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Analytical electromagnetic form factors of the nucleon in a soft-wall AdS/QCD model.

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Abstract

We present analytical expressions for the electromagnetic form factors of the nucleon within a soft-wall AdS/QCD framework. The Dirac and Pauli form factors are derived from bulk interaction terms and expressed in closed form using Beta functions. The resulting Sachs form factors exhibit the expected scaling behavior and reproduce the correct normalization at zero momentum transfer. We further compute the electric and magnetic radii and discuss their dependence on the holographic scale parameter. Our results provide a compact and predictive description of nucleon structure.

Keywords: AdS/QCD, electromagnetic form factors, nucleon structure

1. Introduction

Understanding the internal structure of the nucleon remains one of the central challenges in hadronic and nuclear physics. Since nucleons are composite bound states of quarks and gluons governed by Quantum Chromodynamics (QCD), their nonperturbative dynamics cannot be fully described by perturbative methods at low energies. Electromagnetic form factors provide fundamental information about the spatial distributions of charge and magnetization inside the nucleon and therefore constitute an essential tool for probing hadron structure. Experimental measurements from elastic electron–nucleon scattering have revealed a rich internal dynamics, including nontrivial scaling behavior and flavor-dependent structure effects [1, 2].

In the framework of QCD, nucleon electromagnetic structure is encoded in the Dirac and Pauli form factors, $F_1(Q^2)$ and $F_2(Q^2)$, or equivalently in the Sachs form factors, $G_E(Q^2)$ and $G_M(Q^2)$. These observables are directly related to important static properties such as the nucleon charge radii and magnetic moments. At large momentum transfer, perturbative QCD predicts specific power-law scaling relations for the form factors [3, 4], while in the low-energy region their behavior is dominated by strongly coupled nonperturbative dynamics. Developing theoretical approaches capable of consistently describing both regimes remains an important open problem.

Gauge/gravity duality, inspired by the AdS/CFT correspondence [5, 6, 7], has provided a new nonperturbative framework for studying strongly coupled

hadronic systems. In particular, holographic QCD models have achieved considerable success in describing hadron spectra, decay constants, and form factors [8, 9, 10, 11, 12]. Among these approaches, the soft-wall AdS/QCD model is especially attractive because the introduction of a quadratic dilaton background naturally generates confinement and linear Regge trajectories while preserving analytical tractability. This framework has been widely applied to the study of meson and baryon properties as well as electromagnetic and axial form factors [10, 11, 12, 13, 14].

In this work, we investigate the nucleon electromagnetic form factors within the soft-wall AdS/QCD model. Starting from the holographic action for bulk fermion fields interacting with the vector sector, we derive analytical closed-form expressions for the Dirac and Pauli form factors. The obtained results provide a transparent description of the momentum-transfer dependence of the nucleon form factors and reproduce the expected scaling behavior at large Q^2 . Furthermore, the analytical structure of the model allows us to study static observables such as magnetic moments and charge radii in a simple and consistent manner. The resulting expressions may also serve as a useful basis for future investigations of generalized parton distributions, transition form factors, and spin-dependent nucleon structure within holographic QCD.

2. Model setup

In the soft-wall AdS/QCD approach, baryons are described by bulk Dirac fields propagating in a five-dimensional Anti-de Sitter (AdS) space with a background dilaton field that introduces confinement [8, 10, 12].

The AdS metric is given by

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \quad (1)$$

where z is the holographic coordinate and $\eta_{\mu\nu}$ is the Minkowski metric.

Consider a Dirac field ψ propagating in the five-dimensional bulk of AdS space, described by the action

$$S = \int d^5x \sqrt{g} e^{-\phi(z)} (i \bar{\psi} e_A^M \Gamma^A D_M \psi - (M + \phi(z)) \bar{\psi} \psi), \quad (2)$$

where M denotes the bulk spinor “mass”, while $e_A^M = z \delta_A^M$ is the inverse vielbein, satisfying $g_{MN} = e_A^M e_B^N \eta_{AB}$, with $\eta_{AB} = \text{diag}(1, -1, -1, -1, -1)$,

and $g = |\det g_{MN}|$. The Dirac matrices Γ^A ($A = 0, 1, 2, 3, 5$) obey the Clifford algebra $\{\Gamma^A \Gamma^B\} = 2\eta^{AB}$. The covariant derivative is defined as $D_M = \partial_M + \frac{1}{8}\omega_{MAB} [\Gamma^A \Gamma^B]$. The non-vanishing components of the spin connection ω_{MAB} are $\omega_{\mu 5\nu} = -\omega_{\mu\nu 5} = (1/z)\delta_\mu^\nu$. The corresponding equation of motion for the bulk spinor, obtained from the action (2), is

$$\left[ie_A^M \Gamma^A D_M - \frac{i}{2}(\partial_M \phi) e_A^M \Gamma^A - (M + \phi(z)) \right] \psi = 0. \quad (3)$$

The normalizable profile functions $\psi_{L,R}^{(n)}$ associated with the n -th Kaluza-Klein mode, which are holographically dual to the left- and right-handed nucleon fields on the boundary, are obtained as solutions of this equation and are given by [10]:

$$\psi_L^{(n)} = \sqrt{\frac{2\Gamma[n+1]}{\Gamma[n+3]}} k^3 z^4 L_n^{(2)}(k^2 z^2), \quad \psi_R^{(n)} = \sqrt{\frac{2\Gamma[n+1]}{\Gamma[n+3]}} \sqrt{n+2} k^2 z^3 L_n^{(1)}(k^2 z^2), \quad (4)$$

where $L_n^{(m)}$ denotes the associated Laguerre polynomial. For the ground state, corresponding to the quantum number $n = 0$, these profile functions reduce to the simple expressions

$$\psi_L = k^3 z^4, \quad \psi_R = \sqrt{2} k^2 z^3. \quad (5)$$

The electromagnetic field is described by a bulk vector field $V_M(x, z)$, which is dual to the conserved vector current on the boundary. Its dynamics is governed by the action

$$S_V = -\frac{1}{4} \int d^5x \sqrt{g} e^{-\kappa^2 z^2} F_{MN} F^{MN}, \quad (6)$$

where $F_{MN} = \partial_M V_N - \partial_N V_M$.

From this action (6) is obtained the bulk equation of motion for the vector field:

$$\partial_z \left(\frac{e^{-\kappa^2 z^2}}{z} \partial_z V(Q, z) \right) + Q^2 \frac{e^{-\kappa^2 z^2}}{z} V(Q, z) = 0, \quad (7)$$

with boundary condition $V(Q, 0) = 1$.

The solution can be written in an integral representation

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{x^a dx}{(1-x)^2} \exp \left[-\frac{\kappa^2 z^2 x}{1-x} \right], \quad (8)$$

where

$$a = \frac{Q^2}{4\kappa^2}. \quad (9)$$

This representation is particularly convenient for evaluating holographic matrix elements and allows for analytic computation of the form factors.

3. Electromagnetic form factors

In Quantum Chromodynamics (QCD), the electromagnetic structure of the nucleon is encoded in the matrix element of the electromagnetic current between nucleon states,

$$\langle N(p') | J^\mu(0) | N(p) \rangle = \bar{u}(p') \left[\gamma^\mu F_1^N(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2^N(Q^2) \right] u(p), \quad (10)$$

where $F_1^N(Q^2)$ and $F_2^N(Q^2)$ ($N = p, n$ correspond to proton and neutron) are the Dirac and Pauli form factors, respectively, $q^\mu = p'^\mu - p^\mu$, and $Q^2 = -q^2$ [1, 2]. The Pauli and Dirac form factors, $F_1^N(Q^2)$ and $F_2^N(Q^2)$ are normalized at $Q^2 = 0$ to the nucleon electric charge and anomalous magnetic moment respectively; that is, $F_1^N(0) = e^N$ and $F_2^N(0) = k_N$.

Within the holographic framework, the electromagnetic interaction is described by the coupling of the bulk fermion field to a vector field dual to the electromagnetic current. The interaction action is given by

$$S_{int} = \int d^5x \sqrt{g} e^{-k^2 z^2} \text{Tr} \left[e^N \bar{\psi} e_B^M \Gamma^B V_M \psi + \eta^N \frac{i}{2} \bar{\psi} e_A^M e_B^N [\Gamma^A, \Gamma^B] F_{MN} \psi \right], \quad (11)$$

where e^N denote the proton and neutron electric charges, and η^N are parameters related to the anomalous magnetic moments. Here, e_A^M are the vielbeins, Γ^A are the Dirac matrices in curved space, and $F_{MN} = \partial_M V_N - \partial_N V_M$ is the field strength tensor associated with the bulk vector field.

Following the standard AdS/CFT correspondence[5, 6, 7], the five-dimensional action is reduced to four-dimensional momentum space, and the nucleon form factors are obtained by evaluating the corresponding bulk matrix elements. Taking into account the isospin structure of the nucleon, one obtains

$$\begin{aligned} F_1^N(Q^2) &= e^N G_1(Q^2) + \eta^N G_2(Q^2), \\ F_2^N(Q^2) &= \eta^N G_3(Q^2), \end{aligned} \quad (12)$$

where the structure functions are defined as

$$\begin{aligned}
G_1(Q^2) &= \int_0^\infty dz \frac{e^{-k^2 z^2}}{2z^3} V(Q, z) (\psi_R^2 + \psi_L^2), \\
G_2(Q^2) &= \int_0^\infty dz \frac{e^{-k^2 z^2}}{2z^2} \frac{\partial V(Q, z)}{\partial z} (\psi_L^2 - \psi_R^2), \\
G_3(Q^2) &= \int_0^\infty dz \frac{2m_N e^{-k^2 z^2}}{2z^2} V(Q, z) (\psi_R \psi_L).
\end{aligned} \tag{13}$$

These expressions provide the holographic representation of the nucleon electromagnetic form factors in terms of the bulk profiles ψ_L , ψ_R and the vector bulk-to-boundary propagator V_M . Using the integral representation (8) of the bulk-to-boundary propagator, the structure functions can be written as

$$\begin{aligned}
G_1(Q^2) &= \int_0^1 \frac{x^a dx}{(1-x)^2} \int_0^\infty e^{-\beta z^2} \left(k^6 z^5 + \frac{k^8 z^7}{2} \right) dz, \\
G_2(Q^2) &= \int_0^1 \frac{x^a dx}{(1-x)^2} \int_0^\infty e^{-\beta z^2} \left(\frac{k^8 (1+x)}{1-x} z^7 - 2k^6 z^5 - \frac{k^{10} x}{1-x} z^9 \right) dz, \\
G_3(Q^2) &= \sqrt{2} m_N k^7 \int_0^1 \frac{x^a dx}{(1-x)^2} \int_0^\infty e^{-\beta z^2} z^7 dz,
\end{aligned} \tag{14}$$

where $\beta = \frac{k^2}{1-x}$. If the structure functions in Eq. (14) are taken into account in Eq. (12), the resulting expressions for the Dirac and Pauli form factors can be written as follows.

$$\begin{aligned}
F_1^{(p)}(Q^2) &= \int_0^1 \frac{x^a dx}{(1-x)^2} \Phi_1^{(p)}(x), \\
F_1^{(n)}(Q^2) &= \eta^n G_2(Q^2) = \int_0^1 \frac{x^a dx}{(1-x)^2} \Phi_1^{(n)}(x), \\
F_2^{(p,n)}(Q^2) &= \int_0^1 \frac{x^a dx}{(1-x)^2} \Phi_2^{(p,n)},
\end{aligned} \tag{15}$$

The functions $\Phi_1^{(p)}(x)$, $\Phi_1^{(n)}(x)$, and $\Phi_2^{(p,n)}(x)$ entering Eq. (15) are obtained by evaluating Gaussian-type

$$\int_0^\infty e^{-\beta z^2} z^n dz = \frac{1}{2} \beta^{-(n+1)/2} \Gamma\left(\frac{n+1}{2}\right), \tag{16}$$

integrals appearing in the structure functions G_1 , G_2 , and G_3 (see details in Appendix A).

$$\begin{aligned}\Phi_1^{(p)}(x) &= \frac{\alpha_5}{\beta^3} + \frac{3\alpha_7}{\beta^4} - \frac{12\alpha_9}{\beta^5}, \\ \Phi_1^{(n)}(x) &= \frac{3a_7}{\beta^4} - \frac{a_5}{\beta^3} - \frac{12a_9}{\beta^5}, \\ \Phi_2^{(p,n)}(x) &= \frac{3\sqrt{2}\eta^{p,n}m_N k^7}{\beta^4}.\end{aligned}\tag{17}$$

The constants entering Eq. (17) are defined as follows:

$$\begin{aligned}\alpha_5 &= k^6(1 - 2\eta^p), \quad \alpha_7 = k^8 \frac{(2\eta^p - 1)x + 2\eta^p + 1}{2(1 - x)}, \quad \alpha_9 = \frac{k^{10}\eta^p x}{1 - x}, \\ a_7 &= \eta^n \frac{k^8(1 + x)}{1 - x}, \quad a_5 = 2\eta^n k^6, \quad a_9 = \eta^n \frac{k^{10}x}{1 - x}.\end{aligned}\tag{18}$$

By eliminating the x -dependent integrals in $F_1^N(Q^2)$ and $F_2^N(Q^2)$ given in Eq. (15) to obtain explicit expressions depending only on Q^2 , and using the definition

$$\beta = \frac{k^2}{1 - x},$$

together with the constants given in Eq. (37) in the expressions of Eq. (17), we obtain the following x -dependent expressions for the functions $\Phi_1^{(p)}(x)$, $\Phi_1^{(n)}(x)$, and $\Phi_2^{(p,n)}(x)$:

$$\begin{aligned}\Phi_1^{(p)}(x) &= (1 - x)^3 \left[12\eta^p x^2 - (9\eta^p + \frac{3}{2})x + \eta^p + \frac{5}{2} \right], \\ \Phi_1^{(n)}(x) &= (1 - x)^3 \left[12\eta^n x^2 - 9\eta^n x + \eta^n \right], \\ \Phi_2^{(p,n)}(x) &= \frac{3\sqrt{2}m_N \eta^{p,n}}{k} (1 - x)^4.\end{aligned}\tag{19}$$

3.1. Dirac and Pauli form factors

Taking into account Eq. (19) in (15), the Dirac form factors are obtained as

$$\begin{aligned} F_1^{(p)}(Q^2) &= \int_0^1 dx x^a (1-x) \left[12\eta^p x^2 - (9\eta^p + \frac{3}{2})x + \eta^p + \frac{5}{2} \right], \\ F_1^{(n)}(Q^2) &= \int_0^1 dx x^a (1-x) \left[12\eta^n x^2 - 9\eta^n x + \eta^n \right], \\ F_2^{(p,n)}(Q^2) &= \frac{3\sqrt{2}m_N\eta^{p,n}}{k} \int_0^1 x^a (1-x)^2 dx. \end{aligned} \quad (20)$$

Using the Beta function identity

$$B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}, \quad (21)$$

the integrals in Eq. (20) can be evaluated analytically.

We obtain (for detailed derivations, see Appendix B)

$$F_1^{(p)}(Q^2) = \frac{(4\eta^p + 1)a^2 + (10 - 2\eta^p)a + 24}{(a+4)(a+3)(a+2)(a+1)}, \quad (22)$$

$$F_1^{(n)}(Q^2) = \frac{4\eta^n a^2 - 2\eta^n a}{(a+4)(a+3)(a+2)(a+1)}. \quad (23)$$

The Pauli form factors read

$$F_2^{(p,n)}(Q^2) = \frac{6\sqrt{2}m_N\eta^{p,n}}{k(a+3)(a+2)(a+1)}. \quad (24)$$

3.2. Sachs form factors

The Sachs form factors are defined as (a detailed derivation of the results presented in this section is given in Appendix C):

$$G_E^N = F_1^N - \frac{Q^2}{4m_N^2} F_2^N, \quad G_M^N = F_1^N + F_2^N. \quad (25)$$

Taking Eqs. (22) and (24) into account, the electric Sachs form factor of the proton can be written in the following more compact form:

$$G_E^p(Q^2) = \frac{E_1^p a^2 + E_2^p(Q^2)a + E_3^p(Q^2)}{(a+4)(a+3)(a+2)(a+1)}, \quad (26)$$

where

$$E_1^p = 1 + 4\eta^p, \quad E_2^p = 10 - 2\eta^p - \frac{3\sqrt{2}\eta^p Q^2}{2m_N k}, \quad E_3^p = 24 - \frac{6\sqrt{2}\eta^p Q^2}{m_N k}.$$

Similarly, for the neutron

$$G_E^n(Q^2) = \frac{E_1^n a^2 - E_2^n(Q^2)a - E_3^n(Q^2)}{(a+4)(a+3)(a+2)(a+1)}, \quad (27)$$

with

$$E_1^n = 4\eta^n, \quad E_2^n = 2\eta^n + \frac{3\sqrt{2}\eta^n Q^2}{2m_N k}, \quad E_3^n = \frac{6\sqrt{2}\eta^n Q^2}{m_N k}.$$

The magnetic form factors are obtained from the Dirac and Pauli form factors as

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2). \quad (28)$$

Using the explicit expressions (22),(23), (24) for $F_1^N(Q^2)$ and $F_2^N(Q^2)$, one finds more compact forms for the $G_M^p(Q^2), G_M^n(Q^2)$:

$$\begin{aligned} G_M^p(Q^2) &= \frac{M_1^p a^2 + M_2^p a + M_3^p}{(a+4)(a+3)(a+2)(a+1)}, \\ M_1^p &= 4\eta^p + 1, \\ M_2^p &= \frac{6\sqrt{2}m_N \eta^p}{k} - 2\eta^p + 10, \\ M_3^p &= \frac{6\sqrt{2}m_N \eta^p}{k} + 24, \end{aligned} \quad (29)$$

$$\begin{aligned} G_M^n(Q^2) &= \frac{M_1^n a^2 + M_2^n a + M_3^n}{(a+4)(a+3)(a+2)(a+1)}, \\ M_1^n &= 4\eta^n, \\ M_2^n &= \frac{6\sqrt{2}m_N \eta^n}{k} - 2\eta^n, \\ M_3^n &= \frac{24\sqrt{2}m_N \eta^n}{k}. \end{aligned} \quad (30)$$

Static properties

The electric and magnetic radii are defined through the slopes of the corresponding form factors at $Q^2 = 0$,

$$\langle r_E^2 \rangle^N = -\frac{1}{G_E^N(0)} \left. \frac{6 dG_E^N(Q^2)}{dQ^2} \right|_{Q^2=0}, \quad \langle r_M^2 \rangle^N = -\frac{1}{G_M^N(0)} \left. \frac{6 dG_M^N(Q^2)}{dQ^2} \right|_{Q^2=0}. \quad (31)$$

Using the analytical expressions derived above, we obtain

$$\langle r_E^2 \rangle^p = \frac{3\sqrt{2}\eta^p}{2m_N k} + \frac{20 + \eta^p}{8k^2}, \quad \langle r_E^2 \rangle^n = \frac{\eta^n}{8k^2} + \frac{3\sqrt{2}\eta^n}{2m_N k}, \quad (32)$$

$$\langle r_M^2 \rangle^p = \frac{20 + \eta^p}{8\mu_p k^2} + \frac{11\sqrt{2}m_N \eta^p}{4\mu_p k^3}, \quad \langle r_M^2 \rangle^n = \frac{\eta^n}{8\mu_n k^2} + \frac{11\sqrt{2}m_N \eta^n}{4\mu_n k^3}. \quad (33)$$

$$\langle r_M^2 \rangle^p = \frac{20 + \eta^p}{8\mu_p k^2} + \frac{11\sqrt{2}m_N \eta^p}{4\mu_p k^3}, \quad \langle r_M^2 \rangle^n = \frac{\eta^n}{8\mu_n k^2} + \frac{11\sqrt{2}m_N \eta^n}{4\mu_n k^3}. \quad (34)$$

Notice that in the context of AdS/QCD charge radii have been discussed before in[10]. Our numerical results for the slopes compared rather well with data[15, 16] :

Quantity	This work	Experiment
$\langle r_E^2 \rangle^p$	$0.932 fm^2$	$\sim 0.77 fm^2$
$\langle r_E^2 \rangle^n$	$-0.147 fm^2$	$\sim -0.116 fm^2$
$\langle r_M^2 \rangle^p$	$0.86 fm^2$	$\sim 0.73 fm^2$
$\langle r_M^2 \rangle^n$	$0.88 fm^2$	$\sim 0.76 fm^2$

Table 1: Numerical results for the nucleon radii compared with experimental data.

4. Results and discussion

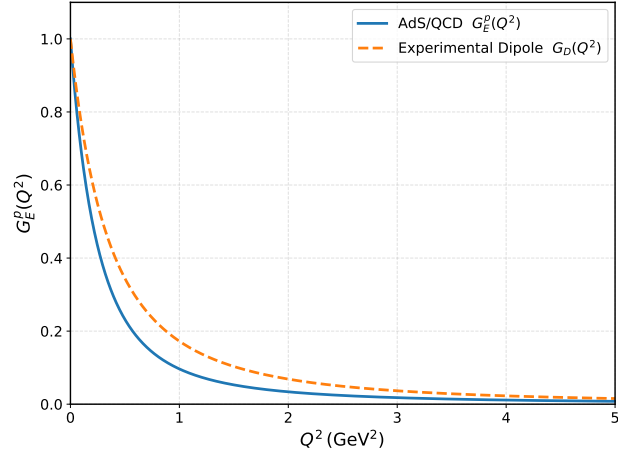


Figure 1: Proton electric form factor.

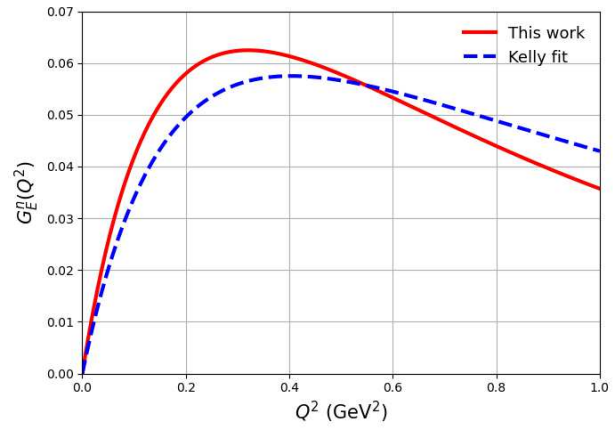


Figure 2: Neutron electric form factor.

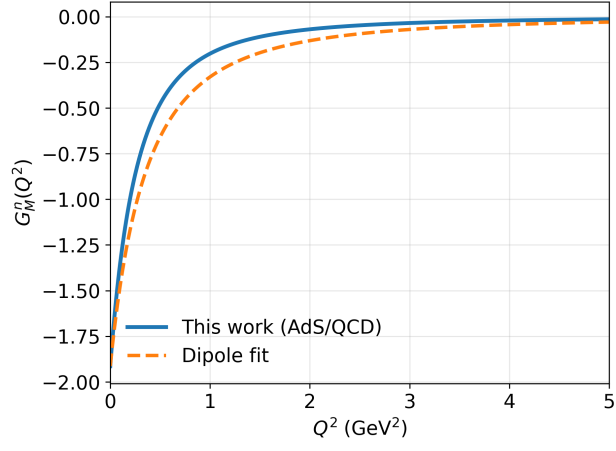


Figure 3: Neutron magnetic form factor.

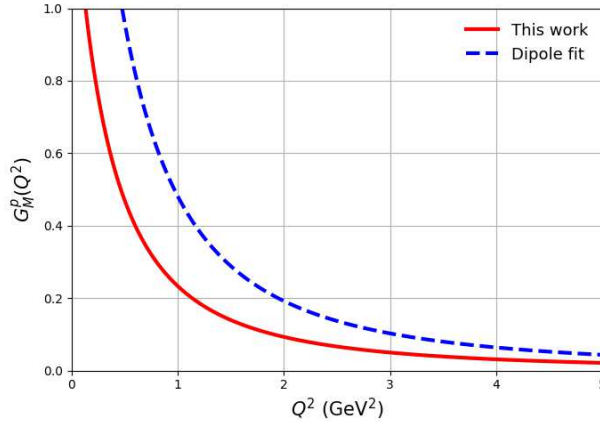


Figure 4: Proton magnetic form factor

The obtained form factors reproduce the correct normalization:

$$F_1^p(0) = 1, \quad F_1^n(0) = 0. \quad (35)$$

The analytic structure reflects the underlying holographic dynamics and exhibits the expected large- Q^2 scaling behavior, with the electromagnetic form factor asymptotically falling as $F(Q^2) \sim 1/Q^4$ as $Q^2 \rightarrow \infty$.

5. Conclusion

We have derived analytical expressions for nucleon electromagnetic form factors in a soft-wall AdS/QCD model. The closed-form results allow for a clear interpretation of nucleon structure and provide a useful basis for further studies. It is worth emphasizing that the main contribution of the present work is methodological rather than the introduction of an entirely new phenomenological framework. Although closely related calculations have been performed previously within holographic QCD, the present analysis employs a different algebraic realization of the relevant quantities and provides an explicit and systematic derivation of the results from the underlying soft-wall AdS/QCD framework. In particular, the intermediate analytical steps leading from the five-dimensional action and bulk profiles to the corresponding form factors and generalized distributions are presented explicitly, rather than being stated only in their final form. In this sense, the present work can also serve as a pedagogical reference for researchers and students who are

learning the technical methods of AdS/QCD and wish to follow the analytical construction of hadronic observables in a transparent manner.

6. Appendix A

By inserting Eq. (14) into Eq. (12), the Gaussian-type integral representations of $\Phi_1^N(x)$ and $\Phi_2^N(x)$ are obtained:

$$\begin{aligned}\Phi_1^{(p)}(x) &= \int_0^\infty e^{-\beta z^2} (\alpha_5 z^5 + \alpha_7 z^7 - \alpha_9 z^9) dz, \\ \Phi_1^{(n)}(x) &= \int_0^\infty e^{-\beta z^2} (a_7 z^7 - a_5 z^5 - a_9 z^9) dz, \\ \Phi_2^{(p,n)}(x) &= \sqrt{2} m_N k^7 \eta^{p,n} \int_0^\infty e^{-\beta z^2} z^7 dz\end{aligned}\tag{36}$$

It is straightforward to see that the constants appearing in Eq. (36) are determined as follows:

$$\begin{aligned}\alpha_5 &= k^6(1 - 2\eta^p), \quad \alpha_7 = k^8 \frac{(2\eta^p - 1)x + 2\eta^p + 1}{2(1 - x)}, \quad \alpha_9 = \frac{k^{10}\eta^p x}{1 - x}, \\ a_7 &= \eta^n \frac{k^8(1 + x)}{1 - x}, \quad a_5 = 2\eta^n k^6, \quad a_9 = \eta^n \frac{k^{10}x}{1 - x}.\end{aligned}\tag{37}$$

Taking into account that the Gaussian-type integrals appearing in Eq. (36) are given by

$$\begin{aligned}\int_0^\infty e^{-\beta z^2} z^5 dz &= \frac{1}{\beta^3}, \\ \int_0^\infty e^{-\beta z^2} z^7 dz &= \frac{3}{\beta^4}, \\ \int_0^\infty e^{-\beta z^2} z^9 dz &= \frac{12}{\beta^5}\end{aligned}\tag{38}$$

we obtain

$$\begin{aligned}\Phi_1^{(p)}(x) &= \frac{\alpha_5}{\beta^3} + \frac{3\alpha_7}{\beta^4} - \frac{12\alpha_9}{\beta^5} \\ \Phi_1^{(n)}(x) &= \frac{3a_7}{\beta^4} - \frac{a_5}{\beta^3} - \frac{12a_9}{\beta^5} \\ \Phi_2^{(p,n)}(x) &= \frac{3\sqrt{2}\eta^{p,n}m_N k^7}{\beta^4}.\end{aligned}\tag{39}$$

7. Appendix B

Applying the Beta integral

$$B(p, q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

to each contribution to the proton form factor $F_1^{(p)}(Q^2)$ in Eq. (20),

$$\begin{aligned} F_1^{(p)}(Q^2) = \int_0^1 dx x^a(1-x) \left[12\eta^p x^2 - (9\eta^p + \frac{3}{2})x + \eta^p + \frac{5}{2} \right] = 12\eta^p \int_0^1 x^{a+2}(1-x) dx - \\ - (9\eta^p + \frac{3}{2}) \int_0^1 x^{a+1}(1-x) dx + (\eta^p + \frac{5}{2}) \int_0^1 x^a(1-x) dx \end{aligned} \quad (40)$$

the integration over x is performed analytically,

$$\begin{aligned} 12\eta^p \int_0^1 x^{a+2}(1-x) dx &= \frac{12\eta^p \Gamma(a+3)\Gamma(2)}{\Gamma(a+5)} = \frac{12\eta^p \Gamma(a+3)}{(a+4)(a+3)\Gamma(a+3)} = \frac{12\eta^p}{(a+4)(a+3)} \\ (9\eta^p + \frac{3}{2}) \int_0^1 x^{a+1}(1-x) dx &= \frac{(9\eta^p + \frac{3}{2})\Gamma(a+2)}{\Gamma(a+4)} = \frac{(9\eta^p + \frac{3}{2})\Gamma(a+2)}{(a+3)(a+2)\Gamma(a+2)} = \frac{(9\eta^p + \frac{3}{2})}{(a+3)(a+2)} \\ (\eta^p + \frac{5}{2}) \int_0^1 x^a(1-x) dx &= \frac{\eta^p + \frac{5}{2}}{(a+2)(a+1)} \end{aligned} \quad (41)$$

For convenience, the integral contributions obtained above can be written in a compact form as

$$\begin{aligned} F_1^{(p)}(Q^2) &= \frac{12\eta^p}{(a+4)(a+3)} - \frac{(9\eta^p + \frac{3}{2})}{(a+3)(a+2)} + \frac{\eta^p + \frac{5}{2}}{(a+2)(a+1)} = \\ &= \frac{12\eta^p(a^2 + 3a + 2) - (9\eta^p + \frac{3}{2})(a^2 + 5a + 4) + (\eta^p + \frac{5}{2})(a^2 + 7a + 12)}{(a+4)(a+3)(a+2)(a+1)} = \\ &= \frac{(4\eta^p + 1)a^2 + (10 - 2\eta^p)a + 24}{(a+4)(a+3)(a+2)(a+1)} \end{aligned} \quad (42)$$

Following the same procedure, the neutron form factor $F_1^{(n)}(Q^2)$ can be calculated analytically.

$$\begin{aligned}
F_1^{(n)}(Q^2) &= \int_0^1 dx x^a (1-x) [12\eta^n x^2 - 9\eta^n x + \eta^n] = 12\eta^n \int_0^1 x^{a+2} (1-x) dx \\
&\quad - 9\eta^n \int_0^1 x^{a+1} (1-x) dx + \eta^n \int_0^1 x^a (1-x) dx \\
12\eta^n \int_0^1 x^{a+2} (1-x) dx &= \frac{12\eta^n \Gamma(a+3) \Gamma(2)}{\Gamma(a+5)} = \frac{12\eta^n \Gamma(a+3)}{(a+4)(a+3)\Gamma(a+3)} = \frac{12\eta^n}{(a+4)(a+3)} \\
9\eta^n \int_0^1 x^{a+1} (1-x) dx &= \frac{9\eta^n \Gamma(a+2)}{\Gamma(a+4)} = \frac{9\eta^n \Gamma(a+2)}{(a+3)(a+2)\Gamma(a+2)} = \frac{9\eta^n}{(a+3)(a+2)} \\
\eta^n \int_0^1 x^a (1-x) dx &= \frac{\eta^n}{(a+2)(a+1)} \tag{43}
\end{aligned}$$

$$\begin{aligned}
F_1^{(n)}(Q^2) &= \frac{12\eta^n}{(a+4)(a+3)} - \frac{9\eta^n}{(a+3)(a+2)} + \frac{\eta^n}{(a+2)(a+1)} = \\
&= \frac{4\eta^n a^2 - 2\eta^n a}{(a+4)(a+3)(a+2)(a+1)} \tag{44}
\end{aligned}$$

Following the same procedure, we now derive the Pauli form factor $F_2^{(p,n)}(Q^2)$:

$$\begin{aligned}
F_2^{(p,n)}(Q^2) &= \frac{3\sqrt{2}m_N \eta^{p,n}}{k} \int_0^1 x^a (1-x)^2 dx = \frac{3\sqrt{2}m_N \eta^{p,n}}{k} \frac{\Gamma(a+1)\Gamma(3)}{\Gamma(a+4)} = \\
&= \frac{6\sqrt{2}m_N \eta^{p,n}}{k} \frac{1}{(a+3)(a+2)(a+1)} \tag{45}
\end{aligned}$$

We also note that the parameters $\eta^{p,n}$ entering Eq. (11) can be determined from Eq. (45) by requiring the normalization condition

$$F_2^{(p,n)}(0) = \kappa_{p,n},$$

where $\kappa_{p,n}$ denote the anomalous magnetic moments of the proton and neutron:

$$F_2^{(p,n)}(0) = k_{p,n} = \frac{\sqrt{2}m_N \eta^{p,n}}{k}, \eta^{p,n} = \frac{k k_{p,n}}{\sqrt{2}m_N}. \tag{46}$$

Using the experimental values of the anomalous magnetic moments,

$$\kappa_p = 1.792847, \quad \kappa_n = -1.913043,$$

and taking $k = 350 \text{ MeV}$ and $m_N = 0.938 \text{ GeV}$, Eq. (46) yields

$$\eta^p \simeq 0.473, \quad \eta^n \simeq -0.505.$$

8. Appendix C

In this section, we evaluate the Sachs form factors. The electric and magnetic Sachs form factors can be expressed in terms of the $F_1^N(Q^2)$ and $F_2^N(Q^2)$, as

$$\begin{aligned} G_E^p(Q^2) &= F_1^p(Q^2) - \frac{Q^2 F_2^p(Q^2)}{4m_N^2} = \frac{(4\eta^p + 1)a^2 + (10 - 2\eta^p)a + 24}{(a+4)(a+3)(a+2)(a+1)} - \\ &- \frac{Q^2}{4m_N^2} \frac{6\sqrt{2}m_N\eta^p}{k} \frac{1}{(a+3)(a+2)(a+1)} = \frac{E_1^p a^2 + E_2^p(Q^2)a + E_3^p(Q^2)}{(a+4)(a+3)(a+2)(a+1)}. \\ E_1^p &= 1 + 4\eta^p, E_2^p(Q^2) = 10 - 2\eta^p - \frac{3\sqrt{2}\eta^p Q^2}{2m_N k}, \\ E_3^p(Q^2) &= 24 - \frac{6\sqrt{2}\eta^p Q^2}{m_N k} \end{aligned} \quad (47)$$

$$\begin{aligned} G_M^p(Q^2) &= F_1^p(Q^2) + F_2^p(Q^2) = \frac{(4\eta^p + 1)a^2 + (10 - 2\eta^p)a + 24}{(a+4)(a+3)(a+2)(a+1)} + \\ &+ \frac{6\sqrt{2}m_N\eta^p}{k} \frac{1}{(a+3)(a+2)(a+1)} \\ G_M^p(Q^2) &= \frac{M_1^p a^2 + M_2^p a + M_3^p}{(a+4)(a+3)(a+2)(a+1)}, \\ M_1^p &= 4\eta^p + 1, \\ M_2^p &= \frac{6\sqrt{2}m_N\eta^p}{k} - 2\eta^p + 10, M_3^p = \frac{6\sqrt{2}m_N\eta^p}{k} + 24 \end{aligned} \quad (48)$$

$$\begin{aligned}
G_E^n(Q^2) &= F_1^n(Q^2) - \frac{Q^2 F_2^n(Q^2)}{4m_N^2} = \frac{4\eta^n a^2 - 2\eta^n a}{(a+4)(a+3)(a+2)(a+1)} - \\
&\quad - \frac{6\sqrt{2}m_N\eta^n Q^2}{4m_N^2 k} \frac{1}{(a+3)(a+2)(a+1)} = \\
&= \frac{4\eta^n a^2 - 2\eta^n a - \frac{3\sqrt{2}\eta^n Q^2}{2m_N k}(a+4)}{(a+4)(a+3)(a+2)(a+1)} \\
G_E^n(Q^2) &= \frac{E_1^n a^2 - E_2^n(Q^2)a - E_3^n(Q^2)}{(a+4)(a+3)(a+2)(a+1)} \quad (49) \\
E_1^n &= 4\eta^n \\
E_2^n(Q^2) &= 2\eta^n + \frac{3\sqrt{2}\eta^n Q^2}{2m_N k} \\
E_3^n(Q^2) &= \frac{6\sqrt{2}\eta^n}{m_N k} Q^2
\end{aligned}$$

$$\begin{aligned}
G_M^n(Q^2) &= F_1^n(Q^2) + F_2^n(Q^2) = \frac{4\eta^n a^2 - 2\eta^n a}{(a+4)(a+3)(a+2)(a+1)} + \\
&\quad + \frac{6\sqrt{2}m_N\eta^n}{k} \frac{1}{(a+3)(a+2)(a+1)} = \\
&= \frac{4\eta^n a^2 - 2\eta^n a + \frac{6\sqrt{2}m_N\eta^n}{k}(a+4)}{(a+4)(a+3)(a+2)(a+1)} = \\
&= \frac{4\eta^n a^2 + (\frac{6\sqrt{2}m_N\eta^n}{k} - 2\eta^n)a + \frac{24\sqrt{2}m_N\eta^n}{k}}{(a+4)(a+3)(a+2)(a+1)} \quad (50) \\
G_M^n(Q^2) &= \frac{M_1^n a^2 + M_2^n a + M_3^n}{(a+4)(a+3)(a+2)(a+1)}, \\
M_1^n &= 4\eta^n, \\
M_2^n &= \frac{6\sqrt{2}m_N\eta^n}{k} - 2\eta^n, \quad M_3^n = \frac{24\sqrt{2}m_N\eta^n}{k}.
\end{aligned}$$

Declarations

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Availability of data and materials

The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Competing interests

The author declares that there are no competing interests.

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Authors contributions

The author conceived the study, performed the theoretical calculations, analyzed the results, and wrote the manuscript. The author read and approved the final manuscript.

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