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# Energy Management of Chaotic Brushless DC Motors Using a Lyapunov - Stable Digital State Feedback Controller

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**Abstract -** This article proposes a digital linear feedback controller based on Lyapunov stability. with the aim of improving the modeling and control of chaotic nonlinear systems .in general and in particular for the case of Brushless DC Motor (BLDCM) in the energy management process for industrial instrumentation chains. methodological approach consists of an Analysis of the evolution of the motor chaotic state mathematical function (BLDCM). by demonstrating existence of chaos for certain nonlinear variables of the motor and especially controlling the appearance of phenomena or chaotic movements based on Lyapunov stability theory. three cases of chaotic operation are therefore developed and analyzed with our controller K therefore the parameters are determined by the theory of Lyapunov stability with the conditions of Lorenz and Rossler; the quadratic currents Iq, the angular positions and speeds, the currents Id are also analyzed for its three cases of chaotic operation in the MATLAB SIMULINK 2018.b environment; the results obtained with the values of digital controllers K1=6; .K2=10;K3=4; offer chaotic stability for case 2 (K2=10) with a speed of convergence towards stable equilibrium from only 20 seconds for the Iq currents and chaotic equilibrium in instability for case 3 (K3=4) with currents Id; even if a return to stable equilibrium is not observed in case 3; and finally a chaotic undulation in position is also noted for case 1 (k1=6); the originalities in this article revolve around this methodology for obtaining digital linear feedback controller by the theory of Lyapunov stability on nonlinear chaotic systems.

**Keywords** - brushless DC motor; mathematical model; chaotic nonlinear system; control; Id and IQ currents; dynamic angular velocity; Lyapunov Stability;

## I. Introduction:

Chaotic systems exist in several industrial applications especially in energy production mechanisms, servomotor and motion transmission systems robotic industrial control and instrumentation (CIR), and flying and rolling vehicles for transport activities [1-2]. Therefore, he issues linked to chaotic control are of great importance for several researchers with objectives such as the search for dynamic stability over time based on the theory of Lyapunov stability [3-5].

### I.1 context and background:

The DC motor (BLDCM) which offers many. advantages [6-8] As a less complex system than others, for precise operational safety with less difficult maintenance; high efficiency, almost zero excitation losses, good speed regulation due to its architecture, can unfortunately in certain conditions undergo the chaotic impact at the level of its quadratic model in current. Modeling and Control of DC Motor Model (BLDCM). This remains a nonlinear system with several solutions known in literature over time. However, nonlinear chaotic systems of fractional order were proposed in [9]. Terminal attractors are designed. with fixed schedules, implementation is carried out using a dedicated synergistic observer, non-linear differential equations on the macro-variables of the system are also used, the fusion of the dynamic evolution of chaotic states is established with a synchronization process that uses a single channel to avoid overloads, and the precision of this synchronization offers satisfactory results depending on the initial conditions of the system. However, one of the weaknesses of this approach lies in the variation of the attractors which are directly linked to the nonlinear differential equations of the system; performances or even chaotic degenerations will be linked to the estimates made by the LYAPUNOV stability approach ; In [10] the control of chaotic systems is also developed with a sliding mode controller for an associated state observer; the state parameters, the disturbance observation module and the controller are responsible for optimizing the synchronism of the chaotic system; this estimation device is analyzed in a comparative study to finite-time and asymptotic control with performance indicators such as: IAE, ITAE and ISV. The method is positioned as being robust in the face of uncertainties and disruptions; the virtual validation experimentation platform is the MATLAB/Simulink environment;

### I.2 previous works:

In [11-13] a predefined time sliding mode control supervision model is proposed; the stability of LYAPUNOV is implemented for the synchronization of complex chaotic systems; the simulation results obtained in the MATLAB Simulink environment show that the control is optimized compared to the previous finite-time model, and the synchronization duration is smaller; additionally, the controller parameters can be defined in advance in this configuration; however, the mastery of differential mathematical models remains closely associated with attractors; it is therefore important that the controller is configured according to the constraints of the model; this reality established in the scientific literature crystalizes the multiple research challenges in optimizing the control of nonlinear chaotic systems; multiple additional limitations are also generally described; limits such as [14-17]:

- The ripple of the propagation variables mainly a function of the quadratic current (id) and (iq) and the derivative of the angular position to the non-linearity of the quadratic state model of the system.
- Chaotic instability in position in Lorentz and Rossels time noted and not evaluated
- Significant errors between theoretical and experimental results; difficult control in position over time.

This work proposes and develops the application of Lyapunov stability theory to the design of a linear feedback digital controller for a BLDCM under chaotic conditions. Although Lyapunov stability is a

well-established concept, its adapted implementation for this particular system and the detailed analysis of different chaotic scenarios contribute to its novelty. The in-depth analysis of three distinct cases with varying controller parameters and their corresponding chaotic behaviors allows a detailed understanding of the system response under different conditions. This comparative approach enriches the research. The presentation of phase portraits and timing diagrams for each case allows a clear visualization of the chaotic dynamics and the controller efficiency. The detailed simulation results, including convergence times for different variables, provide valuable information. And the major challenge of BLDCM control highlights: the presence of chaotic instabilities that can degrade performance. By proposing a control strategy based on Lyapunov stability, we propose an optimal development of robust and reliable BLDCM systems. The simulation results provide practical insights into the selection of controller parameters to achieve stability. In addition, the comparative analysis of different cases helps identify the optimal control strategy for specific operating conditions. The study therefore contributes to the advancement of control techniques for nonlinear chaotic systems, particularly in the context of electromechanical devices. And the use of Lyapunov stability theory provides a solid theoretical basis for the proposed control strategy. The results are based on MATLAB/Simulink simulations, which are widely used and accepted by the engineering community. The detailed simulation results, including phase portraits and timing diagrams, are consistent with theoretical predictions. And are similar to those of [18-22]. This strengthens the credibility of the results. Search engines such as Science direct, Google Scholar, Web of Science, Taylor and Francis were used in the documentary research methodology with a permutation of keywords carried out in each search engine, and the selection of the quantity and quality of the number of articles published on the chaotic issues of nonlinear systems, such as the DC motor (BLDCM), made it possible to present Figure 1, which illustrates the current documentary trend on the subject and its importance.

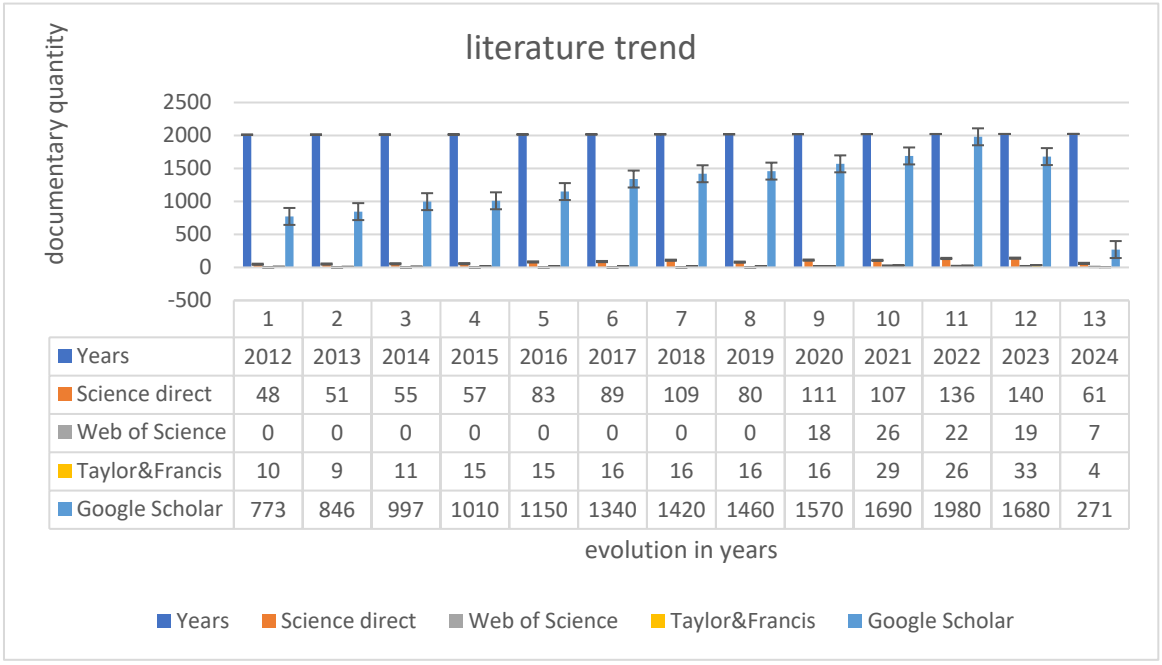



Figure 1: Literature trend on BLDCM motor modeling and chaotic control

**I.3 paper contributions:**

The major contributions of this study are as follows.

**(a)** A summary of the literature on recent work on the modeling and control of DC motors, each time presenting the limits due to the chaotic phenomena of certain non-linear variables of their state model.

**(b)** Show by a dynamic characteristic study evolution of the mathematical function of the chaotic state of the motor (BLDCM), in which variables are non-linear and are at the origin of chaotic phenomena.

**(c)** A digital controller is proposed with the mission of stopping chaotic instability phenomena or movements based on Lyapunov equilibrium conditions with Lorentz and Rossel conditions.

**(d)** Analyzer: Variable current and different ripples during transmission connection and optimal operation with minimization of ripples.

**I.4 paper organization:**

In the rest of this article, in section 2 an in-depth presentation of the current literature on the strategies of Modeling and controls of nonlinear systems such as the DC motor (BLDCM) is presented with emphasis on the chaotic disadvantages of the system; in section 3 a schematic and mathematical modeling of the DC motor (BLDCM) is carried out for the different operating times; in section 4 a formulation of the problem on nonlinearity with the introduction of the lyapunov theory is developed with three possible scenarios by the methods of Lorentz and Rossels; in section 5 the simulations of the results and discussions of the digital controller are carried out demonstrating that the digital controller based on the Lyapunov theory succeeds in stabilizing the system and the chaotic effect at the equilibrium point; in section 6 a conclusion with recommendations and perspectives based on the results obtained in chaotic conditions;

## 2. Recent progress in the literature on modeling and control of (BLDCM):

Table 1 presents recent developments in Modeling and Control (BLDCM) with the advantages and limitations of the different studies over time. this in-depth analysis of the results of the literature offers the possibility of better appreciating the new contributions in this work; with a positioning in relation to this literature.

Table 1: recent progress on modeling an control (BLDCM)

| References | Themes | Types of modeling | Order strategies | Contribution | Disadvantages |
|---|---|---|---|---|---|
| **[23].** | Implementation and Analysis of Digital Performance Controllers for brushless DC motor drives | Matrix approach with simplification of the nonlinearity of the model. | PID controller; fuzzy control | Quick check; Simplicity of experimentation; | Large margins of error between theoretical and experimental results; Control in difficult position over time. |
| [24] | Brushless DC motor torque ripple reduction based on adaptation input-output feedback linearization | Nonlinear mathematical models using the Lyapunov function; | Feedback Linearization (AIOFBL) control ; PI controleur; | Better control of the position parameter over time; good control of quadratic currents; elimination of spectral harmonics; | No in-depth study on the linearization criteria according to the evolution of the chaotic spectrum; |
| [25] | Techniques for controlling a(BLDCM) system with attenuation of parameter ripple in position | Matrix approach with simplification of the nonlinearity of the model. | The technique is a combination of Golden Eagle Optimization (GEO) and Radial Basis | The THD value varies respectively from 1.26; 0.9951; and 7.4 for the quantities of current, power and derivative of the position vector for | transmission is a function of the derivative of the angular position which is chaotic over time |

| | | | | | |
|---|---|---|---|---|---|
| | | | Function Neural Network (RBFNN), hence its name GEO-RPFNN | the strategy developed in the article | |
| [26] | optimize motor speed performance using a DC-DC converter | Matrix model with association of the Laplace transformation for quadratic currents | PI controleur | An improvement in the voltage profile is obtained over a period by associating a variation in the number of turns of the inductor one coil is fully linked to the other | unstable speed control and changes to regulation instructions |
| **[27]** | motor control with a fuzzy PID optimizer | Matrix approach with simplification of the nonlinearity of the model. | Fuzzy PID | control and prediction of chaotic states are developed as solutions in MATLAB | Large margins of error between theoretical and experimental results; Difficult control in position over time. |
| [28] | Brushless low-speed sensorless control method DC motor based on high frequency pulsed voltage injection | Matrix approach with simplification of the nonlinearity of the model. | PID regulator | an evaluation of positions for low speeds makes it possible to propose a model of controllable chaotic evolution | Chaotic instability in position over time |
| [29] | Brushless DC motor speed control using an optimized bat algorithm Adaptive neuro-fuzzy inference system | Matrix approach with simplification of the nonlinearity of the model. | optimized algorithm Adaptive neuro-fuzzy inference | performance indicators are evaluated to test the robustness of the controller | Chaotic instability in position over time |
| [30] | evaluation of motor parameters in terms of stability | Matrix approach with simplification of the nonlinearity of the model. | Experimental approach | improvements in the evolution of the magnetic field on the motor axes with chaotic stability | Chaotic instability in position over time |
| [31] | Brushless DC Chaos System Nonlinear State Feedback Control engine | Matrix approach with Lyapunov for nonlinear systems | Lyapunov Stability theory | In-depth control of quadratic current and position; price taking into account non-linearity. | Chaotic instability of Lorentz and Rossels observed and not evaluated |

In view of the different issues noted in the literature on the chaotic phenomena of non-linear systems, such as DC motors (BLDCM), it is therefore important to begin section three with a schematic and mathematical illustration with the ultimate aim of better understanding the birth of the chaotic state at the model level of the system studied

## 3. Schematic and mathematical modeling of DC motor (BLDCM):

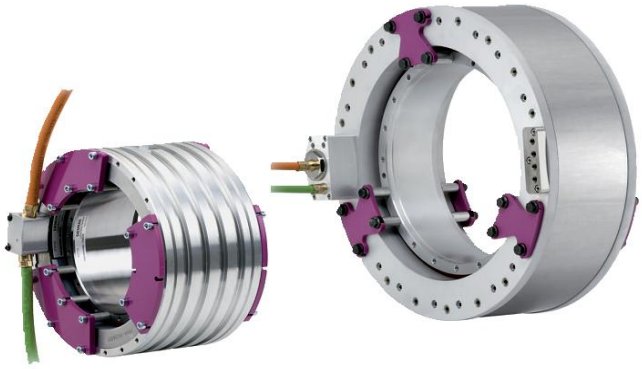


**Figure 2: 1FW6 motor torque from Siemens Drive Technologies. [8]**

Figure 2 presents a motor model (BLDCM) regularly known for drive cases with a reduced shaft and having a very high torque. This category of motor is widely used in industry, reaching maximum powers of 61.3KW and minimum powers of 17 kW, with oscillating rotation speeds up to 650 rpm as the maximum speed and 38 rpm as the minimum speed, and the supply voltage for this prototype is 400/480V; with a significant overload capacity of 2/Mn. Figure 3 presents a view of the power supply instrumentation chain using a converter that stabilizes the regime of the input variables at the level of current, voltage, and especially frequency; in its internal architecture for the motor (BLDCM), manufacturers generally position a recessed winding at the stator level; [32-35].

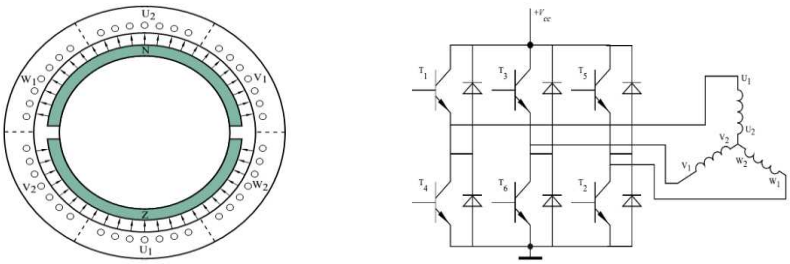


**Figure 3: front view and power module for a (BLDCM)**

### 3.1 mathematical models:

Several modeling approaches have been developed in the literature, and the most common are those that neglect chaotic phenomena in operating hypotheses. Our objectives in this part are to first present the very simplified mathematical models associated with the hypotheses; second, a non-linear formulation closer to reality will be developed and exploited for the design of a digital controller based on the theory of the stability of the *Lyapunov*, he most common of which is three phases, four poles, connected in a trapezoidal Y DC type (BLDCM) counter-electromotive force is modeled; a mutual inductance is added between the stator and the rotor [14] Several operating conditions or constraints are defined for optimal operation: the first concerns the module or the magnetic field circuit which is simply neglected among the main constraints; then as a second constraint the value of the resistance at the stator level is internal and mutual. A third constraint would like a balance to be established at the phase level with continuous stability over time, with cancellation of ripples including leakage currents. The mathematical models of the evolution of the permanent electrical regime are defined according to the following details [15]:

BLDCM Motor for the voltage and phase regime:

$$\begin{cases} V_a = Ri_a + (L - M)\frac{di_a}{dt} + E_a \\ V_b = Ri_b + (L - M)\frac{di_b}{dt} + E_b \\ V_c = Ri_c + (L - M)\frac{di_c}{dt} + E_c \end{cases} \quad (1)$$

DC Motor Back EMF (BLDCM) Equations:

$$\begin{cases} E_a = K_e\omega_m F(\theta_e) \\ E_b = K_e\omega_m F\left(\theta_e - \frac{2\pi}{3}\right) \\ E_c = K_e\omega_m F\left(\theta_e - \frac{4\pi}{3}\right) \end{cases} \quad (2)$$

Torque equations for each phase of DC motor (BLDCM)

$$\begin{cases} T_a = K_t i_a F(\theta_e) \\ T_b = K_t i_b F\left(\theta_e - \frac{2\pi}{3}\right) \\ T_c = K_t i_c F\left(\theta_e - \frac{4\pi}{3}\right) \end{cases} \quad (3)$$

The electromagnetic torque is: $T_e = T_a + T_b + T_c$ (4)

**3.2 Dynamic operation:**

In this section the main objective is to establish the main switching sequences for the motor (BLDCM) during its dynamic operating regime over time; Figure 4 therefore illustrates the six steps of the different sequences; a modeling of the rotor is carried out using a magnetic bar and a simplification with a single winding at the stator level is shown.

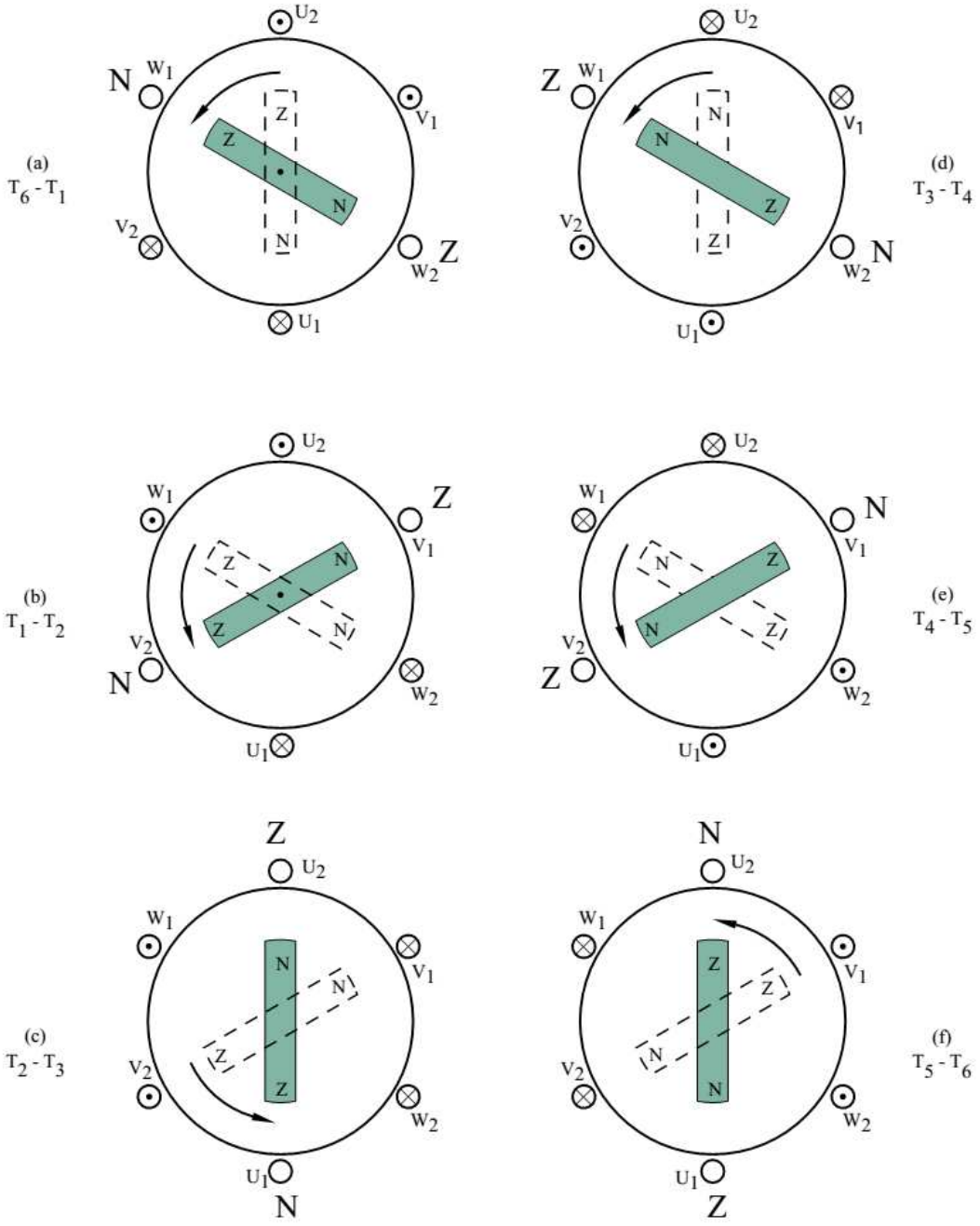



**Figure 4: Dynamic evolution of the six stages of motor operation (BLDCM)**

**4. Formulation of the problem on the nonlinearity of the DC motor (BLDCM):**

The objective of this part is to establish that the DC motor (BLDCM) is a non-linear system that is unstable and chaotic for certain conditions at the level of its variables. A physical approach to the model allows us to propose figure 1, which can be described by Hemati et al (1994) and Chang et al. (2004).

$$\frac{d}{dt}I(t) = L^{-1}(\theta)[\,V(T) - RI(t) - \left(\frac{dL(\theta)}{d\theta}I(t) + \frac{d\delta(\theta)}{d\theta}\right)\frac{d\theta}{dt}], \qquad (5)$$

where I(t) is the instantaneous current value, L(θ) is the matrix value for inductance, V(t) is the instantaneous voltage value, R is the matrix value of the resistance winding, δ(θ) is the magnitude of the flux generated by the magnets, and (θ is) angular displacement. The mathematical evolution in this case over time is defined as:

$$\frac{d}{dt}w = \frac{1}{J}[\,T(I,\theta) - T_l(t)] \quad (6)$$

where w is the angular speed of the rotator, is the inertia of the rotator $T(I,\theta)$ electromagnetic parameter, and $T_l(t)$ is the external parameter influencing the motor shaft. Considering viscous damping friction, the external torques can be described by

$$T_l(t) = bw + T_L \quad (7)$$

where b is the viscous damping coefficient, and $T_L$ is the torque due to the external load, notch effect, and Coulomb friction; thus far, equations (5) and (6) clearly depend on (θ). This already presents strong nonlinearity in the system. Therefore, the following park transformation is used:

$$\frac{d}{dt}i_q = \frac{1}{L_q}[-Ri_q - n\omega(L_d i_d + K_t) + v_q] \quad (8)$$

$$\frac{d}{dt}i_d = \frac{1}{L_d}[-Ri_d + nL_q\omega i_q + v_d] \quad (9)$$

And the electromagnetic torque is described by:

$$T(i_q, i_d) = n\big[\,k_t i_q + (L_d - L_q) i_q i_d\big], \quad (10)$$

On the other hand on a:

$$\delta = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix};\ E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix};\quad (11)$$

### *4.1 Equations of state with chaotic variables:*

The motor model (BLDCM) presents a balance between voltage and torque [9], and simplification assumptions are made if the dimensions are the same. Therefore, the mathematical model is as follows:

$$\begin{cases} \dot{x}_1 = v_q - x_1 - x_2x_3 + \rho x_3 \\ \dot{x}_2 = v_d - x_2 + x_1x_3 \\ \dot{x}_1 = \sigma(x_1 - x_3) - T_L \end{cases} \quad (12)$$

where the state variables $x_1x_2, x_3$ are the current iq equivalent to the motor on the axis q, the current id of the d axis, and the speed ω, ϱ, and σ are the values belonging to the model. as additional constraints, the motor is empty, that is, no consideration of loads and no voltage values ; $v_q$ = 0; and $v_d$=0, with $T_L$=0. This operating condition is similar to a perfect balance, similar to stopping the motor, and the mathematical model is therefore:

$$\begin{cases} \dot{x}_1 = -x_1 - x_2x_3 + \rho x_3 \\ \dot{x}_2 = -x_2 + x_1x_3 \\ \dot{x}_1 = \sigma(x_1 - x_3) \end{cases} \quad (13)$$

The variables (ϱ, σ) in the mathematical model (12) are disturbed by the temperature and chaotic instability in the motor (BLDCM); researchers Hémati et al. (1994) and Wang Mufeng et al. (2016) demonstrated that for certain windows of values at the level of the variables, the engine evolves in a chaotic regime; the mathematical model (12) and globally invariant symmetrically to X2. Therefore, the mathematical model does not change for (x1, x2, x3)→(-x1, x2, -x3); therefore, the mathematical model

(12) gives up or releases energy when time converges towards 'infinity. ' Chaotic exponential undulation in the form of a spiral is created for elementary displacements in volumes; therefore, it becomes essential to control this chaotic spiral by acting on the components.

or variables of the volume displacement over time; extremities are therefore defined on the basis of a scanning balance calculation algorithm; thus, for the values ϱ=20, σ=5.46, with the starting conditions (x1, x2, x3) = (0.1.10, -1.5), the coefficients based on the Lyapunov equilibrium are established numerically in the literature owing to the synthesis methods of Briggs et al. (1990) ; E1 = 0.463, E2 = 0, E3 = -7.923, and the index n =2.059 is also calculated for the dimension level; this configuration therefore confirms the unstable chaotic state of the system. The reasons for the equilibrium conditions in the sense of Lapunov stability are developed in the following section for our case study:

### *4.2 Lyapunov theory of stability*

The exponential coefficients of Lyapunov cumulatively constitute chaotic spirals; thus, when a system loses energy, it will present at least a negative exponential coefficient. With the equilibrium constraint a convergence towards o, we observe a limit in the volume element for an extremum [14]. The propagation of the exponential Lyapunov coefficients is opposed to the spectral displacement of extrema. Each exponential coefficient indicates a trajectory with a well-defined direction. Therefore, the system develops repetitions around this condition. A series of undulations, therefore, arises by causing uncertainty in the process of evolution or forecasts; it is chaos [15-16]. Therefore, the exponential coefficients are present and offer a qualitative vision of the chaotic evolution of the system. [18-20]; dimensional levels are defined for stable orbitals. They are counted negatively for a regime of periodic evolution; in this configuration the couples (+,0, -), (0,0,-), (0, -, -), (-, -, -), are the only ones possible at a fixed point [13-14] ; Table 2 provides a summary of the exponential coefficients for which the differential mathematical models are known.

$$\gamma_i = \lim_{t\to\infty} \frac{1}{t} \log \frac{P_i(t)}{P_i(0)} \quad (14)$$

**Table 2**: Summary of the numerical values calculated for the exponential laypunov coefficients corresponding to the defined differential models [13]

| System | Parameter values | Lyapunov spectrum (bits/s) | Lyapunov dimension |
|---|---|---|---|
| • **Henon et al ( 1976)** $\begin{cases} X_{n+1} = 1 - aX_n^2 + Y_n \\ Y_{n+1} = bX_n \end{cases}$ | $\begin{cases} a = 1.4 \\ b = 0.3 \end{cases}$ | $\begin{cases} \gamma_1 = 0.603 \\ \gamma_2 = -2.34 \end{cases}$ | 1.26 |
| • **Rossler –chaos et al (1976)** $\begin{cases} \dot{X} = -(Y + Z) \\ \dot{Y} = X + aY \\ \dot{Z} = b + Z(X - C) \end{cases}$ | $\begin{cases} a = 0.15 \\ b = 0.20 \\ c = 10 \end{cases}$ | $\begin{cases} \gamma_1 = 0.13 \\ \gamma_2 = 0.00 \\ \gamma_3 = -14.1 \end{cases}$ | 2.01 |
| • **Lorenz et al (1983)** $\begin{cases} \dot{X} = \sigma(Y - X) \\ \dot{Y} = X(R - Z) - Y \\ \dot{Z} = XY - bZ \end{cases}$ | $\begin{cases} \sigma = 16.00 \\ R = 45.92 \\ b = 4.0 \end{cases}$ | $\begin{cases} \gamma_1 = 2.16 \\ \gamma_2 = 0.00 \\ \gamma_3 = -32.4 \end{cases}$ | 2.07 |

| | | | |
|---|---|---|---|
| • **Rossler-hyperchaos et al (1979):** $\begin{cases} \dot{X} = -(Y + Z) \\ \dot{Y} = X + aY + W \\ \dot{Z} = b + XZ \\ \dot{W} = cW - dZ \end{cases}$ | $\begin{cases} a = 0.25 \\ b = 3 \\ c = 0.05 \\ d = 0.5 \end{cases}$ | $\begin{cases} \gamma_1 = 0.16 \\ \gamma_2 = 0.03 \\ \gamma_3 = 0.00 \\ \gamma_4 = -39.0 \end{cases}$ | 3.005 |
| • **Mackey-Glass et al (1977) :** $\dot{X} = \frac{aX(t+s)}{1 + [X(t+s)]^c} - bX(t)$ | $\begin{cases} a = 0.2 \\ b = 0.10 \\ c = 10 \\ s = 31.18 \end{cases}$ | $\begin{cases} \gamma_1 = 6.30.10^{-3} \\ \gamma_2 = 2.62.10^{-3} \\ \gamma_3 = 8.10^{-6} \\ \gamma_4 = -1.39.10^{-2} \end{cases}$ | 3.64 |

**4.3 new non-linear model of the digital state controller:**

In view of the above, it is therefore established that the DC motor (BLDCM), the chaotic instability caused for certain ranges of values, requires control over time with the main aim of controlling the system and ensuring the stability of the system quickly. The reason why a DC motor feedback controller (BLDCM) is proposed and translated by the mathematical model is as follows:

$$\begin{cases} U_1 = x_2x_3 - \rho x_3 \\ U_2 = -x_1x_3 \\ U_3 = -\sigma x_1 + Kx_3 \end{cases} \quad (15)$$

With k the parameter controller (15) is introduced (12), the controller (15) is introduced, and The DC (BLDCM) mathematical equation thus obtained is (16):

$$\begin{cases} \dot{x}_1 = -x_1 - x_2x_3 + \rho x_3 + U_1 \\ \dot{x}_2 = -x_2 + x_1x_3 + U_2 \\ \dot{x}_3 = \sigma(x_1 - x_3) + U_3 \end{cases} \quad (16)$$

The Lyapunov stability theory allows us to construct a Lyapunov function $L\,(x_{1,}x_2,x_3)$ appropriate which is based on the chaotic system, with the possibility of gradually stabilizing the chaotic state for the following conditions:

$$L\big(x_{1,}x_2,x_3\big) = \tfrac{1}{2}(x_1^2 + x_2^2 + x_3^2) \quad (17)$$

With $L\,(x_{1,}x_2,x_3) > 0$, by deriving equation (5):

$$\dot{L}\big(x_{1,}x_2,x_3\big) = \dot{x}_1x_1 + \dot{x}_2x_2 + \dot{x}_3x_3 = -\big[x_{1,}x_2,x_3\big]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sigma - K \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_{3,} \end{bmatrix} = X^TPX \quad (18)$$

To obtain the positive matrix P, it is important to satisfy the following constraints: K > $\sigma$ ;

On the other hand, for K > $\sigma$ and $\dot{L}\big(x_{1,}x_2,x_3\big) < 0$ the digital controller defined for feedback tomodel (15) will have the main mission of stabilizing the chaotic evolutions towards equilibrium under the conditions of the Laypunov theory.

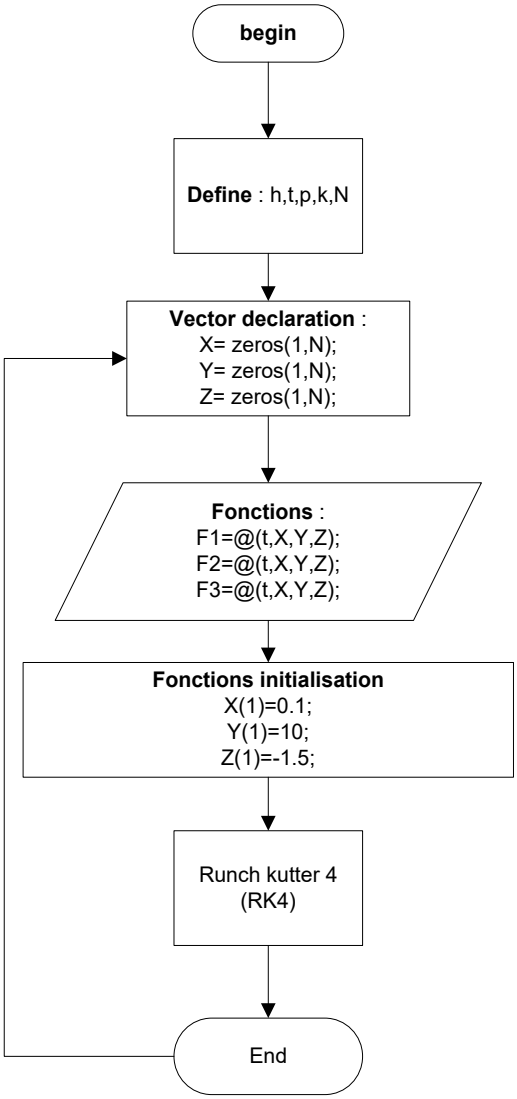



**Figure 5: Flowchart of the proposed digital controller program**

Figure 5 therefore proposes the program algorithm of the digital controller of the system, it is carried out in several stages as presented in Figure 5; it integrates Table 2: Summary of the numerical values calculated for the exponential laypunov coefficients corresponding to the defined differential models [13] according to the chaotic nonlinear mathematical model obtained; the parameters and dimensions of lyapunov are therefore calculated numerically in accordance with the chaotic model in the presence

## 5. Results and discussions:

Figure 6 presents the spectrum of state values in currents, voltage, and position for the dynamic operation of the DC motor (BLDCM); therefore, it becomes essential to analyze in depth the chaotic behavior of the quadratic currents Iq and Id, as well as the undulations in positions for chaotic speeds in the sense of Lyapunov.

### *5.1 spectrum of output dimmers:*

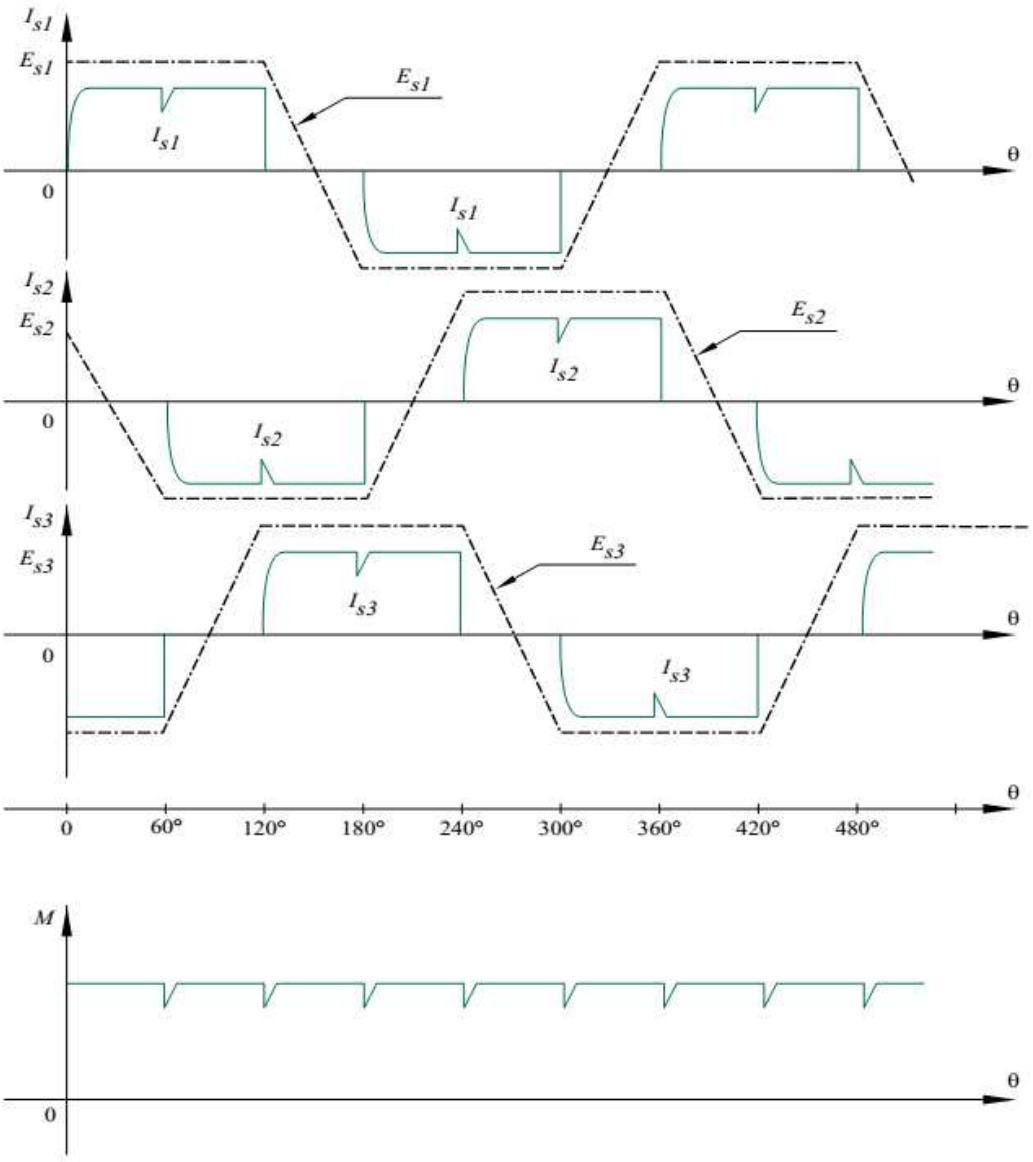



**Figure 6; track of system parameters in dynamic mode**

***5.2 results of chaotic variables of the system*** (BLDCM) ***case 1:***

This section presents the results for the portrait and chronogram phases for case 1, in which the value of the controller K1= 6, and the values ϱ=20; = 5.46.

a) ***The phase portrait case 1***:

Figure 7 presents case 1 for the Lyapunov stability theory with our digital controller, which has the values K1=6, p = 20, and = 5.46, in which the chaotic phenomenon is analyzed between x and z by the curve z=f(x); this is the chaotic state between the quadratic current Iq and the angular velocity; the same is true for the curve y=f(x), which displays the chaotic state between currents Iq and Id; finally, the curves Z=f(y) and curves (z,y,x), which, respectively, analyze the chaotic state between the speed and the currents Id, then a global interaction between the currents Id, Iq, and the speed in position.

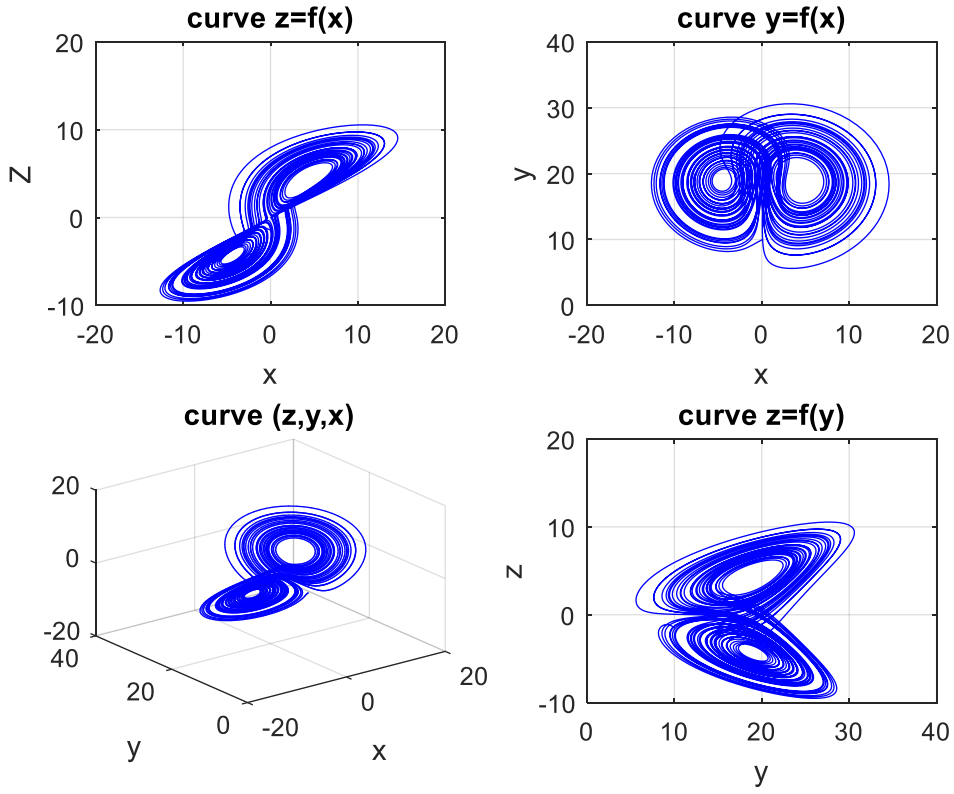



*Figure 7: phase portrait of case 1 (K1= 6; and values ρ=20; σ=5.46 ;)*

*a) timing diagram of the system case 1 :*

Figure 8 also shows the results obtained with our K1 controller for the variables x,y, and z, which represent the currents Iq, Id, and the speed in position; a very strong chaotic instability is noted for the currents (Iq) represented by the variable X. A strong oscillation around the zero-equilibrium position is noted; the digital controller fails to stabilize the system in the Lyapunov sense; and the alternation of signs of the amplitudes in this case1 already signals a very unstable chaotic state. On the other hand, the currents (Id), although not finding a stable equilibrium position at zero, present a relatively stable chaotic oscillation with a practically constant positive amplitude variation. Therefore, the seat of a controlled chaotic state is less dangerous for the system. with amplitudes between 10A and 30 A; Finally, the z component, which represents the undulations in speed, also reflects an unstable chaotic state that is dangerous for the system because the oscillations take place around the zero-equilibrium position with strong variations of unequal amplitudes (-10 a 10 rad/s)

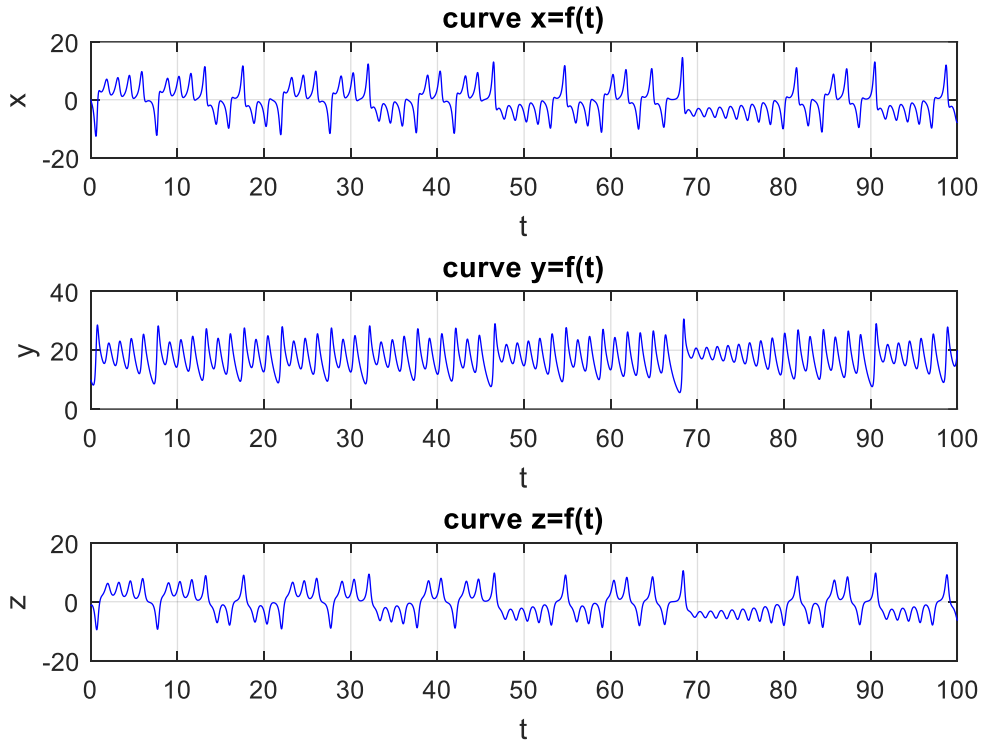



*Figure 8: timing diagram of the system case 1(K1= 6; and values ρ=20; σ=5.46)*

***5.3 results of chaotic variables of the system*** (BLDCM) ***case 2:***

This section presents the results for the portrait and chronogram phases for Case 2, in which the value of the controller K2= 10 and the values ϱ=0.15; σ=0.20.

***a) The phase portrait case 2:***

Figure 9 presents case 2 for the Lyapunov stability theory with our digital controller, which has the values K2 = 6, ϱ=0.15, and =0.20; in this case2 the constraints of Rossler –Chaos et al. (1976) in table 2 are integrated at the level of the digital controller. in this simulation result the chaotic phenomenon is analyzed between x and z by the curve z=f(x); this is the chaotic state between the quadratic current Iq and the angular speed; we note the achievement of equilibrium in zeros by the system; the digital controller successfully stabilized the system for the evolution of the variables present ; the same for the curve y=f(x) which displays the chaotic state between the currents Iq and Id; a convergence towards zero balance is noted for the two currents; the digital controller therefore succeeds in stabilizing the system present according to the behavior of the currents Id and Iq; finally the curves Z = f(y), which also reach its equilibrium in zeros and curves (z,y,x) which respectively analyze the chaotic state between the speed and the currents Id then a global interaction between the currents Id, Iq, and the speed in position; the Lyapunov conditions are verified in this case 2 with satisfactory stability at the level of the digital controller;

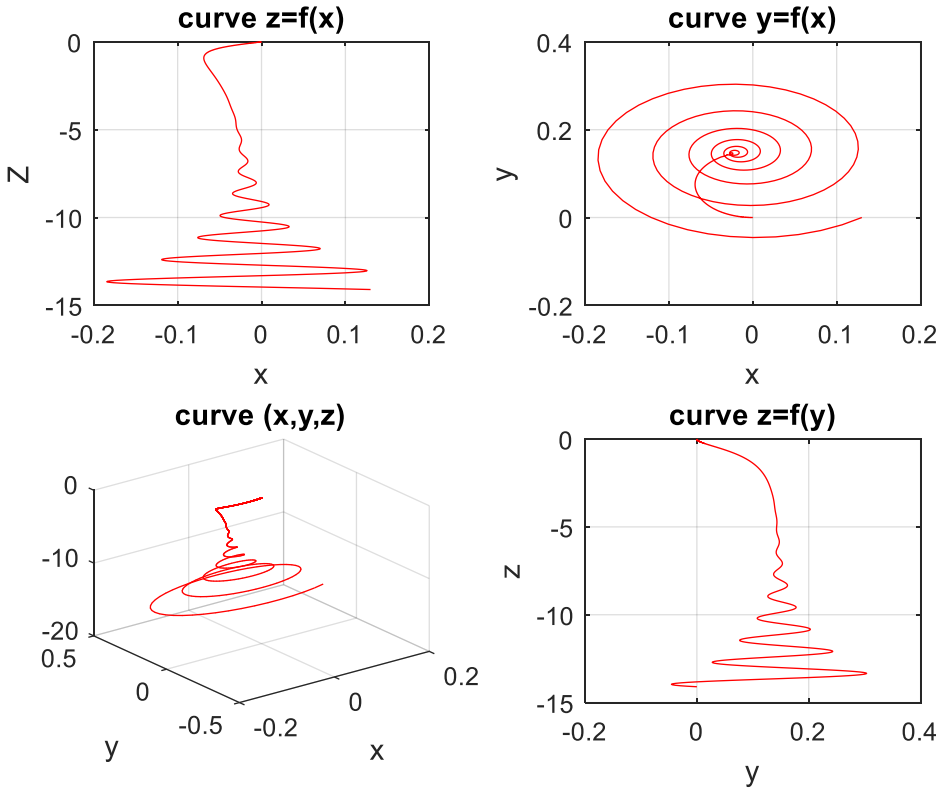



**Figure 9: phase portrait case 2 (K2 = 10;$\rho = 0.15$ ; $\sigma = 0.20$ )**

*a) timing diagram of the system case 2 :*

Figure 10 also analyzes the results obtained with our K2 controller; the constraints of Rossler – Chaos et al. (1976) in Table 2 are integrated at the level of the digital controller; for the variables x,y, and z, which represent the currents Iq, Id, and the speed in position, we see a very strong chaotic stability for the currents (Iq) represented by the variable X; because beyond 30 s, the digital controller succeeds in bringing the system back to zero for a stable balance. Similarly, the currents (Id) find a stable equilibrium position in zero, presenting a stable chaotic oscillation with a practically constant positive amplitude variation. And this from just 20 seconds proves that the digital controller offers better stability in this configuration; It is therefore the seat of a chaotic state completely controlled after only 20 seconds; and less dangerous for the system; finally the z component which represents the undulations in speeds, also translates a stable chaotic state for the system; because the oscillations reach the zero equilibrium position after only 20 seconds. In this configuration, the chaotic state is very well controlled.

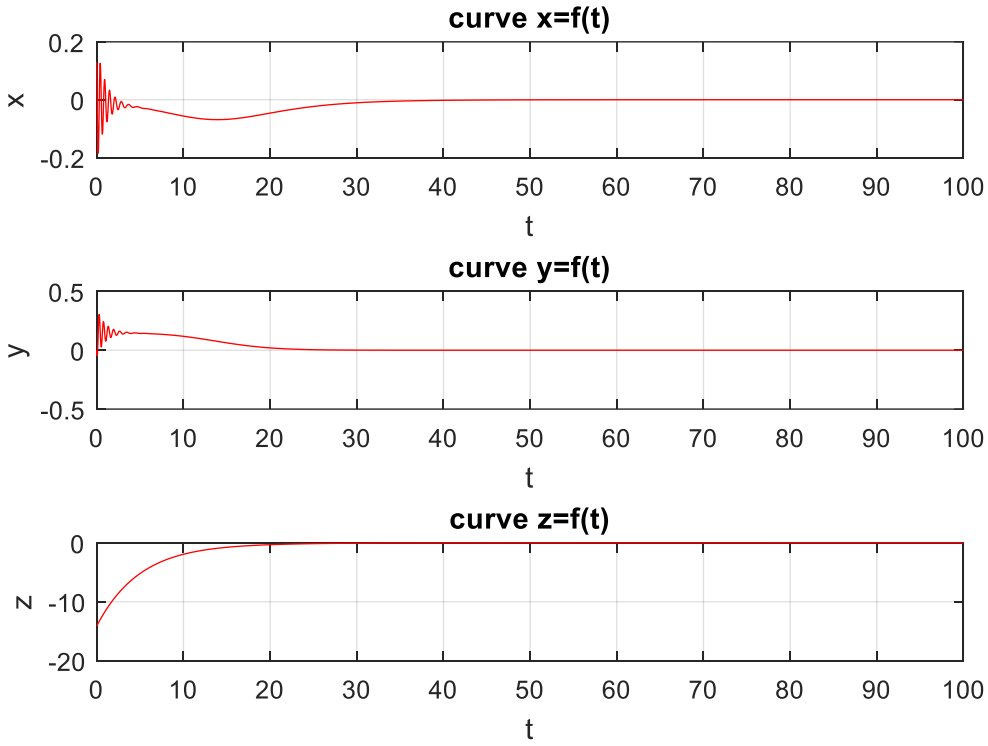



**Figure 10: timing diagram of the system case 2(K2 = 10;$\boldsymbol{\rho = 0.15}$ ; $\boldsymbol{\sigma = 0.20}$ )**

***5.4 results of chaotic variables of the system*** (BLDCM) ***case 3:***

This section presents the results for the portrait and chronogram phases for case 3, in which the value of the controller K3= 4, and the values ϱ=16 and σ=45.92.

***a) The phase portrait case 3 :***

Figure 11 presents case 3 for the Lyapunov stability theory with our digital controller, which has the values K3= 4, ϱ=16, and = 45.92; in this case3 the Lorenz et al. (1983) constraints in table 2 are integrated at the level of the digital controller. in this simulation result the chaotic phenomenon is analyzed between x and z by the curve z=f(x); this is the chaotic state between the quadratic current Iq and the angular speed; we note the achievement of equilibrium in zeros by the system; the digital controller successfully stabilized the system for the evolution of the variables present ; the same for the curve y=f(x) which displays the chaotic state between the currents Iq and Id; a convergence towards zero balance is noted for the two currents; the digital controller therefore succeeds in stabilizing the system present according to the behavior of the currents Id and Iq; finally the curves Z = f(y), which also reach its equilibrium in zeros and curves (z,y,x) which respectively analyze the chaotic state between the speed and the currents Id then a global interaction between the currents Id, Iq, and the speed in position; the Lyapunov conditions are verified in this case 3 also with satisfactory stability at the level of the digital controller;

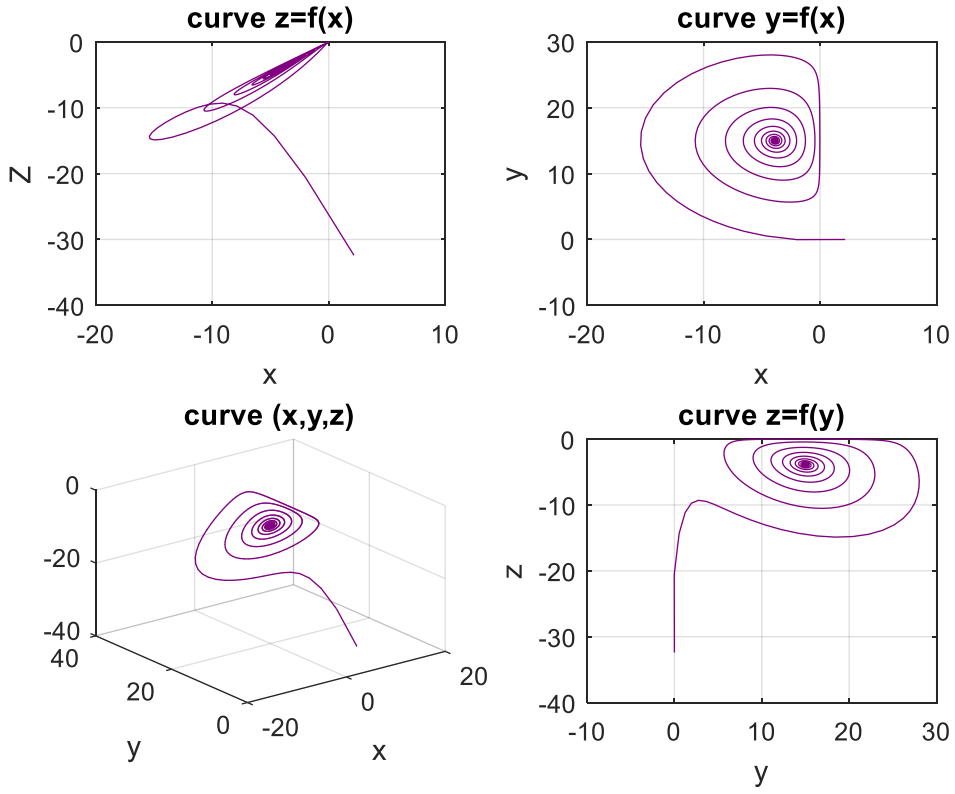



*Figure 11: phase portrait of case 3 (K3= 4; and values ρ=16; σ=45.92)*

*a) timing diagram of the system case 3:*

Figure 12 also analyzes the results obtained with our K3 controller; the Lorenz et al (1983) constraints in table 2 are integrated at the level of the digital controller; for the variables x,y,z which represent the currents Iq, Id, and the speed in position; we see a very strong chaotic stability for the currents (Iq) represented by the variable X; because beyond 10 seconds the digital controller succeeds in bringing the system back to (-4A) for a stable equilibrium. It is therefore a totally controlled chaotic state. Similarly, the currents (Id) find a stable chaotic position from 10 for a continuous value of 15 A, presenting a stable chaotic oscillation with a practically constant positive amplitude variation. proof that the digital controller offers better stability in this configuration; It is therefore the seat of a chaotic state completely controlled after only 10 seconds; and less dangerous for the system; finally the z component which represents the undulations in speeds, also translates a stable chaotic state for the system; because the oscillations reach the chaotic equilibrium position after only 10 seconds. In this configuration, the chaotic state is very well controlled.

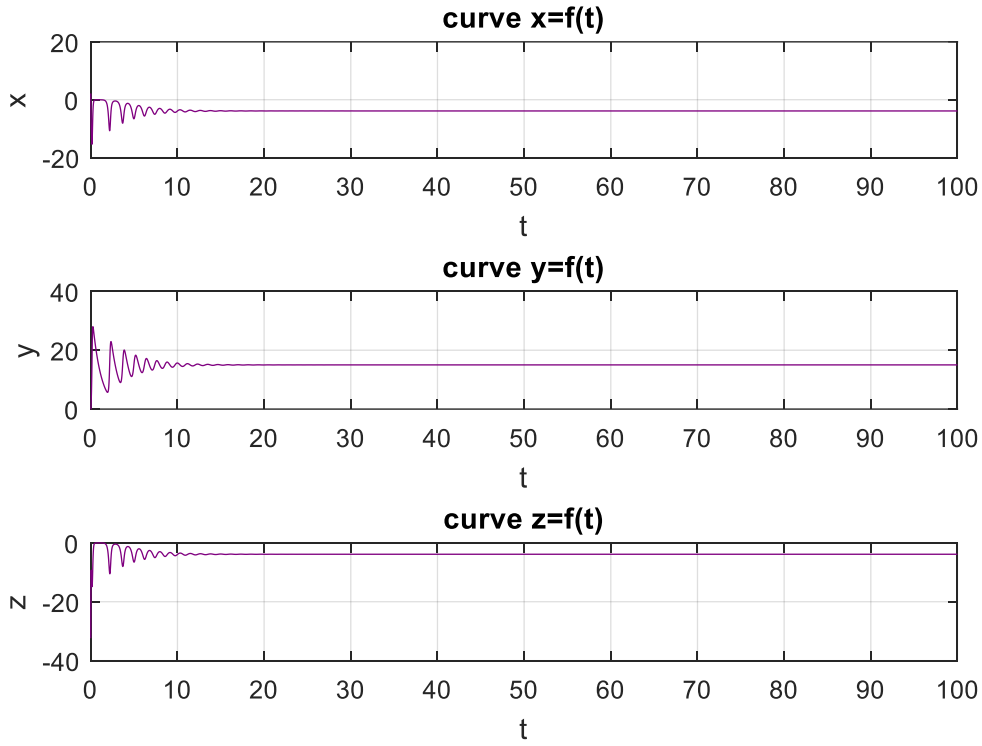



**Figure 12: chronogram of system case 3 (K3=4; and values ϱ=16; σ=45.92)**

In view of the above, cross-analysis allows us to better analyze the behavior of our digital controller in the three operating cases, with the aim of ultimately classifying the chaotic states of the system according to Lyapunov's stability theory.

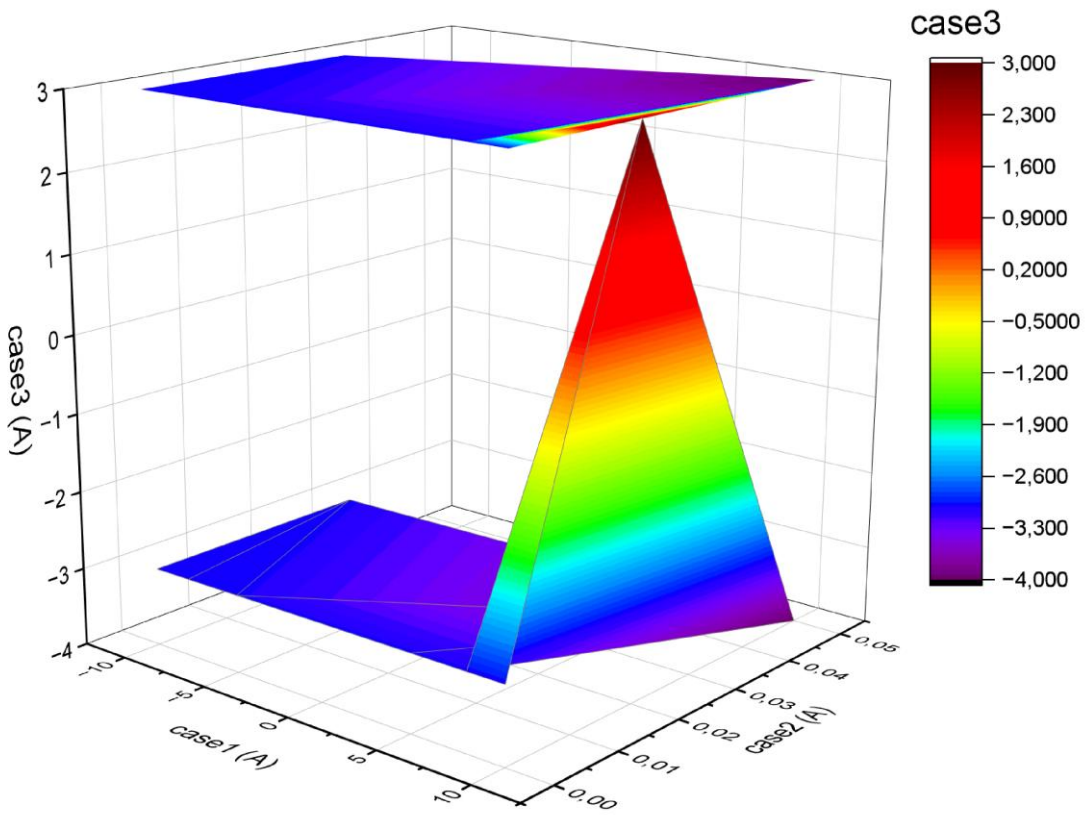



Figure 13: Analysis of the cross-evolution of the quadratic currents X1 for the three cases of the controller K

Figure 13 shows the behavior of x1, which represents the evolution of the quadratic current (Iq). Corrector K varies with the values K1 =6, K2=10, and K3=4; for the three simulation cases, we see that case 2 is the most stable because the quadratic current is almost zero beyond 40 s. The chaos quickly stabilized. However, we see stability in chaos for case 3; although the current does not cancel out, it manages to stabilize beyond 40 s. Therefore, the system is stable for cases2 and cases3. On the other hand, Case 1 presents an uncontrollable chaotic state for the system. Therefore, the K1 corrector does not succeed in stabilizing case1. Therefore, Lyapunov's theory of stability has not been established for this scenario.

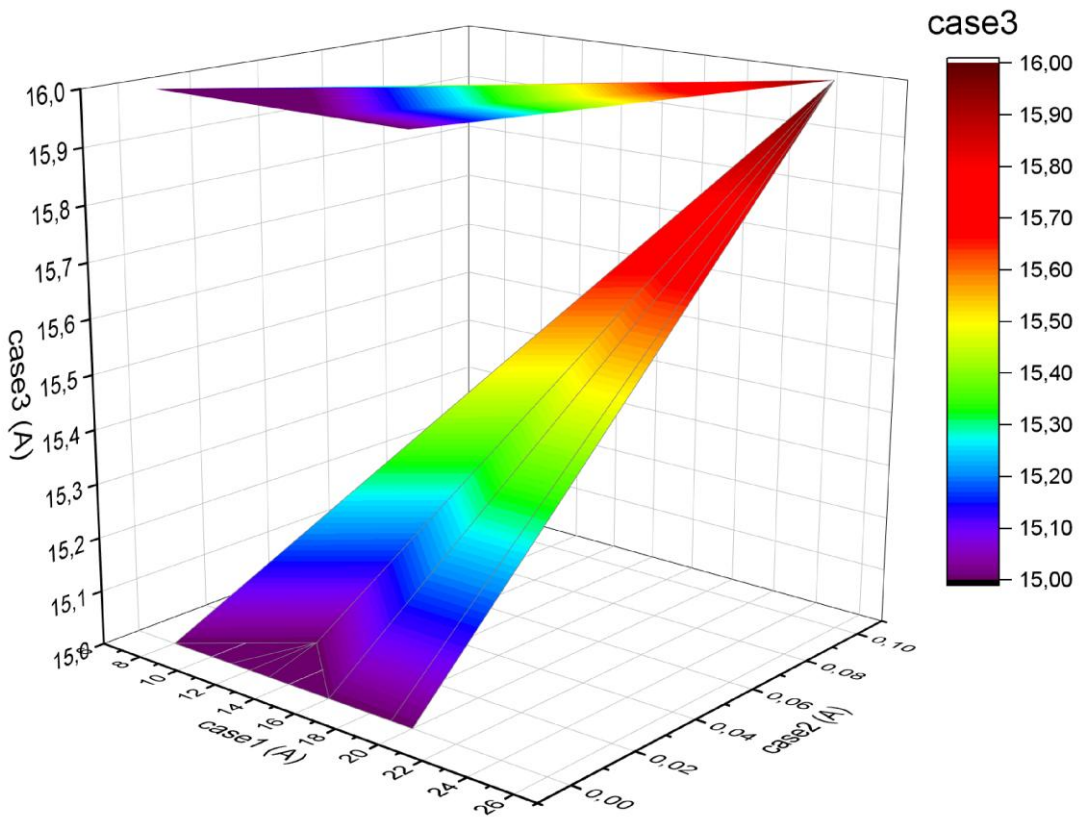



Figure 14: Analysis of the cross-evolution of currents X2(Id) for the three cases of controller K

Figure 14 also presents a cross synthesis for the three cases of the controller (K1, K2, K3) case2 is the most stable because beyond 20 s, the current X2 (Id) is zero, stability in the sense of Lyapunov is established, balance is quickly achieved, and case 3 is also stable beyond 20 at 15 A, although equilibrium is not achieved. Therefore, we have an equilibrium of the instability of the chaotic state in the Lyapunov sense, and Case 1 presents an instability in the sense of Lyapunov for the value of the corrector K1.

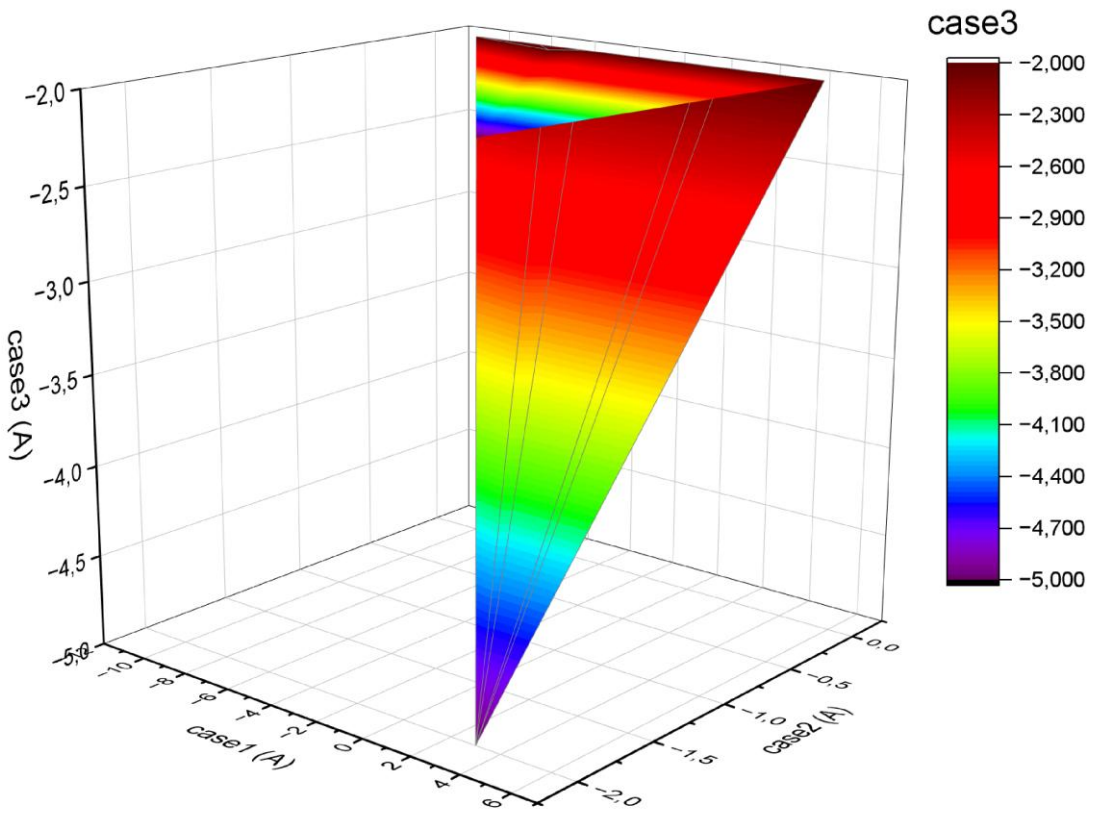



**Figure 15: speed positions**

Figure 15 also presents chaotic stability for case2 of the K2 corrector for the position in speed with rapid convergence to equilibrium beyond 20 s. Case 3 also offers chaotic stability from 20 s for the K3 corrector, although balance is not achieved. and case1 remains in a permanent and unstable chaotic state. The results obtained in this study conform to the requirements of the stability of Lyapunov theory; moreover, they are similar to the work of Onyebuchi et al. (2023) in terms of the variations in the undulations in positions for chaotic conditions and moments; they also conform to the different scenarios of Ge et al. (2006) for chaotic spectra for certain values.

**Challenges and future scope**:

• Future work involving real-time implementation will require a Lyapunov controller—deployed on a DSP, FPGA, or TI C2000 platform with a sampling frequency exceeding 20 kHz—to ensure robust experimental validation.

• Couple state feedback with an Extended Kalman Filter (EKF) or a sliding mode observer to estimate unknown load torque, thereby testing robustness against parameter variations (R, L, J) and measurement noise.

• Hybridize the controller with LSTM or Reinforcement Learning (RL) to adapt Lyapunov gains online and optimize energy consumption while guaranteeing system stability.

• Extend the application of these results to drones/UAVs—addressing issues such as flight instability or limited autonomy—or to industrial pumps and fans.

## 6. Conclusion :

In this article the aim was to contribute to the modeling and control of non-linear systems for the case of the brushless motor through a linear state control methodological approach making it possible to eliminate and It is imperative to stabilize the chaotic evolution inside the system by suitable control which integrates the theory of Lyapunov stability; an analytical study of the dynamic evolution of the engine (BLDCM) demonstrates the existence of chaos for some nonlinear motor variables; the reasons why the linear feedback controller was proposed and simulated based on Lyapunov stability; because the successful control method asymptotically stabilizes the system up to the equilibrium point. An in-depth analysis of the literature was also proposed to demonstrate the challenges in the study of chaotic nonlinear systems. The main contributions of this study are as follows.

(i) show by a dynamic characteristic study of the mathematical state model for the BLDCM motor system; therefore, the variables are non-linear and are at the origin of chaotic phenomena for certain values.

(ii) proposed a digital controller with the mission of stopping the birth of chaotic phenomena or movements based on the theory of Lyapunov stability under the conditions of Lorentz and Rossel.

(iii) Analyzer: Variable current and different ripples during transmission connection and optimal operation with minimization of ripples.

This work is therefore positioned as a major contribution in the modeling and control of nonlinear systems in general and that of the BLDCM engine, in particular in energy instrumentation chains with multiple advantages for sustainable development. Therefore, it is important in this work to design a meta-heuristic algorithm that can automatically optimize the determination of the coefficients K of the corrector beyond the numerically known values for certain mathematical equations, as shown in table 2.

**CRediT author statement:**

**Luc Vivien Assiene Mouodo** : Conceptualization, Methodology, Software, Writing-original draft preparation Paune Félix: Validity tests, Data curation, Visualization, Investigation and supervision ; Ekemb Gabriel : Conceptualization, Methodology and supervision; **Petros Axaopoulos** : Conceptualization, Methodology and supervision ; English language proofreading and monitoring

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability statement**

There were no data to support this study. We agree to share the data upon reasonable request.

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‘Ethics, Consent to Participate, and Consent to Publish declarations: not applicable ;

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