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Comparative optimization of multi-objective algorithm for mixing dynamics and thermal protection on exhaust pipe

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Abstract

The temperature of the exhaust pipe wall and exhaust gas is the main source of generating infrared signals, which increases the exposure risk of military bases. To reduce the infrared signals, this study investigated the effects of the outer diameter of the inlet, temperature difference, and mass ratio on the uniformity index of temperature mixture and the increase in the maximum temperature of the pipe wall. Subsequently, based on non-dominated sorting and crowding distance, the sparrow search algorithm, particle swarm optimization, whale optimization algorithm, and grey wolf optimizer were rewritten as multi-objective algorithms. These multi-objective algorithms were compared in terms of their advantages and disadvantages on a newly proposed multi-objective test problem. The optimal algorithm was selected for the optimization of heat protection of the exhaust pipe. The research results on the thermal protection law indicate that the velocity ratio between the hot fluid and the cold fluid is the main factor controlling the mixing efficiency. When the velocity ratio is close to 1, the air flow entrainment effect is the weakest, resulting in the poorest temperature and mass mixture uniformity, and the maximum increase in the pipe wall temperature. By reducing the area of the cold fluid inlet or increasing the mass flow ratio, the deviation between the speed ratio and 1 can be significantly reduced, thereby effectively enhancing the air flow entrainment diffusion and improving the mixture uniformity. On the contrary, an increase in the temperature difference will enhance the buoyancy lift of the hot fluid, thereby worsening the mixing

uniformity in the vertical direction. Subsequently, polynomial fitting was performed on the temperature mixture uniformity index, CO₂ mixture uniformity index, and the increase in the pipe wall temperature. Finally, based on these relationships, a multi-objective optimization was carried out to achieve the best mixture uniformity and minimize the increase in the pipe wall temperature. When the mass ratio $MR=5$, the outer radius $R=100$ mm, and the temperature difference $\Delta T=40$ K, the overall insulation effect is the best. Compared with extreme conditions, this scheme can maximize the temperature uniformity by 94.34% and reduce the increase in the pipe wall temperature by 80.39%.

Keywords: Non-isothermal coaxial jets; Mixing index; Active and passive scalar; Film cooling; Exhaust thermal insulation

1. Introduction

When diesel engines are used for military power generation, the hot exhaust gas and the heated exhaust pipes will emit infrared signals, which is not conducive to concealment [1, 2]. By adopting certain cooling methods and reducing the temperature, the infrared signals can be reduced and the concealment ability can be enhanced [3, 4]. Therefore, in this study, a stream of cold air is introduced at the inlet of the exhaust pipe to mix with the hot exhaust gas, thereby reducing the heating effect on the exhaust pipe.

From a fluid dynamics perspective, this specific cooling strategy is physically realized through the mechanism of coaxial jetting. This phenomenon is not unique to exhaust systems but is a fundamental process prevalent in numerous industrial applications, including combustor, nuclear reactors, and film cooling [5-7]. Understanding the mixing and heat transfer characteristics when hot and cold fluids converge within enclosed channels or vessels is of paramount importance for engineering applications [8, 9]. For instance, mixing components in nuclear reactors are highly susceptible to thermal fatigue cracking induced by significant temperature fluctuations, which poses a threat to the reliable operation of the facility [10, 11]. Consequently, a comprehensive understanding of the flow characteristics and heat transfer processes associated with the mixing of hot and cold fluids is essential for establishing design guidelines to ensure engineering reliability.

Among various injection designs, coaxial injection has garnered significant attention as an efficient method for mixing hot and cold fluids due to its superior geometric configuration [12, 13]. In a coaxial arrangement, two fluid streams are discharged along the same axis in the form of a central jet and an

annular jet, where the velocity difference between the inner and outer flows generates complex shear and entrainment characteristics [14, 15]. Furthermore, the dominant parameters influencing the process vary across different flow stages, resulting in diverse flow structures in the near-field and fully mixed regions, a subject that has attracted extensive research interest [16, 17]. Dahm et al. [18] conducted experiments and numerical simulations to study the near-field flow characteristics of coaxial circular jets discharged into a static environment. They used laser-induced fluorescence technology to record the flow visualization. The results showed that the morphologies of the near-field vortices and their interaction dynamics exhibited rich diversity, which depended not only on the velocity ratio but also on the absolute velocity of the jet. During the flow process, different vortex structures would also be generated in the two fluids, which was caused by the instability of the coaxial flow layer. Moreover, the flow structure was also significantly influenced by the thickness of the vortex layer and the flow wake. Ko and Kwan [19] measured the average velocities and turbulence intensities of the coaxial jet under different average velocity ratios by using pressure measurements, hot-wire and microphone spectra. They found that the relationship between velocity and turbulence intensity was similar in both the initial mixing zone and the fully mixed zone, and was consistent with the results of a single jet. Villermaux and Rehab [20] conducted a systematic study on the generation of interfaces under conditions of large outer-to-inner velocity ratios, the scale-dependent fractal characteristics, and the evolution laws of component compositions after the mixing transition, using the planar laser-induced fluorescence technique.

However, when jets transport fluids at different temperatures, the density differences resulting from thermal gradients alter the hydrodynamic stability of the shear layer, thereby complicating the hydrodynamic characteristics of coaxial jets [21]. Kok et al. [22] employed a combination of experimental measurements and Large Eddy Simulation to systematically analyze the thermal mixing behavior of hot and cold fluid coaxial jets. The results indicated that thermal mixing efficiency improves with an increasing temperature difference between the hot and cold jets. Furthermore, increasing the hot jet flow rate in the first half of the channel enhances mixing performance, whereas a mass flow ratio of 2 yields the optimal mixing effect in the latter half of the channel. The study also identified a dominant frequency of 5 Hz for temperature fluctuations, which aligns closely with findings in existing literature regarding thermal mixing [23, 24]. Zhou et al. [25] utilized large eddy simulation to reveal the formation mechanisms of temperature fluctuations during the mixing of hot and cold fluids. Their results demonstrated that the oscillation of the cold fluid center, the expansion and contraction of its control region, and changes in the

region's shape contribute to flow field instability. Srivastava and Patel [26] investigated coaxial swirling air jets in gas turbine combustors, examining the effects of heating the central jet and placing bluff bodies at various locations on the mixing process. Varol et al. [27] experimentally studied the thermal mixing of dual jets at different temperatures within a narrow channel, specifically investigating the influence of channel inclination angle. Comparative results showed that the highest thermal mixing efficiency within the channel was achieved at an inclination angle of 60° . Pasa et al. [28] experimentally examined the effects of the frequency ratio and mass flux ratio of inner and outer actuators on the flow and heat transfer characteristics of coaxial jets. The results indicated that under conditions of lower frequency ratio and lower mass flux ratio, the peak centerline velocity of the jet increased by approximately 27% compared to the baseline case, while the peak Nusselt number increased by about 135.18%. Liu et al. [29] captured flow structures using two-dimensional particle image velocimetry and reconstructed the downstream three-dimensional velocity field via linear interpolation and velocity filling methods. Their study revealed that momentum exchange between the stages of a triple swirler forms a unified clockwise rotating jet, with the downstream flow exhibiting a three-layer zonal structure consisting of an annular recirculation zone, a high-speed swirling shear layer, and a central recirculation zone.

When relying on coaxial jet mixing mechanisms for cooling in thermal protection applications, it is essential to accurately predict the heat transfer characteristics of the mixing process while also focusing on the impact of the mixed jet on the surrounding solids [30, 31]. Research indicates that the characteristics of the mixing of hot and cold fluids have a significant impact on the temperature of surrounding walls. The literature [32, 33] all mentioned that during the mixing of hot and cold fluids, the temperature of the solid wall experienced fluctuations. This phenomenon has been identified as being caused by thermal striping resulting from insufficient mixing of hot and cold fluids [34]. Therefore, when using cold fluids for thermal protection of exhaust pipes, it is extremely necessary to ensure that the cold and hot fluids are mixed uniformly, which also reduces the infrared signals generated by the exhaust gas. However, when the cold flow volume is insufficient, the mixed flow will heat the pipe wall surface. This causes the pipe wall surface to become the main emitter of infrared signals. Inspired by film cooling, utilizing the cold fluid as a thermal barrier film represents another viable protection strategy [35, 36]. The effectiveness of this approach depends primarily on the relative velocity between the hot and cold fluids. Wang et al. [37] investigated the influence of different cold flow rates on the temperature of the pipe wall after the coaxial mixing of hot fluid and cold fluid. The results showed that as cold flow rate gradually increased, the

thermal insulation film formed by the cold fluid became increasingly difficult to be breached, and the temperature of the pipe wall showed a monotonically decreasing trend. In research on flow of cold-hot coaxial jets within a tube, Li et al. [38] also took into account the burden on the flow resistance caused by increasing the flow rate, and determined the optimal flow rate that provided the optimal cooling and pump consumption through multi-objective optimization.

Based on the aforementioned flow and heat transfer characteristics of coaxial jets, the application of coaxial jet mixing for thermal protection in exhaust ducts is highly appropriate. When there is a significant speed difference between the cold fluid and the hot exhaust gas, rapid mixing occurs to cool the hot exhaust gas. This reduces the heat radiation signal released by the discharged hot exhaust gas at outlet of exhaust. However, excessively increasing the speed difference to enhance the diffusion of hot smoke gas within the pipeline will increase the heating of the pipeline wall by the hot exhaust gas. Moreover, adjusting the speed difference by changing the inlet area and flow rate of cold fluid will affect the interaction between the hot smoke gas and the cold fluid, thereby influencing the heating effect on the pipeline wall. This will also increase the infrared signals released by the pipeline. Therefore, it is necessary to reveal the laws governing the influence of adjusting the inlet outer radius, the mass ratio, and the temperature difference on the mixing uniformity and temperature rise of the pipe wall. Finally, guidelines are established for the overall minimum infrared radiation signal. To gain a more comprehensive understanding of the coaxial jet mixing, this study not only considered the influence of the three factors on the active temperature transfer, but also took into account the influence on the passive mass transfer.

2. Model description

2.1. Physical and numerical model

This study investigates coaxial jetting of hot and cold fluids within a pipe. As illustrated in **Figure 1**, the pipe under investigation has a length of 14 m and a radius of 300 mm. The hot and cold fluids enter the pipe inlet in a concentric configuration, with the hot fluid entering through the center and the cold fluid entering through the outer annulus surrounding the hot fluid. The physical properties of the fluids are presented in **Table 1**. To reveal the influence of different velocity ratios on the coaxial jet within the pipe, the study varied the inlet outer radius of cold fluid while maintaining a consistent mass flow rate for both fluids. Consequently, the inlet areas of the hot and cold fluids changed, generating diverse velocity ratios. The research examines not only the heat transfer mechanisms during the coaxial jet process but also the transport characteristics of CO₂. Species in hot fluid at initial inlet is detailed in **Table 2**. To simulate this

model, a numerical model depicted in **Figure 2** was constructed. Both the hot and cold fluids were assigned velocity inlets, with the corresponding velocities calculated based on the ratio of flow rate to inlet area. The pipe outlet was defined as a pressure outlet. Natural convective heat transfer between the pipe wall and the surrounding air was considered. Mesh refinement was applied at the interface between two different zones. The study primarily focuses on non-isothermal coaxial jets, considering the effects of the inlet outer radius R , the mass flow ratio MR , and the temperature difference ΔT . The research variables and their corresponding ranges are presented in **Table 3**. Herein, the inlet outer radius of cold fluid R is shown in **Figure 1**, the mass flow ratio is defined as $MR=Q_c/Q_h$, and the temperature difference is defined as $\Delta T=T_h-T_c$.

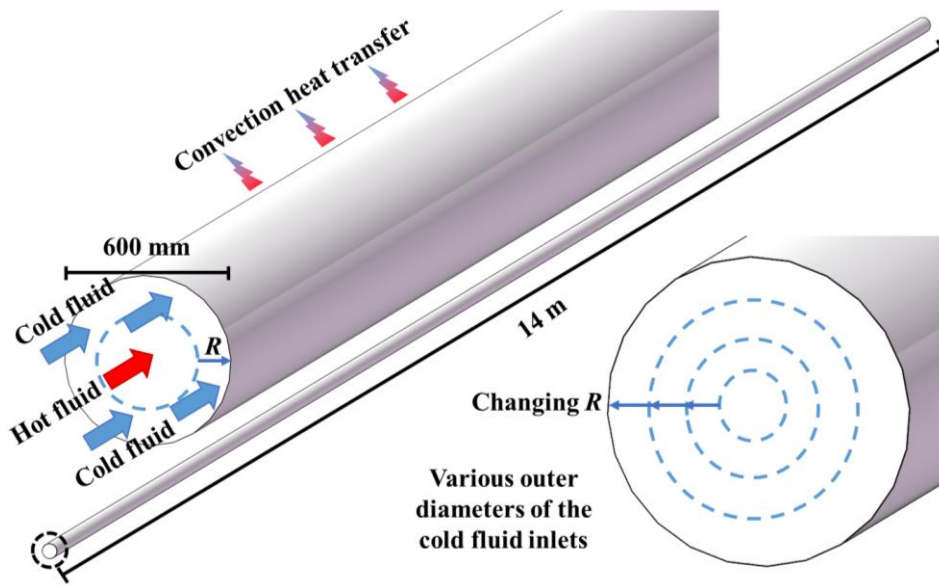


Figure 1. Physical overall figure of research subject.

Table 1. Physical properties of iron and air.

Materials	Density ($\text{kg}\cdot\text{m}^{-3}$)	Conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)	Specific heat capacity ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)	Viscosity ($\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$)
Iron	8030	16	502	
Air	1.342	0.0236	1009	0.0000167

Table 2. Species of hot fluid.

Species	H ₂ O	O ₂	N ₂	CO ₂
Proportion [%]	3.5	12.0	74.0	10.5

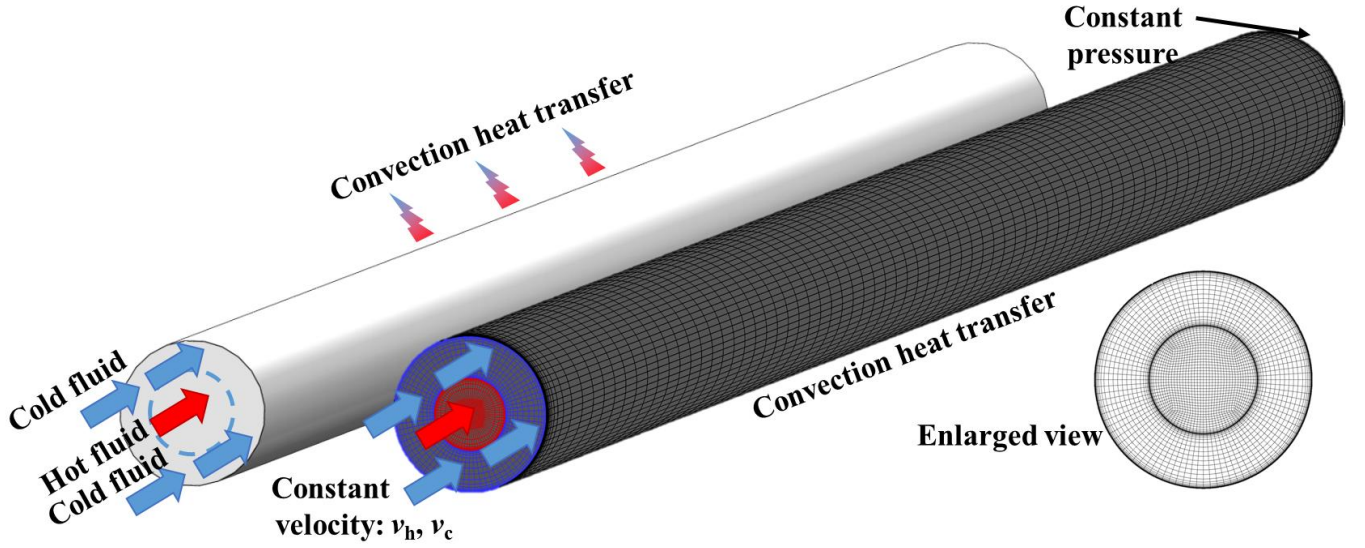


Figure 2. Display of established numerical model.

Table 3. Research parameters and corresponding variation ranges.

R [mm]	ΔT [K]	MR
100	40	1
120	50	2
140	60	3
160	70	4
180	80	5
200		

2.2. Governing equations

The process of coaxial jet flow of non-isothermal fluids within a tube involves the exchange of momentum and heat. This mainly follows the laws of conservation of mass, conservation of momentum, and conservation of energy.

Mass conservation equation [39]:

$$\frac{1}{r} \frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0 \quad (1)$$

Momentum conservation equation [39]:

r direction:

$$\begin{aligned} u_r \frac{\partial u_r}{\partial r} + u_\theta \frac{1}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = \\ -\frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{\mu}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right] + \rho g \beta (T - T_0)_r \end{aligned} \quad (2)$$

θ direction:

$$\begin{aligned}
& u_r \frac{\partial u_\theta}{\partial r} + u_\theta \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = \\
& -\frac{1}{\rho} \frac{1}{r} \frac{\partial P}{\partial \theta} + \frac{\mu}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} - \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} \right] + \rho g \beta (T - T_0)_\theta
\end{aligned} \tag{3}$$

z direction:

$$\begin{aligned}
& u_r \frac{\partial u_z}{\partial r} + u_\theta \frac{1}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = \\
& -\frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{\mu}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] + \rho g \beta (T - T_0)_z
\end{aligned} \tag{4}$$

where β is thermal expansion coefficient, μ represents the dynamic viscosity, P denotes the pressure on this cell.

Energy conservation equation [40]:

$$\begin{aligned}
& u_r \frac{\partial T}{\partial r} + u_\theta \frac{1}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} = \\
& \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\lambda \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) - p \left[\frac{1}{r} \frac{\partial (r u_\theta)}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \right]
\end{aligned} \tag{5}$$

where λ denotes fluid thermal conductivity.

The heat fluid in the study is composed of CO₂ and other components. Therefore, during the non-isothermal transfer process of hot and cold fluids, there will be a process of mass transfer [41].

$$\frac{u_\theta}{r u_{av}} \frac{\partial Y_i}{\partial \theta} + U \frac{\partial Y_i}{\partial z} = \frac{u_{av} R}{\alpha} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial Y_i}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 Y_i}{\partial \theta^2} + \frac{\partial^2 Y_i}{\partial z^2} \right] \tag{6}$$

where Y_i denotes the mass fraction of component i and α is the diffusion coefficient.

2.3. Solution setting

The research focuses more on the heat and mass diffusion results when the non-isothermal fluid mixing jet reaches stability. Therefore, the calculation adopts a steady-state approach. The calculations were carried out using the pressure-based solution method for the equations of conservation of mass, momentum and energy. The flow conditions involved in the study were all in turbulent state, so the turbulence model was selected for calculation. Calculate and simultaneously set up multi-component transport models to capture the transfer process of CO₂ in hot and cold fluids. During the calculation process, the convergence accuracies of the mass, momentum and energy conservation equations were set at 10^{-6} , 10^{-6} and 10^{-12} respectively.

2.4. Data processing

This study focuses on the coaxial jet of non-isothermal fluids, mainly concentrating on the uniformity of the mixture and the temperature rise on pipe wall. Therefore, uniformity of outlet cross-sectional temperature, uniformity of CO₂ mixture, and temperature rise on wall were set. Temperature uniformity index MI_T is calculated using the method described in the literature [42, 43],

$$MI_T = \frac{\sqrt{\left[\sum_{i=1}^n (T_i - T_{ave})^2 / (n-1) \right]}}{T_h - T_c} \quad (7)$$

where, subscript h and c present the hot and cold fluid. Following the concept of temperature uniformity index, this study proposes the mass diffusion uniformity index MI_c ,

$$MI_c = \frac{\sqrt{\left[\sum_{i=1}^n (c_i - c_{ave})^2 / (n-1) \right]}}{c_h - c_c} \quad (8)$$

In addition to studying the uniformity index of coaxial jet mixing, the study also considered the thermal protection index. The maximum temperature rise (TR) is proposed to be,

$$TR = T_{max} - T_c \quad (9)$$

where, T_{max} means the maximum temperature on the pipe wall, and T_c denotes the cold fluid temperature.

3. Model validation

3.1. Independence validation

For a pipeline that is 14 meters long, the division of the grid has a significant impact on the calculation accuracy and the consumption of computing resources. Therefore, before conducting the batch calculation, we carried out a verification of the grid independence. Three different grid sizes, namely 3124464, 4685312 and 6048024, were respectively used to conduct the independence verification. Using three different grids, the mixing index of temperature and CO₂ concentration at the outlet cross-section were calculated under the working conditions of MR=3, $\Delta T=50K$, and outer radius $R=160mm$. **Figure 3(a)** and **(b)** presents the calculation results. As can be seen from the figure, using a grid size of 4685312 yields better calculation results and saves a significant amount of computing resources.

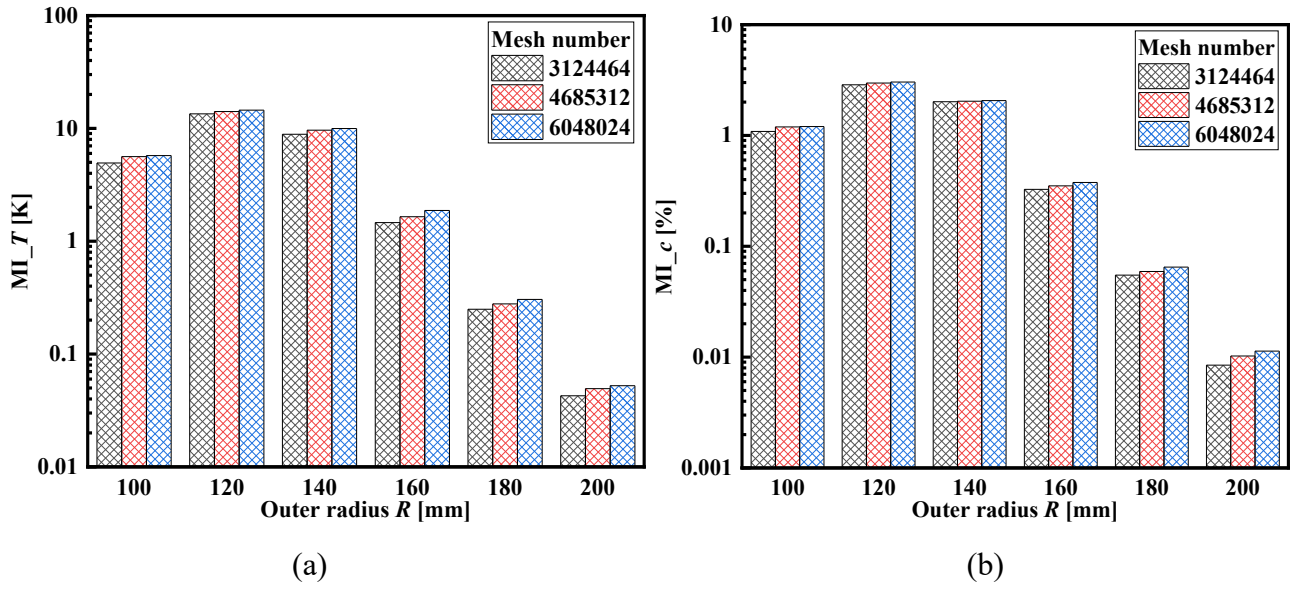


Figure 3. Comparisons of mixing indices of temperature and CO₂ under different mesh numbers.

3.2. Accuracy validation

To validate the accuracy of the established model, this study selected an experimental article addressing a similar problem for comparative calculation [44]. In the literature, an experimental setup for non-isothermal fluid coaxial jet mixing within a length of 1 m tube was established, and temperature measurement points were installed on the upper pipe wall to capture the temperature variation patterns. To verify the accuracy of the computational model, the established model was employed to calculate the influence of the hot and cold fluid mixing jet on the pipe wall temperature within this 1 m long pipe, and the results are presented in **Figure 4**. As can be observed from the figure, the pipe wall temperature rise calculated by the established model can accurately capture the experimentally measured pipe wall temperature variation, thereby validating the accuracy of the established model. During the verification process, the RNG $k-\epsilon$ and SST $k-\omega$ models are compared. The results show that RNG $k-\epsilon$ has higher computational accuracy, which is due to its excellent performance in simulating the turbulent kinetic energy of the jet core [45, 46].

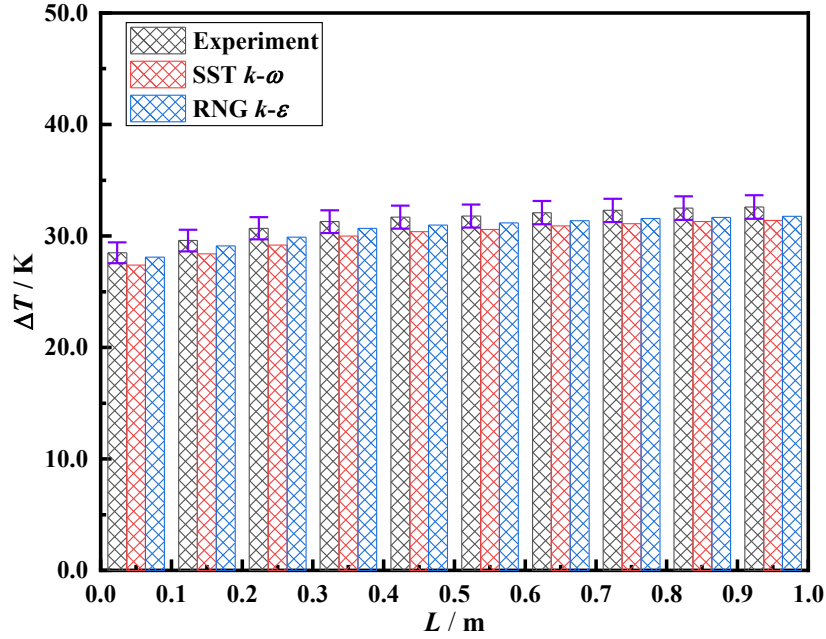


Figure 4. Validation of established numerical model against experimental results [44].

4. Comparison of multi-objective algorithms

This research requires multi-objective optimization on the exhaust temperature and the wall temperature, to obtain the combination with the minimum values for both. This process relies on an excellent multi-objective optimization algorithm, which must be capable of solving a wide range of pareto sets and having high accuracy. Therefore, a variety of multi-objective optimization algorithms were selected for testing, with the aim of obtaining the optimal multi-objective optimization algorithm. The multi-objective sparrow search algorithm (MOSSA), multi-objective particle swarm optimization (MOPSO), multi-objective whale optimization algorithm (MOWOA), multi-objective grey wolf optimizer (MOGWO), non-dominated sorting genetic algorithm II (NSGA-II) are selected for test.

4.1. Multi-objective sparrow search algorithm

The multi-objective sparrow search algorithm is a swarm intelligence optimization algorithm developed based on the standard sparrow search algorithm. This algorithm mimics the hunting behavior of the sparrow and conducts iterative optimization through producers, followers, and scouters [47].

The update for producers is calculated follows

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t \cdot \exp\left(-\frac{i}{\alpha \cdot iter_{\max}}\right) & \text{when } R_2 < ST \\ X_{i,j}^t + Q \cdot L & \text{when } R_2 > ST \end{cases} \quad (10)$$

The update for followers is calculated follows

$$X_{i,j}^{t+1} = \begin{cases} Q \cdot \exp\left(\frac{X_{worst} - X_{i,j}^t}{i^2}\right) & \text{when } i > n/2 \\ X_p^{t+1} + |X_{i,j}^t - X_p^{t+1}| \cdot A^+ \cdot L & \text{otherwise} \end{cases} \quad (11)$$

The update for scouts is calculated follows

$$X_{i,j}^{t+1} = \begin{cases} X_{best}^t + \beta \cdot |X_{i,j}^t - X_{best}^t| & \text{when } f_i > f_g \\ X_{i,j}^t + K \left(\frac{|X_{i,j}^t - X_{worst}^t|}{(f_i - f_w) + \varepsilon} \right) & \text{when } f_i = f_g \end{cases} \quad (12)$$

4.2. Multi-objective particle swarm optimization

The multi-objective particle swarm optimization algorithm was proposed based on the standard particle swarm algorithm. The standard particle swarm optimization algorithm updates each particle by itself using its own search inertia, its own search results, and the search results of the entire swarm of particles. The update rules for the velocity and position of each particle are as follows [48]:

$$\begin{aligned} v_{id} &= w \cdot v_{id} + c_1 \cdot rand() \cdot (p_{id} - x_{id}) + c_2 \cdot Rand() \cdot (p_{gd} - x_{id}) \\ s_{id} &= s_{id} + v_{id} \cdot n \end{aligned} \quad (13)$$

4.3. Multi-objective whale optimization algorithm

The whale optimization algorithm imitates the unique bubble-net feeding strategy of humpback whales during their hunting process to conduct optimization calculations. Through the synergy of three behaviors, encircling prey, bubble-net attack, and search for prey is accomplished [49]. The corresponding calculation formula is presented as follows:

Encircling prey is calculated as

$$\begin{aligned} \vec{D} &= |\vec{C} \cdot \vec{X}^*(t) - \vec{X}(t)| \\ \vec{X}(t+1) &= \vec{X}^*(t) - \vec{A} \cdot \vec{D} \end{aligned} \quad (14)$$

Bubble-net attack is calculated as

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} & \text{if } p < 0.5 \\ \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) & \text{if } p \geq 0.5 \end{cases} \quad (15)$$

Search for prey is calculated as

$$\begin{aligned} \vec{D} &= |\vec{C} \cdot \vec{X}_{rand} - \vec{X}| \\ \vec{X}(t+1) &= \vec{X}_{rand} - \vec{A} \cdot \vec{D} \end{aligned} \quad (16)$$

4.4. Multi-objective grey wolf optimizer

The design of the grey wolf optimizer is inspired by the strict social hierarchy and group cooperative hunting behavior of the wolf population. This algorithm combines the entire optimization solution through the division of ranks and three stages of hunting. During the rank division process, the wolf pack is divided into four groups: alpha (α), beta (β), delta (δ), and omega (ω). Subsequently, the wolves in each group hunt in accordance with the steps of encircling prey, hunting, attacking prey, and searching for prey [50]. Encircling prey is calculated as

$$\begin{aligned}\vec{D} &= \left| \vec{C} \cdot \vec{X}^*(t) - \vec{X}(t) \right| \\ \vec{X}(t+1) &= \vec{X}^*(t) - \vec{A} \cdot \vec{D}\end{aligned}\quad (17)$$

Hunting is calculated as

$$\begin{aligned}\vec{D}_\alpha &= \left| \vec{C}_1 \cdot \vec{X}_\alpha - \vec{X} \right|, \vec{D}_\beta = \left| \vec{C}_2 \cdot \vec{X}_\beta - \vec{X} \right|, \vec{D}_\delta = \left| \vec{C}_3 \cdot \vec{X}_\delta - \vec{X} \right| \\ \vec{X}_1 &= \vec{X}_\alpha - \vec{A}_1 \cdot (\vec{D}_\alpha), \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta), \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta) \\ \vec{X}(t+1) &= \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}\end{aligned}\quad (18)$$

4.5. Non-dominated sorting genetic algorithm II

The Non-Dominant Sorting Genetic Algorithm II is an iterative optimization method for the solution set based on genetic algorithms. The iterative process includes genetic operations, offspring generation, merging of parent and offspring generations, and selection and retention. The Pareto solution set is solved through fast non-dominated sorting and crowding distance calculation. This algorithm adopts an elite retention strategy for the iterative screening of individuals. In each generation, the parent and offspring populations are merged, and the best N individuals are selected to form the next generation.

Non-dominated sorting:

$$\forall i \in \{1, 2, \dots, n\}, f_i(A) \leq f_i(B) \text{ and } \exists j \in \{1, 2, \dots, n\}, f_j(A) < f_j(B) \quad (19)$$

Crowding distance calculation:

$$d_i = \sum_{m=1}^M \left(\frac{f_{i+1}^m - f_{i-1}^m}{f_{\max}^m - f_{\min}^m} \right) \quad (20)$$

4.6. Evaluation of optimization

To evaluate the solved Pareto solution sets by different multi-objective algorithms, this study selects eight multi-objective optimization problems mentioned in the literature [51] for testing. These eight multi-

objective problems are presented in **Table 4**. Among them, MOP1 are bi-objective optimization problems, while MOP2 to MOP4 are tri-objective optimization problems.

To quantitatively evaluate the Pareto solution sets obtained by different algorithms, the inverted generational distance (IGD), select spacing (SP), and maximum spread (MS) were introduced to assess the three dimensions of convergence, uniformity, and coverage breadth [52]. IGD calculates the average of the nearest Euclidean distances from each reference point on the true Pareto frontier to the algorithm's solution set, simultaneously measuring the convergence accuracy and distribution integrity of the solution set in approaching the true frontier [53].

$$IGD(PF, PF^*) = \frac{\sqrt{\sum_{i=1}^{|PF^*|} d_i^2}}{|PF^*|}, \quad d_i = \min_{X_j \in PF} \|F(X_i^*) - F(X_j)\|, \quad X_i^* \in PF^* \quad (21)$$

The smaller the value is, the more it indicates that the solution set is close to the actual frontier and there are no obvious missing areas.

SP measures the standard deviation of the spacing between adjacent solutions to specifically evaluate the uniformity of the distribution of the solution set in the target space [54].

$$SP(PF) = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (\bar{d} - d_i)^2}, \quad d_i = \min_{X_j \in PF, X_j \neq X_i} \left(\sum_{r=1}^g |f_r(X_i) - f_r(X_j)| \right), \quad X_i \in PF \quad (22)$$

The smaller the value is, the more evenly the solution is distributed along the front.

MS calculates the diagonal length or range of the hypercube formed by the extreme points of each target dimension in the solution set to reflect the coverage and expansiveness of the solution set in the target space [52].

$$MS(PF, PF^*) = \sqrt{\frac{1}{g} \sum_{r=1}^g \delta_r^2}, \quad (23)$$

$$\delta_r = \frac{\min_{X_i \in PF} \left(\max_{X_i \in PF} f_r(X_i) - \max_{X_j^* \in PF^*} f_r(X_j^*) \right) - \max_{X_i \in PF} \left(\min_{X_i \in PF} f_r(X_i) - \min_{X_j^* \in PF^*} f_r(X_j^*) \right)}{\max_{X_i^* \in PF^*} f_r(X_i^*) - \min_{X_j^* \in PF^*} f_r(X_j^*)}$$

The larger the value is, the more extensive the frontier areas that the algorithm can explore.

Table 4. The definitions of eight multi-objective problems.

Problem	Expression
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	$X = [x_1, \dots, x_{n_1}, \dots, x_{n_1+n_2}] \in [0, 1]^{n_1+n_2}, n_1 = n_2 = 5,$ $a_1 = 0.05, a_2 = 0.05, a_3 = 10,$
Common	$y_1 = \left(\frac{1}{n_1} \sum_{i=1}^{n_1} x_i \right)^{a_1}, y_2 = \left(\frac{1}{[n_1/2]} \sum_{i=1}^{[n_1/2]} x_i \right)^{a_2},$ $y_3 = \left(\frac{1}{[n_1/2]} \sum_{i=[n_1/2]+1}^{n_1} x_i \right)^{a_3}, g = \sum_{i=n_1+1}^{n_1+n_2} (x_i - 0.5)^2$
MOP1	$f_1 = g + \cos^{0.5} \left(\frac{\pi}{2} y_1 \right), f_2 = g + \sin^{0.5} \left(\frac{\pi}{2} y_1 \right)$
MOP2	$f_1 = (g+1)y_1, f_2 = (1+g) \left(y_1 + \frac{1}{10} \sin(10\pi y_1) \right),$ $f_3 = (g+1)(1-y_1)$
MOP3	$r_1 = \max \{ 0, \min \{ \sin^2(3\pi y_2), \sin^2(3\pi y_3) \} - 0.05 \}$ $f_1 = (1+g)y_2 + r_1, f_2 = (1+g)y_3 + r_1,$ $f_3 = (0.5+g)(2-y_2-y_3) + r_1$
MOP4	$f_1 = y_2, f_2 = y_3, f_3 = (1+g) \left[3 - \sum_{i=2}^3 \frac{y_i (1 + \sin(19\pi y_i))}{1+g} \right]$

4.7. Display of improvement

Tables 5-7 quantitatively compare the Pareto solution sets obtained by different multi-objective optimization algorithms for 4 multi-objective problems through IGD, SP and MS. IGD reflects the convergence of multi-objective algorithms in solving the Pareto solution set. As can be seen from **Table 5**, MOSSA shows the best IGD index in all 4 multi-objective optimization problems. SP reflects the uniformity of the multi-objective algorithm in solving the Pareto solution set. The calculation results in **Table 6** show that the MOSSA algorithm has the best SP index. This proves that the MOSSA has the best uniformity for the Pareto solution set. The MS index reflects the coverage of the value range of the solution set. From the comparison of MS in **Table 7**, among all the algorithms, MOSSA has the largest coverage ratio for calculated Pareto solution set. **Figure 5** plots the distribution of the Pareto solution sets obtained by different multi-objective optimization algorithms for 4 multi-objective problems compared to the real Pareto solution set. From the comparison in **Figure 5**, it can also be seen that MOSSA outperforms other algorithms in solving the Pareto solution set. This is reflected in the optimal and widest coverage range of the Pareto solution set calculated by MOSSA, and the high degree of fitting with the analytical solution.

Table 5. Comparison of IGD for Pareto solution sets of 4 multi-objective problems.

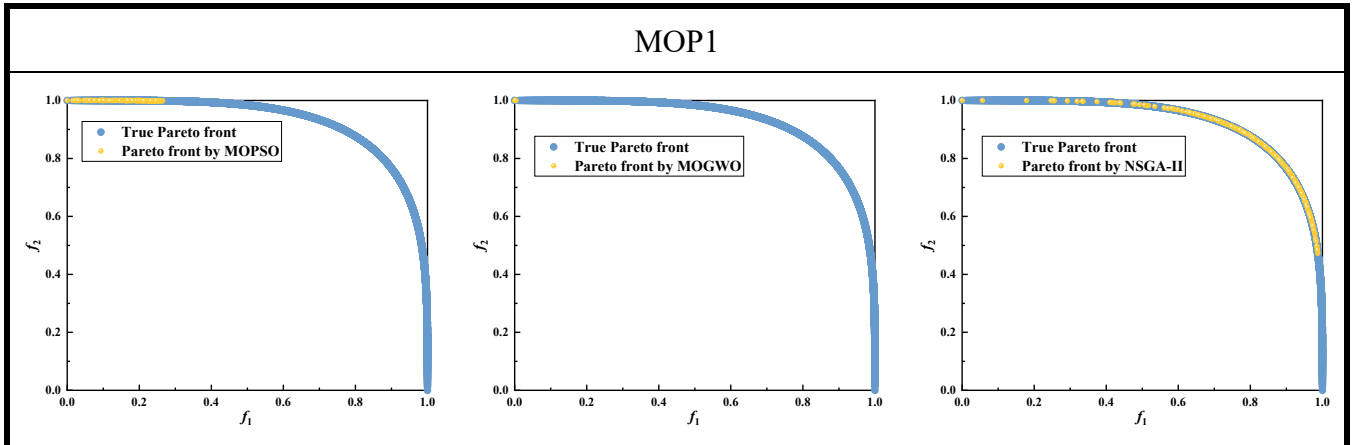
	MOSSA	MOWOA	NSGA	MOPSO	MOGWO
MOP1	2.66×10^{-4}	3.78×10^{-3}	7.38×10^{-4}	3.20×10^{-3}	3.98×10^{-3}
MOP2	4.86×10^{-4}	1.02×10^{-3}	7.50×10^{-4}	3.90×10^{-3}	8.65×10^{-4}
MOP3	9.03×10^{-4}	3.78×10^{-3}	8.82×10^{-4}	3.02×10^{-3}	5.14×10^{-4}
MOP4	1.51×10^{-3}	3.76×10^{-3}	1.88×10^{-3}	3.51×10^{-3}	2.77×10^{-3}

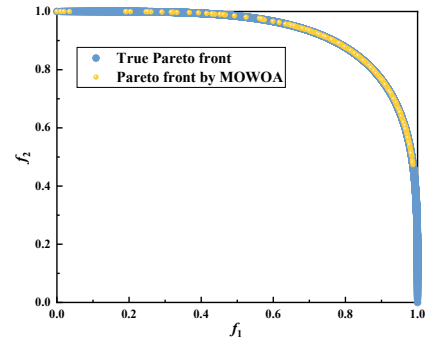
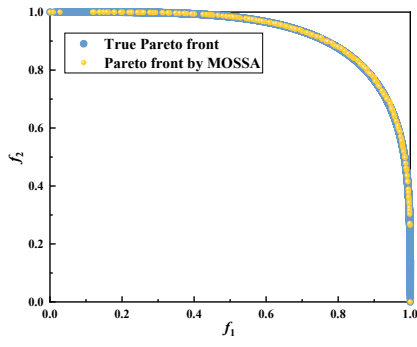
Table 6. Comparison of SP for Pareto solution sets of 4 multi-objective problems.

	MOSSA	MOWOA	NSGA	MOPSO	MOGWO
MOP1	4.43×10^{-2}	2.79×10^{-1}	7.92×10^{-2}	2.39×10^{-1}	2.92×10^{-1}
MOP2	7.19×10^{-2}	1.23×10^{-1}	7.63×10^{-2}	4.42×10^{-1}	9.18×10^{-2}
MOP3	2.53×10^{-1}	2.67×10^{-1}	2.78×10^{-1}	2.58×10^{-1}	2.94×10^{-1}
MOP4	6.46×10^{-1}	6.83×10^{-1}	8.77×10^{-1}	6.87×10^{-1}	6.67×10^{-1}

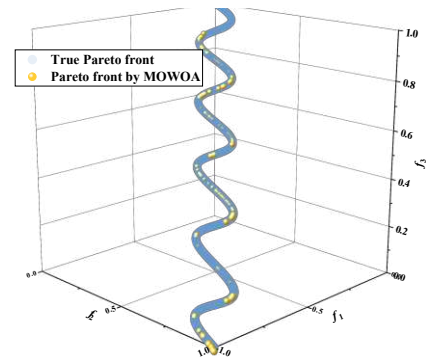
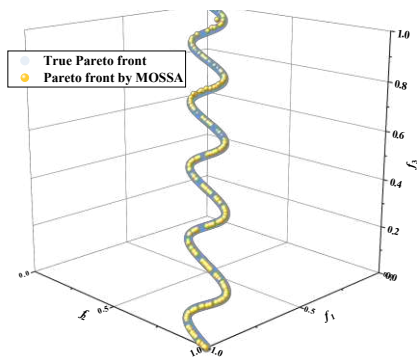
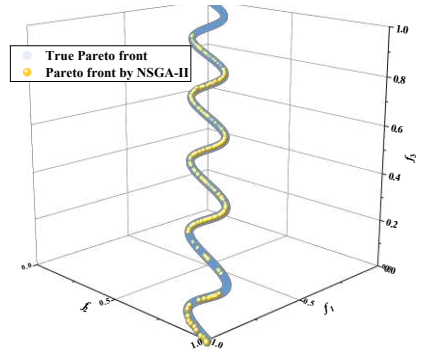
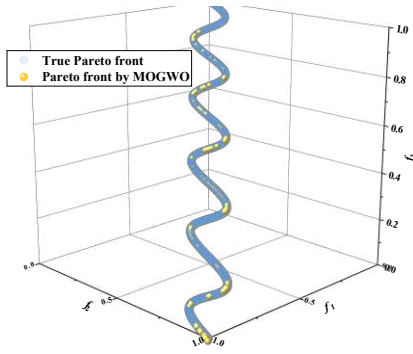
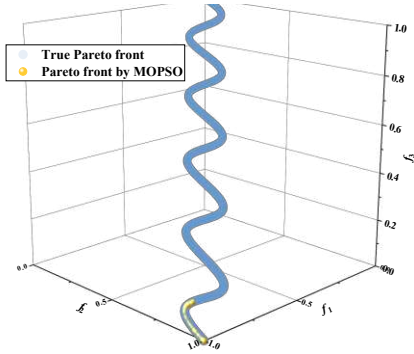
Table 7. Comparison of MS for Pareto solution sets of 4 multi-objective problems.

	MOSSA	MOWOA	NSGA	MOPSO	MOGWO
MOP1	0.92	0.76	0.79	0.19	0.00
MOP2	1.00	0.91	0.90	0.11	0.92
MOP3	0.96	0.78	0.91	0.76	0.82
MOP4	0.93	0.63	0.61	0.91	0.60

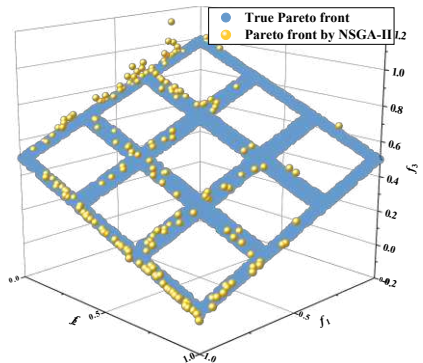
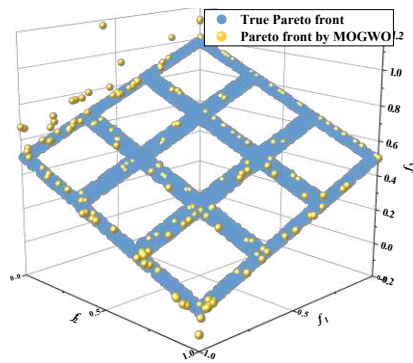
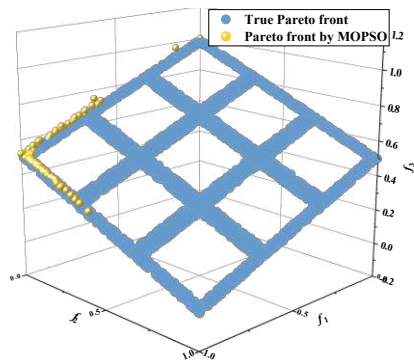




MOP2



MOP3



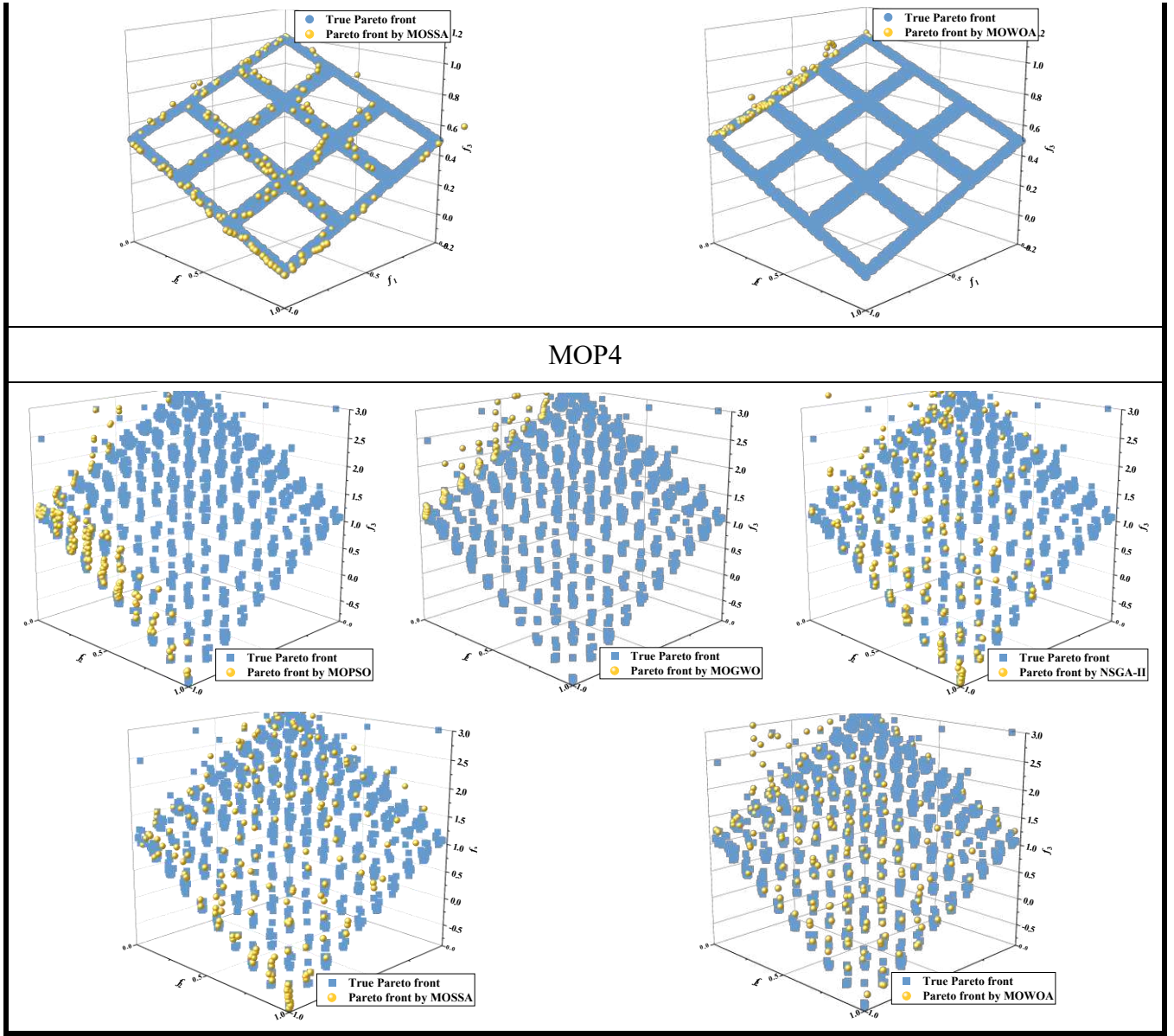


Figure 5. Comparison of the pareto solutions by different multi-objective optimization algorithm for different benchmark tests.

5. Results and discussions

5.1. Influence of inlet area ratio and mass ratio on jet mixing

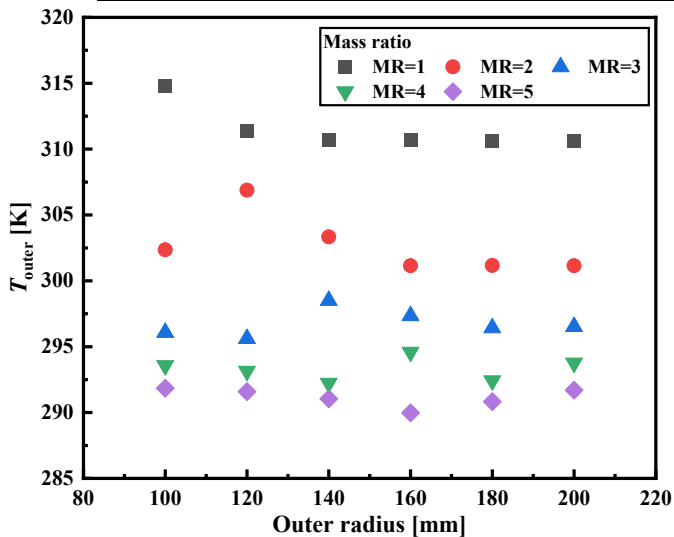
When two fluids undergo coaxial jetting within a pipe, their heat and mass transfer processes are significantly influenced by the velocity ratio. This study focuses on the impact of varying inlet outer radius under identical mass flow rates, which essentially reflects the influence of the velocity ratio between the two streams. The velocity ratios corresponding to different inlet outer radii are presented in **Table 8**. **Figure 6** compares the heat and mass transfer results for the two fluids across different inlet outer radii, while also contrasting the effects of varying mass flow ratios. **Figures 6(a)** and **5(b)** represent the outer layer temperature at the outlet and the outlet mixing index at steady-state, respectively. A general

comparison of **Figure 6(a)** reveals that for the same inlet outer radius, a higher mass ratio of cold to hot fluid results in a lower temperature of the outer layer fluid at the outlet. This occurs because a higher proportion of cold fluid significantly dilutes the heat transferred from the hot fluid, thereby reducing the temperature of the cold fluid in the outer layer at the outlet. However, the influence of different inlet outer radii on the outer layer temperature varies distinctly across different mass ratios. When the mass ratio is 1, the outer layer temperature is highest at an outer radius of 100 mm and gradually decreases as the inlet outer radius increases. When the mass ratio is 2, the highest outer layer temperature occurs at an outer radius of 120 mm. As the mass ratio continues to increase, the inlet outer radius corresponding to the highest outer layer temperature also increases. For mass ratios of 3 and 4, the highest points appear at outer radius of 140 mm and 160 mm, respectively. Observing the conditions for mass ratios of 3 and 4 reveals that prior to the highest point in outer layer temperature, the temperature decreases as the inlet outer radius increases, whereas it increases with the outer radius after the highest point. When the mass ratio is 5, no highest point in outer layer temperature is observed during the variation of inlet outer radius, although the descending trend to the left of the theoretical peak and the ascending trend to the right are both observed. This indicates that increasing the outer radius in 20 mm increments did not capture the exact point of maximum outer layer temperature. Regarding the aforementioned outer layer temperatures generated by different mass ratios and outer radii, a combination with the velocity ratios in **Table 8** reveals that the peak outer layer temperature consistently occurs when the ratio of cold to hot fluid velocity is closest to unity. This is because when the velocity ratio between the inner and outer layers is either significantly large or small, the high-velocity outer cold fluid or inner hot fluid entrains the other stream during flow, causing the two fluids to mix more rapidly. **Figure 6(b)** displays the temperature mixing uniformity index at the outlet cross-section. A larger index indicates poorer mixing uniformity between the two streams. The figure shows that at an outer radius of 100 mm, the temperature mixing uniformity index gradually decreases as MR increases. Because increased cold fluid flow improves diffusion mixing. This is primarily because as the cold fluid flow increases, the deviation of the cold-to-hot fluid velocity ratio from unity grows larger, generating stronger entrainment that causes hot fluid to diffuse more rapidly. When the outer radius is changed to 120 mm, 140 mm, and 160 mm, the temperature mixing uniformity index initially increases and subsequently decreases as MR increases. Because reduced hot fluid inlet area increases the hot fluid velocity, causing the diffusion effect to first weaken and then improve. When outer radius increases to 180 mm and 200 mm, the hot fluid velocity becomes quite high. During the gradual

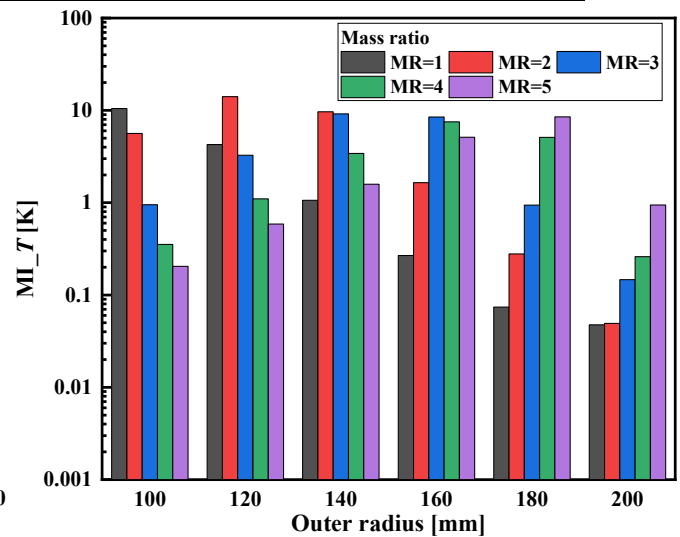
increase of MR, cold fluid velocity approaches hot fluid velocity, which weakens diffusion phenomenon. Consequently, increasing MR results in poorer temperature mixing uniformity. **Figures 6(c) and 6(d)** present the comparison results of the outer layer CO₂ fraction at the outlet and the outlet cross-sectional CO₂ mixing uniformity under different inlet outer radii and mass ratios. The figures indicate that under different hot-to-cold fluid ratios, the patterns of mass transfer in non-isothermal coaxial jets are very similar to those of heat transfer, and the CO₂ mixing uniformity parameters at the outlet cross-section closely resemble the temperature mixing uniformity parameters. This suggests that in the study of varying velocity ratios, the influencing mechanisms for heat transfer and mass transfer are consistent.

Table 8. Velocities and velocity ratios of the cold and hot fluids under different outer radii of the inlet and different quality ratios.

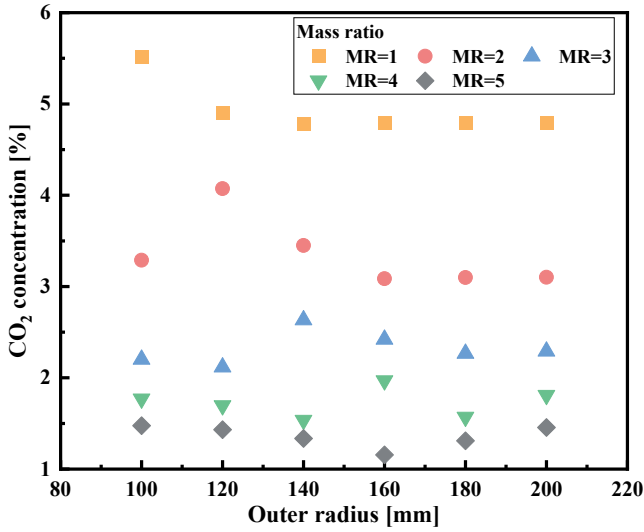
MR		Outer radius / mm					
		100	120	140	160	180	200
v_h		6.63	8.19	10.36	13.53	18.42	26.53
v_c	1	5.31	4.61	4.12	3.77	3.51	3.32
	2	10.61	9.21	8.24	7.54	7.02	6.63
	3	15.92	13.82	12.36	11.30	10.53	9.95
	4	21.22	18.42	16.48	15.07	14.03	13.26
	5	26.53	23.03	20.59	18.84	17.54	16.58
v_{ratio}	1	0.80	0.56	0.40	0.28	0.19	0.13
	2	1.60	1.13	0.80	0.56	0.38	0.25
	3	2.40	1.69	1.19	0.84	0.57	0.38
	4	3.20	2.25	1.59	1.11	0.76	0.50
	5	4.00	2.81	1.99	1.39	0.95	0.63



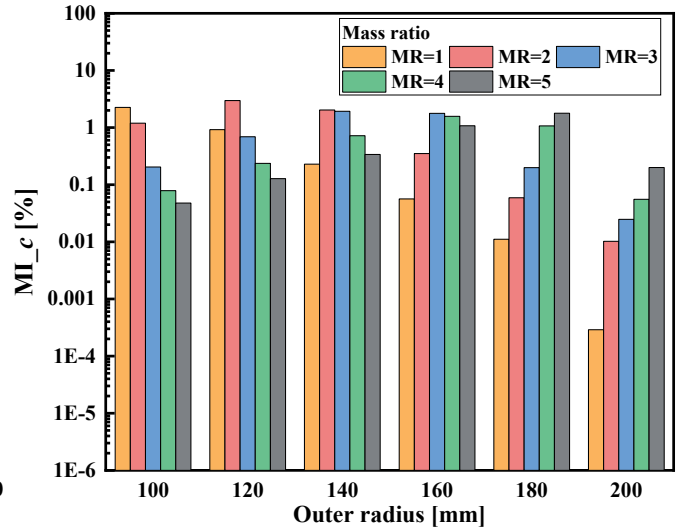
(a)



(b)



(c)



(d)

Figure 6. Influence of MR on heat and mass transfer under different outer radii for: (a) Outer temperature, (b) MI of temperature, (c) Outer CO₂, (d) MI of CO₂.

Figure 7 illustrates the distributions of temperature, CO₂ concentration, and velocity corresponding to an increase in the cold fluid outer radii from 100 mm to 200 mm. The conditions correspond to a mass flow ratio of 3 and a temperature difference of 50 K between the hot and cold fluids. The visualization provided by the contours effectively confirms the influence of varying flow ratios and outer radii on the temperature and CO₂ mixing uniformity at the outlet cross-section. As observed in the temperature and velocity contours in Figures (a) and (b), when the cold fluid outer radius is 100 mm, the cold fluid velocity is high while the hot fluid velocity is low. This relatively small velocity difference causes strong entrainment of hot fluid by cold fluid during propagation, thereby accelerating the diffusion of the hot fluid throughout the pipe. As the cold fluid inlet radius increases, its velocity decreases, which reduces the entrainment of the hot fluid. Consequently, the contours show that the hot fluid jet core gradually extends as the cold fluid channel width increases. When the cold fluid inlet width reaches 140 mm and 160 mm, the hot fluid jet core can penetrate the entire length of the pipe. Upon further increasing the cold fluid inlet width to 180 mm and 200 mm, the progressively decreasing hot fluid inlet area results in an increase in hot fluid velocity. This enhances the entrainment effect of the hot fluid on the cold fluid, with the intensity of entrainment strengthening as the hot fluid velocity rises. Therefore, at a width of 200 mm, the preserved length of the hot fluid core is shorter. Observing the CO₂ distributions in Figure (c) reveals that, under the influence of the relative velocity difference, the mass diffusion laws governing the mixing of the coaxial jets are consistent with the thermal diffusion laws.

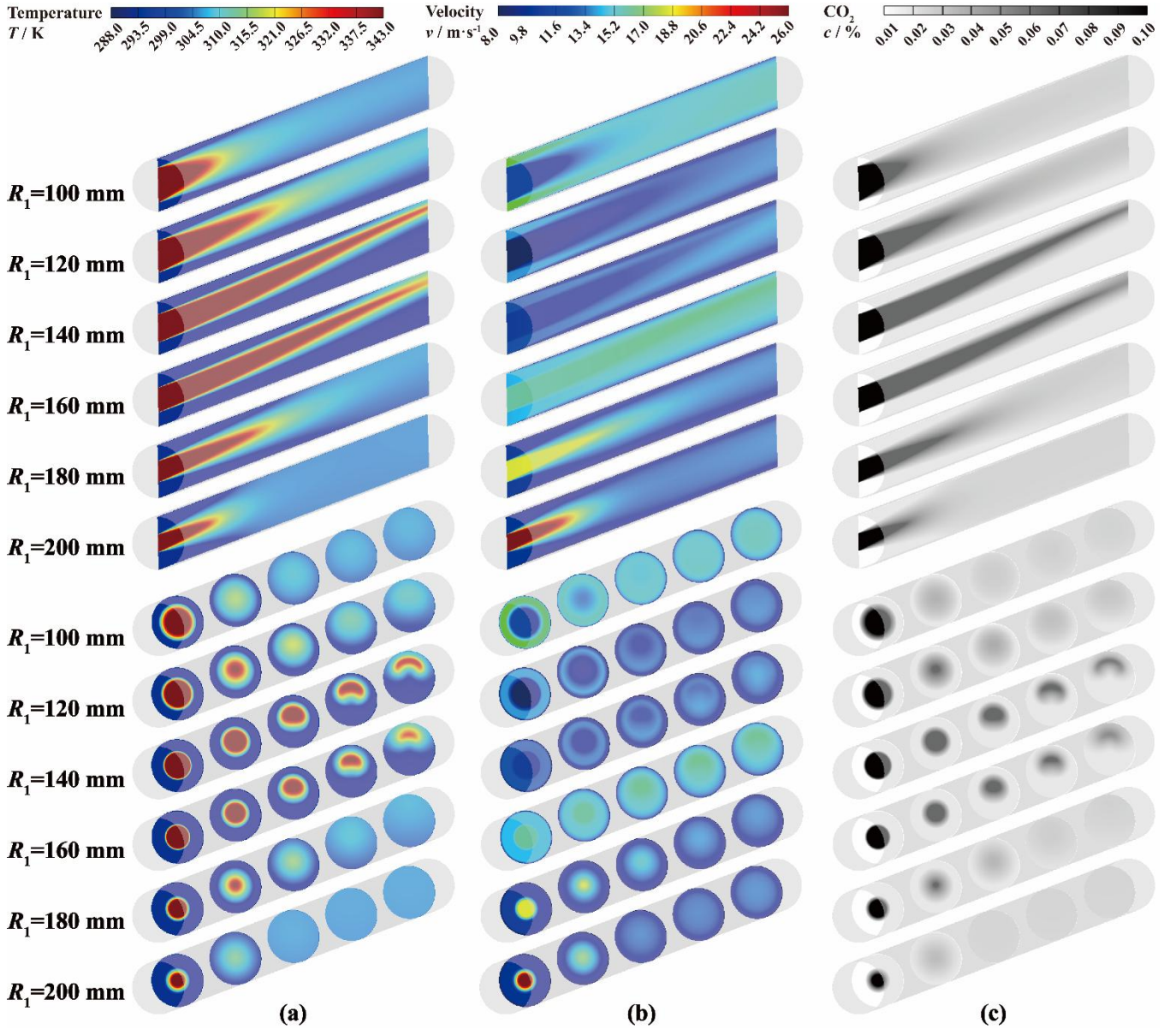


Figure 7. Comparison of (a) temperature, (b) velocity distribution and (c) CO₂ under difference outer radii.

Figure 8 compares the temperature, velocity, and CO₂ concentration distributions at steady-state for a coaxial jet with an outer radius of 160 mm at mass flow ratios of 1, 2, 3, 4 and 5. As observed in the temperature and velocity contours in Figures (a) and (b), when the mass flow ratio is 1, the velocity of the hot fluid is significantly higher than that of the cold fluid. Consequently, the hot fluid continuously entrains the surrounding cold fluid and mixes with it during propagation. When the mass flow ratio increases to 2, the velocity difference between the cold and hot fluids becomes 0.56, which is closer to a ratio of unity compared to the mass flow ratio of 1 case. Therefore, the entrainment effect of the hot fluid is relatively reduced, resulting in an extended hot fluid jet core. As the mass flow ratio increases to 3 and 4, the velocity ratios of the cold to hot fluids reach 0.84 and 1.11, respectively. At these values, the velocity ratios

approach unity, and the mutual entrainment between the two jets is minimized. Under these conditions, both jets can penetrate through the entire length of the pipe. When the mass flow ratio is 5, the velocity of the cold fluid is significantly higher than that of the hot fluid. Thus, during coaxial propagation, the cold fluid continuously entrains and mixes with the hot fluid. Furthermore, the mixing uniformity of the two fluids at a mass flow ratio of 1 is higher than that at a mass flow ratio of 5, primarily because the velocity difference between the fluids is much larger in the former case. At a mass flow ratio of 3, the velocities of the two fluids are relatively similar; consequently, no significant entrainment phenomenon occurs, and the mixing is the least uniform. As the flow propagates to the outlet, the hot fluid rises to the cold fluid region. The CO₂ concentration distributions in Figure (c) indicate that mass diffusion follows a transfer process similar to that of thermal diffusion, confirming that the mechanisms of mass transfer and heat transfer under the influence of velocity ratios are consistent.

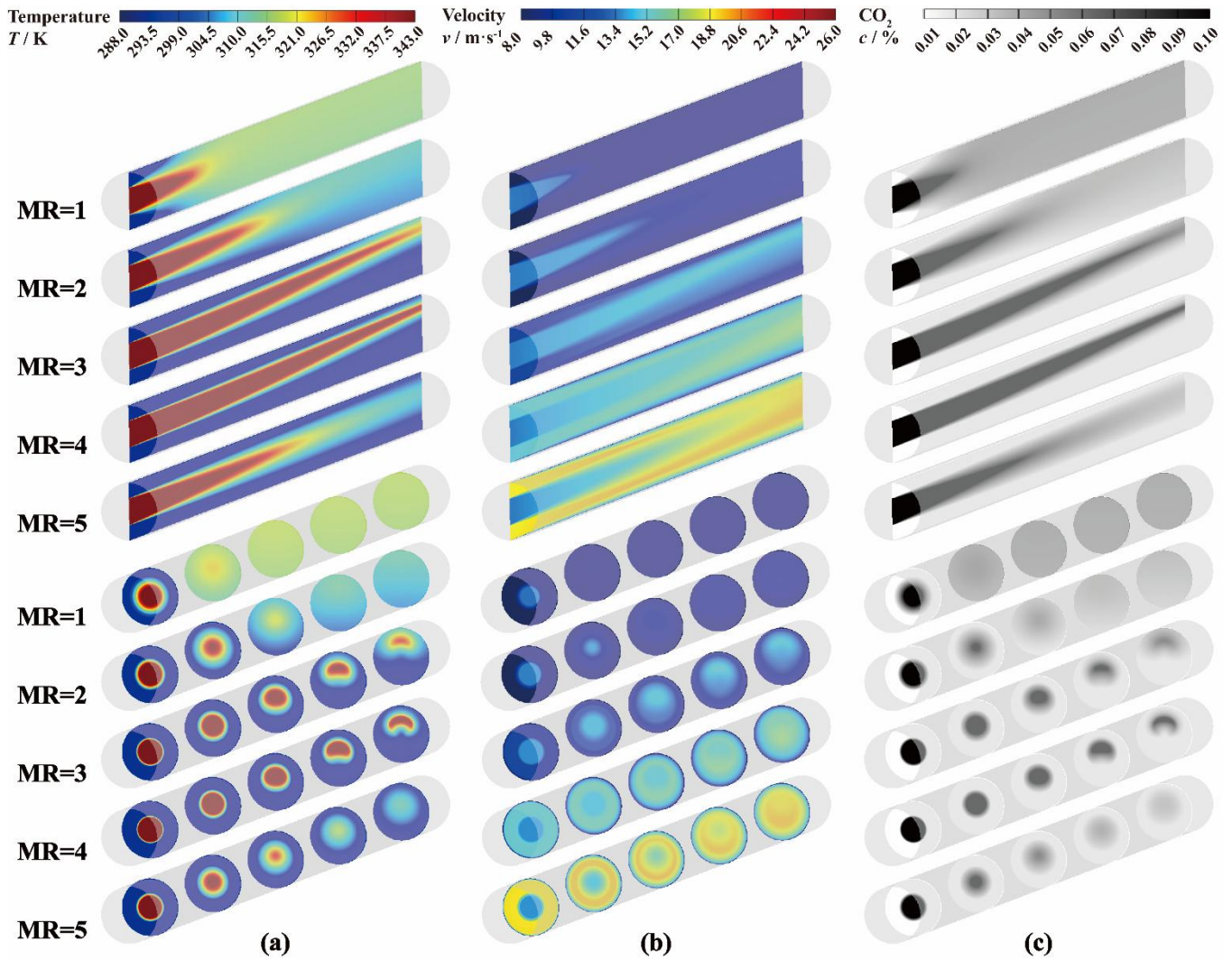


Figure 8. Comparison of (a) temperature, (b) velocity distribution and (c) CO₂ under difference mass ratios.

5.2. Influence of temperature difference on jet mixing

Figure 9 compares the effects of outer radius on heat and mass transfer during the coaxial jet process under varying temperature differences. Figures (a) and (b) display the outer layer temperature at the outlet and the temperature mixing uniformity of the outlet cross-section. As shown in Figure (a), the outer layer temperature at the outlet gradually increases as the temperature difference decreases. Furthermore, the difference between the maximum outer layer temperature and the corresponding temperatures at other diameters increases with the temperature difference. When the temperature difference is 40 K, the influence of different outer radii on the outlet outer layer temperature is minimal. This occurs because a larger temperature difference generates greater buoyancy lift for hot fluid, increasing hot fluid proportion in outer layer at the outlet and thereby raising its temperature. Figure (b) compares the temperature mixing uniformity at the outlet cross-section. The figure indicates that mixing is more uniform at smaller temperature differences and poorer at larger ones, regardless of the cold fluid width. This is also caused by the intensified buoyant rise of the hot fluid due to a larger temperature difference, which increases vertical mixing non-uniformity within the pipe. Figures (c) and (d) present the CO₂ concentration in the outer layer and the CO₂ mixing uniformity index at the outlet. The CO₂ concentration at the outlet outer layer exhibits trends inconsistent with the temperature distribution under different temperature differences. This discrepancy arises because changing the temperature difference actively influences the mixing temperature by altering buoyancy magnitude, whereas CO₂ concentration acts as a passive scalar in this study. A comparison of the outer CO₂ concentrations reveals that at 100 mm and 120 mm, the concentration is higher at lower temperature differences. At 140 mm and 160 mm, the concentration is higher at larger temperature differences. For outer radii of 180 mm, 200 mm, and 220 mm, the concentration again increases as the temperature difference decreases. This phenomenon is primarily attributed to the fact that at 140 mm and 160 mm, the inner and outer fluids possess very similar velocities. Consequently, entrainment between the two streams is weak, and CO₂ in the hot jet relies on buoyancy to penetrate the outer fluid. Therefore, a larger temperature difference results in higher outer CO₂ concentrations at the outlet. However, for inlet outer radii of 100 mm, 120 mm, 180 mm, 200 mm, and 220 mm, the velocity difference between the fluids is significant, allowing the hot fluid to diffuse across the entire pipe before reaching the outlet. In these cases, the larger temperature difference causes the hot fluid located in the lower part of the pipe to rise out of the outer ring region, thereby reducing the CO₂ concentration in the outer ring. Nevertheless, the CO₂ mixing uniformity index in Figure (d) consistently

indicates that smaller temperature differences yield better mixing. The buoyant rise of the hot plume caused by large temperature differences significantly degrades the CO₂ mixing uniformity at the outlet.

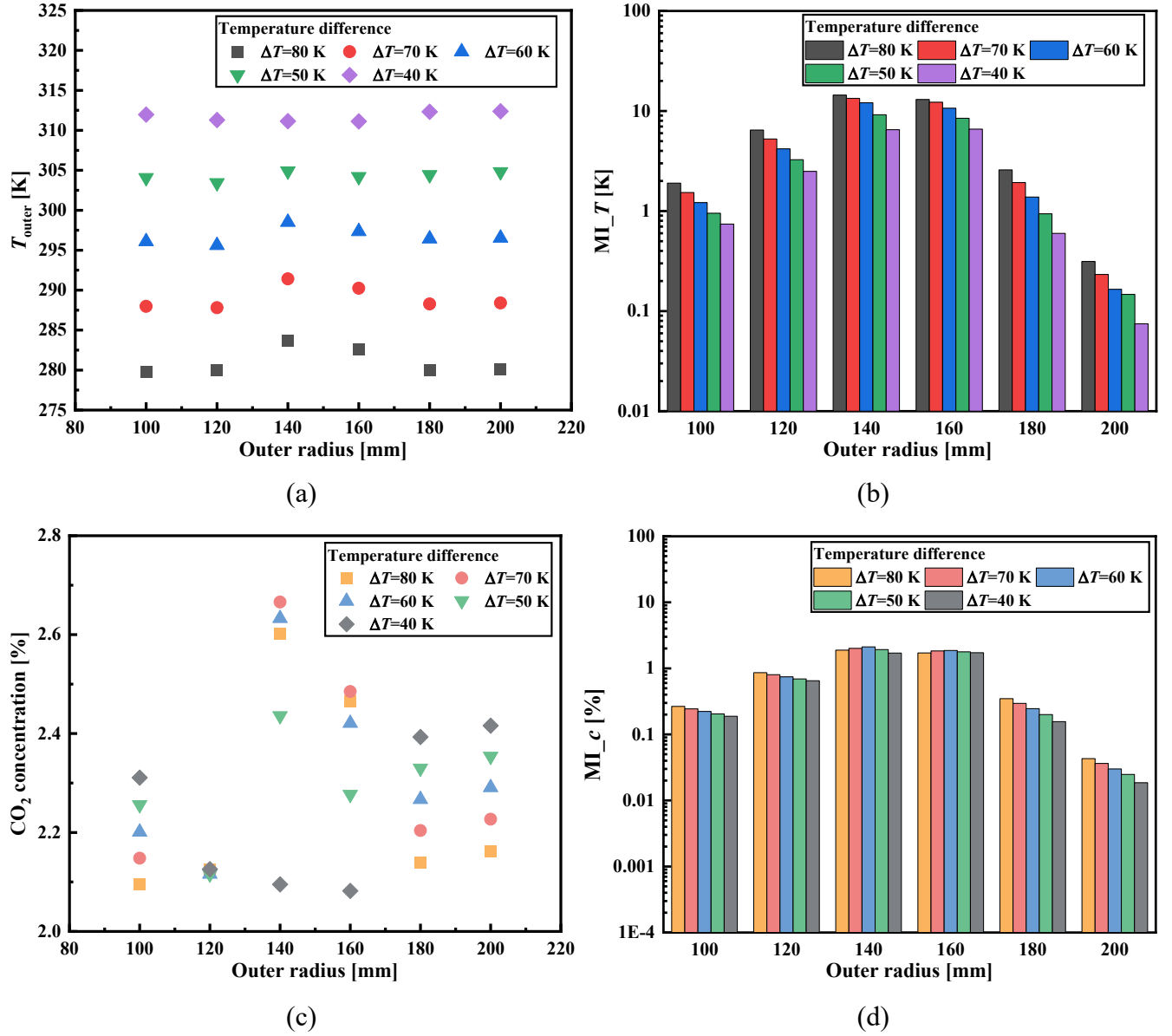


Figure 9. Influence of temperature difference on heat and mass transfer under different outer radii for: (a) Outer temperature, (b) MI of temperature, (c) Outer CO₂, (d) MI of CO₂.

Figure 10 illustrates the mixing and entrainment processes of coaxial jets under varying temperature differences, specifically with a mass flow ratio of 3 and an outer radius of 160 mm. As observed in the temperature contours in Figure (a), at a temperature difference of 80 K, the hot fluid core exhibits a higher buoyant rise and maintains a compact structure as it travels toward the outlet. When the temperature difference decreases to 70 K, the hot fluid jet core displays a wider diffusion pattern compared to the 80 K case. At 60 K, the buoyant rise height of the hot fluid core decreases, and signs of entrainment and diffusion by the cold fluid become evident at the outlet. This phenomenon is also visible in the outlet

temperature cross-sections. At a temperature difference of 50 K, the hot fluid occupies nearly the entire upper half of the pipe at the outlet. As the temperature difference is reduced to 40 K, the buoyant rise height further decreases, and the hot fluid core undergoes more significant diffusion due to cold fluid entrainment. This results in a more uniform distribution of high-temperature fluid at the outlet, explaining the improved temperature uniformity observed at lower temperature differences. A comparison with the velocity distribution contours in Figure (b) reveals that under large temperature differences, the hot fluid maintains its core and moves upward within the pipe. Conversely, a reduction in the temperature difference causes the high-temperature fluid to diffuse downwards at higher velocities, eventually filling the pipe. Observation in Figure (c) shows that CO₂ concentration distributions is similar to the temperature distribution. This similarity elucidates why the uniformity of CO₂ concentration distribution is also superior at lower temperature differences.

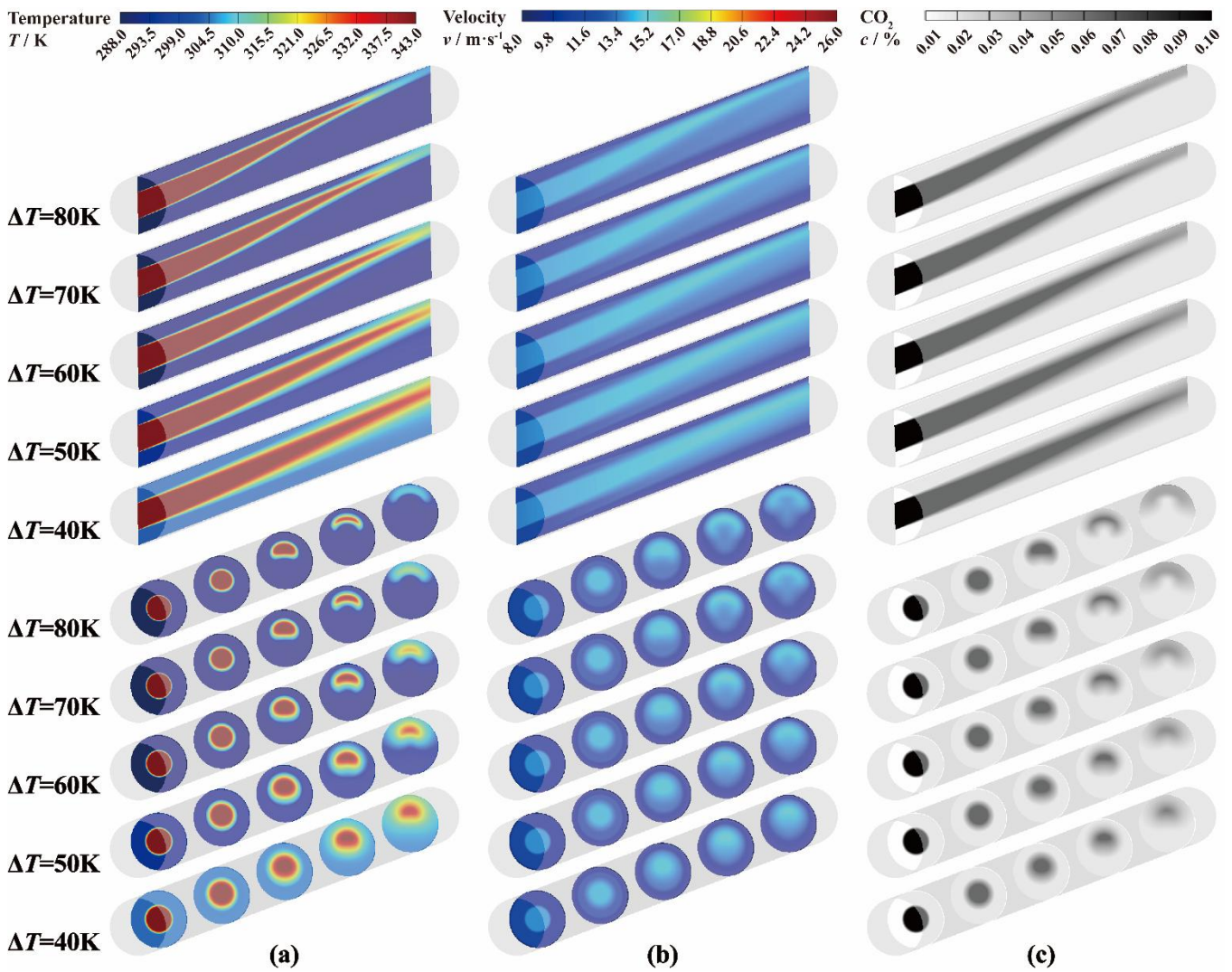


Figure 10. Comparison of (a) temperature, (b) velocity distribution and (c) CO₂ under various temperature differences.

5.3. Coupling influence mechanism

Figure 11 illustrates the interactive effects of the temperature difference, inlet outer radius, and mass flow ratio on the temperature mixing uniformity index at the outlet cross-section. In this context, a higher index value indicates poorer mixing uniformity. The figure reveals that as the temperature difference increases, the uniformity index at the outlet rises, signifying a deterioration in mixing performance. This outcome results from large temperature differences promoting the buoyant rise of the hot fluid, which adversely affects temperature uniformity. Furthermore, as the inlet outer radius increases, the influence of the temperature difference on mixing uniformity diminishes. This is because a smaller inlet diameter increases the hot fluid inlet velocity, thereby enhancing the entrainment effect. Regarding the influence of the cold fluid inlet radius, it is observed that an intermediate radius consistently yields the poorest temperature mixing uniformity, regardless of the temperature difference or flow rate. This occurs because an intermediate radius results in similar velocities for both fluids, minimizing the diffusion effects typically driven by large velocity differences. An analysis of the cold fluid flow rate shows that a rate of $6000 \text{ m}^3/\text{h}$ corresponds to the poorest mixing uniformity across different temperature differences. For influence of the inlet radius, a flow rate of $15000 \text{ m}^3/\text{h}$ provides the best uniformity for radii of 100 mm, 120 mm, and 140 mm. Conversely, for radii between 160 mm and 200 mm, $3000 \text{ m}^3/\text{h}$ yields lowest uniformity. These trends are governed by the necessity to maximize the velocity difference between the fluids to enhance diffusion during the coaxial jet process.

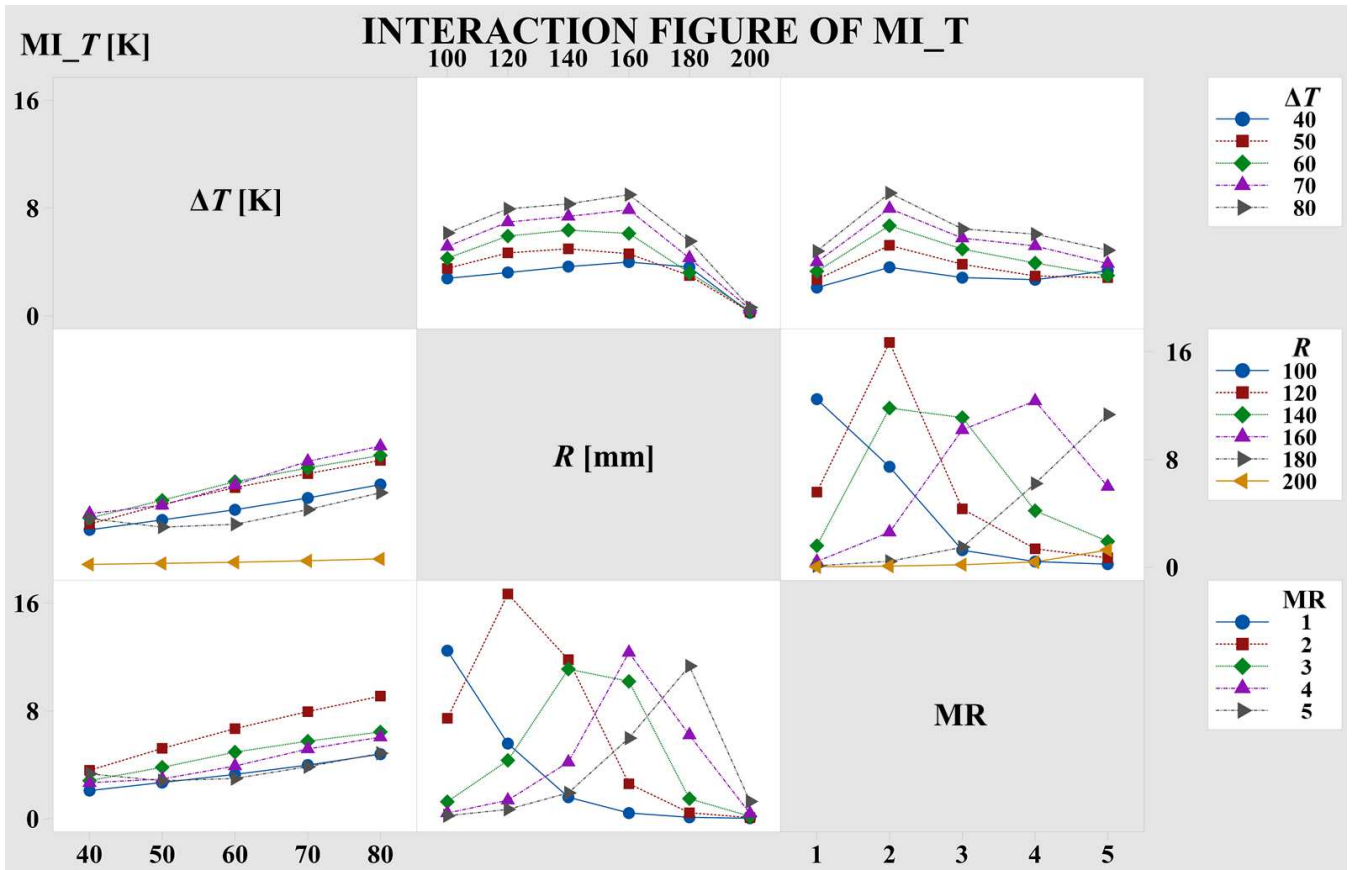


Figure 11. Interactive influence of temperature difference, outer radius and cold fluid flow on the uniformity index of temperature mixing.

Figure 12 presents a comparison of the interactive effects of the temperature difference, inlet outer radius, and mass flow ratio on the CO₂ mixing uniformity at the outlet cross-section. The figure indicates that as the temperature difference increases, the CO₂ mixing uniformity index at the outlet exhibits a slight increase across various inlet outer radii and flow rates. This phenomenon is attributed to the strong buoyancy effects induced by large temperature differences. Regarding the influence of the inlet outer radius, it is observed that at intermediate cold fluid flow rates, an outer radius of 200 mm yields the optimal CO₂ mixing uniformity across all investigated temperature differences. This improvement occurs because the 200 mm diameter configuration generates a significant velocity difference between the hot and cold fluids, which enhances shear diffusion and facilitates the uniform dispersion of CO₂. An analysis of the inlet outer radius under varying flow rates reveals that, except for the 3000 m³/h condition, the 200 mm diameter consistently provides superior mixing uniformity. Furthermore, at intermediate inlet outer radii, both high and low cold fluid flow rates result in better CO₂ uniformity. The underlying mechanism governing these trends is that maintaining a large velocity difference between the fluids significantly enhances the entrainment and diffusion effects.

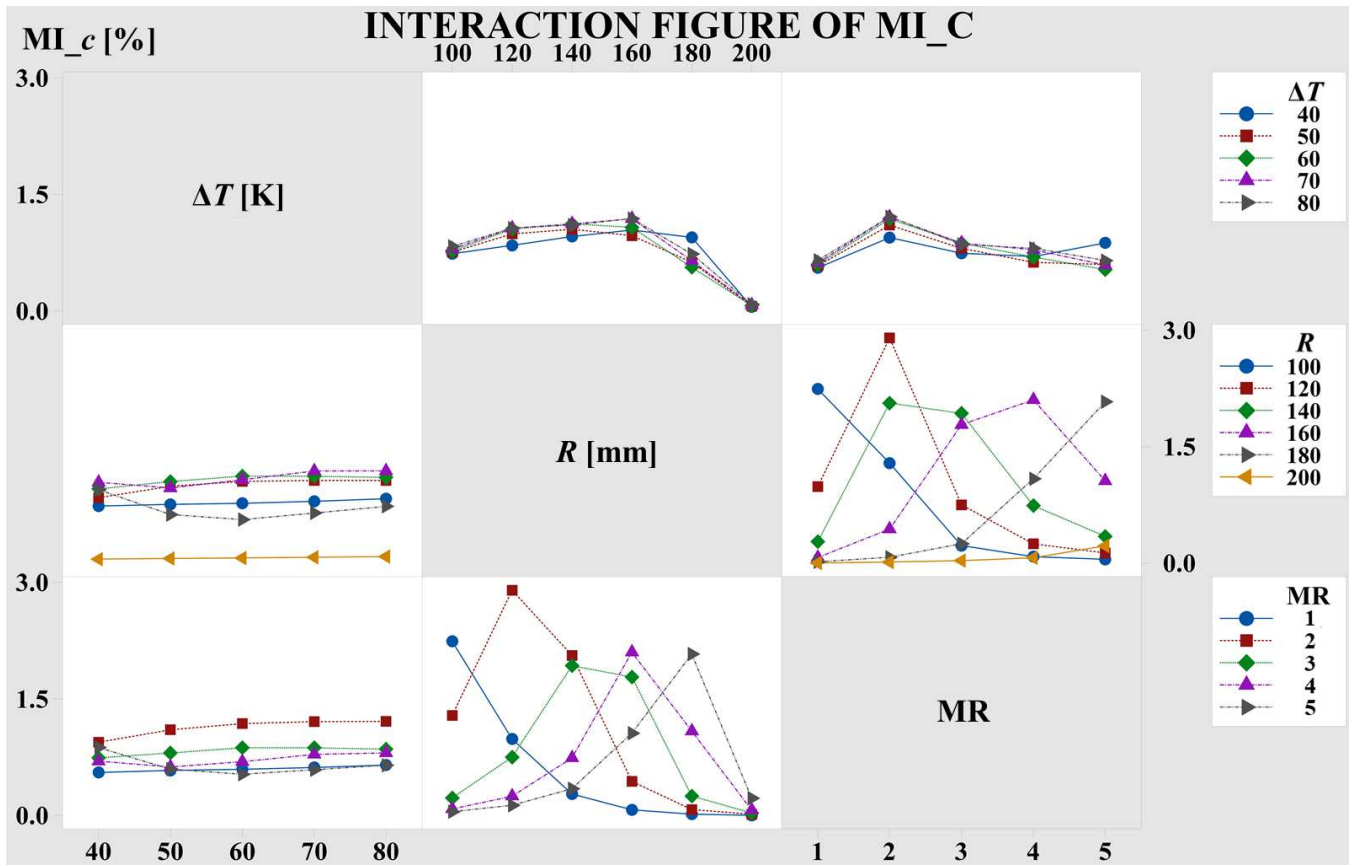


Figure 12. Interactive influence of temperature difference, outer radius and cold fluid flow on the uniformity index of CO₂ concentration.

5.4. Influence on the maximum pipe temperature

Figure 13(a) illustrates the effects of inlet outer radius and mass flow ratio on the maximum pipe wall temperature rise. It is observed that at a mass flow ratio of 1, temperature rise decreases as inlet outer radius increases. This reduction is primarily attributed to the increased velocity difference between the fluids resulting from the varying inlet areas, which enhances hot fluid diffusion and prevents extensive heating of the pipe wall. When the mass flow ratio increases to 2 and 3, the maximum wall temperature rise initially increases and subsequently decreases with the expansion of the inlet outer radius. This indicates that a large velocity difference promotes strong diffusion, facilitating uniform mixing and reducing wall heating. Conversely, a smaller velocity difference allows the hot fluid core to persist to the pipe exit, resulting in more intense heating. At mass flow ratios of 4 and 5, the maximum temperature rise also exhibits first increasing and then decreasing as radius expands. This behavior arises because higher mass flow ratios generate a wider range of velocity ratios, eventually leading to an optimal ratio that minimizes wall temperature rise. **Figure 13(b)** compares the effects of inlet outer radius and temperature difference on the maximum wall temperature rise. Overall, the temperature rise initially increases and then

decreases with the increasing diameter across all temperature differences. This phenomenon occurs because at intermediate diameters, the hot fluid core extends to the pipe outlet, where buoyancy forces cause it to rise and heat the solid wall. Furthermore, an increase in the temperature difference leads to a higher maximum wall temperature rise. This is because larger temperature difference enhances the buoyancy of hot fluid, thereby intensifying heating on pipe wall.

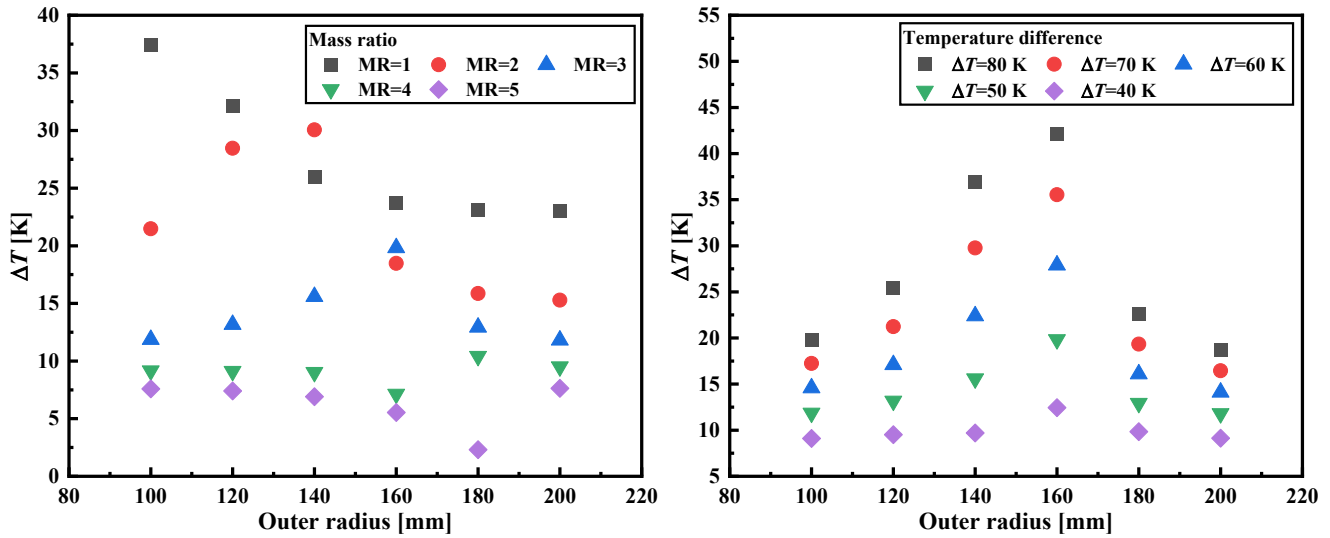


Figure 13. Influence of outer radius, temperature difference and MR on pipe wall temperature rise.

Figure 14 illustrates interactive effects of temperature difference, inlet outer radius, and mass flow ratio on the pipe wall temperature rise. An analysis of the temperature difference reveals that, regardless of the inlet outer radius or mass flow ratio, a larger temperature difference consistently results in a higher wall temperature rise. Regarding the inlet outer radius, the maximum wall temperature rise initially increases and subsequently decreases as the diameter increases under any given temperature difference, reaching a peak value at 140 mm. However, the variation trends with respect to the inlet outer radius differ across various mass flow ratios. When the mass flow ratio is 1, the wall temperature rise decreases monotonically as the inlet outer radius increases. For mass flow ratios of 2, 3, and 4, the wall temperature rise exhibits a trend of increasing followed by decreasing. Conversely, at a mass flow ratio of 5, the temperature rise first decreases and then increases with the expansion of the inlet outer radius. Finally, regarding the mass flow ratio, the wall temperature rise consistently decreases as the mass flow ratio increases, a trend that remains consistent across different temperature differences and inlet outer radii.

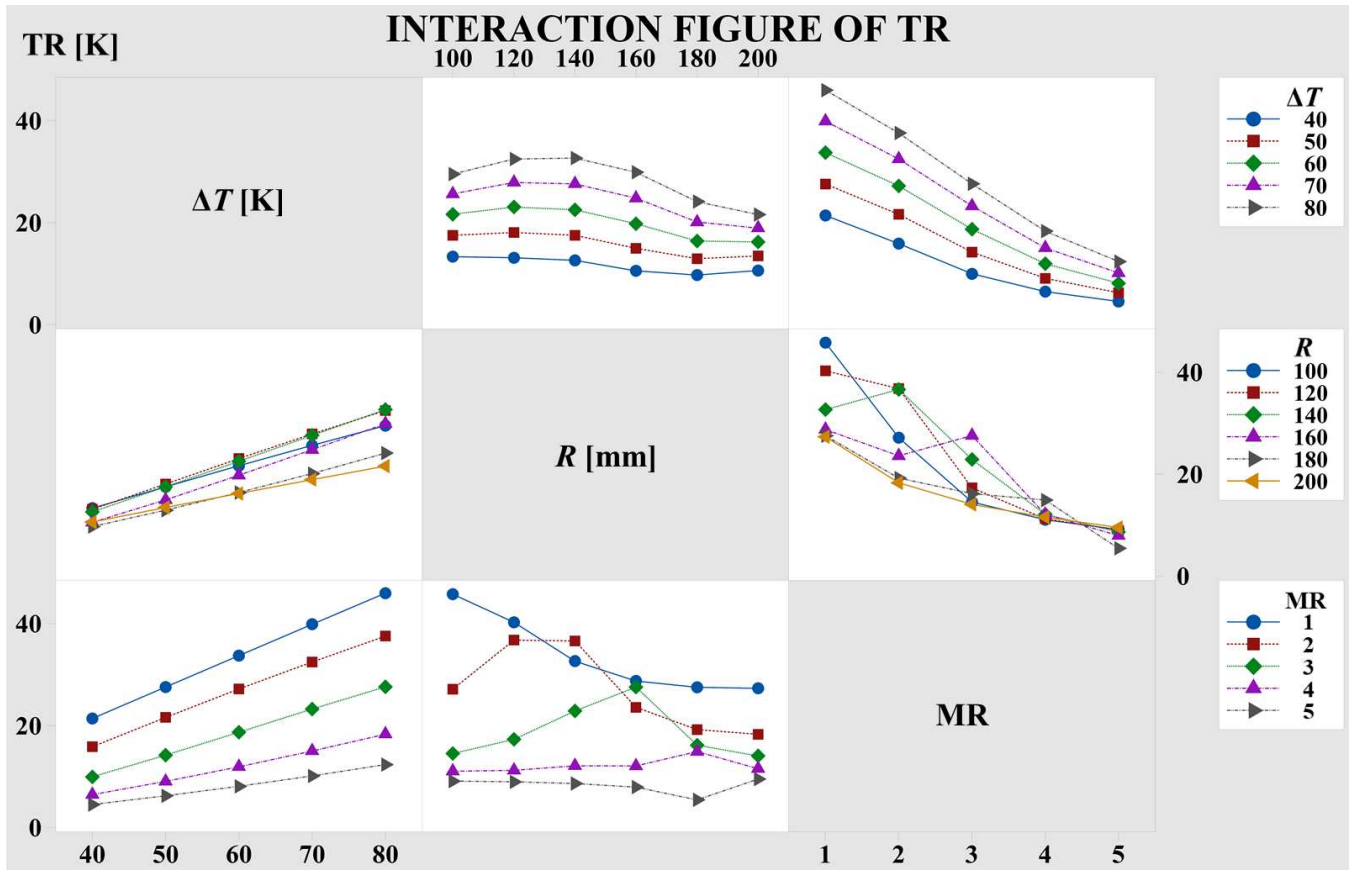


Figure 14. Interactive influence of temperature difference, outer radius and MR on pipe wall temperature rise.

5.5. Fitting correlations

Following the analysis of the interactive effects of the three variables on the temperature mixing uniformity index, CO₂ mixing uniformity index, and pipe wall temperature rise, three quartic polynomial correlations were fitted for each response variable. This was done to facilitate the rapid and accurate prediction of these indices in subsequent applications. The data utilized for the fitting process, which were obtained from the calculations in the preceding sections, are presented in **Table 9**. The parametric ranges investigated include a cold fluid inlet outer radius of 100 mm to 200 mm, a temperature difference between hot and cold fluids ranging from 40 K to 80 K, and a mass flow ratio varying from 1 to 5. The resulting fitted correlations are expressed as follows:

$$\begin{aligned}
\lg(\text{MI}_T) = & -5.5 - 0.32a + 0.27b + 1.42c + 3.6 \times 10^{-3}ab - 0.022ac - 0.032bc \\
& + 5.3 \times 10^{-3}aa - 3.2 \times 10^{-3}bb - 0.46cc - 1.1 \times 10^{-5}abc - 4.0 \times 10^{-5}aab \\
& + 7.8 \times 10^{-4}aac - 7.77 \times 10^{-6}abb - 8.2 \times 10^{-3}acc + 6.89 \times 10^{-4}bbc \\
& - 0.014bcc - 4.70 \times 10^{-5}aaa + 1.1 \times 10^{-5}bbb + 0.32ccc + 6.98 \times 10^{-8}aabb \\
& - 2.09 \times 10^{-6}aabc + 6.6 \times 10^{-5}aacc + 1.78 \times 10^{-6}abbc - 3.8 \times 10^{-5}abcc \\
& + 8.3 \times 10^{-5}bbcc - 1.56 \times 10^{-7}aaab - 4.85 \times 10^{-6}aaac - 1.58 \times 10^{-8}abbb \\
& + 5.15 \times 10^{-4}accc - 3.05 \times 10^{-6}bbbc - 9.7 \times 10^{-4}bccc + 1.70 \times 10^{-7}aaaa \\
& - 5.28 \times 10^{-9}bbbb - 0.014cccc
\end{aligned} \tag{10}$$

$$\begin{aligned}
\lg(\text{MI}_c) = & -13.35 + 4.9 \times 10^{-3}a + 0.39b + 0.49c + 1.9 \times 10^{-3}ab - 7.2 \times 10^{-3}ac \\
& - 8.4 \times 10^{-3}bc + 2.2 \times 10^{-3}aa - 4.5 \times 10^{-3}bb - 0.81cc - 1.4 \times 10^{-4}abc \\
& - 1.2 \times 10^{-5}aab + 2.7 \times 10^{-4}aac - 6.7 \times 10^{-6}abb - 4.5 \times 10^{-3}acc \\
& + 4.5 \times 10^{-4}bbc - 0.013bcc - 2.70 \times 10^{-5}aaa + 1.9 \times 10^{-5}bbb + 0.37ccc \\
& + 4.3 \times 10^{-8}aabb - 6.8 \times 10^{-7}aabc + 4.3 \times 10^{-5}aacc + 8.2 \times 10^{-7}abbc \\
& - 7.6 \times 10^{-6}abcc + 4.8 \times 10^{-5}bbcc - 1.8 \times 10^{-8}aaab - 2.3 \times 10^{-6}aaac \\
& - 9.0 \times 10^{-10}abbb + 2.7 \times 10^{-5}accc - 1.9 \times 10^{-6}bbbc - 2.5 \times 10^{-4}bccc \\
& + 9.5 \times 10^{-8}aaaa - 2.8 \times 10^{-8}bbbb - 0.023cccc
\end{aligned} \tag{11}$$

$$\begin{aligned}
\sqrt{\text{TR}} = & 29.09 + 0.32a - 0.77b - 8.44c - 3.2 \times 10^{-3}ab - 0.028ac + 0.14bc \\
& - 1.5 \times 10^{-3}aa + 8.2 \times 10^{-3}bb + 0.29cc + 4.5 \times 10^{-4}abc + 1.6 \times 10^{-6}aab \\
& + 8.5 \times 10^{-6}aac + 2.1 \times 10^{-5}abb - 7.4 \times 10^{-3}acc - 4.9 \times 10^{-4}bbc \\
& - 0.024bcc - 8.3 \times 10^{-6}aaa - 4.1 \times 10^{-5}bbb + 0.36ccc + 5.8 \times 10^{-9}aabb \\
& - 2.6 \times 10^{-7}aabc + 2.8 \times 10^{-5}aacc - 1.3 \times 10^{-6}abbc + 1.1 \times 10^{-5}abcc \\
& + 1.3 \times 10^{-4}bbcc - 5.5 \times 10^{-9}aaab - 4.6 \times 10^{-7}aaac - 5.3 \times 10^{-8}abbb \\
& + 1.1 \times 10^{-4}accc - 4.0 \times 10^{-7}bbbc - 1.8 \times 10^{-3}bccc - 1.9 \times 10^{-8}aaaa \\
& + 8.2 \times 10^{-8}bbbb - 4.7 \times 10^{-3}cccc
\end{aligned} \tag{12}$$

where, a represents the ΔT , b denotes the outer radius of the MR, and c presents the MR.

Table 9. The display of the CO₂ mixing index, temperature mixing index and temperature rise under various inlet outer radii, temperature differences and mass ratios between hot and cold fluid.

ΔT [K]	R [mm]	MR	MI _c [%]	MI _T [K]	TR [K]	ΔT [K]	R [mm]	MR	MI _c [%]	MI _T [K]	TR [K]
50	100	1	10.46	2.26	37.46	80	160	3	13.03	1.71	42.12
50	120	1	4.27	0.92	32.12	80	180	3	2.58	0.35	22.65
50	140	1	1.06	0.23	26.01	80	200	3	0.31	0.04	18.70
50	160	1	0.27	0.06	23.74	80	100	4	0.57	0.09	14.80
50	180	1	0.07	0.01	23.09	80	120	4	1.93	0.27	15.45
50	200	1	0.05	0.00	23.00	80	140	4	5.80	0.77	18.27
50	100	2	5.62	1.19	21.48	80	160	4	18.95	2.49	21.96
50	120	2	14.11	2.97	28.46	80	180	4	8.48	1.12	23.80

50	140	2	9.65	2.03	30.06	80	200	4	0.67	0.09	15.61
50	160	2	1.65	0.35	18.48	80	100	5	0.27	0.05	12.17
50	180	2	0.28	0.06	15.86	80	120	5	0.91	0.13	12.12
50	200	2	0.05	0.01	15.29	80	140	5	2.60	0.35	12.08
50	100	3	0.95	0.20	11.86	80	160	5	7.76	1.03	12.71
50	120	3	3.27	0.69	13.17	80	180	5	15.67	2.05	11.76
50	140	3	9.17	1.92	15.59	80	200	5	1.95	0.26	13.28
50	160	3	8.47	1.78	19.83	40	100	1	8.66	2.32	29.02
50	180	3	0.94	0.20	12.93	40	120	1	2.87	0.76	23.88
50	200	3	0.15	0.02	11.81	40	140	1	0.67	0.18	19.91
50	100	4	0.35	0.08	9.16	40	160	1	0.19	0.04	18.68
50	120	4	1.10	0.24	9.11	40	180	1	0.09	0.01	18.39
50	140	4	3.41	0.72	9.02	40	200	1	0.07	0.00	18.51
50	160	4	7.52	1.57	7.13	40	100	2	3.99	1.04	15.43
50	180	4	5.10	1.07	10.42	40	120	2	9.29	2.44	19.48
50	200	4	0.26	0.06	9.51	40	140	2	7.07	1.85	21.78
50	100	5	0.20	0.05	7.59	40	160	2	1.02	0.27	13.90
50	120	5	0.59	0.13	7.40	40	180	2	0.18	0.04	12.42
50	140	5	1.59	0.34	6.91	40	200	2	0.04	0.01	12.18
50	160	5	5.12	1.08	5.54	40	100	3	0.74	0.19	9.10
50	180	5	8.51	1.78	2.31	40	120	3	2.50	0.65	9.52
50	200	5	0.94	0.20	7.63	40	140	3	6.51	1.70	9.69
60	100	1	12.09	2.17	45.84	40	160	3	6.60	1.72	12.45
60	120	1	5.65	1.02	40.42	40	180	3	0.60	0.16	9.84
60	140	1	1.53	0.28	32.37	40	200	3	0.07	0.02	9.13
60	160	1	0.39	0.07	28.76	40	100	4	0.31	0.08	7.10
60	180	1	0.09	0.01	27.66	40	120	4	0.88	0.23	6.93
60	200	1	0.02	0.00	27.36	40	140	4	2.72	0.71	6.39
60	100	2	7.43	1.32	27.46	40	160	4	7.94	2.07	3.91
60	120	2	17.73	3.12	37.41	40	180	4	4.05	1.05	7.00
60	140	2	12.10	2.13	37.28	40	200	4	0.18	0.05	7.37
60	160	2	2.44	0.44	23.33	40	100	5	0.19	0.05	5.89
60	180	2	0.41	0.07	19.26	40	120	5	0.49	0.13	5.69
60	200	2	0.07	0.01	18.39	40	140	5	1.28	0.33	5.18
60	100	3	1.21	0.22	14.58	40	160	5	4.18	1.09	3.70
60	120	3	4.19	0.74	17.10	40	180	5	13.18	3.46	0.89
60	140	3	12.09	2.11	22.40	40	200	5	0.70	0.18	5.77
60	160	3	10.67	1.87	27.90	70	100	1	14.22	2.18	54.17
60	180	3	1.38	0.25	16.10	70	120	1	6.94	1.08	48.55
60	200	3	0.17	0.03	14.11	70	140	1	2.07	0.32	39.02
60	100	4	0.41	0.08	11.13	70	160	1	0.55	0.09	33.77
60	120	4	1.34	0.25	11.26	70	180	1	0.12	0.02	32.09
60	140	4	4.15	0.74	11.88	70	200	1	0.01	0.00	31.69
60	160	4	11.08	1.93	11.24	70	100	2	9.26	1.41	33.02
60	180	4	6.16	1.08	14.40	70	120	2	20.17	3.04	45.58

60	200	4	0.37	0.07	11.61	70	140	2	14.26	2.16	43.85
60	100	5	0.22	0.05	9.21	70	160	2	3.40	0.52	28.44
60	120	5	0.69	0.13	9.05	70	180	2	0.58	0.09	22.62
60	140	5	1.91	0.34	8.64	70	200	2	0.11	0.02	21.39
60	160	5	6.01	1.06	7.65	70	100	3	1.53	0.24	17.24
60	180	5	7.90	1.38	4.51	70	120	3	5.26	0.80	21.23
60	200	5	1.23	0.22	9.50	70	140	3	13.39	2.01	29.76
80	100	1	16.99	2.27	62.45	70	160	3	12.26	1.84	35.54
80	120	1	8.19	1.12	56.42	70	180	3	1.93	0.30	19.34
80	140	1	2.67	0.37	45.94	70	200	3	0.23	0.04	16.45
80	160	1	0.74	0.10	38.78	70	100	4	0.48	0.08	13.01
80	180	1	0.17	0.02	36.40	70	120	4	1.62	0.26	13.38
80	200	1	0.03	0.00	35.99	70	140	4	4.94	0.75	14.96
80	100	2	10.98	1.47	38.33	70	160	4	16.29	2.44	16.21
80	120	2	22.20	2.93	52.93	70	180	4	7.28	1.10	18.89
80	140	2	16.05	2.13	50.14	70	200	4	0.51	0.08	13.64
80	160	2	4.51	0.61	33.80	70	100	5	0.24	0.05	10.74
80	180	2	0.79	0.11	25.93	70	120	5	0.80	0.13	10.62
80	200	2	0.15	0.02	24.25	70	140	5	2.25	0.35	10.36
80	100	3	1.90	0.27	19.81	70	160	5	6.88	1.04	10.03
80	120	3	6.46	0.86	25.45	70	180	5	11.49	1.72	7.63
80	140	3	14.42	1.89	36.90	70	200	5	1.57	0.24	11.39

Figure 15 illustrates the three-dimensional graphs generated from the fitted correlations. These plots provide an intuitive visualization of the effects exerted by inlet outer radius of cold fluid, temperature difference, and the mass flow ratio on three key response variables, the outlet temperature uniformity, the CO₂ concentration uniformity, and the temperature rise of pipe wall. Subsequently, **Figure 16** presents a detailed comparison between the values predicted by the fitted correlations and the actual simulation results. As evidenced by the figure, the proposed correlations demonstrate high predictive accuracy for the response variables. Utilizing these established correlations, an operation guidance to balance MI_T, MI_c, and TR was conducted. The lower the uniformity index of the outlet temperature mixture, the smaller the temperature difference between the outlet flue gas and the surrounding air will be. And the lower the temperature rise on the pipe wall, the closer the pipe is to the surrounding air. Therefore, to achieve the lowest overall radiation of infrared signals through the entire pipeline, the parameters should be selected to minimize the temperature uniformity at outlet and the temperature rise on the pipe wall. From the three-dimensional images of these indicators, to achieve the lowest value of the MI_T, the focus should be on the lower outer radius at a high mass ratio and the larger outer radius at a low mass ratio. All of these are conducive to the rapid diffusion of the hot fluid within the pipeline. To minimize the temperature rise on the pipe wall, the focus should be on the operating conditions with a relatively large

mass ratio. When simultaneously aiming to reduce the MI_T and shorten the TR, a smaller outer radius and a higher mass ratio should be chosen.

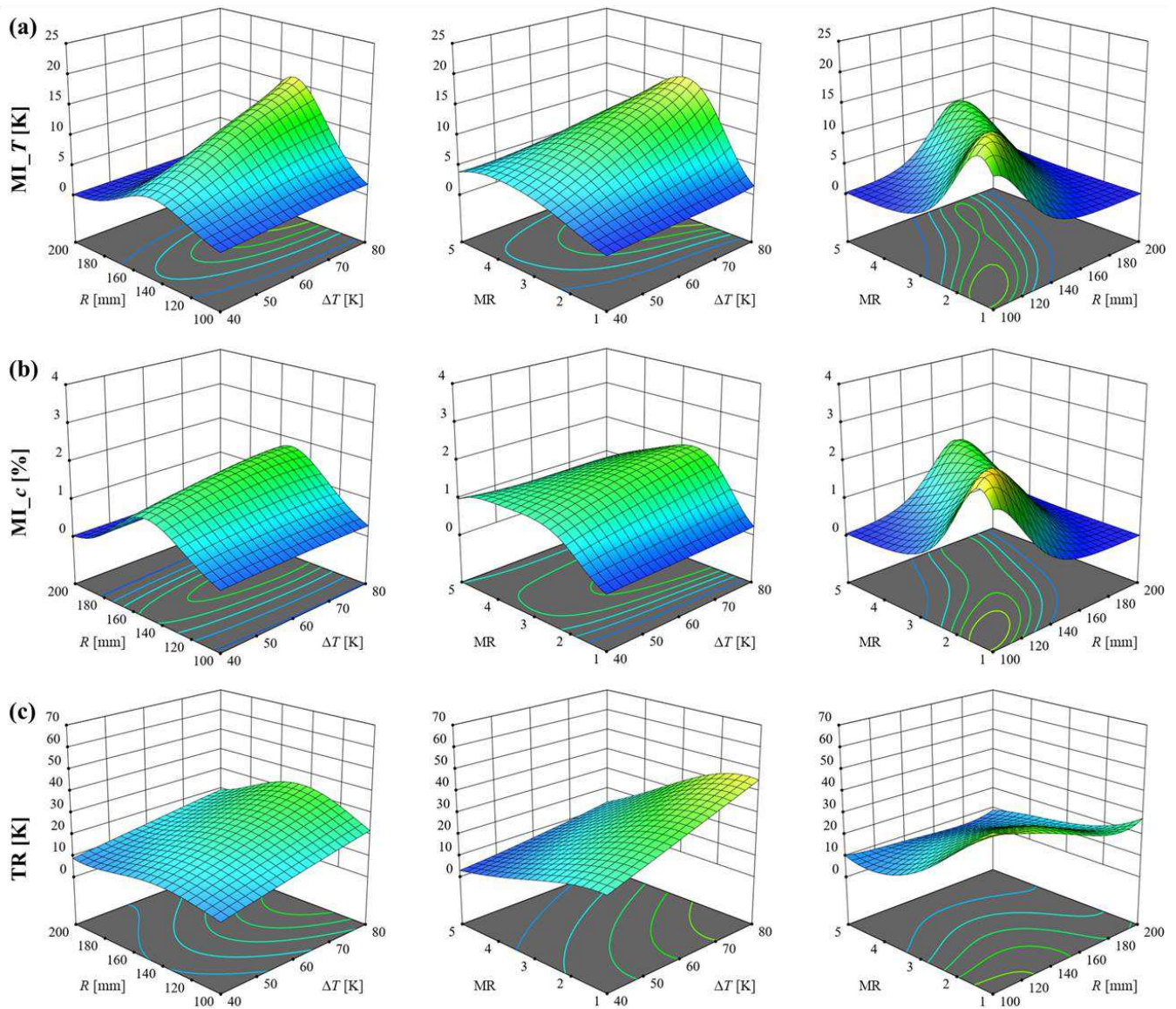


Figure 15. Graphs of the temperature mixing index, CO₂ mixing index, and temperature rise, under different temperature differences, inlet radii, and mass ratios.

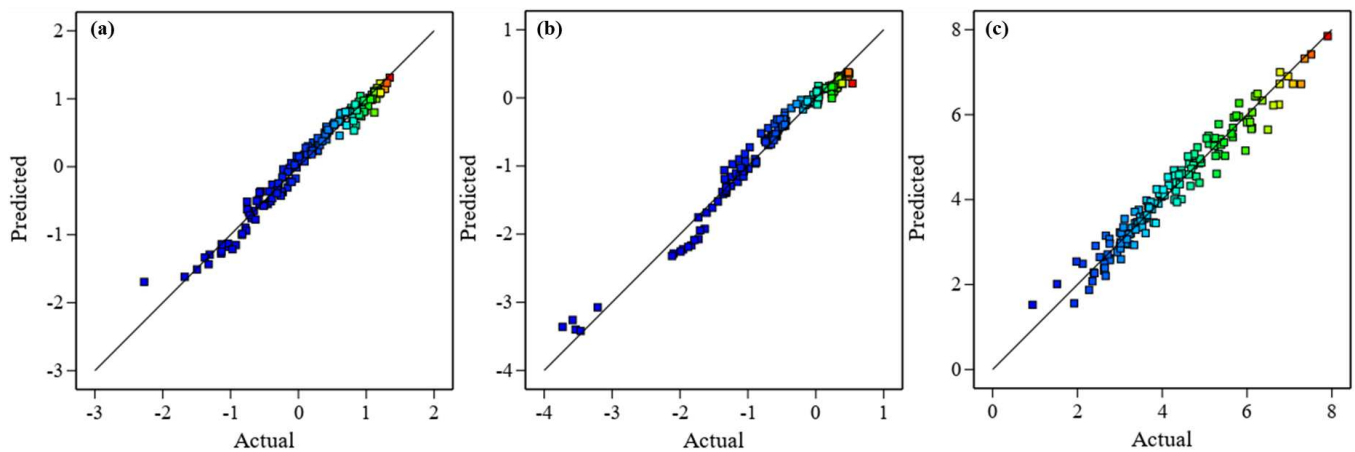


Figure 16. Comparison between predicted value and actual value of $\lg(MI_T)$, $\lg(MI_c)$ and $\sqrt{TR}$.

Subsequently, the MOSSA was employed to conduct multi-objective optimization for the outlet temperature uniformity and the temperature rise on pipe wall. The obtained Pareto sets are shown in **Figure 17(a)**. Results show that the temperature mixing uniformity and the temperature rise on pipe wall has negative relation. Therefore, it is necessary to make a trade-off choice between both indexes. In this study, the ideal point method was employed to select the optimal result from the Pareto sets, which is illustrated in **Figure 17(b)**. From the results, the optimal operating condition corresponds to a temperature difference of 40 K, an outer diameter of the inlet of 120 mm, and a mass ratio of 5. **Table 10** shows the comparison among the optimized results and the results under the extreme conditions. The results indicate that the optimized operating conditions have achieved the maximum improvement of 94.34% on temperature uniformity, and 80.39% on tube wall temperature rise.

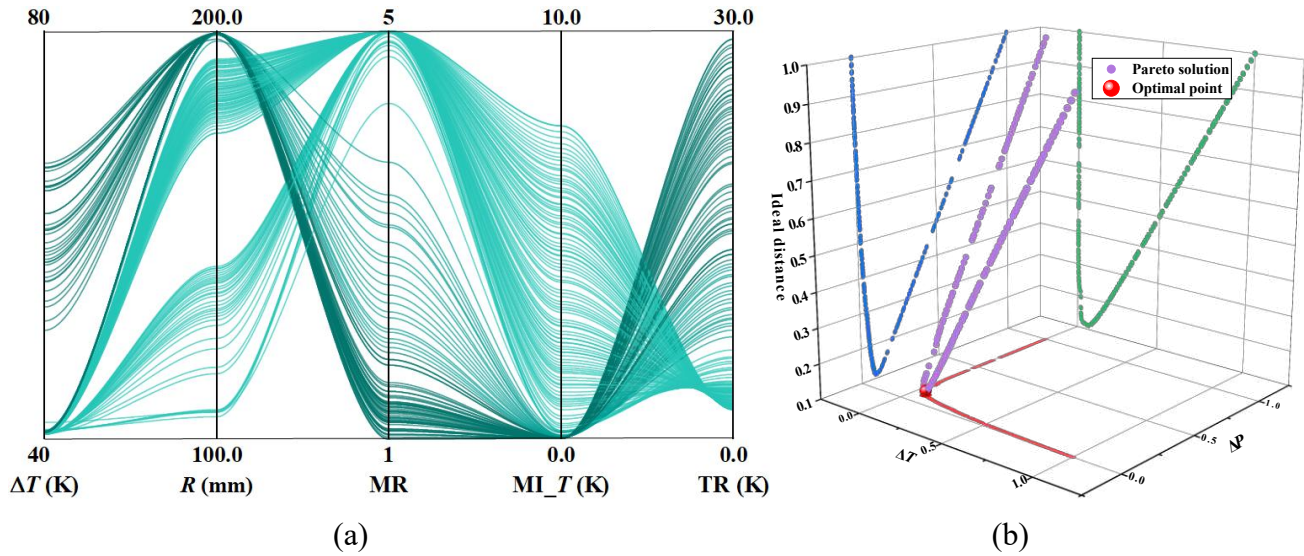


Figure 17. (a) Multi-objective optimization results for the MI_T at outlet and the TR of the pipe wall, (b) determination of the optimal operating point by ideal point method.

Table 10. Comparison of optimized results with the baseline case at extreme levels.

	R (mm)	MR	ΔT (K)	MI_T (K)	TR (K)	Improvement	
						MI_T	TR
Case 1	100	5	40	0.19	5.89	157.89%	-3.40%
Case 2	200	5	40	0.70	5.77	-30.00%	-1.39%
Case 3	100	1	40	8.66	29.02	-94.34%	-80.39%
Case 4	200	1	40	0.07	18.51	600.00%	-69.26%

Case 5	100	5	80	0.27	12.17	81.48%	-53.25%
Case 6	200	5	80	1.96	13.28	-75.00%	-57.15%
Optimized	120	5	40	0.49	5.69		

6. Conclusion

This study systematically investigates the heat and mass transfer characteristics of non-isothermal coaxial jets within a cylindrical pipe. Specifically, the effects of the inlet outer radius, temperature difference between cold and hot fluid, and mass flow ratio between cold and hot fluid are analyzed. The primary conclusions are as follows:

1. The velocity ratio between the cold and hot fluids is the governing parameter for mixing uniformity. As the velocity ratio approaches unity, the entrainment effect between the two jets is minimized, resulting in the longest potential core of the hot fluid jet. Consequently, the mixing uniformity of temperature and CO₂ concentration at the outlet cross-section is worst, and the pipe wall temperature rise is maximum. Conversely, any deviation from a velocity ratio of unity enhances entrainment and promotes mixing, such as changing the inlet outer radius or mass ratio.
2. As the inlet outer radius increases from 100 mm to 200 mm, the cold fluid velocity decreases while the hot fluid velocity increases. At low mass flow ratios, increasing inlet outer radius reduces the velocity ratio, driving it further from unity. This enhances mixing uniformity and reduces temperature rise of pipe wall. However, at high mass flow ratios, increasing the inlet outer radius causes the velocity ratio to initially approach unity and subsequently deviate from it. This non-monotonic behavior results in a deterioration followed by an improvement in mixing uniformity.
3. The temperature difference influences the mixing process via buoyancy force, exhibiting distinct mechanisms for temperature and CO₂ transport within pipe. This discrepancy arises because temperature acts as an active scalar, whereas CO₂ concentration functions as a passive scalar. Regarding the mixing uniformity at the outlet, a larger temperature difference leads to poorer uniformity. This is attributed to the intensified buoyancy-driven rise of the hot fluid under large temperature gradients, which deteriorates vertical mixing uniformity.
4. Among various multi-objective optimization algorithms, MOSSA demonstrates the best optimization performance. By applying this algorithm to optimize the multi-objective problem, it can be observed that when the quality ratio $MR=5$, the outer radius $R=100\text{mm}$, and the temperature difference $\Delta T=40\text{K}$, the overall insulation effect is the best. Compared with extreme conditions, under this condition, the

temperature uniformity can be maximally optimized by 94.34%, and the increase in the tube wall temperature can be reduced by 80.39%.

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Data availability

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Competing interests

The authors declare no competing interests.

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