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Thermodynamic properties of anisotropic diluted semimagnetic semiconductor quantum dots

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Abstract

This study investigates the thermodynamic and magnetic properties of electrons in diluted magnetic semiconductor quantum dots with anisotropic harmonic potential under the influence of a magnetic field. The dependence of the specific heat capacity and magnetic moment on the magnetic field and temperature is investigated. At low temperatures, a Schottky anomaly is observed in the temperature dependence of the specific heat capacity. Calculations show that the Mn ion exhibits antiferromagnetic properties as an electron gas at x = 0 and becomes paramagnetic at x = 0.115.

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Introduction

Recently, interest in Diluted magnetic semiconductor (DSM) nanostructures has increased as they are being implemented in spintronic applications. Diluted magnetic semiconductors are alloys made from semiconductors with and without magnetic properties. The presence of magnetic ions in diluted magnetic semiconductors creates various exchange interactions. As a result of these interactions, the band parameters change depending on the magnetic field and temperature [1,

2].

In the papers [3-5], the thermodynamic properties of DMS nanostructures have been investigated considering the Rashba spin-orbit interaction.
The magnetization of electrons in diluted magnetic semiconductors has been calculated by considering the exchange interaction in Ref. [6].
In Ref. [7], the effect of exchange interaction on polaron binding energy in diluted magnetic semiconductor nanotubes is investigated.
The energy spectrum of electrons in diluted magnetic quantum heterostructures, surrounded by a non-magnetic semiconductor and placed in an external magnetic field, was calculated in [8].
The effect of Rashba spin-orbital interaction on tunneling conductivity in diluted magnetic semiconductor nanostructures was investigated in paper [9].

Theory

The harmonic oscillation potential is one of the most widely used potentials in quantum mechanics, solid-state physics, and nanostructures. This is because the Schrödinger equation for this potential can be solved analytically. Harmonic and anisotropic oscillator potentials are used as confinement potentials in nanostructures. Rebane [10,11] developed an analytical model for a two- and three-dimensional anisotropic oscillator potential in a magnetic field. The author calculated the energy spectrum and wave function of a 3-dimensional anisotropic oscillator potential in an arbitrarily oriented external magnetic field using a shift operator.

Chakraborty [12] studied the electronic properties of anisotropic quantum

dots in a magnetic field.

Galkin et al. [13] investigated the variation of photoconductivity and ballistic conductivity depending on the external magnetic field using an anisotropic potential.

The super statistical thermodynamic properties of an anisotropic oscillator in a non-commutative plane are calculated in Ref. [14,15].

By using canonical ensemble statistics, as investigated in [16], the magnetic properties of the electron system in quantum-dot superlattices with an anisotropic parabolic potential in a magnetic field. In [17], the thermodynamic properties of a two-dimensional electron gas in semi-magnetic semiconductor superlattices have been investigated.

Faraday rotators are made from diluted-magnetic semiconductors used in optical telecommunications [18].

The authors of article [19] investigated the magnetic properties of cylindrical nano-shells by considering the Rashba spin-orbit interaction. The authors [20] investigated the effect of the Rashba spin-orbit interaction on the thermodynamic properties of parabolic quantum dots in magnetic fields.

In this study, the energy spectrum, magnetic and thermodynamic properties of electrons in two-dimensional diluted magnetic semiconductor DMS quantum dots with an anisotropic harmonic potential in an external magnetic field oriented in the OZ direction were investigated. The variation of specific heat and magnetic momentum with magnetic field and temperature was analyzed. The Hamiltonian of the considered system is

$$H = \frac{1}{2m}\left(\vec{P} - \frac{e\vec{A}}{c}\right)^2 + \frac{m}{2}(\omega_1^2 x^2 + \omega_2^2 y^2) + H_{ex} + H_z \qquad (1)$$

In the mean field approximation, the exchange Hamiltonian term can be

written as [5]:

$$H_{ex} = \frac{1}{2} < S_z > N_0 x J_{s-d} \sigma_z = 3A_r \sigma_z \quad (2)$$

where J_{s-d} is a constant which describes the exchange interaction, N_0-is the density of the unit cells. The thermodynamic average $< S_z >$ of the localized Mn spin is determined by the expression

$$< S_z >= -S_0 B_{5/2}\left(\frac{Sg_{Mn}\mu_B H}{k_B T}\right) \quad (3)$$

where $B_{5/2}\left(\frac{Sg_{Mn}\mu_B H}{k_B T}\right)$ is the Brillouin function, $g_{Mn} = 2$ is the g factor of Mn ions, S=5/2 and k_B is the Boltzmann constant.

The Zeeman term as

$$H_Z = \frac{1}{2} g \mu_B H \sigma_z \quad (4)$$

Where g-Lande factor and μ_BBohr magneton.

The sp-d exchange interaction between the magnetic moment of the Mn ion and the charge carriers of the conduction and valence bands significantly alters the Lande g-factor of the electrons,

$$g^* = g + {}^{6A_r}/_{\mu_B H} \quad (5)$$

For a uniform magnetic field, the vector potentials have the components [11]:

$$\vec{A} = H \frac{x\omega_1 \overrightarrow{e_y} - y\omega_2 \overrightarrow{e_x}}{\omega_1 + \omega_2} \quad (6)$$

and Schrödinger equation is

$$\left\{\frac{1}{2m}\left(\left(P_x + \frac{eH\omega_2}{\omega_1+\omega_2} y\right)^2 + \left(P_y - \frac{eH\omega_1}{\omega_1+\omega_2} x\right)^2\right) I + \frac{m}{2}(\omega_1^2 x^2 + \omega_2^2 y^2) I + \frac{1}{2} g^* \sigma_z \mu_B H - EI\right\} \begin{pmatrix} \psi_1(x,y) \\ \psi_2(x,y) \end{pmatrix} = 0 \quad (7)$$

Where I 2-dimensional unit matrix, $\psi_{1,2}(x, y)$ spin up and spin down wave functions.

After a simple transformation, we write equation (7) as follows:

$$\frac{1}{2m}\Big\{P_x^2 I + P_y^2 I + \frac{2m\omega_c}{\omega_1+\omega_2}\big(\omega_2 y P_x - \omega_1 x P_y\big)I + I(\Omega_1^2 x^2 + \Omega_2^2 y^2) + g^* \mu_B H m \sigma_z - 2mEI\Big\}\begin{pmatrix}\psi_1(x,y)\\ \psi_2(x,y)\end{pmatrix} = 0 \quad (8)$$

where

$$\Omega_1^2 = m^2\omega_1^2\left(1+\left(\frac{\omega_c}{\omega_1+\omega_2}\right)^2\right), \Omega_2^2 = m^2\omega_2^2\left(1+\left(\frac{\omega_c}{\omega_1+\omega_2}\right)^2\right) \quad (9)$$

After performing the coordinate transformations [13];

$$x = q_1\cos\chi - \frac{\chi_2}{\chi}p_2\sin\chi\,, y = q_2\cos\chi - \frac{\chi_2}{\chi}p_1\sin\chi \quad (10)$$

$$P_x = p_1\cos\chi + \frac{\chi_1}{\chi}q_2\sin\chi\,, P_y = p_1\cos\chi + \frac{\chi_1}{\chi}q_1\sin\chi \quad (11)$$

$$\tan 2\chi = \frac{2m\omega_c}{(\Omega_1^2-\Omega_2^2)}\sqrt{\frac{(\omega_2\Omega_1^2+\omega_1\Omega_2^2)}{\omega_1+\omega_2}} \quad (12)$$

The equation (8) becomes as follows:

$$\frac{1}{2m}(p_1^2\alpha_1^2 + p_2^2\alpha_2^2 + q_1^2\beta_1^2 + q_2^2\beta_2^2)\psi_{1,2}(x,y) = E1\psi_{1,2}(x,y) \quad (13)$$

where

$$E1I = -3A_r\sigma_z - g\sigma_z\mu_B HI + EI \quad (14)$$

$$\alpha_1^2 = \frac{2\omega_1\Omega_2^2+\omega_2(\Omega_1^2+\Omega_2^2+\Omega_3^2)}{2(\Omega_1^2+\Omega_2^2)}; \alpha_2^2 = \frac{2\omega_2\Omega_1^2+\omega_1(\Omega_1^2+\Omega_2^2-\Omega_3^2)}{2(\Omega_1^2+\Omega_2^2)} \quad (15)$$

$$\beta_1^2 = \frac{2\omega_2\Omega_1^2+\omega_1(\Omega_1^2+\Omega_2^2+\Omega_3^2)}{2(\omega_1+\omega_2)}; \beta_2^2 = \frac{2\omega_1\Omega_2^2+\omega_2(\Omega_1^2+\Omega_2^2-\Omega_3^2)}{2(\omega_1+\omega_2)} \quad (16)$$

$$\Omega_3^2 = \sqrt{(\Omega_1^2-\Omega_2^2) + \frac{4m^2\omega_c^2}{\omega_1+\omega_2}(\omega_2\Omega_1^2+\omega_1\Omega_2^2)}; \quad (17)$$

In the case $\omega_1 = \omega_2$, these expressions are the same as in [13].

$$EI = 3A_r\sigma_z + \frac{1}{2}g\sigma_z\mu_B H + \hbar\omega_{11}\left(n_1 + \frac{1}{2}\right)I + I\hbar\omega_{22}\left(n_2 + \frac{1}{2}\right)(18)$$

$$\omega_{11} = \frac{\alpha_1\beta_1}{m}, \omega_{22} = \frac{\alpha_2\beta_2}{m} \quad (19)$$

To calculate the thermodynamic properties of non-degenerate electrons,we need to calculate the distribution function.

$$Z = \sum_{n_1 n_2 \sigma} e^{-\beta E_{n_1 n_2 \sigma}} \quad (20)$$

where $E_{n_1 n_2 \sigma}$is the energy spectrum of the considered system, $\beta = \frac{1}{k_B T}$

k_B is the Boltzmann constant, and T is the thermodynamic equilibrium temperature. The heat capacity can be obtained by [3,4]:

$$c = k_B\beta^2\frac{\partial^2 LnZ}{\partial\beta^2} \quad (21)$$

The free energy of a non-degenerate system can be obtained using the partition function as

$$F = -k_B T LnZ \quad (22)$$

The magnetization of the electron gas can be written as:

$$M = -\frac{\partial F}{\partial H} == -\frac{\partial F}{\partial\alpha}\frac{\partial\alpha}{\partial H} = \frac{2\mathrm{m}0\xi\mu_B}{m}\frac{\partial LnZ}{\partial\alpha} \quad (23)$$

Rezults

We introduce dimensionless parameters $\alpha = \frac{\hbar\omega_c}{\hbar\omega_1}$, the magnetic field strength, and $\xi = (\beta\hbar\omega_1)^{-1}$to represent the measured temperature,and $t = \frac{\omega_2}{\omega_1}, \gamma = \frac{m}{m_0} = 0.096, y = \frac{N_0 J_{s-d}}{\hbar\omega_1} = 9$ [21].

Figure 1 shows the variation of electron energy in Cd1-xMnxTe DSM quantum dots with magnetic field dependence for Mn concentrations x=0.115, spin-up and spin-down cases for quantum numbers n=1 and m=5. As seen in the figure, Zeeman splitting increases as the magnetic field energy increases.

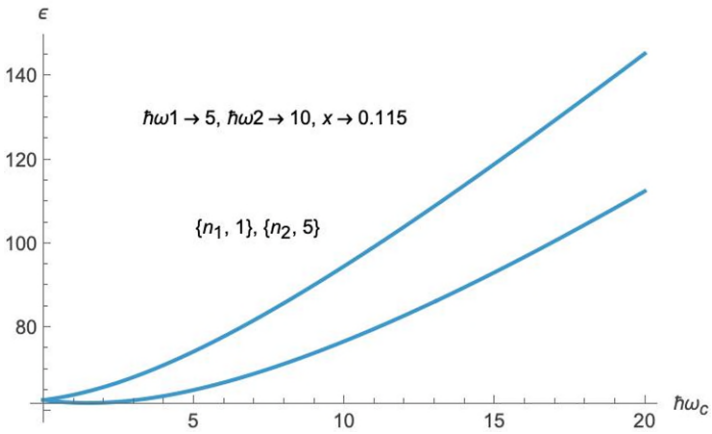


Fig.1.Energy levels of an Cd_{1-x} Mn_x Te anisotropic quantum dots as a function of the magnetic field $\hbar\omega_c$ for $\hbar\omega_1 = 5meV, \hbar\omega_2 = 10meV, x = 0.015$.

In Fig. 2 we show the reduced specific heat $\frac{C}{k_B}$ of the DMS quantum dot in the presence of Zeeman and exchange terms as a function of the reduced temperature $\xi = (\beta\hbar\omega_1)^{-1}$ for two different values of Mn concentrations x=0;0.005 at dimensionless parameters $\alpha = 1.5. t = \frac{\omega_2}{\omega_1} = 1.5$.

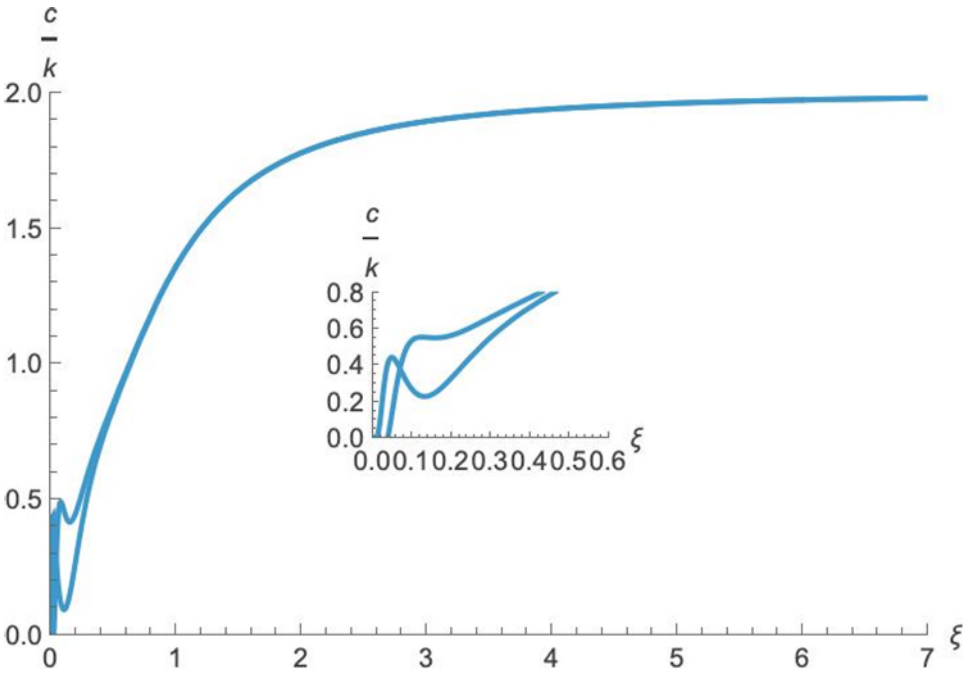


Fig. 2 The dimensionless specific heat $\frac{C}{k_B}$ of a DMS quantum dot as a function of dimensionless temperature for two different values of Mn concentrations x=0,0.005 at dimensionless parameters $\alpha = 1.5. t = \frac{\omega_2}{\omega_1} = 1.5, \gamma = \frac{m}{m_0} = 0.096, y = \frac{N_0 J_{s-d}}{\hbar \omega_1} = 9$

As can be seen from the figure, at low temperatures, the specific heat initially increases, reaches a maximum value, and then decreases, resulting in a characteristic peak-like structure. This behavior corresponds to the well-known Schottky anomaly in the heat capacity, which is characteristic of systems in which only a few low-lying energy states are thermally accessible at low temperatures. In the present case, the thermal energy is sufficient to populate primarily the two lowest energy levels. As the temperature increases further, higher energy levels become thermally accessible, and the specific heat approaches the value of $2k_B$. Since the system under consideration is a two-dimensional oscillator, each spatial degree of freedom contributes k_B to the

specific heat in the high-temperature limit.

Figure 3 shows the dependence of the dimensionless specific heat $\frac{C}{k_B}$ on the dimensionless magnetic field for two different values of the dimensionless temperature. At low temperatures, the specific heat increases at low magnetic field values, reaches a peak, and then decreases; at high magnetic field values, the specific heat approaches k_B. At high temperatures, the specific heat is less than $2k_B$ in zero magnetic field (due to the concentration of Mn ions) and approaches k_B at high magnetic fields. In a strong magnetic field, the two-dimensional system is transformed into a one-dimensional harmonic structure, and its specific heat is k_B.

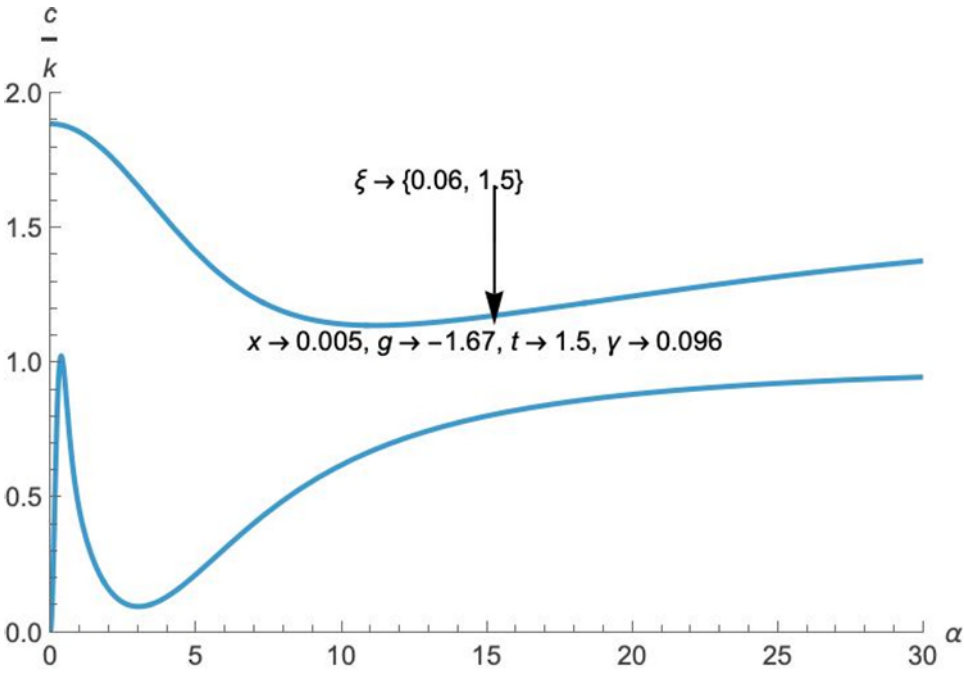


Fig. 3 The reduced specific heat $\frac{C}{k_B}$ of an DSM quantum dot has been

plotted as a function of the dimensionless parameter α at two different values of the dimensionless parameter $\xi = 0.06; 1.5$.

Figure 4 shows the change in the dimensionless magnetic moment of electrons in $Cd_{1-x}Mn_xTe$ DSM quantum dots as a function of the dimensionless magnetic field at two different Mn concentrations, x = 0, 0.115, and a constant dimensionless temperature $\xi = 0.5$. As can be seen in the figure, at a Mn concentration of x = 0, the magnetic moment changes from negative to positive as the magnetic field shifts from right to left from zero. This is known to indicate the antiferromagnetic properties of the system [22].

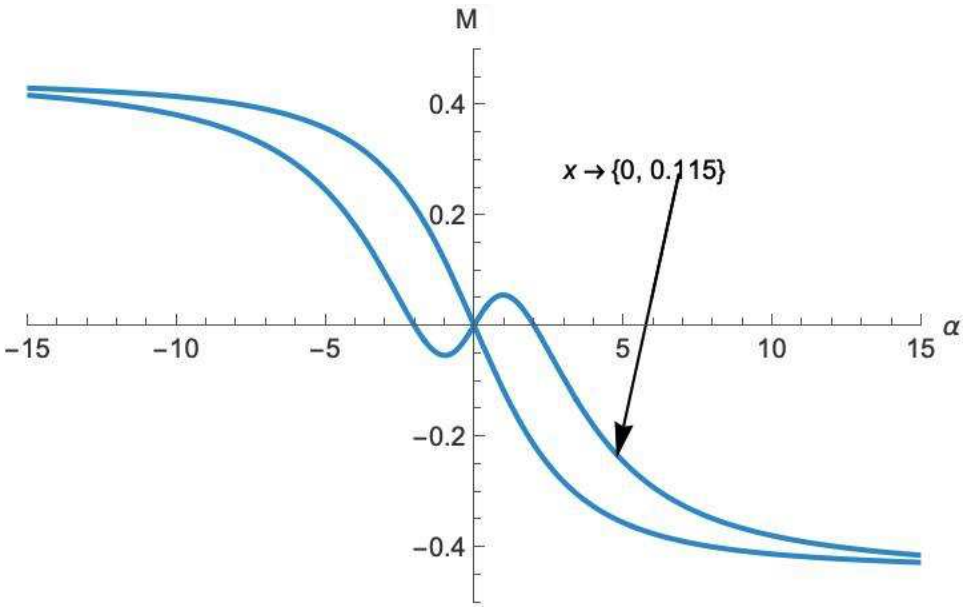


Fig.4. The reduced magnetic moment (in $2\mu_B\gamma^{-1}$units) as a function of magnetic field at a constant dimensionless temperature $\xi = 0.5$.

At a Mn concentration of x = 0.115, the magnetic moment changes from positive to negative as the magnetic field shifts from right to left from zero. This is known to indicate the paramagnetic properties of the system

[16]. Figure 5 shows a plot of the dimensionless magnetic moment versus the dimensionless magnetic field in anisotropic Cd_{1-x} Mn_x Te DMS quantum dots at two different dimensionless temperatures $\xi = \{0.5,1\}$ and a constant Mn concentration $x = 0.15$.

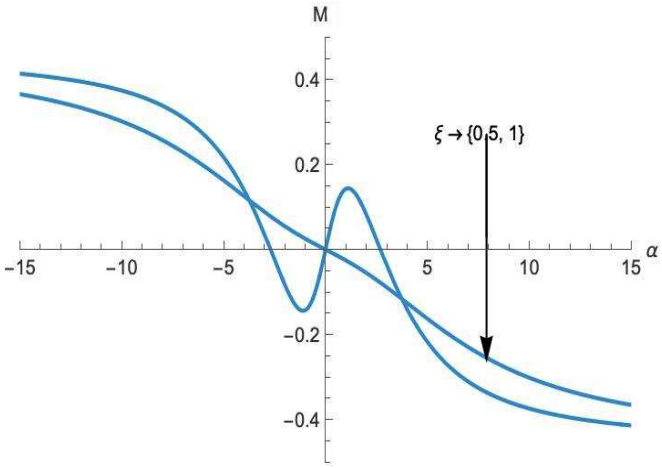


Fig.5. The reduced magnetic moment as a function of magnetic field a constant Mn concentration $x = 0.15$.

As can be seen from the figure, antiferromagnetic properties at low temperatures transform into paramagnetic properties at high temperatures. Figures 6 and 7 show the dependence of the dimensionless magnetic moment on the dimensionless temperature for anisotropic $Cd_{1-x}Mn_xTe$ DMS quantum dots. The calculations are performed for two different values of the dimensionless magnetic field, α = −0.5 and 0.5, at fixed Mn concentrations of x = 0 and x = 0.115.

As shown in the figure, at a Mn concentration of x=0, the magnetic moment is either positive or negative throughout the temperature range. In a dimensionless magnetic field with a negative value, the magnetic moment takes a positive value; the direction of the magnetic moment changes when the direction of the magnetic field changes.

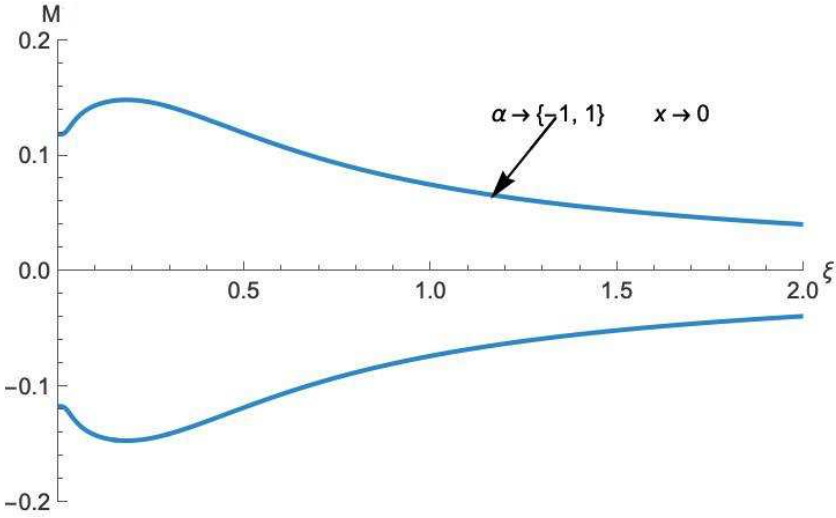


Fig.6. The dimensionless magnetic moment as a function of the dimensionless temperature in anisotropic $Cd_{1-x}Mn_xTe$ DMS quantum dots at two different values of the dimensionless magnetic field α={-0.5,0.5} and constant Mn concentrations x=0 and x=0.115.

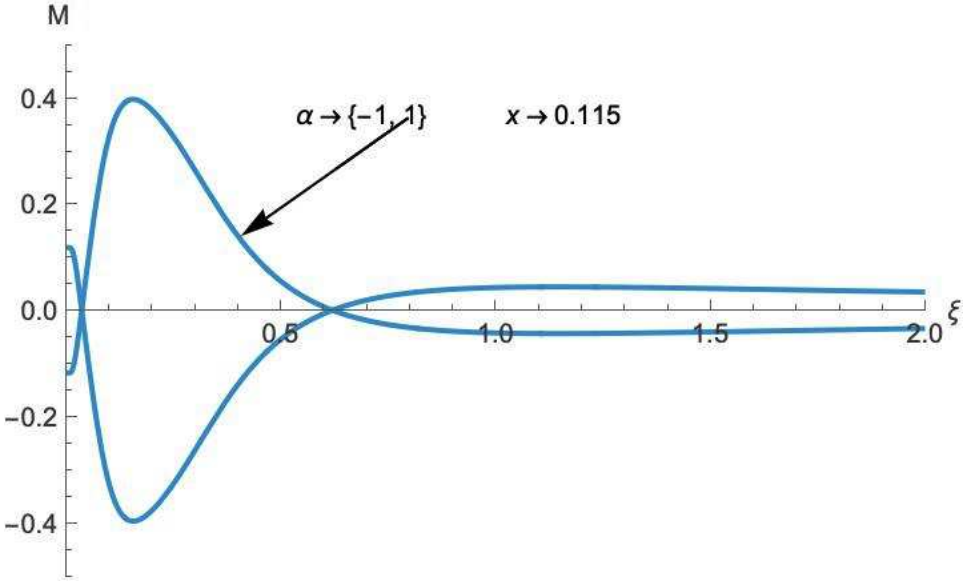


Fig.7. The dimensionless magnetic moment as a function of the dimensionless temperature in anisotropic $Cd_{1-x}Mn_xTe$ DMS quantum

dots at two different values of the dimensionless magnetic field α={-0.5,0.5} and constant Mn concentrations x=0.115.When the concentration of Mn ions is x = 0.115, the magnetic moment changes sign twice with temperature variation, both at positive and negative magnetic field values.

Conclusions

We have studied, theoretically, the thermodynamic properties of electron gas on the anisotropic DMS quantum dots in the presence of the externally applied static magnetic field. The dependence of specific heat capacity on magnetic field in two-dimensional anisotropic diluted magnetic semiconductor quantum dots was studied. At low magnetic fields and temperatures, the specific heat capacity increases, reaches a peak, and then decreases again. At high magnetic fields, it approaches k_B. The dependence of magnetization on temperature and magnetic flux was also studied.

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