

A unified marginal value of water across competing services

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A unified marginal value of water across competing services

Abstract

Water can provide multiple services within the same managed system, but the economic value of a unit of water depends on which service it enables and when it is available. This makes the marginal value of water difficult to quantify when competing uses must be assessed on a common economic scale. Existing approaches generally value individual water services within separate sectoral frameworks, limiting direct comparison of the marginal contribution of the same water stock across uses. Moreover, when these services are represented as hierarchically weighted objectives in an optimization model, redundant differences in objective-weight magnitudes can distort shadow prices without changing the underlying optimal solution. Here we develop a compact-weight commensuration framework that removes this redundant scaling while preserving the priority structure and optimal operating solution of the original model. The resulting shadow prices provide a common economic metric for the marginal value of water across competing services and system states. Applied to the Xiaowan cascaded-reservoir system, the framework substantially reduces orders-of-magnitude disparities in conventional shadow prices without altering the optimal operating state, and reveals distinct seasonal patterns in the value of inflow, storage, energy and capacity. The results show that reservoir water is better understood not as a fixed-volume commodity, but as a portfolio of competing services whose marginal value changes with scarcity, timing and system requirements. This common valuation framework provides a quantitative basis for comparing alternative water uses and informing integrated reservoir allocation and cross-sectoral water-service markets.

Keywords: reservoir water valuation; multifunctional water services; shadow price; compact weights; water allocation; water–energy nexus

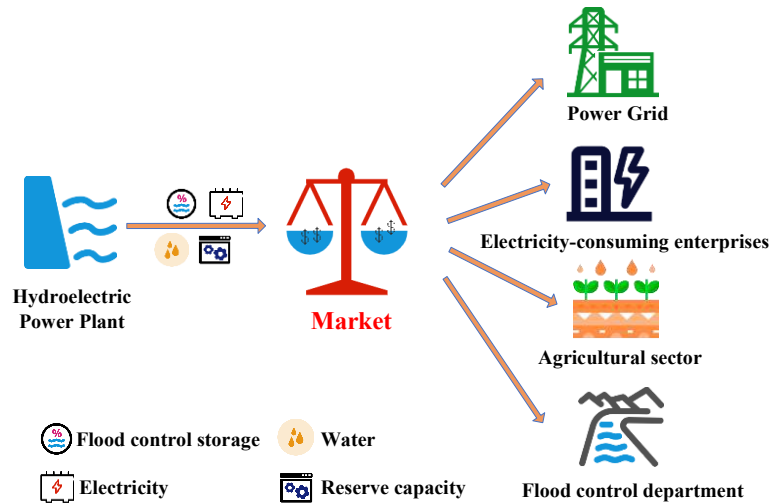
Introduction

Water scarcity is increasingly a problem of allocation rather than availability alone. As climate variability, competing demands and environmental constraints reshape water systems, the same unit of water can generate fundamentally different benefits depending on when, where and how it is used^{1,2}. A cubic meter of water in reservoirs can support irrigation, hydropower generation, municipal and industrial supply, or flood-risk reduction, depending on when and where it is released or retained³. Its economic value is therefore not an intrinsic property of water volume alone. It emerges from the set of services that the stored water can provide under prevailing hydrological conditions, infrastructure constraints and market demands^{2,4,5}. This raises a fundamental question for multipurpose reservoir management: how much is one additional cubic meter of stored water worth when it can serve different purposes in different markets?

Existing approaches generally value these services within separate sectoral frameworks. Irrigation water may be valued from crop productivity or water-rights prices, hydropower water from electricity revenues and opportunity costs, municipal water from tariffs or marginal supply costs, and flood-control storage from avoided damages^{6–9}. Such approaches are useful for quantifying individual services, but they do not necessarily provide a common metric for comparing the marginal contribution of the same physical water stock across competing uses. This limitation becomes particularly important in multipurpose reservoirs because allocating water to one service necessarily changes the quantity, timing or opportunity to provide other services. Recent hydro-economic studies have increasingly attempted to integrate economic, environmental and operational considerations, but most remain focused on optimizing allocation or operation under multiple objectives rather than establishing a common economic measure of the marginal value of the underlying stored water^{10–12}. A critical gap persists: no unified framework exists to equitably value and trade multidimensional energy resources within integrated markets. This

limitation stems from three fundamental barriers: (1) the heterogeneous physical units and temporal-spatial characteristics of resources; (2) conflicting stakeholder priorities across sectors (agricultural, industrial, environmental); and (3) the inability of conventional cost-demand models to internalize externalities like ecological impacts and intergenerational equity.

Fig.1: Different energy sources involved in the process of hydropower scheduling.



Consider the paradigmatic case in Fig. 1: A hydropower facility simultaneously manages water for irrigation, electricity generation, flood control reserves, and grid stability reserves. Each resource dimension presents distinct valuation challenges:

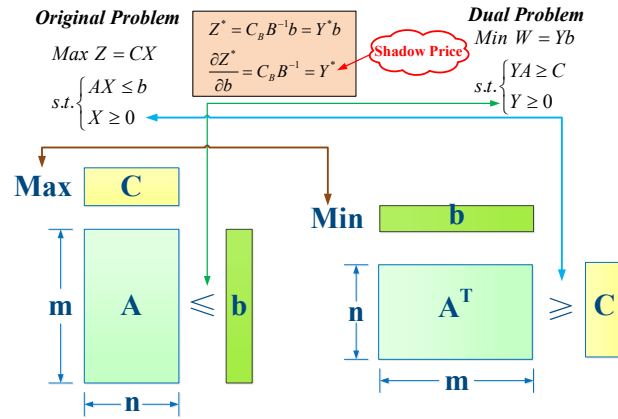
- How should water pricing reflect agricultural productivity versus hydropower revenue tradeoffs?
- What premium ensures sufficient reserve capacity for grid resilience during peak demand?
- Can flood control reserves be monetized to reflect downstream risk mitigation benefits?

The difficulty is compounded by the fact that the value of stored water is dynamic¹³. The same volume can have substantially different values across seasons, reservoir locations and hydrological conditions. Water retained upstream can generate benefits through multiple downstream hydropower stations, whereas the same volume stored downstream may have fewer opportunities for reuse. During dry periods, water may become scarce and highly valuable for irrigation or electricity generation, whereas during wet periods additional inflows may have little value or even impose costs when storage constraints lead to spillage. These interactions mean that water value cannot be adequately represented by a fixed tariff, an average production cost or a static estimate of opportunity cost. Instead, its value should be defined by the marginal contribution of water to the total economic benefits of the managed water system.

Here we develop a unified marginal valuation framework that translates heterogeneous reservoir services into a common economic metric. The central quantity is the marginal economic value of stored water, defined as the incremental system-wide economic benefit generated by one additional unit of available water, conditional on the prevailing hydrological, operational and market conditions. By expressing the marginal benefits of water across irrigation, hydropower, water supply and risk-management services in a common monetary numeraire, the framework enables direct comparison of otherwise heterogeneous water uses. Importantly, the resulting value is not a fixed price of water. It is a dynamic measure of the opportunity cost and scarcity of water at a particular reservoir and time, reflecting the best alternative uses that are displaced when water is allocated to a given service.

Shadow prices provide a theoretically attractive starting point because they quantify the marginal effect of relaxing a constraint within an optimized system¹⁴⁻¹⁶. They have therefore been widely used to represent scarcity values in water and energy models. However, in hierarchical multi-objective reservoir optimization, shadow prices are also affected by the numerical representation of objective priorities^{17,18}. Large differences in objective weights can propagate into the dual problem and produce shadow prices that differ by several orders of magnitude even when the underlying optimal operating strategy is unchanged, as illustrated in Fig.2. Such values may consequently reflect the scaling of the optimization formulation rather than the marginal contribution of the underlying resource. This creates a methodological barrier to comparing the economic value of multifunctional reservoir water.

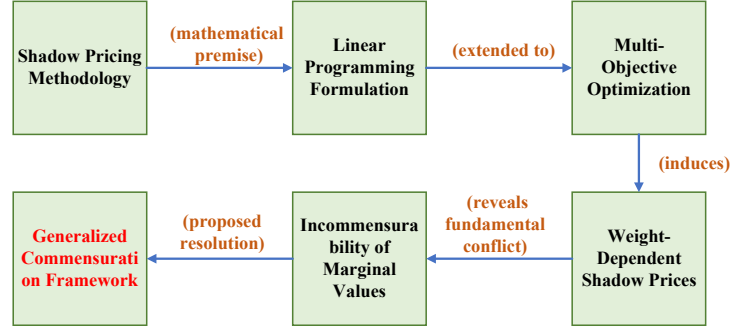
Fig.2: Shadow price concept diagram.



Thus, the developed framework introduces compact objective weights that preserve the priority hierarchy and optimal operating solution of a multi-objective reservoir model while removing redundant differences in objective-weight magnitudes. Shadow prices derived under the compact weights can therefore be used to express otherwise incommensurable water services on a common monetary scale. Applied to a cascaded-reservoir hydropower system, the framework simultaneously values water inflow, storage, energy supply, generation capacity and reserve capacity. We demonstrate the framework using a weekly hydropower scheduling model of cascaded reservoirs with hourly power and capacity balances, as illustrated in Fig.3. Results show that conventional objective weights can generate shadow prices differing by several orders of magnitude despite identical optimal operating solutions, whereas compact weights substantially reduce these artificial disparities. The resulting values reveal how the marginal value of reservoir water varies with season, storage state and system requirements. By converting the multifunctional benefits of reservoir water into a common value metric, this framework provides a quantitative basis for integrated water allocation and potential cross-sectoral water-service markets.

The central conceptual shift is to treat reservoir storage not simply as a volume of water, but as a portfolio of competing water services. A common physical stock can have several simultaneous opportunity costs and benefits. A monetary numeraire does not imply that these services are identical or that all of them have established market prices; rather, it provides a consistent basis for comparing their marginal contributions within the optimized system. This distinction is important for integrated water management because it allows the economic role of stored water to be evaluated across sectors and through time.

Fig.3: Overview of Research Ideas.



Methods

Objective-weight scaling can distort marginal resource values

We first examine why conventional hierarchical weights can produce non-comparable shadow prices. In a multi-objective optimization problem, the priority structure determines which objectives dominate the solution, whereas the numerical magnitude of the weights also enters the objective coefficient matrix (C_B) used to construct the dual problem ($z^* = C_B B^{-1} b = Y^* b$). When the optimal solution remains unchanged over a range of weight ratios, the additional magnitude assigned to a higher-priority objective contains no additional information about the operating decision. Nevertheless, that magnitude can be transmitted to the associated shadow prices.

We refer to this interval of redundant weight scaling as weight redundancy. The key implication is that two weight vectors can represent the same priority ordering and yield the same optimal operating solution while producing different dual prices. The problem becomes more pronounced when several hierarchical objectives are present, because redundancy can occur in multiple directions in the feasible objective space. An objective that is effectively insensitive to weight changes is treated as non-elastic, whereas an objective whose optimal value responds to weight changes provides the flexibility needed to identify a meaningful trade-off.

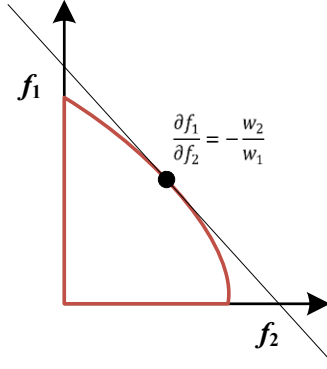
For a multi-objective optimization problem where one sub-objective is clearly prioritized over the others, it can typically be formulated as follows:

$$\max_X \sum_i w_i f_i(X) \quad (1)$$

$$s.t. \begin{cases} h_i(X) = 0 \\ g_i(X) \leq 0 \\ w_{i+1} \square w_i \geq 0 \end{cases} \quad (2)$$

- (1) On the continuously differentiable Pareto frontier of a multi-objective optimization problem, trade-off relationships can be accurately estimated.

Fig.4: Illustration of Trade-off Relationships Between Objectives.



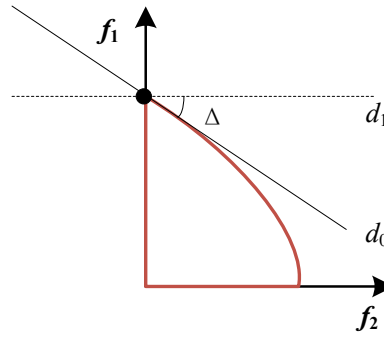
As illustrated in Fig.4, when considering a multi-objective optimization problem with two sub-objectives, the position of the optimal solution on the Pareto frontier is determined by the weight vector assigned to the objectives, which reflects the decision-maker's preferences between the sub-objectives.

At the trade-off decision point, the derivative represents the transactional relationship between the two objectives, that is,

$$\frac{\partial f_1}{\partial f_2} = -\frac{w_2}{w_1} \quad (3)$$

(2) In multi-objective optimization problem with priority hierarchy, the 'weight redundancy' emerges.

Fig.5: Illustration of " Weight Ratio Redundancy " Between Objectives.



As illustrated in Fig.5, when the first objective has priority, the optimal solution can be attained across a range of derivative/weight ratios from d_0 to d_1 :

$$d_0 = -\frac{w_1^{[0]}}{w_2^{[0]}} \quad (4)$$

$$d_1 = -\frac{w_1^{[1]}}{w_2^{[1]}} \quad (5)$$

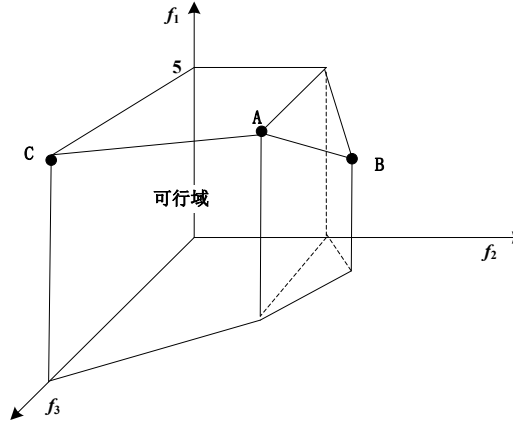
Therefore, the ratio of weights possesses a degree of redundancy or margin:

$$\Delta = d_1 - d_0 \quad (6)$$

When determining the optimal weight ratio for establishing the trade-off between objectives, d_0 is the most precise choice. An excessively high w_1 would significantly underestimate the importance of f_2 . Therefore, the trade-off relationship between objectives is best assessed by eliminating "weight ratio redundancy ". The adjusted weights, known as 'compact weights,' must be specifically calibrated for each objective, enabling accurate evaluation of trade-offs between different sub-objectives.

(3) "Multiple redundancies" exists within linear constraint space:

Fig.6: Illustration of "Multiple Redundancies".



As illustrated in Fig.6, When the priorities of the sub-objectives f_1 to f_3 decrease successively, the optimal solution is located at point A, where f_1 is maximized and subsequently, followed by maximizing f_2 and f_3 as much as possible under the constraints. In this scenario, if the weight of f_2 is increased until the "weight ratio redundancy " in that direction (denoted as Δ_1) is "exhausted", the optimal solution shifts to point B, sacrificing f_1 to enhance f_2 . Similarly, if the weight of f_2 is decreased until the " weight ratio redundancy " in that direction (denoted as Δ_2) is "exhausted", the optimal solution moves to point C, sacrificing f_2 to boost f_3 . This clearly shows that different directions of weight change result in different "weight ratio redundancies. " Therefore, it is essential to carefully consider from which direction to eliminate these redundancies to accurately determine the trade-off relationships between objectives.

(4) Elastic and non-elastic sub- objectives: trading should be oriented towards elastic ones.

If the following constraint (7) is introduced to Fig.6:

$$f_1 \geq 5 \quad (7)$$

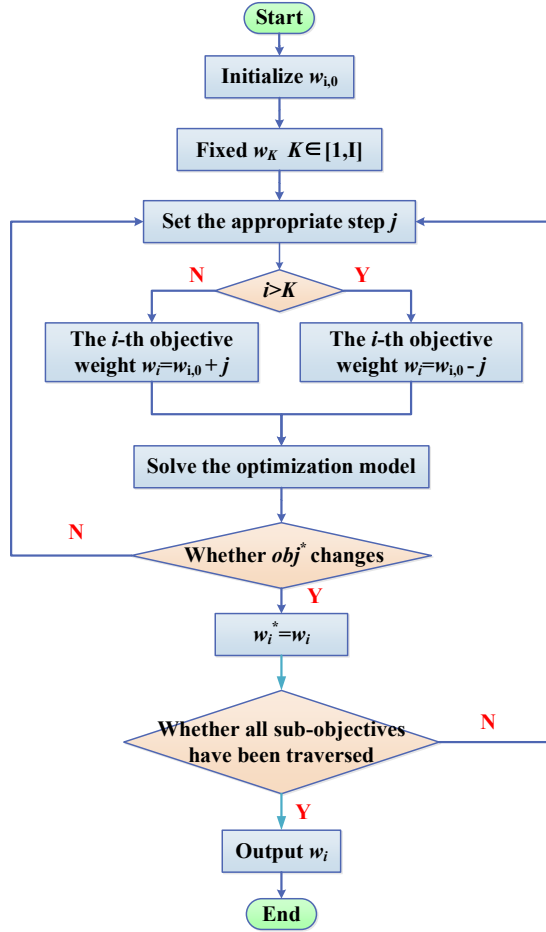
Then regardless of how much the weight of f_2 increases, f_1 remains unchanged, indicating that f_1 is inelastic. Economically, f_1 does not respond to market information, likely due to resource limitations. As a result, f_2 cannot eliminate "weight redundancy" by simply increasing its weight.

In this scenario, f_3 exhibits the greatest elasticity (range of values), followed by f_2 . Given that f_1 is inelastic and does not influence the optimal solution even with a weight of 0, its compact weight should be set to 0. Meanwhile, f_2 and f_3 possess the flexibility to trade off with each other. The compact weight of f_3 , being the most elastic, remains stable, whereas the weight of f_2 can be reduced towards f_3 to eliminate "weight redundancy", obtaining its compact weight at the critical point.

Compact weights preserve the operating solution while removing redundant scaling

The compact-weight procedure starts from a hierarchical weight vector and selects an elastic reference objective. The weights of adjacent higher- and lower-priority objectives are then adjusted towards the reference objective until a change in the optimal solution is detected. The boundary at which the current redundancy is exhausted defines a convergence weight. The process is repeated across adjacent objectives and iterated until the complete vector converges, as seen in Fig. 7. The resulting compact weights preserve the hierarchy of priorities but remove unnecessary numerical separation between objectives.

Fig.7: Flowchart of The Compact Weight Determination Method.



To ensure the rigor of the method for determining compact weights, two mathematical theorems are presented and proved in this work.

[Theorem 1] When the weight of a specific objective decreases/increases, the corresponding sub-objective function value is bound to be monotonically decreasing/increasing.

[Theorem 2] Considering an optimization problem with non-negative weighted priority objectives, if there is a one-to-one mapping relationship between its optimal solution and the objectives as follows:

$$\begin{cases} f_i^* = \text{optm}(X^*) \\ X^* = \text{optm}^{-1}(f_1^*, f_2^*, \dots) \end{cases} \quad (8)$$

Denote two optimization problems with only one unequal objective weight:

Problem 1:

$$P1: \begin{cases} \max_X (w_1 f_1 + \dots + w_i^{[1]} f_i + \dots + w_N f_N) \\ s.t. \quad g_j(X) = 0 \end{cases} \quad (9)$$

Problem 2:

$$P2: \begin{cases} \max_X (w_1 f_1 + \dots + w_i^{[2]} f_i + \dots + w_N f_N) \\ s.t. \quad g_j(X) = 0 \end{cases} \quad (10)$$

Their optimal solutions are respectively denoted as $(X^{*[1]}; f_1^{*[1]}, f_2^{*[1]}, \dots)$ and $(X^{*[2]}; f_1^{*[2]}, f_2^{*[2]}, \dots)$.

If:

$$f_i^{*[1]} = f_i^{*[2]} \quad (11)$$

Then,

$$f_k^{*[1]} = f_k^{*[2]} \quad (\forall k) \quad (12)$$

Furthermore, the two optimization problems have the same optimal solution.

The two theorems proposed in this study are respectively proved mathematically, see **appendix A** for details, which ensure the scientific rigor of the proposed method, wherein:

[Theorem 1] guarantees that the "convergent" weights can be obtained by monotonically increasing/decreasing the weight of a particular objective, thereby providing the potential for convergence of the final weight vector.

[Theorem 2] ensures that the optimal solution remains unchanged under the obtained "convergent" weights, ensuring that the derived marginal values still represent market information under the system's optimal state.

Results

Numerical validation of the compact-weight concept

To validate the method for determining the compact weights proposed in the previous section and to provide an intuitive understanding of its principles, two numerical cases are given as following.

$$\begin{aligned} \max f &= 100x_1 + 10x_2 + x_3 + 0.1x_4 + 0.01x_5 \\ s.t. \quad &\begin{cases} 0.3x_1 + 0.7x_2 \leq 300 \\ 0.4x_2 + 0.3x_3 \leq 200 \\ 0.3x_3 + 0.3x_4 \leq 150 \\ 0.2x_4 + 0.1x_5 \leq 50 \\ x_1 \leq 500 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases} \end{aligned} \quad (13)$$

Formula (13) introduces a linear programming (LP) example with five sub-objectives. By fixing the weight of the elastic objective x_2 and sequentially optimizing the convergence weights of the remaining ones, Fig.8(a) illustrates the iterative process of compact weights for different sub-objectives during the optimization. As shown in the figure, the compact weight of x_1 reaches convergence by the fifth iteration, while those of x_3 and x_4 converge by the fourth iteration. Notably, the compact weight of x_5 achieves convergence as early as the third iteration. This demonstrates that the proposed method exhibits a fast convergence rate and high solution efficiency.

Fig.8: The Iteration of Compact Weights

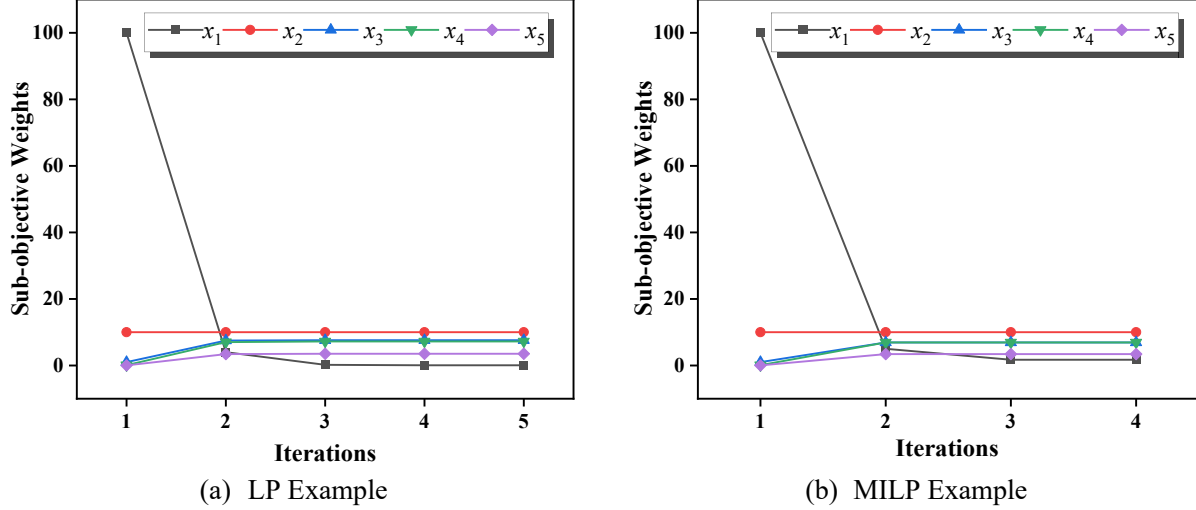


Table-1 summarizes the optimization results for all sub-objectives, based on both the initial weights and the compact weights. The optimal weight vector is determined as: $w^* = [0.05, 1.0, 7.6, 7.2, 3.51]$. The relatively small magnitude of the converged weight for the first-level objective x_1 can be attributed to its nature as a non-elastic sub-objective in this LP problem, indicating that x_1 offers limited value information and is heavily constrained by resources. In contrast, the similar magnitudes of the compact weights for x_3 and x_4 suggest that these two objectives have approximately equal "trading" values.

Table-1 Weight Result of LP Example.

Variables	x_1	x_2	x_3	x_4	x_5
Initial Weights	100	10	1	0.1	0.01
Compact Weights	0.05	10	7.6	7.2	3.51

As shown in Table-2, after optimizing the compact weights of individual sub-objectives within the objective function of the LP example, substituting the optimized compact weights for the initial weights and re-optimizing the problem does not alter the optimal solution of the original problem, which verifies Theorem 2. Consequently, the shadow prices based on compact weights are consistent with the shadow prices of the original problem, ensuring that the marginal values still represent market information under the system's optimal state.

Table-2 Optimal Solution Result of LP Example.

Variables	x_1	x_2	x_3	x_4	x_5
Optimal Solution of The Original Problem	500	214.286	380.952	119.048	261.905
Optimal Solution under The Compact Weights	500	214.286	380.952	119.048	261.905

Formula (14) presents a Mixed Integer Linear Programming (MILP) example comprising five sub-objectives. With the weight of the elastic sub-objective x_2 fixed, the compact weights of the remaining sub-objectives are optimized sequentially. Fig.8(b) illustrates the iterative process of calculating compact weights of each sub-objective during the optimization. As shown in the figure, the compact weight of x_1 converges by the fourth iteration, while those of x_3 , x_4 , and x_5 all converge by the third iteration, demonstrating that the proposed method exhibits a comparatively fast convergence rate in the iterative process of the MILP example.

$$\begin{aligned} \max f &= 100x_1 + 10x_2 + x_3 + 0.1x_4 + 0.01x_5 \\ \text{s.t.} \quad &\begin{cases} 0.3x_1 + 0.7x_2 \leq 300 \\ 0.4x_2 + 0.3x_3 \leq 200 \\ 0.3x_3 + 0.3x_4 \leq 150 \\ 0.2x_4 + 0.1x_5 \leq 50 \\ x_1 \leq 500 \\ x_1, x_2, x_3, x_4, x_5 \text{ are integers} \end{cases} \end{aligned} \quad (14)$$

Table-3 summarizes the optimization results for the compact weights of all objectives, yielding the optimal compact weight vector as $w^* = [1.7, 10, 6.9, 6.9, 3.51]$. It is noteworthy that, despite identical objective functions and constraint sets, significant variations emerge when the decision variables are transformed into integers. Specifically, the convergence of compact weights for x_3 and x_4 results in equal values, suggesting a roughly equivalent "trading" value between these two objectives. Conversely, the smaller "trading" value associated with x_5 implies a limitation in resources, which in turn reduces the system's flexibility.

Table-3 Weight Result of MILP Example.

Variables	x_1	x_2	x_3	x_4	x_5
Initial Weights	100	10	1	0.1	0.01
Compact Weights	1.7	10	6.9	6.9	3.41

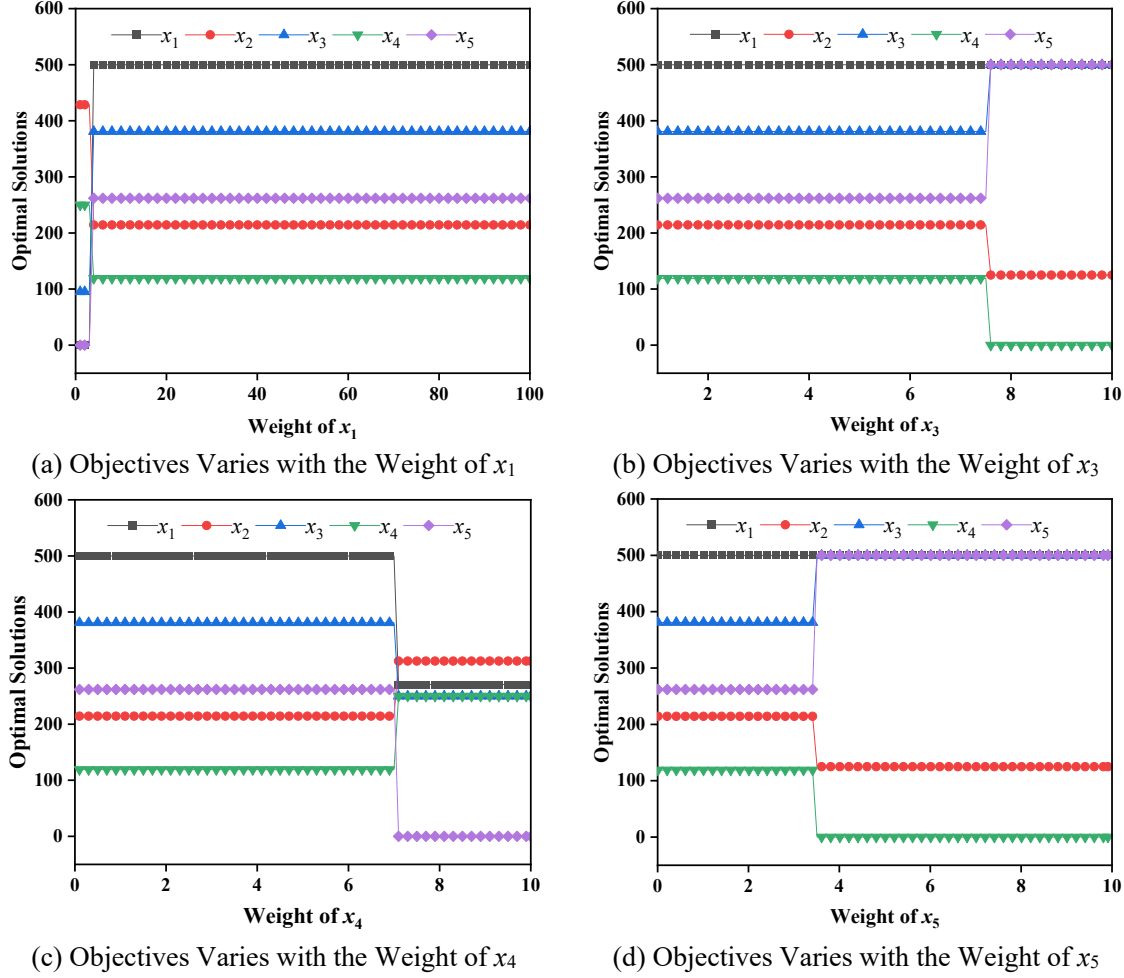
Similarly, as illustrated in Table-4, after optimizing the weights of individual sub-objectives within the objective function of the MILP example and substituting the optimized target weight vector back into the original planning problem, the optimal solution of the original problem remains unchanged. This result confirms that the shadow prices derived from the planning problem based on this weight vector continue to accurately represent the marginal values of market information under the system's optimal state. Furthermore, by eliminating "weight redundancy," the valuation of different resources becomes more equitable, allowing for a fairer assessment of their respective values.

Table-4 Optimal Solution Result of MILP Example.

Variables	x_1	x_2	x_3	x_4	x_5
Optimal Solution of The Original Problem	500	214	381	119	262
Optimal Solution under The Compact Weights	500	214	381	119	262

Fig.9 illustrates the process of eliminating "weight redundancy" between adjacent objectives during the first iteration when determining the compact weights in the LP example. This process depicts how the objectives evolve as their respective weights are adjusted, ensuring a prioritized consideration of all objectives without excessive weighting bias.

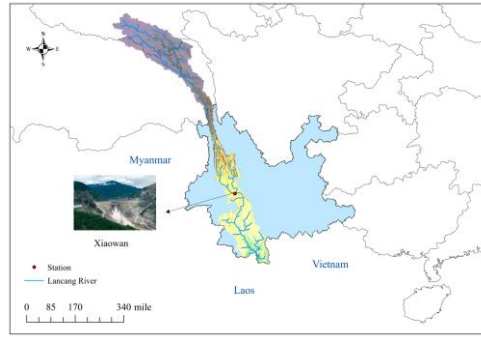
Fig.9: Optimal Objectives with Weight Changes During the First Iteration



Multifunctional water valuation in the Xiaowan reservoir

The engineering case uses the weekly cascaded-reservoir hydropower scheduling model with hourly power and capacity balances. The optimization was solved using Gurobi 10.0.1 on a computer equipped with an AMD Ryzen 7 3700X processor at 3.60 GHz. The Xiaowan hydropower station on the Lancang River was selected as the case study. The complete model formulation and data structure follow our scheduling model reported previously¹⁹, see **appendix A** for details. The case study is the Xiaowan hydropower station on the Lancang River in southwest China, with the geographical locations depicted in Fig.10. The model includes six hierarchical sub-objectives: total spillage, guaranteed energy yield, total energy produced, guaranteed spare capacity, storage volume and total spare capacity. Total energy produced is selected as the reference objective because it is sufficiently elastic to changes in the objective formulation and has directly quantifiable production value.

Fig.10: Geographic Locations of Studied reservoir.



The compact weights converge rapidly. The weight associated with total spillage decreases to zero by the second iteration, indicating that spillage does not provide additional marginal information under the studied operating conditions. The compact weight for guaranteed energy yield converges to 79.5, whereas the compact weights for guaranteed spare capacity, storage volume and total spare capacity converge to 1.1, 0.0039 and 0.87, respectively. The resulting compact vector is $[0, 79.5, 10, 1.1, 0.0039, 0.87]$.

Fig.11: Iterative Process of Sub-objective Weights in the Scheduling Model

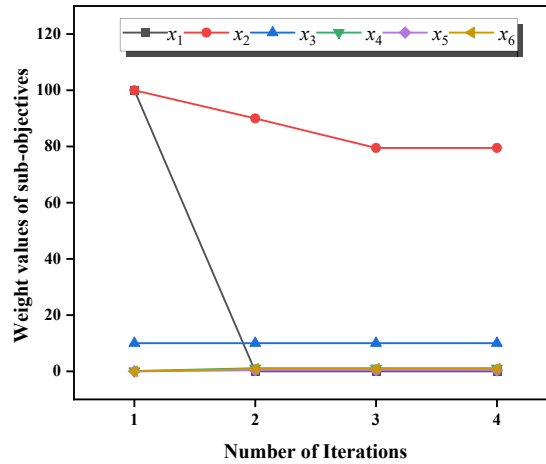


Table-5 Compact Weight Results for Each Sub-objective.

The i -th Sub-objective	x_1	x_2	x_3	x_4	x_5	x_6
	Total Spillage	Guaranteed Energy Yield	Total Energy Produced	Guaranteed Spare Capacity	Storage Volume	Total Spare Capacity
Initial Weight	-1000	100	10	0.01	0.0001	0.00001
Compact Weight	0	79.5	10	1.1	0.0039	0.87

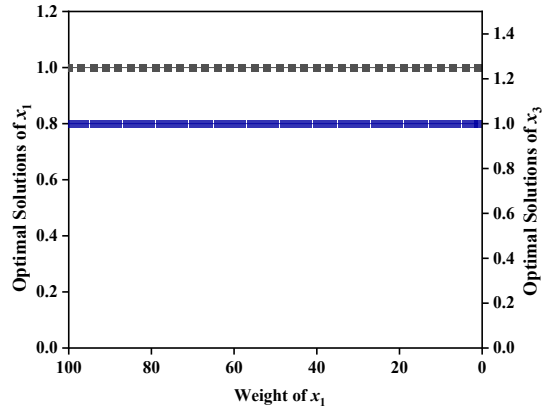
Replacing the initial weights with the compact vector leaves the optimal values of all six sub-objectives unchanged: total spillage remains 0; guaranteed energy yield remains 3.69 GWh; total energy production remains 18,212 GWh; guaranteed spare capacity remains 0 MW; storage volume remains 492,000 million m^3 as reported in the original model; and total spare capacity remains 2,433.57 MW. The unchanged solution demonstrates that compact-weight valuation is performed at the same system optimum rather than by changing the operating policy.

Table-6 Optimal Solutions of the Model Under Different Weight Vectors.

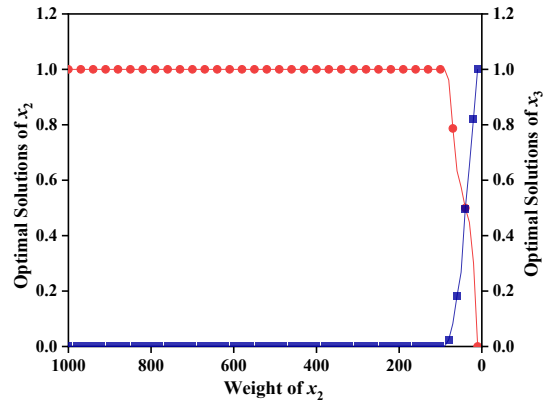
The i -th Sub-objective	Optimal Solutions with Initial Weight	Optimal Solutions with Compact Weight
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x_1	Total Spillage (million m ³)	0	0
x_2	Guaranteed Energy Yield (GWh)	3.69	3.69
x_3	Total Energy Produced (GWh)	18212	18212
x_4	Guaranteed Spare Capacity (MW)	0	0
x_5	Storage Volume (million m ³)	492000	492000
x_6	Total Spare Capacity (MW)	2433.57	2433.57

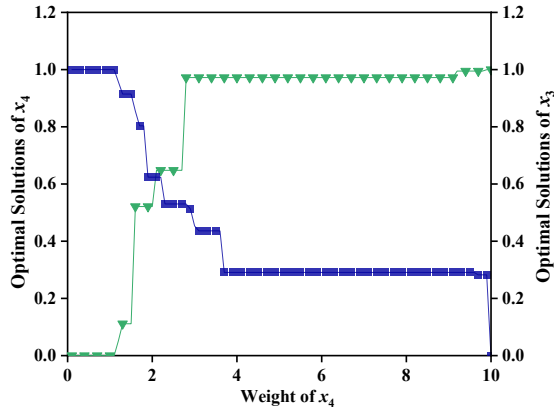
Fig.12: Optimal Objectives with Weight Changes During the First Iteration.



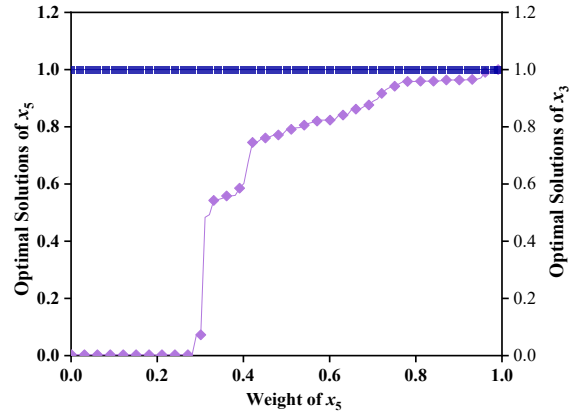
(a) Result of x_1



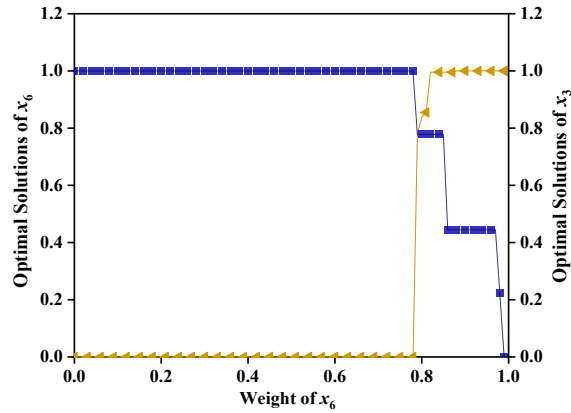
(b) Result of x_2



(c) Result of x_4



(d) Result of x_5

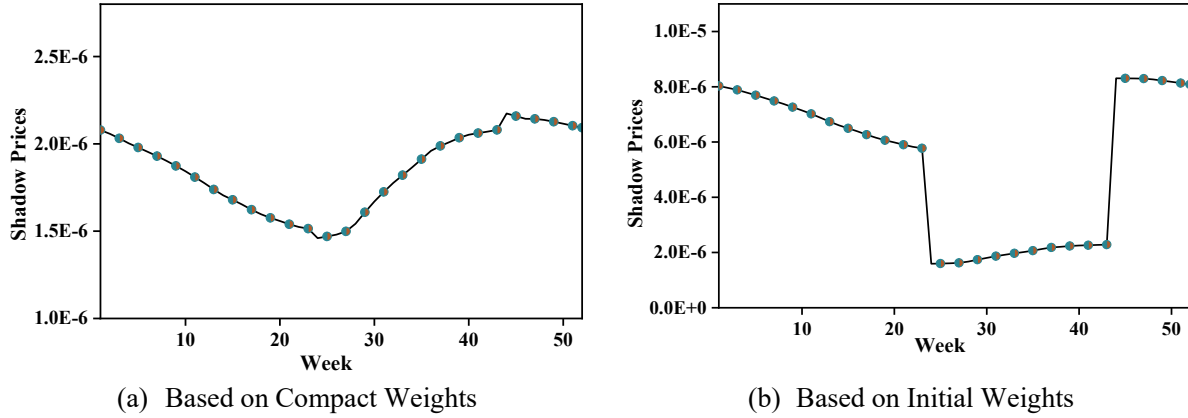


(e) Result of x_6

A common monetary scale reveals the seasonal value of reservoir water

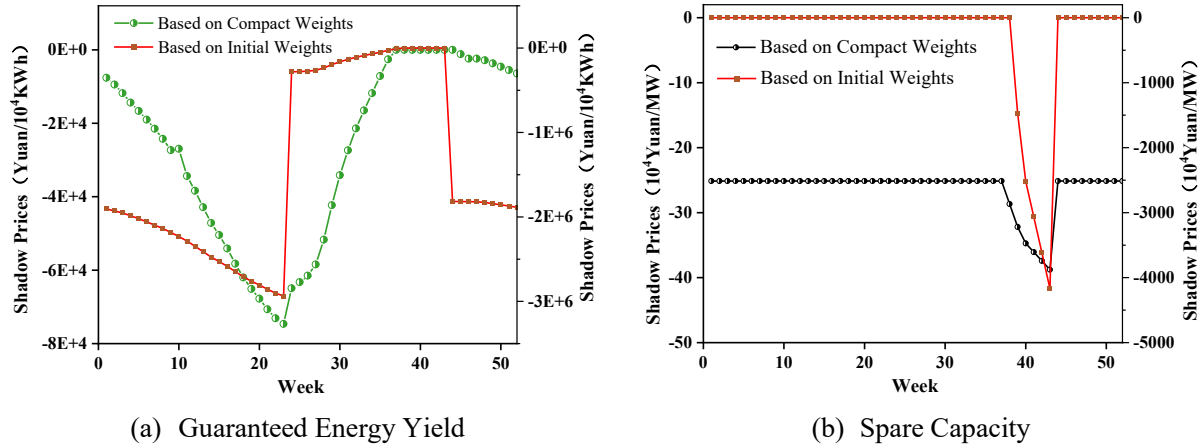
The principal application of the framework is the conversion of marginal values associated with different reservoir services into a common monetary scale. Under the initial weights, the shadow price of inflow varies strongly across seasons and between resource types. These variations partly arise from the numerical scale of the hierarchical weights and make direct comparison difficult. Under the compact weights, the shadow prices are substantially more commensurable while retaining physically meaningful seasonal patterns: inflow has greater marginal value in the dry season than in the flood season, when water is comparatively abundant.

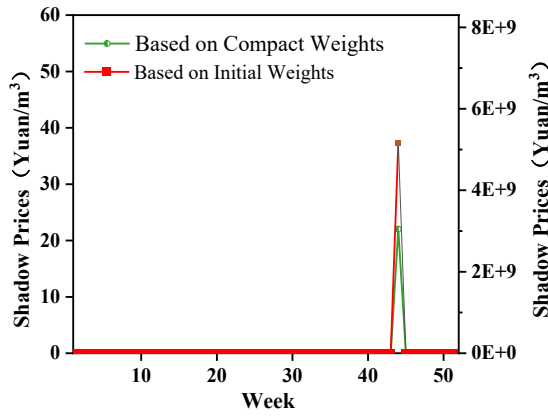
Fig.13: Shadow Prices of Inflow.



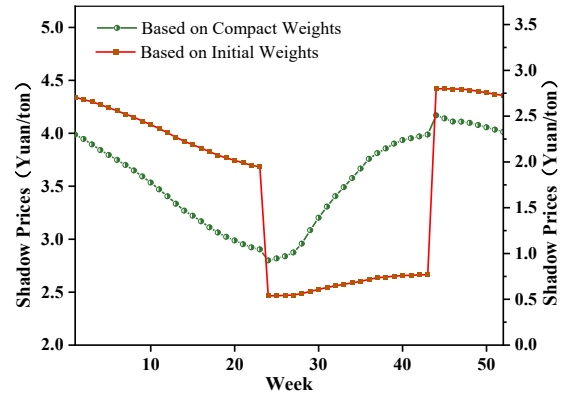
Using the reported water price of 2.8 yuan per tonne as the reference numeraire, the compact-weight shadow prices are converted into monetary values for the different services represented in the scheduling model. The resulting values show that the same physical water stock has different marginal values depending on the service it enables and the time at which it is available. Guaranteed outgoing energy has greater value in the middle of the year than at the beginning or end, the upper limit of storage has a pronounced value around week 44, and inflow is more valuable during the dry season than during the flood season.

Fig.14: Commensurated Shadow Prices.

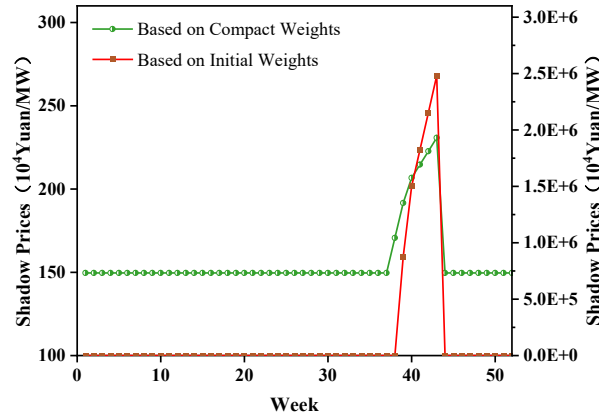




(c) Upper Limit of Storage



(d) Inflow



(e) Installed Capacity

The conventional formulation can create orders-of-magnitude valuation distortions

The contrast between the two weight formulations is particularly pronounced in week 44. Under the initial weights, the reported prices are 1,816,450 yuan per 10,000 kWh for guaranteed energy yield, 5.2 billion yuan per m³ for the upper limit of storage, 0.12 yuan per kW for spare capacity and 73.49 yuan per kW for installed capacity. These values span an extremely wide range and are not suitable as a common basis for comparing competing services.

After compact-weight commensuration, the corresponding reported values are 1,172 yuan per 10,000 kWh, 22.02 yuan per m³, 25.13 yuan per kW and 149.59 yuan per kW, respectively. Importantly, these changes occur without changing the optimal reservoir operating solution. The comparison therefore indicates that the extreme differences in the original shadow prices were not solely a consequence of physical scarcity or system operation; they were also induced by redundant objective-weight scaling.

This result is central to the proposed interpretation of reservoir water. The compact-weight shadow price is not intended to replace observed market prices or sector-specific tariffs. Instead, it provides a common marginal-value metric within the optimized reservoir system. This makes it possible to compare the opportunity cost of retaining, releasing or allocating water to different services without treating their original physical units as directly comparable.

Table-7 Comparison of Shadow Prices Under Two Scenarios.

Resources	Guaranteed Energy Yield	Upper Limit of Storage	Inflow (10 ⁴ ton)	Spare Capacity	Installed Capacity
-----------	-------------------------	------------------------	------------------------------	----------------	--------------------

	(10 ⁸ kWh)	(10 ⁴ m ³)		(10 ⁴ kW)	(10 ⁴ kW)
Prices Under Compact Weights/10 ⁴ Yuan	1172	22.02	4.17	25.13	149.59
Prices Under Initial Weights/10 ⁴ Yuan	1816450	5.2E9	6.11	0.12	73.49

Discussion

From reservoir volume to a multifunctional economic asset

The results suggest a broader interpretation of reservoir storage. A storage volume is physically measured in cubic metres, but its economic significance is generated by the set of services that the stored water can provide. The marginal value of storage is therefore state dependent. Water retained near a critical seasonal transition can have a different opportunity value from the same volume retained when inflows are abundant. Similarly, the value of inflow depends on the reservoir's ability to transform that inflow into future services.

This perspective moves reservoir valuation beyond a single water price. A single tariff may be appropriate for a specific transaction, but it cannot by itself describe the opportunity value of water when the same stock simultaneously supports several system objectives. A common numeraire is therefore useful not because it eliminates the diversity of water services, but because it makes that diversity quantitatively comparable.

Implications for integrated water allocation and markets

The framework provides a valuation basis rather than a complete market mechanism. The commensurated marginal values could be used as inputs to market clearing, bidding or compensation mechanisms in settings where water services are traded or jointly allocated. Such applications would additionally require institutional definitions of rights, participants, demand, supply, transaction rules and environmental safeguards, none of which are established by the present optimization framework.

The approach may nevertheless be useful for identifying the opportunity cost of alternative water allocations. For example, an additional unit of stored water can be compared with its marginal contribution to energy production, reserve provision or future scarcity mitigation. This can help decision-makers identify when water should be released for immediate benefits and when its option value as stored water is sufficiently high to justify retention.

Limitations and future directions

Several limitations should be considered. First, the case study focuses on a hydropower-oriented reservoir scheduling problem and therefore does not explicitly represent all possible water services, such as municipal supply, irrigation productivity, ecosystem services or flood-risk reduction. Second, the monetary conversion uses the reported water price as a reference numeraire; alternative reference prices should be tested in future work to assess robustness. Third, the compact-weight theory is demonstrated within the stated optimization assumptions, and extensions to strongly non-convex, stochastic or multi-agent market settings require further investigation. Finally, a unified valuation metric should not be interpreted as evidence that all services have identical social value: environmental and distributional constraints may impose values that are not captured by a purely operational objective.

Conclusions

This study shows that the value of reservoir water cannot be adequately represented by physical volume or by sector-specific prices alone when the same stored water simultaneously supports multiple services. Reservoir

storage is a multifunctional asset whose marginal value depends on the services that can be generated, the system state and the timing of water availability.

We developed a compact-weight commensuration framework to address a specific but fundamental obstacle to such valuation: redundant differences in objective-weight magnitude can distort shadow prices without changing the optimal operating solution. The framework preserves the hierarchy of objectives and the optimal solution while generating shadow prices that are more suitable for cross-service comparison. In the Xiaowan case study, conventional weights produced extreme differences among the reported marginal values of energy, storage and capacity resources. Compact weights reduced these artificial disparities while retaining the same optimal operating state. The resulting monetary values also revealed meaningful seasonal patterns, including higher inflow value during dry periods and distinct periods of high storage and energy value.

The main implication is therefore conceptual as well as methodological: a reservoir should be valued not only by how much water it stores, but by the portfolio of services that each unit of stored water can provide. Expressing these marginal contributions through a common economic metric provides a quantitative bridge between reservoir operation, integrated water allocation and potential cross-sectoral water-service markets.

Acknowledgements

Conflict of Interest Statement

The authors have no conflicts of interest to disclose.

Appendix

Appendix A

Proof for [Theorem 1]: Without loss of generality, consider the following multi-objective optimization problem:

$$\begin{cases} F = w_1 f_1(X) + w_2 f_2(X) + w_3 f_3(X) \\ X = [x_1, x_2, \dots, x_n]^T \\ x_i, \Delta \geq 0 \quad i \in [1, n] \end{cases} \quad (\text{A-1})$$

For any weight vector $[a, b, c]$:

$$\begin{cases} F_1 = a \cdot f_1(X) + (b + \Delta) \cdot f_2(X) + c \cdot f_3(X) \\ F_2 = a \cdot f_1(X) + b \cdot f_2(X) + c \cdot f_3(X) \end{cases} \quad (\text{A-2})$$

Assume Δ is a small positive number. Let $X_1 = [x_1^1, x_2^1, \dots, x_n^1]^T$ and $X_2 = [x_1^2, x_2^2, \dots, x_n^2]^T$ be the optimal solutions to maximize F_1 and F_2 , respectively. Then,

$$\begin{aligned} a \cdot f_1(X_1) + (b + \Delta) \cdot f_2(X_1) + c \cdot f_3(X_1) \\ \geq a \cdot f_1(X_2) + (b + \Delta) \cdot f_2(X_2) + c \cdot f_3(X_2) \end{aligned} \quad (\text{A-3})$$

Further,

$$a \cdot (f_1(X_1) - f_1(X_2)) + (b + \Delta) \cdot (f_2(X_1) - f_2(X_2)) + c \cdot (f_3(X_1) - f_3(X_2)) \geq 0 \quad (\text{A-4})$$

Similarly,

$$a^* (f_1(X_2) - f_1(X_1)) + b^* (f_2(X_2) - f_2(X_1)) + c^* (f_3(X_2) - f_3(X_1)) \geq 0 \quad (\text{A-5})$$

Adding the inequalities (1-4) and (1-5):

$$\Delta \cdot (f_2(X_1) - f_2(X_2)) \geq 0 \quad (\text{A-6})$$

This implies:

$$f_2(X_1) \geq f_2(X_2) \quad (\text{A-7})$$

Therefore, a decrease in the weight of f_2 leads to a decrease in $f_2(X)$, proving the monotonicity. The same logic can be applied when the weight increases to show that the sub-objective function value increases.

Proof complete.

Proof for [Theorem 2]: At the optimal solution of P_1 , it is evident that:

$$[\sum_{k|k \neq i} w_k \square f_k^{*[1]} + w_i^{[1]} \square f_i^{*[1]}] - [\sum_{k|k \neq i} w_k \square f_k^{*[2]} + w_i^{[1]} \square f_i^{*[2]}] \geq 0 \quad (\text{A-8})$$

This simplifies to:

$$\sum_{k|k \neq i} w_k \square (f_k^{*[1]} - f_k^{*[2]}) - w_i^{[1]} \square (f_i^{*[1]} - f_i^{*[2]}) \geq 0 \quad (\text{A-9})$$

Based on $f_i^{*[1]} = f_i^{*[2]}$, it can be inferred that:

$$\sum_{k|k \neq i} w_k \square (f_k^{*[1]} - f_k^{*[2]}) \geq 0 \quad (\text{A-10})$$

Similarly, by analogous reasoning:

$$\sum_{k|k \neq i} w_k \square (f_k^{*[2]} - f_k^{*[1]}) \geq 0 \quad (\text{A-11})$$

This implies:

$$\sum_{k|k \neq i} w_k \square (f_k^{*[1]} - f_k^{*[2]}) \leq 0 \quad (\text{A-12})$$

Thus,

$$\sum_{k|k \neq i} w_k \square (f_k^{*[1]} - f_k^{*[2]}) = 0 \quad (\text{A-13})$$

Given the equality of the objective functions, the optimal solutions of both problems coincide with the optimal solution of P_1 . According to the assumption of uniqueness of the optimal solution, it can be concluded that:

$$X^{*[1]} = X^{*[2]} \quad (\text{A-14})$$

Based on $\begin{cases} f_i^* = \text{optm}(X^*) \\ X^* = \text{optm}^{-1}(f_1^*, f_2^*, \dots) \end{cases}$, it can be inferred that:

$$f_i^{*[1]} = f_i^{*[2]} \quad (\forall k) \quad (\text{A-15})$$

Proof complete.

Appendix B

Symbol	Category	definition	Description
w	Subscript	week	Represents the week in the planning horizon.
i	Subscript	hydropower reservoir / hydropower station	Represents reservoir or hydropower station.
j	Subscript	unit	Represents generating unit of hydropower station.
k	Subscript	load level	Represents the load level used to determine the peak-shaving position of a hydropower station.
m	Subscript	load-rank index	Represents the m -th largest load period
h	Subscript	hour / time interval	Represents an hourly time interval.
t	Subscript	hour / time	Represents hour t during a typical day of week w .
P_i	Parameter	objective weight	Weights with different magnitudes introduced to prioritize the objectives sequentially.
spl_{iw}	Decision variable	spillage	Spillage from reservoir i in week w , measured in m^3/s .
g^+/g^-	Decision variable	guaranteed energy yield/guaranteed energy deficit	Energy yield guaranteed for transmission to external power systems /the guaranteed energy deficit at most for internal power systems in any week, measured in MWh.
E_{iw}	Decision variable	energy produced	Energy produced by hydropower station i in week w , measured in MWh.
e	Decision variable	guaranteed spare capacity	Spare capacity guaranteed by the system, measured in MW.
V_{iw}	Decision variable	storage volume	Storage volume of hydropower reservoir i at the beginning of week w , measured in million m^3 .
XS_{iw}	Decision variable	spare capacity	Spare capacity provided by hydropower station i in week w , measured in MW.
$\lambda_i^{\text{OE}}, \lambda_i^{\text{VE}}$	Parameter	energy-conversion coefficients	Coefficients convert a discharge during a week in m^3/s and a storage volume in million m^3 into energy production in MWh respectively.
$\bar{\eta}_i$	Parameter	average water rate	Average water rate of hydropower station i , measured in m^3/kWh ; used to estimate the energy-conversion coefficients.
I_{iw}	Parameter	local inflow	Local inflow into reservoir i in week w , measured in m^3/s .
Q_{iw}	Decision variable	release	Release from reservoir i in week w , measured in m^3/s .
q_{iw}	Decision variable	generating discharge	Generating discharge of hydropower station i in week w , measured in m^3/s .
$\Omega(i)$	Set	upstream reservoirs	Set of reservoirs directly upstream of reservoir i .
$V_{iw}^{\min} / V_{iw}^{\max}$	Parameter	storage lower and upper bounds	Lower and upper bounds on storage of reservoir i , measured in million m^3 .
$Q_{iw}^{\min} / Q_{iw}^{\max}$	Parameter	release lower and upper bounds	Lower and upper bounds on release from reservoir i , measured in m^3/s .
$N_{iw}^{\min} / N_{iw}^{\max}$	Parameter	power lower and upper bounds	Lower and upper bounds on power of hydropower station i , measured in MW.
$GH^{\max}(\cdot)$	Variable / function	water head	Water head used to determine generating-discharge capacity, measured in meters.
$Z^u(\cdot)$	Function	forebay water level	Forebay water level related to reservoir storage volume.
$Z^d(\cdot)$	Function	tailwater level	Tailwater level related to reservoir release.
$\eta(\cdot)$	Parameter / function	generating efficiency	Generating efficiency related to water head; converts generating discharge into power output.
C_i	Parameter	hydropower-station capacity	Capacity of hydropower station i .
XM_{iw}	Decision variable	maintenance capacity	Capacity assigned to maintenance of the week w .
XR_{iw}	Decision variable	reserved capacity	Capacity reserved for system requirements of the week w .
XD_{iw}	Decision variable	disabled capacity	Capacity partly disabled of the week w , including capacity restricted by lower water heads.

XW_{iw}	Decision variable	working capacity	Capacity available for normal operation/peak shaving of the week w .
B_w	Parameter	minimum reserved capacity	Minimum capacity reserved for handling unexpected demands during each day of week w .
x_{ijw}	Binary variable	maintenance-completion indicator	Indicates whether unit j of hydropower station i completes maintenance in week $w-1$, used to ensure maintenance continuity.
M_{ij}	Parameter	maintenance duration	Number of weeks required to maintain unit j of hydropower station i .
C_{ij}	Parameter	installed capacity	Installed capacity of unit j of hydropower station i , measured in MW.
Y_{wt}	Parameter	internal power demand	Power demand from internal power systems in hour t during a typical day in week w .
$E_{iw,h(m)}$	Decision variable	power generation at the m -th largest load period	Power generation of hydropower station i during the m -th largest load period in week w .
$D_{iw,h(m)}$	Parameter	hydropower-station load	Load associated with hydropower station i during the m -th largest load period in week w , as defined in the source.
$Dbase_w$	Variable	base load	Base load in week w .
D_{wt}	Parameter	power load	Power load in hour t during the typical day of week w .
S_{wt}	Parameter	solar power	Solar power in hour t during the typical day of week w .
W_{wt}	Parameter	wind power	Wind power in hour t during the typical day of week w .
HS_{wt}	Parameter	small-hydropower generation	Hydropower generation from small hydropower plants in hour t during the typical day of week w .
C	Constant	sufficiently large constant	A sufficiently large constant used to construct the hourly power-load curve and allow energy production beyond internal demand for transmission to external power systems.
N	Parameter	number of hydropower reservoirs / plants	Number of hydropower reservoirs/plants.
V_{ikw}	Binary variable	load-level assignment	Indicates whether hydropower station i is scheduled at load level k to start peak shaving in week w .
$\underline{y}_{ikw} / \bar{y}_{ikw}$	Auxiliary variables	working-zone division variables	Variables representing the divisions of the lower and upper bounds of the working zone of hydropower station i into load levels k and $k+1$, as described in the source.
$f_w(x)$	Function	area under the load curve	Area below load level x of the typical hourly load curve in week w ; used to represent energy.
$h(m)$	Mapping/notation	time corresponding to load rank m	Maps the m -th largest load to its corresponding hourly time interval; $h(1)$ is the absolute peak hour.

Multiple objectives mathematically expressed as:

$$\max[-P_1 \sum_{i,w} (\lambda_i^{QE} \cdot spl_{iw}) + P_2 \cdot (g^+ + g^-) + P_3 \sum_{i,w} E_{iw} + P_4 \cdot (7 \times 24) \cdot e + P_5 \sum_{i,w} (\lambda_i^{VE} \cdot V_{iw}) + P_6 \sum_{i,w} (7 \times 24) \cdot XS_{iw}] \quad (B-1)$$

$$\lambda_i^{QE} = \frac{3.6 \times 7 \times 24}{\bar{\eta}_i} \quad (B-2)$$

$$\lambda_i^{VE} = \frac{1000}{\bar{\eta}_i} \quad (B-3)$$

Constraints include:

$$V_{i,w+1} = V_{iw} + (I_{iw} - Q_{iw} + \sum_{r \in \Omega(i)} Q_{rw}) \times \frac{7 \times 24 \times 3600}{10^6} \quad (B-4)$$

$$q_{iw} + spl_{iw} = Q_{iw} \quad (B-5)$$

$$V_{iw}^{\min} \leq V_{iw} \leq V_{iw}^{\max} \quad (B-6)$$

$$Q_{iw}^{\min} \leq Q_{iw} \leq Q_{iw}^{\max} \quad (B-7)$$

$$N_{iw}^{\min} \leq N_{iw} \leq N_{iw}^{\max} \quad (B-8)$$

$$q_{iw} \leq GH^{\max}(h_{iw}) \quad (B-9)$$

$$h_{iw}=Z^u(\frac{V_{iw}+V_{i,w+1}}{2})-Z^d(Q_{iw}) \quad (B-10)$$

$$E_{iw}=N_{iw}\times (7\times 24) \quad (B-11)$$

$$N_{iw}=\eta(h_{iw})\cdot q_{iw} \quad (B-12)$$

$$XM_{iw}+XR_{iw}+XD_{iw}+XW_{iw}+XS_{iw}=C_i \quad (B-13)$$

$$\sum_i XR_{iw} \geq B_w \quad (B-14)$$

$$\sum_w s_{ijw} = 1 \quad (B-15)$$

$$\sum_j \sum_{t=w+1}^{w+M_j} (s_{ijt} \cdot C_{ij}) = XM_{iw} \quad (B-16)$$

$$XD_{iw}=(C_i-XM_{iw})\cdot f(h_{iw}) \quad (B-17)$$

$$\sum_i XS_{iw} \geq e \quad (B-18)$$

$$E_{iw,h(1)}=XW_{iw}\cdot 1hr \quad (B-19)$$

$$E_{iw,h(m)}\geq E_{iw,h(m+1)} \quad (B-20)$$

$$\sum_i E_{iw,h(m)}=(D_{w,h(m)}-Dbase_w)\cdot 1hr \quad (B-21)$$

$$\sum_i E_{iw}-7\times \sum_h Y_{wh}\geq (g^+-g^-) \quad (B-22)$$

$$x_{0,w}=0\leq x_{1,w}\leq \ldots \leq x_{N,w}=D_w \quad (B-23)$$

$$D_w=\max_t\{D_{wt}\} \quad (B-24)$$

$$D_{wt}=Y_{wt}+C \quad (B-25)$$

$$Y_{wt}=YN_{wt}-(S_{wt}+W_{wt}+HS_{wt}) \quad (B-26)$$

$$\sum_k v_{ikw}=1 \quad (B-27)$$

$$\sum_i v_{ikw}=1 \quad (B-28)$$

$$XW_{iw}=\sum_k(\bar{y}_{ikw}-\underline{y}_{ikw}) \quad (B-29)$$

$$\bar{y}_{i,k,w}=\begin{cases} x_{k+1,w} & (v_{ikw}=1) \\ 0 & \text{otherwise} \end{cases} \quad (B-30)$$

$$\underline{y}_{i,k,w}=\begin{cases} x_{k,w} & (v_{ikw}=1) \\ 0 & \text{otherwise} \end{cases} \quad (B-31)$$

$$\frac{E_{iw}}{7}=f_w(\sum_k \bar{y}_{ikw})-f_w(\sum_k \underline{y}_{ikw}) \quad (B-32)$$

$$\sum_m \bar{y}_{imw}=\bar{y}_{ikw}=x_{k+1,w} \quad (B-33)$$

$$\sum_m \underline{y}_{imw}=\underline{y}_{ikw}=x_{k,w} \quad (B-34)$$

If there are any questions about this model, you can refer to the detailed explanations of each formula in the previous work¹⁹.

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