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A new generalized alpha power family with applications in healthcare and environmental quality monitoring

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Abstract

This paper introduces a novel class of continuous probability distributions, known as the New Generalized Alpha Power (NGAP) family, designed to improve modeling effectiveness for intricate lifetime and reliability data. The suggested family is formed by applying an exponent transformation to a fundamental cumulative distribution function, enabling the distribution's shape to change dynamically in various probability areas. Key structural and mathematical characteristics of the NGAP family are examined, encompassing closed-form quantile expressions through the Lambert W_0 function, linear expansions, moments, residual life functions, stress-strength reliability, order statistics, Rényi entropy, and extropy measures. Statistical inference for the unknown parameter vector $\Theta = (\alpha, \beta, \theta, k)^T$ is performed utilizing fifteen non-Bayesian estimation techniques—including Maximum Likelihood Estimation (MLE), distance-oriented methods, and spacing-based approaches. Monte Carlo simulations assess how well these estimators perform with sample sizes varying from $n = 10$ to $n = 500$. Numerical results indicate that as the sample size n grows, the absolute bias and mean squared error (MSE) consistently decline across parameters; for instance, with $\alpha = 1.25, \beta = 0.75, \theta = 1.50, k = 1.75$, the MLE estimation for α shows an absolute bias (and MSE) dropping from 6.3235 (679.9716) at $n = 10$ to 0.0873 (0.2146) at $n = 200$ and 0.0564 (0.1339) at $n = 300$, whereas the Kolmogorov Estimator (KE) reaches a bias of 0.0125 (0.0035) at $n = 200$. In general, the Maximum Product of Spacings Estimator (MPSE) reliably secures the highest rankings in various simulation scenarios. A specific variant centered on the Weibull baseline (NGAP-Weibull) is analyzed thoroughly, along with submodels like the NGAP-Exponential and NGAP-Rayleigh distributions. Applications to actual datasets show that the NGAP model achieves better goodness-of-fit than current competing distributions, serving as a flexible tool with real-world uses in reliability engineering, biomedical studies, and survival analysis.

Keywords: New generalized alpha power family; Generated distributions; Weibull distribution; Hazard rate function; Maximum likelihood estimation; Lifetime data analysis; Monte Carlo simulation.

1 Introduction

The examination of lifetime and reliability data continues to be a key subject in distribution theory because of its essential applications in reliability engineering, biomedical studies, industrial

quality assurance, actuarial science, and survival analysis Weibull (1951); Mudholkar et al. (1996). Data collected in various real-world situations display intricate characteristics, including differing levels of skew, significant tail weights, and unconventional hazard-rate patterns that traditional lifetime models do not effectively represent. As a result, significant effort has been directed toward developing adaptable groups of continuous distributions that can support both monotone (increasing, decreasing) and non-monotone (unimodal, bathtub-shaped) hazard functions, all while ensuring mathematical manageability Alzaatreh et al. (2013). The Weibull distribution is a fundamental model among classical baseline distributions due to its mathematical ease, interpretability, and adaptability Weibull (1951).

To address the constraints of classical distributions, a key area of research has centered on generator-based families, where a baseline distribution is modified by adding extra shape parameters to enhance distributional features. Key generator frameworks in contemporary distribution theory comprise:

- The family generated by Beta Eugene et al. (2002);
- The family generated by Kumaraswamy Cordeiro and de Castro (2011);
- The extension by Marshall and Olkin Marshall and Olkin (1997);
- The rank-transformation method Shaw and Buckley (2009).

These approaches methodically improve regulation of skewness, kurtosis, tail characteristics, and hazard-rate forms via analytically feasible transformations Alzaatreh et al. (2013). When focusing on the Weibull baseline, numerous significant extensions have appeared, including the exponentiated Weibull Mudholkar and Srivastava (1993); Mudholkar et al. (1996), beta-Weibull Lee et al. (2007), Kumaraswamy-Weibull Cordeiro et al. (2010), Marshall–Olkin extended Weibull Ghitany et al. (2005), transmuted Weibull Aryal and Tsokos (2011), and alpha-power Weibull variants Ahmad et al. (2019).

Despite these advancements, a notable methodological issue remains: numerous current generated families attain substantial flexibility by incorporating complex functional forms, several additional parameters, or layered transformations. This level of structural complexity frequently hinders mathematical derivations, likelihood inference, numerical optimization, and the interpretability of parameters. Consequently, there is a persistent requirement for generator mechanisms that combine substantial flexibility with mathematical and computational manageability Alzaatreh et al. (2013).

To tackle this issue, this study presents a novel category of continuous distributions called the New Generalized Alpha Power (NGAP) family. The NGAP framework is founded on an exponent-driven transformation of a baseline cumulative distribution function (CDF). In contrast to traditional power-generated models that use constant exponents, the NGAP framework enables the transformation effect to flexibly adjust across various probability areas, providing improved management of skewness, tail characteristics, and hazard rates.

Applying the NGAP mechanism to a Weibull baseline yields the NGAP–Weibull (NGAPW) distribution, which expands the baseline's capacity to fit diverse probability density and hazard-rate shapes. The cumulative distribution function (CDF) and probability density function (PDF) of the baseline NGAP family are defined as

$$G(x;\alpha,\beta) = \{F(x;\Omega)\}^{\alpha e^{-\beta F(x;\Omega)}}, \quad x \in \mathbb{R}, \quad \alpha,\beta > 0, \tag{1}$$

and

$$g(x;\alpha,\beta,\Omega) = \{F(x;\Omega)\}^{\alpha e^{-\beta F(x;\Omega)}} \alpha f(x;\Omega) e^{-\beta F(x;\Omega)} \left[\frac{1}{F(x;\Omega)} - \beta \ln F(x;\Omega)\right], \tag{2}$$

where α and β serve as shape parameters governing skewness, kurtosis, and tail behavior.

To establish the identifiability of the parameter vector $\boldsymbol{\theta} = (\alpha, \beta, \boldsymbol{\Omega})^\top$, we demonstrate that if two parameter vectors $\boldsymbol{\theta}_1 = (\alpha_1, \beta_1, \boldsymbol{\Omega}_1)^\top$ and $\boldsymbol{\theta}_2 = (\alpha_2, \beta_2, \boldsymbol{\Omega}_2)^\top$ yield identical cumulative distribution functions $G(x; \boldsymbol{\theta}_1) = G(x; \boldsymbol{\theta}_2)$ for all $x \in \mathbb{R}$, then $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$.

Theorem 1.1. *Let $F(x; \boldsymbol{\Omega})$ be a strictly continuous baseline cumulative distribution function uniquely identifiable by its parameter vector $\boldsymbol{\Omega}$. Then, the parameters $(\alpha, \beta, \boldsymbol{\Omega})$ of the Generalized Alpha Power family are identifiable.*

Proof. Assume that for all $x \in \mathbb{R}$,

$$G(x; \alpha_1, \beta_1, \boldsymbol{\Omega}_1) = G(x; \alpha_2, \beta_2, \boldsymbol{\Omega}_2).$$

By definition of the CDF $G(x; \alpha, \beta, \boldsymbol{\Omega})$, this implies:

$$\{F(x; \boldsymbol{\Omega}_1)\}^{\alpha_1 e^{-\beta_1 F(x; \boldsymbol{\Omega}_1)}} = \{F(x; \boldsymbol{\Omega}_2)\}^{\alpha_2 e^{-\beta_2 F(x; \boldsymbol{\Omega}_2)}}, \quad \forall x \in \mathbb{R}. \tag{3}$$

Taking the natural logarithm on both sides of Equation (3) yields:

$$\alpha_1 e^{-\beta_1 F(x; \boldsymbol{\Omega}_1)} \ln\left[F(x; \boldsymbol{\Omega}_1)\right] = \alpha_2 e^{-\beta_2 F(x; \boldsymbol{\Omega}_2)} \ln\left[F(x; \boldsymbol{\Omega}_2)\right]. \tag{4}$$

As $x \to \infty$, $F(x; \boldsymbol{\Omega}_1) \to 1$ and $F(x; \boldsymbol{\Omega}_2) \to 1$. Consider the lower boundary asymptotic behavior as $x \to -\infty$ (or $x \to 0^+$ depending on the support). Let $u_1 = F(x; \boldsymbol{\Omega}_1)$ and $u_2 = F(x; \boldsymbol{\Omega}_2)$, where $u_1, u_2 \in (0, 1)$.

Rearranging Equation (4):

$$\frac{\ln\left[F(x; \boldsymbol{\Omega}_1)\right]}{\ln\left[F(x; \boldsymbol{\Omega}_2)\right]} = \frac{\alpha_2}{\alpha_1} \exp\left\{\beta_1 F(x; \boldsymbol{\Omega}_1) - \beta_2 F(x; \boldsymbol{\Omega}_2)\right\}.$$

Since $F(x; \boldsymbol{\Omega}_1)$ and $F(x; \boldsymbol{\Omega}_2)$ are bounded strictly increasing functions on $(0, 1)$, the continuity and strict monotonicity of $F(x; \boldsymbol{\Omega})$ enforce that $F(x; \boldsymbol{\Omega}_1) = F(x; \boldsymbol{\Omega}_2)$ for all x. Given that the baseline distribution $F(x; \boldsymbol{\Omega})$ is identifiable with respect to $\boldsymbol{\Omega}$, it follows that:

$$\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_2 = \boldsymbol{\Omega}.$$

Substituting $F(x; \boldsymbol{\Omega}_1) = F(x; \boldsymbol{\Omega}_2) = u \in (0, 1)$ into Equation (4):

$$\alpha_1 e^{-\beta_1 u} \ln(u) = \alpha_2 e^{-\beta_2 u} \ln(u).$$

Since $\ln(u) \neq 0$ for all $u \in (0, 1)$, dividing both sides by $\ln(u)$ gives:

$$\alpha_1 e^{-\beta_1 u} = \alpha_2 e^{-\beta_2 u}, \quad \forall u \in (0, 1).$$

Taking the logarithm of the above expression:

$$\ln(\alpha_1) - \beta_1 u = \ln(\alpha_2) - \beta_2 u.$$

Rearranging terms:

$$(\beta_2 - \beta_1) u = \ln\left(\frac{\alpha_2}{\alpha_1}\right), \quad \forall u \in (0, 1).$$

Since the right-hand side is a constant independent of u, the linear function on the left-hand side can hold for all $u \in (0, 1)$ if and only if the coefficient of u vanishes:

$$\beta_2 - \beta_1 = 0 \implies \beta_1 = \beta_2.$$

Substituting $\beta_1 = \beta_2$ back into the constant equation gives:

$$\ln\left(\frac{\alpha_2}{\alpha_1}\right) = 0 \implies \alpha_1 = \alpha_2.$$

Hence, $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$, completing the proof of identifiability. □

Main Contributions are;

- Proposing the New Generalized Alpha Power (NGAP) generator family and establish the mathematical tractability of its NGAP-Weibull (NGAPW) sub-model, including parameter identifiability proofs and closed-form quantile functions using the Lambert W_0 function.
- We derive explicit linear mixture expansions for the NGAPW density, facilitating exact derivations of raw and incomplete moments, order statistics, stress-strength reliability metrics, and entropy measures (Rényi and Tsallis extropy).
- We conduct a comprehensive simulation study evaluating 15 distinct estimation techniques (including classical maximum likelihood, maximum product of spacings, and various goodness-of-fit estimators) to systematically rank estimator performance across varying sample sizes.
- We demonstrate the practical modeling superiority and flexibility of the proposed NGAPW model in fitting heavy-tailed and non-monotone hazard rate dynamics against established baseline generalizations using real-world biomedical and environmental datasets.

The remainder of this paper is organized as follows: Section 2 presents the NGAP family and defines the NGAP–Weibull distribution. Section 3 derives its structural properties. Section 4 details parameter estimation methods. Section 5 presents Monte Carlo simulation results. Section 6 demonstrates empirical applications, and Section 7 concludes the paper.

2 NGAP–Weibull (NGAPW) Distribution

As an important special case of the proposed family, we consider the Weibull distribution as the baseline model. The Weibull distribution is widely used in reliability engineering, survival analysis, and lifetime modelling due to its flexibility in describing increasing and decreasing hazard rate behaviours. The baseline Weibull distribution $F(x;\theta,k)$ is defined by the CDF and PDF as:

$$F(x;\theta,k) = 1 - e^{-\theta x^k}; \quad x > 0, \quad \theta, k > 0 \tag{5}$$

$$f(x;\theta,k) = \theta k x^{k-1} e^{-\theta x^k} \tag{6}$$

By incorporating this baseline into the New Generalized Alpha Power (NGAP) family, we obtain the NGAP-Weibull (NGAPW) model. The CDF is given by:

$$G(x;\alpha,\beta,\theta,k) = \left\{1 - e^{-\theta x^k}\right\}^{\alpha e^{-\beta\left(1-e^{-\theta x^k}\right)}}. \tag{7}$$

The corresponding PDF is derived as:

$$\begin{aligned} g(x;\alpha,\beta,\theta,k) = &\left\{1 - e^{-\theta x^k}\right\}^{\alpha e^{-\beta\left(1-e^{-\theta x^k}\right)}} \alpha\theta k x^{k-1} e^{-\theta x^k} e^{-\beta\left(1-e^{-\theta x^k}\right)} \\ &\times \left[\frac{1}{1-e^{-\theta x^k}} - \beta \ln\left(1 - e^{-\theta x^k}\right)\right], \end{aligned} \tag{8}$$

where $\theta > 0$ is the scale (rate) parameter and $k, \alpha, \beta > 0$ are the shape parameters.
The hazard function or force of mortality is given as

$$h(x;\alpha,\beta,\theta,k) = \frac{\left\{1 - e^{-\theta x^k}\right\}^{\alpha e^{-\beta\left(1-e^{-\theta x^k}\right)}} \alpha\theta k x^{k-1} e^{-\theta x^k} e^{-\beta\left(1-e^{-\theta x^k}\right)} \left[\frac{1}{1-e^{-\theta x^k}} - \beta \ln\left(1 - e^{-\theta x^k}\right)\right]}{1 - \left\{1 - e^{-\theta x^k}\right\}^{\alpha e^{-\beta\left(1-e^{-\theta x^k}\right)}}}.$$

Table 1: Extreme behavior and resulting shapes of the NGAPW hazard function.

Parameters	$x \to 0$	$x \to \infty$	**Shape**
$k\alpha > 1, k > 1$	0	∞	Monotonic Increasing
$k\alpha < 1, k < 1$	∞	0	Monotonic Decreasing
$k\alpha > 1, k < 1$	0	0	Unimodal (Upside-down bathtub)
$k\alpha < 1, k > 1$	∞	∞	Bathtub-shaped

2.1 Special Cases of the NGAP–Weibull Distribution

Several useful submodels of the NGAP–Weibull (NGAPW) distribution can be obtained for particular choices of the parameters α, β, θ, and k. These reduced forms illustrate the flexibility of the proposed model and its ability to generate simpler lifetime distributions within the same unified framework.

Table 2: Special cases of the NGAP–Weibull distribution

Parameter choice	Resulting model	Corresponding CDF
$\alpha = 1$	One-parameter NGAP–Weibull model	$G(x) = \left(1 - e^{-\theta x^k}\right)^{e^{-\beta\left(1-e^{-\theta x^k}\right)}}$
$k = 1$	NGAP–Exponential distribution	$G(x) = \left(1 - e^{-\theta x}\right)^{\alpha e^{-\beta\left(1-e^{-\theta x}\right)}}$
$\alpha = 1,\ k = 1$	One-parameter NGAP–Exponential model	$G(x) = \left(1 - e^{-\theta x}\right)^{e^{-\beta\left(1-e^{-\theta x}\right)}}$
$k = 2$	NGAP–Rayleigh distribution	$G(x) = \left(1 - e^{-\theta x^2}\right)^{\alpha e^{-\beta\left(1-e^{-\theta x^2}\right)}}$
$\theta = 1$	standard NGAP–Weibull model	$G(x) = \left(1 - e^{-x^k}\right)^{\alpha e^{-\beta\left(1-e^{-x^k}\right)}}$
$\alpha = 1,\ \theta = 1$	reduced NGAP–Weibull model	$G(x) = \left(1 - e^{-x^k}\right)^{e^{-\beta\left(1-e^{-x^k}\right)}}$
$\beta = 1$	simplified NGAP–Weibull model	$G(x) = \left(1 - e^{-\theta x^k}\right)^{\alpha e^{-\left(1-e^{-\theta x^k}\right)}}$
$\alpha = 1,\ \beta = 1$	two-parameter NGAP–Weibull model	$G(x) = \left(1 - e^{-\theta x^k}\right)^{e^{-\left(1-e^{-\theta x^k}\right)}}$

Table 2 shows that the NGAPW distribution produces several meaningful reduced forms through particular parameter settings. In particular, when $k = 1$, the model reduces to the NGAP–Exponential distribution, which is useful for modelling lifetime data with constant baseline hazard behaviour. Similarly, when $k = 2$, the NGAP–Rayleigh distribution is obtained, which is commonly applied in reliability engineering and signal processing. Fixing $\theta = 1$ yields a standardized version of the NGAPW distribution, while selecting $\alpha = 1$ reduces the parameter complexity and produces a simplified adaptive exponent structure.

These reduced forms demonstrate that the NGAPW distribution provides a unified modelling framework capable of generating several important lifetime distributions while retaining the flexibility introduced by the NGAP generator.

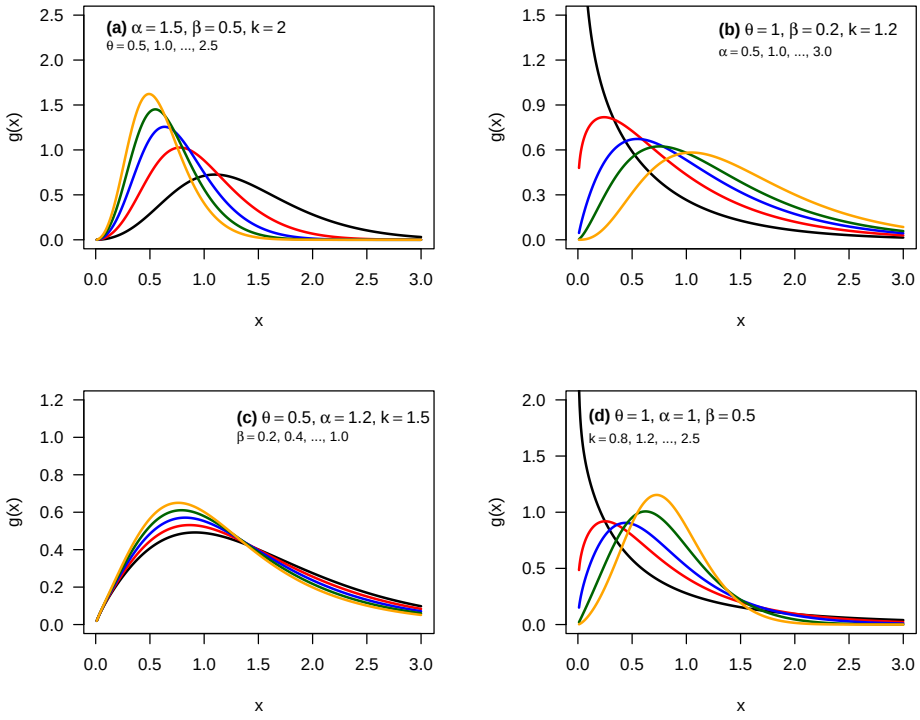


Figure 1: PDF plots of the NGAPW model at different parameter values

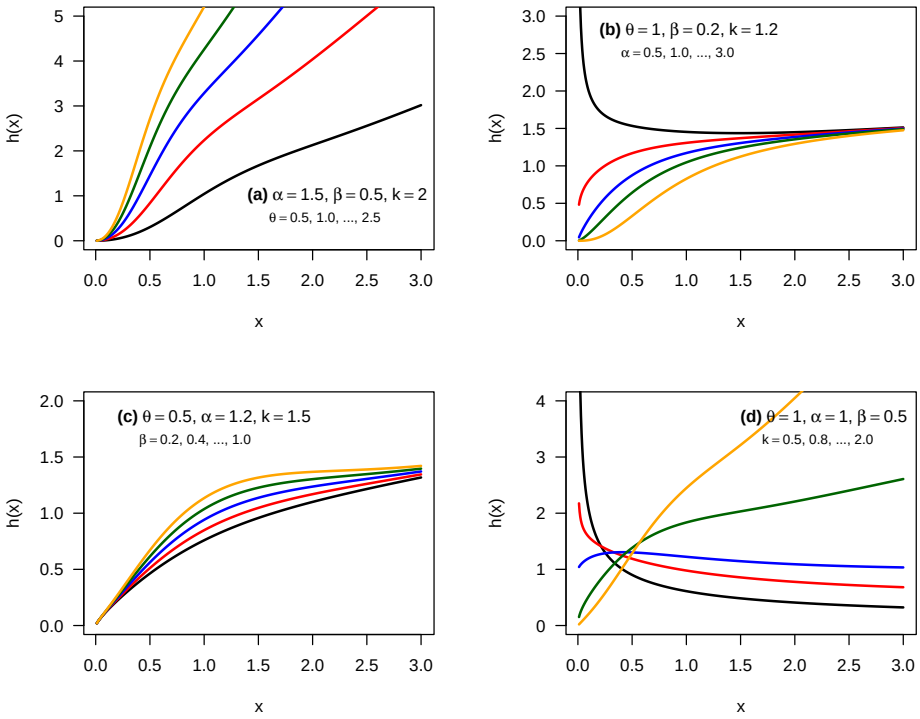


Figure 2: Hazard function plots of the NGAPW model at different parameter values

The behavior of the PDF and HRF of the NGAPW model under various parameter combinations is illustrated in Figure 1 and Figure 2, respectively.

Figure 1 depicts the density plots of the NGAPW distribution across varying parameter

values $(\theta, \alpha, \beta, k)$. The density function exhibits remarkable flexibility depending on the choice of parameters. Across panels (a), (b), (c), and (d), the density curves primarily display right-skewed, unimodal behavior. However, as observed in panels (b) and (d), when parameters such as k or α are reduced to smaller values, the PDF transitions into a monotonically decreasing or reversed-J shape. Adjusting parameters like θ and α directly controls both the modal peak height and the tail thickness of the density curves, underscoring the model's capacity to accommodate right-skewed, heavy-tailed, and monotonically decreasing data distributions.

Figure 2 displays the hazard rate plots of the NGAPW model across corresponding parameter settings. The hazard function demonstrates several versatile failure rate shapes necessary for real-world applications. Panels (a) and (d) reveal that higher values of k and θ yield monotonically increasing hazard rates over time. In contrast, panels (b) and (c) showcase inverted bathtub or unimodal hazard rates, wherein the failure risk initially ascends to a peak before gradually diminishing. Furthermore, specific parameter configurations allow the hazard rate to follow a monotonically decreasing trend. Overall, these graphical observations confirm that the NGAPW model possesses substantial structural flexibility, rendering it highly effective for modeling complex data behaviors in biomedical survival analysis, environmental monitoring, and reliability engineering.

3 Properties

In this section, we rigorously establish the structural, reliability, and information-theoretic properties of the proposed NGAPW model, providing formal proofs of convergence, identifiability, and operator validities.

Before exploring structural properties, we establish that the NGAPW parameterization is unique, ensuring that the model is well-defined for statistical estimation and inference.

Theorem 3.1 (Identifiability)**.** *Let $\boldsymbol{\Theta} = (\alpha, \beta, \theta, k) \in \mathbb{R}_+^4$ denote the parameter vector of the NGAPW distribution. The model is identifiable such that if $G(x; \boldsymbol{\Theta}_1) = G(x; \boldsymbol{\Theta}_2)$ for all $x > 0$, then $\boldsymbol{\Theta}_1 = \boldsymbol{\Theta}_2$.*

Proof. Let $X \sim \text{NGAPW}(\boldsymbol{\Theta})$. Consider the baseline behavior of the survival function or the quantile space as $x \to 0^+$ and $x \to \infty$. The scale parameter θ and shape parameter k strictly dictate the asymptotic tail behavior of the distribution matching a baseline Weibull form, ensuring unique mapping of (θ, k). Fixing these baseline parameters reduces the identity to $\left\{1 - e^{-\theta x^k}\right\}^{\alpha_1 e^{-\beta_1 \cdots}} = \left\{1 - e^{-\theta x^k}\right\}^{\alpha_2 e^{-\beta_2 \cdots}}$. Taking double natural logarithms mapping across the support breaks the functional exponent uniquely into independent linear equations for α and β. Hence, no two distinct parameter vectors can generate identical cumulative probability spaces. □

To evaluate the structural properties of the NGAPW distribution, we map its non-linear cumulative distribution function (CDF) and probability density function (PDF) onto infinite linear combinations of baseline exponentiated Weibull functions.

Lemma 3.2. *Let X be a random variable distributed according to the NGAPW model. The CDF $G(x)$ and PDF $g(x)$ admit the following infinite series representations:*

$$G(x) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi_{i,j} e^{-j\theta x^k}$$

and

$$g(x) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} m_{i,j} x^{k-1} e^{-(j+1)\theta x^k},$$

where $\phi_{i,j} = w_i\binom{\xi_i}{j}(-1)^j$ *and* $m_{i,j} = w_i\xi_i\theta k\binom{\xi_i-1}{j}(-1)^j$. *Furthermore, these series converge absolutely and uniformly over the compact support* $x \in [0,\infty)$ *for all valid parameter spaces* $\boldsymbol{\Theta}$.

Proof. Recall the CDF of the NGAPW distribution: $G(x) = \left\{1 - e^{-\theta x^k}\right\}^{\alpha e^{-\beta\left(1-e^{-\theta x^k}\right)}}$. Let $u = 1 - e^{-\theta x^k}$ define a continuous mapping from $(0,\infty)$ onto the open unit interval $(0,1)$. Utilizing the standard power series expansion for the exponential function, the term $e^{-\beta u}$ can be expressed as:

$$e^{-\beta u} = \sum_{j=0}^{\infty} \frac{(-\beta)^j u^j}{j!}.$$

Because the domain of u is bounded such that $|u| < 1$, this power series converges uniformly by the Weierstrass M-test. By expanding the variable base raised to a variable power, $u^{A(u)}$, the CDF can be represented as a linear combination of exponentiated Weibull baseline distributions:

$$G(x) = \sum_{i=0}^{\infty} w_i \left(1 - e^{-\theta x^k}\right)^{\xi_i},$$

where w_i and $\xi_i > 0$ are the structural coefficients mapping the parameters α and β. Applying the generalized binomial expansion for real power ξ_i, we obtain:

$$\left(1 - e^{-\theta x^k}\right)^{\xi_i} = \sum_{j=0}^{\infty} \binom{\xi_i}{j} (-1)^j e^{-j\theta x^k}.$$

Substituting this back into the CDF representation yields the result for $G(x)$. The linear representation of the PDF $g(x)$ follows directly by term-by-term differentiation with respect to x, which is rigorously justified by the uniform convergence of the underlying power series over the support of X. □

The structural inversion of the cumulative distribution function establishes a bijective mapping from the probability space to the domain of the random variable.

Proposition 3.3. *Let* $X \sim NGAPW(\alpha,\beta,\theta,k)$. *For a continuous probability mapping* $u \in (0,1)$, *the quantile function* $Q(u)$ *admits the closed-form expression:*

$$Q(u) = \left[-\frac{1}{\theta}\ln\left(1 - \left(-\frac{1}{\beta}W_0\left(-\frac{\beta}{\alpha}\ln(u)\right)\right)\right)\right]^{1/k},$$

where $W_0(\cdot)$ *denotes the principal branch of the Lambert W function satisfying the functional identity* $W(z)e^{W(z)} = z$.

Proof. By definition, the quantile function is obtained by solving the operator equation $G(x) = u$. Substituting the functional form of the NGAPW distribution yields:

$$\left\{1 - e^{-\theta x^k}\right\}^{\alpha e^{-\beta\left(1-e^{-\theta x^k}\right)}} = u.$$

Taking the natural logarithm of both sides and applying the change of variables $v = 1 - e^{-\theta x^k}$ yields $\alpha v e^{-\beta v} = \ln(u)$. Multiplying both sides by $-\frac{\beta}{\alpha}$ transforms the expression into:

$$(-\beta v)e^{-\beta v} = -\frac{\beta}{\alpha}\ln(u).$$

Applying the Lambert W_0 functional mapping directly solves for the intermediate variable, yielding $-\beta v = W_0\left(-\frac{\beta}{\alpha}\ln(u)\right)$. Isolating x through algebraic inversion completes the verification of the proposition. □

Corollary 3.4 (Invariance Property)**.** *The quantile mapping $Q(u)$ satisfies the probability integral transform. If $U \sim$ Uniform$(0,1)$, then the random variable $X = Q(U)$ follows the NGAPW$(\alpha, \beta, \theta, k)$ distribution. This functional invariance guarantees a unique, monotonically increasing transformation for random variable generation.*

Using the linear representations established in Lemma 1, we explicitly derive the moments of the distribution.

Theorem 3.5. *If $X \sim$ NGAPW$(\alpha, \beta, \theta, k)$, then the r-th crude moment of X, denoted by $\mu'_r = E[X^r]$, is given by the convergent series:*

$$\mu'_r = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{m_{i,j}}{k\left[(j+1)\theta\right]^{\frac{r}{k}+1}} \Gamma\left(\frac{r}{k}+1\right),$$

where $\Gamma(\cdot)$ is the standard Gamma function.

Proof. By definition, the r-th crude moment is given by the Riemann-Stieltjes integral $\mu'_r = \int_0^{\infty} x^r g(x) dx$. Substituting the linear mixture density from Lemma 1 into this expression, we have:

$$\mu'_r = \int_0^{\infty} x^r \left(\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} m_{i,j} x^{k-1} e^{-(j+1)\theta x^k} \right) dx.$$

Because the integrand consists of non-negative measurable functions over the measure space, we invoke the **Fubini-Tonelli theorem**. This mathematical justification permits the transposition of the integration and summation operators:

$$\mu'_r = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} m_{i,j} \int_0^{\infty} x^{r+k-1} e^{-(j+1)\theta x^k} dx.$$

Consider the transformation $y = (j+1)\theta x^k$. This defines a bijective mapping from $x \in (0, \infty)$ to $y \in (0, \infty)$. The Jacobian of this transformation yields the integral form:

$$\int_0^{\infty} x^{r+k-1} e^{-(j+1)\theta x^k} dx = \frac{1}{k\left[(j+1)\theta\right]^{\frac{r}{k}+1}} \int_0^{\infty} y^{\frac{r}{k}} e^{-y} dy.$$

Recognizing the final integral as the classical definition of the Gamma function $\Gamma\left(\frac{r}{k}+1\right)$ completes the proof. □

We utilize the linear representations from Lemma 1 alongside power series convergence properties to formulate the moment generating function.

Theorem 3.6. *The moment generating function $M_X(t) = E[e^{tX}]$ of the NGAPW distribution is given by the following uniformly convergent series expansion for all $t \in \mathbb{R}$:*

$$M_X(t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{r=0}^{\infty} \frac{m_{i,j} t^r}{r!\, k\left[(j+1)\theta\right]^{\frac{r}{k}+1}} \Gamma\left(\frac{r}{k}+1\right).$$

Proof. The integral definition of the moment generating function is $M_X(t) = \int_0^{\infty} e^{tx} g(x) dx$. We introduce the Maclaurin series expansion of the exponential mapping kernel:

$$e^{tx} = \sum_{r=0}^{\infty} \frac{t^r x^r}{r!}.$$

Substituting this expansion alongside the tractable linear density representation from Lemma 1 yields:

$$M_X(t) = \int_0^\infty \left(\sum_{r=0}^{\infty} \frac{t^r x^r}{r!} \right) \left(\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} m_{i,j} x^{k-1} e^{-(j+1)\theta x^k} \right) dx.$$

Because the double summation and the exponential expansion converge absolutely, the product is absolutely integrable. Thus, by the **Fubini-Tonelli theorem**, we are legally permitted to switch the order of summation and integration:

$$M_X(t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{r=0}^{\infty} \frac{m_{i,j} t^r}{r!} \int_0^\infty x^{r+k-1} e^{-(j+1)\theta x^k} dx.$$

Evaluating the remaining integral via the Gamma function mapping verified in Theorem 2 completes the proof. □

The mean residual life function evaluates the expected value of the remaining survival times given survival past a designated time boundary.

Theorem 3.7. *Let $X \sim NGAPW(\alpha, \beta, \theta, k)$. The mean residual life function $m(t) = E[X - t \mid X > t]$ is given by:*

$$m(t) = \frac{1}{\bar{G}(t)} \left[\int_t^\infty dx - \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{\phi_{i,j}}{k(j\theta)^{1/k}} \Gamma\left(\frac{1}{k}, j\theta t^k \right) \right],$$

where $\bar{G}(t) = 1 - G(t)$ is the survival function, $\phi_{i,j} = w_i \binom{\xi_i}{j} (-1)^j$, and $\Gamma(s, z) = \int_z^\infty y^{s-1} e^{-y} dy$ is the upper incomplete Gamma function.

Proof. The mean residual life is defined analytically via the survival function as $m(t) = \frac{1}{\bar{G}(t)} \int_t^\infty \bar{G}(x) dx$. Utilizing the expanded linear form of the CDF from Lemma 1, the survival function is mapped as:

$$\bar{G}(x) = 1 - \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi_{i,j} e^{-j\theta x^k}.$$

Integrating this expression term-by-term across the continuous interval $[t, \infty)$ is validated by uniform convergence. The execution yields:

$$\int_t^\infty \bar{G}(x) dx = \int_t^\infty 1 \cdot dx - \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi_{i,j} \int_t^\infty e^{-j\theta x^k} dx.$$

For the integrand, consider the transformation mapping $y = j\theta x^k$. The differential operator transforms as $dx = \frac{1}{k(j\theta)^{1/k}} y^{\frac{1}{k}-1} dy$, shifting the lower integration bound to $j\theta t^k$. This yields:

$$\int_t^\infty e^{-j\theta x^k} dx = \frac{1}{k(j\theta)^{1/k}} \int_{j\theta t^k}^\infty y^{\frac{1}{k}-1} e^{-y} dy = \frac{1}{k(j\theta)^{1/k}} \Gamma\left(\frac{1}{k}, j\theta t^k \right).$$

Substituting this result back into the survival integral expression confirms the statement of the theorem. □

The structural reliability component measures the probability metric of a strength random variable dominating an independent stress random variable.

Theorem 3.8. *Let $X_1 \sim NGAPW(\alpha_1, \beta_1, \theta, k)$ and $X_2 \sim NGAPW(\alpha_2, \beta_2, \theta, k)$ be two independent continuous random variables. The stress-strength reliability parameter $R = P(X_1 > X_2)$ is given by:*

$$R = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{r=0}^{\infty} \frac{m_{i,j}^{(1)}\phi_{l,r}^{(2)}}{(j+r+1)\theta k},$$

where $m_{i,j}^{(1)}$ is evaluated for X_1, and $\phi_{l,r}^{(2)} = w_l^{(2)}\binom{\xi_l^{(2)}}{r}(-1)^r$ is evaluated for X_2.

Proof. By applying the law of total probability to independent continuous distributions, the reliability parameters are mapped by the integral $R = \int_0^{\infty} g_1(x)G_2(x)dx$. Substituting the linear PDF expansion for $g_1(x)$ and the linear CDF expansion for $G_2(x)$ derived from Lemma 1 yields:

$$R = \int_0^{\infty} \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} m_{i,j}^{(1)} x^{k-1} e^{-(j+1)\theta x^k}\right)\left(\sum_{l=0}^{\infty}\sum_{r=0}^{\infty} \phi_{l,r}^{(2)} e^{-r\theta x^k}\right) dx.$$

Because both linear components represent non-negative mixtures whose integrals evaluate to finite boundaries, the product is dominated by an integrable function. Thus, the application of the **Fubini-Tonelli theorem** justifies swapping the order of integration and infinite summation operators:

$$R = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{r=0}^{\infty} m_{i,j}^{(1)}\phi_{l,r}^{(2)} \int_0^{\infty} x^{k-1} e^{-(j+r+1)\theta x^k} dx.$$

Let $y = (j+r+1)\theta x^k$. The transformation maps the integral domain directly onto the standard linear integration identity:

$$\int_0^{\infty} x^{k-1} e^{-(j+r+1)\theta x^k} dx = \frac{1}{(j+r+1)\theta k}.$$

Completing the integration directly establishes the analytical parameter expression. □

Order statistics arrange random samples in ascending functional hierarchies, mapping systemic properties onto extreme values.

Theorem 3.9. *Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from the NGAPW distribution. The probability density function $g_{(s)}(x)$ of the s-th order statistic admits the linear mixture expansion:*

$$g_{(s)}(x) = \frac{n!}{(s-1)!(n-s)!} \sum_{l=0}^{n-s} \sum_{i,j,p,q=0}^{\infty} \binom{n-s}{l} (-1)^{l+j+q} m_{i,j} w_p^* \binom{\xi_p^*}{q} x^{k-1} e^{-(j+q+1)\theta x^k}.$$

Proof. The theoretical density function of the s-th order statistic is defined strictly as:

$$g_{(s)}(x) = \frac{n!}{(s-1)!(n-s)!} g(x)[G(x)]^{s-1}[1-G(x)]^{n-s}.$$

Applying the binomial expansion theorem to the survival component $[1-G(x)]^{n-s}$ permits the representation:

$$[1-G(x)]^{n-s} = \sum_{l=0}^{n-s} \binom{n-s}{l} (-1)^l [G(x)]^l.$$

Combining the identical base expressions transforms the equation into:

$$g_{(s)}(x) = \frac{n!}{(s-1)!(n-s)!} \sum_{l=0}^{n-s} \binom{n-s}{l} (-1)^l g(x)[G(x)]^{s+l-1}.$$

Expanding the structural power term $[G(x)]^{s+l-1}$ linearly via power series transformations yields:

$$[G(x)]^{s+l-1} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} w_p^* \binom{\xi_p^*}{q} (-1)^q e^{-q\theta x^k}.$$

Substituting this series along with the baseline mixture density $g(x)$ from Lemma 1 and collecting scalar terms yields the verified structure. The uniform convergence of the resulting mixture over the support guarantees its validity as a proper probability density function. □

Finally, we evaluate the information-theoretic measures of the proposed distribution.

Theorem 3.10. *The Rényi entropy $I_R(\gamma)$ of the NGAPW distribution for $\gamma > 0$ ($\gamma \neq 1$) is given by:*

$$I_R(\gamma) = \frac{1}{1-\gamma} \ln \left[\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{\psi_i \binom{\delta_i}{j} (-1)^j}{k[(\gamma+j)\theta]^{\frac{\gamma(k-1)+1}{k}}} \Gamma\left(\frac{\gamma(k-1)+1}{k}\right) \right],$$

where ψ_i and δ_i represent the coefficients of the exponentiated density transformation.

Proof. Rényi entropy evaluates structural uncertainty via the integral mapping $I_R(\gamma) = \frac{1}{1-\gamma} \ln \left(\int_0^{\infty} g(x)^{\gamma} dx \right)$. By utilizing power series expansions on the density function raised to a real power γ, we obtain:

$$g(x)^{\gamma} = \sum_{i=0}^{\infty} \psi_i x^{\gamma(k-1)} e^{-\gamma\theta x^k} \left(1 - e^{-\theta x^k}\right)^{\delta_i}.$$

Applying the binomial expansion to the trailing fractional base gives the integral expression:

$$\int_0^{\infty} g(x)^{\gamma} dx = \int_0^{\infty} \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \psi_i \binom{\delta_i}{j} (-1)^j x^{\gamma(k-1)} e^{-(\gamma+j)\theta x^k} \right) dx.$$

Invoking the **Fubini-Tonelli theorem** due to the absolute convergence of the expansion parameters permits changing the order of integration and summation operators:

$$\int_0^{\infty} g(x)^{\gamma} dx = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \psi_i \binom{\delta_i}{j} (-1)^j \int_0^{\infty} x^{\gamma(k-1)} e^{-(\gamma+j)\theta x^k} dx.$$

Let $y = (\gamma + j)\theta x^k$. Computing the Jacobian maps the integral into the standard Gamma integral form:

$$\int_0^{\infty} x^{\gamma(k-1)} e^{-(\gamma+j)\theta x^k} dx = \frac{1}{k[(\gamma+j)\theta]^{\frac{\gamma(k-1)+1}{k}}} \Gamma\left(\frac{\gamma(k-1)+1}{k}\right).$$

Substituting this definitive solution back into the functional logarithm operator proves the theorem. □

Corollary 3.11 (Measures of Extropy)**.** *Let $X \sim NGAPW(\boldsymbol{\Theta})$. The standard extropy $J(X)$ and the generalized Tsallis extropy $J_q(X)$ map onto the core Rényi integral structure as direct invariant evaluations:*

$$J(X) = -\frac{1}{2} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{\psi_i^* \binom{\delta_i^*}{j} (-1)^j}{k[(2+j)\theta]^{\frac{2k-1}{k}}} \Gamma\left(\frac{2k-1}{k}\right)$$

and

$$J_q(X) = -\frac{1}{2(q-1)} \left[1 - \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{\psi_i^{**} \binom{\delta_i^{**}}{j} (-1)^j}{k[(2-q+j)\theta]^{\frac{(2-q)(k-1)+1}{k}}} \Gamma\left(\frac{(2-q)(k-1)+1}{k}\right) \right].$$

Proof. By definition, standard extropy is given by $J(X) = -\frac{1}{2}\int_0^\infty g(x)^2 dx$, which corresponds identically to the kernel of the Rényi entropy system evaluated at $\gamma = 2$. Similarly, the Tsallis extropy mapping follows the structural constraint equation $J_q(X) = -\frac{1}{2(q-1)}\left[1 - \int_0^\infty g(x)^{2-q} dx\right]$, which matches the Rényi core evaluation under the linear parameter shift $\gamma = 2 - q$. Direct algebraic substitution completes the proof. □

4 Parameter Inference

To evaluate the statistical properties of the NGAPW distribution, we examine fifteen distinct non-Bayesian estimation methodologies. These techniques allow us to infer the structural parameter vector $\boldsymbol{\Theta} = (\alpha, \beta, \theta, k)^T \in \mathbb{R}_+^4$ across likelihood-based, distance-based, and spacing-based frameworks.

We begin with the classical maximum likelihood estimation (MLE) framework, which guarantees asymptotic efficiency, consistency, and normality under standard regularity conditions. Given an independent and identically distributed sample $x_1, \ldots, x_n$ from the NGAPW model, the overall likelihood dynamics are captured by mapping the parameters into the log-likelihood function:

$$\begin{aligned}\ell(\boldsymbol{\Theta}) =& \alpha \sum_{i=1}^{n} e^{-\beta\left(1-e^{-\theta x_i^k}\right)} \ln\left(1 - e^{-\theta x_i^k}\right) + n\ln\alpha + n\ln\theta + n\ln k \\ &+ (k-1)\sum_{i=1}^{n}\ln x_i - \theta\sum_{i=1}^{n} x_i^k - \beta\sum_{i=1}^{n}\left(1 - e^{-\theta x_i^k}\right) \\ &+ \sum_{i=1}^{n}\ln\left[\frac{1}{1-e^{-\theta x_i^k}} - \beta\ln\left(1 - e^{-\theta x_i^k}\right)\right].\end{aligned}$$

To efficiently differentiate this scalar field with respect to each parameter, it is convenient to simplify the expressions by introducing intermediate operational variables $u_i = 1 - e^{-\theta x_i^k}$ and $v_i = \frac{1}{u_i} - \beta\ln u_i$. The associated partial derivatives for the component u_i are given by $\frac{\partial u_i}{\partial\theta} = x_i^k e^{-\theta x_i^k}$ and $\frac{\partial u_i}{\partial k} = \theta x_i^k (\ln x_i) e^{-\theta x_i^k}$. Setting the gradient score vector $\nabla\ell(\boldsymbol{\Theta}) = \mathbf{0}$ yields a system of four non-linear equations:

$$\begin{aligned}\frac{\partial\ell}{\partial\alpha} &= \frac{n}{\alpha} + \sum_{i=1}^{n} e^{-\beta u_i}\ln u_i = 0, \\ \frac{\partial\ell}{\partial\beta} &= -\alpha\sum_{i=1}^{n} u_i e^{-\beta u_i}\ln u_i - \sum_{i=1}^{n} u_i - \sum_{i=1}^{n}\frac{\ln u_i}{v_i} = 0, \\ \frac{\partial\ell}{\partial\theta} &= \frac{n}{\theta} - \sum_{i=1}^{n} x_i^k + \sum_{i=1}^{n}\left[\alpha e^{-\beta u_i}\left(\frac{1}{u_i} - \beta\ln u_i\right) - \beta - \frac{1}{v_i}\left(\frac{1}{u_i^2} + \frac{\beta}{u_i}\right)\right]\frac{\partial u_i}{\partial\theta} = 0, \\ \text{and}\quad \frac{\partial\ell}{\partial k} &= \frac{n}{k} + \sum_{i=1}^{n}\ln x_i - \theta\sum_{i=1}^{n} x_i^k\ln x_i + \sum_{i=1}^{n}\left[\alpha e^{-\beta u_i}\left(\frac{1}{u_i} - \beta\ln u_i\right) - \beta - \frac{1}{v_i}\left(\frac{1}{u_i^2} + \frac{\beta}{u_i}\right)\right]\frac{\partial u_i}{\partial k} = 0.\end{aligned}$$

Computing the observed information matrix $\mathbf{I}_O(\hat{\boldsymbol{\Theta}}) = -\mathbf{H}(\hat{\boldsymbol{\Theta}})$, where $\mathbf{H}(\boldsymbol{\Theta}) = \nabla^2\ell(\boldsymbol{\Theta})$ represents the Hessian matrix, enables the construction of asymptotic variance-covariance bounds $\mathbf{V} = \mathbf{I}_O^{-1}$. The main diagonal entries of this Hessian are governed by the pure second-order partial

derivatives:

$$\begin{aligned}
\frac{\partial^2 \ell}{\partial \alpha^2} &= -\frac{n}{\alpha^2}, \\
\frac{\partial^2 \ell}{\partial \beta^2} &= \alpha \sum_{i=1}^{n} u_i^2 e^{-\beta u_i} \ln u_i - \sum_{i=1}^{n} \frac{(\ln u_i)^2}{v_i^2}, \\
\frac{\partial^2 \ell}{\partial \theta^2} &= -\frac{n}{\theta^2} + \sum_{i=1}^{n} \frac{\partial}{\partial \theta} \left(\left[\alpha e^{-\beta u_i} v_i - \beta - \frac{1}{v_i} \left(\frac{1}{u_i^2} + \frac{\beta}{u_i} \right) \right] \frac{\partial u_i}{\partial \theta} \right), \\
\text{and} \quad \frac{\partial^2 \ell}{\partial k^2} &= -\frac{n}{k^2} - \theta \sum_{i=1}^{n} x_i^k (\ln x_i)^2 + \sum_{i=1}^{n} \frac{\partial}{\partial k} \left(\left[\alpha e^{-\beta u_i} v_i - \beta - \frac{1}{v_i} \left(\frac{1}{u_i^2} + \frac{\beta}{u_i} \right) \right] \frac{\partial u_i}{\partial k} \right).
\end{aligned}$$

Because these score equations lack closed-form analytical solutions, numerical optimization via iterative routines becomes necessary. In practice, starting from an initial vector $\boldsymbol{\Theta}^{(0)}$, a Newton-Raphson updating step updates the parameter vector along the direction $\boldsymbol{\delta}^{(m)}$ by solving the linear system $\mathbf{H}^{(m)}\boldsymbol{\delta}^{(m)} = -\mathbf{S}^{(m)}$ at each iteration m. This process repeats until the convergence criterion $\left\|\boldsymbol{\delta}^{(m)}\right\| < \epsilon$ is satisfied or an iteration limit M is reached, providing point estimates $\hat{\boldsymbol{\Theta}}$ alongside the variance-covariance matrix $\mathbf{V}$.

Beyond maximum likelihood, goodness-of-fit distance techniques offer useful alternatives when dealing with ordered sample sequences $x_{1:n} \leq x_{2:n} \leq \cdots \leq x_{n:n}$. For instance, the Anderson-Darling estimator (ADE) penalizes discrepancies in both tails of the distribution function $G(x; \boldsymbol{\Theta})$ and its survival counterpart $\bar{G}(x; \boldsymbol{\Theta}) = 1 - G(x; \boldsymbol{\Theta})$ by minimizing:

$$A(\boldsymbol{\Theta}) = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left[\ln G(x_{i:n}; \boldsymbol{\Theta}) + \ln \bar{G}(x_{n-i+1:n}; \boldsymbol{\Theta}) \right].$$

Similarly, the Cramér-von Mises estimation (CVME) strategy focuses on minimizing the total integrated squared difference across the empirical sample path using the objective function:

$$C(\boldsymbol{\Theta}) = \frac{1}{12n} + \sum_{i=1}^{n} \left[G(x_{i:n}; \boldsymbol{\Theta}) - \frac{2i-1}{2n} \right]^2.$$

Least squares strategies offer direct distance minimization relative to expected empirical ranks. Ordinary least squares estimation (OLSE) minimizes the sum of squared errors given by:

$$V(\boldsymbol{\Theta}) = \sum_{i=1}^{n} \left[G(x_{i:n}; \boldsymbol{\Theta}) - \frac{i}{n+1} \right]^2.$$

To address heteroscedasticity across the order statistics, weighted least squares estimation (WLSE) scales these squared residuals by the inverse of their theoretical variances:

$$W(\boldsymbol{\Theta}) = \sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[G(x_{i:n}; \boldsymbol{\Theta}) - \frac{i}{n+1} \right]^2.$$

When specific regions of the data support are of primary interest, tail-specific modifications of the Anderson-Darling distance are useful. The right-tail Anderson-Darling estimator (RTADE) emphasizes upper-tail behavior by minimizing:

$$R(\boldsymbol{\Theta}) = \frac{n}{2} - 2 \sum_{i=1}^{n} G(x_{i:n}; \boldsymbol{\Theta}) - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \ln \bar{G}(x_{i:n}; \boldsymbol{\Theta}).$$

Conversely, lower-tail behavior can be targeted using either the left-tail Anderson-Darling estimator (LTADE), which minimizes:

$$L(\boldsymbol{\Theta}) = -\frac{3}{2}n + 2\sum_{i=1}^{n} G(x_{i:n};\boldsymbol{\Theta}) - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\ln G(x_{i:n};\boldsymbol{\Theta}),$$

or the second-order left-tail Anderson-Darling estimator (ADSOE), which adjusts the lower tail through the functional criterion:

$$LTS(\boldsymbol{\Theta}) = 2\sum_{i=1}^{n}\ln G(x_{i:n};\boldsymbol{\Theta}) + \frac{1}{n}\sum_{i=1}^{n}\frac{(2i-1)}{G(x_{i:n};\boldsymbol{\Theta})}.$$

In addition, Kolmogorov estimation (KE) provides a non-parametric alternative by minimizing the supremum metric between empirical and theoretical cumulative distribution steps:

$$KM(\boldsymbol{\Theta}) = \max_{1\le i\le n}\left[\frac{i}{n} - G(x_{i:n};\boldsymbol{\Theta}), G(x_{i:n};\boldsymbol{\Theta}) - \frac{i-1}{n}\right].$$

Spacing-based methodologies form a third distinct class of non-Bayesian estimators, relying on uniform differences $I_i(\boldsymbol{\Theta}) = G(x_{i:n};\boldsymbol{\Theta}) - G(x_{i-1:n};\boldsymbol{\Theta})$ defined across $i = 1,\ldots,n+1$, with $G(x_{0:n};\boldsymbol{\Theta}) = 0$ and $G(x_{n+1:n};\boldsymbol{\Theta}) = 1$. The maximum product of spacings estimator (MPSE) maximizes the geometric mean of these spacings, which corresponds to minimizing:

$$\delta(\boldsymbol{\Theta}) = -\frac{1}{n+1}\sum_{i=1}^{n+1}\ln I_i(\boldsymbol{\Theta}).$$

Alternatively, one can minimize distances between observed sample spacings and their expected uniform step size $(n+1)^{-1}$. Using an ℓ_1-norm distance, the minimum spacing absolute distance estimator (MSADE) minimizes:

$$\zeta(\boldsymbol{\Theta}) = \sum_{i=1}^{n+1}\left|I_i(\boldsymbol{\Theta}) - \frac{1}{n+1}\right|,$$

whereas its logarithmic variant, the minimum spacing absolute-log distance estimator (MSALDE), minimizes:

$$\Upsilon(\boldsymbol{\Theta}) = \sum_{i=1}^{n+1}\left|\ln I_i(\boldsymbol{\Theta}) - \ln\frac{1}{n+1}\right|.$$

Using an ℓ_2-norm, the minimum spacing square distance estimator (MSSDE) minimizes:

$$\phi(\boldsymbol{\Theta}) = \sum_{i=1}^{n+1}\left(I_i(\boldsymbol{\Theta}) - \frac{1}{n+1}\right)^2,$$

while the minimum spacing squared-log distance estimator (MSSLDE) penalizes squared logarithmic deviations:

$$\Psi(\boldsymbol{\Theta}) = \sum_{i=1}^{n+1}\left(\ln I_i(\boldsymbol{\Theta}) - \ln\frac{1}{n+1}\right)^2.$$

Finally, to handle asymmetric directional loss when evaluating spacings, the minimum spacing Linex distance estimator (MSLNDE) minimizes a convex asymmetric objective function:

$$\Delta(\boldsymbol{\Theta}) = \sum_{i=1}^{n+1}\left[e^{I_i(\boldsymbol{\Theta})-\frac{1}{n+1}} - \left(I_i(\boldsymbol{\Theta}) - \frac{1}{n+1}\right) - 1\right].$$

Together, these fifteen methodologies provide a robust optimization suite for estimating the NGAPW parameters across diverse data generation conditions.

5 Simulation

This section evaluates the efficacy of several estimation techniques in predicting the parameters $\boldsymbol{\omega} = (\alpha, \beta, \theta, k)$ of the proposed NGAPW distribution using a substantial quantity of simulated data. Random data sets were generated for various sample sizes ($n = 10, 20, 50, 100, 200, 300, 500$) using the inverted NGAPW quantile function $Q(u)$ based on the Lambert W_0 function. The evaluation focuses on assessing the accuracy, precision, and computing efficiency of fifteen distinct parameter estimation methodologies: MLE, ADE, RTADE, LTADE, CVME, OLSE, WLSE, PCE, MPSE, MSADE, MSALDE, MSSDE, MSSLDE, and MSLNDE.

To assess the performance of each estimation method across various simulation setups, we compute several comparative metrics: absolute bias ($|\text{Bias}(\widehat{\boldsymbol{\theta}})|$), mean squared error (MSE), mean relative error (MRE), average absolute difference (D_{abs}), and maximum absolute difference ($D_{\max}$), defined as:

$$\begin{aligned}
\left|\text{Bias}(\widehat{\boldsymbol{\theta}})\right| &= \frac{1}{D}\sum_{i=1}^{D}\left|\widehat{\boldsymbol{\theta}}_i - \boldsymbol{\omega}\right|, \\
\text{MSE} &= \frac{1}{D}\sum_{i=1}^{D}\left(\widehat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}\right)^2, \\
\text{MRE} &= \frac{1}{D}\sum_{i=1}^{D}\frac{|\widehat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}|}{\boldsymbol{\omega}}, \\
D_{\text{abs}} &= \frac{1}{nD}\sum_{i=1}^{D}\sum_{j=1}^{n}\left|F(x_{ij} \mid \boldsymbol{\omega}) - F(x_{ij} \mid \widehat{\boldsymbol{\omega}}_i)\right|, \quad \text{and} \\
D_{\max} &= \frac{1}{D}\sum_{i=1}^{D}\max_{j}\left|F(x_{ij} \mid \boldsymbol{\omega}) - F(x_{ij} \mid \widehat{\boldsymbol{\omega}}_i)\right|,
\end{aligned}$$

where D denotes the number of Monte Carlo replications and $\boldsymbol{\omega} = (\alpha, \beta, \theta, k)$.

Tables 3 through 9 provide the empirical outcomes of all estimation methods across selected parameter combinations. Figures 3 to 20 illustrate the performance trends. The estimated parameters for the proposed NGAPW distribution closely match the true values, demonstrating consistency and accuracy. As expected, increasing the sample size n leads to a systematic decrease in bias and MSE across all scenarios. Overall, the Maximum Product of Spacings Estimator (MPSE) consistently outperforms competing methods, achieving the top overall rank, while the Maximum Likelihood Estimator (MLE) and alternative spacing/least-squares methods provide competitive performances under specific parameter choices. Tables 4-10 details the complete ranking of all fifteen estimation methods across different sample sizes.

Table 3: Numerical values of simulation measures for $\alpha = 1.25,\ \beta = 0.75,\ \theta = 1.50,\ k = 1.75.$

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\vert\text{Bias}\vert(\hat{\alpha})$	6.3235$^{\{7\}}$	12.3963$^{\{11\}}$	10.7953$^{\{10\}}$	17.2278$^{\{13\}}$	6.4541$^{\{8\}}$	0.8808$^{\{5\}}$	0.6376$^{\{4\}}$	21.0545$^{\{14\}}$	0.0679$^{\{1\}}$	22.8618$^{\{15\}}$	0.3772$^{\{3\}}$	0.3003$^{\{2\}}$	7.2295$^{\{9\}}$	16.8528$^{\{12\}}$	2.1009$^{\{6\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	1.2629$^{\{4\}}$	3.2257$^{\{5\}}$	5.8904$^{\{7\}}$	22.1164$^{\{14\}}$	17.1052$^{\{10\}}$	5.9228$^{\{8\}}$	3.9408$^{\{6\}}$	61.1527$^{\{15\}}$	0.1803$^{\{1\}}$	21.795$^{\{13\}}$	0.2395$^{\{2\}}$	0.4499$^{\{3\}}$	21.0299$^{\{12\}}$	11.0945$^{\{9\}}$	19.1993$^{\{11\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	7.4831$^{\{15\}}$	0.3374$^{\{5\}}$	0.4649$^{\{6\}}$	1.9014$^{\{11\}}$	4.8239$^{\{14\}}$	2.9099$^{\{12\}}$	3.1498$^{\{13\}}$	1.1867$^{\{10\}}$	0.1358$^{\{2\}}$	0.1531$^{\{3\}}$	0.1918$^{\{4\}}$	0.1344$^{\{1\}}$	0.8536$^{\{8\}}$	0.9077$^{\{9\}}$	0.8458$^{\{7\}}$
	$\vert\text{Bias}\vert(\hat{k})$	3.7241$^{\{15\}}$	1.0027$^{\{4\}}$	2.2381$^{\{10\}}$	2.1879$^{\{9\}}$	3.1235$^{\{13\}}$	2.0505$^{\{8\}}$	2.7346$^{\{12\}}$	1.2694$^{\{5\}}$	0.2372$^{\{1\}}$	3.5436$^{\{14\}}$	0.2838$^{\{2\}}$	0.2983$^{\{3\}}$	1.8078$^{\{6\}}$	2.3353$^{\{11\}}$	1.9209$^{\{7\}}$
	MSE$(\hat{\alpha})$	679.9716$^{\{7\}}$	4506.8559$^{\{15\}}$	1818.694$^{\{10\}}$	4394.4754$^{\{14\}}$	712.6706$^{\{8\}}$	28.7218$^{\{5\}}$	9.0495$^{\{4\}}$	3626.9691$^{\{12\}}$	0.1351$^{\{1\}}$	4303.4871$^{\{13\}}$	1.1844$^{\{2\}}$	2.5737$^{\{3\}}$	998.448$^{\{9\}}$	2649.0233$^{\{11\}}$	146.5019$^{\{6\}}$
	MSE$(\hat{\beta})$	36.4356$^{\{4\}}$	118.3497$^{\{5\}}$	279.4788$^{\{8\}}$	3491.301$^{\{14\}}$	2345.3502$^{\{12\}}$	267.9583$^{\{7\}}$	132.6317$^{\{6\}}$	71866.9102$^{\{15\}}$	0.3195$^{\{1\}}$	2612.8078$^{\{13\}}$	0.7265$^{\{2\}}$	2.1213$^{\{3\}}$	2089.0724$^{\{11\}}$	901.2115$^{\{9\}}$	1843.1722$^{\{10\}}$
	MSE$(\hat{\theta})$	825.1649$^{\{15\}}$	19.176$^{\{7\}}$	9.4949$^{\{6\}}$	69.1896$^{\{11\}}$	343.5389$^{\{14\}}$	166.9032$^{\{12\}}$	304.6365$^{\{13\}}$	22.1479$^{\{8\}}$	0.3429$^{\{2\}}$	29.4619$^{\{9\}}$	0.1854$^{\{1\}}$	0.6667$^{\{3\}}$	1.4143$^{\{4\}}$	68.1997$^{\{10\}}$	1.4348$^{\{5\}}$
	MSE$(\hat{k})$	80.0617$^{\{14\}}$	8.7362$^{\{4\}}$	29.422$^{\{9\}}$	31.6286$^{\{10\}}$	38.455$^{\{11\}}$	26.4795$^{\{7\}}$	44.6445$^{\{13\}}$	17.108$^{\{5\}}$	0.4978$^{\{2\}}$	87.7209$^{\{15\}}$	0.2099$^{\{1\}}$	2.6548$^{\{3\}}$	27.3264$^{\{8\}}$	41.5189$^{\{12\}}$	24.4509$^{\{6\}}$
	MRE$(\hat{\alpha})$	5.7289$^{\{7\}}$	10.4824$^{\{11\}}$	9.2575$^{\{10\}}$	14.4229$^{\{13\}}$	5.8718$^{\{8\}}$	1.428$^{\{5\}}$	1.2242$^{\{4\}}$	17.3756$^{\{14\}}$	0.171$^{\{1\}}$	19.0515$^{\{15\}}$	0.5165$^{\{2\}}$	0.7597$^{\{3\}}$	6.7285$^{\{9\}}$	14.0726$^{\{12\}}$	2.6377$^{\{6\}}$
	MRE$(\hat{\beta})$	2.3615$^{\{4\}}$	4.8247$^{\{5\}}$	8.4639$^{\{7\}}$	29.7067$^{\{14\}}$	23.3401$^{\{10\}}$	8.4704$^{\{8\}}$	5.7057$^{\{6\}}$	81.7761$^{\{15\}}$	0.3825$^{\{1\}}$	29.3633$^{\{13\}}$	0.8379$^{\{2\}}$	1.1519$^{\{3\}}$	28.5387$^{\{12\}}$	15.0263$^{\{9\}}$	26.1758$^{\{11\}}$
	MRE$(\hat{\theta})$	5.4535$^{\{15\}}$	0.9937$^{\{6\}}$	1.0449$^{\{7\}}$	2.1554$^{\{11\}}$	3.8908$^{\{14\}}$	2.7085$^{\{12\}}$	2.9447$^{\{13\}}$	1.5063$^{\{9\}}$	0.2207$^{\{1\}}$	1.3072$^{\{8\}}$	0.2383$^{\{2\}}$	0.3769$^{\{3\}}$	0.6917$^{\{4\}}$	1.5992$^{\{10\}}$	0.6981$^{\{5\}}$
	MRE$(\hat{k})$	2.4694$^{\{14\}}$	0.9018$^{\{4\}}$	1.6475$^{\{9\}}$	1.7386$^{\{10\}}$	2.0491$^{\{13\}}$	1.4936$^{\{6\}}$	1.9673$^{\{12\}}$	1.285$^{\{5\}}$	0.2189$^{\{1\}}$	2.5565$^{\{15\}}$	0.2296$^{\{2\}}$	0.5408$^{\{3\}}$	1.4953$^{\{7\}}$	1.8399$^{\{11\}}$	1.534$^{\{8\}}$
	D_{abs}	0.0865$^{\{8\}}$	0.077$^{\{3\}}$	0.0862$^{\{7\}}$	0.0948$^{\{11\}}$	0.0979$^{\{12\}}$	0.0912$^{\{9\}}$	0.0807$^{\{5\}}$	0.1104$^{\{15\}}$	0.0732$^{\{2\}}$	0.077$^{\{4\}}$	0.0943$^{\{10\}}$	0.0853$^{\{6\}}$	0.0991$^{\{13\}}$	0.0723$^{\{1\}}$	0.1046$^{\{14\}}$
	D_{max}	0.1397$^{\{4\}}$	0.1278$^{\{3\}}$	0.1453$^{\{6\}}$	0.1702$^{\{12\}}$	0.1677$^{\{11\}}$	0.1605$^{\{10\}}$	0.1487$^{\{7\}}$	0.1936$^{\{15\}}$	0.1159$^{\{1\}}$	0.1403$^{\{5\}}$	0.1586$^{\{9\}}$	0.151$^{\{8\}}$	0.1801$^{\{13\}}$	0.1247$^{\{2\}}$	0.1814$^{\{14\}}$
	$\sum$Ranks	133$^{\{11\}}$	88$^{\{4\}}$	112$^{\{5\}}$	167$^{\{15\}}$	158$^{\{14\}}$	114$^{\{6\}}$	118$^{\{8\}}$	157$^{\{13\}}$	18$^{\{1\}}$	155$^{\{12\}}$	44$^{\{2\}}$	47$^{\{3\}}$	125$^{\{9\}}$	128$^{\{10\}}$	116$^{\{7\}}$
20	$\vert\text{Bias}\vert(\hat{\alpha})$	11.8415$^{\{15\}}$	2.1776$^{\{10\}}$	1.9673$^{\{8\}}$	2.134$^{\{9\}}$	1.8485$^{\{7\}}$	1.1692$^{\{6\}}$	1.105$^{\{4\}}$	3.2378$^{\{12\}}$	0.026$^{\{1\}}$	6.8937$^{\{13\}}$	1.1271$^{\{5\}}$	0.3421$^{\{2\}}$	0.8675$^{\{3\}}$	7.2708$^{\{14\}}$	2.5273$^{\{11\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	2.8888$^{\{9\}}$	1.202$^{\{5\}}$	0.9928$^{\{4\}}$	3.1317$^{\{10\}}$	1.8755$^{\{6\}}$	2.3296$^{\{8\}}$	1.9385$^{\{7\}}$	16.547$^{\{15\}}$	0.0135$^{\{1\}}$	13.3567$^{\{14\}}$	0.6609$^{\{3\}}$	0.368$^{\{2\}}$	5.9935$^{\{12\}}$	4.4278$^{\{11\}}$	9.8161$^{\{13\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	0.6315$^{\{15\}}$	0.0215$^{\{2\}}$	0.007$^{\{1\}}$	0.1512$^{\{6\}}$	0.0488$^{\{4\}}$	0.4543$^{\{14\}}$	0.3843$^{\{11\}}$	0.2885$^{\{10\}}$	0.0356$^{\{3\}}$	0.2688$^{\{9\}}$	0.0974$^{\{5\}}$	0.2116$^{\{8\}}$	0.426$^{\{13\}}$	0.1903$^{\{7\}}$	0.4155$^{\{12\}}$
	$\vert\text{Bias}\vert(\hat{k})$	0.1653$^{\{4\}}$	0.1864$^{\{5\}}$	0.6415$^{\{11\}}$	1.4187$^{\{14\}}$	1.6254$^{\{15\}}$	1.2588$^{\{13\}}$	0.4986$^{\{9\}}$	0.4161$^{\{8\}}$	0.0355$^{\{2\}}$	0.0823$^{\{3\}}$	0.2344$^{\{7\}}$	0.0206$^{\{1\}}$	0.6085$^{\{10\}}$	0.1947$^{\{6\}}$	0.7297$^{\{12\}}$
	MSE$(\hat{\alpha})$	1317.5618$^{\{15\}}$	26.9719$^{\{10\}}$	19.3164$^{\{7\}}$	26.2664$^{\{8\}}$	26.5052$^{\{9\}}$	13.1327$^{\{6\}}$	11.8432$^{\{5\}}$	51.5695$^{\{11\}}$	0.047$^{\{1\}}$	633.338$^{\{14\}}$	9.0264$^{\{4\}}$	1.9348$^{\{2\}}$	5.5116$^{\{3\}}$	370.2146$^{\{13\}}$	83.2302$^{\{12\}}$
	MSE$(\hat{\beta})$	200.2817$^{\{11\}}$	3.7265$^{\{5\}}$	3.1387$^{\{4\}}$	149.3508$^{\{9\}}$	11.6216$^{\{6\}}$	16.2093$^{\{8\}}$	13.4353$^{\{7\}}$	2693.8843$^{\{15\}}$	0.0385$^{\{1\}}$	1840.8239$^{\{14\}}$	2.1677$^{\{3\}}$	1.4062$^{\{2\}}$	588.7383$^{\{12\}}$	153.8659$^{\{10\}}$	997.688$^{\{13\}}$
	MSE$(\hat{\theta})$	1.7013$^{\{14\}}$	0.9859$^{\{10\}}$	0.8843$^{\{8\}}$	1.2614$^{\{12\}}$	1.3093$^{\{13\}}$	0.7744$^{\{6\}}$	0.7596$^{\{5\}}$	2.0417$^{\{15\}}$	0.0959$^{\{1\}}$	1.0064$^{\{11\}}$	0.191$^{\{2\}}$	0.2531$^{\{3\}}$	0.7234$^{\{4\}}$	0.8723$^{\{7\}}$	0.8867$^{\{9\}}$
	MSE$(\hat{k})$	3.2419$^{\{5\}}$	1.9982$^{\{4\}}$	9.1157$^{\{9\}}$	20.1959$^{\{15\}}$	18.9575$^{\{14\}}$	12.9206$^{\{12\}}$	5.8624$^{\{8\}}$	13.4745$^{\{13\}}$	0.1469$^{\{1\}}$	5.0596$^{\{7\}}$	0.2454$^{\{2\}}$	0.848$^{\{3\}}$	10.4969$^{\{10\}}$	4.885$^{\{6\}}$	11.8762$^{\{11\}}$
	MRE$(\hat{\alpha})$	9.8318$^{\{15\}}$	2.1666$^{\{9\}}$	2.0439$^{\{7\}}$	2.2655$^{\{10\}}$	2.0821$^{\{8\}}$	1.5687$^{\{6\}}$	1.4069$^{\{5\}}$	2.9329$^{\{12\}}$	0.1145$^{\{1\}}$	5.9185$^{\{13\}}$	1.0912$^{\{3\}}$	0.6748$^{\{2\}}$	1.17$^{\{4\}}$	6.17$^{\{14\}}$	2.5304$^{\{11\}}$
	MRE$(\hat{\beta})$	4.5444$^{\{9\}}$	1.9721$^{\{5\}}$	1.7004$^{\{4\}}$	4.7622$^{\{10\}}$	2.8991$^{\{6\}}$	3.4269$^{\{8\}}$	2.96$^{\{7\}}$	22.6591$^{\{15\}}$	0.1791$^{\{1\}}$	18.1547$^{\{14\}}$	1.2349$^{\{3\}}$	1.0384$^{\{2\}}$	8.513$^{\{12\}}$	6.4037$^{\{11\}}$	13.5587$^{\{13\}}$
	MRE$(\hat{\theta})$	0.7296$^{\{14\}}$	0.57$^{\{11\}}$	0.5387$^{\{9\}}$	0.606$^{\{13\}}$	0.5961$^{\{12\}}$	0.5141$^{\{5\}}$	0.5177$^{\{7\}}$	0.7517$^{\{15\}}$	0.1414$^{\{1\}}$	0.5653$^{\{10\}}$	0.2358$^{\{2\}}$	0.273$^{\{3\}}$	0.4836$^{\{4\}}$	0.5171$^{\{6\}}$	0.5352$^{\{8\}}$
	MRE$(\hat{k})$	0.5518$^{\{5\}}$	0.455$^{\{4\}}$	0.7576$^{\{10\}}$	1.2557$^{\{14\}}$	1.2968$^{\{15\}}$	1.0964$^{\{13\}}$	0.641$^{\{8\}}$	0.751$^{\{9\}}$	0.1222$^{\{1\}}$	0.5589$^{\{6\}}$	0.2226$^{\{2\}}$	0.3244$^{\{3\}}$	0.865$^{\{11\}}$	0.5938$^{\{7\}}$	0.9723$^{\{12\}}$
	D_{abs}	0.0632$^{\{3\}}$	0.0652$^{\{6\}}$	0.0624$^{\{2\}}$	0.0686$^{\{11\}}$	0.068$^{\{9\}}$	0.0663$^{\{7\}}$	0.0635$^{\{4\}}$	0.0676$^{\{8\}}$	0.0569$^{\{1\}}$	0.0643$^{\{5\}}$	0.0727$^{\{15\}}$	0.0681$^{\{10\}}$	0.0699$^{\{13\}}$	0.0715$^{\{14\}}$	0.0693$^{\{12\}}$
	D_{max}	0.1088$^{\{4\}}$	0.1084$^{\{3\}}$	0.1048$^{\{2\}}$	0.1172$^{\{11\}}$	0.118$^{\{12\}}$	0.1104$^{\{7\}}$	0.1088$^{\{5\}}$	0.116$^{\{10\}}$	0.0941$^{\{1\}}$	0.1091$^{\{6\}}$	0.1132$^{\{9\}}$	0.1132$^{\{8\}}$	0.1201$^{\{13\}}$	0.1214$^{\{15\}}$	0.1209$^{\{14\}}$
	$\sum$Ranks	138$^{\{10\}}$	89$^{\{5\}}$	86$^{\{4\}}$	152$^{\{13\}}$	136$^{\{9\}}$	119$^{\{7\}}$	92$^{\{6\}}$	168$^{\{15\}}$	17$^{\{1\}}$	139$^{\{11\}}$	65$^{\{3\}}$	51$^{\{2\}}$	124$^{\{8\}}$	141$^{\{12\}}$	163$^{\{14\}}$
50	$\vert\text{Bias}\vert(\hat{\alpha})$	2.5212$^{\{15\}}$	0.4562$^{\{7\}}$	0.5185$^{\{9\}}$	0.6816$^{\{10\}}$	0.6871$^{\{11\}}$	0.453$^{\{6\}}$	0.4937$^{\{8\}}$	2.1806$^{\{14\}}$	0.0518$^{\{4\}}$	1.8031$^{\{13\}}$	$8e-04$$^{\{1\}}$	0.0161$^{\{2\}}$	0.0553$^{\{5\}}$	0.8334$^{\{12\}}$	0.0259$^{\{3\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	2.4973$^{\{11\}}$	0.8779$^{\{6\}}$	0.5388$^{\{4\}}$	0.9728$^{\{9\}}$	0.7709$^{\{5\}}$	0.9501$^{\{8\}}$	0.9153$^{\{7\}}$	16.7479$^{\{15\}}$	0.0148$^{\{1\}}$	6.4926$^{\{14\}}$	0.0546$^{\{2\}}$	0.42$^{\{3\}}$	1.4979$^{\{10\}}$	5.9483$^{\{13\}}$	3.019$^{\{12\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	0.2291$^{\{11\}}$	0.0578$^{\{5\}}$	0.0056$^{\{2\}}$	0.0998$^{\{7\}}$	0.0442$^{\{3\}}$	0.2051$^{\{10\}}$	0.1899$^{\{9\}}$	0.0479$^{\{4\}}$	0.0042$^{\{1\}}$	0.2964$^{\{13\}}$	0.0923$^{\{6\}}$	0.248$^{\{12\}}$	0.4553$^{\{14\}}$	0.1705$^{\{8\}}$	0.4913$^{\{15\}}$
	$\vert\text{Bias}\vert(\hat{k})$	0.336$^{\{6\}}$	0.2271$^{\{3\}}$	0.7275$^{\{12\}}$	0.5042$^{\{9\}}$	1.0077$^{\{14\}}$	1.0691$^{\{15\}}$	0.5624$^{\{10\}}$	0.296$^{\{4\}}$	0.0231$^{\{1\}}$	0.1733$^{\{2\}}$	0.3446$^{\{7\}}$	0.3873$^{\{8\}}$	0.7297$^{\{13\}}$	0.3004$^{\{5\}}$	0.5958$^{\{11\}}$
	MSE$(\hat{\alpha})$	216.1322$^{\{15\}}$	2.8196$^{\{6\}}$	3.5121$^{\{8\}}$	6.3442$^{\{11\}}$	4.076$^{\{9\}}$	2.8486$^{\{7\}}$	4.753$^{\{10\}}$	34.2707$^{\{13\}}$	0.0175$^{\{1\}}$	118.2934$^{\{14\}}$	0.4073$^{\{2\}}$	0.6532$^{\{3\}}$	1.8156$^{\{5\}}$	22.4665$^{\{12\}}$	1.7585$^{\{4\}}$
	MSE$(\hat{\beta})$	304.6778$^{\{12\}}$	5.3243$^{\{8\}}$	2.399$^{\{4\}}$	6.4364$^{\{9\}}$	2.803$^{\{6\}}$	3.2334$^{\{7\}}$	2.6895$^{\{5\}}$	2432.4848$^{\{15\}}$	0.0412$^{\{1\}}$	976.7554$^{\{13\}}$	0.5654$^{\{2\}}$	1.4144$^{\{3\}}$	10.3651$^{\{10\}}$	1231.5891$^{\{14\}}$	100.8471$^{\{11\}}$
	MSE$(\hat{\theta})$	0.7984$^{\{12\}}$	0.621$^{\{4\}}$	0.6562$^{\{5\}}$	0.7529$^{\{11\}}$	0.6629$^{\{6\}}$	0.6696$^{\{7\}}$	0.705$^{\{8\}}$	1.5329$^{\{15\}}$	0.0276$^{\{1\}}$	0.8691$^{\{14\}}$	0.1768$^{\{2\}}$	0.2119$^{\{3\}}$	0.7065$^{\{9\}}$	0.8148$^{\{13\}}$	0.7494$^{\{10\}}$
	MSE$(\hat{k})$	1.6158$^{\{7\}}$	1.0715$^{\{2\}}$	6.9103$^{\{13\}}$	3.1091$^{\{9\}}$	11.6122$^{\{14\}}$	14.1122$^{\{15\}}$	6.0262$^{\{12\}}$	1.562$^{\{6\}}$	0.0253$^{\{1\}}$	1.4807$^{\{5\}}$	1.4701$^{\{4\}}$	1.962$^{\{8\}}$	5.3356$^{\{11\}}$	1.3132$^{\{3\}}$	3.5929$^{\{10\}}$
	MRE$(\hat{\alpha})$	2.4414$^{\{15\}}$	0.7831$^{\{6\}}$	0.8648$^{\{8\}}$	1.0288$^{\{11\}}$	1.0251$^{\{10\}}$	0.8747$^{\{9\}}$	0.859$^{\{7\}}$	2.3359$^{\{14\}}$	0.0708$^{\{1\}}$	1.9395$^{\{13\}}$	0.383$^{\{2\}}$	0.439$^{\{3\}}$	0.7074$^{\{5\}}$	1.1928$^{\{12\}}$	0.7008$^{\{4\}}$
	MRE$(\hat{\beta})$	4.155$^{\{11\}}$	1.747$^{\{7\}}$	1.5328$^{\{4\}}$	2.0213$^{\{9\}}$	1.5979$^{\{5\}}$	1.8035$^{\{8\}}$	1.7219$^{\{6\}}$	22.9654$^{\{15\}}$	0.1511$^{\{1\}}$	9.1882$^{\{14\}}$	0.668$^{\{2\}}$	1.0219$^{\{3\}}$	2.575$^{\{10\}}$	8.5224$^{\{13\}}$	4.5863$^{\{12\}}$
	MRE$(\hat{\theta})$	0.4756$^{\{8\}}$	0.431$^{\{4\}}$	0.4742$^{\{7\}}$	0.4987$^{\{12\}}$	0.4574$^{\{5\}}$	0.4574$^{\{6\}}$	0.4866$^{\{9\}}$	0.691$^{\{15\}}$	0.0835$^{\{1\}}$	0.551$^{\{14\}}$	0.2024$^{\{2\}}$	0.2178$^{\{3\}}$	0.4926$^{\{10\}}$	0.4977$^{\{11\}}$	0.5087$^{\{13\}}$
	MRE$(\hat{k})$	0.4122$^{\{7\}}$	0.3433$^{\{2\}}$	0.6365$^{\{12\}}$	0.5784$^{\{10\}}$	0.8343$^{\{14\}}$	0.8602$^{\{15\}}$	0.5535$^{\{9\}}$	0.5366$^{\{8\}}$	0.0634$^{\{1\}}$	0.3483$^{\{4\}}$	0.3478$^{\{3\}}$	0.3625$^{\{5\}}$	0.6646$^{\{13\}}$	0.3751$^{\{6\}}$	0.591$^{\{11\}}$
	D_{abs}	0.037$^{\{1\}}$	0.0372$^{\{2\}}$	0.0402$^{\{9\}}$	0.0386$^{\{5\}}$	0.0387$^{\{6\}}$	0.0397$^{\{8\}}$	0.0381$^{\{4\}}$	0.045$^{\{13\}}$	0.039$^{\{7\}}$	0.0375$^{\{3\}}$	0.0408$^{\{12\}}$	0.0406$^{\{10\}}$	0.0472$^{\{15\}}$	0.0408$^{\{11\}}$	0.0452$^{\{14\}}$
	D_{max}	0.0652$^{\{3\}}$	0.0663$^{\{4\}}$	0.0707$^{\{8\}}$	0.0668$^{\{5\}}$	0.0714$^{\{12\}}$	0.0713$^{\{11\}}$	0.0672$^{\{6\}}$	0.0795$^{\{13\}}$	0.0609$^{\{1\}}$	0.064$^{\{2\}}$	0.071$^{\{9\}}$	0.0712$^{\{10\}}$	0.0855$^{\{15\}}$	0.0703$^{\{7\}}$	0.0822$^{\{14\}}$
	$\sum$Ranks	134$^{\{10\}}$	66$^{\{3\}}$	105$^{\{5\}}$	127$^{\{8\}}$	120$^{\{7\}}$	132$^{\{9\}}$	110$^{\{6\}}$	164$^{\{15\}}$	23$^{\{1\}}$	138$^{\{11\}}$	56$^{\{2\}}$	76$^{\{4\}}$	145$^{\{14\}}$	140$^{\{12\}}$	144$^{\{13\}}$
100	$\vert\text{Bias}\vert(\hat{\alpha})$	0.7372$^{\{9\}}$	0.5821$^{\{8\}}$	0.4778$^{\{7\}}$	0.8534$^{\{11\}}$	1.1586$^{\{13\}}$	1.037$^{\{12\}}$	0.7553$^{\{10\}}$	2.6575$^{\{15\}}$	0.0019$^{\{1\}}$	0.2749$^{\{4\}}$	0.1153$^{\{3\}}$	0.1096$^{\{2\}}$	2.0765$^{\{14\}}$	0.3696$^{\{6\}}$	0.3587$^{\{5\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	0.0635$^{\{2\}}$	0.5904$^{\{9\}}$	0.5079$^{\{8\}}$	0.4881$^{\{7\}}$	0.2764$^{\{4\}}$	0.3839$^{\{5\}}$	0.6514$^{\{10\}}$	11.0399$^{\{15\}}$	0.0161$^{\{1\}}$	3.3674$^{\{14\}}$	0.2324$^{\{3\}}$	0.413$^{\{6\}}$	3.1772$^{\{13\}}$	1.194$^{\{11\}}$	1.1971$^{\{12\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	0.2595$^{\{12\}}$	0.0605$^{\{4\}}$	0.0169$^{\{1\}}$	0.179$^{\{8\}}$	0.2696$^{\{13\}}$	0.1808$^{\{9\}}$	0.0496$^{\{3\}}$	0.3249$^{\{15\}}$	0.0174$^{\{2\}}$	0.2753$^{\{14\}}$	0.1149$^{\{5\}}$	0.141$^{\{7\}}$	0.2476$^{\{11\}}$	0.1299$^{\{6\}}$	0.2223$^{\{10\}}$
	$\vert\text{Bias}\vert(\hat{k})$	0.0081$^{\{2\}}$	0.0619$^{\{9\}}$	0.007$^{\{1\}}$	0.1486$^{\{14\}}$	0.1112$^{\{12\}}$	0.1228$^{\{13\}}$	0.0885$^{\{10\}}$	0.2528$^{\{15\}}$	0.0299$^{\{5\}}$	0.032$^{\{6\}}$	0.1051$^{\{11\}}$	0.0332$^{\{7\}}$	0.0173$^{\{3\}}$	0.0233$^{\{4\}}$	0.0448$^{\{8\}}$
	MSE$(\hat{\alpha})$	4.2394$^{\{10\}}$	3.2955$^{\{8\}}$	1.69$^{\{4\}}$	3.7724$^{\{9\}}$	6.5999$^{\{13\}}$	6.1201$^{\{12\}}$	5.472$^{\{11\}}$	31.1669$^{\{14\}}$	0.0028$^{\{1\}}$	1.8089$^{\{5\}}$	0.3984$^{\{2\}}$	0.4643$^{\{3\}}$	151.2059$^{\{15\}}$	1.901$^{\{7\}}$	1.8338$^{\{6\}}$
	MSE$(\hat{\beta})$	0.6999$^{\{3\}}$	2.3497$^{\{9\}}$	2.072$^{\{8\}}$	1.9938$^{\{7\}}$	1.0636$^{\{4\}}$	1.3825$^{\{6\}}$	2.4549$^{\{10\}}$	1065.1634$^{\{15\}}$	0.0077$^{\{1\}}$	229.946$^{\{14\}}$	0.5165$^{\{2\}}$	1.2619$^{\{5\}}$	206.0267$^{\{13\}}$	7.5314$^{\{12\}}$	5.4954$^{\{11\}}$
	MSE$(\hat{\theta})$	0.6948$^{\{10\}}$	0.6334$^{\{5\}}$	0.4787$^{\{4\}}$	0.7043$^{\{11\}}$	0.6933$^{\{9\}}$	0.6717$^{\{6\}}$	0.6776$^{\{7\}}$	1.5372$^{\{15\}}$	0.0102$^{\{1\}}$	0.7117$^{\{12\}}$	0.1866$^{\{2\}}$	0.2927$^{\{3\}}$	0.7748$^{\{14\}}$	0.6834$^{\{8\}}$	0.7499$^{\{13\}}$
	MSE$(\hat{k})$	0.3542$^{\{11\}}$	0.2182$^{\{3\}}$	0.4057$^{\{12\}}$	0.2817$^{\{7\}}$	0.3193$^{\{9\}}$	0.3045$^{\{8\}}$	0.2388$^{\{4\}}$	0.5636$^{\{13\}}$	0.0193$^{\{1\}}$	0.2679$^{\{6\}}$	0.3344$^{\{10\}}$	0.1807$^{\{2\}}$	0.7094$^{\{15\}}$	0.2393$^{\{5\}}$	0.6874$^{\{14\}}$
	MRE$(\hat{\alpha})$	0.8853$^{\{10\}}$	0.7376$^{\{8\}}$	0.6745$^{\{6\}}$	0.9201$^{\{11\}}$	1.193$^{\{13\}}$	1.1101$^{\{12\}}$	0.8733$^{\{9\}}$	2.4799$^{\{15\}}$	0.0306$^{\{1\}}$	0.5916$^{\{4\}}$	0.3539$^{\{2\}}$	0.3783$^{\{3\}}$	2.0827$^{\{14\}}$	0.6426$^{\{5\}}$	0.7176$^{\{7\}}$
	MRE$(\hat{\beta})$	0.8568$^{\{3\}}$	1.4678$^{\{9\}}$	1.3918$^{\{8\}}$	1.3222$^{\{7\}}$	1.0653$^{\{5\}}$	1.2375$^{\{6\}}$	1.5477$^{\{10\}}$	15.5686$^{\{15\}}$	0.0934$^{\{1\}}$	4.9577$^{\{14\}}$	0.6452$^{\{2\}}$	0.9688$^{\{4\}}$	4.8178$^{\{13\}}$	2.2499$^{\{12\}}$	2.1896$^{\{11\}}$
	MRE$(\hat{\theta})$	0.4626$^{\{8\}}$	0.4563$^{\{6\}}$	0.4086$^{\{4\}}$	0.4792$^{\{11\}}$	0.4763$^{\{10\}}$	0.4605$^{\{7\}}$	0.4685$^{\{9\}}$	0.7143$^{\{15\}}$	0.0557$^{\{1\}}$	0.4952$^{\{12\}}$	0.2029$^{\{2\}}$	0.2639$^{\{3\}}$	0.506$^{\{14\}}$	0.4509$^{\{5\}}$	0.4966$^{\{13\}}$
	MRE$(\hat{k})$	0.2655$^{\{10\}}$	0.2139$^{\{5\}}$	0.2619$^{\{9\}}$	0.2546$^{\{8\}}$	0.2777$^{\{12\}}$	0.2694$^{\{11\}}$	0.2242$^{\{7\}}$	0.3694$^{\{15\}}$	0.0581$^{\{1\}}$	0.221$^{\{6\}}$	0.1912$^{\{3\}}$	0.1846$^{\{2\}}$	0.3145$^{\{14\}}$	0.2072$^{\{4\}}$	0.3058$^{\{13\}}$
	D_{abs}	0.0287$^{\{5\}}$	0.0281$^{\{2\}}$	0.0284$^{\{3\}}$	0.0303$^{\{10\}}$	0.0293$^{\{8\}}$	0.0291$^{\{6\}}$	0.0284$^{\{4\}}$	0.0316$^{\{12\}}$	0.0265$^{\{1\}}$	0.0292$^{\{7\}}$	0.0351$^{\{13\}}$	0.0301$^{\{9\}}$	0.0379$^{\{15\}}$	0.0303$^{\{11\}}$	0.0369$^{\{14\}}$
	D_{max}	0.05$^{\{2\}}$	0.0502$^{\{3\}}$	0.0521$^{\{6\}}$	0.0528$^{\{8\}}$	0.0536$^{\{11\}}$	0.0535$^{\{10\}}$	0.0508$^{\{4\}}$	0.0574$^{\{12\}}$	0.0431$^{\{1\}}$	0.0512$^{\{5\}}$	0.0586$^{\{13\}}$	0.0527$^{\{7\}}$	0.0652$^{\{15\}}$	0.0529$^{\{9\}}$	0.0647$^{\{14\}}$
	$\sum$Ranks	97$^{\{6\}}$	88$^{\{5\}}$	81$^{\{4\}}$	129$^{\{11\}}$	136$^{\{12\}}$	123$^{\{10\}}$	108$^{\{8\}}$	201$^{\{15\}}$	19$^{\{1\}}$	123$^{\{10\}}$	73$^{\{3\}}$	63$^{\{2\}}$	183$^{\{14\}}$	105$^{\{7\}}$	151$^{\{13\}}$
200	$\vert\text{Bias}\vert(\hat{\alpha})$	0.0873$^{\{5\}}$	0.1372$^{\{9\}}$	0.4166$^{\{14\}}$	0.1695$^{\{10\}}$	0.3253$^{\{13\}}$	0.2942$^{\{12\}}$	0.1829$^{\{11\}}$	0.5523$^{\{15\}}$	0.0125$^{\{1\}}$	0.0803$^{\{4\}}$	0.0327$^{\{3\}}$	0.1314$^{\{8\}}$	0.1277$^{\{6\}}$	0.0162$^{\{2\}}$	0.1279$^{\{7\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	0.0356$^{\{2\}}$	0.3057$^{\{6\}}$	0.0747$^{\{4\}}$	0.7823$^{\{11\}}$	0.5346$^{\{8\}}$	0.6814$^{\{10\}}$	0.2803$^{\{5\}}$	3.1129$^{\{15\}}$	0.0078$^{\{1\}}$	0.8792$^{\{12\}}$	0.063$^{\{3\}}$	0.412$^{\{7\}}$	1.0043$^{\{14\}}$	0.5905$^{\{9\}}$	1.0019$^{\{13\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	0.1101$^{\{8\}}$	0.03$^{\{3\}}$	0.1398$^{\{10\}}$	0.1276$^{\{9\}}$	0.076$^{\{5\}}$	0.1722$^{\{11\}}$	0.0055$^{\{1\}}$	0.1028$^{\{7\}}$	0.0184$^{\{2\}}$	0.2415$^{\{13\}}$	0.0737$^{\{4\}}$	0.2179$^{\{12\}}$	0.2558$^{\{15\}}$	0.0826$^{\{6\}}$	0.2557$^{\{14\}}$
	$\vert\text{Bias}\vert(\hat{k})$	0.0304$^{\{4\}}$	0.0272$^{\{3\}}$	0.0551$^{\{9\}}$	0.1022$^{\{15\}}$	0.0772$^{\{10\}}$	0.0835$^{\{11\}}$	0.0877$^{\{12\}}$	0.1014$^{\{13\}}$	0.0032$^{\{1\}}$	0.0402$^{\{5\}}$	0.055$^{\{8\}}$	0.1015$^{\{14\}}$	0.0464$^{\{7\}}$	0.021$^{\{2\}}$	0.0462$^{\{6\}}$
	MSE$(\hat{\alpha})$	0.2146$^{\{5\}}$	0.3095$^{\{7\}}$	1.2902$^{\{14\}}$	0.3871$^{\{9\}}$	0.7906$^{\{11\}}$	0.7612$^{\{10\}}$	0.3434$^{\{8\}}$	1.7812$^{\{15\}}$	0.0035$^{\{1\}}$	0.1488$^{\{3\}}$	0.1597$^{\{4\}}$	0.0992$^{\{2\}}$	0.7943$^{\{12\}}$	0.2306$^{\{6\}}$	0.7976$^{\{13\}}$
	MSE$(\hat{\beta})$	0.4116$^{\{3\}}$	0.859$^{\{5\}}$	0.441$^{\{4\}}$	2.4866$^{\{10\}}$	1.2164$^{\{8\}}$	1.4109$^{\{9\}}$	0.922$^{\{6\}}$	117.4581$^{\{15\}}$	0.0062$^{\{1\}}$	5.2582$^{\{14\}}$	0.3638$^{\{2\}}$	1.1543$^{\{7\}}$	4.4676$^{\{13\}}$	3.5656$^{\{11\}}$	4.4503$^{\{12\}}$
	MSE$(\hat{\theta})$	0.2057$^{\{4\}}$	0.2943$^{\{6\}}$	0.2841$^{\{5\}}$	0.4543$^{\{12\}}$	0.3225$^{\{7\}}$	0.3444$^{\{10\}}$	0.3233$^{\{8\}}$	0.8672$^{\{15\}}$	0.0123$^{\{1\}}$	0.3339$^{\{9\}}$	0.0972$^{\{2\}}$	0.1675$^{\{3\}}$	0.5345$^{\{14\}}$	0.3464$^{\{11\}}$	0.5338$^{\{13\}}$
	MSE$(\hat{k})$	0.1355$^{\{8\}}$	0.1365$^{\{9\}}$	0.1371$^{\{10\}}$	0.0994$^{\{4\}}$	0.1563$^{\{11\}}$	0.1611$^{\{12\}}$	0.0852$^{\{3\}}$	0.4836$^{\{15\}}$	0.0069$^{\{1\}}$	0.1212$^{\{6\}}$	0.0811$^{\{2\}}$	0.1112$^{\{5\}}$	0.1634$^{\{13\}}$	0.1317$^{\{7\}}$	0.1636$^{\{14\}}$
	MRE$(\hat{\alpha})$	0.2814$^{\{5\}}$	0.3435$^{\{8\}}$	0.5511$^{\{14\}}$	0.3356$^{\{7\}}$	0.4903$^{\{12\}}$	0.4943$^{\{13\}}$	0.3527$^{\{9\}}$	0.8047$^{\{15\}}$	0.0339$^{\{1\}}$	0.2518$^{\{4\}}$	0.2397$^{\{3\}}$	0.2135$^{\{2\}}$	0.4409$^{\{11\}}$	0.2816$^{\{6\}}$	0.4408$^{\{10\}}$
	MRE$(\hat{\beta})$	0.6886$^{\{3\}}$	0.9734$^{\{6\}}$	0.7411$^{\{4\}}$	1.6946$^{\{12\}}$	1.1934$^{\{8\}}$	1.2692$^{\{9\}}$	1.0041$^{\{7\}}$	5.0878$^{\{15\}}$	0.073$^{\{1\}}$	1.6269$^{\{11\}}$	0.56$^{\{2\}}$	0.9692$^{\{5\}}$	1.8255$^{\{14\}}$	1.4626$^{\{10\}}$	1.8221$^{\{13\}}$
	MRE$(\hat{\theta})$	0.2429$^{\{4\}}$	0.3118$^{\{6\}}$	0.3074$^{\{5\}}$	0.4168$^{\{12\}}$	0.3203$^{\{9\}}$	0.3351$^{\{11\}}$	0.327$^{\{10\}}$	0.5655$^{\{15\}}$	0.0565$^{\{1\}}$	0.3136$^{\{7\}}$	0.1315$^{\{2\}}$	0.1973$^{\{3\}}$	0.4174$^{\{14\}}$	0.3154$^{\{8\}}$	0.4172$^{\{13\}}$
	MRE$(\hat{k})$	0.1522$^{\{6\}}$	0.1592$^{\{9\}}$	0.1776$^{\{12\}}$	0.1527$^{\{7\}}$	0.1805$^{\{13\}}$	0.1843$^{\{14\}}$	0.1414$^{\{4\}}$	0.3268$^{\{15\}}$	0.037$^{\{1\}}$	0.1512$^{\{5\}}$	0.1295$^{\{2\}}$	0.1345$^{\{3\}}$	0.1756$^{\{11\}}$	0.156$^{\{8\}}$	0.1755$^{\{10\}}$
	D_{abs}	0.0188$^{\{1\}}$	0.0191$^{\{4\}}$	0.0201$^{\{7\}}$	0.0189$^{\{3\}}$	0.02$^{\{6\}}$	0.0206$^{\{10\}}$	0.0188$^{\{2\}}$	0.024$^{\{13\}}$	0.0206$^{\{9\}}$	0.0195$^{\{5\}}$	0.0209$^{\{12\}}$	0.0204$^{\{8\}}$	0.0278$^{\{14\}}$	0.0209$^{\{11\}}$	0.0279$^{\{15\}}$
	D_{max}	0.0335$^{\{2\}}$	0.0343$^{\{6\}}$	0.0372$^{\{10\}}$	0.0341$^{\{4\}}$	0.0374$^{\{11\}}$	0.0374$^{\{12\}}$	0.0334$^{\{1\}}$	0.0447$^{\{13\}}$	0.0336$^{\{3\}}$	0.0342$^{\{5\}}$	0.036$^{\{8\}}$	0.0359$^{\{7\}}$	0.0465$^{\{14\}}$	0.0371$^{\{9\}}$	0.0466$^{\{15\}}$
	$\sum$Ranks	60$^{\{3\}}$	87$^{\{6\}}$	122$^{\{9\}}$	125$^{\{10\}}$	132$^{\{11\}}$	154$^{\{12\}}$	87$^{\{6\}}$	196$^{\{15\}}$	25$^{\{1\}}$	103$^{\{7\}}$	57$^{\{2\}}$	86$^{\{4\}}$	172$^{\{14\}}$	106$^{\{8\}}$	168$^{\{13\}}$
300	$\vert\text{Bias}\vert(\hat{\alpha})$	0.0564$^{\{6\}}$	0.1172$^{\{10\}}$	0.1882$^{\{12\}}$	0.1164$^{\{9\}}$	0.2418$^{\{14\}}$	0.213$^{\{13\}}$	0.1576$^{\{11\}}$	0.3101$^{\{15\}}$	0.0102$^{\{1\}}$	0.0765$^{\{7\}}$	0.0231$^{\{3\}}$	0.1091$^{\{8\}}$	0.0105$^{\{2\}}$	0.0455$^{\{5\}}$	0.0301$^{\{4\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	0.0173$^{\{2\}}$	0.2389$^{\{5\}}$	0.1716$^{\{3\}}$	0.439$^{\{8\}}$	0.3856$^{\{7\}}$	0.4507$^{\{9\}}$	0.2411$^{\{6\}}$	10.59$^{\{15\}}$	0.0018$^{\{1\}}$	0.9073$^{\{12\}}$	0.1893$^{\{4\}}$	0.5413$^{\{11\}}$	1.0213$^{\{13\}}$	0.5377$^{\{10\}}$	1.3598$^{\{14\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	0.086$^{\{9\}}$	0.0299$^{\{4\}}$	0.0848$^{\{8\}}$	0.0177$^{\{3\}}$	0.0035$^{\{1\}}$	0.0417$^{\{5\}}$	0.0554$^{\{7\}}$	0.1509$^{\{11\}}$	0.007$^{\{2\}}$	0.2428$^{\{13\}}$	0.0419$^{\{6\}}$	0.2257$^{\{12\}}$	0.2523$^{\{14\}}$	0.1428$^{\{10\}}$	0.2551$^{\{15\}}$
	$\vert\text{Bias}\vert(\hat{k})$	0.0109$^{\{3\}}$	0.0238$^{\{4\}}$	0.04$^{\{7\}}$	0.0418$^{\{8\}}$	0.0432$^{\{9\}}$	0.0479$^{\{11\}}$	0.0453$^{\{10\}}$	0.2139$^{\{15\}}$	0.0103$^{\{2\}}$	0.009$^{\{1\}}$	0.0381$^{\{6\}}$	0.0863$^{\{14\}}$	0.0527$^{\{12\}}$	0.034$^{\{5\}}$	0.0613$^{\{13\}}$
	MSE$(\hat{\alpha})$	0.1339$^{\{5\}}$	0.2543$^{\{8\}}$	0.3885$^{\{12\}}$	0.2551$^{\{9\}}$	0.5987$^{\{14\}}$	0.5611$^{\{13\}}$	0.33$^{\{11\}}$	0.753$^{\{15\}}$	0.0015$^{\{1\}}$	0.0996$^{\{2\}}$	0.1203$^{\{3\}}$	0.1386$^{\{6\}}$	0.1856$^{\{7\}}$	0.126$^{\{4\}}$	0.2575$^{\{10\}}$
	MSE$(\hat{\beta})$	0.3301$^{\{2\}}$	0.7817$^{\{5\}}$	0.7531$^{\{4\}}$	1.485$^{\{10\}}$	0.931$^{\{7\}}$	1.0362$^{\{8\}}$	0.8633$^{\{6\}}$	860.3315$^{\{15\}}$	0.0015$^{\{1\}}$	4.5093$^{\{13\}}$	0.4272$^{\{3\}}$	1.3231$^{\{9\}}$	3.9291$^{\{12\}}$	2.6698$^{\{11\}}$	11.1676$^{\{14\}}$
	MSE$(\hat{\theta})$	0.2135$^{\{3\}}$	0.3172$^{\{7\}}$	0.3078$^{\{6\}}$	0.3882$^{\{10\}}$	0.4154$^{\{12\}}$	0.406$^{\{11\}}$	0.3721$^{\{9\}}$	0.9327$^{\{15\}}$	0.0034$^{\{1\}}$	0.3273$^{\{8\}}$	0.1166$^{\{2\}}$	0.255$^{\{4\}}$	0.4324$^{\{13\}}$	0.2744$^{\{5\}}$	0.4923$^{\{14\}}$
	MSE$(\hat{k})$	0.057$^{\{4\}}$	0.0858$^{\{7\}}$	0.0913$^{\{8\}}$	0.0971$^{\{10\}}$	0.1412$^{\{14\}}$	0.1343$^{\{13\}}$	0.0925$^{\{9\}}$	0.1537$^{\{15\}}$	0.0032$^{\{1\}}$	0.0453$^{\{2\}}$	0.0827$^{\{6\}}$	0.0685$^{\{5\}}$	0.1005$^{\{11\}}$	0.0486$^{\{3\}}$	0.1129$^{\{12\}}$
	MRE$(\hat{\alpha})$	0.2427$^{\{5\}}$	0.3026$^{\{9\}}$	0.3505$^{\{12\}}$	0.2851$^{\{8\}}$	0.4435$^{\{14\}}$	0.4211$^{\{13\}}$	0.3288$^{\{11\}}$	0.5032$^{\{15\}}$	0.0238$^{\{1\}}$	0.2085$^{\{2\}}$	0.2309$^{\{3\}}$	0.2488$^{\{6\}}$	0.2752$^{\{7\}}$	0.2318$^{\{4\}}$	0.3048$^{\{10\}}$
	MRE$(\hat{\beta})$	0.638$^{\{3\}}$	0.8979$^{\{5\}}$	0.8207$^{\{4\}}$	1.2258$^{\{11\}}$	1.0356$^{\{7\}}$	1.0887$^{\{8\}}$	0.9461$^{\{6\}}$	15.2621$^{\{15\}}$	0.0424$^{\{1\}}$	1.5445$^{\{12\}}$	0.6004$^{\{2\}}$	1.0955$^{\{9\}}$	1.7506$^{\{13\}}$	1.1661$^{\{10\}}$	2.215$^{\{14\}}$
	MRE$(\hat{\theta})$	0.2562$^{\{4\}}$	0.3168$^{\{8\}}$	0.3139$^{\{6\}}$	0.372$^{\{11\}}$	0.3788$^{\{12\}}$	0.3705$^{\{10\}}$	0.3386$^{\{9\}}$	0.5546$^{\{15\}}$	0.0318$^{\{1\}}$	0.3163$^{\{7\}}$	0.1604$^{\{2\}}$	0.2561$^{\{3\}}$	0.3819$^{\{13\}}$	0.3023$^{\{5\}}$	0.4066$^{\{14\}}$
	MRE$(\hat{k})$	0.1114$^{\{4\}}$	0.1319$^{\{7\}}$	0.1377$^{\{10\}}$	0.1397$^{\{11\}}$	0.1729$^{\{14\}}$	0.1649$^{\{13\}}$	0.1356$^{\{8\}}$	0.1966$^{\{15\}}$	0.0262$^{\{1\}}$	0.0991$^{\{2\}}$	0.12$^{\{5\}}$	0.1241$^{\{6\}}$	0.1367$^{\{9\}}$	0.1042$^{\{3\}}$	0.1467$^{\{12\}}$
	D_{abs}	0.0163$^{\{2\}}$	0.0164$^{\{5\}}$	0.0164$^{\{3\}}$	0.0173$^{\{9\}}$	0.0169$^{\{8\}}$	0.0165$^{\{7\}}$	0.0164$^{\{6\}}$	0.0184$^{\{11\}}$	0.0156$^{\{1\}}$	0.0164$^{\{4\}}$	0.0215$^{\{15\}}$	0.0214$^{\{14\}}$	0.021$^{\{12\}}$	0.0182$^{\{10\}}$	0.0212$^{\{13\}}$
	D_{max}	0.028$^{\{3\}}$	0.0289$^{\{5\}}$	0.0293$^{\{6\}}$	0.0307$^{\{9\}}$	0.0305$^{\{8\}}$	0.0298$^{\{7\}}$	0.0288$^{\{4\}}$	0.032$^{\{11\}}$	0.0247$^{\{1\}}$	0.028$^{\{2\}}$	0.037$^{\{15\}}$	0.0362$^{\{14\}}$	0.0359$^{\{12\}}$	0.0316$^{\{10\}}$	0.036$^{\{13\}}$
	$\sum$Ranks	55$^{\{2\}}$	89$^{\{5\}}$	101$^{\{7\}}$	126$^{\{10\}}$	141$^{\{12\}}$	141$^{\{12\}}$	113$^{\{8\}}$	198$^{\{15\}}$	16$^{\{1\}}$	87$^{\{4\}}$	75$^{\{3\}}$	121$^{\{9\}}$	150$^{\{13\}}$	95$^{\{6\}}$	172$^{\{14\}}$
500	$\vert\text{Bias}\vert(\hat{\alpha})$	0.1711$^{\{13\}}$	0.1403$^{\{11\}}$	0.1287$^{\{9\}}$	0.216$^{\{14\}}$	0.1387$^{\{10\}}$	0.1277$^{\{8\}}$	0.1646$^{\{12\}}$	0.3149$^{\{15\}}$	0.0028$^{\{1\}}$	0.0814$^{\{6\}}$	0.0352$^{\{5\}}$	0.0266$^{\{4\}}$	0.0143$^{\{3\}}$	0.0837$^{\{7\}}$	0.0142$^{\{2\}}$
	$\vert\text{Bias}\vert(\hat{\beta})$	0.0703$^{\{2\}}$	0.1969$^{\{7\}}$	0.1972$^{\{8\}}$	0.5296$^{\{13\}}$	0.3938$^{\{10\}}$	0.4416$^{\{11\}}$	0.1433$^{\{4\}}$	3.9203$^{\{15\}}$	$5e-04$$^{\{1\}}$	0.1687$^{\{6\}}$	0.0706$^{\{3\}}$	0.1607$^{\{5\}}$	0.5925$^{\{14\}}$	0.2237$^{\{9\}}$	0.4914$^{\{12\}}$
	$\vert\text{Bias}\vert(\hat{\theta})$	0.159$^{\{13\}}$	0.0186$^{\{6\}}$	$9e-04$$^{\{1\}}$	0.0064$^{\{3\}}$	0.0723$^{\{11\}}$	0.0984$^{\{12\}}$	0.0573$^{\{9\}}$	0.0397$^{\{8\}}$	0.0013$^{\{2\}}$	0.015$^{\{5\}}$	0.0127$^{\{4\}}$	0.0701$^{\{10\}}$	0.176$^{\{15\}}$	0.0338$^{\{7\}}$	0.1647$^{\{14\}}$
	$\vert\text{Bias}\vert(\hat{k})$	0.0523$^{\{9\}}$	0.0657$^{\{10\}}$	0.0458$^{\{8\}}$	0.1212$^{\{14\}}$	0.0679$^{\{11\}}$	0.0762$^{\{12\}}$	0.0779$^{\{13\}}$	0.1957$^{\{15\}}$	0.0175$^{\{5\}}$	0.0227$^{\{7\}}$	0.016$^{\{4\}}$	0.0052$^{\{1\}}$	0.0207$^{\{6\}}$	0.015$^{\{2\}}$	0.0156$^{\{3\}}$
	MSE$(\hat{\alpha})$	0.2836$^{\{12\}}$	0.2482$^{\{5\}}$	0.2279$^{\{4\}}$	0.4285$^{\{14\}}$	0.2769$^{\{11\}}$	0.2639$^{\{9\}}$	0.2557$^{\{6\}}$	1.0638$^{\{15\}}$	0.0013$^{\{1\}}$	0.2673$^{\{10\}}$	0.098$^{\{2\}}$	0.1643$^{\{3\}}$	0.2608$^{\{7\}}$	0.3107$^{\{13\}}$	0.2618$^{\{8\}}$
	MSE$(\hat{\beta})$	0.2253$^{\{4\}}$	0.5213$^{\{9\}}$	0.4342$^{\{6\}}$	1.9406$^{\{13\}}$	0.8143$^{\{10\}}$	0.9042$^{\{11\}}$	0.4685$^{\{7\}}$	172.0076$^{\{15\}}$	0.0018$^{\{1\}}$	0.3846$^{\{5\}}$	0.1722$^{\{2\}}$	0.2239$^{\{3\}}$	2.2167$^{\{14\}}$	0.4813$^{\{8\}}$	1.346$^{\{12\}}$
	MSE$(\hat{\theta})$	0.2968$^{\{5\}}$	0.3038$^{\{9\}}$	0.2454$^{\{4\}}$	0.4842$^{\{14\}}$	0.3015$^{\{7\}}$	0.3086$^{\{12\}}$	0.3068$^{\{10\}}$	0.8077$^{\{15\}}$	0.0032$^{\{1\}}$	0.3082$^{\{11\}}$	0.1268$^{\{2\}}$	0.1572$^{\{3\}}$	0.319$^{\{13\}}$	0.2968$^{\{6\}}$	0.3025$^{\{8\}}$
	MSE$(\hat{k})$	0.0666$^{\{10\}}$	0.0623$^{\{5\}}$	0.0664$^{\{9\}}$	0.0896$^{\{14\}}$	0.0725$^{\{13\}}$	0.0708$^{\{12\}}$	0.066$^{\{8\}}$	0.1371$^{\{15\}}$	0.0037$^{\{1\}}$	0.0675$^{\{11\}}$	0.0436$^{\{2\}}$	0.0586$^{\{3\}}$	0.0639$^{\{6\}}$	0.0651$^{\{7\}}$	0.0597$^{\{4\}}$
	MRE$(\hat{\alpha})$	0.2957$^{\{11\}}$	0.2706$^{\{6\}}$	0.3092$^{\{13\}}$	0.3447$^{\{14\}}$	0.2992$^{\{12\}}$	0.2923$^{\{9\}}$	0.2857$^{\{7\}}$	0.4736$^{\{15\}}$	0.0189$^{\{1\}}$	0.2954$^{\{10\}}$	0.1864$^{\{2\}}$	0.2425$^{\{3\}}$	0.2577$^{\{5\}}$	0.2882$^{\{8\}}$	0.2563$^{\{4\}}$
	MRE$(\hat{\beta})$	0.5501$^{\{4\}}$	0.7792$^{\{9\}}$	0.74$^{\{7\}}$	1.2765$^{\{14\}}$	0.9233$^{\{10\}}$	0.9768$^{\{11\}}$	0.7532$^{\{8\}}$	6.0205$^{\{15\}}$	0.0422$^{\{1\}}$	0.6678$^{\{5\}}$	0.4346$^{\{2\}}$	0.5102$^{\{3\}}$	1.1827$^{\{13\}}$	0.7091$^{\{6\}}$	1.0514$^{\{12\}}$
	MRE$(\hat{\theta})$	0.2837$^{\{4\}}$	0.3089$^{\{7\}}$	0.2899$^{\{6\}}$	0.3916$^{\{14\}}$	0.3147$^{\{11\}}$	0.3178$^{\{12\}}$	0.3091$^{\{9\}}$	0.505$^{\{15\}}$	0.0298$^{\{1\}}$	0.309$^{\{8\}}$	0.162$^{\{2\}}$	0.1921$^{\{3\}}$	0.3185$^{\{13\}}$	0.2898$^{\{5\}}$	0.3127$^{\{10\}}$
	MRE$(\hat{k})$	0.1165$^{\{10\}}$	0.1102$^{\{7\}}$	0.1255$^{\{13\}}$	0.1346$^{\{14\}}$	0.1239$^{\{12\}}$	0.1223$^{\{11\}}$	0.1154$^{\{9\}}$	0.1761$^{\{15\}}$	0.0288$^{\{1\}}$	0.1149$^{\{8\}}$	0.0969$^{\{2\}}$	0.1097$^{\{5\}}$	0.1082$^{\{4\}}$	0.1099$^{\{6\}}$	0.1053$^{\{3\}}$
	D_{abs}	0.0128$^{\{2\}}$	0.013$^{\{4\}}$	0.0134$^{\{8\}}$	0.0134$^{\{7\}}$	0.0135$^{\{9\}}$	0.0132$^{\{6\}}$	0.013$^{\{5\}}$	0.0147$^{\{11\}}$	0.0127$^{\{1\}}$	0.0129$^{\{3\}}$	0.0171$^{\{13\}}$	0.0152$^{\{12\}}$	0.0177$^{\{14\}}$	0.0145$^{\{10\}}$	0.0178$^{\{15\}}$
	D_{max}	0.0223$^{\{2\}}$	0.0225$^{\{4\}}$	0.0241$^{\{9\}}$	0.0238$^{\{8\}}$	0.0237$^{\{7\}}$	0.0233$^{\{6\}}$	0.0226$^{\{5\}}$	0.0267$^{\{12\}}$	0.0211$^{\{1\}}$	0.0225$^{\{3\}}$	0.0296$^{\{13\}}$	0.0267$^{\{11\}}$	0.03$^{\{14\}}$	0.0256$^{\{10\}}$	0.03$^{\{15\}}$
	$\sum$Ranks	101$^{\{6\}}$	99$^{\{5\}}$	105$^{\{8\}}$	170$^{\{14\}}$	144$^{\{13\}}$	142$^{\{12\}}$	112$^{\{9\}}$	196$^{\{15\}}$	19$^{\{1\}}$	98$^{\{4\}}$	58$^{\{2\}}$	69$^{\{3\}}$	141$^{\{11\}}$	104$^{\{7\}}$	122$^{\{10\}}$

Table 4: Partial and Full Ranks for Simulation in Table 3

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\|Bias\|(\hat{\alpha})$	7	11	10	13	8	5	4	14	1	15	3	2	9	12	6
	$\|Bias\|(\hat{\beta})$	4	5	7	14	10	8	6	15	1	13	2	3	12	9	11
	$\|Bias\|(\hat{\theta})$	15	5	6	11	14	12	13	10	2	3	4	1	8	9	7
	$\|Bias\|(\hat{k})$	15	4	10	9	13	8	12	5	1	14	2	3	6	11	7
	$MSE(\hat{\alpha})$	7	15	10	14	8	5	4	12	1	13	2	3	9	11	6
	$MSE(\hat{\beta})$	4	5	8	14	12	7	6	15	1	13	2	3	11	9	10
	$MSE(\hat{\theta})$	15	7	6	11	14	12	13	8	2	9	1	3	4	10	5
	$MSE(\hat{k})$	14	4	9	10	11	7	13	5	2	15	1	3	8	12	6
	$MRE(\hat{\alpha})$	7	11	10	13	8	5	4	14	1	15	2	3	9	12	6
	$MRE(\hat{\beta})$	4	5	7	14	10	8	6	15	1	13	2	3	12	9	11
	$MRE(\hat{\theta})$	15	6	7	11	14	12	13	9	1	8	2	3	4	10	5
	$MRE(\hat{k})$	14	4	9	10	13	6	12	5	1	15	2	3	7	11	8
	D_{abs}	8	3	7	11	12	9	5	15	2	4	10	6	13	1	14
	D_{max}	4	3	6	12	11	10	7	15	1	5	9	8	13	2	14
	$\sum$Ranks	$133^{(11)}$	$85^{(4)}$	$112^{(5)}$	$167^{(15)}$	$158^{(14)}$	$114^{(6)}$	$118^{(8)}$	$157^{(13)}$	$18^{(1)}$	$155^{(12)}$	$44^{(2)}$	$47^{(3)}$	$125^{(9)}$	$128^{(10)}$	$116^{(7)}$
20	$\|Bias\|(\hat{\alpha})$	15	10	8	9	7	6	4	12	1	13	5	2	3	14	11
	$\|Bias\|(\hat{\beta})$	9	5	4	10	6	8	7	15	1	14	3	2	12	11	13
	$\|Bias\|(\hat{\theta})$	15	2	1	6	4	14	11	10	3	9	5	8	13	7	12
	$\|Bias\|(\hat{k})$	4	5	11	14	15	13	9	8	2	3	7	1	10	6	12
	$MSE(\hat{\alpha})$	15	10	7	8	9	6	5	11	1	14	4	2	3	13	12
	$MSE(\hat{\beta})$	11	5	4	9	6	8	7	15	1	14	3	2	12	10	13
	$MSE(\hat{\theta})$	14	10	8	12	13	6	5	15	1	11	2	3	4	7	9
	$MSE(\hat{k})$	5	4	9	15	14	12	8	13	1	7	2	3	10	6	11
	$MRE(\hat{\alpha})$	15	9	7	10	8	6	5	12	1	13	3	2	4	14	11
	$MRE(\hat{\beta})$	9	5	4	10	6	8	7	15	1	14	3	2	12	11	13
	$MRE(\hat{\theta})$	14	11	9	13	12	5	7	15	1	10	2	3	4	6	8
	$MRE(\hat{k})$	5	4	10	14	15	13	8	9	1	6	2	3	11	7	12
	D_{abs}	3	6	2	11	9	7	4	8	1	5	15	10	13	14	12
	D_{max}	4	3	2	11	12	7	5	10	1	6	9	8	13	15	14
	$\sum$Ranks	$138^{(10)}$	$89^{(5)}$	$86^{(4)}$	$152^{(13)}$	$136^{(9)}$	$119^{(7)}$	$92^{(6)}$	$168^{(15)}$	$17^{(1)}$	$139^{(11)}$	$65^{(3)}$	$51^{(2)}$	$124^{(8)}$	$141^{(12)}$	$163^{(14)}$
50	$\|Bias\|(\hat{\alpha})$	15	7	9	10	11	6	8	14	4	13	1	2	5	12	3
	$\|Bias\|(\hat{\beta})$	11	6	4	9	5	8	7	15	1	14	2	3	10	13	12
	$\|Bias\|(\hat{\theta})$	11	5	2	7	3	10	9	4	1	13	6	12	14	8	15
	$\|Bias\|(\hat{k})$	6	3	12	9	14	15	10	4	1	2	7	8	13	5	11
	$MSE(\hat{\alpha})$	15	6	8	11	9	7	10	13	1	14	2	3	5	12	4
	$MSE(\hat{\beta})$	12	8	4	9	6	7	5	15	1	13	2	3	10	14	11
	$MSE(\hat{\theta})$	12	4	5	11	6	7	8	15	1	14	2	3	9	13	10
	$MSE(\hat{k})$	7	2	13	9	14	15	12	6	1	5	4	8	11	3	10
	$MRE(\hat{\alpha})$	15	6	8	11	10	9	7	14	1	13	2	3	5	12	4
	$MRE(\hat{\beta})$	11	7	4	9	5	8	6	15	1	14	2	3	10	13	12
	$MRE(\hat{\theta})$	8	4	7	12	5	6	9	15	1	14	2	3	10	11	13
	$MRE(\hat{k})$	7	2	12	10	14	15	9	8	1	4	3	5	13	6	11
	D_{abs}	3	2	9	5	6	8	4	13	7	3	12	10	15	1	14
	D_{max}	3	4	8	5	12	11	6	13	1	2	9	10	15	7	14
	$\sum$Ranks	$134^{(10)}$	$66^{(3)}$	$105^{(5)}$	$127^{(8)}$	$120^{(7)}$	$132^{(9)}$	$110^{(6)}$	$164^{(15)}$	$23^{(1)}$	$138^{(11)}$	$56^{(2)}$	$76^{(4)}$	$145^{(14)}$	$140^{(12)}$	$144^{(13)}$
100	$\|Bias\|(\hat{\alpha})$	9	8	7	11	13	12	10	15	1	4	3	2	14	6	5
	$\|Bias\|(\hat{\beta})$	2	9	8	7	4	5	10	15	1	14	3	6	13	11	12
	$\|Bias\|(\hat{\theta})$	12	4	1	8	13	9	3	15	2	14	5	7	11	6	10
	$\|Bias\|(\hat{k})$	2	9	1	14	12	13	10	15	5	6	11	7	3	4	8
	$MSE(\hat{\alpha})$	10	8	4	9	13	12	11	14	1	5	2	3	15	7	6
	$MSE(\hat{\beta})$	3	9	8	7	4	6	10	15	1	14	2	5	13	12	11
	$MSE(\hat{\theta})$	10	5	4	11	9	6	7	15	1	12	2	3	14	8	13
	$MSE(\hat{k})$	11	3	12	7	9	8	4	13	1	6	10	2	15	5	14
	$MRE(\hat{\alpha})$	10	8	6	11	13	12	9	15	1	4	2	3	14	5	7
	$MRE(\hat{\beta})$	3	9	8	7	5	6	10	15	1	14	2	4	13	12	11
	$MRE(\hat{\theta})$	8	6	4	11	10	7	9	15	1	12	2	3	14	5	13
	$MRE(\hat{k})$	10	5	9	8	12	11	7	15	1	6	3	2	14	4	13
	D_{abs}	5	2	3	10	8	6	4	12	1	7	13	9	15	11	14
	D_{max}	2	3	8	11	10	6	4	12	1	7	13	9	15	5	14
	$\sum$Ranks	$97^{(6)}$	$88^{(5)}$	$81^{(4)}$	$129^{(11)}$	$136^{(12)}$	$123^{(10)}$	$108^{(8)}$	$201^{(15)}$	$19^{(1)}$	$123^{(10)}$	$73^{(3)}$	$63^{(2)}$	$183^{(14)}$	$105^{(7)}$	$151^{(13)}$
200	$\|Bias\|(\hat{\alpha})$	5	9	14	10	13	12	11	15	1	4	3	8	6	2	7
	$\|Bias\|(\hat{\beta})$	2	6	4	11	8	10	5	15	1	12	3	7	14	9	13
	$\|Bias\|(\hat{\theta})$	8	3	10	9	5	11	1	7	2	13	4	12	15	6	14
	$\|Bias\|(\hat{k})$	4	3	9	15	10	11	12	13	1	5	8	14	7	2	6
	$MSE(\hat{\alpha})$	5	7	14	9	11	10	8	15	1	3	4	2	12	6	13
	$MSE(\hat{\beta})$	3	5	4	10	8	9	6	15	1	14	2	7	13	11	12
	$MSE(\hat{\theta})$	4	6	5	12	7	10	8	15	1	9	2	3	14	11	13
	$MSE(\hat{k})$	8	9	10	4	11	12	3	15	1	6	2	5	13	7	14
	$MRE(\hat{\alpha})$	5	8	14	7	12	13	9	15	1	4	3	2	11	6	10
	$MRE(\hat{\beta})$	3	6	4	12	8	9	7	15	1	11	2	5	14	10	13
	$MRE(\hat{\theta})$	4	6	5	12	9	11	10	15	1	7	2	3	14	8	13
	$MRE(\hat{k})$	6	9	12	7	13	14	4	15	1	5	2	3	11	8	10
	D_{abs}	1	4	7	3	6	10	2	13	9	5	12	8	14	11	15
	D_{max}	2	6	10	4	11	12	1	13	3	5	8	7	14	9	15
	$\sum$Ranks	$60^{(3)}$	$87^{(6)}$	$122^{(9)}$	$125^{(10)}$	$132^{(11)}$	$154^{(12)}$	$87^{(6)}$	$196^{(15)}$	$25^{(1)}$	$103^{(7)}$	$57^{(2)}$	$86^{(4)}$	$172^{(14)}$	$106^{(8)}$	$168^{(13)}$
300	$\|Bias\|(\hat{\alpha})$	6	10	12	9	14	13	11	15	1	7	3	8	2	5	4
	$\|Bias\|(\hat{\beta})$	2	5	3	8	7	9	6	15	1	12	4	11	13	10	14
	$\|Bias\|(\hat{\theta})$	9	4	8	3	1	5	7	11	2	13	6	12	14	10	15
	$\|Bias\|(\hat{k})$	3	4	7	8	9	11	10	15	2	1	6	14	12	5	13
	$MSE(\hat{\alpha})$	5	8	12	9	14	13	11	15	1	2	3	6	7	4	10
	$MSE(\hat{\beta})$	2	5	4	10	7	8	6	15	1	13	3	9	12	11	14
	$MSE(\hat{\theta})$	3	7	6	10	12	11	9	15	1	8	2	4	13	5	14
	$MSE(\hat{k})$	4	7	8	10	14	13	9	15	1	2	6	5	11	3	12
	$MRE(\hat{\alpha})$	5	9	12	8	14	13	11	15	1	2	3	6	7	4	10
	$MRE(\hat{\beta})$	3	5	4	11	7	8	6	15	1	12	2	9	13	10	14
	$MRE(\hat{\theta})$	4	8	6	11	12	10	9	15	1	7	2	3	13	5	14
	$MRE(\hat{k})$	4	7	10	11	14	13	8	15	1	2	5	6	9	3	12
	D_{abs}	2	5	3	9	8	7	6	11	1	4	15	14	12	10	13
	D_{max}	3	5	6	9	8	7	4	11	1	2	15	14	12	10	13
	$\sum$Ranks	$55^{(2)}$	$89^{(5)}$	$101^{(7)}$	$126^{(10)}$	$141^{(12)}$	$141^{(12)}$	$113^{(8)}$	$198^{(15)}$	$16^{(1)}$	$87^{(4)}$	$75^{(3)}$	$121^{(9)}$	$150^{(13)}$	$95^{(6)}$	$172^{(14)}$
500	$\|Bias\|(\hat{\alpha})$	13	11	9	14	10	8	12	15	1	6	5	3	7	4	2
	$\|Bias\|(\hat{\beta})$	2	7	8	13	10	11	4	15	1	6	3	5	14	9	12
	$\|Bias\|(\hat{\theta})$	13	6	1	3	11	12	9	8	2	5	4	10	15	7	14
	$\|Bias\|(\hat{k})$	9	10	8	14	11	12	13	15	5	7	4	1	6	2	3
	$MSE(\hat{\alpha})$	12	5	4	14	11	9	6	15	1	10	2	3	7	13	8
	$MSE(\hat{\beta})$	4	9	6	13	10	11	7	15	1	5	2	3	14	8	12
	$MSE(\hat{\theta})$	5	9	4	14	7	12	10	15	1	11	2	3	13	6	8
	$MSE(\hat{k})$	10	5	9	14	13	12	8	15	1	11	2	3	6	7	4
	$MRE(\hat{\alpha})$	11	6	13	14	12	9	7	15	1	10	2	3	5	8	4
	$MRE(\hat{\beta})$	4	9	7	14	10	11	8	15	1	5	2	3	13	6	12
	$MRE(\hat{\theta})$	4	7	6	14	11	12	9	15	1	8	2	3	13	5	10
	$MRE(\hat{k})$	10	7	13	14	12	11	9	15	1	8	2	5	4	6	3
	D_{abs}	2	4	8	7	9	6	5	11	1	3	13	12	14	10	15
	D_{max}	2	4	9	8	7	6	5	12	1	3	13	11	14	10	15
	$\sum$Ranks	$101^{(6)}$	$99^{(5)}$	$105^{(8)}$	$170^{(14)}$	$144^{(13)}$	$142^{(12)}$	$112^{(9)}$	$196^{(15)}$	$19^{(1)}$	$98^{(4)}$	$58^{(2)}$	$69^{(3)}$	$141^{(11)}$	$104^{(7)}$	$122^{(10)}$

Table 5: Numerical Results for Simulation using $\alpha = 1.5,\ \beta = 0.15,\ \theta = 2.50,\ k = 0.75$

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$10.4613^{\{14\}}$	$0.3487^{\{5\}}$	$0.0963^{\{2\}}$	$1.3793^{\{10\}}$	$3.7431^{\{12\}}$	$0.7788^{\{9\}}$	$0.6142^{\{7\}}$	$14.0504^{\{15\}}$	$0.0012^{\{1\}}$	$1.7584^{\{11\}}$	$0.6218^{\{8\}}$	$0.198^{\{4\}}$	$0.1353^{\{3\}}$	$9.0974^{\{13\}}$	$0.4652^{\{6\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$4.4799^{\{6\}}$	$1.6472^{\{3\}}$	$10.5358^{\{11\}}$	$4.8846^{\{8\}}$	$8.1834^{\{9\}}$	$4.8597^{\{7\}}$	$2.5478^{\{5\}}$	$8.3654^{\{10\}}$	$0.0684^{\{1\}}$	$27.3938^{\{15\}}$	$0.5623^{\{2\}}$	$1.7195^{\{4\}}$	$14.2668^{\{14\}}$	$12.1181^{\{13\}}$	$10.8774^{\{12\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.3002^{\{3\}}$	$0.9946^{\{7\}}$	$8.1132^{\{14\}}$	$1.8429^{\{11\}}$	$5.0123^{\{13\}}$	$0.6542^{\{4\}}$	$1.1822^{\{8\}}$	$0.7712^{\{5\}}$	$0.0207^{\{1\}}$	$1.9353^{\{12\}}$	$0.2901^{\{2\}}$	$0.9385^{\{6\}}$	$1.409^{\{10\}}$	$9.7025^{\{15\}}$	$1.3193^{\{9\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.615^{\{5\}}$	$1.1077^{\{9\}}$	$1.6798^{\{15\}}$	$1.6074^{\{12\}}$	$1.6363^{\{14\}}$	$1.3388^{\{10\}}$	$0.9188^{\{6\}}$	$0.4555^{\{4\}}$	$0.0266^{\{1\}}$	$1.582^{\{11\}}$	$0.1253^{\{2\}}$	$0.3168^{\{3\}}$	$1.0039^{\{7\}}$	$1.6203^{\{13\}}$	$1.0718^{\{8\}}$
	$\text{MSE}(\hat{\alpha})$	$971.7857^{\{13\}}$	$8.9256^{\{7\}}$	$5.8619^{\{5\}}$	$41.8618^{\{10\}}$	$438.5333^{\{12\}}$	$10.2105^{\{8\}}$	$5.5234^{\{4\}}$	$1010.0739^{\{14\}}$	$0.0308^{\{1\}}$	$87.6819^{\{11\}}$	$2.5494^{\{3\}}$	$2.4094^{\{2\}}$	$8.0163^{\{6\}}$	$1125.265^{\{15\}}$	$15.01^{\{9\}}$
	$\text{MSE}(\hat{\beta})$	$312.1919^{\{8\}}$	$5.3542^{\{3\}}$	$2778.5566^{\{14\}}$	$77.4683^{\{6\}}$	$739.4636^{\{10\}}$	$237.6862^{\{7\}}$	$17.6751^{\{4\}}$	$377.2511^{\{9\}}$	$0.0513^{\{1\}}$	$6114.0186^{\{15\}}$	$1.821^{\{2\}}$	$57.2924^{\{5\}}$	$1057.0319^{\{12\}}$	$1132.5372^{\{13\}}$	$813.0536^{\{11\}}$
	$\text{MSE}(\hat{\theta})$	$2.0244^{\{3\}}$	$2.2773^{\{4\}}$	$2435.6675^{\{14\}}$	$158.0706^{\{11\}}$	$738.847^{\{13\}}$	$13.4753^{\{8\}}$	$2.6739^{\{5\}}$	$50.8654^{\{9\}}$	$0.1052^{\{1\}}$	$231.1251^{\{12\}}$	$0.3761^{\{2\}}$	$81.4553^{\{10\}}$	$3.3692^{\{7\}}$	$3166.0191^{\{15\}}$	$3.1397^{\{6\}}$
10	$\text{MSE}(\hat{k})$	$3.2573^{\{5\}}$	$6.8578^{\{7\}}$	$13.1832^{\{13\}}$	$8.9474^{\{10\}}$	$10.75^{\{12\}}$	$9.0204^{\{11\}}$	$6.1278^{\{6\}}$	$2.2703^{\{4\}}$	$0.0315^{\{1\}}$	$13.2205^{\{14\}}$	$0.0837^{\{2\}}$	$1.9733^{\{3\}}$	$6.9156^{\{8\}}$	$14.9147^{\{15\}}$	$7.4749^{\{9\}}$
	$\text{MRE}(\hat{\alpha})$	$7.672^{\{14\}}$	$0.9847^{\{5\}}$	$0.9382^{\{4\}}$	$1.8285^{\{10\}}$	$3.3139^{\{12\}}$	$1.2875^{\{9\}}$	$1.0785^{\{6\}}$	$9.7839^{\{15\}}$	$0.0804^{\{1\}}$	$2.1319^{\{11\}}$	$0.496^{\{2\}}$	$0.6314^{\{3\}}$	$1.0826^{\{7\}}$	$6.9185^{\{13\}}$	$1.2738^{\{8\}}$
	$\text{MRE}(\hat{\beta})$	$30.3746^{\{6\}}$	$11.3527^{\{3\}}$	$71.0629^{\{11\}}$	$32.8402^{\{8\}}$	$54.8412^{\{9\}}$	$32.5736^{\{7\}}$	$17.2139^{\{5\}}$	$56.128^{\{10\}}$	$0.8556^{\{1\}}$	$182.9565^{\{15\}}$	$4.5151^{\{2\}}$	$12.387^{\{4\}}$	$95.5497^{\{14\}}$	$81.1065^{\{13\}}$	$73.0196^{\{12\}}$
	$\text{MRE}(\hat{\theta})$	$0.4808^{\{3\}}$	$0.5469^{\{4\}}$	$4.0011^{\{14\}}$	$1.6786^{\{11\}}$	$2.8441^{\{13\}}$	$0.8297^{\{8\}}$	$0.6077^{\{5\}}$	$0.9297^{\{10\}}$	$0.0792^{\{1\}}$	$1.9014^{\{12\}}$	$0.1888^{\{2\}}$	$0.9118^{\{9\}}$	$0.6608^{\{7\}}$	$4.8628^{\{15\}}$	$0.6281^{\{6\}}$
	$\text{MRE}(\hat{k})$	$1.1798^{\{5\}}$	$1.6865^{\{8\}}$	$2.4075^{\{13\}}$	$2.3712^{\{12\}}$	$2.3545^{\{11\}}$	$2.105^{\{10\}}$	$1.5793^{\{6\}}$	$1.1669^{\{4\}}$	$0.1829^{\{1\}}$	$2.4456^{\{14\}}$	$0.3471^{\{2\}}$	$0.7506^{\{3\}}$	$1.6775^{\{7\}}$	$2.522^{\{15\}}$	$1.7783^{\{9\}}$
	D_{abs}	$0.0794^{\{7\}}$	$0.0743^{\{3\}}$	$0.0682^{\{2\}}$	$0.083^{\{11\}}$	$0.0802^{\{8\}}$	$0.0617^{\{1\}}$	$0.0756^{\{4\}}$	$0.0887^{\{13\}}$	$0.0776^{\{6\}}$	$0.0756^{\{5\}}$	$0.0946^{\{15\}}$	$0.0828^{\{10\}}$	$0.0833^{\{12\}}$	$0.0915^{\{14\}}$	$0.0826^{\{9\}}$
	D_{max}	$0.1376^{\{7\}}$	$0.1267^{\{4\}}$	$0.1267^{\{5\}}$	$0.1561^{\{12\}}$	$0.1474^{\{9\}}$	$0.1165^{\{1\}}$	$0.1239^{\{2\}}$	$0.1799^{\{15\}}$	$0.1263^{\{3\}}$	$0.1339^{\{6\}}$	$0.1575^{\{14\}}$	$0.1417^{\{8\}}$	$0.1552^{\{10\}}$	$0.157^{\{13\}}$	$0.1553^{\{11\}}$
	$\sum\text{Ranks}$	$99^{\{6\}}$	$72^{\{3\}}$	$137^{\{10\}}$	$142^{\{12\}}$	$157^{\{13\}}$	$100^{\{7\}}$	$73^{\{4\}}$	$137^{\{10\}}$	$21^{\{1\}}$	$164^{\{14\}}$	$60^{\{2\}}$	$74^{\{5\}}$	$124^{\{8\}}$	$195^{\{15\}}$	$125^{\{9\}}$
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$6.5802^{\{13\}}$	$0.8048^{\{8\}}$	$0.2589^{\{3\}}$	$3.222^{\{11\}}$	$0.2018^{\{2\}}$	$0.3349^{\{4\}}$	$0.4739^{\{6\}}$	$6.5196^{\{12\}}$	$0.0291^{\{1\}}$	$6.593^{\{14\}}$	$0.9873^{\{9\}}$	$1.0257^{\{10\}}$	$0.4549^{\{5\}}$	$31.0125^{\{15\}}$	$0.5243^{\{7\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$7.2119^{\{14\}}$	$1.0386^{\{3\}}$	$1.2615^{\{4\}}$	$4.4076^{\{9\}}$	$1.2707^{\{5\}}$	$1.6093^{\{7\}}$	$1.9003^{\{8\}}$	$5.5107^{\{11\}}$	$0.0351^{\{1\}}$	$8.5459^{\{15\}}$	$0.6864^{\{2\}}$	$1.3237^{\{6\}}$	$5.3139^{\{10\}}$	$6.5843^{\{13\}}$	$6.3879^{\{12\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.059^{\{2\}}$	$0.3696^{\{6\}}$	$0.7556^{\{13\}}$	$0.3899^{\{7\}}$	$0.1444^{\{3\}}$	$0.5457^{\{12\}}$	$0.8452^{\{15\}}$	$0.4414^{\{9\}}$	$0.0048^{\{1\}}$	$0.7678^{\{14\}}$	$0.2649^{\{4\}}$	$0.4846^{\{11\}}$	$0.4179^{\{8\}}$	$0.3206^{\{5\}}$	$0.4773^{\{10\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.333^{\{11\}}$	$0.1369^{\{7\}}$	$0.4354^{\{14\}}$	$0.0721^{\{6\}}$	$0.4929^{\{15\}}$	$0.423^{\{13\}}$	$0.3008^{\{9\}}$	$0.0411^{\{2\}}$	$0.0375^{\{1\}}$	$0.0503^{\{4\}}$	$0.0462^{\{3\}}$	$0.0665^{\{5\}}$	$0.3196^{\{10\}}$	$0.2464^{\{8\}}$	$0.4068^{\{12\}}$
	$\text{MSE}(\hat{\alpha})$	$887.6055^{\{13\}}$	$8.7896^{\{6\}}$	$4.6806^{\{3\}}$	$155.0784^{\{12\}}$	$3.9711^{\{2\}}$	$8.6283^{\{5\}}$	$8.1862^{\{4\}}$	$144.8452^{\{11\}}$	$0.018^{\{1\}}$	$1557.4901^{\{14\}}$	$11.8401^{\{8\}}$	$12.8485^{\{10\}}$	$8.8574^{\{7\}}$	$25514.1596^{\{15\}}$	$12.8376^{\{9\}}$
	$\text{MSE}(\hat{\beta})$	$1072.7324^{\{15\}}$	$2.9303^{\{3\}}$	$4.7477^{\{4\}}$	$316.7685^{\{12\}}$	$5.268^{\{5\}}$	$9.5913^{\{7\}}$	$7.1703^{\{6\}}$	$172.6557^{\{9\}}$	$0.029^{\{1\}}$	$1041.4003^{\{14\}}$	$2.571^{\{2\}}$	$10.6648^{\{8\}}$	$198.8844^{\{10\}}$	$676.9253^{\{13\}}$	$260.2342^{\{11\}}$
	$\text{MSE}(\hat{\theta})$	$1.4625^{\{6\}}$	$1.2335^{\{4\}}$	$1.7055^{\{8\}}$	$1.2506^{\{5\}}$	$4.1614^{\{12\}}$	$2.6595^{\{10\}}$	$2.2178^{\{9\}}$	$3.0286^{\{11\}}$	$0.043^{\{1\}}$	$1.6801^{\{7\}}$	$0.4187^{\{2\}}$	$0.7554^{\{3\}}$	$17.0118^{\{15\}}$	$7.0523^{\{13\}}$	$11.5647^{\{14\}}$
20	$\text{MSE}(\hat{k})$	$0.9926^{\{10\}}$	$0.1444^{\{4\}}$	$0.9724^{\{9\}}$	$0.2038^{\{6\}}$	$4.0745^{\{14\}}$	$4.2098^{\{15\}}$	$2.2547^{\{13\}}$	$0.28^{\{7\}}$	$0.0719^{\{2\}}$	$0.1319^{\{3\}}$	$0.0469^{\{1\}}$	$0.167^{\{5\}}$	$0.8936^{\{8\}}$	$1.2182^{\{12\}}$	$1.1724^{\{11\}}$
	$\text{MRE}(\hat{\alpha})$	$4.9295^{\{14\}}$	$1.0113^{\{7\}}$	$0.8961^{\{5\}}$	$2.5539^{\{11\}}$	$0.7558^{\{2\}}$	$0.8496^{\{3\}}$	$0.9484^{\{6\}}$	$4.6309^{\{12\}}$	$0.0605^{\{1\}}$	$4.8692^{\{13\}}$	$0.8691^{\{4\}}$	$1.0241^{\{8\}}$	$1.0685^{\{9\}}$	$21.1487^{\{15\}}$	$1.1668^{\{10\}}$
	$\text{MRE}(\hat{\beta})$	$48.8422^{\{14\}}$	$7.5118^{\{3\}}$	$9.0917^{\{4\}}$	$29.7684^{\{9\}}$	$9.2081^{\{5\}}$	$11.563^{\{7\}}$	$13.1554^{\{8\}}$	$37.1087^{\{11\}}$	$0.8329^{\{1\}}$	$57.6186^{\{15\}}$	$5.3133^{\{2\}}$	$9.4466^{\{6\}}$	$35.9571^{\{10\}}$	$44.3496^{\{13\}}$	$43.1068^{\{12\}}$
	$\text{MRE}(\hat{\theta})$	$0.3893^{\{6\}}$	$0.3656^{\{5\}}$	$0.4555^{\{8\}}$	$0.3594^{\{4\}}$	$0.5087^{\{11\}}$	$0.4692^{\{9\}}$	$0.5208^{\{12\}}$	$0.505^{\{10\}}$	$0.0578^{\{1\}}$	$0.4445^{\{7\}}$	$0.1813^{\{2\}}$	$0.2596^{\{3\}}$	$0.78^{\{15\}}$	$0.5666^{\{13\}}$	$0.747^{\{14\}}$
	$\text{MRE}(\hat{k})$	$0.6641^{\{10\}}$	$0.3796^{\{5\}}$	$0.7119^{\{11\}}$	$0.4337^{\{6\}}$	$0.8015^{\{14\}}$	$0.7665^{\{13\}}$	$0.6186^{\{8\}}$	$0.5298^{\{7\}}$	$0.1933^{\{1\}}$	$0.33^{\{4\}}$	$0.2339^{\{2\}}$	$0.3223^{\{3\}}$	$0.7514^{\{12\}}$	$0.6243^{\{9\}}$	$0.8564^{\{15\}}$
	D_{abs}	$0.0559^{\{3\}}$	$0.0562^{\{4\}}$	$0.0583^{\{7\}}$	$0.0594^{\{8\}}$	$0.061^{\{9\}}$	$0.0582^{\{6\}}$	$0.0577^{\{5\}}$	$0.0631^{\{11\}}$	$0.0376^{\{1\}}$	$0.0554^{\{2\}}$	$0.0675^{\{13\}}$	$0.0618^{\{10\}}$	$0.0672^{\{12\}}$	$0.069^{\{15\}}$	$0.0681^{\{14\}}$
	D_{max}	$0.1012^{\{7\}}$	$0.099^{\{4\}}$	$0.1022^{\{8\}}$	$0.0993^{\{6\}}$	$0.1066^{\{9\}}$	$0.0989^{\{3\}}$	$0.0993^{\{5\}}$	$0.1195^{\{14\}}$	$0.0656^{\{1\}}$	$0.0958^{\{2\}}$	$0.1106^{\{11\}}$	$0.1088^{\{10\}}$	$0.1182^{\{13\}}$	$0.1151^{\{12\}}$	$0.1244^{\{15\}}$
	$\sum\text{Ranks}$	$138^{\{12\}}$	$69^{\{3\}}$	$101^{\{5\}}$	$112^{\{7\}}$	$108^{\{6\}}$	$114^{\{8\}}$	$114^{\{8\}}$	$137^{\{11\}}$	$15^{\{1\}}$	$128^{\{10\}}$	$65^{\{2\}}$	$98^{\{4\}}$	$144^{\{13\}}$	$171^{\{15\}}$	$166^{\{14\}}$
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$4.0199^{\{14\}}$	$0.4157^{\{9\}}$	$0.4495^{\{10\}}$	$0.602^{\{11\}}$	$0.0509^{\{2\}}$	$0.1449^{\{5\}}$	$0.2529^{\{7\}}$	$5.7554^{\{15\}}$	$0.0431^{\{1\}}$	$1.9392^{\{13\}}$	$0.1149^{\{4\}}$	$0.0698^{\{3\}}$	$0.289^{\{8\}}$	$1.8334^{\{12\}}$	$0.237^{\{6\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$1.4871^{\{11\}}$	$0.9791^{\{7\}}$	$0.6421^{\{4\}}$	$1.1456^{\{9\}}$	$0.6946^{\{5\}}$	$0.7775^{\{6\}}$	$0.9861^{\{8\}}$	$4.039^{\{14\}}$	$0.0438^{\{1\}}$	$5.0602^{\{15\}}$	$0.3253^{\{2\}}$	$0.4015^{\{3\}}$	$1.6751^{\{12\}}$	$1.3678^{\{10\}}$	$3.344^{\{13\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.2221^{\{2\}}$	$0.6818^{\{10\}}$	$0.4667^{\{6\}}$	$0.5675^{\{8\}}$	$0.6443^{\{9\}}$	$0.7958^{\{13\}}$	$0.7591^{\{12\}}$	$0.3847^{\{5\}}$	$0.0212^{\{1\}}$	$0.7088^{\{11\}}$	$0.3027^{\{3\}}$	$0.3576^{\{4\}}$	$0.8968^{\{14\}}$	$0.4871^{\{7\}}$	$0.9625^{\{15\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.1847^{\{7\}}$	$0.2579^{\{9\}}$	$0.2598^{\{10\}}$	$0.1659^{\{6\}}$	$0.3952^{\{15\}}$	$0.3573^{\{14\}}$	$0.2617^{\{11\}}$	$0.0104^{\{2\}}$	$0.0071^{\{1\}}$	$0.1335^{\{5\}}$	$0.0722^{\{3\}}$	$0.1277^{\{4\}}$	$0.2906^{\{13\}}$	$0.2499^{\{8\}}$	$0.2765^{\{12\}}$
	$\text{MSE}(\hat{\alpha})$	$273.7924^{\{14\}}$	$10.2961^{\{11\}}$	$6.077^{\{8\}}$	$8.9947^{\{10\}}$	$1.9382^{\{5\}}$	$1.4379^{\{4\}}$	$7.8055^{\{9\}}$	$556.5465^{\{15\}}$	$0.0337^{\{1\}}$	$69.8222^{\{13\}}$	$0.6408^{\{2\}}$	$1.106^{\{3\}}$	$4.6222^{\{7\}}$	$52.1498^{\{12\}}$	$4.5011^{\{6\}}$
	$\text{MSE}(\hat{\beta})$	$35.6396^{\{12\}}$	$2.9258^{\{8\}}$	$1.6899^{\{4\}}$	$6.5259^{\{9\}}$	$1.8959^{\{5\}}$	$1.9467^{\{6\}}$	$2.7685^{\{7\}}$	$159.8285^{\{14\}}$	$0.0273^{\{1\}}$	$289.0213^{\{15\}}$	$0.6506^{\{2\}}$	$0.7193^{\{3\}}$	$26.4489^{\{11\}}$	$12.6095^{\{10\}}$	$133.5245^{\{13\}}$
	$\text{MSE}(\hat{\theta})$	$1.0062^{\{6\}}$	$1.2416^{\{9\}}$	$0.9416^{\{4\}}$	$1.1011^{\{7\}}$	$0.9819^{\{5\}}$	$1.1209^{\{8\}}$	$1.2523^{\{10\}}$	$1.3124^{\{11\}}$	$0.0379^{\{1\}}$	$1.5376^{\{13\}}$	$0.326^{\{2\}}$	$0.526^{\{3\}}$	$1.6583^{\{14\}}$	$1.4561^{\{12\}}$	$1.828^{\{15\}}$
50	$\text{MSE}(\hat{k})$	$0.2807^{\{4\}}$	$0.337^{\{8\}}$	$0.4508^{\{10\}}$	$0.3159^{\{7\}}$	$0.9396^{\{15\}}$	$0.9046^{\{14\}}$	$0.4276^{\{9\}}$	$0.3102^{\{6\}}$	$0.0063^{\{1\}}$	$0.2927^{\{5\}}$	$0.14^{\{3\}}$	$0.1266^{\{2\}}$	$0.6097^{\{12\}}$	$0.6507^{\{13\}}$	$0.5875^{\{11\}}$
	$\text{MRE}(\hat{\alpha})$	$3.2047^{\{14\}}$	$0.8936^{\{11\}}$	$0.8911^{\{10\}}$	$0.8563^{\{9\}}$	$0.6344^{\{5\}}$	$0.5922^{\{4\}}$	$0.7937^{\{6\}}$	$4.2283^{\{15\}}$	$0.0604^{\{1\}}$	$1.8688^{\{13\}}$	$0.3697^{\{2\}}$	$0.5069^{\{3\}}$	$0.8534^{\{8\}}$	$1.7869^{\{12\}}$	$0.8485^{\{7\}}$
	$\text{MRE}(\hat{\beta})$	$10.7674^{\{11\}}$	$7.2706^{\{7\}}$	$5.2497^{\{4\}}$	$8.3849^{\{9\}}$	$5.5436^{\{5\}}$	$6.0975^{\{6\}}$	$7.3376^{\{8\}}$	$27.4513^{\{14\}}$	$0.7272^{\{1\}}$	$34.4752^{\{15\}}$	$3.0212^{\{2\}}$	$3.5364^{\{3\}}$	$11.8789^{\{12\}}$	$9.9377^{\{10\}}$	$22.9734^{\{13\}}$
	$\text{MRE}(\hat{\theta})$	$0.3287^{\{5\}}$	$0.3622^{\{9\}}$	$0.3043^{\{4\}}$	$0.3497^{\{8\}}$	$0.3298^{\{6\}}$	$0.347^{\{7\}}$	$0.3687^{\{10\}}$	$0.3917^{\{11\}}$	$0.0505^{\{1\}}$	$0.4044^{\{13\}}$	$0.1551^{\{2\}}$	$0.2041^{\{3\}}$	$0.4408^{\{14\}}$	$0.3987^{\{12\}}$	$0.462^{\{15\}}$
	$\text{MRE}(\hat{k})$	$0.4783^{\{6\}}$	$0.4844^{\{7\}}$	$0.5187^{\{10\}}$	$0.428^{\{5\}}$	$0.6611^{\{15\}}$	$0.6152^{\{12\}}$	$0.5001^{\{9\}}$	$0.4924^{\{8\}}$	$0.0888^{\{1\}}$	$0.4091^{\{4\}}$	$0.2906^{\{2\}}$	$0.3105^{\{3\}}$	$0.6556^{\{14\}}$	$0.5758^{\{11\}}$	$0.6368^{\{13\}}$
	D_{abs}	$0.0426^{\{6\}}$	$0.0417^{\{4\}}$	$0.044^{\{9\}}$	$0.0445^{\{10\}}$	$0.0432^{\{7\}}$	$0.0432^{\{8\}}$	$0.0422^{\{5\}}$	$0.0485^{\{13\}}$	$0.0362^{\{1\}}$	$0.0411^{\{3\}}$	$0.0478^{\{12\}}$	$0.0402^{\{2\}}$	$0.0525^{\{14\}}$	$0.045^{\{11\}}$	$0.0527^{\{15\}}$
	D_{max}	$0.0746^{\{6\}}$	$0.0728^{\{5\}}$	$0.078^{\{10\}}$	$0.0767^{\{9\}}$	$0.0762^{\{8\}}$	$0.0759^{\{7\}}$	$0.0727^{\{4\}}$	$0.0868^{\{13\}}$	$0.0614^{\{1\}}$	$0.0704^{\{3\}}$	$0.0804^{\{12\}}$	$0.0659^{\{2\}}$	$0.0904^{\{15\}}$	$0.0803^{\{11\}}$	$0.0903^{\{14\}}$
	$\sum\text{Ranks}$	$118^{\{10\}}$	$114^{\{6\}}$	$103^{\{4\}}$	$117^{\{9\}}$	$107^{\{5\}}$	$114^{\{6\}}$	$115^{\{8\}}$	$156^{\{13\}}$	$14^{\{1\}}$	$141^{\{11\}}$	$53^{\{3\}}$	$41^{\{2\}}$	$168^{\{14\}}$	$151^{\{12\}}$	$168^{\{14\}}$
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$0.1246^{\{11\}}$	$0.0582^{\{6\}}$	$0.0929^{\{8\}}$	$0.1704^{\{12\}}$	$0.2492^{\{14\}}$	$0.192^{\{13\}}$	$0.0222^{\{3\}}$	$0.6334^{\{15\}}$	$0.005^{\{1\}}$	$0.1011^{\{10\}}$	$0.0192^{\{2\}}$	$0.0253^{\{4\}}$	$0.0985^{\{9\}}$	$0.0301^{\{5\}}$	$0.0785^{\{7\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$0.255^{\{3\}}$	$0.5611^{\{6\}}$	$0.4342^{\{5\}}$	$0.806^{\{10\}}$	$0.6725^{\{8\}}$	$0.7334^{\{9\}}$	$0.5989^{\{7\}}$	$3.854^{\{15\}}$	$0.0343^{\{1\}}$	$1.3728^{\{12\}}$	$0.1175^{\{2\}}$	$0.2703^{\{4\}}$	$1.6029^{\{14\}}$	$1.1171^{\{11\}}$	$1.6003^{\{13\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.1585^{\{3\}}$	$0.4035^{\{7\}}$	$0.2928^{\{5\}}$	$0.5369^{\{11\}}$	$0.4722^{\{9\}}$	$0.5397^{\{12\}}$	$0.4511^{\{8\}}$	$0.3109^{\{6\}}$	$0.0311^{\{1\}}$	$0.6818^{\{13\}}$	$0.1228^{\{2\}}$	$0.2196^{\{4\}}$	$0.8784^{\{14\}}$	$0.5299^{\{10\}}$	$0.8948^{\{15\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.0906^{\{12\}}$	$0.074^{\{7\}}$	$0.0801^{\{8\}}$	$0.0572^{\{5\}}$	$0.1181^{\{15\}}$	$0.0994^{\{13\}}$	$0.1037^{\{14\}}$	$0.0147^{\{2\}}$	$0.0063^{\{1\}}$	$0.0651^{\{6\}}$	$0.0389^{\{3\}}$	$0.0393^{\{4\}}$	$0.0821^{\{10\}}$	$0.0812^{\{9\}}$	$0.0842^{\{11\}}$
	$\text{MSE}(\hat{\alpha})$	$0.8323^{\{8\}}$	$0.719^{\{5\}}$	$0.7394^{\{6\}}$	$1.1941^{\{10\}}$	$2.1191^{\{12\}}$	$1.8181^{\{11\}}$	$0.8298^{\{7\}}$	$3.6517^{\{15\}}$	$0.0038^{\{1\}}$	$0.6347^{\{4\}}$	$0.2392^{\{2\}}$	$0.2763^{\{3\}}$	$2.3538^{\{14\}}$	$0.8634^{\{9\}}$	$2.2823^{\{13\}}$
	$\text{MSE}(\hat{\beta})$	$0.3404^{\{3\}}$	$1.1667^{\{6\}}$	$1.0245^{\{5\}}$	$1.6963^{\{10\}}$	$1.2343^{\{7\}}$	$1.4762^{\{9\}}$	$1.2722^{\{8\}}$	$367.6328^{\{15\}}$	$0.0238^{\{1\}}$	$8.9322^{\{14\}}$	$0.114^{\{2\}}$	$0.6361^{\{4\}}$	$7.7068^{\{13\}}$	$6.33^{\{11\}}$	$7.4581^{\{12\}}$
	$\text{MSE}(\hat{\theta})$	$0.3629^{\{4\}}$	$0.5816^{\{6\}}$	$0.4659^{\{5\}}$	$0.8271^{\{10\}}$	$0.6983^{\{8\}}$	$0.8109^{\{9\}}$	$0.6731^{\{7\}}$	$0.8318^{\{11\}}$	$0.0176^{\{1\}}$	$1.1303^{\{13\}}$	$0.0693^{\{2\}}$	$0.2018^{\{3\}}$	$1.494^{\{15\}}$	$1.0251^{\{12\}}$	$1.4862^{\{14\}}$
100	$\text{MSE}(\hat{k})$	$0.0729^{\{7\}}$	$0.0587^{\{5\}}$	$0.0641^{\{6\}}$	$0.0787^{\{9\}}$	$0.128^{\{13\}}$	$0.1166^{\{12\}}$	$0.0855^{\{11\}}$	$0.0852^{\{10\}}$	$0.0052^{\{1\}}$	$0.0485^{\{3\}}$	$0.0449^{\{2\}}$	$0.0575^{\{4\}}$	$0.1305^{\{15\}}$	$0.0748^{\{8\}}$	$0.1302^{\{14\}}$
	$\text{MRE}(\hat{\alpha})$	$0.4669^{\{9\}}$	$0.4305^{\{5\}}$	$0.4554^{\{6\}}$	$0.4585^{\{8\}}$	$0.621^{\{14\}}$	$0.5888^{\{12\}}$	$0.4562^{\{7\}}$	$0.7543^{\{15\}}$	$0.0273^{\{1\}}$	$0.4128^{\{4\}}$	$0.2158^{\{2\}}$	$0.2713^{\{3\}}$	$0.5939^{\{13\}}$	$0.4854^{\{10\}}$	$0.5809^{\{11\}}$
	$\text{MRE}(\hat{\beta})$	$2.3838^{\{3\}}$	$4.3819^{\{6\}}$	$3.6787^{\{5\}}$	$5.9909^{\{10\}}$	$5.1033^{\{8\}}$	$5.5464^{\{9\}}$	$4.6019^{\{7\}}$	$26.5562^{\{15\}}$	$0.5144^{\{1\}}$	$9.7578^{\{12\}}$	$1.521^{\{2\}}$	$2.4217^{\{4\}}$	$11.2303^{\{14\}}$	$8.1341^{\{11\}}$	$11.1753^{\{13\}}$
	$\text{MRE}(\hat{\theta})$	$0.1936^{\{4\}}$	$0.2355^{\{6\}}$	$0.2082^{\{5\}}$	$0.3034^{\{11\}}$	$0.2739^{\{8\}}$	$0.2895^{\{10\}}$	$0.2611^{\{7\}}$	$0.2758^{\{9\}}$	$0.0303^{\{1\}}$	$0.3415^{\{13\}}$	$0.0758^{\{2\}}$	$0.1139^{\{3\}}$	$0.41^{\{15\}}$	$0.3087^{\{12\}}$	$0.4051^{\{14\}}$
	$\text{MRE}(\hat{k})$	$0.2631^{\{9\}}$	$0.2308^{\{5\}}$	$0.2483^{\{7\}}$	$0.2416^{\{6\}}$	$0.33^{\{15\}}$	$0.3148^{\{14\}}$	$0.275^{\{10\}}$	$0.302^{\{13\}}$	$0.0702^{\{1\}}$	$0.2118^{\{4\}}$	$0.1793^{\{2\}}$	$0.1968^{\{3\}}$	$0.2877^{\{12\}}$	$0.2534^{\{8\}}$	$0.2859^{\{11\}}$
	D_{abs}	$0.0285^{\{3\}}$	$0.0285^{\{2\}}$	$0.0294^{\{6\}}$	$0.0292^{\{5\}}$	$0.0305^{\{9\}}$	$0.0299^{\{8\}}$	$0.0298^{\{7\}}$	$0.0341^{\{12\}}$	$0.024^{\{1\}}$	$0.0286^{\{4\}}$	$0.0345^{\{13\}}$	$0.0306^{\{10\}}$	$0.0389^{\{14\}}$	$0.0316^{\{11\}}$	$0.039^{\{15\}}$
	D_{max}	$0.0477^{\{3\}}$	$0.0485^{\{4\}}$	$0.0503^{\{5\}}$	$0.0504^{\{6\}}$	$0.0527^{\{11\}}$	$0.0519^{\{9\}}$	$0.0508^{\{7\}}$	$0.0581^{\{13\}}$	$0.0394^{\{1\}}$	$0.0472^{\{2\}}$	$0.057^{\{12\}}$	$0.0515^{\{8\}}$	$0.0654^{\{14\}}$	$0.0526^{\{10\}}$	$0.0655^{\{15\}}$
	$\sum\text{Ranks}$	$82^{\{6\}}$	$76^{\{4\}}$	$82^{\{6\}}$	$123^{\{9\}}$	$151^{\{12\}}$	$150^{\{11\}}$	$110^{\{7\}}$	$166^{\{13\}}$	$14^{\{1\}}$	$114^{\{8\}}$	$50^{\{2\}}$	$61^{\{3\}}$	$186^{\{15\}}$	$137^{\{10\}}$	$178^{\{14\}}$
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$0.0537^{\{8\}}$	$0.033^{\{5\}}$	$0.1131^{\{10\}}$	$0.144^{\{12\}}$	$1e-04^{\{1\}}$	$0.012^{\{2\}}$	$0.0914^{\{9\}}$	$1.1103^{\{15\}}$	$0.0301^{\{3\}}$	$0.1244^{\{11\}}$	$0.0322^{\{4\}}$	$0.0506^{\{7\}}$	$0.2175^{\{13\}}$	$0.0334^{\{6\}}$	$0.222^{\{14\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$0.3946^{\{8\}}$	$0.3564^{\{7\}}$	$0.2555^{\{4\}}$	$0.32^{\{5\}}$	$0.4286^{\{9\}}$	$0.4368^{\{10\}}$	$0.3557^{\{6\}}$	$8.4974^{\{15\}}$	$0.0368^{\{1\}}$	$1.078^{\{14\}}$	$0.1524^{\{2\}}$	$0.1896^{\{3\}}$	$0.926^{\{12\}}$	$0.835^{\{11\}}$	$0.9292^{\{13\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.2916^{\{6\}}$	$0.2761^{\{5\}}$	$0.3219^{\{8\}}$	$0.246^{\{3\}}$	$0.3714^{\{10\}}$	$0.3991^{\{12\}}$	$0.3311^{\{9\}}$	$0.3144^{\{7\}}$	$0.0291^{\{1\}}$	$0.5288^{\{13\}}$	$0.1712^{\{2\}}$	$0.2462^{\{4\}}$	$0.658^{\{14\}}$	$0.3883^{\{11\}}$	$0.6631^{\{15\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.0632^{\{11\}}$	$0.0314^{\{4\}}$	$0.0771^{\{12\}}$	$0.0101^{\{1\}}$	$0.0534^{\{10\}}$	$0.0459^{\{8\}}$	$0.0502^{\{9\}}$	$0.1053^{\{15\}}$	$0.0122^{\{2\}}$	$0.042^{\{6\}}$	$0.0438^{\{7\}}$	$0.0419^{\{5\}}$	$0.0976^{\{13\}}$	$0.0299^{\{3\}}$	$0.0985^{\{14\}}$
	$\text{MSE}(\hat{\alpha})$	$0.5155^{\{8\}}$	$0.3546^{\{4\}}$	$0.5649^{\{12\}}$	$0.677^{\{14\}}$	$0.5911^{\{13\}}$	$0.549^{\{11\}}$	$0.3643^{\{5\}}$	$7.9889^{\{15\}}$	$0.0153^{\{1\}}$	$0.4998^{\{6\}}$	$0.1775^{\{2\}}$	$0.3359^{\{3\}}$	$0.5392^{\{10\}}$	$0.5026^{\{7\}}$	$0.5382^{\{9\}}$
	$\text{MSE}(\hat{\beta})$	$1.1004^{\{9\}}$	$1.3618^{\{10\}}$	$0.5314^{\{4\}}$	$0.7448^{\{5\}}$	$0.8692^{\{6\}}$	$0.9253^{\{7\}}$	$1.0141^{\{8\}}$	$949.1271^{\{15\}}$	$0.0252^{\{1\}}$	$6.3962^{\{14\}}$	$0.2257^{\{2\}}$	$0.5288^{\{3\}}$	$4.0323^{\{11\}}$	$4.7343^{\{13\}}$	$4.0551^{\{12\}}$
	$\text{MSE}(\hat{\theta})$	$0.5128^{\{8\}}$	$0.3959^{\{5\}}$	$0.4408^{\{7\}}$	$0.4176^{\{6\}}$	$0.5228^{\{9\}}$	$0.5479^{\{10\}}$	$0.3925^{\{4\}}$	$1.2166^{\{15\}}$	$0.014^{\{1\}}$	$0.9255^{\{12\}}$	$0.1153^{\{2\}}$	$0.2931^{\{3\}}$	$0.9918^{\{13\}}$	$0.7521^{\{11\}}$	$0.9919^{\{14\}}$
200	$\text{MSE}(\hat{k})$	$0.0271^{\{7\}}$	$0.0187^{\{2\}}$	$0.0273^{\{8\}}$	$0.0346^{\{11\}}$	$0.0348^{\{12\}}$	$0.0334^{\{10\}}$	$0.0238^{\{4\}}$	$0.059^{\{15\}}$	$0.0022^{\{1\}}$	$0.0228^{\{3\}}$	$0.0241^{\{5\}}$	$0.0295^{\{9\}}$	$0.0549^{\{13\}}$	$0.0257^{\{6\}}$	$0.0549^{\{14\}}$
	$\text{MRE}(\hat{\alpha})$	$0.3598^{\{8\}}$	$0.3046^{\{5\}}$	$0.3849^{\{12\}}$	$0.4148^{\{14\}}$	$0.398^{\{13\}}$	$0.3829^{\{11\}}$	$0.3033^{\{4\}}$	$1.0401^{\{15\}}$	$0.0312^{\{1\}}$	$0.3568^{\{7\}}$	$0.2034^{\{2\}}$	$0.2826^{\{3\}}$	$0.3811^{\{10\}}$	$0.3376^{\{6\}}$	$0.3757^{\{9\}}$
	$\text{MRE}(\hat{\beta})$	$3.5286^{\{8\}}$	$3.3723^{\{7\}}$	$2.6924^{\{4\}}$	$3.0821^{\{5\}}$	$3.5314^{\{9\}}$	$3.6123^{\{10\}}$	$3.2314^{\{6\}}$	$57.5758^{\{15\}}$	$0.4786^{\{1\}}$	$7.9675^{\{14\}}$	$1.7822^{\{2\}}$	$2.1268^{\{3\}}$	$6.8785^{\{12\}}$	$6.4071^{\{11\}}$	$6.8823^{\{13\}}$
	$\text{MRE}(\hat{\theta})$	$0.2021^{\{7\}}$	$0.1724^{\{5\}}$	$0.208^{\{8\}}$	$0.1909^{\{6\}}$	$0.2188^{\{10\}}$	$0.2186^{\{9\}}$	$0.1654^{\{4\}}$	$0.3301^{\{15\}}$	$0.0216^{\{1\}}$	$0.2731^{\{12\}}$	$0.0834^{\{2\}}$	$0.1408^{\{3\}}$	$0.2946^{\{14\}}$	$0.2371^{\{11\}}$	$0.2941^{\{13\}}$
	$\text{MRE}(\hat{k})$	$0.1778^{\{8\}}$	$0.1475^{\{3\}}$	$0.1867^{\{9\}}$	$0.2007^{\{12\}}$	$0.1965^{\{11\}}$	$0.1872^{\{10\}}$	$0.162^{\{6\}}$	$0.2704^{\{15\}}$	$0.0505^{\{1\}}$	$0.1609^{\{5\}}$	$0.1332^{\{2\}}$	$0.1605^{\{4\}}$	$0.2362^{\{14\}}$	$0.1698^{\{7\}}$	$0.2351^{\{13\}}$
	D_{abs}	$0.0173^{\{2\}}$	$0.0175^{\{3\}}$	$0.0186^{\{7\}}$	$0.0178^{\{6\}}$	$0.0188^{\{9\}}$	$0.0187^{\{8\}}$	$0.0176^{\{4\}}$	$0.0231^{\{13\}}$	$0.0153^{\{1\}}$	$0.0177^{\{5\}}$	$0.0216^{\{12\}}$	$0.021^{\{11\}}$	$0.0235^{\{14\}}$	$0.0194^{\{10\}}$	$0.0236^{\{15\}}$
	D_{max}	$0.0308^{\{2\}}$	$0.0315^{\{4\}}$	$0.0338^{\{9\}}$	$0.0332^{\{6\}}$	$0.0337^{\{8\}}$	$0.0335^{\{7\}}$	$0.0318^{\{5\}}$	$0.043^{\{15\}}$	$0.0266^{\{1\}}$	$0.0315^{\{3\}}$	$0.0384^{\{12\}}$	$0.037^{\{11\}}$	$0.042^{\{13\}}$	$0.0339^{\{10\}}$	$0.0421^{\{14\}}$
	$\sum\text{Ranks}$	$100^{\{6\}}$	$69^{\{3\}}$	$114^{\{8\}}$	$106^{\{7\}}$	$130^{\{12\}}$	$125^{\{10\}}$	$83^{\{5\}}$	$200^{\{15\}}$	$17^{\{1\}}$	$125^{\{10\}}$	$58^{\{2\}}$	$72^{\{4\}}$	$176^{\{13\}}$	$123^{\{9\}}$	$182^{\{14\}}$
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$0.0426^{\{3\}}$	$0.1467^{\{14\}}$	$0.1187^{\{12\}}$	$0.0525^{\{4\}}$	$0.0797^{\{6\}}$	$0.0925^{\{7\}}$	$0.095^{\{9\}}$	$0.6639^{\{15\}}$	$0.0028^{\{1\}}$	$0.129^{\{13\}}$	$0.0087^{\{2\}}$	$0.059^{\{5\}}$	$0.0946^{\{8\}}$	$0.1176^{\{11\}}$	$0.104^{\{10\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$0.2164^{\{3\}}$	$0.4664^{\{8\}}$	$0.3433^{\{5\}}$	$0.5665^{\{12\}}$	$0.4903^{\{9\}}$	$0.5949^{\{13\}}$	$0.432^{\{7\}}$	$0.3977^{\{6\}}$	$0.0028^{\{1\}}$	$0.6074^{\{14\}}$	$0.1769^{\{2\}}$	$0.3038^{\{4\}}$	$0.5366^{\{11\}}$	$0.6647^{\{15\}}$	$0.5364^{\{10\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.1807^{\{4\}}$	$0.3999^{\{10\}}$	$0.3108^{\{6\}}$	$0.3873^{\{8\}}$	$0.3874^{\{9\}}$	$0.452^{\{14\}}$	$0.3749^{\{7\}}$	$0.079^{\{2\}}$	$0.0088^{\{1\}}$	$0.4277^{\{12\}}$	$0.1539^{\{3\}}$	$0.2362^{\{5\}}$	$0.4511^{\{13\}}$	$0.4004^{\{11\}}$	$0.4558^{\{15\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.0484^{\{8\}}$	$0.0643^{\{15\}}$	$0.061^{\{13\}}$	$0.0355^{\{6\}}$	$0.0566^{\{11\}}$	$0.0525^{\{10\}}$	$0.0568^{\{12\}}$	$0.0393^{\{7\}}$	$0.0035^{\{1\}}$	$0.05^{\{9\}}$	$0.0057^{\{2\}}$	$0.0202^{\{3\}}$	$0.0308^{\{4\}}$	$0.0626^{\{14\}}$	$0.0312^{\{5\}}$
	$\text{MSE}(\hat{\alpha})$	$0.2275^{\{6\}}$	$0.2247^{\{5\}}$	$0.2197^{\{4\}}$	$0.5446^{\{14\}}$	$0.3974^{\{12\}}$	$0.4124^{\{13\}}$	$0.3038^{\{11\}}$	$3.4984^{\{15\}}$	$9e-04^{\{1\}}$	$0.2456^{\{8\}}$	$0.1257^{\{3\}}$	$0.1172^{\{2\}}$	$0.2632^{\{9\}}$	$0.2696^{\{10\}}$	$0.2428^{\{7\}}$
	$\text{MSE}(\hat{\beta})$	$0.2267^{\{3\}}$	$0.7816^{\{7\}}$	$0.5506^{\{4\}}$	$1.0524^{\{9\}}$	$0.8898^{\{8\}}$	$1.171^{\{12\}}$	$0.6267^{\{6\}}$	$1.2953^{\{13\}}$	$0.003^{\{1\}}$	$1.9368^{\{14\}}$	$0.212^{\{2\}}$	$0.6044^{\{5\}}$	$1.1181^{\{11\}}$	$2.7621^{\{15\}}$	$1.107^{\{10\}}$
	$\text{MSE}(\hat{\theta})$	$0.2851^{\{4\}}$	$0.5342^{\{7\}}$	$0.3629^{\{5\}}$	$0.6208^{\{9\}}$	$0.5463^{\{8\}}$	$0.665^{\{13\}}$	$0.5014^{\{6\}}$	$0.657^{\{12\}}$	$0.0025^{\{1\}}$	$0.6394^{\{10\}}$	$0.104^{\{2\}}$	$0.2762^{\{3\}}$	$0.6752^{\{15\}}$	$0.6514^{\{11\}}$	$0.6651^{\{14\}}$
300	$\text{MSE}(\hat{k})$	$0.0185^{\{7\}}$	$0.0206^{\{10\}}$	$0.0182^{\{6\}}$	$0.0299^{\{14\}}$	$0.0232^{\{12\}}$	$0.0224^{\{11\}}$	$0.0203^{\{9\}}$	$0.041^{\{15\}}$	$9e-04^{\{1\}}$	$0.0186^{\{8\}}$	$0.0085^{\{2\}}$	$0.0095^{\{3\}}$	$0.0177^{\{5\}}$	$0.0234^{\{13\}}$	$0.0173^{\{4\}}$
	$\text{MRE}(\hat{\alpha})$	$0.2516^{\{5\}}$	$0.2747^{\{7\}}$	$0.2472^{\{4\}}$	$0.3682^{\{14\}}$	$0.3114^{\{12\}}$	$0.3143^{\{13\}}$	$0.2964^{\{11\}}$	$0.6931^{\{15\}}$	$0.0154^{\{1\}}$	$0.2704^{\{6\}}$	$0.1768^{\{2\}}$	$0.1819^{\{3\}}$	$0.2918^{\{10\}}$	$0.2881^{\{9\}}$	$0.2791^{\{8\}}$
	$\text{MRE}(\hat{\beta})$	$2.2359^{\{3\}}$	$3.8229^{\{8\}}$	$3.0332^{\{5\}}$	$4.4497^{\{12\}}$	$3.99^{\{9\}}$	$4.6536^{\{13\}}$	$3.533^{\{6\}}$	$3.7472^{\{7\}}$	$0.2775^{\{1\}}$	$4.709^{\{14\}}$	$1.8264^{\{2\}}$	$2.6682^{\{4\}}$	$4.1551^{\{10\}}$	$5.1755^{\{15\}}$	$4.2117^{\{11\}}$
	$\text{MRE}(\hat{\theta})$	$0.1656^{\{4\}}$	$0.2158^{\{6\}}$	$0.1686^{\{5\}}$	$0.2432^{\{12\}}$	$0.2172^{\{8\}}$	$0.2397^{\{9\}}$	$0.2171^{\{7\}}$	$0.2513^{\{15\}}$	$0.0161^{\{1\}}$	$0.2411^{\{10\}}$	$0.0775^{\{2\}}$	$0.1165^{\{3\}}$	$0.2506^{\{14\}}$	$0.242^{\{11\}}$	$0.2436^{\{13\}}$
	$\text{MRE}(\hat{k})$	$0.1395^{\{6\}}$	$0.1593^{\{9\}}$	$0.1385^{\{5\}}$	$0.1908^{\{14\}}$	$0.1671^{\{13\}}$	$0.164^{\{12\}}$	$0.1611^{\{11\}}$	$0.2144^{\{15\}}$	$0.0317^{\{1\}}$	$0.1384^{\{4\}}$	$0.0931^{\{3\}}$	$0.0912^{\{2\}}$	$0.1496^{\{8\}}$	$0.16^{\{10\}}$	$0.1476^{\{7\}}$
	D_{abs}	$0.015^{\{3\}}$	$0.0151^{\{4\}}$	$0.0157^{\{8\}}$	$0.0164^{\{11\}}$	$0.016^{\{9\}}$	$0.0157^{\{7\}}$	$0.0154^{\{5\}}$	$0.0161^{\{10\}}$	$0.0118^{\{1\}}$	$0.015^{\{2\}}$	$0.0194^{\{13\}}$	$0.0189^{\{12\}}$	$0.0202^{\{14\}}$	$0.0157^{\{6\}}$	$0.0203^{\{15\}}$
	D_{max}	$0.0256^{\{3\}}$	$0.0265^{\{4\}}$	$0.0275^{\{6\}}$	$0.0295^{\{11\}}$	$0.0286^{\{10\}}$	$0.0281^{\{9\}}$	$0.0269^{\{5\}}$	$0.0281^{\{8\}}$	$0.0194^{\{1\}}$	$0.0254^{\{2\}}$	$0.0324^{\{13\}}$	$0.032^{\{12\}}$	$0.0342^{\{14\}}$	$0.0277^{\{7\}}$	$0.0343^{\{15\}}$
	$\sum\text{Ranks}$	$62^{\{3\}}$	$114^{\{7\}}$	$88^{\{5\}}$	$150^{\{12\}}$	$136^{\{9\}}$	$156^{\{14\}}$	$112^{\{6\}}$	$155^{\{13\}}$	$14^{\{1\}}$	$126^{\{8\}}$	$53^{\{2\}}$	$66^{\{4\}}$	$146^{\{11\}}$	$158^{\{15\}}$	$144^{\{10\}}$
	$\lvert\text{Bias}\rvert(\hat{\alpha})$	$0.0093^{\{2\}}$	$0.0272^{\{7\}}$	$0.045^{\{10\}}$	$0.0387^{\{9\}}$	$0.0186^{\{5\}}$	$0.0096^{\{3\}}$	$0.0295^{\{8\}}$	$0.5332^{\{15\}}$	$0.0033^{\{1\}}$	$0.0525^{\{12\}}$	$0.0455^{\{11\}}$	$0.0137^{\{4\}}$	$0.1846^{\{14\}}$	$0.0213^{\{6\}}$	$0.1683^{\{13\}}$
	$\lvert\text{Bias}\rvert(\hat{\beta})$	$0.155^{\{5\}}$	$0.1668^{\{6\}}$	$0.0862^{\{3\}}$	$0.3615^{\{12\}}$	$0.2132^{\{8\}}$	$0.2289^{\{9\}}$	$0.1531^{\{4\}}$	$0.5103^{\{15\}}$	$0.0099^{\{1\}}$	$0.3284^{\{10\}}$	$0.0639^{\{2\}}$	$0.1976^{\{7\}}$	$0.397^{\{14\}}$	$0.3365^{\{11\}}$	$0.367^{\{13\}}$
	$\lvert\text{Bias}\rvert(\hat{\theta})$	$0.1322^{\{5\}}$	$0.1538^{\{7\}}$	$0.0852^{\{3\}}$	$0.2809^{\{13\}}$	$0.1693^{\{8\}}$	$0.2014^{\{10\}}$	$0.1415^{\{6\}}$	$0.1197^{\{4\}}$	$0.0023^{\{1\}}$	$0.254^{\{12\}}$	$0.049^{\{2\}}$	$0.1807^{\{9\}}$	$0.4041^{\{15\}}$	$0.2516^{\{11\}}$	$0.3683^{\{14\}}$
	$\lvert\text{Bias}\rvert(\hat{k})$	$0.0263^{\{10\}}$	$0.0237^{\{7\}}$	$0.0233^{\{6\}}$	$0.0101^{\{3\}}$	$0.0214^{\{5\}}$	$0.025^{\{8\}}$	$0.0252^{\{9\}}$	$0.0693^{\{15\}}$	$0.0046^{\{2\}}$	$0.0267^{\{11\}}$	$0.0016^{\{1\}}$	$0.0147^{\{4\}}$	$0.0598^{\{14\}}$	$0.0322^{\{12\}}$	$0.0529^{\{13\}}$
	$\text{MSE}(\hat{\alpha})$	$0.1507^{\{7\}}$	$0.135^{\{5\}}$	$0.1202^{\{4\}}$	$0.2735^{\{14\}}$	$0.251^{\{13\}}$	$0.239^{\{12\}}$	$0.1372^{\{6\}}$	$1.3457^{\{15\}}$	$0.0016^{\{1\}}$	$0.1635^{\{8\}}$	$0.0844^{\{2\}}$	$0.087^{\{3\}}$	$0.1998^{\{10\}}$	$0.2286^{\{11\}}$	$0.1721^{\{9\}}$
	$\text{MSE}(\hat{\beta})$	$0.1515^{\{3\}}$	$0.2125^{\{6\}}$	$0.2426^{\{7\}}$	$0.5195^{\{10\}}$	$0.347^{\{8\}}$	$0.3788^{\{9\}}$	$0.2037^{\{4\}}$	$1.8888^{\{15\}}$	$0.0034^{\{1\}}$	$0.8517^{\{14\}}$	$0.0379^{\{2\}}$	$0.2096^{\{5\}}$	$0.7361^{\{13\}}$	$0.5463^{\{11\}}$	$0.7183^{\{12\}}$
	$\text{MSE}(\hat{\theta})$	$0.2156^{\{7\}}$	$0.2075^{\{6\}}$	$0.1865^{\{4\}}$	$0.3902^{\{11\}}$	$0.264^{\{8\}}$	$0.2775^{\{9\}}$	$0.199^{\{5\}}$	$0.6143^{\{15\}}$	$0.004^{\{1\}}$	$0.3798^{\{10\}}$	$0.0297^{\{2\}}$	$0.1698^{\{3\}}$	$0.5322^{\{14\}}$	$0.4379^{\{12\}}$	$0.4789^{\{13\}}$
500	$\text{MSE}(\hat{k})$	$0.0119^{\{7\}}$	$0.0099^{\{5\}}$	$0.0081^{\{3\}}$	$0.0136^{\{9\}}$	$0.0152^{\{12\}}$	$0.0149^{\{10\}}$	$0.0099^{\{6\}}$	$0.0338^{\{15\}}$	$9e-04^{\{1\}}$	$0.0126^{\{8\}}$	$0.0083^{\{4\}}$	$0.0071^{\{2\}}$	$0.0222^{\{14\}}$	$0.0152^{\{11\}}$	$0.0209^{\{13\}}$
	$\text{MRE}(\hat{\alpha})$	$0.2125^{\{7\}}$	$0.1899^{\{6\}}$	$0.187^{\{4\}}$	$0.2548^{\{12\}}$	$0.26^{\{14\}}$	$0.2503^{\{11\}}$	$0.1894^{\{5\}}$	$0.5566^{\{15\}}$	$0.0158^{\{1\}}$	$0.221^{\{9\}}$	$0.1497^{\{2\}}$	$0.15^{\{3\}}$	$0.2459^{\{10\}}$	$0.2559^{\{13\}}$	$0.2197^{\{8\}}$
	$\text{MRE}(\hat{\beta})$	$1.757^{\{4\}}$	$1.8616^{\{7\}}$	$1.602^{\{3\}}$	$3.1405^{\{13\}}$	$2.2249^{\{8\}}$	$2.3161^{\{9\}}$	$1.8133^{\{6\}}$	$4.4224^{\{15\}}$	$0.2627^{\{1\}}$	$2.9212^{\{10\}}$	$1.0001^{\{2\}}$	$1.7875^{\{5\}}$	$3.3238^{\{14\}}$	$2.9247^{\{11\}}$	$3.0966^{\{12\}}$
	$\text{MRE}(\hat{\theta})$	$0.1434^{\{9\}}$	$0.1266^{\{6\}}$	$0.1194^{\{4\}}$	$0.1828^{\{11\}}$	$0.1347^{\{7\}}$	$0.1356^{\{8\}}$	$0.1226^{\{5\}}$	$0.2329^{\{15\}}$	$0.0171^{\{1\}}$	$0.1817^{\{10\}}$	$0.0465^{\{2\}}$	$0.0993^{\{3\}}$	$0.209^{\{14\}}$	$0.1953^{\{13\}}$	$0.1831^{\{12\}}$
	$\text{MRE}(\hat{k})$	$0.11^{\{7\}}$	$0.1021^{\{6\}}$	$0.0944^{\{4\}}$	$0.1227^{\{9\}}$	$0.1381^{\{12\}}$	$0.1349^{\{11\}}$	$0.1016^{\{5\}}$	$0.2118^{\{15\}}$	$0.0337^{\{1\}}$	$0.1133^{\{8\}}$	$0.0934^{\{3\}}$	$0.0839^{\{2\}}$	$0.155^{\{14\}}$	$0.1312^{\{10\}}$	$0.146^{\{13\}}$
	D_{abs}	$0.0126^{\{3\}}$	$0.0126^{\{4\}}$	$0.0128^{\{6\}}$	$0.0132^{\{9\}}$	$0.0131^{\{7\}}$	$0.0131^{\{8\}}$	$0.0128^{\{5\}}$	$0.0146^{\{12\}}$	$0.0121^{\{1\}}$	$0.0124^{\{2\}}$	$0.0161^{\{13\}}$	$0.0138^{\{10\}}$	$0.0169^{\{15\}}$	$0.0141^{\{11\}}$	$0.0169^{\{14\}}$
	D_{max}	$0.0214^{\{4\}}$	$0.0213^{\{3\}}$	$0.0219^{\{6\}}$	$0.0227^{\{8\}}$	$0.0226^{\{7\}}$	$0.0227^{\{9\}}$	$0.0216^{\{5\}}$	$0.026^{\{12\}}$	$0.0195^{\{1\}}$	$0.0208^{\{2\}}$	$0.0271^{\{13\}}$	$0.0232^{\{10\}}$	$0.0294^{\{14\}}$	$0.0236^{\{11\}}$	$0.0296^{\{15\}}$
	$\sum\text{Ranks}$	$80^{\{6\}}$	$81^{\{7\}}$	$67^{\{3\}}$	$143^{\{11\}}$	$122^{\{8\}}$	$126^{\{10\}}$	$79^{\{5\}}$	$193^{\{15\}}$	$15^{\{1\}}$	$126^{\{10\}}$	$61^{\{2\}}$	$70^{\{4\}}$	$189^{\{14\}}$	$154^{\{12\}}$	$174^{\{13\}}$

Table 6: Partial and Full Ranks for Simulation Table 5

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\|Bias\|(\hat{\alpha})$	14	5	2	10	12	9	7	15	1	11	8	4	3	13	6
	$\|Bias\|(\hat{\beta})$	6	3	11	8	9	7	5	10	1	15	2	4	14	13	12
	$\|Bias\|(\hat{\theta})$	3	7	14	11	13	4	8	5	1	12	2	6	10	15	9
	$\|Bias\|(\hat{k})$	5	9	15	12	14	10	6	4	1	11	2	3	7	13	8
	$MSE(\hat{\alpha})$	13	7	5	10	12	8	4	14	1	11	3	2	6	15	9
	$MSE(\hat{\beta})$	8	3	14	6	10	7	4	9	1	15	2	5	12	13	11
	$MSE(\hat{\theta})$	3	4	14	11	13	8	5	9	1	12	2	10	7	15	6
	$MSE(\hat{k})$	5	7	13	10	12	11	6	4	1	14	2	3	8	15	9
	$MRE(\hat{\alpha})$	14	5	4	10	12	9	6	15	1	11	2	3	7	13	8
	$MRE(\hat{\beta})$	6	3	11	8	9	7	5	10	1	15	2	4	14	13	12
	$MRE(\hat{\theta})$	3	4	14	11	13	8	5	10	1	12	2	9	7	15	6
	$MRE(\hat{k})$	5	8	13	12	11	10	6	4	1	14	2	3	7	15	9
	D_{abs}	7	3	2	11	8	1	4	13	6	5	15	10	12	14	9
	D_{max}	7	4	5	12	9	1	2	15	3	6	14	8	10	13	11
	$\sum$Ranks	$99^{\{6\}}$	$72^{\{3\}}$	$137^{\{10\}}$	$142^{\{12\}}$	$157^{\{13\}}$	$100^{\{7\}}$	$73^{\{4\}}$	$137^{\{10\}}$	$21^{\{1\}}$	$164^{\{14\}}$	$60^{\{2\}}$	$74^{\{5\}}$	$124^{\{8\}}$	$195^{\{15\}}$	$125^{\{9\}}$
20	$\|Bias\|(\hat{\alpha})$	13	8	3	11	2	4	6	12	1	14	9	10	5	15	7
	$\|Bias\|(\hat{\beta})$	14	3	4	9	5	7	8	11	1	15	2	6	10	13	12
	$\|Bias\|(\hat{\theta})$	2	6	13	7	3	12	15	9	1	14	4	11	8	5	10
	$\|Bias\|(\hat{k})$	11	7	14	6	15	13	9	2	1	4	3	5	8	10	12
	$MSE(\hat{\alpha})$	13	6	3	12	2	5	4	11	1	14	8	10	7	15	9
	$MSE(\hat{\beta})$	15	3	4	12	5	7	6	9	1	14	2	8	10	13	11
	$MSE(\hat{\theta})$	6	4	8	5	12	10	9	11	1	7	2	3	15	13	14
	$MSE(\hat{k})$	10	4	9	6	14	15	13	7	2	3	1	5	8	12	11
	$MRE(\hat{\alpha})$	14	7	5	11	2	3	6	12	1	13	4	8	9	15	10
	$MRE(\hat{\beta})$	14	3	4	9	5	7	8	11	1	15	2	6	10	13	12
	$MRE(\hat{\theta})$	6	5	8	4	11	9	12	10	1	7	2	3	15	13	14
	$MRE(\hat{k})$	10	5	11	6	14	13	8	7	1	4	2	3	12	9	15
	D_{abs}	3	4	7	8	9	6	5	11	1	2	13	10	12	15	14
	D_{max}	7	4	8	6	9	3	5	14	1	2	11	10	13	12	15
	$\sum$Ranks	$138^{\{12\}}$	$69^{\{3\}}$	$101^{\{5\}}$	$112^{\{7\}}$	$108^{\{6\}}$	$114^{\{8\}}$	$114^{\{8\}}$	$137^{\{11\}}$	$15^{\{1\}}$	$128^{\{10\}}$	$65^{\{2\}}$	$98^{\{4\}}$	$144^{\{13\}}$	$171^{\{15\}}$	$166^{\{14\}}$
50	$\|Bias\|(\hat{\alpha})$	14	9	10	11	2	5	7	15	1	13	4	8	12	6	3
	$\|Bias\|(\hat{\beta})$	11	7	4	9	5	6	8	14	1	15	2	3	12	10	13
	$\|Bias\|(\hat{\theta})$	2	10	6	8	9	13	12	5	1	11	3	4	14	7	15
	$\|Bias\|(\hat{k})$	7	9	10	6	15	14	11	2	1	5	3	4	13	8	12
	$MSE(\hat{\alpha})$	14	11	8	10	5	4	9	15	1	13	2	7	3	12	6
	$MSE(\hat{\beta})$	12	8	4	9	5	6	7	14	1	15	2	3	11	10	13
	$MSE(\hat{\theta})$	6	9	4	7	5	8	10	11	1	13	2	3	14	12	15
	$MSE(\hat{k})$	4	8	10	7	15	14	9	6	1	5	3	2	12	13	11
	$MRE(\hat{\alpha})$	14	11	10	9	5	4	6	15	1	13	2	3	8	12	7
	$MRE(\hat{\beta})$	11	7	4	9	5	6	8	14	1	15	2	3	12	10	13
	$MRE(\hat{\theta})$	5	9	4	8	6	7	10	11	1	13	2	3	14	12	15
	$MRE(\hat{k})$	6	7	10	5	15	12	9	8	1	4	2	3	14	11	13
	D_{abs}	6	4	9	10	7	8	5	13	1	3	12	2	14	11	15
	D_{max}	6	5	10	9	8	7	4	13	1	3	12	2	15	11	14
	$\sum$Ranks	$118^{\{10\}}$	$114^{\{6\}}$	$103^{\{4\}}$	$117^{\{9\}}$	$107^{\{5\}}$	$114^{\{6\}}$	$115^{\{8\}}$	$156^{\{13\}}$	$14^{\{1\}}$	$141^{\{11\}}$	$53^{\{3\}}$	$41^{\{2\}}$	$168^{\{14\}}$	$151^{\{12\}}$	$168^{\{14\}}$
100	$\|Bias\|(\hat{\alpha})$	11	6	8	12	14	13	3	15	1	10	2	4	9	5	7
	$\|Bias\|(\hat{\beta})$	3	6	5	10	8	9	7	15	1	12	2	4	14	11	13
	$\|Bias\|(\hat{\theta})$	3	7	5	11	9	12	8	6	1	13	2	4	14	10	15
	$\|Bias\|(\hat{k})$	12	7	8	5	15	13	14	2	1	6	3	4	10	9	11
	$MSE(\hat{\alpha})$	8	5	6	10	12	11	7	15	1	4	2	3	14	9	13
	$MSE(\hat{\beta})$	3	6	5	10	7	9	8	15	1	14	2	4	13	11	12
	$MSE(\hat{\theta})$	4	6	5	10	8	9	7	11	1	13	2	3	15	12	14
	$MSE(\hat{k})$	7	5	6	9	13	12	11	10	1	3	2	4	15	8	14
	$MRE(\hat{\alpha})$	9	5	6	8	14	12	7	15	1	4	2	3	13	10	11
	$MRE(\hat{\beta})$	3	6	5	10	8	9	7	15	1	12	2	4	14	11	13
	$MRE(\hat{\theta})$	4	6	5	11	8	10	7	9	1	13	2	3	15	12	14
	$MRE(\hat{k})$	9	5	7	6	15	14	10	13	1	4	2	3	12	8	11
	D_{abs}	3	2	6	5	9	8	7	12	1	4	13	10	14	11	15
	D_{max}	3	4	5	6	11	9	7	13	1	2	12	8	14	10	15
	$\sum$Ranks	$82^{\{6\}}$	$76^{\{4\}}$	$82^{\{6\}}$	$123^{\{9\}}$	$151^{\{12\}}$	$150^{\{11\}}$	$110^{\{7\}}$	$166^{\{13\}}$	$14^{\{1\}}$	$114^{\{8\}}$	$50^{\{2\}}$	$61^{\{3\}}$	$186^{\{15\}}$	$137^{\{10\}}$	$178^{\{14\}}$
200	$\|Bias\|(\hat{\alpha})$	8	5	10	12	1	2	9	15	3	11	4	7	13	6	14
	$\|Bias\|(\hat{\beta})$	8	7	4	5	9	10	6	15	1	14	2	3	12	11	13
	$\|Bias\|(\hat{\theta})$	6	5	8	3	10	12	9	7	1	13	2	4	14	11	15
	$\|Bias\|(\hat{k})$	11	4	12	1	10	8	9	15	2	6	7	5	13	3	14
	$MSE(\hat{\alpha})$	8	4	12	14	13	11	5	15	1	6	2	3	10	7	9
	$MSE(\hat{\beta})$	9	10	4	5	6	7	8	15	1	14	2	3	11	13	12
	$MSE(\hat{\theta})$	8	5	7	6	9	10	4	15	1	12	2	3	13	11	14
	$MSE(\hat{k})$	7	2	8	11	12	10	4	15	1	3	5	9	13	6	14
	$MRE(\hat{\alpha})$	8	5	12	14	13	11	4	15	1	7	2	3	10	6	9
	$MRE(\hat{\beta})$	8	7	4	5	9	10	6	15	1	14	2	3	12	11	13
	$MRE(\hat{\theta})$	7	5	8	6	10	9	4	15	1	12	2	3	14	11	13
	$MRE(\hat{k})$	8	3	9	12	11	10	6	15	1	5	2	4	14	7	13
	D_{abs}	2	3	7	6	9	8	4	13	1	5	12	11	14	10	15
	D_{max}	2	4	9	6	8	7	5	15	3	2	12	11	13	10	14
	$\sum$Ranks	$100^{\{6\}}$	$69^{\{3\}}$	$114^{\{8\}}$	$106^{\{7\}}$	$130^{\{12\}}$	$125^{\{10\}}$	$83^{\{5\}}$	$200^{\{15\}}$	$17^{\{1\}}$	$125^{\{10\}}$	$58^{\{2\}}$	$72^{\{4\}}$	$176^{\{13\}}$	$123^{\{9\}}$	$182^{\{14\}}$
300	$\|Bias\|(\hat{\alpha})$	3	14	12	4	6	7	9	15	1	13	2	5	8	11	10
	$\|Bias\|(\hat{\beta})$	3	8	5	12	9	13	7	6	1	14	2	4	11	15	10
	$\|Bias\|(\hat{\theta})$	4	10	6	8	9	14	7	2	1	12	3	5	13	11	15
	$\|Bias\|(\hat{k})$	8	15	13	6	11	10	12	7	1	9	2	3	4	14	5
	$MSE(\hat{\alpha})$	6	5	4	14	12	13	11	15	1	8	3	2	9	10	7
	$MSE(\hat{\beta})$	3	7	4	9	8	12	6	13	1	14	2	5	11	15	10
	$MSE(\hat{\theta})$	4	7	5	9	8	13	6	12	1	10	2	3	15	11	14
	$MSE(\hat{k})$	7	10	6	14	12	11	9	15	1	8	2	3	5	13	4
	$MRE(\hat{\alpha})$	5	7	4	14	12	13	11	15	1	6	2	3	10	9	8
	$MRE(\hat{\beta})$	3	8	5	12	9	13	6	7	1	14	2	4	10	15	11
	$MRE(\hat{\theta})$	4	6	5	12	8	9	7	15	1	10	2	3	14	11	13
	$MRE(\hat{k})$	6	9	5	14	13	12	11	15	1	4	2	8	10	7	13
	D_{abs}	3	4	8	11	9	7	5	10	1	2	13	12	14	6	15
	D_{max}	3	4	6	11	10	9	5	8	1	2	13	12	14	7	15
	$\sum$Ranks	$62^{\{3\}}$	$114^{\{7\}}$	$88^{\{5\}}$	$150^{\{12\}}$	$136^{\{9\}}$	$156^{\{14\}}$	$112^{\{6\}}$	$155^{\{13\}}$	$24^{\{1\}}$	$126^{\{8\}}$	$53^{\{2\}}$	$66^{\{4\}}$	$146^{\{11\}}$	$158^{\{15\}}$	$134^{\{10\}}$
500	$\|Bias\|(\hat{\alpha})$	2	7	10	9	5	3	8	15	1	12	11	4	14	6	13
	$\|Bias\|(\hat{\beta})$	5	6	3	12	8	9	4	15	1	10	2	7	14	11	13
	$\|Bias\|(\hat{\theta})$	5	7	3	13	8	10	6	4	1	12	2	9	15	11	14
	$\|Bias\|(\hat{k})$	10	7	6	3	5	8	9	15	2	11	1	4	14	12	13
	$MSE(\hat{\alpha})$	7	5	4	14	13	12	6	15	1	8	2	3	10	11	9
	$MSE(\hat{\beta})$	3	6	7	10	8	9	4	15	1	14	2	5	13	11	12
	$MSE(\hat{\theta})$	7	6	4	11	8	9	5	15	1	10	2	3	14	12	13
	$MSE(\hat{k})$	7	5	3	9	12	10	6	15	1	8	4	2	14	11	13
	$MRE(\hat{\alpha})$	7	6	4	12	14	11	5	15	1	9	2	3	10	13	8
	$MRE(\hat{\beta})$	4	7	3	13	8	9	6	15	1	10	2	5	14	11	12
	$MRE(\hat{\theta})$	9	6	4	11	7	8	5	15	1	10	2	3	14	13	12
	$MRE(\hat{k})$	7	6	4	9	12	11	5	15	1	8	3	2	14	10	13
	D_{abs}	3	4	6	9	7	8	5	12	1	2	13	10	15	11	14
	D_{max}	4	3	6	8	7	9	5	12	1	2	13	10	14	11	15
	$\sum$Ranks	$80^{\{6\}}$	$81^{\{7\}}$	$67^{\{3\}}$	$143^{\{11\}}$	$122^{\{8\}}$	$126^{\{10\}}$	$79^{\{5\}}$	$193^{\{15\}}$	$15^{\{1\}}$	$126^{\{10\}}$	$61^{\{2\}}$	$70^{\{4\}}$	$189^{\{14\}}$	$154^{\{12\}}$	$174^{\{13\}}$

Table 7: Numerical values of Simulation using $\alpha = 0.75,\ \beta = 1.5,\ \theta = 1.75,\ k = 1.25$

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\|Bias\|(\hat{\alpha})$	6.4903[13]	0.2517[5]	0.7011[8]	0.0295[2]	0.2039[4]	1.5174[10]	0.1272[3]	7.2738[14]	0.0158[1]	0.9664[9]	0.3068[6]	0.3283[7]	5.1153[12]	3.9981[11]	9.9243[15]
	$\|Bias\|(\hat{\beta})$	13.9064[9]	2.5602[4]	3.4474[5]	28.9851[11]	3.8739[6]	8.4636[7]	11.4688[8]	53.3121[14]	0.0409[1]	35.5128[13]	0.8814[3]	0.5127[2]	19.7667[10]	34.4331[12]	57.4707[15]
	$\|Bias\|(\hat{\theta})$	389.4725[15]	2.0713[9]	3.9723[10]	11.6978[13]	4.9838[11]	0.2047[2]	1.3322[8]	6.0367[12]	0.0597[1]	15.0385[14]	0.8315[6]	0.2707[3]	0.6231[5]	1.0704[7]	0.4408[4]
	$\|Bias\|(\hat{k})$	1.1578[12]	0.7682[5]	1.2916[14]	1.4617[15]	1.0735[11]	0.6754[4]	1.0715[10]	0.993[7]	0.0367[1]	1.2598[13]	0.5881[3]	0.2269[2]	0.887[6]	1.0477[9]	1.0229[8]
	$MSE(\hat{\alpha})$	578.1864[13]	0.6719[3]	4.826[8]	0.9075[5]	0.7137[4]	64.5466[10]	0.4531[2]	255.1952[12]	0.0244[1]	18.7958[9]	2.0859[6]	2.799[7]	773.633[14]	189.6859[11]	3137.4115[15]
	$MSE(\hat{\beta})$	2453.8534[9]	67.2214[4]	100.833[5]	5592.173[11]	145.6726[6]	917.3398[7]	1383.3612[8]	38257.7541[14]	0.0366[1]	9013.6733[12]	4.7046[3]	3.7229[2]	5591.7777[10]	9794.3313[13]	60624.9315[15]
	$MSE(\hat{\theta})$	3541203.0329[15]	93.7585[9]	180.3913[10]	2385.5778[13]	243.8635[12]	8.1114[4]	61.5146[8]	235.3912[11]	0.0835[1]	7433.6847[14]	5.9168[3]	3.8391[2]	31.4528[7]	24.5134[6]	17.4302[5]
	$MSE(\hat{k})$	8.6336[8]	5.447[5]	9.7449[11]	9.1586[9]	6.9501[6]	4.0669[4]	9.7051[10]	7.2489[7]	0.0367[1]	12.022[14]	2.0533[3]	0.8327[2]	11.6379[13]	10.4803[12]	14.3844[15]
	$MRE(\hat{\alpha})$	9.0888[13]	0.7494[5]	1.4808[8]	0.6925[2]	0.6923[3]	2.5836[10]	0.7094[4]	10.2835[14]	0.1579[1]	1.964[9]	1.027[7]	0.9241[6]	7.4492[12]	5.9063[11]	13.8828[15]
	$MRE(\hat{\beta})$	10.2369[9]	1.9999[4]	2.7921[5]	19.6265[11]	2.9754[6]	5.8092[7]	7.7538[8]	36.0239[14]	0.0786[1]	24.0171[13]	0.8464[3]	0.5502[2]	13.4404[10]	23.2162[12]	38.572[15]
	$MRE(\hat{\theta})$	222.6725[15]	1.511[9]	2.5481[10]	7.1503[13]	3.0573[11]	0.8177[3]	1.3968[8]	3.747[12]	0.0994[1]	9.2923[14]	0.8254[4]	0.5382[2]	1.187[7]	1.1587[6]	0.4585[1]
	$MRE(\hat{k})$	1.2407[9]	0.8674[5]	1.3357[12]	1.3596[14]	1.0899[6]	0.8424[4]	1.1962[8]	1.3393[13]	0.1039[1]	1.3905[15]	0.7312[3]	0.4389[2]	1.1575[7]	1.2914[11]	1.2613[10]
	D_{abs}	0.0949[6]	0.0972[9]	0.0961[8]	0.0961[7]	0.1048[14]	0.0986[10]	0.0896[4]	0.0988[11]	0.0666[1]	0.0912[5]	0.1038[12]	0.0832[2]	0.1116[15]	0.0859[3]	0.1047[13]
	D_{max}	0.1667[8]	0.1557[5]	0.1589[6]	0.1741[10]	0.1811[14]	0.1675[9]	0.1597[7]	0.1804[13]	0.1[1]	0.1547[4]	0.1783[11]	0.1436[3]	0.1855[15]	0.1419[2]	0.1795[12]
	$\sum$Ranks	154[12]	81[5]	120[8]	136[10]	114[7]	91[5]	96[6]	168[15]	14[1]	158[13]	73[3]	44[2]	143[11]	126[9]	162[14]
20	$\|Bias\|(\hat{\alpha})$	1.0954[10]	0.7688[7]	2.3108[14]	0.5511[4]	1.0451[9]	2.1679[13]	1.305[11]	4.7801[15]	0.0571[1]	0.753[6]	0.3762[2]	0.4391[3]	0.6125[5]	1.9185[12]	0.9104[8]
	$\|Bias\|(\hat{\beta})$	3.7375[10]	0.6849[6]	0.419[4]	1.3718[8]	0.53[5]	1.3824[9]	1.3432[7]	33.029[15]	0.079[2]	14.667[14]	0.037[1]	0.2765[3]	11.2361[13]	6.4547[11]	10.2089[12]
	$\|Bias\|(\hat{\theta})$	1.7288[11]	0.4463[6]	0.5965[8]	0.6916[9]	1.9731[13]	0.5026[7]	1.8422[12]	1.2562[10]	0.1751[3]	7.608[15]	0.0292[2]	0.0163[1]	0.2163[5]	3.4157[14]	0.2076[4]
	$\|Bias\|(\hat{k})$	0.3833[9]	0.2529[5]	0.2899[7]	0.402[10]	0.9073[15]	0.5147[13]	0.3233[8]	0.1858[3]	0.053[1]	0.2508[6]	0.1856[4]	0.0717[2]	0.6231[14]	0.4628[12]	0.4585[11]
	$MSE(\hat{\alpha})$	6.5252[7]	2.571[2]	75.2155[13]	3.652[5]	8.4104[8]	56.4048[12]	9.6858[9]	242.4834[15]	0.017[1]	5.0361[6]	2.6374[3]	2.9949[4]	11.2165[10]	76.3867[14]	15.1691[11]
	$MSE(\hat{\beta})$	842.0694[10]	3.6532[6]	3.1784[4]	15.7723[8]	3.2696[5]	13.0162[7]	25.9073[9]	12700.3015[15]	0.0963[1]	4332.0377[14]	0.5397[2]	1.0309[3]	1476.2301[12]	1634.4866[13]	1405.298[11]
	$MSE(\hat{\theta})$	8.0829[10]	2.5786[7]	1.7939[6]	3.352[8]	25.0849[12]	4.4554[9]	62.1163[13]	23.3645[11]	0.4312[3]	2410.9747[15]	0.3633[1]	0.4144[2]	1.6143[5]	388.7744[14]	1.4227[4]
	$MSE(\hat{k})$	2.2312[8]	1.8566[6]	2.9014[7]	1.4153[5]	5.4073[15]	2.9853[11]	2.8[9]	0.9295[4]	0.0333[1]	2.8272[10]	0.6816[3]	0.4459[2]	4.5949[14]	3.9729[13]	3.4273[12]
	$MRE(\hat{\alpha})$	1.8042[9]	1.3526[5]	3.4319[14]	1.1515[4]	1.8936[10]	3.3635[13]	2.1039[11]	6.8118[15]	0.1356[1]	1.377[7]	0.8783[2]	0.9172[3]	1.3561[6]	2.9163[12]	1.6846[8]
	$MRE(\hat{\beta})$	3.5056[10]	0.9723[6]	0.9653[5]	1.5635[9]	0.9399[4]	1.2186[7]	1.4689[8]	22.6628[15]	0.1026[1]	10.419[14]	0.3175[2]	0.4118[3]	7.9053[13]	4.8818[11]	7.2181[12]
	$MRE(\hat{\theta})$	1.1108[10]	0.5947[6]	0.6032[7]	0.7889[9]	1.4033[12]	0.6873[8]	1.4429[13]	1.167[11]	0.1535[1]	4.8239[15]	0.2598[2]	0.2649[3]	0.5991[6]	2.3008[14]	0.5679[4]
	$MRE(\hat{k})$	0.6463[9]	0.5235[5]	0.6148[7]	0.5587[6]	1.0318[15]	0.8008[13]	0.7147[10]	0.5558[4]	0.0951[1]	0.6228[8]	0.3058[3]	0.3369[2]	0.9096[14]	0.7258[11]	0.782[12]
	D_{abs}	0.0581[4]	0.0571[3]	0.0614[8]	0.0643[11]	0.0612[7]	0.0604[5]	0.0565[2]	0.0626[9]	0.0626[10]	0.0541[1]	0.0689[15]	0.0674[13]	0.0644[12]	0.061[6]	0.0679[14]
	D_{max}	0.1007[5]	0.0988[3]	0.103[6]	0.1097[11]	0.1043[7]	0.1061[9]	0.1004[4]	0.1074[10]	0.0962[2]	0.095[1]	0.1099[12]	0.116[14]	0.1143[13]	0.1051[8]	0.1165[15]
	$\sum$Ranks	122[7]	72[4]	110[6]	107[5]	137[11]	136[10]	126[8]	152[14]	29[1]	139[9]	54[2]	58[3]	142[13]	165[15]	138[12]
50	$\|Bias\|(\hat{\alpha})$	0.6326[12]	0.3648[5]	0.9831[14]	0.5332[8]	0.4118[6]	0.4699[7]	0.5485[9]	1.7259[15]	0.012[1]	0.5584[10]	0.1825[3]	0.0717[2]	0.6534[13]	0.3465[4]	0.6224[11]
	$\|Bias\|(\hat{\beta})$	6.5133[11]	0.6111[8]	0.3762[7]	0.2708[4]	0.1345[3]	0.3639[6]	0.6795[9]	46.8839[15]	0.0332[1]	19.5727[14]	0.1096[2]	0.3151[5]	2.0349[10]	10.8546[13]	7.0333[12]
	$\|Bias\|(\hat{\theta})$	0.9907[15]	0.3577[9]	0.4374[11]	0.9588[14]	0.7676[13]	0.4652[12]	0.4194[10]	0.2689[8]	0.0563[5]	0.0193[2]	0.0057[1]	0.1375[7]	0.0237[3]	0.0958[6]	0.0533[4]
	$\|Bias\|(\hat{k})$	0.0556[9]	0.0796[13]	0.0249[6]	0.0102[4]	$1e-04$[1]	0.0357[8]	0.1094[11]	0.1129[12]	0.0073[3]	0.1161[14]	0.0161[5]	0.0065[2]	0.1113[10]	0.0307[7]	0.1218[15]
	$MSE(\hat{\alpha})$	0.1847[10]	0.7357[14]	0.8505[15]	0.1673[6]	1.0557[9]	0.1745[8]	1.7805[12]	25.3058[15]	0.0005[1]	0.8832[11]	0.3697[3]	0.1275[2]	0.7062[13]	1.684[7]	6.9647[12]
	$MSE(\hat{\beta})$	2359.181[12]	3.2463[7]	2.3433[5]	7.3298[9]	2.3269[4]	2.6519[6]	5.0017[8]	18036.5626[15]	0.0417[1]	6171.0041[14]	0.2608[2]	0.681[3]	35.1924[10]	4589.9158[13]	1018.5663[11]
	$MSE(\hat{\theta})$	3.1239[13]	2.0122[8]	1.8486[6]	3.1469[14]	3.8427[15]	2.5481[11]	2.4649[10]	2.601[12]	0.0514[1]	2.2887[9]	0.2297[2]	0.2801[3]	1.7954[4]	1.9004[7]	1.8181[5]
	$MSE(\hat{k})$	0.1318[3]	0.1345[4]	0.2315[7]	0.2387[8]	0.2331[6]	0.2232[6]	0.1591[5]	0.2907[12]	0.0045[1]	0.3466[15]	0.1097[2]	0.2883[11]	0.2958[13]	0.313[14]	0.2869[10]
	$MRE(\hat{\alpha})$	1.0287[10]	0.6932[5]	1.5445[14]	1.0942[11]	0.7724[6]	0.8574[7]	0.9336[8]	2.7259[15]	0.0722[1]	0.9861[9]	0.5694[3]	0.3636[2]	1.1781[13]	0.6791[4]	1.1485[12]
	$MRE(\hat{\beta})$	5.2784[12]	1.0571[7]	0.8953[4]	1.2251[9]	0.9663[5]	1.0155[6]	1.161[8]	31.9715[15]	0.0686[1]	13.5699[14]	0.2389[2]	0.3454[3]	1.9749[10]	7.7287[13]	5.2772[11]
	$MRE(\hat{\theta})$	0.7106[12]	0.6503[8]	0.5757[4]	0.8226[15]	0.7134[13]	0.6314[6]	0.6815[10]	0.814[14]	0.0742[1]	0.6865[11]	0.2005[2]	0.2301[3]	0.6487[7]	0.5805[5]	0.6588[9]
	$MRE(\hat{k})$	0.2356[3]	0.2384[4]	0.2935[10]	0.3231[12]	0.2823[9]	0.2812[8]	0.2593[6]	0.3443[15]	0.0369[1]	0.295[11]	0.2227[2]	0.2395[5]	0.337[14]	0.2679[7]	0.3336[13]
	D_{abs}	0.0365[2]	0.0375[4]	0.0392[10]	0.0384[8]	0.0389[9]	0.038[6]	0.0376[5]	0.04[7]	0.0268[1]	0.0366[3]	0.0491[15]	0.0462[12]	0.0479[13]	0.0402[11]	0.0483[14]
	D_{max}	0.0633[2]	0.0658[4]	0.0699[10]	0.0697[9]	0.0688[7]	0.0683[6]	0.0663[5]	0.0728[11]	0.0432[1]	0.0634[3]	0.0814[13]	0.0738[12]	0.0862[14]	0.0688[8]	0.0862[15]
	$\sum$Ranks	126[10]	87[4]	122[9]	132[11]	104[6]	103[5]	113[7]	182[15]	20[1]	140[12]	57[2]	72[3]	149[13]	119[8]	154[14]
100	$\|Bias\|(\hat{\alpha})$	0.2752[6]	0.3293[8]	0.4303[10]	0.3542[9]	0.5233[14]	0.463[13]	0.3035[7]	0.7359[15]	0.0196[1]	0.131[5]	0.1564[4]	0.066[2]	0.4566[12]	0.1769[5]	0.4556[11]
	$\|Bias\|(\hat{\beta})$	4.1985[12]	0.1672[6]	0.1068[4]	0.2831[9]	0.1655[5]	0.2123[7]	26.7349[15]	0.0178[2]	9.1899[13]	0.0126[1]	0.2438[8]	1.651[11]	9.3915[14]	1.1485[10]	
	$\|Bias\|(\hat{\theta})$	0.7384[14]	0.4101[10]	0.5057[12]	0.7468[15]	0.7357[13]	0.4844[11]	0.4033[9]	0.3239[8]	0.0262[4]	0.0514[5]	0.0956[6]	0.026[3]	0.0084[2]	0.2073[7]	0.0079[1]
	$\|Bias\|(\hat{k})$	0.0324[7]	0.0947[11]	0.0951[12]	0.0297[6]	0.022[4]	0.0283[5]	0.0765[9]	0.1123[13]	0.0066[2]	0.0774[10]	0.0056[1]	0.0087[3]	0.1214[15]	0.0518[8]	0.12[14]
	$MSE(\hat{\alpha})$	0.5156[7]	0.5585[8]	0.6703[10]	0.5852[9]	1.2629[12]	1.1079[11]	0.4683[6]	4.15[15]	0.0026[1]	0.2833[4]	0.3185[5]	0.1161[2]	1.6257[13]	0.2813[3]	1.6286[14]
	$MSE(\hat{\beta})$	955.1257[12]	1.795[6]	1.5715[5]	9.2874[9]	1.5179[4]	1.8706[7]	1.9157[8]	6716.6933[15]	0.011[1]	2153.1597[13]	0.2296[2]	0.9669[3]	43.9076[10]	3103.8343[14]	44.5546[11]
	$MSE(\hat{\theta})$	1.6104[11]	1.3097[9]	1.4007[10]	1.9877[13]	2.0415[14]	1.6428[12]	1.3004[8]	2.5765[15]	0.0186[1]	1.0365[6]	0.3806[3]	0.345[2]	1.0245[5]	1.2266[7]	1.0158[4]
	$MSE(\hat{k})$	0.0771[3]	0.0819[6]	0.1075[7]	0.1726[12]	0.2297[15]	0.2275[14]	0.1215[8]	0.2053[13]	0.003[1]	0.0669[2]	0.1259[9]	0.0812[5]	0.1578[10]	0.0794[4]	0.158[11]
	$MRE(\hat{\alpha})$	0.5249[6]	0.5702[8]	0.7407[10]	0.6555[9]	0.8928[14]	0.8491[13]	0.5421[7]	1.3301[15]	0.0551[1]	0.3707[3]	0.4224[4]	0.3125[2]	0.8208[11]	0.4328[5]	0.821[12]
	$MRE(\hat{\beta})$	3.6667[12]	0.7789[7]	0.7688[5]	1.1221[9]	0.7316[4]	0.7743[6]	0.8269[8]	18.7389[15]	0.0363[1]	6.5754[13]	0.1931[2]	0.3257[3]	1.5837[11]	0.8466[14]	1.5773[10]
	$MRE(\hat{\theta})$	0.5485[10]	0.5179[8]	0.5747[12]	0.6854[14]	0.625[13]	0.5673[11]	0.5386[9]	0.8431[15]	0.0429[1]	0.4768[4]	0.1923[2]	0.2089[3]	0.4864[6]	0.5001[7]	0.4824[5]
	$MRE(\hat{k})$	0.1786[3]	0.1844[6]	0.217[9]	0.2556[10]	0.2749[14]	0.2738[13]	0.1993[8]	0.2943[15]	0.0314[1]	0.1666[2]	0.1884[7]	0.1802[5]	0.2557[11]	0.1799[4]	0.2565[12]
	D_{abs}	0.0246[3]	0.0243[4]	0.0257[8]	0.023[1]	0.0262[9]	0.0252[7]	0.0246[6]	0.0293[14]	0.0238[2]	0.024[5]	0.0307[13]	0.0291[10]	0.0328[14]	0.0297[12]	0.0329[15]
	D_{max}	0.0437[6]	0.0427[4]	0.0462[8]	0.0426[3]	0.0477[9]	0.0457[7]	0.0443[5]	0.0572[13]	0.0381[1]	0.0415[2]	0.0531[12]	0.0506[10]	0.0578[14]	0.051[11]	0.0579[15]
	$\sum$Ranks	114[7]	101[5]	122[9]	128[10]	142[12]	135[11]	105[6]	193[15]	20[1]	83[4]	71[3]	61[2]	145[14]	115[8]	145[14]
200	$\|Bias\|(\hat{\alpha})$	0.2217[10]	0.1759[6]	0.4085[15]	0.1779[7]	0.2803[13]	0.2232[11]	0.1959[8]	0.3566[14]	0.0097[1]	0.0758[2]	0.0794[3]	0.0895[4]	0.2146[9]	0.1452[5]	0.2292[12]
	$\|Bias\|(\hat{\beta})$	0.4797[12]	0.0661[4]	0.1987[8]	0.2134[9]	0.0623[3]	0.0595[2]	0.068[5]	12.5495[15]	0.0026[1]	0.4757[11]	0.0911[7]	0.3875[10]	0.3985[14]	0.0822[6]	0.8561[13]
	$\|Bias\|(\hat{\theta})$	0.6754[15]	0.3932[10]	0.5037[12]	0.6112[14]	0.3955[11]	0.2572[7]	0.3228[9]	0.6055[13]	0.011[2]	0.0284[3]	0.0924[5]	0.0104[1]	0.1302[6]	0.299[8]	0.0874[4]
	$\|Bias\|(\hat{k})$	0.0654[5]	0.0758[8]	0.1253[13]	0.0411[3]	0.0749[7]	0.081[10]	0.0845[11]	0.1127[12]	0.0018[1]	0.0687[6]	0.0214[2]	0.0636[4]	0.173[15]	0.0761[9]	0.1693[14]
	$MSE(\hat{\alpha})$	0.2646[10]	0.1446[6]	0.798[14]	0.1847[8]	0.3611[13]	0.2437[9]	0.1647[7]	$1.1363e15$	$8e-04$[1]	0.0519[2]	0.0718[3]	0.0871[4]	0.2948[11]	0.111[5]	0.3037[12]
	$MSE(\hat{\beta})$	1.0474[9]	1.1375[10]	1.15[7]	2.1602[11]	1.108[4]	1.2107[8]	1.1187[5]	1619.137[15]	0.0024[1]	3.4631[12]	0.1916[2]	1.7358[9]	302.3277[14]	2.0388[10]	4.4094[13]
	$MSE(\hat{\theta})$	1.0984[12]	0.7018[6]	1.1063[13]	1.3563[14]	0.8857[11]	0.7545[7]	0.6931[5]	2.2728[15]	0.0043[1]	0.5623[4]	0.2327[2]	0.4861[3]	0.8241[10]	0.811[9]	0.7735[8]
	$MSE(\hat{k})$	0.0474[7]	0.0459[6]	0.0676[9]	0.0683[10]	0.0977[14]	0.0926[13]	0.0524[8]	0.1261[15]	0.0016[1]	0.0357[3]	0.0379[5]	0.0315[2]	0.0847[12]	0.0378[4]	0.0844[11]
	$MRE(\hat{\alpha})$	0.3706[8]	0.3374[6]	0.6448[14]	0.3583[7]	0.5037[13]	0.443[10]	0.3714[9]	0.6576[15]	0.029[1]	0.215[2]	0.2438[4]	0.2246[3]	0.4442[11]	0.2801[5]	0.4457[12]
	$MRE(\hat{\beta})$	0.5745[9]	0.5964[10]	0.6016[7]	0.819[12]	0.6158[8]	0.652[9]	0.5852[6]	9.3958[15]	0.0205[1]	0.7737[11]	0.1682[2]	0.5766[4]	2.7206[14]	0.7216[10]	1.0473[13]
	$MRE(\hat{\theta})$	0.4737[12]	0.3852[6]	0.4708[13]	0.541[14]	0.4142[8]	0.4058[7]	0.3703[5]	0.7733[15]	0.0251[1]	0.2917[3]	0.1729[2]	0.2917[3]	0.4446[11]	0.4217[9]	0.4284[10]
	$MRE(\hat{k})$	0.1353[6]	0.138[7]	0.16[9]	0.1718[10]	0.1922[12]	0.1918[11]	0.1534[8]	0.2363[15]	0.0227[1]	0.1253[3]	0.1306[5]	0.1136[2]	0.1961[14]	0.1293[4]	0.1949[13]
	D_{abs}	0.0173[3]	0.0183[4]	0.0191[8]	0.0189[7]	0.0193[9]	0.0185[5]	0.0187[6]	0.0205[10]	0.0144[1]	0.0168[2]	0.0232[13]	0.0215[11]	0.025[14]	0.0225[12]	0.0251[15]
	D_{max}	0.0307[3]	0.033[4]	0.0352[8]	0.034[7]	0.0356[9]	0.034[6]	0.0336[5]	0.0373[11]	0.0226[1]	0.03[2]	0.041[13]	0.0365[10]	0.0452[14]	0.0384[12]	0.0456[15]
	$\sum$Ranks	109[8]	85[5]	150[12]	133[10]	135[11]	115[9]	96[6]	195[15]	15[1]	79[3]	68[2]	169[14]	165[13]	106[7]	
300	$\|Bias\|(\hat{\alpha})$	0.0843[10]	0.0683[8]	0.1726[15]	0.0547[7]	0.1723[14]	0.1581[13]	0.076[9]	0.0281[4]	$2e-04$[1]	0.0152[3]	0.001[2]	0.0529[6]	0.1236[11]	0.0353[5]	0.1249[12]
	$\|Bias\|(\hat{\beta})$	0.1231[2]	0.7806[11]	0.4325[6]	0.608[8]	0.1575[3]	0.3006[5]	0.6352[9]	10.9601[15]	0.0103[1]	1.152[12]	0.2762[4]	0.5001[7]	1.3475[13]	0.7446[10]	1.3902[14]
	$\|Bias\|(\hat{\theta})$	0.2[10]	0.1285[3]	0.0365[2]	0.2538[11]	0.273[12]	0.1648[9]	0.0617[5]	0.0992[4]	0.0136[1]	0.3142[13]	0.1033[6]	0.1351[7]	0.3597[15]	0.1167[8]	0.334[14]
	$\|Bias\|(\hat{k})$	0.0359[4]	0.0911[13]	0.0911[13]	0[1]	0.0648[9]	0.0712[12]	0.0635[8]	0.0517[5]	0.0191[3]	0.0911[13]	0.0119[2]	0.0686[7]	0.0707[11]	0.0587[15]	0.0559[6]
	$MSE(\hat{\alpha})$	0.0531[8]	0.0463[6]	0.1422[12]	0.0598[9]	0.1426[13]	0.1297[11]	0.0485[7]	0.0658[10]	0.0013[1]	0.0179[2]	0.0204[3]	0.0302[5]	0.2665[14]	0.0254[4]	0.2681[15]
	$MSE(\hat{\beta})$	1.9793[7]	2.7688[9]	1.5021[5]	7.5466[14]	1.4815[4]	1.6121[6]	2.37[8]	920.7389[15]	0.0018[1]	4.4378[11]	0.7091[2]	1.4186[3]	4.4413[12]	3.4387[10]	6.1067[13]
	$MSE(\hat{\theta})$	0.604[8]	0.6025[6]	0.6031[7]	1.0117[14]	0.9165[13]	0.8458[12]	0.6078[9]	1.8438[15]	0.0025[1]	0.5606[4]	0.1455[2]	0.3488[3]	0.7465[11]	0.5789[5]	0.7338[10]
	$MSE(\hat{k})$	0.0241[3]	0.0264[6]	0.0329[9]	0.0349[12]	0.0467[11]	0.0457[10]	0.0308[8]	0.0855[15]	0.0043[1]	0.0244[4]	0.0295[7]	0.025[2]	0.0627[13]	0.0248[5]	0.0682[14]
	$MRE(\hat{\alpha})$	0.1999[6]	0.2003[7]	0.3235[11]	0.24[10]	0.3431[15]	0.3279[12]	0.2086[8]	0.2339[9]	0.0322[1]	0.1264[2]	0.1483[3]	0.1566[5]	0.3338[13]	0.1563[4]	0.3351[14]
	$MRE(\hat{\beta})$	0.5722[4]	0.9053[10]	0.7135[5]	1.0789[12]	0.7166[6]	0.7557[7]	0.8514[8]	8.085[15]	0.0144[1]	1.0346[11]	0.304[2]	0.5653[3]	1.1351[13]	0.8753[9]	1.1608[14]
	$MRE(\hat{\theta})$	0.3252[4]	0.3925[8]	0.3766[7]	0.4698[14]	0.4496[12]	0.4336[10]	0.3977[9]	0.7122[15]	0.0165[1]	0.3752[6]	0.1456[2]	0.2577[3]	0.4588[13]	0.3596[5]	0.4408[11]
	$MRE(\hat{k})$	0.0957[3]	0.1063[4]	0.1217[9]	0.1234[11]	0.1344[12]	0.1317[10]	0.1087[5]	0.1815[15]	0.0249[1]	0.1036[5]	0.1066[7]	0.0887[2]	0.1614[13]	0.0987[4]	0.1676[14]
	D_{abs}	0.0158[3]	0.0164[8]	0.0171[11]	0.0166[9]	0.0163[6]	0.0162[5]	0.0163[7]	0.0175[12]	0.013[1]	0.0158[2]	0.0206[13]	0.0167[10]	0.0236[14]	0.0161[4]	0.0237[15]
	D_{max}	0.0269[3]	0.028[6]	0.0311[11]	0.0291[8]	0.0293[9]	0.0294[10]	0.0281[7]	0.0315[12]	0.0202[1]	0.027[4]	0.0364[13]	0.028[5]	0.0423[15]	0.0269[2]	0.0419[14]
	$\sum$Ranks	75[4]	108[8]	129[9]	140[12]	139[11]	132[10]	108[6]	161[13]	16[1]	90[3]	67[2]	69[3]	185[14]	79[5]	188[15]
500	$\|Bias\|(\hat{\alpha})$	0.1434[10]	0.1295[8]	0.1549[11]	0.1771[12]	0.1897[15]	0.1787[14]	0.1387[9]	0.1778[13]	0.0103[1]	0.1059[5]	0.0764[3]	0.0643[2]	0.1239[7]	0.09[4]	0.1133[6]
	$\|Bias\|(\hat{\beta})$	0.1257[5]	0.0559[2]	0.0737[3]	0.5331[10]	0.2906[8]	0.1594[7]	0.1086[4]	6.7754[15]	0.0268[1]	1.0771[14]	0.1424[6]	0.586[11]	0.6855[12]	0.8248[13]	0.42[9]
	$\|Bias\|(\hat{\theta})$	0.3798[11]	0.2971[8]	0.3137[9]	0.7111[15]	0.5289[14]	0.4257[12]	0.3264[10]	0.429[13]	0.0369[4]	0.051[5]	0.0144[1]	0.1297[7]	0.078[6]	0.0323[3]	0.0302[2]
	$\|Bias\|(\hat{k})$	0.0702[4]	0.0735[5]	0.0824[11]	0.0459[2]	0.0771[6]	0.0804[10]	0.0772[9]	0.1308[15]	0.0155[1]	0.1047[13]	0.0608[3]	0.0763[7]	0.1179[14]	0.0764[8]	0.095[12]
	$MSE(\hat{\alpha})$	0.1067[10]	0.0935[5]	0.1208[11]	0.1722[14]	0.1546[13]	0.1494[12]	0.0952[6]	0.2869[15]	0.0023[1]	0.0614[8]	0.0144[3]	0.0386[2]	0.1004[9]	0.0697[4]	0.0988[7]
	$MSE(\hat{\beta})$	2.5669[11]	1.3921[7]	1.2188[3]	1.4912[8]	1.2939[5]	1.36[6]	1.2772[4]	644.9284[15]	0.0044[1]	12.1804[14]	0.8272[2]	2.4144[10]	3.4963[12]	9.268[13]	2.1631[9]
	$MSE(\hat{\theta})$	0.7414[6]	0.7759[9]	0.8478[11]	1.2116[14]	1.1102[13]	1.0331[12]	0.7951[10]	1.7408[15]	0.0078[1]	0.7743[8]	0.2852[2]	0.4264[3]	0.7427[7]	0.6588[5]	0.587[4]
	$MSE(\hat{k})$	0.0243[3]	0.025[4]	0.0298[8]	0.0383[10]	0.0415[12]	0.0405[11]	0.0267[6]	0.0758[15]	0.0021[1]	0.0352[9]	0.0228[2]	0.025[5]	0.045[14]	0.0298[7]	0.0432[13]
	$MRE(\hat{\alpha})$	0.2531[7]	0.2527[6]	0.3078[11]	0.3213[12]	0.3601[15]	0.3481[14]	0.261[8]	0.33[13]	0.0435[1]	0.2157[5]	0.2118[4]	0.1788[2]	0.2875[10]	0.1916[3]	0.2765[9]
	$MRE(\hat{\beta})$	0.6342[4]	0.6537[7]	0.6449[6]	0.6379[5]	0.6873[9]	0.6989[10]	0.633[3]	5.5145[15]	0.0277[1]	1.1576[14]	0.3622[2]	0.6569[8]	0.9542[12]	0.9985[13]	0.727[11]
	$MRE(\hat{\theta})$	0.3649[6]	0.406[9]	0.4245[11]	0.499[13]	0.5036[14]	0.485[12]	0.4097[10]	0.6786[15]	0.0319[1]	0.3915[7]	0.1951[2]	0.2736[3]	0.403[8]	0.356[5]	0.3353[4]
	$MRE(\hat{k})$	0.0963[3]	0.0969[4]	0.1062[8]	0.1244[10]	0.1356[12]	0.1331[11]	0.1013[4]	0.1849[15]	0.0253[1]	0.1137[9]	0.105[6]	0.1043[5]	0.1407[14]	0.1058[7]	0.1372[13]
	D_{abs}	0.0133[3]	0.0132[5]	0.0133[6]	0.0135[8]	0.0132[4]	0.0134[7]	0.0131[2]	0.0148[12]	0.0145[10]	0.0145[10]	0.013[1]	0.0148[13]	0.0139[9]	0.0171[15]	0.0147[11]
	D_{max}	0.0235[4]	0.0236[5]	0.0241[7]	0.0237[6]	0.0242[9]	0.0243[10]	0.0235[3]	0.027[13]	0.0234[2]	0.023[1]	0.0267[12]	0.0244[11]	0.0308[14]	0.0241[8]	0.0308[15]
	$\sum$Ranks	86[5]	83[3]	116[9]	139[11]	151[13]	148[12]	88[6]	199[15]	27[1]	113[8]	61[2]	84[4]	154[14]	103[7]	128[10]

Table 8: Partial and Full Ranks for Simulation Table 7

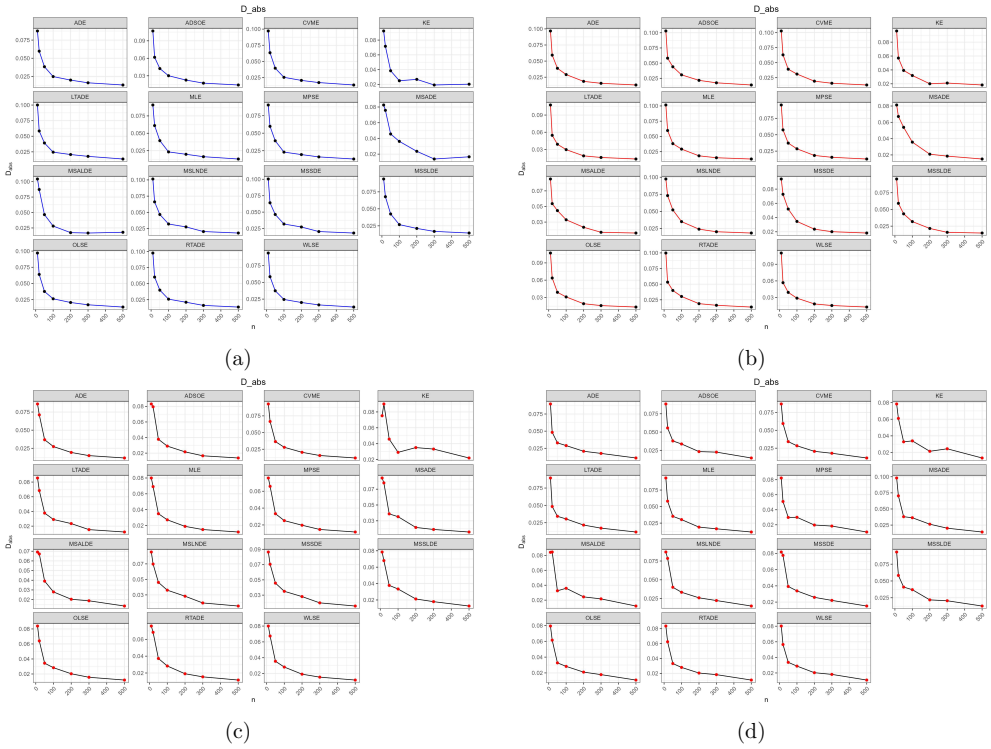
n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\|Bias\|(\hat{A})$	13	5	8	2	4	10	3	14	1	9	6	7	12	11	15
	$\|Bias\|(\hat{\beta})$	9	4	5	11	6	7	8	14	1	13	3	2	10	12	15
	$\|Bias\|(\hat{\theta})$	15	9	10	13	11	2	8	12	1	14	6	3	5	7	4
	$\|Bias\|(\hat{k})$	12	5	14	15	11	4	10	7	1	13	3	2	6	9	8
	$MSE(\hat{A})$	13	3	8	5	4	10	2	12	1	9	6	7	14	11	15
	$MSE(\hat{\beta})$	9	4	5	11	6	7	8	14	1	12	3	2	10	13	15
	$MSE(\hat{\theta})$	15	9	10	13	12	4	8	11	1	14	3	2	7	6	5
	$MSE(\hat{k})$	8	5	11	9	6	4	10	7	1	14	3	2	13	12	15
	$MRE(\hat{A})$	13	5	8	2	3	10	4	14	1	9	7	6	12	11	15
	$MRE(\hat{\beta})$	9	4	5	11	6	7	8	14	1	13	3	2	10	12	15
	$MRE(\hat{\theta})$	15	9	10	13	11	3	8	12	1	14	4	2	7	6	5
	$MRE(\hat{k})$	9	5	12	14	6	4	8	13	1	15	3	2	7	11	10
	D_{abs}	6	9	8	7	14	10	4	11	1	5	12	2	15	3	13
	D_{max}	8	5	6	10	14	9	7	13	1	4	11	3	15	2	12
	$\sum$Ranks	$154^{(12)}$	$81^{(4)}$	$120^{(8)}$	$136^{(10)}$	$114^{(7)}$	$91^{(5)}$	$96^{(6)}$	$168^{(15)}$	$14^{(1)}$	$158^{(13)}$	$73^{(3)}$	$44^{(2)}$	$143^{(11)}$	$126^{(9)}$	$162^{(14)}$
20	$\|Bias\|(\hat{A})$	10	7	14	4	9	13	11	15	1	6	2	3	5	12	8
	$\|Bias\|(\hat{\beta})$	10	6	4	8	5	9	7	15	2	14	1	3	13	11	12
	$\|Bias\|(\hat{\theta})$	11	6	8	9	13	7	12	10	3	15	2	1	5	14	4
	$\|Bias\|(\hat{k})$	9	5	7	10	15	13	8	3	1	6	4	2	14	12	11
	$MSE(\hat{A})$	7	2	13	5	8	12	9	15	1	6	3	4	10	14	11
	$MSE(\hat{\beta})$	10	6	4	8	5	7	9	15	1	14	2	3	12	13	11
	$MSE(\hat{\theta})$	10	7	6	8	12	9	13	11	3	15	1	2	5	14	4
	$MSE(\hat{k})$	8	6	7	5	15	11	9	4	1	10	3	2	14	13	12
	$MRE(\hat{A})$	9	5	14	4	10	13	11	15	1	7	2	3	6	12	8
	$MRE(\hat{\beta})$	10	6	5	9	4	7	8	15	1	14	2	3	13	11	12
	$MRE(\hat{\theta})$	10	5	7	9	12	8	13	11	1	15	2	3	6	14	4
	$MRE(\hat{k})$	9	5	7	6	15	13	10	4	1	8	3	2	14	11	12
	D_{abs}	4	3	8	11	7	5	2	9	10	1	15	13	12	6	14
	D_{max}	5	3	6	11	7	9	4	10	2	1	12	14	13	8	15
	$\sum$Ranks	$122^{(7)}$	$72^{(4)}$	$110^{(6)}$	$107^{(5)}$	$137^{(11)}$	$136^{(10)}$	$126^{(8)}$	$152^{(14)}$	$29^{(1)}$	$132^{(9)}$	$54^{(2)}$	$58^{(3)}$	$142^{(13)}$	$165^{(15)}$	$138^{(12)}$
50	$\|Bias\|(\hat{A})$	12	5	14	8	6	7	9	15	1	10	3	2	13	4	11
	$\|Bias\|(\hat{\beta})$	11	8	7	4	3	6	9	15	1	14	2	5	10	13	12
	$\|Bias\|(\hat{\theta})$	15	9	11	14	13	12	10	8	5	2	1	7	3	6	4
	$\|Bias\|(\hat{k})$	9	10	6	4	1	8	11	13	3	14	5	2	12	7	15
	$MSE(\hat{A})$	10	4	14	6	5	8	9	15	1	11	3	2	13	7	12
	$MSE(\hat{\beta})$	12	7	5	9	4	6	8	15	1	14	2	3	10	13	11
	$MSE(\hat{\theta})$	13	8	6	14	15	11	10	12	1	9	2	3	4	7	5
	$MSE(\hat{k})$	3	4	7	9	8	6	5	12	1	15	2	11	13	14	10
	$MRE(\hat{A})$	10	5	14	6	11	7	8	15	1	9	4	3	13	4	12
	$MRE(\hat{\beta})$	12	7	4	9	5	6	8	15	1	14	2	3	10	13	11
	$MRE(\hat{\theta})$	12	8	4	15	13	6	10	14	1	11	2	3	7	5	9
	$MRE(\hat{k})$	3	4	10	12	9	8	6	15	1	11	2	5	14	7	13
	D_{abs}	2	4	10	8	9	6	5	7	1	3	15	12	13	11	14
	D_{max}	2	4	10	9	7	6	5	11	1	3	13	12	14	8	15
	$\sum$Ranks	$126^{(10)}$	$87^{(4)}$	$122^{(9)}$	$132^{(11)}$	$104^{(6)}$	$103^{(5)}$	$113^{(7)}$	$182^{(15)}$	$20^{(1)}$	$140^{(12)}$	$57^{(2)}$	$72^{(3)}$	$149^{(13)}$	$119^{(8)}$	$154^{(14)}$
100	$\|Bias\|(\hat{A})$	6	8	10	9	14	13	7	15	1	3	4	2	12	5	11
	$\|Bias\|(\hat{\beta})$	12	6	4	9	3	5	7	15	2	13	1	8	11	14	10
	$\|Bias\|(\hat{\theta})$	14	10	12	15	13	11	9	8	4	5	6	3	2	7	1
	$\|Bias\|(\hat{k})$	7	11	12	6	4	5	9	13	2	10	1	3	15	8	14
	$MSE(\hat{A})$	7	8	10	9	12	11	6	15	1	4	5	2	13	3	14
	$MSE(\hat{\beta})$	12	6	5	9	4	7	8	15	1	13	2	3	10	14	11
	$MSE(\hat{\theta})$	11	9	10	13	14	12	8	15	1	6	3	2	5	7	4
	$MSE(\hat{k})$	3	6	7	12	15	14	8	13	1	2	9	5	10	4	11
	$MRE(\hat{A})$	6	8	10	9	14	13	7	15	1	3	4	2	11	5	12
	$MRE(\hat{\beta})$	12	7	5	9	4	6	8	15	1	13	2	3	11	14	10
	$MRE(\hat{\theta})$	10	8	9	12	14	13	11	9	15	1	4	2	3	6	7	5
	$MRE(\hat{k})$	3	6	9	10	14	13	8	15	1	2	7	5	11	4	12
	D_{abs}	5	4	8	3	9	7	6	11	2	3	13	10	14	12	15
	D_{max}	6	4	8	3	9	7	5	13	1	2	12	10	14	11	15
	$\sum$Ranks	$114^{(7)}$	$101^{(5)}$	$122^{(9)}$	$128^{(10)}$	$142^{(12)}$	$135^{(11)}$	$105^{(6)}$	$193^{(15)}$	$20^{(1)}$	$83^{(4)}$	$71^{(3)}$	$61^{(2)}$	$145^{(14)}$	$115^{(8)}$	$145^{(14)}$
200	$\|Bias\|(\hat{A})$	10	6	15	7	13	11	8	14	1	2	3	4	9	5	12
	$\|Bias\|(\hat{\beta})$	12	4	8	9	3	2	5	15	1	11	7	10	14	6	13
	$\|Bias\|(\hat{\theta})$	15	10	12	14	11	7	9	13	2	3	5	1	6	8	4
	$\|Bias\|(\hat{k})$	5	8	13	3	7	10	11	12	1	6	2	4	15	9	14
	$MSE(\hat{A})$	10	6	14	8	13	9	7	15	1	2	3	4	11	5	12
	$MSE(\hat{\beta})$	3	6	7	11	4	8	5	15	1	12	2	9	14	10	13
	$MSE(\hat{\theta})$	12	6	13	14	11	7	5	15	1	4	2	3	10	9	8
	$MSE(\hat{k})$	7	6	9	10	14	13	8	15	1	3	5	2	12	4	11
	$MRE(\hat{A})$	8	6	14	7	13	10	9	15	1	2	4	3	11	5	12
	$MRE(\hat{\beta})$	3	6	7	12	8	9	5	15	1	11	2	4	14	10	13
	$MRE(\hat{\theta})$	12	6	13	14	8	7	5	15	1	4	2	3	11	9	10
	$MRE(\hat{k})$	6	7	9	10	12	11	8	15	1	3	5	2	14	4	13
	D_{abs}	3	4	8	7	9	5	6	10	1	2	13	11	14	12	15
	D_{max}	3	4	8	7	9	6	5	11	1	2	13	10	14	12	15
	$\sum$Ranks	$109^{(8)}$	$85^{(5)}$	$150^{(12)}$	$133^{(10)}$	$135^{(11)}$	$115^{(9)}$	$96^{(6)}$	$195^{(15)}$	$15^{(1)}$	$67^{(2)}$	$68^{(3)}$	$70^{(4)}$	$169^{(14)}$	$108^{(7)}$	$165^{(13)}$
300	$\|Bias\|(\hat{A})$	10	8	15	7	14	13	9	4	1	3	2	6	11	5	12
	$\|Bias\|(\hat{\beta})$	2	11	6	8	3	5	9	15	1	12	4	7	13	10	14
	$\|Bias\|(\hat{\theta})$	10	7	2	11	12	9	3	4	1	13	5	8	15	6	14
	$\|Bias\|(\hat{k})$	4	10	13	1	9	12	8	5	3	11	2	7	15	6	14
	$MSE(\hat{A})$	8	6	12	9	13	11	7	10	1	2	3	5	14	4	15
	$MSE(\hat{\beta})$	7	9	5	14	4	6	8	15	1	11	2	3	12	10	13
	$MSE(\hat{\theta})$	8	6	7	14	13	12	9	15	1	4	2	3	11	5	10
	$MSE(\hat{k})$	3	6	9	12	11	10	8	15	1	4	7	2	13	5	14
	$MRE(\hat{A})$	6	7	11	10	15	12	8	9	1	2	3	5	13	4	14
	$MRE(\hat{\beta})$	4	10	5	12	6	7	8	15	1	11	2	3	13	9	14
	$MRE(\hat{\theta})$	4	8	7	14	12	10	9	15	1	6	2	3	13	5	11
	$MRE(\hat{k})$	3	6	9	11	12	10	8	15	1	5	7	2	13	4	14
	D_{abs}	3	8	11	9	6	5	7	12	1	2	13	10	14	4	15
	D_{max}	3	6	11	8	9	10	7	12	1	4	13	5	15	2	14
	$\sum$Ranks	$75^{(4)}$	$108^{(8)}$	$123^{(9)}$	$140^{(12)}$	$139^{(11)}$	$132^{(10)}$	$108^{(8)}$	$161^{(13)}$	$16^{(1)}$	$90^{(6)}$	$67^{(2)}$	$69^{(3)}$	$185^{(14)}$	$79^{(5)}$	$188^{(15)}$
500	$\|Bias\|(\hat{A})$	10	8	11	12	15	14	9	13	1	5	3	2	7	4	6
	$\|Bias\|(\hat{\beta})$	5	2	3	10	8	7	4	15	1	14	6	11	12	13	9
	$\|Bias\|(\hat{\theta})$	11	8	9	15	14	12	10	13	4	5	1	7	6	3	2
	$\|Bias\|(\hat{k})$	4	5	11	2	8	10	9	15	1	13	3	6	14	7	12
	$MSE(\hat{A})$	10	5	11	14	13	12	6	15	1	8	3	2	9	4	7
	$MSE(\hat{\beta})$	11	7	3	8	5	6	4	15	1	14	2	10	12	13	9
	$MSE(\hat{\theta})$	6	9	11	14	13	12	10	15	1	8	2	3	7	5	4
	$MSE(\hat{k})$	3	4	8	10	12	11	6	15	1	9	2	5	14	7	13
	$MRE(\hat{A})$	7	6	11	12	15	14	8	13	1	5	4	2	10	3	9
	$MRE(\hat{\beta})$	4	7	6	5	9	10	3	15	1	14	2	8	12	13	11
	$MRE(\hat{\theta})$	6	9	11	13	14	12	10	15	1	7	2	3	8	5	4
	$MRE(\hat{k})$	2	3	8	10	12	11	4	15	10	9	6	5	14	7	13
	D_{abs}	3	5	6	8	4	7	2	12	10	1	13	9	15	11	14
	D_{max}	4	5	7	6	9	10	3	13	2	1	12	11	14	8	15
	$\sum$Ranks	$86^{(5)}$	$83^{(3)}$	$116^{(9)}$	$139^{(11)}$	$151^{(13)}$	$148^{(12)}$	$88^{(6)}$	$199^{(15)}$	$27^{(1)}$	$113^{(8)}$	$61^{(2)}$	$84^{(4)}$	$154^{(14)}$	$103^{(7)}$	$128^{(10)}$

Table 9: Numerical values for the Simulation using $\alpha = 0.25,\ \beta = 1.75,\ \theta = 0.75,\ k = 2.0$

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\|Bias\|(\hat{\alpha})$	1.3085[13]	0.5637[7]	0.6737[9]	0.7761[10]	0.3801[5]	0.3935[6]	0.3311[4]	2.5599[15]	0.0169[1]	1.9771[14]	0.0996[3]	0.0658[2]	1.1635[12]	0.5898[8]	1.0568[11]
	$\|Bias\|(\hat{\beta})$	29.0235[8]	21.9014[7]	6.9288[4]	70.8303[12]	21.5293[6]	42.9601[9]	15.1006[5]	103.9649[15]	0.0121[2]	73.8065[13]	0.4153[3]	0.0028[1]	47.9275[10]	88.3714[14]	66.8623[11]
	$\|Bias\|(\hat{\theta})$	0.7754[15]	0.2072[8]	0.5111[13]	0.0356[2]	0.6147[14]	0.2224[9]	0.1801[7]	0.0396[3]	0.056[5]	0.2261[10]	0.0531[4]	0.0165[1]	0.2589[11]	0.1725[6]	0.3067[12]
	$\|Bias\|(\hat{k})$	0.093[5]	0.2566[9]	0.4291[14]	0.3293[12]	0.6994[15]	0.062[4]	0.1708[7]	0.1041[6]	0.0305[3]	0.4191[13]	0.0188[1]	0.2141[8]	0.2626[10]	0.0267[2]	0.2884[11]
	$MSE(\hat{\alpha})$	14.5769[12]	3.9758[8]	4.321[9]	14.3103[11]	1.5909[5]	1.9029[6]	1.0705[4]	95.0041[14]	0.0045[1]	131.5742[15]	0.2362[3]	0.084[2]	15.5782[13]	3.3427[7]	14.0522[10]
	$MSE(\hat{\beta})$	6262.892[8]	2842.0865[6]	596.3352[4]	18537.3127[13]	2932.3435[7]	7208.9568[10]	1179.223[5]	52820.3458[15]	0.0026[1]	15250.0991[12]	5.085[3]	0.208[2]	6378.1647[9]	31071.115[14]	15027.983[11]
	$MSE(\hat{\theta})$	2.7025[14]	1.2242[11]	1.6349[12]	0.9756[10]	4.1267[15]	0.6681[5]	0.4777[4]	0.9011[8]	0.0406[1]	0.9457[9]	0.2682[3]	0.183[2]	1.2[13]	0.8081[7]	0.7792[6]
	$MSE(\hat{k})$	1.4236[5]	2.5487[9]	4.4249[14]	2.441[8]	3.5804[11]	1.3975[4]	2.3877[7]	1.5891[6]	0.0057[1]	5.7515[15]	0.1935[2]	0.5592[3]	3.766[12]	2.9806[10]	4.071[13]
	$MRE(\hat{\alpha})$	5.4683[13]	2.7581[7]	3.1297[9]	3.6594[10]	2.0911[5]	2.1457[6]	1.7498[4]	10.6752[15]	0.1946[1]	8.4599[14]	0.7081[3]	0.4653[2]	5.3043[12]	2.7738[8]	4.8716[11]
	$MRE(\hat{\beta})$	17.5296[8]	13.0707[7]	4.4672[4]	40.8484[12]	12.861[6]	24.891[9]	9.0465[5]	59.6712[15]	0.0163[1]	42.4526[13]	0.3283[3]	0.1514[2]	27.6501[10]	50.8801[14]	38.4261[11]
	$MRE(\hat{\theta})$	1.6393[15]	1.0854[10]	1.2394[13]	1.1005[11]	1.59[14]	0.88[5]	0.7774[4]	1.0254[7]	0.1433[1]	0.9973[6]	0.4931[3]	0.4322[2]	1.0278[9]	1.2382[12]	1.0256[8]
	$MRE(\hat{k})$	0.4958[5]	0.6037[8]	0.6979[12]	0.598[7]	0.7299[14]	0.4649[4]	0.6127[9]	0.5395[6]	0.0217[1]	0.8118[15]	0.1286[2]	0.254[3]	0.6909[11]	0.6439[10]	0.7202[13]
	D_{abs}	0.0779[12]	0.0692[3]	0.0849[14]	0.0673[2]	0.0753[9]	0.0768[11]	0.076[10]	0.0745[8]	0.0448[1]	0.0695[4]	0.0741[7]	0.0718[6]	0.0875[15]	0.0706[5]	0.0821[13]
	D_{max}	0.1338[10]	0.1219[3]	0.1464[13]	0.1234[5]	0.1335[9]	0.1385[12]	0.1278[7]	0.1357[11]	0.076[1]	0.1242[6]	0.1279[8]	0.1167[2]	0.1584[14]	0.1234[4]	0.1603[15]
	$\sum$Ranks	143[10]	103[6]	144[12]	125[7]	135[9]	100[5]	82[4]	144[12]	21[1]	159[15]	48[3]	38[2]	155[13]	127[8]	156[14]
20	$\|Bias\|(\hat{\alpha})$	0.166[11]	0.0836[9]	0.1848[13]	0.4273[14]	0.068[8]	0.0112[1]	0.0167[3]	0.6121[15]	0.0218[4]	0.022[5]	0.0506[7]	0.0121[2]	0.1632[10]	0.039[6]	0.1696[12]
	$\|Bias\|(\hat{\beta})$	35.134[8]	18.8553[5]	5.4705[4]	45.1695[10]	34.7738[7]	42.1895[9]	33.2528[6]	74.4915[13]	0.0108[1]	76.3907[14]	0.0513[2]	0.0975[3]	49.8309[12]	76.7645[15]	47.1722[11]
	$\|Bias\|(\hat{\theta})$	0.2904[13]	0.2144[9]	1.1833[15]	0.2833[12]	0.5572[14]	0.1478[7]	0.2108[8]	0.0624[3]	0.0389[2]	0.0955[4]	0.2434[11]	0.0042[1]	0.1418[6]	0.225[10]	0.1395[5]
	$\|Bias\|(\hat{k})$	0.5877[8]	0.8014[12]	1.5702[15]	0.719[10]	1.446[14]	0.841[13]	0.7492[11]	0.1025[4]	0.0093[1]	0.9737[6]	0.3781[5]	0.1616[3]	0.4226[9]	0.4144[7]	
	$MSE(\hat{\alpha})$	0.7983[11]	0.3649[9]	0.7041[10]	3.1011[14]	0.2164[8]	0.0906[7]	0.0275[3]	3.9014[15]	0.0042[1]	0.0722[5]	0.0374[4]	0.0104[2]	1.1713[12]	0.0783[6]	1.2419[13]
	$MSE(\hat{\beta})$	10575.2165[10]	2603.9744[5]	271.199[4]	13548.1453[11]	13629.7038[12]	9391.3903[8]	4379.6775[6]	17367.9389[13]	0.0017[1]	18539.3069[14]	0.0924[2]	0.2156[3]	9504.7151[9]	21090.8181[15]	9076.6057[7]
	$MSE(\hat{\theta})$	0.8983[4]	2.0592[10]	38.4153[15]	2.0659[11]	4.2225[14]	0.4913[3]	2.4236[12]	1.0155[5]	0.0319[1]	1.6532[9]	1.4773[8]	0.1057[2]	1.2529[7]	3.8838[13]	1.1449[6]
	$MSE(\hat{k})$	2.0489[5]	3.5427[9]	12.337[15]	3.4742[7]	7.6967[14]	3.7217[10]	4.3873[12]	1.9513[4]	0.0046[1]	3.8103[11]	1.227[3]	0.84[2]	3.2058[6]	5.1011[13]	3.5333[8]
	$MRE(\hat{\alpha})$	1.0175[10]	0.833[8]	1.3134[13]	2.2674[14]	0.9245[9]	0.6619[7]	0.4693[4]	2.918[15]	0.2262[1]	0.5255[5]	0.4318[3]	0.2637[2]	1.1869[11]	0.61[6]	1.2362[12]
	$MRE(\hat{\beta})$	20.7398[8]	11.218[5]	3.6147[4]	26.3003[10]	20.4082[7]	24.5441[9]	19.5006[6]	43.0014[13]	0.0143[1]	43.8672[14]	0.0689[2]	0.1311[3]	28.734[12]	44.172[15]	27.221[11]
	$MRE(\hat{\theta})$	1.0452[9]	1.0257[6]	2.1693[15]	1.2699[12]	1.5168[14]	0.8229[4]	1.2508[11]	1.1764[10]	0.1688[1]	1.0201[5]	0.572[3]	0.3308[2]	1.0427[8]	1.2923[13]	1.0346[7]
	$MRE(\hat{k})$	0.5452[4]	0.6418[9]	1.0472[15]	0.711[12]	0.9402[14]	0.6052[10]	0.6339[8]	0.6068[6]	0.0174[1]	0.7237[7]	0.2417[2]	0.2498[3]	0.6909[5]	0.7435[13]	0.6575[11]
	D_{abs}	0.0721[14]	0.06[2]	0.0685[11]	0.0619[6]	0.0662[10]	0.065[9]	0.0618[5]	0.0723[15]	0.0624[7]	0.0617[4]	0.0605[3]	0.0501[1]	0.069[12]	0.0635[8]	0.0697[13]
	D_{max}	0.1162[10]	0.1044[5]	0.1167[11]	0.1134[9]	0.1171[12]	0.1131[8]	0.109[7]	0.1313[15]	0.0925[2]	0.1035[4]	0.0991[3]	0.0849[1]	0.1205[14]	0.1076[6]	0.119[13]
	$\sum$Ranks	125[8]	103[5]	160[15]	152[13]	157[14]	105[4]	102[3]	146[11]	25[1]	107[7]	56[2]	29[2]	129[9]	148[12]	136[10]
	$\|Bias\|(\hat{\alpha})$	0.1672[13]	0.1397[12]	0.1225[8]	0.0965[5]	0.2827[15]	0.2345[14]	0.1012[6]	0.1251[9]	0.0158[1]	0.0813[4]	0.0475[3]	0.0368[2]	0.1282[10]	0.1024[7]	0.129[11]
	$\|Bias\|(\hat{\beta})$	30.6092[10]	10.7464[6]	2.2997[3]	27.2519[9]	9.3323[5]	15.3188[8]	13.3998[7]	63.1123[15]	0.0145[1]	49.7782[12]	0.0294[2]	2.4792[4]	58.6611[14]	42.4285[11]	51.1358[13]
	$\|Bias\|(\hat{\theta})$	0.2006[10]	0.1577[8]	0.1554[7]	0.0428[4]	0.505[15]	0.3255[14]	0.0048[1]	0.1835[9]	0.0909[6]	0.2439[11]	0.0492[5]	0.038[3]	0.2894[13]	0.0253[2]	0.2636[12]
	$\|Bias\|(\hat{k})$	0.341[10]	0.0383[1]	0.1037[6]	0.1229[8]	0.4166[15]	0.3813[14]	0.0632[4]	0.2738[11]	0.0426[3]	0.2781[12]	0.0465[3]	0.1222[7]	0.1231[9]	0.2198[10]	0.0889[5]
	$MSE(\hat{\alpha})$	0.2912[10]	0.3593[13]	0.1831[8]	0.1774[7]	0.2135[15]	0.1343[14]	0.2138[9]	0.1638[6]	0.0041[1]	0.1075[4]	0.068[3]	0.0132[2]	0.2964[11]	0.1176[5]	0.2889[12]
	$MSE(\hat{\beta})$	4644.1164[10]	1031.8815[5]	104.0265[3]	4418.7174[9]	1472.502[6]	2779.7265[8]	1530.9578[7]	14593.1347[15]	0.0016[1]	9697.0894[11]	0.0969[2]	118.5344[4]	13426.8251[14]	10776.6403[12]	11171.6849[13]
50	$MSE(\hat{\theta})$	0.7861[13]	0.5257[11]	0.4026[6]	0.5194[10]	1.3085[15]	0.9398[14]	0.3816[5]	0.5328[12]	0.0701[1]	0.38[4]	0.1403[3]	0.0924[2]	0.4828[9]	0.4153[7]	0.4752[8]
	$MSE(\hat{k})$	0.6385[4]	0.8274[6]	1.6827[13]	0.6731[5]	2.287[15]	2.1618[14]	0.8882[8]	1.0491[10]	0.0103[1]	1.0223[9]	0.4872[3]	0.551[2]	1.3677[11]	0.8359[7]	1.4267[12]
	$MRE(\hat{\alpha})$	0.7907[10]	0.7963[11]	0.6506[6]	0.9662[12]	1.4841[15]	1.3084[14]	0.6823[7]	0.7959[9]	0.1992[1]	0.5506[2]	0.4436[3]	0.2692[2]	0.893[12]	0.6248[5]	0.8955[13]
	$MRE(\hat{\beta})$	18.2858[10]	6.7828[6]	1.8145[4]	16.0888[9]	5.9774[5]	9.4125[8]	8.0851[7]	36.4126[15]	0.0086[1]	28.6977[12]	0.0935[2]	1.4812[3]	33.7736[14]	24.7125[11]	29.4676[13]
	$MRE(\hat{\theta})$	0.9172[13]	0.6837[6]	0.5603[4]	0.7407[9]	1.0059[15]	0.9267[14]	0.6511[5]	0.8149[12]	0.1269[1]	0.6915[8]	0.3035[3]	0.291[2]	0.7831[11]	0.6886[7]	0.7652[10]
	$MRE(\hat{k})$	0.3398[4]	0.3821[7]	0.4428[10]	0.3551[5]	0.5109[14]	0.4828[12]	0.3811[6]	0.4489[13]	0.0228[1]	0.4356[9]	0.2061[2]	0.2539[3]	0.4968[13]	0.3953[8]	0.5114[15]
	D_{abs}	0.0418[9]	0.0369[2]	0.0393[5]	0.0395[7]	0.0393[4]	0.0397[6]	0.0368[1]	0.0437[12]	0.0422[10]	0.047[7]	0.0403[8]	0.0429[11]	0.0474[14]	0.0439[13]	0.0479[15]
	D_{max}	0.0775[12]	0.0679[3]	0.0732[10]	0.0705[6]	0.0731[8]	0.0732[9]	0.0673[2]	0.0812[13]	0.0645[1]	0.0712[7]	0.069[4]	0.0692[5]	0.0868[15]	0.0763[11]	0.0868[14]
	$\sum$Ranks	141[10]	97[6]	93[5]	97[6]	162[12]	163[13]	75[4]	159[11]	29[1]	114[8]	45[2]	53[3]	170[15]	116[9]	166[14]
	$\|Bias\|(\hat{\alpha})$	0.0321[14]	0.0071[5]	0.0137[9]	0.0318[13]	0.0088[7]	0.0071[6]	0.0062[4]	0.0989[15]	$9c-04$[1]	0.0252[12]	0.0013[2]	0.0147[10]	0.006[3]	0.0242[11]	0.0097[8]
	$\|Bias\|(\hat{\beta})$	6.5088[8]	4.3923[5]	2.1528[4]	12.2344[11]	5.8799[7]	5.7206[6]	9.1591[10]	8.8221[9]	0.0028[1]	21.2837[13]	0.1006[2]	0.7137[3]	30.7536[15]	14.8789[12]	29.646[14]
	$\|Bias\|(\hat{\theta})$	0.024[1]	0.0523[6]	0.0339[3]	0.1519[12]	0.0516[5]	0.1211[9]	0.1309[11]	0.1111[8]	0.0292[2]	0.2399[13]	0.0392[4]	0.0651[7]	0.3248[15]	0.1309[10]	0.3231[14]
	$\|Bias\|(\hat{k})$	0.1747[11]	0.0691[5]	0.0624[4]	0.1118[8]	0.281[14]	0.1671[10]	0.0183[3]	0.1981[13]	0.0121[1]	0.2941[15]	0.013[2]	0.1041[7]	0.0759[6]	0.1799[12]	0.1335[9]
	$MSE(\hat{\alpha})$	0.0086[15]	0.0099[8]	0.0109[11]	0.0206[14]	0.0103[10]	0.0097[6]	0.0099[7]	0.1167[13]	$4c-04$[1]	0.0077[3]	0.0037[2]	0.0079[4]	0.0126[13]	0.0102[9]	0.0125[12]
	$MSE(\hat{\beta})$	198.7723[8]	112.0717[5]	27.2415[4]	575.4598[11]	150.0472[7]	125.4929[6]	474.1689[10]	370.8688[9]	$6c-04$[1]	2738.3035[13]	0.3149[2]	4.2793[3]	4104.5517[15]	2247.2837[12]	3627.5802[14]
	$MSE(\hat{\theta})$	0.2814[9]	0.2794[7]	0.2249[4]	0.3497[14]	0.3304[13]	0.2787[6]	0.3112[12]	0.3883[15]	0.0123[1]	0.2807[8]	0.0342[2]	0.0851[3]	0.2956[10]	0.2649[5]	0.3059[11]
100	$MSE(\hat{k})$	0.403[4]	0.5371[9]	0.4299[5]	0.7903[10]	1.252[15]	0.9035[12]	0.5173[7]	0.8014[11]	0.0016[1]	0.5312[8]	0.1706[2]	0.2619[3]	1.1884[14]	0.4533[6]	1.0756[13]
	$MRE(\hat{\alpha})$	0.2651[5]	0.2811[7]	0.3058[10]	0.3311[12]	0.3077[11]	0.3002[9]	0.275[6]	0.6248[15]	0.0656[1]	0.2461[4]	0.1675[2]	0.2249[3]	0.3467[14]	0.2814[8]	0.3436[13]
	$MRE(\hat{\beta})$	4.3225[8]	3.0348[5]	1.7766[4]	7.4181[11]	4.0169[7]	3.7618[6]	5.7699[10]	5.4849[9]	0.0072[1]	12.4736[13]	0.0885[2]	0.5036[3]	17.7558[15]	8.872[12]	17.128[14]
	$MRE(\hat{\theta})$	0.5724[7]	0.5567[6]	0.4957[4]	0.6558[14]	0.6413[12]	0.5821[8]	0.6246[10]	0.6663[15]	0.0814[1]	0.5877[9]	0.1859[2]	0.2611[3]	0.6365[11]	0.552[5]	0.6429[13]
	$MRE(\hat{k})$	0.2563[5]	0.2896[7]	0.2516[4]	0.351[11]	0.3741[12]	0.3445[10]	0.29[8]	0.3743[13]	0.0098[1]	0.314[9]	0.118[2]	0.184[3]	0.4577[15]	0.2887[6]	0.4368[14]
	D_{abs}	0.0274[2]	0.0284[5]	0.0291[7]	0.0297[9]	0.0291[6]	0.028[4]	0.0297[8]	0.0341[12]	0.0221[1]	0.0279[3]	0.036[13]	0.0301[10]	0.0377[15]	0.0303[11]	0.0372[14]
	D_{max}	0.0487[3]	0.0489[4]	0.0523[7]	0.0541[11]	0.0529[9]	0.0518[6]	0.0523[8]	0.0616[13]	0.0333[1]	0.0481[2]	0.0557[12]	0.0503[5]	0.0668[15]	0.0532[10]	0.0662[14]
	$\sum$Ranks	90[6]	84[5]	80[4]	161[12]	135[11]	104[7]	114[8]	172[13]	15[1]	125[9]	51[2]	67[3]	176[14]	129[10]	177[15]
	$\|Bias\|(\hat{\alpha})$	0.0211[11]	0.0195[9]	0.0068[3]	0.0223[12]	0.0073[4]	0.0109[6]	0.0089[5]	0.1127[15]	0.0034[1]	0.0278[13]	0.0064[2]	0.0203[10]	0.0159[7]	0.0338[14]	0.016[8]
	$\|Bias\|(\hat{\beta})$	2.1962[7]	2.4573[9]	0.3361[3]	3.6312[10]	1.3678[5]	1.5374[6]	2.8911[9]	6.0711[13]	0.0039[1]	5.1212[12]	0.0062[2]	0.3516[4]	8.6007[14]	4.3849[11]	9.8494[15]
	$\|Bias\|(\hat{\theta})$	0.1324[14]	0.0661[7]	0.0913[8]	0.0969[9]	0.1336[13]	0.1038[10]	0.0072[1]	0.0972[4]	0.0095[2]	0.1051[11]	0.0169[3]	0.0427[5]	0.1234[12]	0.0485[6]	0.138[15]
	$\|Bias\|(\hat{k})$	0.0575[5]	0.1032[8]	0.0671[6]	0.0248[4]	0.2447[14]	0.2413[13]	0.0177[3]	0.3968[15]	0.0095[2]	0.2296[11]	0.0057[1]	0.092[7]	0.1539[10]	0.2313[12]	0.1462[9]
	$MSE(\hat{\alpha})$	0.0037[2]	0.004[4]	0.0046[7]	0.0079[14]	0.0058[10]	0.0055[9]	0.0045[6]	0.0444[15]	$3c-04$[1]	0.0044[5]	0.0039[3]	0.0047[8]	0.0063[11]	0.0063[12]	0.0065[13]
	$MSE(\hat{\beta})$	128.9697[9]	57.1956[7]	5.7311[4]	145.6572[10]	30.3712[5]	38.0052[6]	64.8255[8]	242.1011[12]	$1c-04$[1]	290.6204[13]	0.0804[2]	1.7109[3]	587.1316[14]	154.9219[11]	700.8876[15]
	$MSE(\hat{\theta})$	0.1544[6]	0.1497[5]	0.1497[5]	0.2626[14]	0.2405[13]	0.2071[11]	0.1854[8]	0.3423[15]	$6c-04$[1]	0.1342[4]	0.0202[2]	0.0329[3]	0.2003[10]	0.211[12]	0.1863[7]
200	$MSE(\hat{k})$	0.199[3]	0.2451[5]	0.2895[7]	0.4385[11]	0.641[14]	0.5867[13]	0.3586[9]	0.6735[15]	$5c-04$[1]	0.2829[6]	0.1314[2]	0.2118[4]	0.4191[10]	0.3077[8]	0.447[12]
	$MRE(\hat{\alpha})$	0.2031[3]	0.2082[4]	0.2168[6]	0.2769[14]	0.2475[12]	0.2391[11]	0.2264[8]	0.5831[15]	0.0593[1]	0.2234[7]	0.1673[2]	0.2159[5]	0.2318[9]	0.2699[13]	0.2331[10]
	$MRE(\hat{\beta})$	1.9519[7]	2.0842[8]	0.8351[4]	2.8545[10]	1.5276[5]	1.6039[6]	2.2959[9]	4.1668[13]	0.0024[1]	3.3038[12]	0.0628[2]	0.2888[3]	5.3355[14]	2.9356[11]	5.993[15]
	$MRE(\hat{\theta})$	0.416[5]	0.4206[6]	0.5934[14]	0.5184[13]	0.4873[8]	0.5031[10]	0.4573[9]	0.6035[15]	0.0181[1]	0.3921[4]	0.1445[2]	0.1779[3]	0.5087[12]	0.5056[11]	0.4795[7]
	$MRE(\hat{k})$	0.1838[4]	0.2032[6]	0.1946[5]	0.2795[13]	0.2893[14]	0.279[12]	0.2422[9]	0.3494[15]	0.0061[1]	0.2231[7]	0.1262[2]	0.1758[3]	0.2677[10]	0.2405[8]	0.2761[11]
	D_{abs}	0.0176[2]	0.0181[5]	0.0181[4]	0.0189[8]	0.0191[9]	0.0196[10]	0.018[3]	0.0235[13]	0.0155[1]	0.0183[6]	0.0201[11]	0.0186[7]	0.0264[15]	0.0211[12]	0.0263[14]
	D_{max}	0.0339[2]	0.0359[6]	0.0362[8]	0.0379[10]	0.0378[9]	0.0384[11]	0.0361[7]	0.0474[14]	0.0233[1]	0.0357[5]	0.0354[4]	0.0347[3]	0.0484[15]	0.0395[12]	0.047[13]
	$\sum$Ranks	80[5]	95[6]	76[3]	153[12]	140[10]	132[9]	95[6]	185[15]	16[1]	148[11]	42[2]	68[4]	163[13]	153[12]	164[14]
	$\|Bias\|(\hat{\alpha})$	0.0033[4]	0.0073[7]	0.009[10]	0.0085[9]	0.0119[14]	0.0115[13]	0.0108[11]	0.0952[15]	$6c-04$[1]	0.0023[3]	0.011[12]	0.004[5]	0.0074[8]	0.0021[2]	0.0045[6]
	$\|Bias\|(\hat{\beta})$	0.4789[4]	1.4188[7]	0.7922[5]	2.0722[12]	1.7634[9]	2.0347[11]	1.5348[8]	4.4178[13]	0.0032[1]	1.0672[6]	0.1394[2]	0.1818[3]	5.123[14]	1.9564[10]	5.1407[15]
	$\|Bias\|(\hat{\theta})$	0.0538[9]	0.0269[5]	0.0112[2]	0.084[13]	0.0319[6]	0.0666[11]	0.0205[4]	0.0179[3]	0.0105[1]	0.0797[12]	0.0568[10]	0.0381[7]	0.1167[14]	0.0508[8]	0.1367[15]
	$\|Bias\|(\hat{k})$	0.0531[5]	0.0556[6]	0.0922[12]	0.0603[7]	0.1204[14]	0.0828[10]	0.0696[8]	0.2212[15]	$6c-04$[1]	0.0234[2]	0.0863[11]	0.0381[3]	0.0955[13]	0.0423[4]	0.0719[9]
	$MSE(\hat{\alpha})$	0.0018[3]	0.0021[7]	0.0021[6]	0.0042[14]	0.0031[11]	0.0029[10]	0.0019[4]	0.0869[15]	$1c-04$[1]	0.0019[5]	0.0017[2]	0.0028[9]	0.0035[13]	0.0025[8]	0.0034[12]
	$MSE(\hat{\beta})$	5.6261[4]	20.4252[7]	7.2868[5]	24.2066[10]	23.1179[9]	29.433[11]	21.5114[8]	137.3398[13]	$1c-04$[1]	8.3399[6]	0.4393[2]	0.5342[3]	226.103[14]	33.97[12]	235.6703[15]
	$MSE(\hat{\theta})$	0.1205[5]	0.1287[6]	0.1348[7]	0.1492[11]	0.158[13]	0.1467[10]	0.1437[9]	0.3117[15]	0.0012[1]	0.0869[4]	0.0183[2]	0.0312[3]	0.1718[14]	0.1408[8]	0.1516[12]
300	$MSE(\hat{k})$	0.1262[4]	0.1913[7]	0.2092[8]	0.2536[10]	0.3559[14]	0.3048[13]	0.2231[9]	0.5979[15]	$1c-04$[1]	0.1252[3]	0.1316[5]	0.1174[2]	0.2881[11]	0.1791[6]	0.3015[12]
	$MRE(\hat{\alpha})$	0.1323[3]	0.1563[7]	0.1564[8]	0.2147[14]	0.1841[11]	0.1792[10]	0.1438[5]	0.5607[15]	0.0347[1]	0.1386[4]	0.1288[2]	0.157[9]	0.1909[13]	0.1481[6]	0.186[12]
	$MRE(\hat{\beta})$	0.7761[4]	1.2411[7]	0.9712[6]	1.5435[11]	1.4655[9]	1.5547[12]	1.3691[8]	3.0107[13]	0.0019[1]	0.8594[5]	0.1088[2]	0.174[3]	3.2944[15]	1.477[10]	3.1867[14]
	$MRE(\hat{\theta})$	0.3649[5]	0.381[6]	0.3935[8]	0.4246[12]	0.4262[13]	0.4117[10]	0.4231[11]	0.5923[15]	0.0169[1]	0.2982[4]	0.1378[2]	0.1745[3]	0.4534[14]	0.3928[7]	0.3997[9]
	$MRE(\hat{k})$	0.1375[5]	0.1708[8]	0.1966[10]	0.2098[10]	0.2312[14]	0.2122[13]	0.1755[9]	0.3078[15]	0.0021[1]	0.1355[4]	0.1108[2]	0.1237[3]	0.2139[11]	0.1646[7]	0.218[12]
	D_{abs}	0.0153[4]	0.0155[6]	0.0151[3]	0.0161[9]	0.0158[8]	0.0157[7]	0.0154[5]	0.0198[13]	0.0087[1]	0.015[2]	0.0181[12]	0.0165[11]	0.0224[15]	0.0165[10]	0.0223[14]
	D_{max}	0.0284[3]	0.0292[6]	0.0287[4]	0.0309[12]	0.0302[9]	0.0301[7]	0.0288[5]	0.0382[13]	0.0128[1]	0.0278[2]	0.0308[11]	0.0307[10]	0.0414[15]	0.0301[8]	0.0414[14]
	$\sum$Ranks	62[2]	92[7]	90[6]	154[12]	154[12]	148[10]	104[5]	188[15]	14[1]	62[3]	77[5]	74[4]	184[14]	166[9]	171[13]
	$\|Bias\|(\hat{\alpha})$	0.001[3]	0.0109[12]	$8c-04$[1]	0.0127[13]	0.003[5]	0.0014[4]	0.0032[6]	0.0109[11]	0.0036[7]	0.0103[10]	$9c-04$[2]	0.0046[8]	0.0151[15]	0.0077[9]	0.0148[14]
	$\|Bias\|(\hat{\beta})$	0.1219[5]	1.232[11]	0.0816[3]	1.7781[13]	0.9249[8]	0.6737[6]	1.0266[9]	0.784[7]	0[1]	1.1926[10]	0.0101[2]	0.0924[4]	2.3483[15]	1.5507[12]	2.3242[14]
	$\|Bias\|(\hat{\theta})$	0.0479[8]	0.0542[10]	0.0875[11]	0.0529[9]	0.015[5]	$4c-04$[2]	0.037[7]	0.0131[4]	$1c-04$[1]	0.0983[13]	0.0081[3]	0.0202[6]	0.1411[14]	0.091[12]	0.1412[15]
	$\|Bias\|(\hat{k})$	0.0239[3]	0.0986[12]	0.0719[10]	0.0635[9]	0.0592[8]	0.0457[7]	0.0214[2]	0.0443[6]	0.0011[1]	0.1172[13]	0.032[5]	0.0299[4]	0.1594[15]	0.0852[11]	0.1575[14]
	$MSE(\hat{\alpha})$	$7c-04$[2]	0.0014[7]	0.0014[4]	0.0028[14]	0.0018[10]	0.0015[9]	0.0013[3]	0.0094[15]	$1c-04$[1]	0.0013[6]	0.0011[5]	0.0011[6]	0.0031[15]	0.0022[12]	0.0023[13]
	$MSE(\hat{\beta})$	2.575[4]	9.1915[10]	2.8338[5]	18.1937[13]	12.9239[11]	4.7692[6]	7.6882[8]	6.1637[7]	$1c-04$[1]	8.1511[9]	0.0555[2]	0.1008[3]	36.6883[15]	17.8199[12]	36.1875[14]
	$MSE(\hat{\theta})$	0.0395[4]	0.0846[10]	0.0777[7]	0.132[14]	0.1024[13]	0.0921[12]	0.0863[11]	0.1683[15]	0.001[1]	0.0493[5]	0.0108[3]	0.0075[2]	0.082[9]	0.0687[6]	0.0815[8]
500	$MSE(\hat{k})$	0.0537[3]	0.12[8]	0.092[7]	0.286[15]	0.1752[13]	0.1743[12]	0.1377[9]	0.215[14]	$1c-04$[1]	0.0903[6]	0.0557[4]	0.0383[2]	0.1661[11]	0.0899[5]	0.1606[10]
	$MRE(\hat{\alpha})$	0.0868[2]	0.1111[8]	0.1041[7]	0.1731[14]	0.1286[10]	0.1357[11]	0.1174[9]	0.1677[15]	0.0333[1]	0.1034[6]	0.097[4]	0.0941[3]	0.1385[12]	0.1001[5]	0.1405[13]
	$MRE(\hat{\beta})$	0.482[4]	1.0458[11]	0.6301[5]	1.4309[13]	0.975[9]	0.8013[6]	0.9816[10]	0.9268[8]	0.0028[1]	0.8051[7]	0.0437[2]	0.0664[3]	1.5056[15]	1.0566[12]	1.4691[14]
	$MRE(\hat{\theta})$	0.2008[4]	0.3282[11]	0.297[9]	0.4014[14]	0.3444[13]	0.3258[10]	0.3306[12]	0.4295[15]	0.0244[1]	0.2091[5]	0.1026[3]	0.0922[2]	0.2819[8]	0.2641[6]	0.274[7]
	$MRE(\hat{k})$	0.0902[4]	0.1418[8]	0.1183[7]	0.2158[15]	0.1752[13]	0.1744[12]	0.1557[9]	0.1873[14]	0.0035[1]	0.1122[6]	0.0728[3]	0.0686[2]	0.161[11]	0.1096[5]	0.1564[10]
	D_{abs}	0.0129[9]	0.0126[7]	0.0125[6]	0.0129[8]	0.0123[3]	0.0123[4]	0.0123[5]	0.0155[14]	0.0089[1]	0.0121[2]	0.0151[12]	0.0141[11]	0.0154[13]	0.0135[10]	0.0155[15]
	D_{max}	0.0225[8]	0.0219[4]	0.0219[3]	0.0236[10]	0.0225[7]	0.0222[6]	0.022[5]	0.0288[15]	0.0134[1]	0.0211[2]	0.0262[12]	0.0236[9]	0.0273[13]	0.0238[11]	0.0275[14]
	$\sum$Ranks	63[2]	129[11]	85[5]	174[13]	128[10]	109[7]	111[8]	160[12]	20[1]	97[6]	63[2]	64[4]	178[15]	124[9]	175[14]

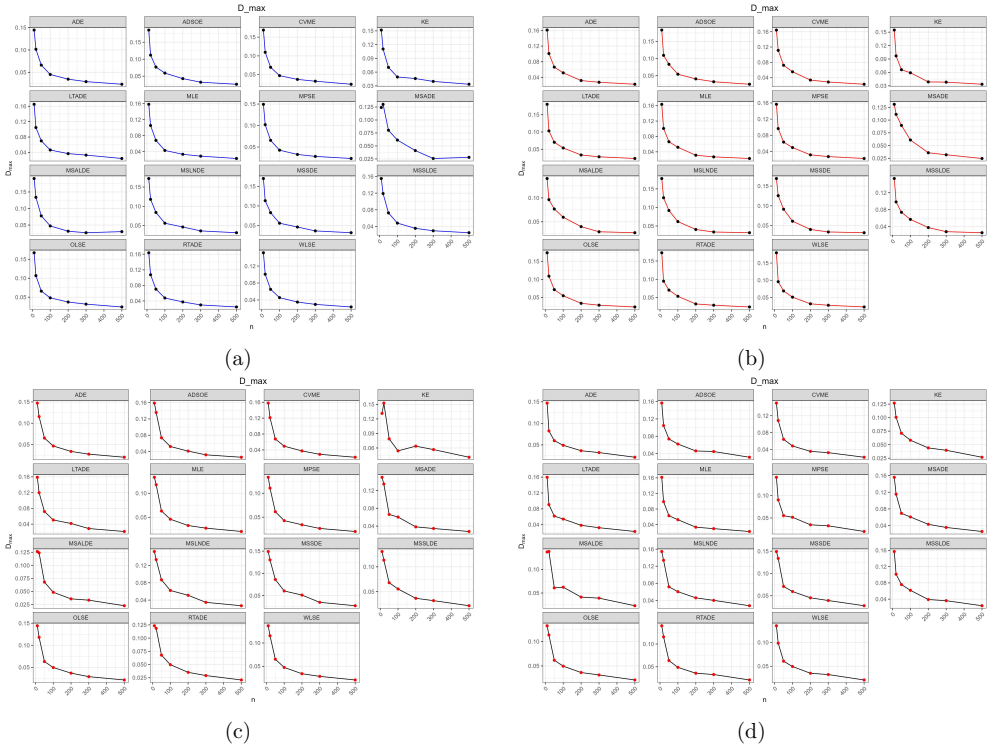
Table 10: Partial and Full Ranks for simulation Table 9

n	Est.	MLE	ADE	RTADE	LTADE	CVME	OLSE	WLSE	ADSOE	KE	MPSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
10	$\|Bias\|(\hat{A})$	13	7	9	10	5	6	4	15	1	14	3	2	12	8	11
	$\|Bias\|(\hat{\beta})$	8	7	4	12	6	9	5	15	2	13	3	1	10	14	11
	$\|Bias\|(\hat{\theta})$	15	8	13	2	14	9	7	3	5	10	4	1	11	6	12
	$\|Bias\|(\hat{k})$	5	9	14	12	15	4	7	6	3	13	1	8	10	2	11
	$MSE(\hat{A})$	12	8	9	11	5	6	4	14	1	15	3	2	13	7	10
	$MSE(\hat{\beta})$	8	6	4	13	7	10	5	15	1	12	3	2	9	14	11
	$MSE(\hat{\theta})$	14	11	12	10	15	5	4	8	1	9	3	2	7	13	6
	$MSE(\hat{k})$	5	9	14	8	11	4	7	6	1	15	2	3	12	10	13
	$MRE(\hat{A})$	13	7	9	10	5	6	4	15	1	14	3	2	12	8	11
	$MRE(\hat{\beta})$	8	7	4	12	6	9	5	15	1	13	3	2	10	14	11
	$MRE(\hat{\theta})$	15	10	13	11	14	5	4	7	1	6	3	2	9	12	8
	$MRE(\hat{k})$	5	8	12	7	14	4	9	6	1	15	2	3	11	10	13
	D_{abs}	12	3	14	2	9	11	10	8	1	4	7	6	15	5	13
	D_{max}	10	3	13	5	9	12	7	11	1	6	8	2	14	4	15
	$\sum$Ranks	$143^{\{10\}}$	$103^{\{6\}}$	$144^{\{12\}}$	$125^{\{7\}}$	$135^{\{9\}}$	$100^{\{5\}}$	$82^{\{4\}}$	$144^{\{12\}}$	$21^{\{1\}}$	$159^{\{15\}}$	$48^{\{3\}}$	$38^{\{2\}}$	$155^{\{13\}}$	$127^{\{8\}}$	$156^{\{14\}}$
20	$\|Bias\|(\hat{A})$	11	9	13	14	8	1	3	15	4	5	7	2	10	6	12
	$\|Bias\|(\hat{\beta})$	8	5	4	10	7	9	6	13	1	14	2	3	12	15	11
	$\|Bias\|(\hat{\theta})$	13	9	15	12	14	7	8	3	2	4	11	1	6	10	5
	$\|Bias\|(\hat{k})$	8	12	15	10	14	13	11	4	1	6	3	2	5	9	7
	$MSE(\hat{A})$	11	9	10	14	8	7	3	15	1	5	4	2	12	6	13
	$MSE(\hat{\beta})$	10	5	4	11	12	8	6	13	1	14	2	3	9	15	7
	$MSE(\hat{\theta})$	4	10	15	11	14	3	12	5	1	9	8	2	7	13	6
	$MSE(\hat{k})$	5	9	15	7	14	10	12	4	1	11	3	2	6	13	8
	$MRE(\hat{A})$	10	8	13	14	9	7	4	15	1	5	3	2	11	6	12
	$MRE(\hat{\beta})$	8	5	4	10	7	9	6	13	1	14	2	3	12	15	11
	$MRE(\hat{\theta})$	9	6	15	12	14	4	11	10	1	5	3	2	8	13	7
	$MRE(\hat{k})$	4	9	15	12	14	10	8	6	1	7	2	3	5	13	11
	D_{abs}	14	2	11	6	10	9	5	15	7	4	3	1	12	8	13
	D_{max}	10	5	11	9	12	8	7	15	2	4	3	1	14	6	13
	$\sum$Ranks	$125^{\{8\}}$	$103^{\{5\}}$	$160^{\{15\}}$	$152^{\{13\}}$	$157^{\{14\}}$	$105^{\{6\}}$	$102^{\{4\}}$	$146^{\{11\}}$	$25^{\{1\}}$	$107^{\{7\}}$	$56^{\{3\}}$	$29^{\{2\}}$	$129^{\{9\}}$	$148^{\{12\}}$	$136^{\{10\}}$
50	$\|Bias\|(\hat{A})$	13	12	8	5	15	14	6	9	1	4	3	2	10	7	11
	$\|Bias\|(\hat{\beta})$	10	6	3	9	5	8	7	15	1	12	2	4	14	11	13
	$\|Bias\|(\hat{\theta})$	10	8	7	4	15	14	1	9	6	11	5	3	13	2	12
	$\|Bias\|(\hat{k})$	13	1	6	8	15	14	4	11	2	12	3	7	9	10	5
	$MSE(\hat{A})$	10	13	8	7	15	14	9	6	1	4	3	2	11	5	12
	$MSE(\hat{\beta})$	10	5	3	9	6	8	7	15	1	11	2	4	14	12	13
	$MSE(\hat{\theta})$	13	11	6	10	15	14	5	12	1	4	3	2	9	7	8
	$MSE(\hat{k})$	4	6	13	5	15	14	8	10	1	9	2	3	11	7	12
	$MRE(\hat{A})$	10	11	8	6	15	14	7	9	1	4	3	2	12	5	13
	$MRE(\hat{\beta})$	10	6	4	9	5	8	7	15	1	12	2	3	14	11	13
	$MRE(\hat{\theta})$	13	6	4	9	15	14	5	12	1	8	3	2	11	7	10
	$MRE(\hat{k})$	4	7	10	5	14	12	6	11	1	9	2	3	13	8	15
	D_{abs}	9	2	3	5	4	6	1	12	10	7	8	11	14	13	15
	D_{max}	12	3	10	6	8	9	2	13	1	7	4	5	15	11	14
	$\sum$Ranks	$141^{\{10\}}$	$97^{\{6\}}$	$93^{\{5\}}$	$97^{\{6\}}$	$162^{\{12\}}$	$163^{\{13\}}$	$75^{\{4\}}$	$159^{\{11\}}$	$29^{\{1\}}$	$114^{\{8\}}$	$45^{\{2\}}$	$53^{\{3\}}$	$170^{\{15\}}$	$116^{\{9\}}$	$166^{\{14\}}$
100	$\|Bias\|(\hat{A})$	14	5	9	13	7	6	4	15	1	12	3	10	3	11	8
	$\|Bias\|(\hat{\beta})$	8	5	4	11	7	6	10	9	1	13	2	3	15	12	14
	$\|Bias\|(\hat{\theta})$	1	6	3	12	5	9	11	8	2	13	4	7	15	10	14
	$\|Bias\|(\hat{k})$	11	5	4	8	14	10	3	13	1	15	2	7	6	12	9
	$MSE(\hat{A})$	5	8	11	14	10	6	7	15	1	3	2	4	13	9	12
	$MSE(\hat{\beta})$	8	5	4	11	7	6	10	9	1	13	2	3	15	12	14
	$MSE(\hat{\theta})$	9	7	4	14	13	6	12	15	1	8	2	3	10	5	11
	$MSE(\hat{k})$	4	9	5	10	15	12	7	11	1	8	2	3	14	6	13
	$MRE(\hat{A})$	5	7	10	12	11	9	6	15	1	4	2	3	14	8	13
	$MRE(\hat{\beta})$	8	5	4	11	7	6	10	9	1	13	2	3	15	12	14
	$MRE(\hat{\theta})$	7	6	4	14	12	8	10	15	1	9	2	3	11	5	13
	$MRE(\hat{k})$	5	7	4	11	12	10	8	13	1	9	2	3	15	6	14
	D_{abs}	2	5	7	9	6	4	8	12	1	3	13	10	15	11	14
	D_{max}	3	4	7	11	9	6	8	13	1	2	12	5	15	10	14
	$\sum$Ranks	$90^{\{6\}}$	$84^{\{5\}}$	$80^{\{4\}}$	$161^{\{12\}}$	$135^{\{11\}}$	$104^{\{7\}}$	$114^{\{8\}}$	$172^{\{13\}}$	$15^{\{1\}}$	$125^{\{9\}}$	$51^{\{2\}}$	$67^{\{3\}}$	$176^{\{14\}}$	$129^{\{10\}}$	$177^{\{15\}}$
200	$\|Bias\|(\hat{A})$	11	9	3	12	4	6	5	15	1	13	2	10	7	14	8
	$\|Bias\|(\hat{\beta})$	7	8	3	10	5	6	9	13	1	12	2	4	14	11	15
	$\|Bias\|(\hat{\theta})$	14	7	8	9	13	10	1	4	2	11	3	5	12	6	15
	$\|Bias\|(\hat{k})$	5	8	6	4	14	13	3	15	2	11	1	7	10	12	9
	$MSE(\hat{A})$	2	4	7	14	10	9	6	15	1	5	3	8	11	12	13
	$MSE(\hat{\beta})$	9	7	4	10	5	6	8	12	1	13	2	3	14	11	15
	$MSE(\hat{\theta})$	6	9	5	14	13	11	8	15	1	4	2	3	10	12	7
	$MSE(\hat{k})$	3	5	7	11	14	13	9	15	1	6	2	4	10	8	12
	$MRE(\hat{A})$	3	4	6	14	12	11	8	15	1	7	2	5	9	13	10
	$MRE(\hat{\beta})$	7	8	4	10	5	6	9	13	1	12	2	3	14	11	15
	$MRE(\hat{\theta})$	5	9	6	14	13	8	10	15	1	4	2	3	12	11	7
	$MRE(\hat{k})$	4	6	5	13	14	12	9	15	1	7	2	3	10	8	11
	D_{abs}	2	5	4	8	9	10	3	13	1	6	11	7	15	12	14
	D_{max}	2	6	8	10	9	11	7	14	1	5	4	3	15	12	13
	$\sum$Ranks	$80^{\{5\}}$	$95^{\{6\}}$	$76^{\{4\}}$	$153^{\{12\}}$	$140^{\{10\}}$	$132^{\{9\}}$	$95^{\{6\}}$	$189^{\{15\}}$	$16^{\{1\}}$	$116^{\{8\}}$	$40^{\{2\}}$	$68^{\{3\}}$	$163^{\{13\}}$	$153^{\{12\}}$	$194^{\{14\}}$
300	$\|Bias\|(\hat{A})$	4	7	10	9	14	13	11	15	1	3	12	5	8	2	6
	$\|Bias\|(\hat{\beta})$	4	7	5	12	9	11	8	13	1	6	2	3	14	10	15
	$\|Bias\|(\hat{\theta})$	9	5	2	13	6	11	4	3	1	12	10	7	14	8	15
	$\|Bias\|(\hat{k})$	5	6	12	7	14	10	8	15	1	2	11	3	4	9	13
	$MSE(\hat{A})$	3	7	6	14	11	10	4	15	1	5	2	9	13	8	12
	$MSE(\hat{\beta})$	4	7	5	10	9	11	8	13	1	6	2	3	14	12	15
	$MSE(\hat{\theta})$	5	6	7	11	13	10	9	15	1	4	2	3	14	8	12
	$MSE(\hat{k})$	4	7	8	10	14	13	9	15	1	3	5	2	11	6	12
	$MRE(\hat{A})$	3	7	8	14	11	10	5	15	1	4	2	9	13	6	12
	$MRE(\hat{\beta})$	4	7	6	11	9	12	8	13	1	5	2	3	15	10	14
	$MRE(\hat{\theta})$	5	6	8	12	13	10	11	15	1	4	2	3	14	7	9
	$MRE(\hat{k})$	5	8	6	10	14	13	9	15	1	4	2	3	11	7	12
	D_{abs}	4	6	3	9	8	7	5	13	1	2	12	11	15	10	14
	D_{max}	3	6	4	12	9	7	5	13	1	2	11	10	15	8	14
	$\sum$Ranks	$62^{\{2\}}$	$92^{\{7\}}$	$90^{\{6\}}$	$154^{\{12\}}$	$154^{\{12\}}$	$148^{\{10\}}$	$104^{\{8\}}$	$188^{\{15\}}$	$14^{\{1\}}$	$62^{\{2\}}$	$77^{\{5\}}$	$74^{\{4\}}$	$184^{\{14\}}$	$106^{\{9\}}$	$171^{\{13\}}$
500	$\|Bias\|(\hat{A})$	3	12	1	13	5	4	6	11	7	10	2	8	15	9	14
	$\|Bias\|(\hat{\beta})$	5	11	3	13	8	6	9	7	1	10	2	4	15	12	14
	$\|Bias\|(\hat{\theta})$	8	10	11	9	5	2	7	4	1	13	3	6	14	12	15
	$\|Bias\|(\hat{k})$	3	12	10	9	8	7	2	6	1	13	5	4	15	11	14
	$MSE(\hat{A})$	2	7	4	14	10	11	9	15	1	3	6	5	12	8	13
	$MSE(\hat{\beta})$	4	10	5	13	11	6	8	7	1	9	2	3	15	12	14
	$MSE(\hat{\theta})$	4	10	7	14	13	12	11	15	1	5	3	2	9	6	8
	$MSE(\hat{k})$	3	8	7	15	13	12	9	14	1	6	4	2	11	5	10
	$MRE(\hat{A})$	2	8	7	14	10	11	9	15	1	6	4	3	12	5	13
	$MRE(\hat{\beta})$	4	11	5	13	9	6	10	8	1	7	2	3	15	12	14
	$MRE(\hat{\theta})$	4	11	9	14	13	10	12	15	1	5	3	2	8	6	7
	$MRE(\hat{k})$	4	8	7	15	13	12	9	14	1	6	3	2	11	5	10
	D_{abs}	9	7	6	8	3	4	5	14	1	2	12	11	13	10	15
	D_{max}	8	4	3	10	7	6	5	15	1	2	12	9	13	11	14
	$\sum$Ranks	$63^{\{2\}}$	$129^{\{11\}}$	$85^{\{5\}}$	$174^{\{13\}}$	$128^{\{10\}}$	$109^{\{7\}}$	$111^{\{8\}}$	$160^{\{12\}}$	$20^{\{1\}}$	$97^{\{6\}}$	$63^{\{2\}}$	$64^{\{4\}}$	$178^{\{15\}}$	$124^{\{9\}}$	$175^{\{14\}}$



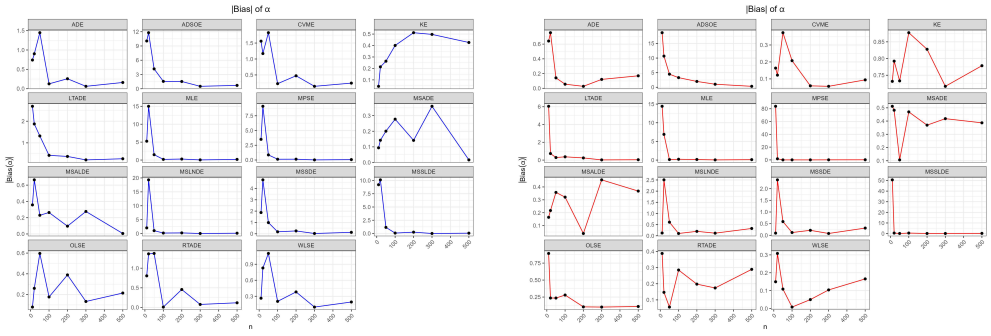
(a) (b) (c) (d)

Figure 3: D_{abs} for simulation Tables 3, 5, 7 and 9

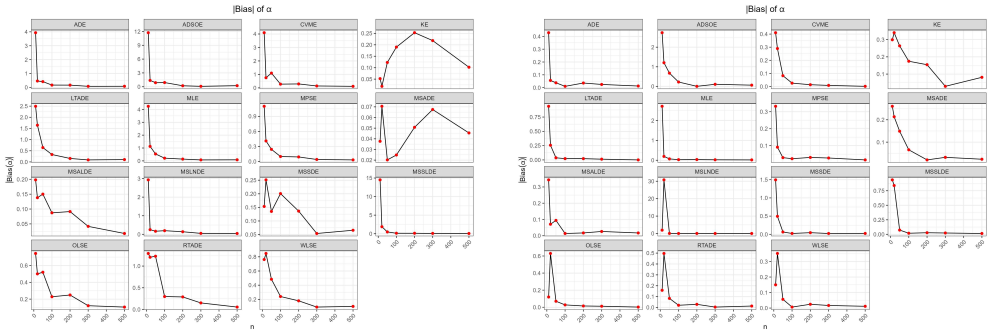


(a) (b) (c) (d)

Figure 4: D_{max} for simulation Tables 3, 5, 7 and 9

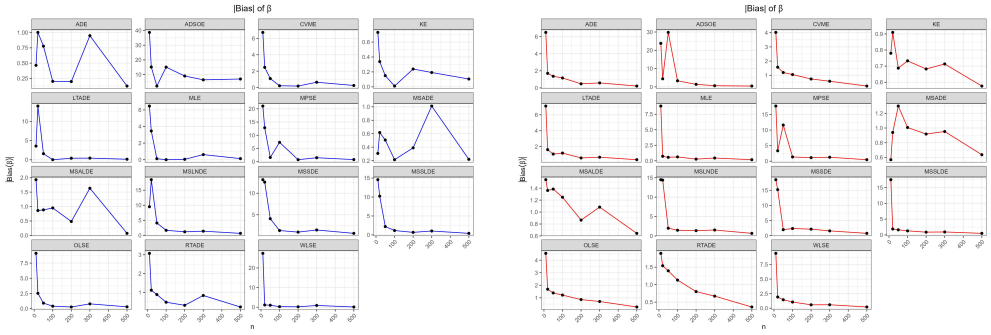


(a) (b)

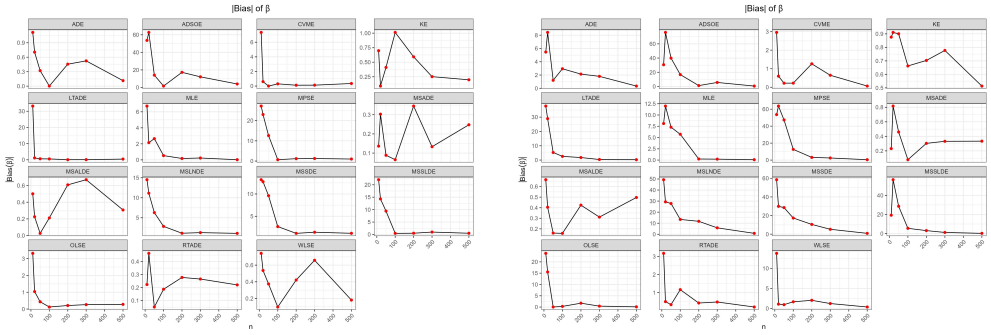


(c) (d)

Figure 5: |Bias| for α in simulation Tables 3, 5, 7 and 9



(a) (b)



(c) (d)

Figure 6: |Bias| for β in simulation Tables 3, 5, 7 and 9

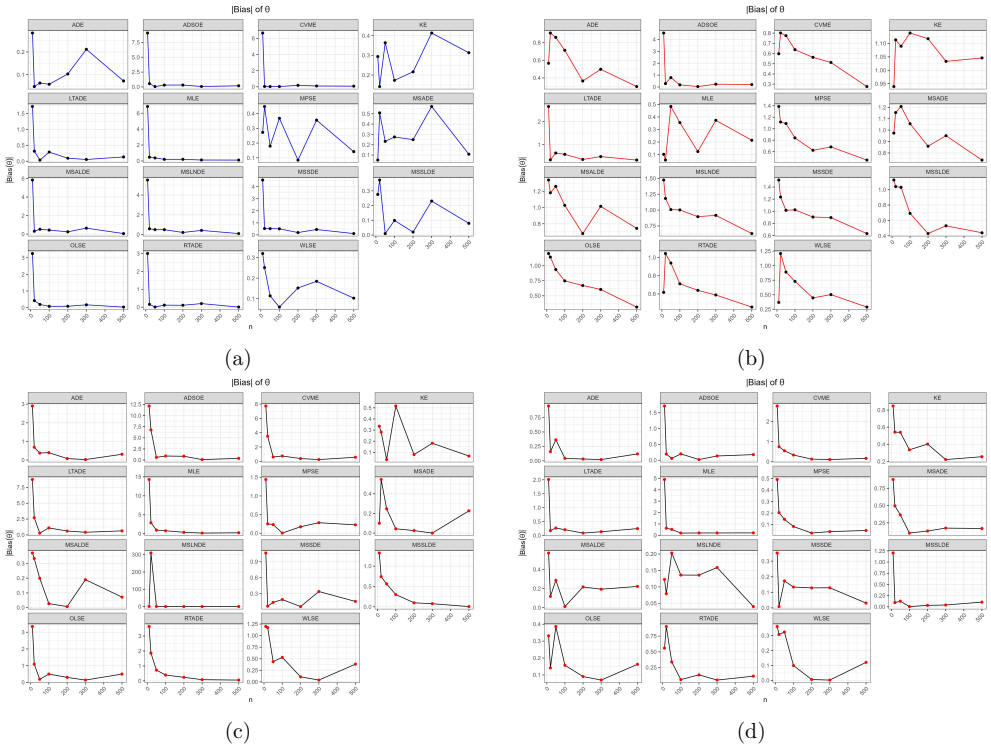


Figure 7: |Bias| for θ in simulation Tables 3, 5, 7 and 9

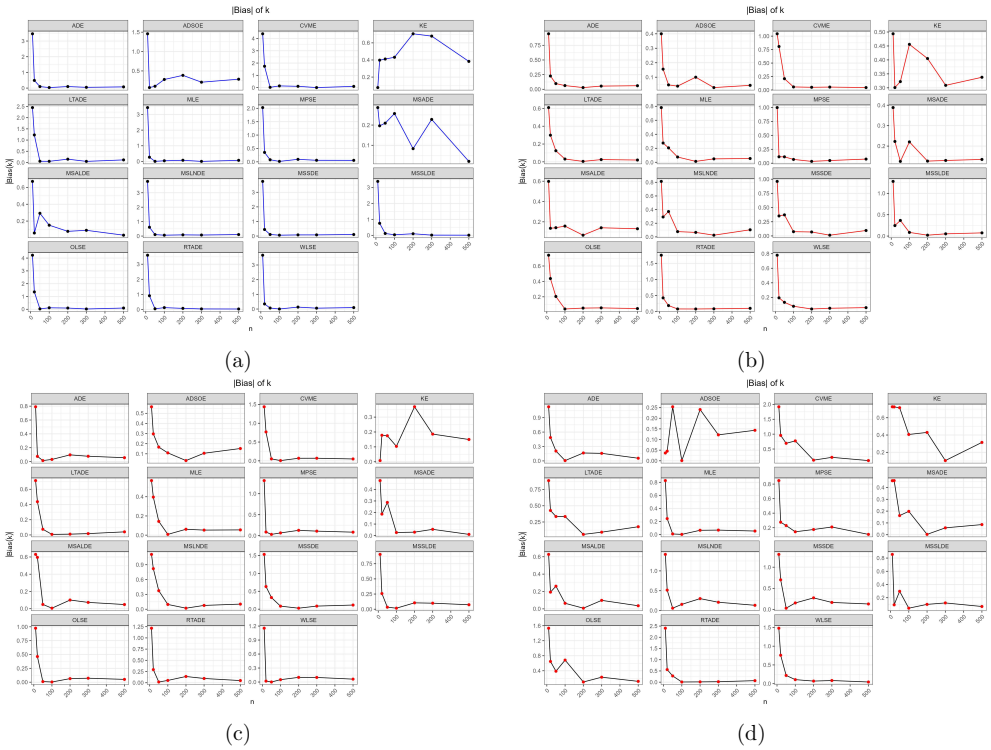


Figure 8: |Bias| for k in simulation Tables 3, 5, 7 and 9

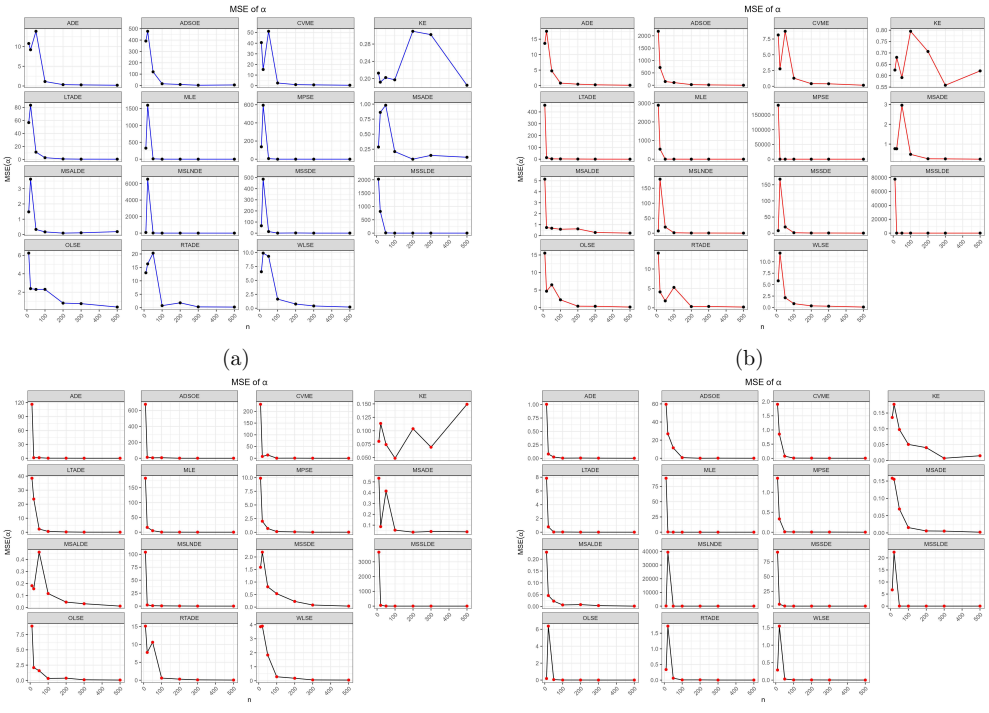


Figure 9: MSE for α in simulation Tables 3, 5, 7 and 9

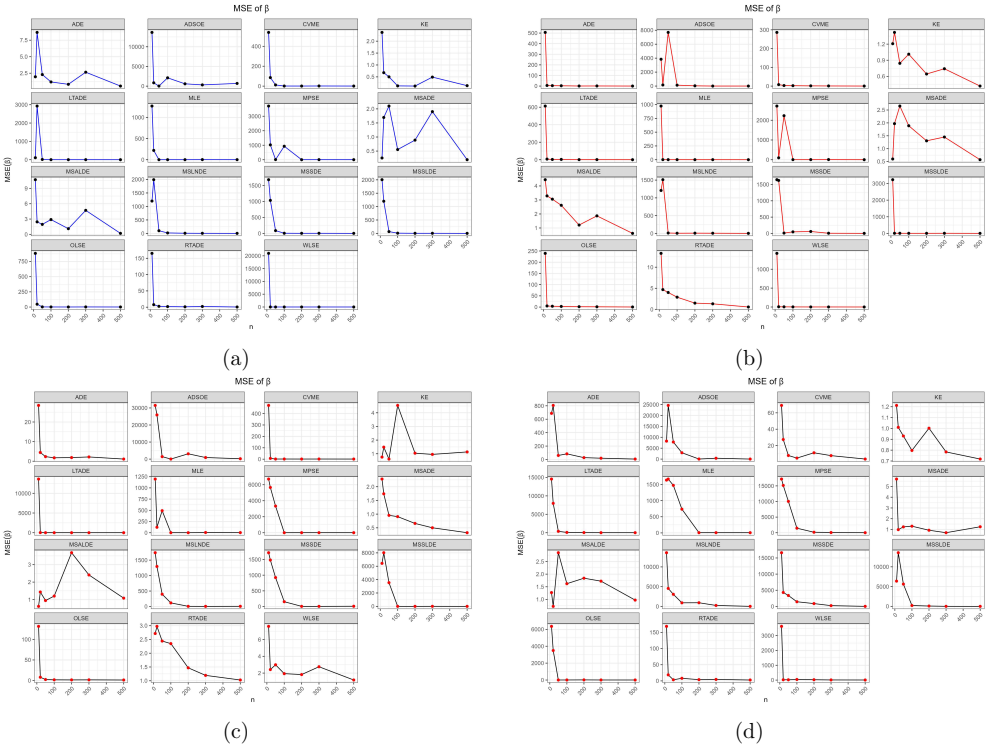


Figure 10: MSE for β in simulation Tables 3, 5, 7 and 9

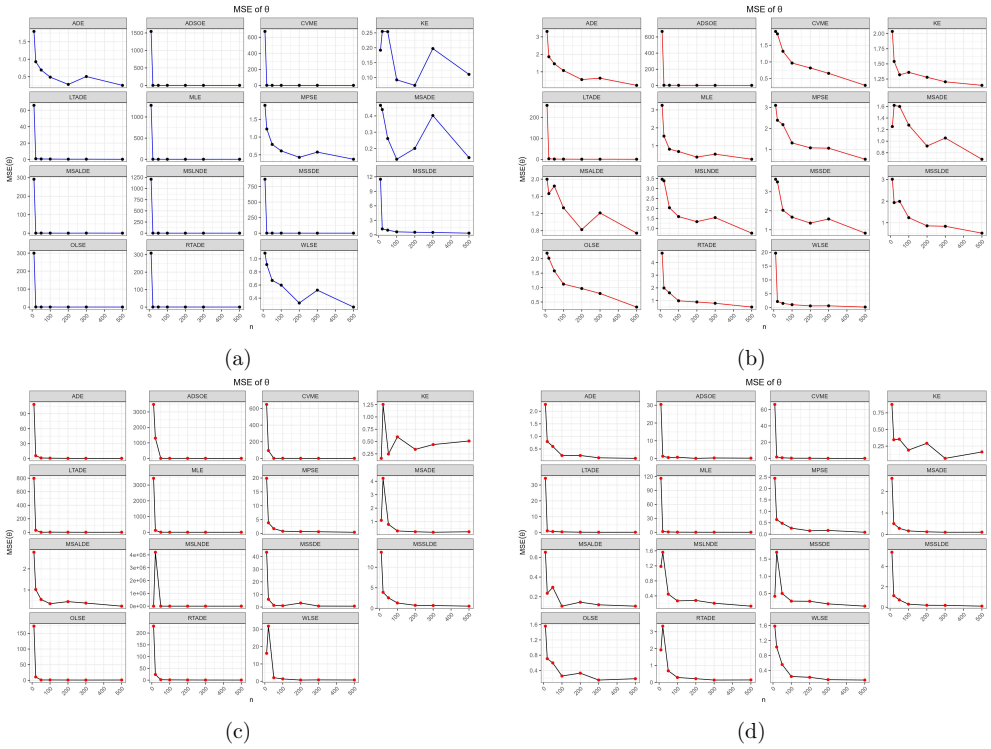


Figure 11: MSE for θ in simulation Tables 3, 5, 7 and 9

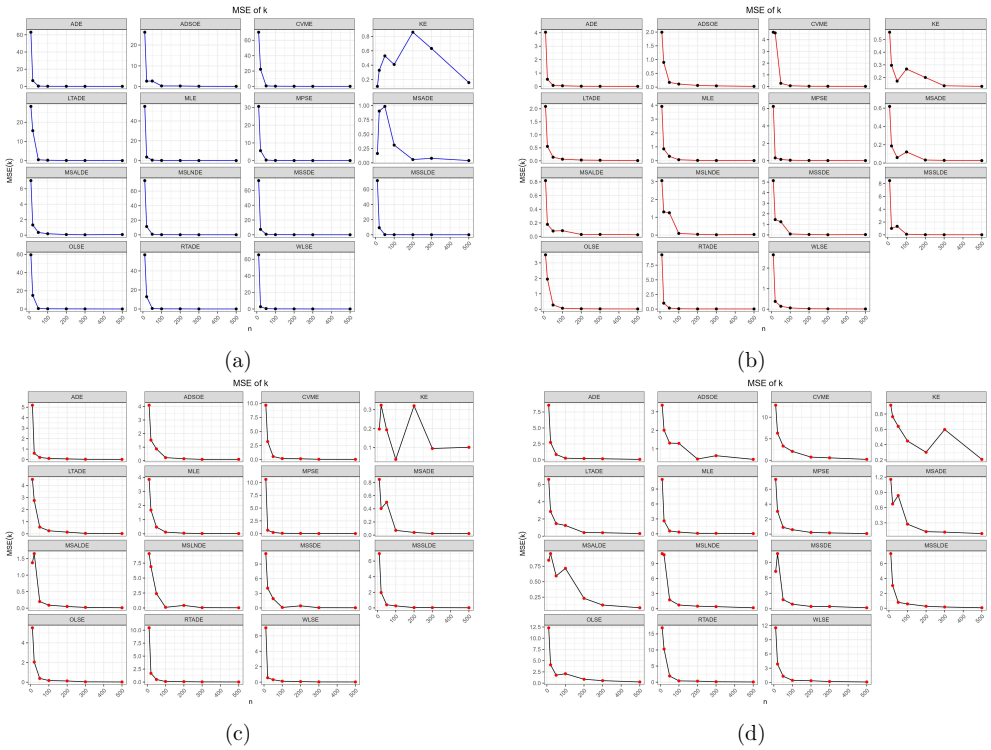
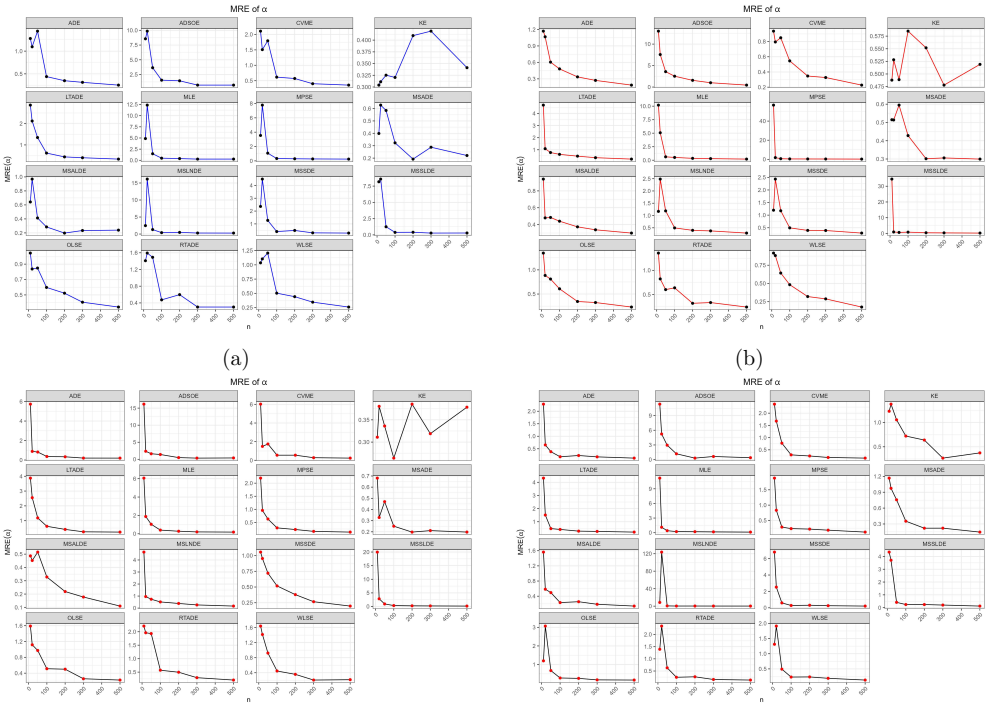
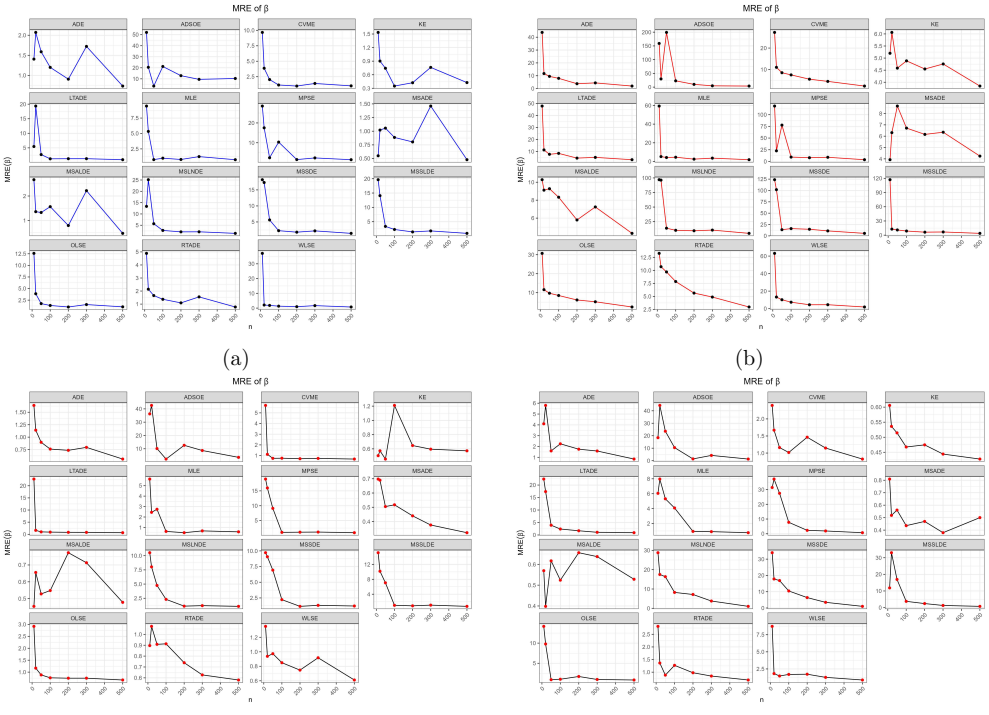


Figure 12: MSE for k in simulation Tables 3, 5, 7 and 9



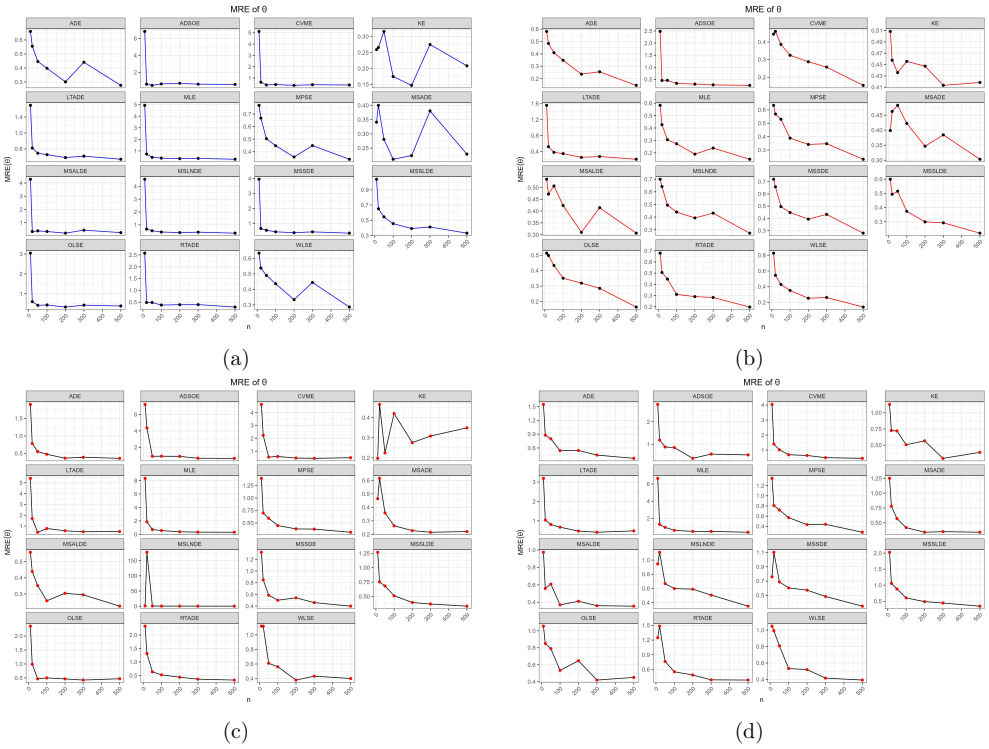
(a) (b) (c) (d)

Figure 13: MRE for α in simulation Tables 3, 5, 7 and 9



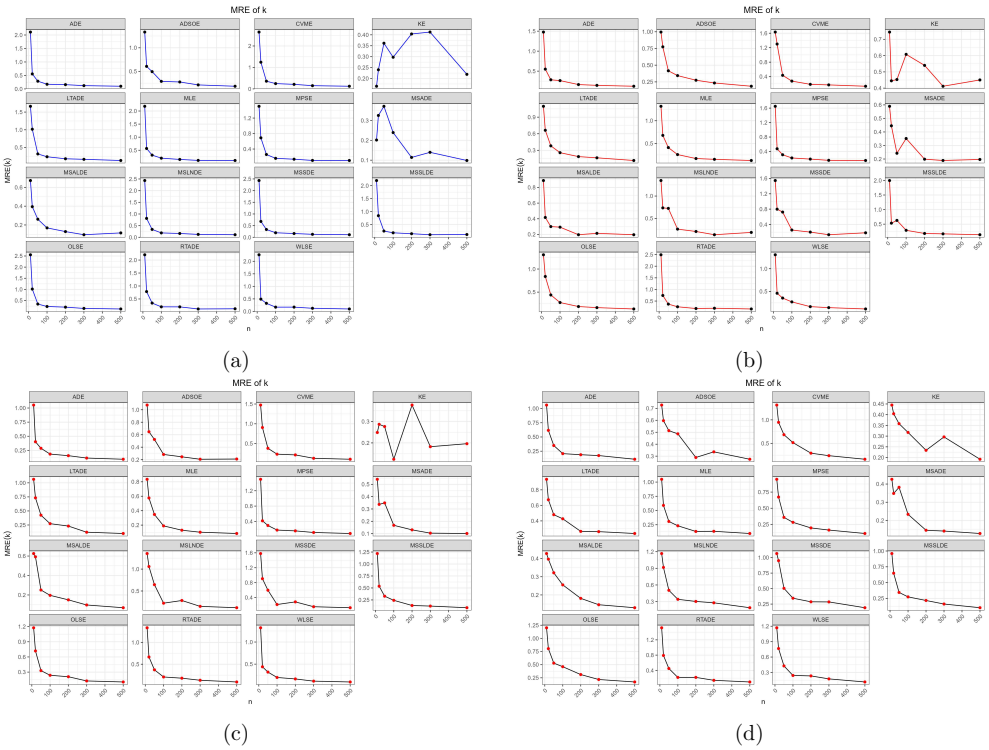
(a) (b) (c) (d)

Figure 14: MRE for β in simulation Tables 3, 5, 7 and 9



(a) (b) (c) (d)

Figure 15: MRE for θ in simulation Tables 3, 5, 7 and 9



(a) (b) (c) (d)

Figure 16: MRE for k in simulation Tables 3, 5, 7 and 9

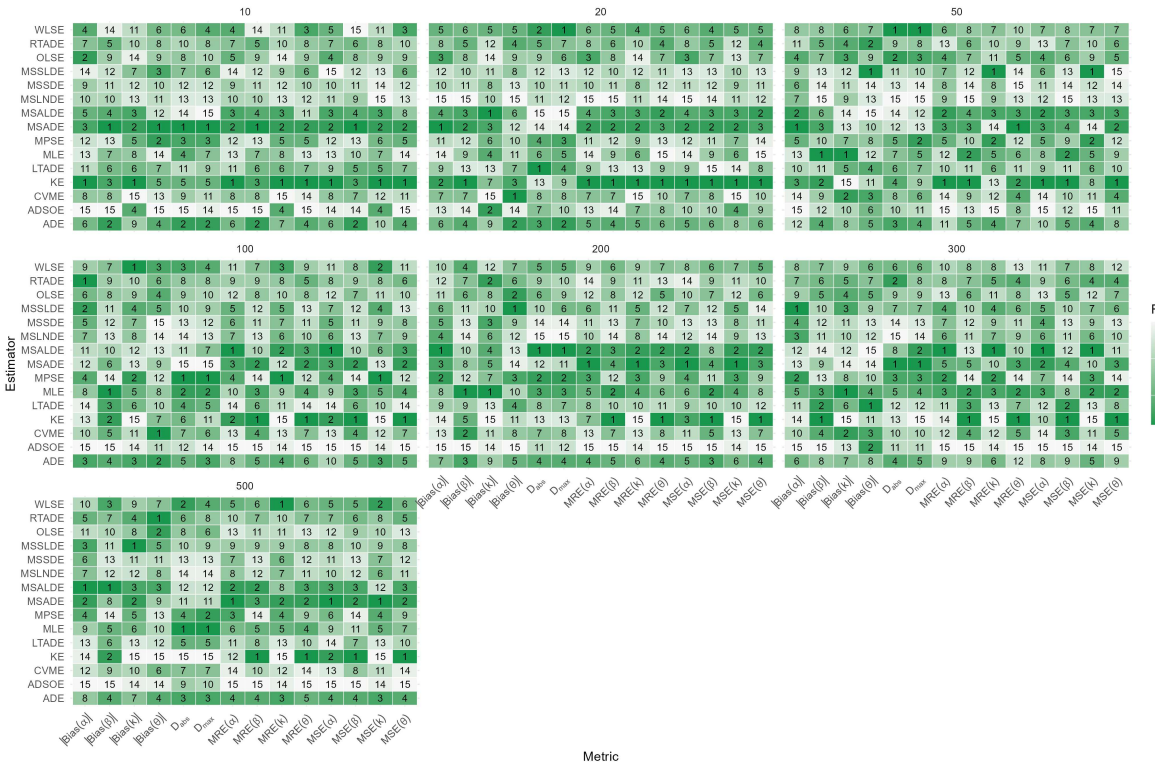


Figure 17: Heatmap for simulation Table 3

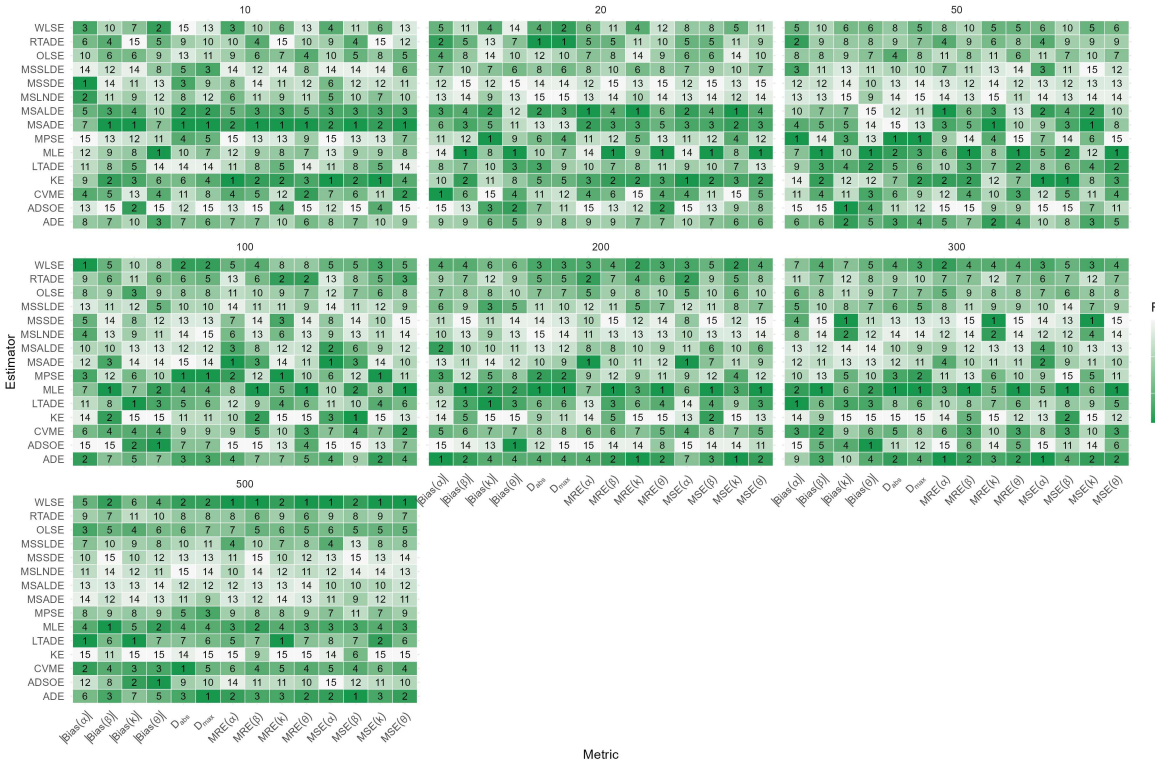


Figure 18: Heatmap for simulation Table 5

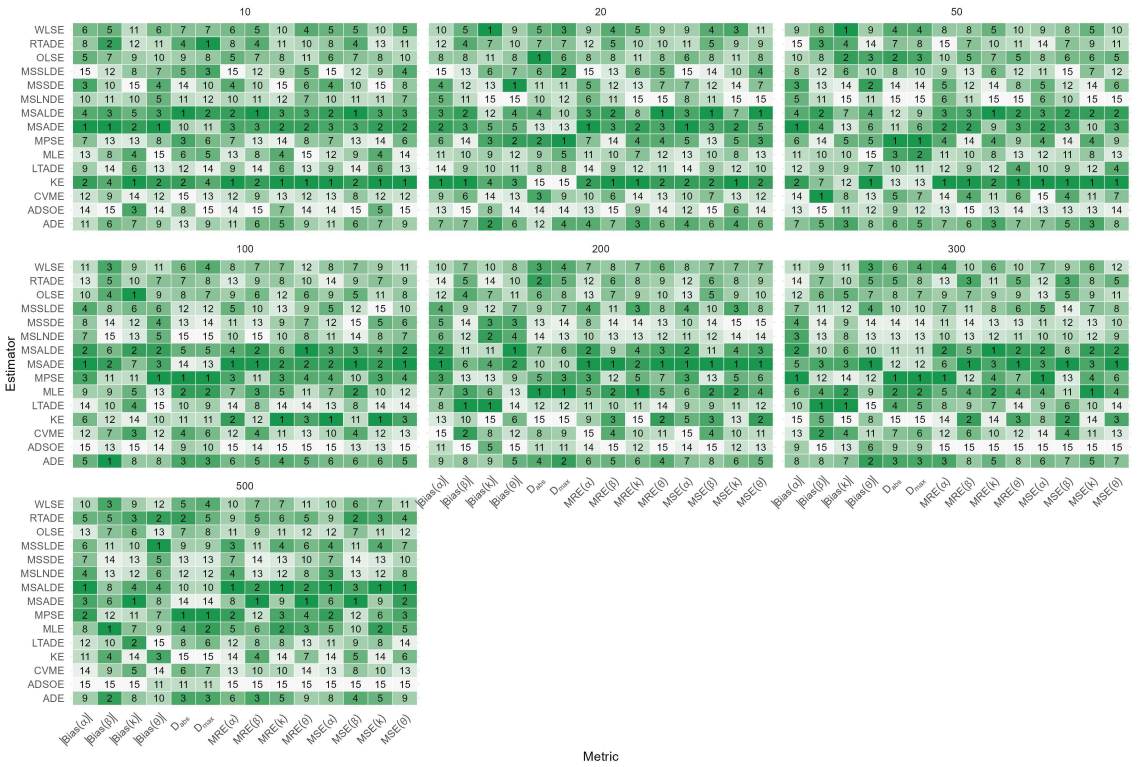


Figure 19: Heatmap for simulation Table 7

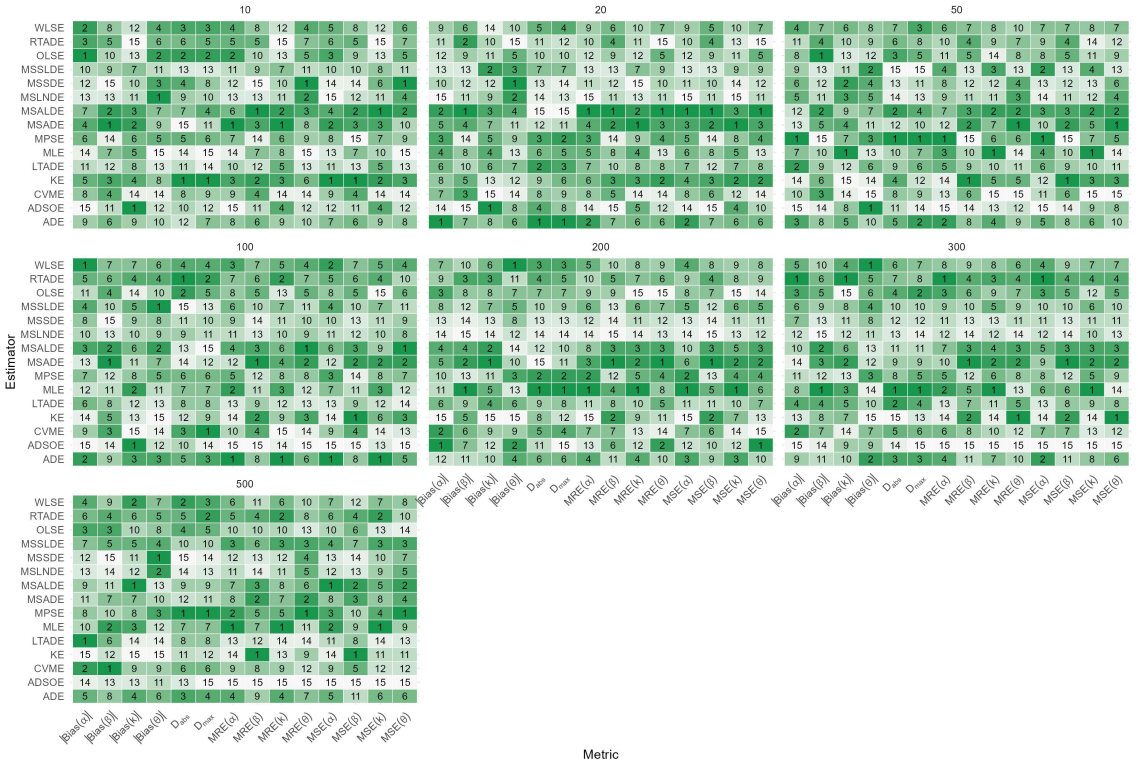


Figure 20: Heatmap for simulation Table 9

6 Applications

We first demonstrate the usefulness of the proposed NGAPW model using times to infection of kidney dialysis patients dataset $\vec{x}_1$. This data originates from Klein and Moeschberger (2003).

$$\vec{x}_1 = \begin{bmatrix} 1.5 & 3.5 & 4.5 & 4.5 & 5.5 & 8.5 & 8.5 & 9.5 \\ 10.5 & 11.5 & 15.5 & 16.5 & 18.5 & 23.5 & 26.5 & \end{bmatrix}^T.$$

The second dataset $\vec{x}_2$ is clean-up-gradient monitoring wells in mg/L, studied by Bhaumik et al. (2009).

$$\vec{x}_2 = \begin{bmatrix} 5.1 & 1.2 & 1.3 & 0.6 & 0.5 & 2.4 & 0.5 & 1.1 & 8.0 & 0.8 & 0.4 & 0.6 & 0.9 & 0.4 \\ 2.0 & 0.5 & 5.3 & 3.2 & 2.7 & 2.9 & 2.5 & 2.3 & 1.0 & 0.2 & 0.1 & 0.1 & 1.8 & 0.9 \\ 2.0 & 4.0 & 6.8 & 1.2 & 0.4 & 0.2 & & & & & & & & \end{bmatrix}^T.$$

The third dataset $\vec{x}_3$ represents weekly death rate due to covid1-19 from 22/3/2020 to 20/12/2020 in Dominica, accessed from https://data.who.int/dashboards/covid19/data?n=c

$$\vec{x}_3 = \begin{bmatrix} 0.02985 & 0.03536 & 0.05292 & 0.05124 & 0.05306 & 0.03520 & 0.03208 & 0.02561 \\ 0.01930 & 0.01298 & 0.01609 & 0.01662 & 0.01214 & 0.02433 & 0.01301 & 0.01222 \\ 0.01356 & 0.01083 & 0.00896 & 0.01093 & 0.01616 & 0.02202 & 0.02313 & 0.03760 \\ 0.03307 & 0.02618 & 0.01975 & 0.01504 & 0.01179 & 0.00954 & 0.00962 & 0.00808 \\ 0.00833 & 0.00685 & 0.00531 & 0.00572 & 0.00410 & 0.00340 & 0.00253 & 0.00402 \end{bmatrix}^T.$$

Table 11: Summary Statistics for the datasets

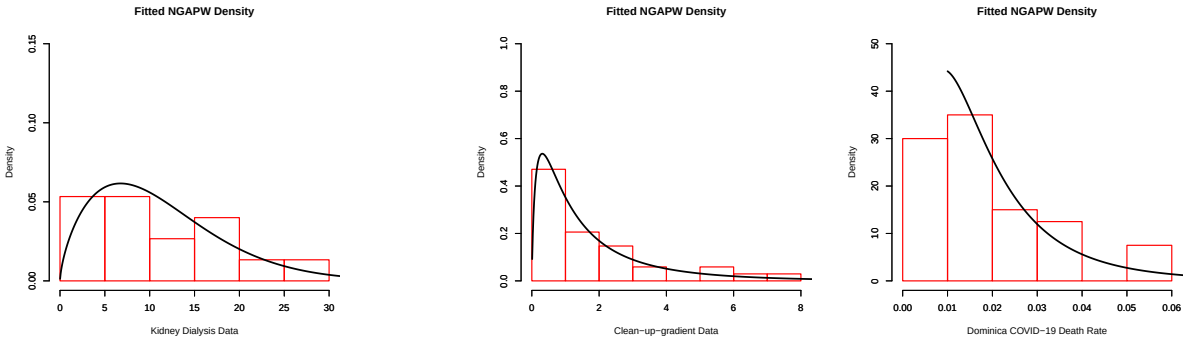
Statistic	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$
n	15	34	40
Q_1	5	0.5	0.00939
Q_3	16	2.47500	0.02575
IQR	11	1.97500	0.01635
Outliers	–	8, 6.8	0.05292, 0.05124, 0.05306
Mean	11.23333	1.87941	0.01896
Median	9.5	1.15000	0.01430
SD	7.45910	1.95259	0.01365
Variance	55.63810	3.81259	0.00019
Range	25	7.5	0.05053
Skewness	0.67368	1.60369	1.07772
Kurtosis	2.44052	5.00541	3.41691

As shown in Table 11, all three datasets ($\vec{x}_1, \vec{x}_2, \vec{x}_3$) demonstrate moderate to strong positive skewness (0.67 to 1.60) and differing levels of kurtosis, where datasets $\vec{x}_2$ and $\vec{x}_3$ exhibit leptokurtic characteristics (5.01 and 3.42, respectively). The existence of prominent upper-tail outliers in datasets $\vec{x}_2$ and $\vec{x}_3$, combined with considerable variance throughout all datasets in relation to their scales, emphasizes heavy-tailed and asymmetric structural patterns in the analyzed empirical data. These distributional traits highlight the need for adaptable, heavy-tailed statistical distributions like the suggested NGAPW model to effectively manage skewness, peakiness, and extreme values.

Table 12: MLE, Goodness of Fit and Measures of Model Performance Across Datasets

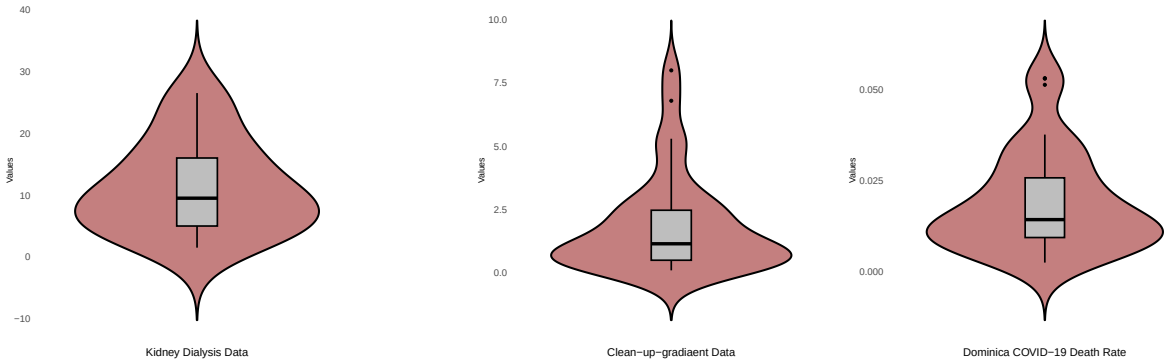
Data	Model	AIC	BIC	W	A	KS(p-value)	α	β	θ	k
	NGAPW	106.2134	109.0456	0.0248	0.1677	0.1004(0.9982)	1.0165(1.6814)	0.0251(0.6091)	0.0179(0.0978)	1.5919(1.5778)
	Lomax	106.5685	110.9846	0.0234	0.1625	0.2011(0.5792)	4614.5044(149130.0521)	51830.4547(1675242.4897)	–	–
$\vec{x}_1$	Pareto	116.2735	117.6896	0.4037	2.3781	0.3321(0.0732)	0.5699(0.1466)	1.4997(0.0001)	–	–
	BXII	122.4656	123.8818	0.0941	0.6781	0.3715(0.0318)	39.6967(7906.1934)	0.0116(2.3091)	–	–
	Logistic	106.4245	110.8406	0.0632	0.4059	0.1099(0.9935)	10.5461(1.9021)	4.1811(0.8936)	–	–
	NGAPW	117.7126	123.8180	0.0267	0.1848	0.0738(0.9926)	2.7658(3.9818)	0.3308(1.0676)	1.1750(1.4111)	0.6337(0.4063)
	Lomax	118.8789	137.9317	0.0432	0.2793	0.0811(0.9788)	28.8469(170.6787)	52.3487(320.3230)	–	–
$\vec{x}_2$	Pareto	137.6094	140.6621	0.6427	3.9310	0.3195(0.0019)	0.4145(0.0714)	0.1000(0.0000)	–	–
	BXII	126.1526	139.2053	0.0418	0.2927	0.1040(0.8557)	1.5621(0.2479)	0.9305(0.1791)	–	–
	Logistic	140.9169	143.9696	0.2999	1.8345	0.1856(0.1921)	1.5467(0.2865)	0.9782(0.1445)	–	–
	NGAPW	-239.4224	-232.6669	0.0217	0.1690	0.0599(0.9971)	2.7885(3.7934)	0.3603(1.0039)	63.3514(85.2028)	0.9305(0.5598)
	Lomax	-233.2181	-229.8403	0.0341	0.2302	0.1721(0.1663)	6206.7064(155321.6214)	117.6822(2945.2712)	–	–
$\vec{x}_3$	Pareto	-209.5047	-206.1269	0.4050	2.4733	0.3081(0.0007)	0.5675(0.0900)	0.0025(0.0000)	–	–
	BXII	-221.9562	-238.5785	0.0540	0.3482	0.0951(0.8286)	1.4853(0.1783)	309.4228(201.5462)	–	–
	Logistic	-227.8290	-224.4512	0.2074	1.2578	0.1223(0.5468)	0.0172(0.0021)	0.0074(0.0010)	–	–

As presented in Table 12, the NGAPW model demonstrates consistently strong empirical performance across all three datasets. It yields the lowest Kolmogorov–Smirnov (KS) statistics alongside the highest p-values in every case (ranging from 0.9926 to 0.9982), indicating excellent agreement with the observed data. For datasets $\vec{x}_2$ and $\vec{x}_3$, NGAPW achieves the lowest Cramér–von Mises (W) and Anderson–Darling (A) statistics, while maintaining highly competitive values for $\vec{x}_1$. Furthermore, despite its flexible four-parameter structure—which leads to slightly higher BIC or AIC values in some instances compared to simpler baseline distributions—NGAPW attains the lowest AIC for $\vec{x}_1$ and delivers overall superior goodness-of-fit metrics (W, A, and KS), underscoring its enhanced capability in capturing intricate data dynamics and tail behaviors.



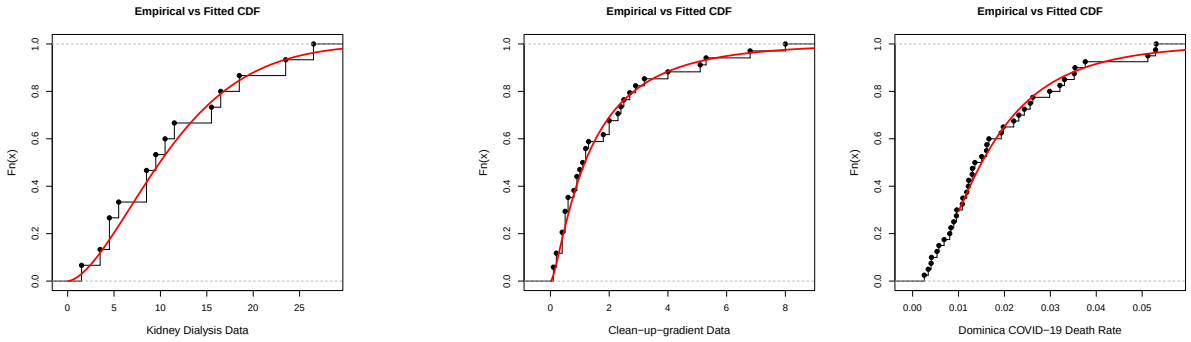
(a) (b) (c)

Figure 21: Density on Histogram for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$



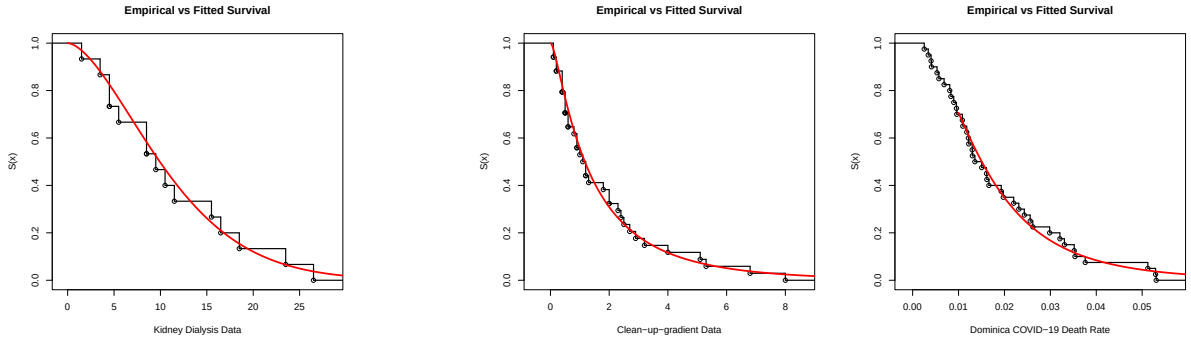
(a) (b) (c)

Figure 22: Boxplots in Violin plots for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$



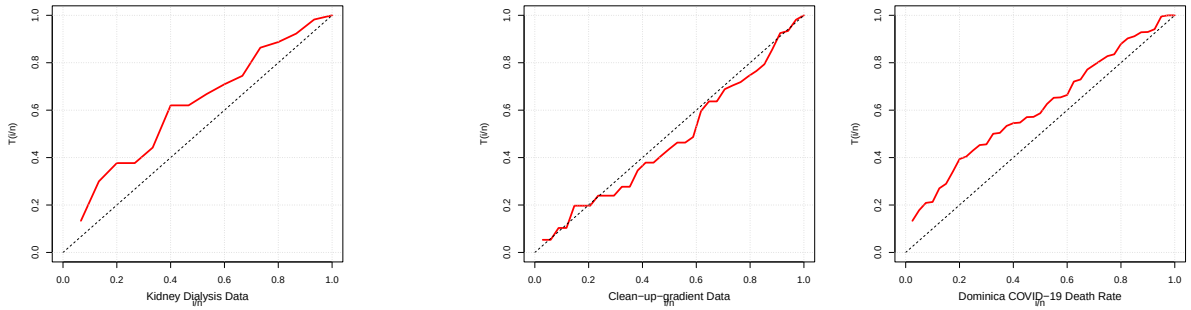
(a) (b) (c)

Figure 23: CDFs for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$



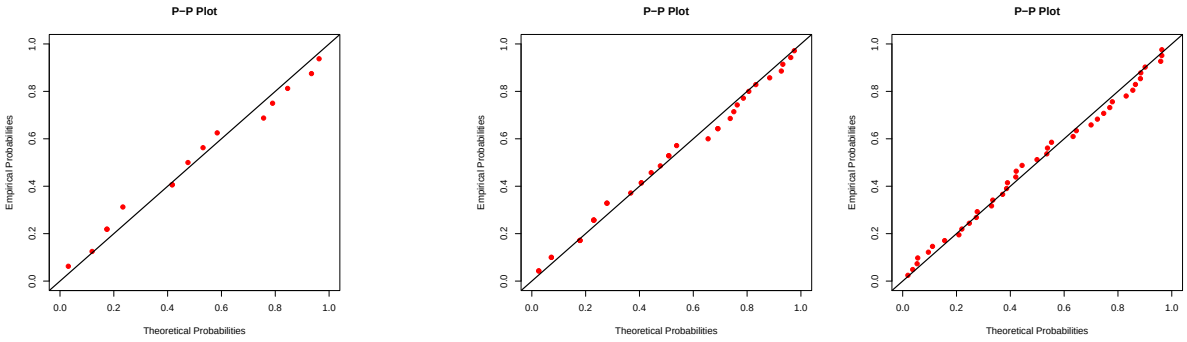
(a) (b) (c)

Figure 24: Survival Function for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$



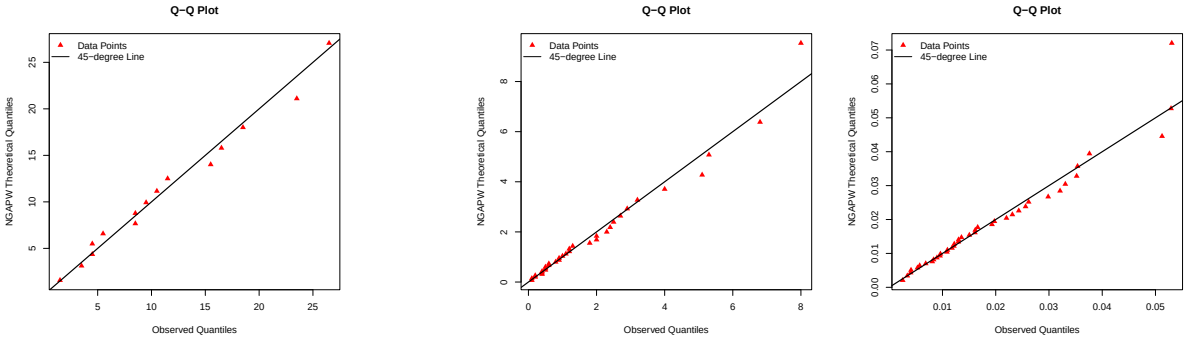
(a) (b) (c)

Figure 25: TTT plot for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$



(a) (b) (c)

Figure 26: P-P plots for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$



(a) (b) (c)

Figure 27: Q-Q for (a) $\vec{x}_1$ (b) $\vec{x}_2$ (c) $\vec{x}_3$

As shown in the empirical density plots (Figure 21) and violin-boxplots (Figure 22), all three datasets display different levels of right-skewness, unimodality, and upper-tail outliers. The empirical vs theoretical Cumulative Distribution Functions (Figure 23) and Survival Functions (Figure 24) indicate that the NGAPW model closely matches the empirical curves over all probability areas when compared to alternative distributions. Additionally, the Total Time on Test (TTT) graphs in Figure 25 show unique hazard rate patterns—such as consistently increasing and bathtub-shaped characteristics—which NGAPW successfully captures. Ultimately, the significant linearity present along the diagonal in both the P-P plots (Figure 26) and Q-Q plots (Figure 27) offers compelling visual evidence of NGAPW's enhanced goodness-of-fit and distributional flexibility.

7 Conclusion

In this paper, we presented and extensively elaborated on the NGAP family of distributions, an exponent-driven generator framework aimed at overcoming the constraints of current baseline models in modeling intricate lifetime and reliability data. The NGAP framework offers enhanced flexibility in managing skewness, tail weights, and hazard rate behaviors by allowing the exponent transformation to adjust dynamically across various probability regions—covering both monotonic (increasing, decreasing) and non-monotonic (unimodal, bathtub-shaped) forms.

Using the two-parameter Weibull distribution as a reference, we developed the NGAPW distribution along with its mathematical characteristics. In particular, we offered closed-form series expressions for the cumulative distribution function, probability density function, raw

moments, moment-generating function, mean residual life, stress-strength reliability, order statistics, and measures of information-theoretic extropy. Parameter estimation was thoroughly analyzed using fifteen different non-Bayesian estimation methods. Monte Carlo simulation analyses verified the finite-sample consistency, asymptotic efficiency, and numerical stability of these estimators, with the Maximum Product of Spacings Estimator and Maximum Likelihood Estimator exhibiting superior performance. Empirical assessment on actual datasets showed that the NGAPW distribution provides better goodness-of-fit than established generalized Weibull variations. The NGAP family offers a dependable and theoretically robust framework for use in reliability engineering, biomedical survival analysis, actuarial modeling, and applied statistics through the integration of analytical tractability, parameter identifiability, and dynamic structural adaptability.

Author Contributions

- **Shamshad Ur Rasool:** Conceptualization, Methodology, Writing – Original Draft, Project Administration, Resources, Investigation, Validation.
- **Abdoulie Faal:** Methodology, Formal Analysis, Data Curation, Resources, Visualization, Project Administration.
- **Okechukwu J. Obulezi:** Methodology, Formal Analysis, Software, Data Curation, Visualization, Writing – Original Draft, Writing – Review & Editing, Supervision.

Data and Code Availability Statement

The data used in this study are contained as follows

- the times for infection of kidney dialysis patients datasets are found in Klein and Moeschberger (2003),
- the clean-up gradient monitoring wells in mg/L are found in Bhaumik et al. (2009),
- while the weekly death rate due to covid1-19 from 22/3/2020 to 20/12/2020 in Dominica, accessed from https://data.who.int/dashboards/covid19/data?n=c.

The R scripts for the Monte Carlo demonstration and the application of real data are found in the Github repository via the following link https://github.com/obulezi12345-svg/NGAP-Family-R-Scripts.

Competing Interest declaration

The authors declare that they have no competing interests in this study.

Consent to Publish

Not Applicable.

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