Calibration Algorithms for Multiscale Geographically and Temporally Weighted Regression: Parameter Recovery, Uncertainty Calibration, and Computational Scaling

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R1 Supplementary Results for the Real Data Application

This section provides additional empirical results and performance evaluations for the real data application in São Paulo described in the main manuscript. Specifically, Table R1 details the complete statistical distribution of the standardised local coefficients estimated via the Top-Down Scale (TDS) algorithm across the 2000, 2010, and 2022 census waves. It supplements the summaries provided in the main text by reporting percentile distributions, conditional association directions, local multicollinearity diagnostics, and spatial dispersion metrics relative to global uncertainty. Furthermore, this supplement visually depicts the spatial-temporal variations of these local coefficients (Figure R1), and provides spatial surface maps comparing the observed (Figure R2) and MGTWR-fitted (Figure R3) log-transformed population densities throughout the study period.

Table R1: Complete distribution of the standardised local coefficients from the TDS algorithm, São Paulo, 2000–2022. P10–P90 indicate percentiles; % Pos./Neg. denote the direction of the conditional association. The absence of local multicollinearity (a Condition Number, % CN OK ≤ 30) was verified in 100% of the cases. The local IQR to global SE ratio (IQR/SE) quantifies the spatial dispersion relative to the global baseline uncertainty.

Covariate	Year	P10	P25	Median	P75	P90	% Pos.	% Neg.	% CN OK	Local IQR	Global SE	IQR/SE
Apartment share	2000	0.071	0.216	0.405	0.676	1.062	94.14	5.86	100.00	0.460	0.010	47.75
	2010	0.070	0.212	0.399	0.676	1.061	93.95	6.05	100.00	0.463	0.010	48.13
	2022	0.064	0.210	0.392	0.674	1.062	93.59	6.41	100.00	0.464	0.010	48.16
Population aged 60+	2000	-0.333	-0.250	-0.160	-0.065	0.008	12.01	87.99	100.00	0.185	0.012	14.88
	2010	-0.436	-0.371	-0.256	-0.214	-0.183	0.00	100.00	100.00	0.157	0.012	12.62
	2022	-0.462	-0.340	-0.230	-0.189	-0.157	0.01	99.99	100.00	0.151	0.012	12.13
Illiteracy rate, age 15+	2000	-0.167	0.015	0.154	0.271	0.419	76.63	23.37	100.00	0.256	0.015	16.53
	2010	-0.167	0.014	0.156	0.272	0.423	76.47	23.53	100.00	0.258	0.015	16.69
	2022	-0.167	0.012	0.156	0.277	0.431	76.47	23.53	100.00	0.265	0.015	17.12
Log distance to operating metro station	2000	-0.544	-0.376	-0.133	0.591	1.175	46.81	53.19	100.00	0.967	0.009	109.89
	2010	-0.474	-0.243	0.123	0.558	1.096	58.18	41.82	100.00	0.802	0.009	91.13
	2022	-0.232	-0.029	0.164	0.516	1.118	69.68	30.32	100.00	0.545	0.009	61.98
Sewage deficit	2000	-0.108	-0.026	0.043	0.138	0.289	66.03	33.97	100.00	0.165	0.009	19.25
	2010	-0.108	-0.026	0.044	0.140	0.293	66.49	33.51	100.00	0.167	0.009	19.49
	2022	-0.110	-0.026	0.046	0.145	0.296	66.88	33.12	100.00	0.171	0.009	19.98

Figure R1: Spatial and temporal distributions of the standardised local coefficients estimated via the TDS algorithm for São Paulo in 2000, 2010, and 2022. The distributions highlight variations in the magnitude, dispersion, and direction of the local associations across the covariates and over the three census waves.

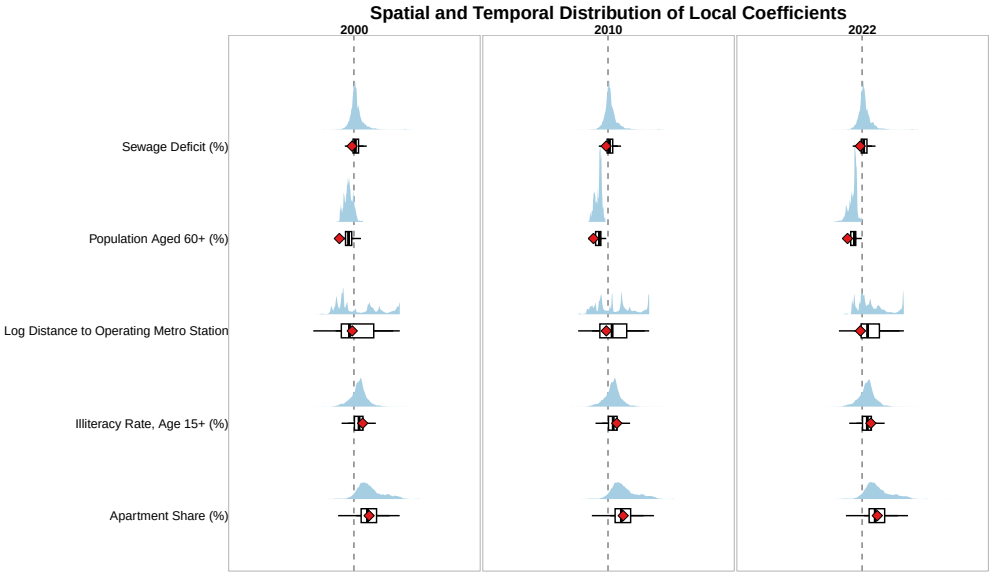

Figure R2: Spatial surface of the observed log-transformed population density in the São Paulo metropolis (2000–2022).

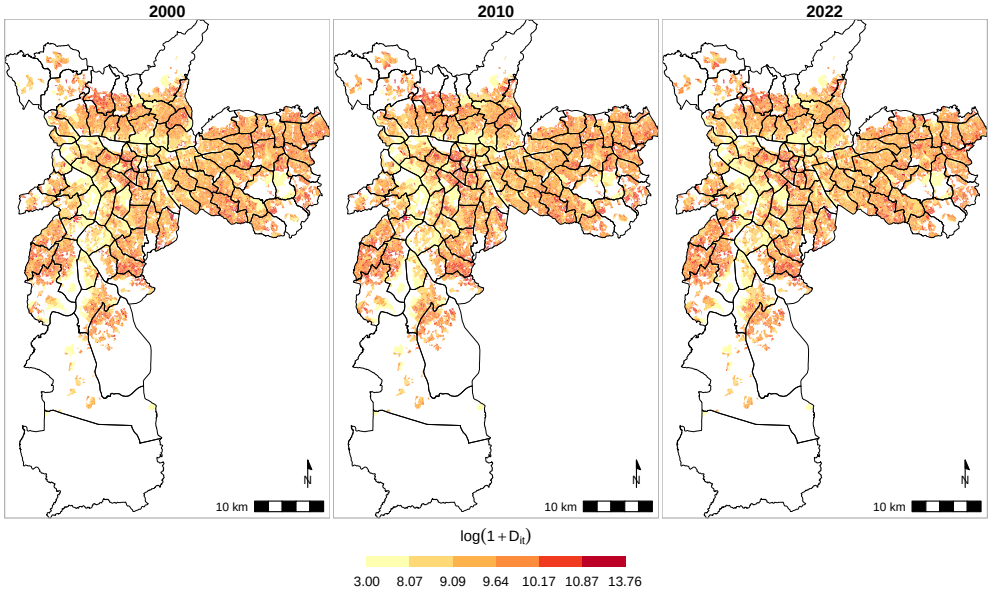

Figure R3: Spatial surface of the log-transformed population density fitted by the MGTWR model (2000–2022).