\Omega > 0$. Then*

$$G'(\tau) = C(1+\tau)^{-\Omega-1}\big[(1-\Omega m)+(1-\Omega)\tau\big], \tag{4}$$

and since the bracket is affine in τ it changes sign at most once. Consequently:

(i) If $\Omega > 1$: when $\Omega m < 1$, G is hump-shaped with unique global maximizer $\tau^* = (1-\Omega m)/(\Omega-1)$; when $\Omega m \geq 1$, G is strictly decreasing on $(0,\infty)$, with $G'(0) = 0$ if and only if $\Omega m = 1$. In either case $\lim_{\tau\to\infty} G = 0$.

(ii) If $\Omega = 1$: G is strictly increasing, constant, or strictly decreasing according as $m < 1$, $m = 1$, or $m > 1$, and $\lim_{\tau\to\infty} G = C$.

(iii) If $\Omega < 1$: when $\Omega m \leq 1$, G is strictly increasing on $(0,\infty)$; when $\Omega m > 1$, G is U-shaped with unique global interior minimizer $\tau^* = (\Omega m - 1)/(1-\Omega)$. In either case $\lim_{\tau\to\infty} G = \infty$.

For $m = 0$ (so $G(0) = 0$): G is hump-shaped if and only if $\Omega > 1$, with peak $1/(\Omega - 1)$; strictly increasing if and only if $\Omega \leq 1$; and bounded if and only if $\Omega \geq 1$, with $G \to C$ when $\Omega = 1$ and $G \to 0$ when $\Omega > 1$. The associated elasticity path is $\varepsilon_c(\tau) = \Omega\,\tau/(1+\tau)$, strictly increasing from 0 with supremum Ω, so the case $\Omega = 1$ has no finite stationary point: $\varepsilon_c < 1$ at every finite τ and $\varepsilon_c \to 1$ only asymptotically.

Proof. Differentiating the product,

$$G' = C(1+\tau)^{-\Omega} - \Omega C(\tau+m)(1+\tau)^{-\Omega-1} = C(1+\tau)^{-\Omega-1}\big[(1+\tau) - \Omega(\tau+m)\big],$$

and the bracket equals $(1-\Omega m) + (1-\Omega)\tau$, which is (4). Being affine in τ, the bracket changes sign at most once, and only from its sign at $\tau = 0$ to the sign of its slope $1-\Omega$. (i) If $\Omega > 1$ the slope is negative. When $\Omega m < 1$ the bracket is positive at 0 and vanishes at $\tau^* = (1-\Omega m)/(\Omega - 1) > 0$, so $G' > 0$ before and $G' < 0$ after: G is hump-shaped with unique global maximizer τ^*. When $\Omega m \geq 1$ the bracket is nonpositive at 0 and strictly negative for $\tau > 0$. (ii) If $\Omega = 1$ the bracket is the constant $1 - m$. (iii) If $\Omega < 1$ the slope is positive; the bracket is nonnegative at 0 if and only if $\Omega m \leq 1$, and otherwise vanishes at $\tau^* = (\Omega m - 1)/(1-\Omega) > 0$ with $G' < 0$ before and $G' > 0$ after. The limits follow from the asymptotic order $G = C(\tau+m)(1+\tau)^{-\Omega} \sim C\tau^{\,1-\Omega}$ as $\tau \to \infty$ for $\Omega \neq 1$ and, for $\Omega = 1$, $G = C(\tau+m)/(1+\tau) \to C$. The $m = 0$ statements are immediate specializations, and the elasticity claim follows from $c = C(1+\tau)^{-\Omega}$: $\varepsilon_c = -\,d\ln c/d\ln\tau = \Omega\tau/(1+\tau)$, with $\varepsilon_c' = \Omega/(1+\tau)^2 > 0$, $\varepsilon_c(0) = 0$, and $\varepsilon_c \to \Omega$. □

Lemma 4. *Let $H(\tau) = C(\tau+m)(a+b\tau)^{-1/(1+\lambda)}$ on $\tau \geq 0$, with $C, a, b > 0$, $m \geq 0$, $\lambda \geq 0$. Then*

$$H'(\tau) = \frac{C}{1+\lambda}\,(a+b\tau)^{-1/(1+\lambda)-1}\big[(1+\lambda)a - bm + \lambda b\tau\big], \tag{5}$$

again with an affine bracket. Consequently:

(i) If $\lambda > 0$: H is strictly increasing on $(0,\infty)$ if $bm \leq (1+\lambda)a$, and U-shaped with unique interior minimizer $\tau^* = [bm - (1+\lambda)a]/(\lambda b)$ otherwise; in either case $H \sim Cb^{-1/(1+\lambda)}\tau^{\lambda/(1+\lambda)} \to \infty$.

(ii) If $\lambda = 0$: H' has the constant sign of $a - bm$, so H is strictly increasing, constant, or strictly decreasing according as $bm < a$, $bm = a$, or $bm > a$; and $\lim_{\tau\to\infty} H = C/b$.

For $m = 0$: H is strictly increasing for all parameter values; it is unbounded if $\lambda > 0$ and converges to $C/b > 0$ if $\lambda = 0$. The associated elasticity path is $\varepsilon_c(\tau) = b\tau/[(1+\lambda)(a+b\tau)]$, strictly increasing from 0 with supremum $1/(1+\lambda) \leq 1$, with equality only when $\lambda = 0$ and only in the limit; hence $\varepsilon_c < 1$ at every finite τ.

Proof. Differentiating the product,

$$H' = C(a+b\tau)^{-\frac{1}{1+\lambda}-1}\Big[(a+b\tau) - \frac{b(\tau+m)}{1+\lambda}\Big] = \frac{C}{1+\lambda}(a+b\tau)^{-\frac{1}{1+\lambda}-1}\big[(1+\lambda)a - bm + \lambda b\tau\big],$$

which is (5), again with an affine bracket; its slope λb is nonnegative, so H' can change sign at most once and only from negative to positive. (i) For $\lambda > 0$ the bracket is nonnegative at 0 if and only if $bm \le (1+\lambda)a$; otherwise it vanishes at $\tau^* = [bm - (1+\lambda)a]/(\lambda b) > 0$, giving the U shape. Asymptotically $H \sim C\, b^{-1/(1+\lambda)}\tau^{1-1/(1+\lambda)} = C\, b^{-1/(1+\lambda)}\tau^{\lambda/(1+\lambda)} \to \infty$. (ii) For $\lambda = 0$ the bracket is the constant $a - bm$ and $H = C(\tau+m)/(a+b\tau) \to C/b$. The $m = 0$ statements follow since the bracket is then $(1+\lambda)a + \lambda b\tau > 0$; the elasticity claim follows from $c = C(a+b\tau)^{-1/(1+\lambda)}$: $\varepsilon_c = b\tau/[(1+\lambda)(a+b\tau)]$, with $\varepsilon_c' = ab/[(1+\lambda)(a+b\tau)^2] > 0$, $\varepsilon_c(0) = 0$, and $\varepsilon_c \to 1/(1+\lambda)$. □

Both proofs run through the same chain: monotone elasticity path, at most one crossing of unity, sign of marginal revenue on each side, and a separate boundedness argument based on the asymptotic order. This chain, and not the pointwise identity of Lemma 1, is what supports the global language of Definition 2.

Characterization. The next proposition maps each combination of utility and scheme into one of the two closed forms; the classification in Table 1 then follows from Lemmas 3 and 4 with $m = 0$. In words: AS and GHH utility deliver the power form under both schemes; KPR utility under Scheme (2) also delivers the power form, with $\Omega = 1$, because equilibrium labor supply turns out to be invariant to the tax rate; KPR utility under Scheme (1) is the one exception, and the only case that needs Lemma 4.

Proposition 1. *Solving $-U_n/U_c = 1/(1+\tau^c)$ together with (2) yields $c(\tau^c)$ in the power form (Lemma 3) or the KPR form (Lemma 4) with $m = 0$ and*

	Scheme (1)	*Scheme (2)*
KPR:	*KPR form,* $a = 1+\eta\lambda,\ b = \eta(1+\lambda)$	*power,* $\Omega = 1$
AS:	*power,* $\Omega = \dfrac{1}{\eta+\lambda}$	*power,* $\Omega = \dfrac{1+\lambda}{\eta+\lambda}$
GHH:	*power,* $\Omega = \dfrac{1}{\lambda}$	*power,* $\Omega = \dfrac{1+\lambda}{\lambda}$

Consequently:

(i) (KPR, Scheme (1)) R_c is strictly increasing; it is unbounded if $\lambda > 0$ and converges to $1/(\kappa\eta)$ if $\lambda = 0$.

(ii) (AS, Scheme (1)) R_c is hump-shaped if and only if $\eta + \lambda < 1$, with unique global maximizer $\tau^c = (\eta+\lambda)/(1-\eta-\lambda)$; otherwise it is strictly increasing. Revenue is bounded if and only if $\eta + \lambda \le 1$.

(iii) (GHH, Scheme (1)) R_c *is hump-shaped if and only if* $\lambda < 1$*, with unique global maximizer* $\tau^c = \lambda/(1-\lambda)$*; otherwise strictly increasing. Revenue is bounded if and only if* $\lambda \leq 1$*.*

(iv) (KPR, Scheme (2)) R_c *is strictly increasing and bounded, for all parameter values.*

(v) (AS, Scheme (2)) R_c *is hump-shaped if and only if* $\eta < 1$*, with unique global maximizer* $\tau^c = (\eta+\lambda)/(1-\eta)$*; otherwise strictly increasing. Revenue is bounded if and only if* $\eta \leq 1$*.*

(vi) (GHH, Scheme (2)) R_c *is hump-shaped for all* $\lambda > 0$*, with unique global maximizer* $\tau^c = \lambda$*; revenue is bounded.*

Proof. The marginal rates of substitution are $-U_n/U_c = \kappa(1+\lambda)c^\eta n^\lambda$ (AS), $\kappa(1+\lambda)n^\lambda$ (GHH), and $\eta(1+\lambda)\kappa c\, n^\lambda/[1-\kappa(1-\eta)n^{1+\lambda}]$ (KPR).

AS. With the link $c = n$ under Scheme (1), the condition $-U_n/U_c = 1/(1+\tau^c)$ becomes $\kappa(1+\lambda)c^{\eta+\lambda} = (1+\tau^c)^{-1}$, so $c = C(1+\tau^c)^{-1/(\eta+\lambda)}$. With the link $(1+\tau^c)c = n$ under Scheme (2), it becomes $\kappa(1+\lambda)c^{\eta+\lambda}(1+\tau^c)^\lambda = (1+\tau^c)^{-1}$, so $c = C(1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}$.

GHH. The condition pins labor supply directly: $n = [\kappa(1+\lambda)(1+\tau^c)]^{-1/\lambda}$ under either link. Then $c = n$ gives $\Omega = 1/\lambda$, and $c = n/(1+\tau^c)$ gives $\Omega = (1+\lambda)/\lambda$.

KPR. Write $z = \kappa n^{1+\lambda}$, so that $cn^\lambda = cz/(\kappa n)$. Under the link $c = n/(1+\tau^c)$ of Scheme (2), the factor $(1+\tau^c)$ cancels between c and the right hand side $1/(1+\tau^c)$, and the condition collapses to $z\big[\eta(1+\lambda)+1-\eta\big] = 1$: equilibrium labor supply is a constant, independent of τ^c, and $c = n/(1+\tau^c) \propto (1+\tau^c)^{-1}$: the power form with $\Omega = 1$ for *all* parameter values. Under the link $c = n$ of Scheme (1) no such cancellation occurs, because the marginal rate of substitution still depends on the level of n; the condition rearranges to $z\big[(1+\eta\lambda)+\eta(1+\lambda)\tau^c\big] = 1$, so $c = n = (z/\kappa)^{1/(1+\lambda)} \propto (a+b\tau^c)^{-1/(1+\lambda)}$ with $a = 1+\eta\lambda$ and $b = \eta(1+\lambda)$: the KPR form.

In every case the constant C collects κ and λ and never enters the exponent. Claims (i)–(vi) then read off Lemmas 3 and 4 with $m = 0$, for each case; the boundary and limit statements are those of the lemmas. The same computations with an existing labor tax $\tau^n > 0$, needed in Section 2.3, are collected in Appendix A. □

Remark 2. *Four equality cases deserve explicit notice, because in each of them revenue is simultaneously strictly increasing and bounded and must not be described as having a peak:* $\eta + \lambda = 1$ *in case (ii),* $\lambda = 1$ *in case (iii),* $\eta = 1$ *in case (v), each of which gives* $\Omega = 1$*, so that* $\varepsilon_c < 1$ *at every finite tax rate and* $\varepsilon_c \to 1$ *only asymptotically, and* $\lambda = 0$ *in case (i), where the KPR form gives the same configuration. Case (iv) is of this type for* all *parameter values.*

Table 1. Consumption tax revenue in the static economy

Scheme	Utility	Ω	Global shape	Boundedness
(1) $s = T$	KPR	(KPR form)	strictly increasing	unbounded unless $\lambda = 0$
	AS	$\frac{1}{\eta+\lambda}$	hump-shaped iff $\eta+\lambda<1$; peak $\frac{\eta+\lambda}{1-\eta-\lambda}$	bounded iff $\eta+\lambda\leq 1$
	GHH	$\frac{1}{\lambda}$	hump-shaped iff $\lambda<1$; peak $\frac{\lambda}{1-\lambda}$	bounded iff $\lambda\leq 1$
(2) $g = T$	KPR	1	strictly increasing	bounded
	AS	$\frac{1+\lambda}{\eta+\lambda}$	hump-shaped iff $\eta<1$; peak $\frac{\eta+\lambda}{1-\eta}$	bounded iff $\eta\leq 1$
	GHH	$\frac{1+\lambda}{\lambda}$	always hump-shaped; peak λ	bounded

Income effects and the use of tax revenue. Why do the three utility functions differ, and why does the use of tax revenue matter? For any regular utility $U(c, n)$, define the local statistics

$$\gamma \equiv -\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n}, \qquad \varphi \equiv \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}, \tag{6}$$

the income effect on labor supply and the inverse of the labor supply elasticity at constant consumption, both evaluated at the equilibrium allocation associated with the prevailing tax rate.[5] Differentiating the intratemporal condition and using (2) gives the exact pointwise decomposition

$$\varepsilon_c(\tau^c) = \frac{\tau^c}{1+\tau^c}\,\Omega(\tau^c), \qquad \Omega(\tau^c) = \begin{cases} \dfrac{1}{\gamma+\varphi} & \text{Scheme (1)}, \\ \dfrac{1+\varphi}{\gamma+\varphi} & \text{Scheme (2)}. \end{cases} \tag{7}$$

(The factor $\Omega(\tau^c)$ in (7) is precisely the gross price elasticity $-d\ln c/d\ln(1+\tau^c)$; the prefactor $\tau^c/(1+\tau^c)$ converts it into the tax rate elasticity of (3).)

Three levels of statement must be distinguished here. *First*, (7) is a pointwise decomposition: because γ and φ are local statistics of $U(c, n)$ rather than primitive parameters, $\Omega(\tau^c)$ may vary with the tax rate, and the observation that $\Omega(\tau^c) > 1$ at some rate implies neither monotonicity of ε_c nor a unique crossing of unity nor a hump. *Second*, for additively separable and GHH utility the statistics are constant along the entire path, with $(\gamma, \varphi) = (\eta, \lambda)$ and $(0, \lambda)$ respectively, so Ω is a constant, (7) integrates to the power form, and only then, in combination with Lemma 3, may the conditions $\gamma + \varphi < 1$ (Scheme (1)) and $\gamma < 1$ (Scheme (2)) be read as necessary and sufficient for a hump. *Third*, KPR utility has $\gamma = 1$ exactly, which is precisely the balanced growth restriction, but φ depends on the allocation. Under Scheme (2) equilibrium labor supply turns out to be invariant to τ^c and, in any case, $\gamma = 1$ makes $\Omega = (1+\varphi)/(1+\varphi) = 1$ whatever the value of φ; under Scheme (1), φ varies along the path, the local condition $\gamma + \varphi < 1$ reads $\varphi < 0$ and cannot be signed a priori, and the global conclusion in Proposition

[5] This representation follows Bilbiie (2020). The KPR expression for φ is derived in Appendix B.

1(i) rests entirely on the KPR form of Lemma 4. The three preference specifications thus differ in more than γ alone, and they cannot be compared by varying γ while holding a common primitive φ fixed.

The economics, however, is transparent. A higher consumption tax lowers the price of leisure relative to consumption; the substitution effect reduces labor supply, while the income effect raises it. Under Scheme (1) the tax proceeds return to the household, income is protected, and only relative prices matter; under Scheme (2) the proceeds are absorbed by government consumption, household income falls, and consumption is compressed further, which is why the hump condition relaxes from $\gamma + \varphi < 1$ to $\gamma < 1$, and why humps and boundedness are more prevalent in the second half of Table 1.

Two boundedness results deserve comment, since unbounded revenue may appear paradoxical. Under Scheme (1) with KPR utility, every yen of tax paid returns as a transfer, so the household can finance an arbitrarily large tax bill and revenue diverges whenever labor supply is not completely inelastic. Under Scheme (2) with additively separable utility and $\eta > 1$, equilibrium labor supply $n = (1+\tau^c)c \propto (1+\tau^c)^{(\eta-1)/(\eta+\lambda)}$ rises without bound as the tax rate grows, so labor income, and with it the feasible tax payment, diverges even though consumption itself converges to zero. Such labor supply divergence is unlikely to be realistic, but it arises for standard preferences at conventional parameter values.

2.3 Total tax revenue curve with labor income tax

Suppose now that a labor income tax at rate $\tau^n \in [0, 1)$ is already in place. The household budget constraint becomes $(1+\tau^c)c \leq (1-\tau^n)wn + s$, and total tax revenue is

$$T = \tau^c\, c + \tau^n\, wn. \tag{8}$$

The total tax revenue curve for consumption tax is the relationship between the consumption tax rate τ^c and total tax revenue T; the labor income tax rate τ^n is held fixed as an existing tax. Schemes (1′) and (2′) are the versions of Schemes (1) and (2) when the existing labor income tax is present and the fiscal rule applies to total tax revenue T: under Scheme (1′) all of T is rebated ($s = T$, $g = 0$), and under Scheme (2′) all of T is absorbed by government consumption ($g = T$, $s = 0$); setting $\tau^n = 0$ reduces them to Schemes (1) and (2). The equilibrium links become $c = n$ and $(1+\tau^c)c = (1-\tau^n)n$, respectively, and the intratemporal condition is $-U_n/U_c = (1-\tau^n)/(1+\tau^c)$.

Substituting the links into (8) (with $w = 1$) gives

$$\text{Scheme (1}'\text{):}\ T = (\tau^c + \tau^n)\, c(\tau^c), \qquad \text{Scheme (2}'\text{):}\ T = \frac{\tau^c + \tau^n}{1-\tau^n}\, c(\tau^c), \tag{9}$$

so in both cases total tax revenue is a *product form* $C(\tau^c + m)(\,\cdot\,)$ with location shift $m = \tau^n$: the existing labor tax enters exactly as the shift in Lemmas 3 and 4. Moreover, τ^n does not alter the exponent:

repeating the derivations of Proposition 1 with the modified condition and links leaves every Ω, and the coefficient b of the KPR form, unchanged, with the labor tax absorbed into the multiplicative constant and, for KPR under Scheme (1′), into $a = (1-\eta)(1-\tau^n) + \eta(1+\lambda) > 0$. The economics of the extension is that an already taxed base is now in play: a higher consumption tax moves labor supply, and with it the existing labor income tax base, so total tax revenue can stop rising, and start falling, before consumption tax revenue does. The complete classification follows from the same two lemmas.

Proposition 2. *Let $\tau^n \in [0,1)$. Under the mappings just described:*

(i) (KPR, Scheme (1′)) T is strictly increasing; it is unbounded if $\lambda > 0$ and bounded if $\lambda = 0$.

(ii) (AS, Scheme (1′)) T is hump-shaped if and only if $\tau^n < \eta + \lambda < 1$, with unique global maximizer $\tau^c = (\eta + \lambda - \tau^n)/(1 - \eta - \lambda)$; it is strictly decreasing on $(0,\infty)$ if $\eta + \lambda \leq \tau^n$; otherwise it is strictly increasing. Revenue is bounded if and only if $\eta + \lambda \leq 1$.

(iii) (GHH, Scheme (1′)) T is hump-shaped if and only if $\tau^n < \lambda < 1$, with unique global maximizer $\tau^c = (\lambda - \tau^n)/(1-\lambda)$; it is strictly decreasing on $(0,\infty)$ if $\lambda \leq \tau^n$; otherwise strictly increasing. Revenue is bounded if and only if $\lambda \leq 1$.

(iv) (KPR, Scheme (2′)) T is strictly increasing and bounded, for all parameter values.

(v) (AS, Scheme (2′)) T is hump-shaped if and only if $(1+\lambda)\tau^n - \lambda < \eta < 1$, with unique global maximizer $\tau^c = \big[(\eta+\lambda) - (1+\lambda)\tau^n\big]/(1-\eta)$; it is strictly decreasing on $(0,\infty)$ if $\eta \leq (1+\lambda)\tau^n - \lambda$; it is strictly increasing if $\eta \geq 1$. Revenue is bounded if and only if $\eta \leq 1$.[6]

(vi) (GHH, Scheme (2′)) T is hump-shaped if and only if $\lambda > (1+\lambda)\tau^n$ (equivalently $\lambda > \tau^n/(1-\tau^n)$), with unique global maximizer $\tau^c = \lambda - (1+\lambda)\tau^n$; otherwise it is strictly decreasing on $(0,\infty)$. Revenue is bounded for all parameter values.

Proof. Each cell applies Lemma 3 or 4 to (9) with $m = \tau^n$ and the exponents of Proposition 1. The threshold conditions are $\Omega\tau^n \lesseqgtr 1$ rearranged in the underlying parameters, and the peak formulas are $\tau^* = (1 - \Omega\tau^n)/(\Omega - 1)$ evaluated for each case; the rearrangements, and the vacuousness argument behind the footnote to case (v), are collected in Appendix A. □

Setting $\tau^n = 0$ recovers Proposition 1 exactly. Two features of Proposition 2 have no counterpart in the analysis of consumption tax revenue. First, total tax revenue can be *decreasing from the outset*: when the existing labor tax is large relative to the labor supply parameters, the erosion of the labor tax base dominates the direct gain at every tax rate. Second, whenever both peaks are interior, the total tax revenue peak lies strictly below the consumption tax revenue peak:

[6] No U-shaped or flat case arises here: it would require $(1+\lambda)\tau^n - \lambda \geq \eta \geq 1$, i.e. $(1+\lambda)(\tau^n - 1) \geq \eta - 1 \geq 0$, which is impossible for $\tau^n < 1$.

Corollary 1. *Fix a scheme and a utility specification in the power form with exponent $\Omega > 1$, and suppose the total tax revenue peak is interior ($\Omega\tau^n < 1$). Then*

$$\tau^*_{R_c} - \tau^*_T = \frac{\Omega\,\tau^n}{\Omega - 1} \;>\; 0.$$

In particular, the gap equals $\tau^n/(1-\eta-\lambda)$ for AS utility under Scheme (1′) and $(1+\lambda)\,\tau^n$ for GHH utility under Scheme (2′).

Proof. By Lemma 3, consumption tax revenue $\tau^c\,c(\tau^c)$ is the product form with $m = 0$ and, for $\Omega > 1$, peaks at $\tau^*_{R_c} = 1/(\Omega-1)$; total tax revenue is the same form with $m = \tau^n$ and, when $\Omega\tau^n < 1$, peaks at $\tau^*_T = (1-\Omega\tau^n)/(\Omega-1)$. Subtracting gives the displayed gap, positive whenever $\tau^n > 0$. The special cases substitute $\Omega = 1/(\eta+\lambda)$ and $\Omega = (1+\lambda)/\lambda$. □

Whenever both peaks are interior, the total tax revenue peak therefore occurs before the consumption tax revenue peak: beyond the total tax revenue peak, a higher consumption tax continues to raise consumption tax revenue while lowering the government's total tax receipts. The consumption tax revenue curve is, in this sense, not a statement about fiscal capacity when other taxes are present.

2.4 Alternative fiscal policy schemes

For the consumption tax revenue analysis of Section 2.2, Schemes (1) and (2) are two polar cases in which all tax revenue is rebated or all of it is absorbed by government consumption. Fixing the ratio g/y (with transfers absorbing the residual) or the ratio s/y (with government consumption absorbing the residual) leaves the equilibrium link (2) unchanged up to a constant of proportionality, so Proposition 1 applies verbatim. Fixing g or s *in level* is genuinely different: the government's share of output then rises as output falls, a rule that nominally rebates the revenue behaves like one that absorbs it, and the hump and boundedness conditions align with those of Scheme (2): on the relevant domain the curve is single-peaked if and only if $\eta < 1$ under AS utility, and never under KPR utility, while the peak location differs. Under GHH utility, single-peakedness holds on the economically admissible domain subject to explicit domain conditions: the GHH requirement $c - \kappa n^{1+\lambda} > 0$ truncates the admissible tax range when government consumption is fixed in level, and nonnegative government consumption defines the feasible interval under the transfer type rule. These results are stated with complete proofs as Propositions C.1 and C.2 in Appendix C; they require separate single-crossing arguments because consumption under rules fixed in levels does not belong to the two closed forms of Section 2.2.

3 Marginal cost of public funds

3.1 Definition and fiscal experiment

We use the definition of Mayshar (1990) for a marginal tax reform, as restated by Slemrod and Yitzhaki (2001): for a marginal increase $d\tau^c > 0$,

$$\text{MCPF} = \frac{\text{welfare loss of the household in money terms}}{\text{increase in the government's total tax revenue}}. \tag{10}$$

Both entries of (10) discipline the exercise. Under the benchmark fund raising experiment used below, the relevant revenue denominator is $dT/d\tau^c$, not $dR_c/d\tau^c$: with $\tau^n > 0$, a higher consumption tax also moves the existing labor income tax base, so the fiscal interaction of Section 2.3 sits *inside* the MCPF. The numerator is a welfare quantity, which ties the concept to a specific fiscal experiment, a sensitivity emphasized by Ballard and Fullerton (1992): under a general fiscal rule, the household's cost of the reform in money terms is $c - ds/d\tau^c$ (Lemma 5), and it equals consumption only when the lump-sum transfer is unchanged by the reform. The MCPF is accordingly evaluated under a marginal fund raising experiment: the marginal revenue in the denominator must be retained for government use, not simultaneously rebated. Scheme $(2')$ is that benchmark experiment, and under the rebate Schemes (1) and $(1')$ the marginal revenue is returned to the household instead; Proposition 3 makes the contrast formal. Under the alternative schemes of Section 2.4, transfers or spending shares move with the reform, so the welfare numerator and the resources retained for government use both change with the fiscal rule: those are different fiscal experiments, not robustness versions of the same formula. Appendix C accordingly establishes the robustness of the revenue curve results under those schemes and makes no MCPF calculation for them; we deliberately confine the MCPF to its benchmark experiment. The dynamic Schemes (3) and (4) are likewise assigned no MCPF interpretation (Remark 3).

We therefore develop the MCPF for the static economy under Scheme $(2')$. Government consumption does not enter the static utility function, so the separability condition under which the MCPF is well defined independently of the spending side is satisfied by construction; the valuation of the marginal public good is a separate object (the marginal benefit of public projects of Slemrod and Yitzhaki, 2001) and is not needed for the cost side.

3.2 Marginal cost of public funds in the static economy

Lemma 5. *In the static economy of Section 2.3, let the fiscal rule determine the transfer as a function $s(\tau^c)$ of the consumption tax rate, let $V(\tau^c)$ be the household's indirect utility, and let $\mu = U_c/(1+\tau^c)$ be its marginal utility of income. Then*

$$\frac{dV}{d\tau^c} = \mu\Big[-c + \frac{ds}{d\tau^c}\Big] :$$

the household's welfare cost of a marginal increase in τ^c, measured in money terms, is $c - ds/d\tau^c$. In particular, under Scheme (2′) the transfer is identically zero, so the cost is c, the mechanical base of the reform; under Scheme (1′) the transfer is total tax revenue, $s = T$, so the cost is $c - dT/d\tau^c$.

Proof. The household maximizes $U(c, n)$ subject to $(1+\tau^c)c = (1-\tau^n)n+s$, taking (τ^c, τ^n, s) as given, with first order conditions $U_c = (1+\tau^c)\mu$ and $-U_n = (1-\tau^n)\mu$ (using $w = 1$), where μ is the multiplier. Along the equilibrium path, $dV/d\tau^c = U_c\, c' + U_n\, n' = \mu\big[(1+\tau^c)c' - (1-\tau^n)n'\big]$. Differentiating the budget constraint gives $(1+\tau^c)c' + c = (1-\tau^n)n' + ds/d\tau^c$, so $dV/d\tau^c = \mu\big[-c + ds/d\tau^c\big]$ under any fiscal rule. Under Scheme (2′), $s = 0$ throughout, so $ds/d\tau^c = 0$ and $dV/d\tau^c = -\mu c$. Under Scheme (1′), the transfer moves with the reform: $s = T = (\tau^c+\tau^n)\, c$ gives $ds/d\tau^c = dT/d\tau^c = c+(\tau^c+\tau^n)\, c'$, so $dV/d\tau^c = \mu(\tau^c + \tau^n)c'$ and the welfare cost in money terms is $c - dT/d\tau^c = (\tau^c + \tau^n)(-c')$, equivalently $c\,\varepsilon_c\,(\tau^c + \tau^n)/\tau^c$ for $\tau^c > 0$; only when $\tau^n = 0$ does this reduce to $c\,\varepsilon_c$. Since $c' < 0$, the cost is positive whenever $\tau^c + \tau^n > 0$, but it is smaller than c exactly where $dT/d\tau^c > 0$ and exceeds c where total tax revenue is falling. □

Lemma 5 is the supporting envelope result. The next proposition, the paper's static MCPF result, applies it to both primed schemes through the government budget $s + g = T$, whose margin $dg/d\tau^c = dT/d\tau^c - ds/d\tau^c$ records the public funds retained at the margin. Because Scheme (2) is the $\tau^n = 0$ special case of Scheme (2′), and Scheme (1) of Scheme (1′), the proposition contains the economy without an existing labor tax as a special case.

Proposition 3. *Consider a marginal increase in τ^c in the static economy of Section 2.3, with government budget $s + g = T$.*

(i) (Scheme (1′)) With $s = T$ and $g = 0$, the transfer moves one for one with total tax revenue, $ds/d\tau^c = dT/d\tau^c$, so the household's welfare cost in money terms is $c - dT/d\tau^c$, and $dg/d\tau^c = dT/d\tau^c - ds/d\tau^c = 0$: the entire marginal change in total tax revenue is returned to the household through the transfer, and the government retains no marginal public funds for government use. Scheme (1′) is therefore not a fund raising experiment, and no MCPF is attached to it.

(ii) (Scheme (2′)) With $s = 0$ and $g = T$, the household's welfare cost in money terms is c, and $dg/d\tau^c = dT/d\tau^c$: the marginal revenue is retained as government consumption. On the domain where $dT/d\tau^c > 0$,

$$\mathrm{MCPF}(\tau^c) = \frac{c}{dT/d\tau^c} = \frac{1}{1 - \varepsilon_c(\tau^c) + \tau^n \dfrac{dn/d\tau^c}{c}}, \qquad \frac{dT}{d\tau^c} = \underbrace{c\,(1-\varepsilon_c)}_{dR_c/d\tau^c} + \underbrace{\tau^n \frac{dn}{d\tau^c}}_{\text{labor tax base}}. \tag{11}$$

For the power form cases with constant exponent Ω,

$$\mathrm{MCPF}(\tau^c) = \frac{(1+\tau^c)(1-\tau^n)}{(1+\tau^c) - \Omega\,(\tau^c + \tau^n)}, \tag{12}$$

which diverges exactly at the total tax revenue peak of Proposition 2 and is not defined as a cost of funds beyond it.

Proof. Part (i) combines Lemma 5 with the government budget: $s = T$ gives $ds/d\tau^c = dT/d\tau^c$, so the welfare cost in money terms is $c - dT/d\tau^c$, and $dg/d\tau^c = dT/d\tau^c - ds/d\tau^c = 0$; with $g = 0$ throughout, no marginal revenue is retained for government use. For part (ii), the numerator of (10) in money units is Lemma 5: dividing the utility loss $-dV/d\tau^c = \mu c$ by the marginal utility of income μ gives c. For the denominator, differentiating the government budget $T = \tau^c c + \tau^n n$ gives $dT/d\tau^c = c + \tau^c\, c' + \tau^n\, n' = c\,(1 - \varepsilon_c) + \tau^n\, dn/d\tau^c$, which is the decomposition in (11), and the first equality of (11) is its reciprocal form. For the power form cases, the link $n = (1+\tau^c)c/(1-\tau^n)$ under Scheme (2′) gives $T = c\,(\tau^c + \tau^n)/(1-\tau^n)$, and with $c = C(1+\tau^c)^{-\Omega}$,

$$\frac{dT}{d\tau^c} = \frac{C}{1-\tau^n}\,(1+\tau^c)^{-\Omega-1}\big[(1+\tau^c) - \Omega(\tau^c + \tau^n)\big],$$

so $\mathrm{MCPF} = c/(dT/d\tau^c)$ is (12). The bracket is that of Lemma 3 with $m = \tau^n$, and it vanishes exactly where Proposition 2 places the total tax revenue peak. □

Corollary 2. *Under KPR preferences and Scheme (2′),*

$$\mathrm{MCPF}(\tau^c) = 1 + \tau^c \qquad \textit{for all } \eta > 0,\ \lambda \geq 0,\ \kappa > 0,\ \tau^n \in [0, 1).$$

Proof. Under KPR preferences and Scheme (2′), $\Omega = 1$ for all parameter values (Proposition 1(iv), unchanged by τ^n), so in (12) of Proposition 3(ii) the bracket equals $(1+\tau^c) - (\tau^c + \tau^n) = 1 - \tau^n$ and $\mathrm{MCPF} = (1+\tau^c)(1-\tau^n)/(1-\tau^n) = 1 + \tau^c$. □

Corollary 2, the KPR special case of Proposition 3(ii), is the standard commodity tax formula for the case in which demand is unit elastic; see Johansson and Kriström (2025). As a formula it is not new. The model supplies the reason it applies: the balanced growth restriction $\gamma = 1$ makes equilibrium labor supply invariant to the consumption tax rate under the static Scheme (2′), so gross consumption expenditure $(1+\tau^c)c$ is constant, demand is unit elastic in the consumer price, and, because the existing labor income tax base does not move, the static labor tax fiscal interaction vanishes identically. Under the KPR preferences used by Trabandt and Uhlig (2011), this static experiment therefore leaves the existing labor tax base unmoved at the margin, and the MCPF equals the consumer price including the tax for every parameter configuration. The neutrality is specific to this experiment: it does not extend to the balanced growth path, where output and the other tax bases move even under KPR preferences (Section 5.2).

A natural objection deserves a direct answer before the interpretation is used. In this static economy, allocations depend on the two tax rates only through the wedge $(1-\tau^n)/(1+\tau^c)$, so at the level of

allocations a consumption tax is equivalent to a labor income tax. The revenue and cost objects studied here are not invariant to that relabelling, however: the two instruments tax different bases, so the split of a given wedge between τ^n and τ^c changes R_c, changes the composition of T, and changes which base the marginal reform erodes or expands. The MCPF in (12) depends on τ^n and τ^c separately, not on the wedge alone; what the wedge equivalence does imply is that the analysis should not be read as a statement about the relative distortiveness of the two instruments, and we do not so read it.

Three further remarks keep the interpretation within its proper bounds. First, the sign of the fiscal interaction term in (11) is the sign of the labor supply response, and hence of $1-\gamma$ in the cases with constant statistics: with GHH preferences ($\gamma = 0$) the labor tax base contracts and the MCPF exceeds the ratio based only on consumption tax revenue, $c/(dR_c/d\tau^c) = 1/(1-\varepsilon_c)$; with AS preferences and $\eta > 1$ the base *expands* and the MCPF falls below it, and can fall below one. An MCPF below one is a standard possibility, not an anomaly, when income effects enlarge taxed bases, as Atkinson and Stern (1974) and Triest (1990) show. This margin of base erosion or base expansion is the familiar tax interaction effect of Bovenberg and de Mooij (1994); we apply it, we do not claim it. Second, the divergence of the MCPF at a total tax revenue peak is likewise standard, as Dahlby and Ferede (2018) and Johansson and Kriström (2025) discuss: at the peak a marginal reform raises no funds, and beyond it a marginal increase in τ^c *reduces* total tax revenue, so we simply report that no funds are raised there rather than attach meaning to a negative ratio. What is specific to this paper is *which* peak is relevant: the total tax revenue peak of Section 2.3, which may lie well before the consumption tax revenue peak. Then $dR_c/d\tau^c > 0$ can coexist with $dT/d\tau^c \approx 0$: the tax can still be on the rising branch of the consumption tax revenue curve while additional public funds have become extremely expensive. Third, the exercise is positive, not normative: we evaluate marginal reforms at observed tax systems, which need not be optimal. Jacobs (2018) shows that the MCPF equals one at an optimal tax system; nothing in this section bears on optimality.

3.3 Quantitative illustration

To gauge how the interaction with the existing labor income tax base affects the government's ability to raise additional revenue, we evaluate the MCPF of Proposition 3(ii) at $\eta = 2$, $\lambda = 0.5$ and $\tau^n = 0.28$, at a consumption tax rate of $\tau^c = 0.05$. For the power form cases (12) gives

$$\mathrm{MCPF}(\tau^c) = \frac{(1+\tau^c)(1-\tau^n)}{(1+\tau^c) - \Omega\,(\tau^c + \tau^n)},$$

with $\Omega = 1$ under KPR utility, $\Omega = (1+\lambda)/(\eta+\lambda) = 3/5$ under AS utility, and $\Omega = (1+\lambda)/\lambda = 3$ under GHH utility.

Table 2 reports the decomposition of marginal total tax revenue in (11) and the resulting MCPF.

Table 2. Marginal cost of public funds in the static economy

Utility	Ω	$(1/c)\, dR_c/d\tau^c$	Cross-tax revenue effect	$(1/c)\, dT/d\tau^c$	MCPF
KPR	1	0.952	0.000	0.952	1.050
AS	$3/5$	0.971	$+0.156$	1.127	0.887
GHH	3	0.857	-0.778	0.079	12.600

Notes: Static economy under Scheme (2′) with $\eta = 2$, $\lambda = 0.5$ and $\tau^n = 0.28$, evaluated at $\tau^c = 5\%$. All entries are per unit of consumption. $(1/c)\, dR_c/d\tau^c = 1 - \varepsilon_c$ is the marginal change in consumption tax revenue per unit of consumption; the cross-tax revenue effect $(\tau^n/c)\, dn/d\tau^c$ is the marginal change in existing labor income tax revenue per unit of consumption; their sum is marginal total tax revenue $(1/c)\, dT/d\tau^c$, the decomposition in (11); and the MCPF is the reciprocal of that marginal total tax revenue under the Scheme (2′) experiment, equal to (12).

The three specifications differ only through the cross-tax revenue effect. Under KPR utility the existing labor income tax base does not move, so that effect is exactly zero and the MCPF equals $1 + \tau^c = 1.05$ (Corollary 2). Under AS utility with $\eta > 1$ the existing labor income tax base expands, the cross-tax revenue effect is positive, marginal total tax revenue exceeds the marginal increase in consumption tax revenue, and the MCPF falls below one. Under GHH utility the existing labor income tax base contracts sharply: the decline in labor income tax revenue offsets about 91% of the marginal increase in consumption tax revenue, marginal total tax revenue is only 0.079 per unit of consumption, and the MCPF is 12.6: raising one additional unit of public revenue entails a welfare cost of 12.6 in money terms.

The GHH case also separates the two revenue objects. Consumption tax revenue is the product form with $\Omega = 3$ and $m = 0$, so by Lemma 3 it peaks at $\tau^c = 1/(\Omega - 1) = 50\%$. Total tax revenue is the same product form with $m = \tau^n$, and by Proposition 2(vi) it peaks at $\tau^c = \lambda - (1 + \lambda)\tau^n = 8\%$. Consumption tax revenue therefore keeps rising until a consumption tax rate of 50%, while total tax revenue is already falling beyond 8%. The MCPF diverges as τ^c approaches 8% from below, because the denominator of (12) vanishes exactly at the total tax revenue peak of Proposition 2. Beyond that rate $dT/d\tau^c < 0$ and the ratio turns negative; we attach no meaning to a negative MCPF and record only that a marginal increase in the consumption tax raises no additional public funds there.

These numbers illustrate how the global and the marginal answers to the paper's question can diverge. Under GHH utility the consumption tax revenue curve is still far from its peak at a 5% tax rate, and yet about 91% of the marginal increase in consumption tax revenue is already offset by the decline in labor income tax revenue, so that the MCPF is 12.6. The gap widens as the rate rises: consumption tax revenue keeps increasing until 50%, whereas total tax revenue peaks at 8%. Ample room for further increases in consumption tax revenue therefore need not mean ample capacity to raise additional total tax revenue at the margin. This is a quantitative illustration of the model, not an empirical estimate, but it is the reason

the global revenue curves and the marginal cost of public funds belong in the same analysis.

4 Extension to the dynamic economy à la Trabandt and Uhlig (2011)

4.1 Model

We now embed the analysis in the neoclassical growth economy of Trabandt and Uhlig (2011). The representative household consumes c_t, invests x_t, holds capital k_{t-1} and government bonds b_{t-1} at the beginning of the period, and supplies labor n_t at wage w_t; capital earns the rental rate d_t, bonds pay the gross interest factor R_t^b, and the household receives a lump-sum government transfer s_t and a transfer from abroad m_t, interpretable as net imports. With δ the depreciation rate of capital and $(\tau^c{}_t, \tau^n{}_t, \tau^k{}_t)$ the consumption, labor, and capital income tax rates, the budget constraint is

$$(1+\tau^c{}_t)c_t + x_t + b_t \le (1-\tau^n{}_t)w_t n_t + (1-\tau^k{}_t)(d_t-\delta)k_{t-1} + \delta k_{t-1} + R_t^b b_{t-1} + s_t + m_t,$$

and capital accumulates according to $k_t = (1-\delta)k_{t-1} + x_t$. Competitive firms produce output y_t with the Cobb–Douglas technology $y_t = \xi^t k_{t-1}^{\theta} n_t^{1-\theta}$, where θ is the capital share and ξ the growth factor of technology, and factor prices are

$$w_t = (1-\theta)\,\frac{y_t}{n_t}, \qquad d_t = \theta\,\frac{y_t}{k_{t-1}}.$$

The government budget constraint is $g_t + s_t + R_t^b b_{t-1} \le b_t + T_t$, where g_t is government consumption and total tax revenue is $T_t = \tau^c{}_t c_t + \tau^n{}_t w_t n_t + \tau^k{}_t (d_t - \delta)k_{t-1}$; the resource constraint is

$$y_t = c_t + x_t + g_t - m_t.$$

The three utility functions are the balanced growth versions of Section 2.1, discounted at factor β, with government consumption entering additively separably and with the preference shifters of Erceg et al. (2006) guaranteeing the existence of a balanced growth path. On that path all trending quantities grow at the factor $\psi = \xi^{1/(1-\theta)}$, and a tilde denotes a detrended variable, $\tilde{a}_t = a_t/\psi^t$, with the predetermined stocks dated so that $\tilde{k} = k_{t-1}/\psi^t$ and $\tilde{b} = b_{t-1}/\psi^t$. The detrended equilibrium system and the technical balanced growth derivations are collected in Appendix D. Revenue objects are compared across balanced growth paths (BGPs), as in Trabandt and Uhlig (2011).

Two schemes are considered, as before:

Definition 3. *Under Scheme (3), the ratios g/y, b/y, and m/y are held constant and the lump-sum transfer absorbs residual changes in total tax revenue. Under Scheme (4), the ratios s/y, b/y, and m/y are held constant and government consumption absorbs the residual changes.*

Holding g/y constant can be read as government control of the expenditure share, as in Hayashi and Prescott (2002). These ratio schemes are the balanced growth counterparts of the static Schemes (1) and (2), and they deliberately differ from the fiscal rule of Trabandt and Uhlig (2011), who hold detrended government spending fixed across balanced growth paths with transfers adjusting; it is the ratio form that keeps the fiscal shares of output constant along the comparison and permits the complete analytical characterization below.

4.2 Consumption tax revenue curve for consumption tax

The dynamic economy adds capital, government debt, growth, and two further tax bases, but on the balanced growth path its dependence on the consumption tax rate is not richer than in the static economy. The argument proceeds in three steps. Lemma 6 establishes that the interest factor, the rental rate, and the principal balanced growth ratios are independent of τ^c. Lemma 7 then shows that detrended consumption responds to τ^c through the same closed forms, with the same exponents, as in the corresponding static schemes. Corollary 3 therefore imports the static classification of the consumption tax revenue curve unchanged, and Proposition 4 in Section 4.3 adds the labor and capital tax bases to characterize total tax revenue.

Lemma 6. *On the balanced growth path, $R^b = \psi^{\eta}/\beta$ under KPR and AS utility and $R^b = \psi^{\bar{\eta}}/\beta$ under GHH utility, and $d = \delta + (R^b - 1)/(1 - \tau^k)$; consequently d, $\tilde{k}/\tilde{y}$, $\tilde{x}/\tilde{y}$, $n/\tilde{y}$, and $\tilde{w}$ are independent of τ^c.*[7] *Moreover:*

(i) under Scheme (3), $\tilde{c}/\tilde{y}$ is independent of τ^c, so $\tilde{c} = \chi n$ with $\chi > 0$ independent of τ^c;

(ii) under Scheme (4), combining the government budget and resource constraints,

$$(1 + \tau^c)\,\frac{\tilde{c}}{\tilde{y}} = 1 - \frac{\tilde{x}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}} - \Theta + (R^b - \psi)\frac{\tilde{b}}{\tilde{y}} + \frac{\tilde{s}}{\tilde{y}} \;\equiv\; K_0, \tag{13}$$

independent of τ^c, where $\Theta \equiv \tau^n(1 - \theta) + \tau^k(d - \delta)\tilde{k}/\tilde{y}$ is revenue from the labor and capital income taxes per unit of output and $\tilde{b} \equiv b_{t-1}/\psi^t$ is detrended debt (the same dating convention as $\tilde{k}$); so $(1 + \tau^c)\tilde{c} = \chi n$ with $\chi > 0$ independent of τ^c.[8]

In both cases the intratemporal condition reduces to $-U_n/U_c = \hat{\Theta}/(1 + \tau^c)$ with $\hat{\Theta} \equiv (1 - \tau^n)\tilde{w} > 0$ independent of τ^c, the balanced growth preference shifters cancelling in detrended form.

[7] The marginal utility of the GHH bundle grows at $\psi^{-\bar{\eta}}$ on the balanced growth path, which is why the GHH curvature replaces η in the Euler condition. Since $\bar{\eta}$ enters only constants that are independent of τ^c, and never an exponent, the classification below is unaffected; the calibration of Section 5.2 sets $\bar{\eta} = \eta = 2$.

[8] Maintaining a constant ratio of debt to output on a path growing at factor ψ requires new issuance $(\psi - 1)\tilde{b}$ each period, so the net drain of debt service on the budget is $(R^b - \psi)\tilde{b}$.

Proof. The two Euler conditions on the balanced growth path, $1 = \beta\psi^{-\eta}\big[1 + (1-\tau^k)(d-\delta)\big]$ and $1 = \beta\psi^{-\eta}R^b$ (with $\bar{\eta}$ in place of η under GHH utility), determine R^b and $d = \delta + (R^b - 1)/(1-\tau^k)$ and involve no τ^c. Then $\tilde{k}/\tilde{y} = \theta/d$, $\tilde{x}/\tilde{y} = [\psi - (1-\delta)]\,\tilde{k}/\tilde{y}$, $n/\tilde{y} = (\tilde{y}/\tilde{k})^{\theta/(1-\theta)}$, and $\tilde{w} = (1-\theta)\tilde{y}/n$ are all independent of τ^c. (i) Under Scheme (3) the resource constraint gives $\tilde{c}/\tilde{y} = 1 - \tilde{x}/\tilde{y} - \tilde{g}/\tilde{y} + \tilde{m}/\tilde{y}$, with each term independent of τ^c, and χ is the ratio of $\tilde{c}/\tilde{y}$ to $n/\tilde{y}$. (ii) Under Scheme (4), detrending the flow budget constraint $g_t + s_t + R^b b_{t-1} = b_t + T_t$ with $\tilde{b} \equiv b_{t-1}/\psi^t$ and using the constancy of b/y gives $\tilde{g} + \tilde{s} + R^b\tilde{b} = \psi\tilde{b} + \tilde{T}$ on the balanced growth path, so $\tilde{g}/\tilde{y} = \tau^c\,\tilde{c}/\tilde{y} + \Theta - \tilde{s}/\tilde{y} - (R^b - \psi)\tilde{b}/\tilde{y}$; substituting into the resource constraint and collecting the $\tilde{c}/\tilde{y}$ terms yields (13), with every term on its right side independent of τ^c. The reduced intratemporal condition follows by dividing the household's two first order conditions, the balanced growth preference shifters cancelling from the ratio. □

Lemma 7. *Consider $-U_n/U_c = \hat{\Theta}/(1+\tau^c)$ together with $\tilde{c} = \chi n$ or $(1+\tau^c)\tilde{c} = \chi n$, where $\hat{\Theta}, \chi > 0$ are independent of τ^c. For AS and GHH utility, $\tilde{c}(\tau^c)$ lies in the power form with the* same *exponent Ω as in the corresponding static scheme, $\hat{\Theta}$ and χ entering only the multiplicative constant. For KPR utility, $(1+\tau^c)\tilde{c} = \chi n$ delivers $\Omega = 1$ independently of all parameters, while $\tilde{c} = \chi n$ delivers the KPR form with $a = \hat{\Theta}(1-\eta) + \chi\eta(1+\lambda)$ and $b = \chi\eta(1+\lambda) > 0$, where $a > 0$ is exactly the condition that the baseline equilibrium exist.*

Proof. For AS utility, the reduced condition with $\tilde{c} = \chi n$ gives $\kappa(1+\lambda)\chi^{-\lambda}\tilde{c}^{\eta+\lambda} = \hat{\Theta}/(1+\tau^c)$, so $\tilde{c} \propto (1+\tau^c)^{-1/(\eta+\lambda)}$; with $(1+\tau^c)\tilde{c} = \chi n$ it gives $\tilde{c} \propto (1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}$. For GHH utility, $\kappa(1+\lambda)n^\lambda = \hat{\Theta}/(1+\tau^c)$ pins $n \propto (1+\tau^c)^{-1/\lambda}$ regardless of $\hat{\Theta}$, and the two links deliver $\Omega = 1/\lambda$ and $(1+\lambda)/\lambda$. For KPR utility, write $z = \kappa n^{1+\lambda}$: under $(1+\tau^c)\tilde{c} = \chi n$ the $(1+\tau^c)$ factors cancel and $z\big[\eta(1+\lambda)\chi + \hat{\Theta}(1-\eta)\big] = \hat{\Theta}$, so n is constant and $\tilde{c} \propto (1+\tau^c)^{-1}$; under $\tilde{c} = \chi n$, $z\big[\hat{\Theta}(1-\eta) + \chi\eta(1+\lambda)(1+\tau^c)\big] = \hat{\Theta}$, the KPR form with the stated a and b, and $a > 0$ is equivalent to $z > 0$ at $\tau^c = 0$, i.e. to existence of the baseline equilibrium. In every case $\hat{\Theta}$ and χ enter only the multiplicative constant (or the coefficient a), never the exponent, so the elasticity path, and with it the entire classification of Lemmas 3 and 4, is that of the corresponding static scheme. □

Because the classification of Section 2.2 depends only on the closed form and its exponent, not on multiplicative constants, the static results transfer:

Corollary 3. *On the balanced growth path, consumption tax revenue $\tilde{R}_c = \tau^c\tilde{c}$ has, under Scheme (3), exactly the properties stated for Scheme (1) in Proposition 1(i)–(iii), and under Scheme (4), exactly the properties stated for Scheme (2) in Proposition 1(iv)–(vi), including all peak formulas, boundedness statements, and boundary cases.*

4.3 Total tax revenue curve for consumption tax

Total tax revenue adds the labor and capital tax bases. By Lemma 6, $\tilde{T} = (\tau^c\,\tilde{c}/\tilde{y} + \Theta)\,\tilde{y}$, and substituting the scheme links puts $\tilde{T}$ in product form. Under Scheme (3), $\tilde{c}/\tilde{y} = \phi_c$ is constant, so

$$\tilde{T} = \phi_c\left(\tau^c + m_3\right)\tilde{y}(\tau^c), \qquad m_3 \equiv \Theta/\phi_c,$$

and, by Lemma 7, $\tilde{y} \propto n \propto \tilde{c}$ lies in the power form (AS, GHH) or the KPR form (KPR), so Lemmas 3–4 apply with $m = m_3$. Under Scheme (4), $(1+\tau^c)\,\tilde{c}/\tilde{y} = K_0$ is constant, so

$$\tilde{T} = \tilde{y}\Big[\frac{K_0\tau^c}{1+\tau^c} + \Theta\Big] = (K_0 + \Theta)\left(\tau^c + m_4\right)\frac{\tilde{y}(\tau^c)}{1+\tau^c}, \qquad m_4 \equiv \frac{\Theta}{K_0+\Theta} \in (0,1),$$

and by Lemma 7 the factor $\tilde{y}/(1+\tau^c)$ lies in the power form with exactly the exponents of Scheme (2) ($\tilde{y}$ constant and $\Omega = 1$ under KPR), so Lemma 3 applies with $m = m_4$. The product forms allow Lemmas 3 and 4 to apply directly; Proposition 4 states the resulting classification, with the case-specific algebra reported in Appendix D.

Proposition 4. *Let $\Theta > 0$, $\phi_c = \tilde{c}/\tilde{y}$, $m_3 \equiv (\tilde{y}/\tilde{c})\Theta$, and K_0 as in (13), all evaluated on the balanced growth path. Then:*

(i) (KPR, Scheme (3)) Let $B(\lambda) \equiv \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) + (1+\lambda)\,\phi_c$. If $\lambda > 0$, $\tilde{T}$ is strictly increasing when $\Theta \le B(\lambda)$ and U-shaped when $\Theta > B(\lambda)$, and revenue is unbounded. If $\lambda = 0$, $\tilde{T}$ is strictly increasing, constant, or strictly decreasing according as $\Theta < B(0)$, $\Theta = B(0)$, or $\Theta > B(0)$, and revenue is bounded.

(ii) (AS, Scheme (3)) With $\Omega = 1/(\eta+\lambda)$: $\tilde{T}$ is hump-shaped if and only if $m_3 < \eta + \lambda < 1$, with unique global maximizer $\tau^c = \big[(\eta+\lambda) - m_3\big]/(1-\eta-\lambda)$; strictly decreasing on $(0,\infty)$ if $\eta+\lambda < 1$ and $m_3 \ge \eta+\lambda$; strictly increasing if $\eta+\lambda > 1$ and $m_3 \le \eta+\lambda$; U-shaped if $\eta+\lambda > 1$ and $m_3 > \eta+\lambda$; and, when $\eta+\lambda = 1$, strictly increasing, constant, or strictly decreasing according as $m_3 < 1$, $m_3 = 1$, or $m_3 > 1$. Revenue is bounded if and only if $\eta+\lambda \le 1$.

(iii) (GHH, Scheme (3)) The same classification with $\eta+\lambda$ replaced by λ throughout; in particular $\tilde{T}$ is hump-shaped if and only if $m_3 < \lambda < 1$, with peak $\tau^c = (\lambda - m_3)/(1-\lambda)$, and bounded if and only if $\lambda \le 1$.

(iv) (KPR, Scheme (4)) $\tilde{T}$ is strictly increasing and bounded, for all parameter values.

(v) (AS, Scheme (4)) $\tilde{T}$ is hump-shaped if and only if $\eta < 1$ and $(1-\eta)\Theta < (\eta+\lambda)K_0$, with unique global maximizer

$$\tau^c = \frac{(\eta+\lambda)\,K_0 - (1-\eta)\,\Theta}{(1-\eta)\,(K_0+\Theta)};$$

it is strictly increasing if $\eta \geq 1$*, and strictly decreasing on* $(0, \infty)$ *if* $\eta < 1$ *and* $(1-\eta)\Theta \geq (\eta+\lambda)K_0$*. Revenue is bounded if and only if* $\eta \leq 1$*.*

(vi) (GHH, Scheme (4)) $\tilde{T}$ *is hump-shaped if and only if* $\lambda K_0 > \Theta$*, with unique global maximizer* $\tau^c = (\lambda K_0 - \Theta)/(K_0 + \Theta)$*; otherwise it is strictly decreasing on* $(0, \infty)$*. Revenue is bounded.*

Proposition 4 is the complete classification: the six cases of (ii)–(iii) are exactly the partition of Lemma 3 applied to the product form, cases (i) and (iv) are the corresponding partition of Lemma 4, whose $\lambda = 0$ branch is what separates the two subcases of (i), and no other global shapes can arise. The capital income tax matters through Θ, which now includes the capital tax base; this is the only place where the dynamic environment changes the substance rather than the constants.

Remark 3. *We deliberately attach no MCPF interpretation to the dynamic results. A comparison across balanced growth paths changes the capital stock, whereas a permanent tax reform starts from a predetermined capital stock and involves a transition; a BGP revenue comparison is therefore not a welfare measure of the reform, and the ratio* $\tilde{c}/(d\tilde{T}/d\tau^c)$ *on the BGP is not an MCPF. The dynamic analysis here reports marginal* revenue*: the marginal change in consumption tax revenue, the cross-tax revenue effect through the labor and capital tax bases, and the total tax revenue curve. The welfare analysis of Section 3 is intentionally confined to the static economy, where the envelope argument applies exactly. Solving the transition is outside the scope of this paper.*

5 Discussion

5.1 Likelihood of a hump-shaped tax revenue curve for consumption tax

The conditions for a hump-shaped consumption tax revenue curve depend on the use of tax revenue, so their empirical plausibility has to be assessed scheme by scheme. Under the rebate Schemes (1) and (3) a hump requires $\eta + \lambda < 1$ under additively separable utility and $\lambda < 1$ under GHH utility. Under the government consumption Schemes (2) and (4) the requirement is weaker: $\eta < 1$ under additively separable utility, and none at all under GHH utility, where the curve is hump-shaped for every $\lambda > 0$. Under KPR utility no scheme delivers a hump. Whether the values that the rebate schemes require are empirically plausible deserves comment, and we summarize. Values of η below one are not standard in business cycle modelling, where $\eta \geq 1$ is typical; Chetty (2006) estimates a mean relative risk aversion of 0.71 (range 0.15–1.78) from labor supply moments, while interpreting η as the inverse intertemporal elasticity of substitution (IES) links it to estimates above one in Mulligan (2002), Vissing-Jørgensen and Attanasio (2003), Sakuragawa and Hosono (2010), and Gruber (2013), but to a mean IES of about 0.5, that is $\eta \approx 2$, in the meta analysis of Havranek et al. (2015). Values of λ below one are common in

representative agent macroeconomics; Prescott (1986) uses $\lambda = 1/2$, and the indivisible labor tradition of Hansen (1985) and Rogerson (1988) rationalizes large aggregate elasticities even when individual Frisch elasticities are small; see Christiano et al. (2010), Keane and Rogerson (2011, 2012), and the structural estimate of 3.8 in Imai and Keane (2004). On balance, both restrictions are contested, so under the rebate schemes a hump-shaped consumption tax revenue curve should be regarded as possible but not typical; this is one reason the frictions studied by Baydur and Yilmaz (2021), Miravete et al. (2018), and Vasilev (2018) matter for generating humps at conventional parameter values. The assessment changes when the revenue is absorbed by government consumption: the additively separable condition weakens to $\eta < 1$, and under GHH utility a hump arises at every λ, so the use of tax revenue is part of the empirical question and not only of the theory. Section 5.2 evaluates Scheme (3), so the comparison across the three utility specifications reported there belongs to the rebate case. By contrast, the total tax revenue and MCPF statements of Sections 2.3 and 3 do not hinge on these contested ranges: the fiscal interaction operates through the response of the existing tax base, which may vanish, as it does under KPR preferences in the static fund raising experiment of Section 3, and the KPR result $\text{MCPF} = 1 + \tau^c$ holds for all parameter values.

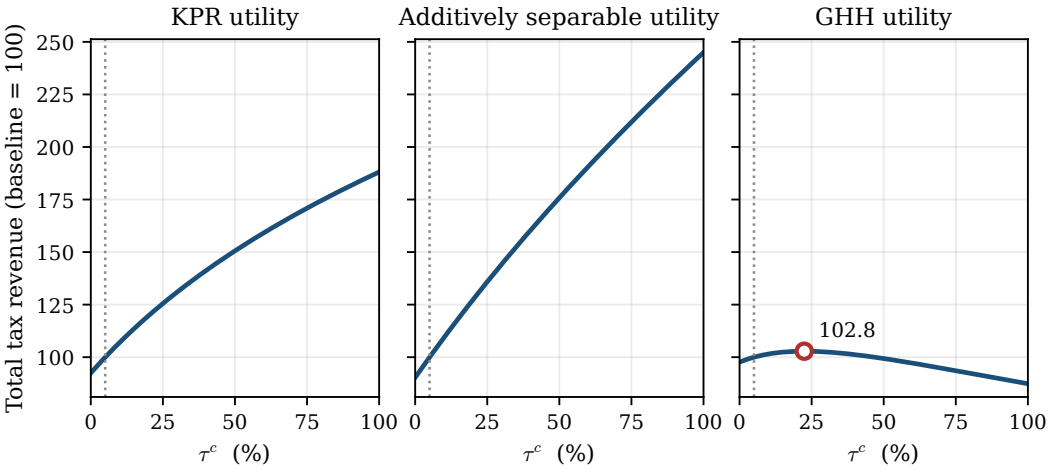
5.2 Quantitative results

We now present quantitative results for the dynamic economy under Scheme (3). The calibration is based on Trabandt and Uhlig (2011) for the US economy: depreciation rate $\delta = 0.07$; steady state ratios $g/y = 0.18$, $b/y = 0.63$, $m/y = 0.04$; balanced growth factor $\psi = 1.02$; steady state labor supply $\bar{n} = 0.25$; tax rates $\tau^k = 0.36$, $\tau^n = 0.28$, a baseline consumption tax rate $\tau^c = 0.05$; and $\eta = 2$. Two parameters are adapted to the present exercise: we set the capital share to $\theta = 0.35$ and the labor supply parameter to $\lambda = 0.5$, a Frisch elasticity of two, whereas their benchmark uses 0.38 and a unit Frisch elasticity. GHH curvature is $\bar{\eta} = \eta = 2$ and $\beta = 1$, so that $R^b = \psi^\eta = 1.0404$ under all three preference specifications. The implied balanced growth objects are $d = 0.133$, $\tilde{k}/\tilde{y} = 2.63$, $\tilde{c}/\tilde{y} = 0.623$, and $\Theta = 0.242$, so that $m_3 = (\tilde{y}/\tilde{c})\Theta = 0.388$; at the baseline, consumption tax revenue is 11.4% of total tax revenue. The numerical procedure is described in Appendix E.

Figure 1 shows total tax revenue on the balanced growth path under Scheme (3). Under KPR and AS utility the curve is strictly increasing over the displayed range, consistent with Proposition 4(i)–(ii) at these parameters. Under GHH utility the curve is hump-shaped: Proposition 4(iii) puts the peak at $\tau^c = (\lambda - m_3)/(1 - \lambda) = 22.4\%$, at which point total tax revenue is only 102.8, less than three percent above its baseline level.

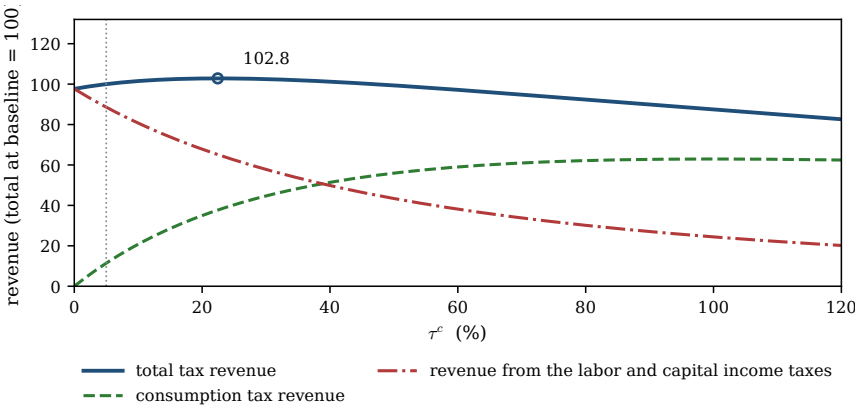
The contrast between the consumption tax revenue curve and the total tax revenue curve is the paper's central quantitative fact. On the same normalization (total tax revenue at the baseline $= 100$),

Figure 1. Total tax revenue on the balanced growth path



Notes: Total tax revenue on the balanced growth path under Scheme (3), for the three utility specifications with $\eta = 2$ and $\lambda = 0.5$. Revenue is normalized to 100 at the baseline $\tau^c = 5\%$; the dotted line marks the baseline tax rate. All three panels use the same vertical scale. The circle marks the GHH peak at $\tau^c = 22.4\%$, where total tax revenue reaches 102.8.

Figure 2. Revenue components on the balanced growth path under GHH utility



Notes: Balanced growth path under Scheme (3) with GHH utility, $\eta = 2$ and $\lambda = 0.5$. All three series use the normalization of the text, total tax revenue at the baseline $\tau^c = 5\%$ equal to 100; the dotted line marks the baseline tax rate. The circle marks the total tax revenue peak at $\tau^c = 22.4\%$, where total tax revenue reaches 102.8.

consumption tax revenue under GHH utility rises from 11.4 at the baseline to 63.0 at the consumption tax revenue peak of $\tau^c = 100\%$, an increase of about 5.5 times, while total tax revenue never exceeds 102.8 and is falling beyond $\tau^c = 22.4\%$. Nearly all of the additional consumption tax revenue is offset by the erosion of the labor and capital income tax bases. Figure 2 shows the two components behind that contrast.

As the consumption tax rate rises, consumption tax revenue climbs while revenue from the labor and capital income taxes falls by almost as much, so that total tax revenue is nearly flat over a wide range and then declines.

Table 3. Marginal revenue on the balanced growth path

τ^c	Utility	$dR_c/d\tau^c$	Cross-tax revenue effect	$dT/d\tau^c$	Offset share
5%	KPR	0.597	−0.202	0.396	0.34
	AS	0.612	−0.092	0.519	0.15
	GHH	0.564	−0.460	0.104	0.82
10%	KPR	0.574	−0.190	0.385	0.33
	AS	0.601	−0.088	0.513	0.15
	GHH	0.510	−0.440	0.070	0.86
15%	KPR	0.554	−0.179	0.375	0.32
	AS	0.591	−0.084	0.507	0.14
	GHH	0.461	−0.420	0.040	0.91
20%	KPR	0.536	−0.170	0.366	0.32
	AS	0.582	−0.081	0.501	0.14
	GHH	0.416	−0.403	0.013	0.97

Notes: $dR_c/d\tau^c = \phi_c(1-\varepsilon_c)$ is the marginal change in consumption tax revenue; the cross-tax revenue effect $-\Theta\,\varepsilon_c/\tau^c$ is the marginal change in revenue from the labor and capital income taxes; $dT/d\tau^c$ is marginal total tax revenue, the sum of the two; the offset share is minus the cross-tax revenue effect divided by $dR_c/d\tau^c$. All quantities are per unit of balanced growth output under Scheme (3), with $\eta = 2$ and $\lambda = 0.5$. These are revenue statistics; no welfare interpretation is attached (Remark 3).

Table 4. Sensitivity to the labor supply parameter under GHH utility

	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 1$	$\lambda = 2$
Consumption tax revenue curve	hump	hump	hump	increasing	increasing
peak τ^c	33%	100%	300%	none	none
boundedness	bounded	bounded	bounded	bounded	unbounded
Total tax revenue curve	decreasing	hump	hump	increasing	increasing
peak τ^c	none	22.4%	144.9%	none	none
revenue at interior peak (baseline = 100)	–	102.8	135.6	–	–
Offset share at $\tau^c = 10\%$	2.22	0.86	0.53	0.39	0.18
Offset share at $\tau^c = 20\%$	3.88	0.97	0.55	0.39	0.18

Notes: GHH utility on the balanced growth path under Scheme (3), varying λ with all other parameters at the values of this section. Shapes and peaks follow Corollary 3 with Proposition 1(iii) and Proposition 4(iii); a dash means the relevant peak is not interior, so no interior-peak revenue is reported. At $\lambda = 1$ both curves are increasing and bounded, the boundary case of Proposition 4(iii) with $m_3 < 1$, and the offset share is constant and equal to m_3. The offset share is defined as in Table 3; a value above one means that revenue from the labor and capital income taxes falls by more than consumption tax revenue rises, so that total tax revenue is decreasing.

Table 3 decomposes marginal total tax revenue at tax rates relevant for policy.

The change in consumption tax revenue $dR_c/d\tau^c$ is similar across preferences, but the cross-tax revenue effect through the other tax bases is not: at the baseline it offsets 15% of the direct gain under

AS utility, 34% under KPR, and 82% under GHH, rising to 97% at $\tau^c = 20\%$. Under GHH utility, at a consumption tax rate of 20%, only about 3 percent of the marginal increase in consumption tax revenue translates into an increase in total tax revenue.

How much of this depends on the labor supply response? Table 4 varies λ under GHH utility, holding the remaining parameters fixed, so that ϕ_c, Θ and $m_3 = 0.388$ are unchanged. The classification follows Corollary 3 and Proposition 4(iii).

A more elastic labor supply (a smaller λ) makes the erosion of the existing bases stronger: at $\lambda = 0.25$ total tax revenue is decreasing from the outset even though consumption tax revenue still peaks at 33%, and the offset share exceeds one. A less elastic labor supply reverses the picture: at $\lambda = 1$ both curves are increasing and bounded, and the offset share is constant at $m_3 = 39\%$, while at $\lambda = 2$ both are increasing and unbounded and the offset is below one fifth. The qualitative message of the paper, that the two revenue curves can give different answers about fiscal capacity, survives across this range; how large the gap is does not. We take no stand here on which λ is correct, and Section 5.1 reviews the evidence.

6 Concluding remarks

This paper has characterized the shape of the Laffer curve for consumption tax in a frictionless neoclassical economy and examined its implications for total tax revenue and the marginal cost of public funds. The curve cannot be hump-shaped under KPR preferences; it can be under additively separable or GHH preferences, with the income effect on labor supply and the use of tax revenue jointly determining the outcome. With an existing labor income tax, the total tax revenue curve peaks strictly before the consumption tax revenue curve whenever both peaks are interior, and with sufficiently strong erosion of the existing base, total tax revenue declines from the outset. In the static economy, the experiment in which revenue is absorbed by government consumption delivers a valid marginal cost of public funds whose denominator is exactly this total margin; it equals $1 + \tau^c$ under KPR preferences because the same restriction on the income effect that rules out a hump also keeps the existing labor tax base fixed. The classification of the consumption tax revenue curve carries over unchanged to the balanced growth path of the dynamic economy à la Trabandt and Uhlig (2011), where the total tax revenue thresholds also involve the capital income tax base. Quantitatively, at the parameter values of Section 5.2 under GHH preferences, the consumption tax revenue curve overstates fiscal capacity by a wide margin: consumption tax revenue can rise more than fivefold while total tax revenue rises by less than three percent.

The model is a frictionless representative agent economy: household heterogeneity, and with it the distributional side of consumption tax reform, is absent; frictions such as home production, market power, and tax evasion, which can generate humps at conventional parameter values, are deliberately excluded; the welfare analysis is confined to the static economy, since comparisons across balanced growth paths

do not measure the welfare effect of a permanent reform with a predetermined capital stock; and nothing here concerns optimal taxation; at an optimum the marginal cost of public funds would equal one.

The Laffer curve for consumption tax is an equilibrium object whose shape is determined by preferences and by the use of tax revenue. Even correctly characterized, however, it need not measure the government's capacity to raise revenue when other distortionary taxes are in place. The fiscal interactions with those existing bases determine marginal total tax revenue, and marginal total tax revenue, not the slope of the consumption tax revenue curve, is the margin at which the cost of public funds is assessed.

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Appendix A Family mappings with an existing labor tax

Throughout, $w = 1$ and the intratemporal condition is $-U_n/U_c = (1-\tau^n)/(1+\tau^c)$; setting $\tau^n = 0$ gives the environment of Section 2.2. The equilibrium links are $c = n$ (Schemes (1), (1′)) and $(1+\tau^c)c = (1-\tau^n)n$ (Schemes (2), (2′)).

Additively separable utility. $-U_n/U_c = \kappa(1+\lambda)c^\eta n^\lambda$. With $c = n$: $\kappa(1+\lambda)c^{\eta+\lambda} = (1-\tau^n)/(1+\tau^c)$, so

$$c = \Big[\frac{1-\tau^n}{\kappa(1+\lambda)}\Big]^{\frac{1}{\eta+\lambda}}(1+\tau^c)^{-\frac{1}{\eta+\lambda}} :$$

the power form with $\Omega = 1/(\eta+\lambda)$. With $(1+\tau^c)c = (1-\tau^n)n$: $\kappa(1+\lambda)c^{\eta+\lambda}\big[(1+\tau^c)/(1-\tau^n)\big]^\lambda = (1-\tau^n)/(1+\tau^c)$, so $c \propto (1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}}$: the power form with $\Omega = (1+\lambda)/(\eta+\lambda)$. In both cases τ^n enters only the multiplicative constant.

GHH utility. $-U_n/U_c = \kappa(1+\lambda)n^\lambda$, so $n = \big[(1-\tau^n)/\{\kappa(1+\lambda)(1+\tau^c)\}\big]^{1/\lambda}$ regardless of the link; then $c = n$ gives $\Omega = 1/\lambda$ and $c = (1-\tau^n)n/(1+\tau^c)$ gives $\Omega = (1+\lambda)/\lambda$.

KPR utility. $-U_n/U_c = \eta(1+\lambda)\kappa c n^{\lambda}/\big[1-\kappa(1-\eta)n^{1+\lambda}\big]$. Write $z = \kappa n^{1+\lambda}$. Under the link $(1+\tau^c)c = (1-\tau^n)n$ the factors $(1+\tau^c)$ and $(1-\tau^n)$ cancel and the condition reduces to $z\big[\eta(1+\lambda)+1-\eta\big] = 1$, i.e. $z = 1/(1+\eta\lambda)$: equilibrium labor supply is a constant, independent of τ^c and of τ^n, and $c = (1-\tau^n)n/(1+\tau^c) \propto (1+\tau^c)^{-1}$, the power form with $\Omega = 1$. Under the link $c = n$ the condition reduces to

$$z\big[(1-\eta)(1-\tau^n)+\eta(1+\lambda)(1+\tau^c)\big] = 1-\tau^n,$$

so $c = n \propto (a+b\tau^c)^{-1/(1+\lambda)}$ with $a = (1-\eta)(1-\tau^n)+\eta(1+\lambda) > 0$ and $b = \eta(1+\lambda) > 0$: the KPR form. For $\tau^n = 0$, $a = 1+\eta\lambda$.

Total tax revenue. Under Scheme (1′), $T = \tau^c c + \tau^n n = (\tau^c+\tau^n)c$ since $c = n$. Under Scheme (2′), $n = (1+\tau^c)c/(1-\tau^n)$ gives $T = \tau^c c + \tau^n(1+\tau^c)c/(1-\tau^n) = c\,(\tau^c+\tau^n)/(1-\tau^n)$. In both cases T is the product form of Lemmas 3–4 with $m = \tau^n$, and Propositions 1 and 2 follow by reading off the classifications, using $\Omega m - 1 = (\tau^n - (\eta+\lambda))/(\eta+\lambda)$ for AS utility under (1′), $\Omega m - 1 = ((1+\lambda)\tau^n - (\eta+\lambda))/(\eta+\lambda)$ under (2′), and the corresponding expressions for GHH utility; the peak formulas are $\tau^* = (1-\Omega m)/(\Omega-1)$, which reproduce the stated closed forms. For the vacuousness claim in Proposition 2(v): $(1+\lambda)\tau^n - \lambda \geq 1$ would require $(1+\lambda)(\tau^n-1) \geq 0$, impossible for $\tau^n < 1$. □

Appendix B General utility and the KPR elasticity

Differentiating $-U_n(c,n) = U_c(c,n)/(1+\tau^c)$ totally and collecting terms,

$$\Big[-\frac{cU_{cc}}{U_c}+\frac{cU_{cn}}{U_n}\Big]\frac{d\ln c}{d\ln\tau^c}+\Big[\frac{nU_{nn}}{U_n}-\frac{nU_{cn}}{U_c}\Big]\frac{d\ln n}{d\ln\tau^c} = -\frac{\tau^c}{1+\tau^c},$$

i.e. $\gamma\, d\ln c/d\ln\tau^c + \varphi\, d\ln n/d\ln\tau^c = -\tau^c/(1+\tau^c)$. Under $c = \chi n$, $d\ln n = d\ln c$ and $\varepsilon_c = [\tau^c/(1+\tau^c)]/(\gamma+\varphi)$; under $(1+\tau^c)c = \chi n$, $d\ln n/d\ln\tau^c = d\ln c/d\ln\tau^c + \tau^c/(1+\tau^c)$ and $\varepsilon_c = [\tau^c/(1+\tau^c)](1+\varphi)/(\gamma+\varphi)$. These are the pointwise expressions (7).

For KPR utility, direct differentiation gives $\gamma = 1$ and

$$\varphi = \frac{\lambda+\kappa(1-\eta)n^{1+\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}.$$

Substituting this expression together with $\gamma = 1$ into the expression for Scheme (1) reproduces the KPR elasticity in closed form. Note that φ for KPR utility depends on the allocation: under schemes of the Scheme (2) type, equilibrium n is constant, so φ is a constant (whose value is immaterial there since $\gamma = 1$ makes $\Omega = 1$), while under schemes of the Scheme (1) type n varies with τ^c and φ varies along the path, where it can be negative at admissible parameter values, so no argument based on a constant Ω applies and the KPR form of Lemma 4 carries the global result.

Appendix C Alternative fiscal schemes

C.1 Ratio schemes

Under Scheme (1*) ($g/y = \bar{\phi}_{gy}$ constant, transfers absorbing the residual), the resource constraint gives $c = (1 - \bar{\phi}_{gy})\, n$, and under Scheme (2*) ($s/y = \bar{\phi}_{sy}$ constant, government consumption absorbing the residual), the household budget gives $(1+\tau^c)c = (1+\bar{\phi}_{sy})\, n$ (at $\tau^n = 0$). In both cases the link between c and n coincides with that of Scheme (1) or (2) up to a constant of proportionality that is independent of τ^c, which, exactly as in Lemma 7, enters only the multiplicative constant of $c(\tau^c)$ and never the exponent. Proposition 1 therefore applies verbatim under (1*) and (2*), including all peak formulas.

C.2 Level-fixed schemes

Two schemes that fix fiscal variables in levels complete the set. Under Scheme (1**), government consumption is fixed at $\bar{g} > 0$ and the transfer absorbs the residual, $s = \tau^c c - \bar{g}$ (negative at low tax rates: a lump-sum tax); the equilibrium link is $n = c + \bar{g}$. Under Scheme (2**), the transfer is fixed at $\bar{s} > 0$ and government consumption absorbs the residual, $g = \tau^c c - \bar{s}$; the link is $n = (1 + \tau^c)c - \bar{s}$, and nonnegative government consumption defines the economically feasible set, which never contains $\tau^c = 0$ (since $g = -\bar{s} < 0$ there). On such a domain the natural notion is *single-peakedness on the feasible interval*, since the curve starts at $R_c = \bar{s}$ where $g = 0$ rather than at zero, and not Definition 2, which anchors a hump at $R_c(0) = 0$.

Proposition C.1. *Under Scheme (1**), with $n = c+\bar{g}$ and the consumption–labor condition $-U_n/U_c = 1/(1+\tau^c)$: (i) for KPR utility, whenever the baseline equilibrium exists, R_c is strictly increasing and bounded, with $R_c \to [1 - \kappa(1-\eta)\bar{g}^{1+\lambda}]/[\eta\kappa(1+\lambda)\bar{g}^{\lambda}]$; (ii) for AS utility, R_c is hump-shaped if and only if $\eta < 1$, and otherwise strictly increasing; revenue is bounded if and only if $\eta \leq 1$; (iii) for GHH utility with curvature $\bar{\eta} > 0$, the admissible domain, on which the GHH bundle $c - \kappa n^{1+\lambda}$ is positive, is $[0, \tilde{\tau})$, where $\tilde{\tau}$ lies strictly below the tax rate at which consumption reaches zero, and an admissible baseline equilibrium exists if and only if $\bar{g} < \bar{g}_{\max} \equiv \frac{\lambda}{1+\lambda}\,[\kappa(1+\lambda)]^{-1/\lambda}$. On $[0, \tilde{\tau})$, R_c is hump-shaped if $\tilde{\tau} > \lambda/(1+\lambda)$ and strictly increasing otherwise; revenue is bounded.*

Proof. (ii) Logarithmic differentiation of $\kappa(1+\lambda)c^{\eta}(c+\bar{g})^{\lambda} = (1+\tau^c)^{-1}$ gives

$$\varepsilon_c(\tau^c) = \frac{\tau^c}{1+\tau^c} \cdot \frac{c+\bar{g}}{\eta(c+\bar{g}) + \lambda c}.$$

The second factor is decreasing in c (its derivative has the sign of $-\lambda\bar{g}$), and c is decreasing in τ^c; both factors of ε_c are therefore increasing, so ε_c is strictly increasing, with $\varepsilon_c(0) = 0$ and, since $c \to 0$ as $\tau^c \to \infty$ is forced by the condition, limit $1/\eta$. Lemma 2(iii) gives a hump if and only if $1/\eta > 1$. For boundedness, $c \sim [\kappa(1+\lambda)\bar{g}^{\lambda}]^{-1/\eta}(1+\tau^c)^{-1/\eta}$, so $R_c \sim \tau^{c\,1-1/\eta}$: bounded if and only if $\eta \leq 1$. (i)

For KPR utility the same computation gives $\varepsilon_c = \left[1 + 1/(\eta\tau^c) + \{(1+\tau^c)/\tau^c\}\,\lambda/(1+\bar{g}/c)\right]^{-1} < 1$ for all τ^c, so R_c is strictly increasing by Lemma 2(i); as $\tau^c \to \infty$, $c \to 0$, $n \to \bar{g}$, and the intratemporal condition gives the stated limit of $\tau^c c$. (For $\eta < 1$, existence of the baseline equilibrium requires $1 - \kappa(1-\eta)n(0)^{1+\lambda} > 0$; since n decreases in τ^c, the condition then holds at every tax rate.) (iii) The condition pins $n = [\kappa(1+\lambda)(1+\tau^c)]^{-1/\lambda}$, strictly decreasing, and $c = n - \bar{g}$. Using $\kappa n^\lambda = 1/[(1+\lambda)(1+\tau^c)]$, the GHH bundle is

$$c - \kappa n^{1+\lambda} = \Phi(\tau^c) - \bar{g}, \qquad \Phi(\tau) \equiv n(\tau)\Big[1 - \frac{1}{(1+\lambda)(1+\tau)}\Big].$$

With $u \equiv (1+\lambda)(1+\tau)$, logarithmic differentiation gives $\Phi'/\Phi = -1/[\lambda(1+\tau)] + (1+\lambda)/[u(u-1)]$, which vanishes at $\tau = 0$ and is strictly negative for $\tau > 0$ (it has the sign of $-(1+\lambda)\tau$): Φ is maximized at $\tau = 0$, where $\Phi(0) = \bar{g}_{\max}$, and strictly decreasing thereafter. Hence an admissible equilibrium exists at $\tau^c = 0$ if and only if $\bar{g} < \bar{g}_{\max}$, and the admissible domain is $[0, \tilde{\tau})$ with $\Phi(\tilde{\tau}) = \bar{g}$; it ends strictly before consumption reaches zero, because the bundle $c - \kappa n^{1+\lambda}$ is strictly smaller than c. On the admissible domain, $\varepsilon_c = [\tau^c/(1+\tau^c)] \cdot (1/\lambda)(1+\bar{g}/c)$ is strictly increasing from $\varepsilon_c(0) = 0$ (both factors increase as c falls). At the admissibility boundary, $n/c = (1+\lambda)(1+\tilde{\tau})$, so $\varepsilon_c \to \tilde{\tau}(1+\lambda)/\lambda$ as $\tau^c \to \tilde{\tau}$. By Lemma 2 applied on $[0, \tilde{\tau})$, R_c is hump-shaped if and only if ε_c crosses one in the interior, that is, if and only if $\tilde{\tau}(1+\lambda)/\lambda > 1$, or $\tilde{\tau} > \lambda/(1+\lambda)$, and strictly increasing otherwise. $R_c \le \tilde{\tau}\, n(0)$: bounded. □

Remark C.1. *For case (iii), positivity of consumption alone is not the relevant admissibility condition: on the larger interval on which $c > 0$, the GHH bundle $c - \kappa n^{1+\lambda}$ turns negative before consumption reaches zero (at the point where $c = 0$ the bundle equals $-\kappa\bar{g}^{1+\lambda} < 0$), so that interval is never admissible in full, and for $\bar{g}$ close to $\bar{g}_{\max}$ the curve has no interior peak on the admissible domain at all. Only for linear GHH utility ($\bar{\eta} = 0$) does the shape statement extend to the full interval on which consumption is positive.*

Proposition C.2. *Under Scheme (2**), with $n = (1+\tau^c)c - \bar{s}$, let $F \equiv \{\tau^c \ge 0 :\ g = \tau^c c - \bar{s} \ge 0\}$ denote the economically feasible set; F is a single interval and never contains $\tau^c = 0$. Then: (i) for KPR utility, equilibrium labor supply is independent of τ^c, $c \propto (1+\tau^c)^{-1}$, and R_c is strictly increasing on $[0, \infty)$, hence on $F = [\bar{s}/n, \infty)$, and bounded, with $R_c \to n + \bar{s}$; (ii) for AS utility: if $\eta > 1$, R_c is strictly increasing and unbounded, with $F = [\tau_1, \infty)$; if $\eta = 1$, labor supply is again independent of τ^c and R_c is strictly increasing and bounded, with limit the solution z of $\kappa(1+\lambda)z(z-\bar{s})^\lambda = 1$; if $\eta < 1$, then whenever F has nonempty interior, R_c is* single-peaked *on F: it rises from $\bar{s}$ at the lower endpoint to a unique interior maximum and declines thereafter, returning to $\bar{s}$ at the upper endpoint when F is bounded and converging to $\bar{s}$ as $\tau^c \to \infty$ when F is unbounded; revenue is bounded. The feasible set is unbounded when $\eta + \lambda > 1$, and is then nonempty for every $\bar{s} > 0$; when $\eta + \lambda = 1$ it is nonempty,*

and then unbounded, if and only if $\bar{s} < 1/[\kappa(1+\lambda)]$; when $\eta + \lambda < 1$ it is bounded, and is a proper interval, a singleton, or empty according as $\bar{s}$ lies below, at, or above a threshold $\bar{s}_{\max}(\eta, \lambda, \kappa)$; (iii) for GHH utility, labor supply is pinned by $n = [\kappa(1+\lambda)(1+\tau^c)]^{-1/\lambda}$, and feasibility is $n\,\tau^c \geq \bar{s}$. For $\lambda > 1$, $F = [\tau_1, \infty)$ has nonempty interior for every $\bar{s} > 0$. For $\lambda = 1$, the supremum of $n\,\tau^c$ is $1/(2\kappa)$, approached but not attained: F has nonempty interior, again equal to $[\tau_1, \infty)$, if and only if $\bar{s} < 1/(2\kappa)$, and is empty otherwise. For $\lambda < 1$, the supremum $M = \frac{\lambda}{1-\lambda}\big[\kappa(1+\lambda)/(1-\lambda)\big]^{-1/\lambda}$ is attained at the unique $\tau^* = \lambda/(1-\lambda)$: F is a proper interval with nonempty interior if $\bar{s} < M$, the degenerate singleton $\{\tau^*\}$ if $\bar{s} = M$, and empty if $\bar{s} > M$. Whenever F has nonempty interior, R_c is single-peaked on F; at the degenerate singleton the only feasible point has $g = 0$ and $R_c = \bar{s}$, and no shape statement applies. Revenue is bounded, with $R_c \to \bar{s}$.

Proof. Write $q = 1 + \tau^c$. The household budget gives $n = qc - \bar{s}$, so $qc = n + \bar{s}$ and

$$R_c = \tau^c c = qc - c = n + \bar{s} - c, \qquad g = \tau^c c - \bar{s} = n - c\,:$$

feasibility is exactly $n \geq c$.

(i) *KPR.* Substituting $c = (n+\bar{s})/q$ into $-U_n/U_c = 1/q$ cancels every occurrence of q: $\eta(1+\lambda)\kappa(n+\bar{s})n^\lambda = 1 - \kappa(1-\eta)n^{1+\lambda}$, an equation that is independent of τ^c, so n is constant and $c = (n+\bar{s})/q$. Then $\varepsilon_c = \tau^c/(1+\tau^c) < 1$ at every tax rate, $R_c = (n+\bar{s})\tau^c/(1+\tau^c)$ is strictly increasing (Lemma 2(i)), and $R_c \to n + \bar{s}$.

(ii) *AS.* For $\eta \neq 1$, eliminating $q = (n+\bar{s})/c$ from $\kappa(1+\lambda)c^\eta n^\lambda = 1/q$ solves for consumption as a function of labor alone,

$$c(n) = \big[\kappa(1+\lambda)(n+\bar{s})\,n^\lambda\big]^{1/(1-\eta)}, \qquad \frac{d\ln q}{dn} = -\frac{1}{1-\eta}\Big[\frac{\eta}{n+\bar{s}} + \frac{\lambda}{n}\Big],$$

so n is a strictly monotone parametrization of τ^c: decreasing for $\eta < 1$, increasing for $\eta > 1$.

If $\eta > 1$: $c(n)$ is decreasing and n increases with τ^c, so $R_c = n + \bar{s} - c$ is strictly increasing; the asymptotic order $qc \sim q^{(\eta-1)/(\eta+\lambda)} \to \infty$ gives unboundedness; and $g = n - c$ increases strictly from $-\bar{s}$, so $F = [\tau_1, \infty)$.

If $\eta = 1$: the condition becomes $\kappa(1+\lambda)(qc)(qc-\bar{s})^\lambda = 1$, so qc, and with it $n = qc - \bar{s}$, is constant; $\varepsilon_c = \tau^c/(1+\tau^c)$, R_c is strictly increasing, and $R_c \to qc = z$.

If $\eta < 1$: differentiating twice, $c' = c\,h$ with $h(n) = [1/(n+\bar{s}) + \lambda/n]/(1-\eta)$, and $c'' = c\,(h^2 + h')$ has the sign of

$$N(t) \equiv \eta t^2 + 2\lambda t + \lambda(\lambda + \eta - 1), \qquad t = \frac{n}{n+\bar{s}} \in (0,1).$$

If $\eta + \lambda \geq 1$, $N > 0$ on $(0,1)$: c is convex, $g = n - c$ is concave, F is an interval, and c' is strictly increasing. If $\eta + \lambda < 1$, N has the unique positive root $t^+ = [\sqrt{\lambda(1-\eta)(\lambda+\eta)} - \lambda]/\eta \in (0,1)$: c is concave for $t < t^+$ and convex for $t > t^+$, so c' first decreases and then increases, and g, with a

single inflection and $g < 0$ at both ends of $(0, \infty)$, is nonnegative on at most one interval. In either case F corresponds to a single interval $[n_a, n_b]$ of labor supplies (with $n_a = 0$ allowed, the case of an unbounded feasible tax range).

Since $dR_c/d\tau^c = [1 - c'(n)]\, dn/d\tau^c$ and $dn/d\tau^c < 0$, the sign of $dR_c/d\tau^c$ is that of $c'(n) - 1$. At a finite feasibility endpoint, $g = 0$ with $g \geq 0$ inside forces $g'(n_a) \geq 0$ and $g'(n_b) \leq 0$, i.e. $c'(n_a) \leq 1 \leq c'(n_b)$; when $n_a = 0$, $c'(n) \to c'_0 < 1$ as $n \to 0$ plays the same role ($c' \to 0$ when $\eta + \lambda > 1$; $c' \to [\kappa(1+\lambda)\bar{s}]^{1/\lambda} < 1$ on the nonempty branch when $\eta + \lambda = 1$). A c' that is increasing, or first decreasing and then increasing, and satisfies these boundary inequalities crosses one exactly once on (n_a, n_b); hence $dR_c/d\tau^c$ changes sign exactly once on F, from positive to negative as τ^c rises: R_c is single-peaked on F. At finite feasibility endpoints $R_c = g + \bar{s} = \bar{s}$; when F is unbounded, $n, c \to 0$ and $R_c \to \bar{s}$; and $R_c \leq n(\tau^c{=}0) + \bar{s}$: bounded.

For the structure of F: near $n = 0$, $c(n)/n \sim [\kappa(1+\lambda)\bar{s}]^{1/(1-\eta)} n^{(\lambda+\eta-1)/(1-\eta)}$, which tends to 0 if $\eta + \lambda > 1$ (so $g > 0$ for small n: F unbounded and nonempty for every $\bar{s}$), to $[\kappa(1+\lambda)\bar{s}]^{1/\lambda}$ if $\eta + \lambda = 1$ (here g is concave with $g(0) = 0$ and $g'(0^+) = 1 - [\kappa(1+\lambda)\bar{s}]^{1/\lambda}$, so F is nonempty, and unbounded, exactly when $\bar{s} < 1/[\kappa(1+\lambda)]$), and to ∞ if $\eta + \lambda < 1$ (so $g < 0$ near both ends and F is bounded; since $c(n)$ is strictly increasing in $\bar{s}$, $\max_n g$ is strictly decreasing in $\bar{s}$, giving the trichotomy of a proper interval, a singleton, or an empty set at the threshold $\bar{s}_{\max}$ where $\max_n g = 0$).

(iii) *GHH.* $n = [\kappa(1+\lambda)q]^{-1/\lambda}$ is pinned by the intratemporal condition, and $c = (n + \bar{s})/q$, so $R_c = (n + \bar{s})(1 - 1/q)$ and

$$\frac{dR_c}{dq} = \frac{Z(q)}{\lambda q^2}, \qquad Z(q) \equiv n(q)\,(1 + \lambda - q) + \lambda\bar{s}, \qquad Z'(q) = -\frac{n}{\lambda q}\big[1 + \lambda + q(\lambda - 1)\big].$$

Feasibility is $g = n - c \geq 0 \iff n\,\tau^c \geq \bar{s}$. For the map $G(q) \equiv (q-1)[\kappa(1+\lambda)q]^{-1/\lambda} = n\,\tau^c$, logarithmic differentiation gives $G'/G = 1/(q-1) - 1/(\lambda q)$, positive if and only if $(\lambda - 1)q > -1$: G is strictly increasing on $(1, \infty)$ for $\lambda \geq 1$, and first increasing and then decreasing with unique maximizer $q^* = 1/(1-\lambda)$ for $\lambda < 1$. Its supremum is $+\infty$ for $\lambda > 1$ (so $F = [\tau_1, \infty)$ is nonempty for every $\bar{s}$); $1/(2\kappa)$ for $\lambda = 1$, approached but not attained (so F is nonempty, and then equal to $[\tau_1, \infty)$, exactly when $\bar{s} < 1/(2\kappa)$); and, for $\lambda < 1$, $(q^* - 1)\, n(q^*) = M$, attained at q^* alone (so $\{G \geq \bar{s}\}$ is a proper interval for $\bar{s} < M$, the singleton $\{q^*\}$ for $\bar{s} = M$, and empty for $\bar{s} > M$). This is the stated structure of F.

For the shape: if $\lambda \geq 1$, $Z' < 0$ strictly and $Z(1) = \lambda[n(1) + \bar{s}] > 0$; for $\lambda > 1$, $Z \to -\infty$, so Z crosses zero exactly once and R_c is single-peaked on $[0, \infty)$, hence on F; for $\lambda = 1$, $Z \to \bar{s} - 1/(2\kappa)$, so R_c is single-peaked when $\bar{s} < 1/(2\kappa)$ and strictly increasing otherwise; F is nonempty exactly in the former case. If $\lambda < 1$, Z is strictly decreasing for $q < (1+\lambda)/(1-\lambda)$ and strictly increasing thereafter, with $Z(1) > 0$ and $Z(\infty) = \lambda\bar{s} > 0$; on a feasible interval $[q_1, q_2]$ with nonempty interior, $R_c(q_1) = R_c(q_2) = \bar{s}$ with $R_c \geq \bar{s}$ between forces $Z(q_1) \geq 0 \geq Z(q_2)$, and the shape of Z, first

decreasing and then increasing, then yields exactly one sign change on $[q_1, q_2]$: R_c is single-peaked on F. In every case $R_c = (n + \bar{s})(1 - 1/q) \to \bar{s}$: bounded. □

Remark C.2. *On the unrestricted domain $\tau^c \geq 0$, that is, ignoring the requirement $g \geq 0$, single-peakedness can fail under Scheme (2**): in the region where $g < 0$, marginal revenue can turn positive again, with R_c converging to $\bar{s}$ from below. For AS utility this occurs when $\eta + \lambda < 1$ (at $\eta = 0.5$, $\lambda = 0.25$, $\bar{s} = 0.1$, marginal revenue turns positive again near $\tau^c \approx 8.1$); for GHH utility, when $\lambda < 1$. In both cases the tax rate elasticity converges to one, so no global single-crossing argument applies on the unrestricted domain. Proposition C.2 is therefore stated on the economically feasible domain, where the single-crossing arguments above are complete.*

Appendix D The dynamic economy

D.1 Detrended equilibrium system

With balanced growth factor $\psi = \xi^{1/(1-\theta)}$ and detrended variables $\tilde{a}_t = a_t/\psi^t$ ($\tilde{k} \equiv k_{t-1}/\psi^t$), and with $\tilde{\mu}$ denoting the detrended multiplier on the household budget constraint, the balanced growth system is

$$
\begin{aligned}
&(1+\tau^c)\tilde{\mu} = U_c(\tilde{c}, n), \qquad \tilde{\mu}(1-\tau^n)\tilde{w} = -U_n(\tilde{c}, n),\\
&1 = \beta\psi^{-\eta}\big[1 + (1-\tau^k)(d-\delta)\big], \qquad 1 = \beta\psi^{-\eta}R^b,\\
&\psi\tilde{k} = (1-\delta)\tilde{k} + \tilde{x}, \qquad \tilde{y} = \tilde{k}^{\theta}n^{1-\theta}, \qquad \tilde{w} = (1-\theta)\tilde{y}/n, \qquad d = \theta\tilde{y}/\tilde{k},\\
&\tilde{y} = \tilde{c} + \tilde{x} + \tilde{g} - \tilde{m}, \qquad \tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d-\delta)\tilde{k},
\end{aligned}
$$

with the AS and GHH preference shifters ($\psi^{t(1-\eta)}$ and ψ^t respectively, following Erceg et al. (2006)) cancelling from the ratio $-U_n/U_c$ in detrended form. For GHH utility the intertemporal conditions hold with the curvature $\bar{\eta}$ in place of η (the marginal utility of the GHH bundle grows at $\psi^{-\bar{\eta}}$ on the balanced growth path), so $R^b = \psi^{\bar{\eta}}/\beta$; $\bar{\eta}$ enters only R^b, d, and the ratios that are independent of τ^c, never an exponent. The calibration sets $\bar{\eta} = \eta = 2$.

D.2 The rental rate

For KPR and AS utility the Euler equation gives $R^b = \psi^{\eta}/\beta$; for GHH utility it gives $R^b = \psi^{\bar{\eta}}/\beta$. In either case the capital Euler equation implies $1 + (1-\tau^k)(d-\delta) = R^b$, hence

$$d = \delta + \frac{R^b - 1}{1 - \tau^k}.$$

At the benchmark calibration, $d = 0.133$.

D.3 Debt service on the balanced growth path

The flow government budget constraint is $g_t + s_t + R^b b_{t-1} = b_t + T_t$. Detrending with $\tilde{b} \equiv b_{t-1}/\psi^t$ (the same dating convention as $\tilde{k}$) gives $b_t/\psi^t = \psi\,\tilde{b}_t$, so on a balanced growth path with b/y constant,

$$\tilde{g} + \tilde{s} + R^b\tilde{b} = \psi\tilde{b} + \tilde{T} \qquad \Longleftrightarrow \qquad \tilde{g} + \tilde{s} = \tilde{T} - (R^b - \psi)\tilde{b}\,:$$

the net drain of debt service on the budget is $(R^b - \psi)\tilde{b}$, because maintaining a constant ratio of debt to output on a path growing at factor ψ requires new issuance $\psi\tilde{b} - \tilde{b}$ each period. (Equivalently, $(R^b/\psi - 1)$ per unit of debt detrended by its own date.) The debt service term is independent of τ^c, so it affects no shape classification; it enters only the constant K_0 and the implied transfer or spending levels. Scheme (3) allocations and total tax revenue do not involve debt service.

D.4 Numerical verification of the constant K_0

For a numerical verification, we keep the remaining parameters at the values of Section 5.2 and set $\eta = 0.5$ and $\tilde{s}/\tilde{y} = 0.05$ under Scheme (4); the expression for K_0 in Lemma 6 then gives $K_0 = 0.511$. A full numerical solution of the balanced growth system confirms $(1+\tau^c)\tilde{c}/\tilde{y} = 0.5106$, constant in τ^c to machine precision, with the resource constraint closing identically.

D.5 Proof of Proposition 4

Under Scheme (3), $\tilde{T} = (\tau^c\,\tilde{c}/\tilde{y} + \Theta)\,\tilde{y} = \phi_c\,(\tau^c + m_3)\,\tilde{y}(\tau^c)$ with $m_3 = \Theta/\phi_c$. For AS and GHH utility, $\tilde{y} \propto n \propto \tilde{c}$ lies in the power form with $\Omega = 1/(\eta+\lambda)$ and $1/\lambda$, so Lemma 3 applies with $m = m_3$; cases (ii)–(iii) read off its classification, with $\Omega m_3 \lessgtr 1$ equivalent to $m_3 \lessgtr \eta + \lambda$ (resp. λ) and peak $(1 - \Omega m_3)/(\Omega - 1) = [(\eta+\lambda) - m_3]/(1 - \eta - \lambda)$ (resp. $[\lambda - m_3]/(1-\lambda)$). For KPR utility, $\tilde{y} \propto (a + b\tau^c)^{-1/(1+\lambda)}$ and Lemma 4 applies with $m = m_3$. The threshold $bm_3 \le (1+\lambda)a$ rearranges, using $\phi_c(1+\lambda)a/b = (1-\eta)(1-\theta)(1-\tau^n)/\eta + (1+\lambda)\phi_c$ (substituting a, b from Lemma 7 and the $\chi\tilde{w}$ identities), to $\Theta \le B(\lambda)$ as displayed in case (i). For $\lambda > 0$ this is Lemma 4(i), which also delivers the U-shaped alternative and unbounded revenue. For $\lambda = 0$ Lemma 4(ii) applies instead: the derivative has the constant sign of $a - bm_3$, so $\tilde{T}$ is strictly increasing, constant, or strictly decreasing according as $\Theta < B(0)$, $\Theta = B(0)$, or $\Theta > B(0)$, and $\tilde{T} \to C/b$, so revenue is bounded.

Under Scheme (4), $\tilde{c} = K_0\tilde{y}/(1+\tau^c)$ gives

$$\tilde{T} = \tilde{y}\Big[\frac{K_0\tau^c}{1+\tau^c} + \Theta\Big] = \tilde{y}\,\frac{(K_0+\Theta)\tau^c + \Theta}{1+\tau^c} = (K_0+\Theta)\,\big(\tau^c + m_4\big)\,\frac{\tilde{y}(\tau^c)}{1+\tau^c},$$
$$m_4 = \frac{\Theta}{K_0+\Theta} \in (0,1).$$

For AS utility, Lemma 7 gives $\tilde{y} \propto (1+\tau^c)^{(\eta-1)/(\eta+\lambda)}$, so $\tilde{y}/(1+\tau^c) \propto (1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}$ and $\tilde{T}$ is the power form product with $\Omega = (1+\lambda)/(\eta+\lambda)$ and $m = m_4$; for GHH utility the same steps

give $\Omega = (1+\lambda)/\lambda$; for KPR utility n is constant, $\tilde{y}$ is constant, and $\Omega = 1$. Lemma 3 then delivers cases (iv)–(vi): since $m_4 < 1$, the case $\Omega \leq 1$ is always strictly increasing; for $\Omega > 1$ the hump condition $\Omega m_4 < 1$ rearranges to $(1-\eta)\Theta < (\eta+\lambda)K_0$ (AS) and $\Theta < \lambda K_0$ (GHH), and the peak $(1-\Omega m_4)/(\Omega - 1)$ simplifies to the displayed expressions.

A direct numerical solution of the balanced growth system at $\eta = 0.5$, $\lambda = 0.5$ locates the Scheme (4) total tax revenue peak at $\tau^c = 1.141$, matching the closed form. □

Appendix E Numerical procedure

All computations use closed forms. The calibration is $\theta = 0.35$, $\delta = 0.07$, $g/y = 0.18$, $b/y = 0.63$, $m/y = 0.04$, $\psi = 1.02$, $\bar{n} = 0.25$, $\tau^k = 0.36$, $\tau^n = 0.28$, baseline $\tau^c = 0.05$, $\eta = 2$, $\lambda = 0.5$, and $\beta = 1$. Balanced growth objects follow the formulas of Appendix D: $d = 0.1331$, $\tilde{k}/\tilde{y} = 2.629$, $\tilde{x}/\tilde{y} = 0.2366$, $\tilde{c}/\tilde{y} = 0.6234$, $n/\tilde{y} = 0.5942$, $\Theta = 0.2417$. Under Scheme (3), total tax revenue is $\tilde{T} \propto (\tau^c \phi_c + \Theta)\, \tilde{y}(\tau^c)$ with $\tilde{y}(\tau^c)$ from Lemma 7; Figure 1 normalizes by the value at $\tau^c = 0.05$. Table 3 reports $\phi_c(1-\varepsilon_c)$ and $-\Theta\varepsilon_c/\tau^c$ with the elasticities for each utility.