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Xiaoyu Hu

Yanshan University

Yongsheng Zhao

Yanshan University

Zongke Liu

Yanshan University

Chencheng Zhang

Yanshan University

Fei Hu

China Academy of Space Technology (Xi'an)

Bo Han

bohan@ysu.edu.cn

Yanshan University

Jiantao Yao

Yanshan University

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Xiaoyu Hu ^{a,+}, Yongsheng Zhao ^{a,b,+}, Zongke Liu ^a, Chencheng Zhang ^a, Fei Hu ^c, Bo Han ^{a,b,*},
Jiantao Yao ^{a,b}

+ These two authors have made equal contributions and are co-first authors

* Corresponding author: Bo Han

Corresponding author E-mail: bohan@ysu.edu.cn

^a School of Mechanical Engineering, Yanshan University, Qinhuangdao 066004, China

^b Hebei Provincial Key Laboratory of Parallel Robot and Mechatronic System, Yanshan University, Qinhuangdao 066004, China

^c China Academy of Space Technology (Xi'an), Xi'an 710100, China

Abstract

Cross-sectional deformation of fully parameterized absolute nodal coordinate formulation (ANCF) beams can affect the representation of revolute joint axes when an orthogonal tangent frame is used for joint modeling. To address this issue, a non-orthogonal cross-section frame that follows the local deformation of the beam cross-section is introduced as the joint coordinate system. The geometric relationship between the tangent and cross-section frames is derived, showing that the tangent frame introduces a joint axis direction error through orthogonalization under xy-shear deformation, whereas the cross-section frame retains the material-defined joint axis direction. The proposed frame is then used to formulate the revolute joint constraints and driving torsion springs, which are subsequently assembled according to the mechanism topology to establish the system-level dynamic equations. Numerical results for a basic deployable module of the truss deployable antenna show that the joint axis direction error of the tangent frame follows the xy-shear deformation predicted by the geometric analysis, whereas the cross-section frame remains aligned with the material-defined joint axis direction. Relative to the Adams/View reference simulation, the cross-section frame formulation reduces the standard deviation of the von Mises stress residual from 26.3 MPa to 2.9 MPa, corresponding to an approximately 89.0% reduction. Comparisons with Adams/View reference simulations under three torsion spring layouts show that the proposed model reproduces the principal position, velocity, and acceleration responses, as well as the deployment-shock-folding sequence. The proposed formulation provides an effective modeling approach for deployment prediction and dynamic design evaluation of flexible deployable structures.

Keywords: Truss deployable antenna; absolute nodal coordinate formulation; rigid-flexible coupling dynamics; revolute joint; non-orthogonal cross-section frame.

1 Introduction

Satellites are widely used in communication, remote sensing, navigation, and Earth observation, and spaceborne antennas are key components for signal transmission and reception in these applications [1]. The growing demand for wider coverage, longer observation range, and improved sensing and communication performance has led to increasingly large and structurally complex space antennas. Since the launch volume is strictly limited by the launch vehicle fairing, large space structures are generally folded during launch and deployed after reaching orbit [2,3].

Truss deployable antennas are attractive for large aperture applications because modular truss structures can provide both a compact folded configuration and a large deployed configuration. Various modular deployable trusses and reconfigurable support structures have been developed for different antenna architectures [4,5]. Non-uniform-height modules and heterogeneous modular mechanisms have also been proposed to improve geometric adaptability and deployment performance [6,7]. Other representative concepts include scissors-based modules, tetrahedral basic modules, and hoop-rib mechanisms [8-10]. Surface partitioning and modular networking methods have been investigated for organizing these modules into large supporting structures [11,12], while double-ring and closed-loop configurations offer alternative structural schemes for large deployable trusses [13,14].

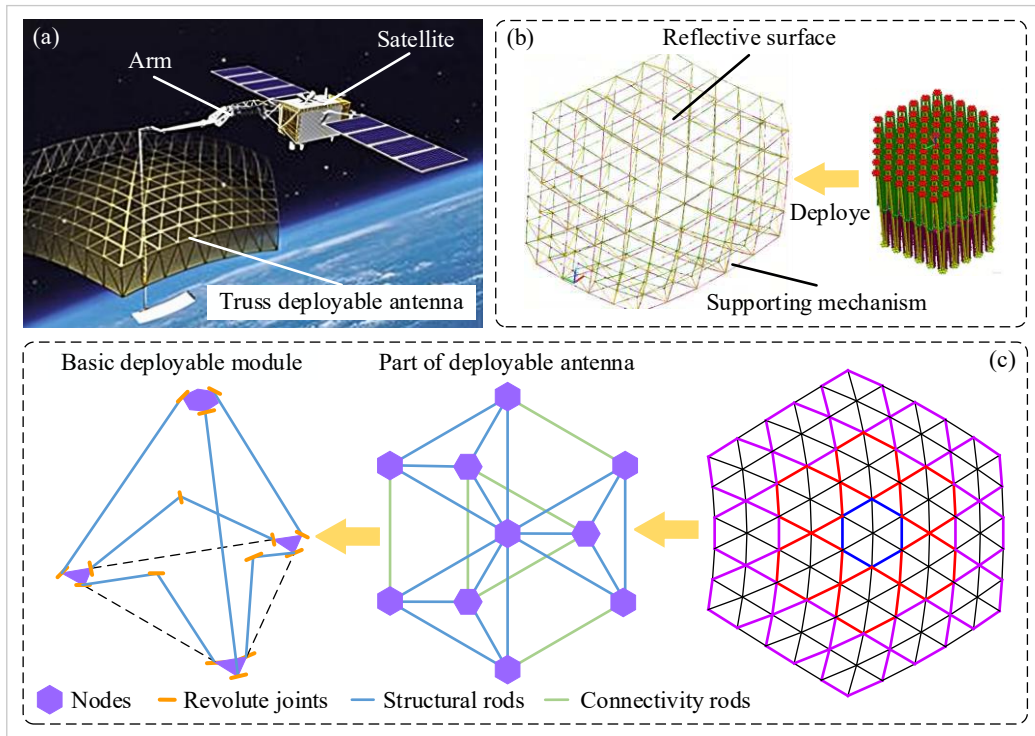


Fig.1 Spaceborne truss deployable antenna and its basic deployable module:
(a) On-orbit configuration; (b) Deployed and folded states; (c) Mechanism components

Fig.1(a) shows the on-orbit configuration of a spaceborne truss deployable antenna, and Fig.1(b) presents its deployed and folded states. After deployment, the supporting truss carries the metallic mesh and forms the prescribed parabolic reflector. As illustrated in Fig.1(c), the antenna is

assembled from multiple basic deployable modules consisting of nodes and rods connected by revolute joints. Adjacent modules share connecting nodes and are linked by additional rods, and the number and arrangement of the modules can be adjusted according to the required aperture and structural configuration. Torsion springs installed at the revolute joints provide the driving torque during deployment.

Reliable deployment is essential for the on-orbit operation of such antennas. Ground deployment tests remain an important means of performance evaluation, but they are expensive and time-consuming, and it is difficult to investigate a wide range of structural and driving parameters experimentally. Moreover, the ground-test environment differs from the orbital condition because of gravity, and gravity compensation is therefore required when ground experiments are used for on-orbit prediction [15]. Numerical simulation provides a practical complement to experiments by allowing structural parameters, driving conditions, and environmental factors to be varied efficiently. For this reason, an accurate rigid-flexible coupling dynamic model is important for deployment prediction and design evaluation.

The deployment dynamics of space antennas has been studied from different perspectives. Multibody models incorporating torsion spring driving forces have been used to analyze deployment motion and control [16,17]. Structural design, dynamic analysis, and prototype experiments have also been combined to evaluate the deployment characteristics of large deployable trusses [18], and dynamic modeling has been integrated with structural optimization and experimental verification [19,20]. The influence of structural flexibility has been considered in deployment models coupling flexible trusses and cable-net systems [21]. For deployed antennas, modal interaction and nonlinear vibration have also received considerable attention [22,23]. These studies show that deployment dynamics has been investigated extensively. However, the accuracy of a rigid-flexible multibody model depends not only on the description of flexible components, but also on how the joints transmit motion and forces between connected bodies.

The Absolute Nodal Coordinate Formulation (ANCF) is widely used for flexible components undergoing large displacement and deformation. Flexible components are described by absolute nodal positions and gradient vectors, leading to a constant mass matrix without the centrifugal and Coriolis inertia terms associated with incremental rotation coordinates [24,25]. Different ANCF element formulations have been developed to improve numerical performance and applicability, including treatments for locking in continuum beam elements [26], improved formulations for slender and variable-cross-section beams [27], and parameterized descriptions of geometrically complex shell structures [28]. High-precision beam formulations based on other flexible multibody approaches have also been developed for large-rotation problems [29].

ANCF has also been extended from individual flexible components to more complex multibody applications. Thermoelastic formulations have been applied to deployable thin plate structures [30], while ALE-ANCF formulations have been developed for cables and beams involving specialized constraints or variable length descriptions [31,32] and have been applied to

tethered-satellite dynamics [33]. Model-reduction and numerical-integration techniques have been introduced to improve the efficiency and stability of nonlinear flexible multibody simulations [34,35]. In space deployable applications, ANCF has been used to model flexible deployable primary mirrors and to predict deployment motion, structural deformation, and vibration [36]. These developments provide a useful basis for modeling flexible components in deployable multibody systems.

However, accurate flexible component modeling alone does not ensure an accurate multibody system model, since the relative motion and force transfer between adjacent components are governed by joints. Sugiyama et al. [37] formulated three-dimensional joint constraints for ANCF flexible multibody systems. Gantoi et al. [38] later introduced ANCF joint coordinate systems based on gradient directions and a tangent frame, providing a direct example of tangent-frame-based joint descriptions in ANCF. Fully parameterized ANCF elements, however, retain transverse gradient vectors and therefore allow deformation of the beam cross-section. Hussein et al. [39] showed that eliminating relative translation and rotation at a connection does not necessarily suppress such cross-sectional deformation. Sugiyama and Yamashita [40] introduced non-generalized intermediate coordinates and imposed orthonormality conditions on the gradient coordinates to define a rigid surface at the joint interface, so that conventional rigid-body joint constraint libraries could be used. This treatment is consistent with formulations in which a rigid joint surface is enforced. In the present work, however, the cross-sectional deformation at the joint node is retained. When shear deformation causes the material directions of the cross-section to become non-orthogonal, reconstructing an orthogonal joint frame may change the direction used to represent a revolute axis embedded in the cross-section. Nonideal joint effects such as clearance, contact, friction, lubrication, and wear have also been included in rigid-flexible multibody models [41,42], and an ANCF-based spatial flexible revolute clearance joint has recently been developed for flexible deployable systems [43]. Despite these developments, the effect of joint frame orthogonalization on the representation of an ideal revolute axis in a deformable fully parameterized ANCF cross-section has not been examined in sufficient detail.

To address this issue, a non-orthogonal cross-section frame is introduced for revolute joint modeling in fully parameterized ANCF beams. The joint axis direction is taken directly from the corresponding cross-sectional material gradient rather than reconstructed through orthogonalization. The geometric relationship between the tangent and cross-section frames is derived to identify the deformation mode responsible for the difference in joint axis direction representation. On this basis, both the revolute joint constraints and the generalized forces and Jacobians of the driving torsion springs are formulated consistently within the proposed cross-section frame, so that joint kinematics and spring torque transmission are described using the same deformation-following coordinate representation. These component- and joint-level formulations are subsequently organized and assembled according to the mechanism topology to obtain the system-level dynamic equations. A basic deployable module is used for numerical verification. The two joint frames are first compared

in terms of joint axis direction representation and local kinematic and mechanical responses, and the assembled model is subsequently compared with an Adams/View reference model under three torsion spring layouts.

The remainder of this paper is organized as follows. Section 2 presents the formulations of the rigid and flexible components, joint coordinate systems, revolute joints, and torsion springs. Section 3 describes the topology analysis and assembly of the system-level dynamic equations. Section 4 gives the numerical solution procedure. Section 5 presents the numerical verification and discussion. Section 6 summarizes the main conclusions and limitations.

2 Modeling of components, joints, and torsion springs

2.1 Modeling of rigid and flexible components

As shown in Fig.2(a), the rigid component has three translational DOFs (X, Y, Z) and three rotational DOFs (U, V, W); the centroid position $\mathbf{r}^c = [x \ y \ z]^T$ and ZYX Euler angles $\boldsymbol{\theta} = [\alpha \ \beta \ \gamma]^T$ are used as generalized coordinates to describe these six DOFs, and the corresponding attitude transformation matrix is:

$$\mathbf{A} = \mathbf{A}(\alpha)\mathbf{A}(\beta)\mathbf{A}(\gamma) \quad (1)$$

The inertia matrix $\mathbf{M}$ and generalized external force vector $\mathbf{Q}$ are then formulated in terms of the generalized coordinates.

The fully parameterized ANCF beam element is shown in Fig.2(b).

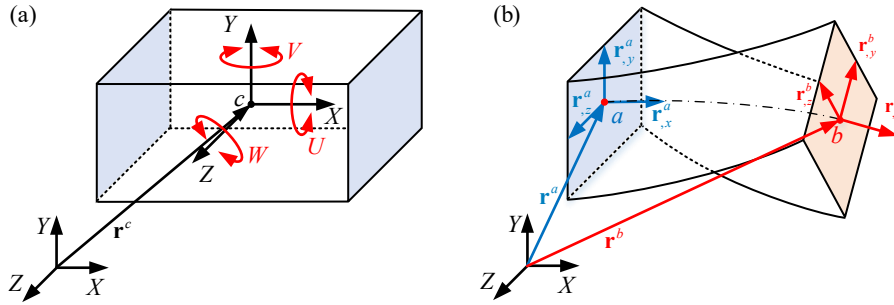


Fig.2 Schematic of rigid component and the fully parameterized ANCF beam element:
(a) Rigid component; (b) Flexible component

The position of an arbitrary particle on the beam element is given by:

$$\mathbf{r} = \mathbf{S}\mathbf{e} \quad (2)$$

The shape function $\mathbf{S}$ can be written as:

$$\mathbf{S} = [S_1\mathbf{I}_3 \ S_2\mathbf{I}_3 \ S_3\mathbf{I}_3 \ S_4\mathbf{I}_3 \ S_5\mathbf{I}_3 \ S_6\mathbf{I}_3 \ S_7\mathbf{I}_3 \ S_8\mathbf{I}_3] \quad (3)$$

where $S_1 = 1 - 3\xi^2 + 2\xi^3$, $S_2 = l(\xi - 2\xi^2 + \xi^3)$, $S_3 = l(\eta - \xi\eta)$, $S_4 = l(\zeta - \xi\zeta)$, $S_5 = 3\xi^2 - 2\xi^3$, $S_6 = l(-\xi^2 + \xi^3)$, $S_7 = l\xi\eta$, $S_8 = l\xi\zeta$ ($\xi = x/l$, $\eta = y/l$, $\zeta = z/l$), $\mathbf{I}_3$ is a 3×3 identity matrix.

The generalized coordinate vector $\mathbf{e}$ of the beam element is composed of absolute nodal

coordinates $\mathbf{e}^a$ and $\mathbf{e}^b$ at endpoints a and b ; each absolute nodal coordinate includes a nodal position vector and its position-gradient terms:

$$\begin{aligned}\mathbf{e}^a &= [\mathbf{r}^a \quad \mathbf{r}_{,x}^a \quad \mathbf{r}_{,y}^a \quad \mathbf{r}_{,z}^a]^T \\ &= \left[r_1^a \quad r_2^a \quad r_3^a \quad \frac{\partial r_1^a}{\partial x} \quad \frac{\partial r_2^a}{\partial x} \quad \frac{\partial r_3^a}{\partial x} \quad \frac{\partial r_1^a}{\partial y} \quad \frac{\partial r_2^a}{\partial y} \quad \frac{\partial r_3^a}{\partial y} \quad \frac{\partial r_1^a}{\partial z} \quad \frac{\partial r_2^a}{\partial z} \quad \frac{\partial r_3^a}{\partial z} \right]^T\end{aligned}\quad (4)$$

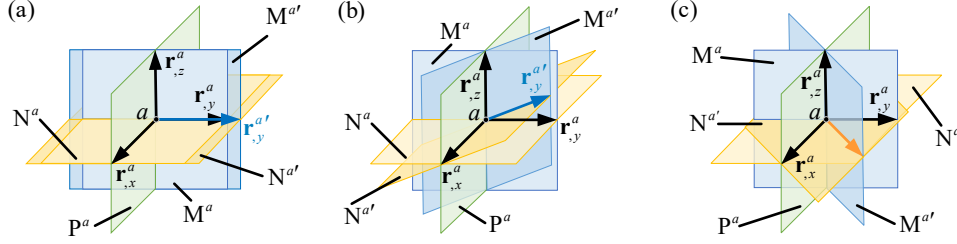


Fig.3 Interpretation of absolute nodal coordinates:

(a) Length change DOF; (b) Rotation DOF about $\mathbf{r}_{,x}^a$; (c) Rotation DOF about $\mathbf{r}_{,z}^a$

Taking the absolute nodal coordinate of node a in Eq.(4) as an example, $\mathbf{r}^a$ denotes the position of the cross-section center at node a . The gradients $\mathbf{r}_{,y}^a$ and $\mathbf{r}_{,z}^a$ define the two cross-sectional material directions, whereas $\mathbf{r}_{,x}^a$ describes the local gradient of the element centerline. $\mathbf{r}_{,x}^a$, $\mathbf{r}_{,y}^a$ and $\mathbf{r}_{,z}^a$ all include a length-change DOF and two rotational DOFs; taking $\mathbf{r}_{,y}^a$ as an example and assuming $\mathbf{r}_{,x}^a$ and $\mathbf{r}_{,z}^a$ are fixed, a length change of $\mathbf{r}_{,y}^a$ in Fig.3(a) causes planes N^a and M^a spanned by $\mathbf{r}_{,y}^a - \mathbf{r}_{,x}^a$ and $\mathbf{r}_{,y}^a - \mathbf{r}_{,z}^a$ to undergo stretching or compression along direction $\mathbf{r}_{,y}^a$. In Fig.3(b), $\mathbf{r}_{,y}^a$ rotates about $\mathbf{r}_{,x}^a$, and this motion induces torsional warping of M^a and rotation of N^a . In Fig.3(c), $\mathbf{r}_{,y}^a$ rotates about $\mathbf{r}_{,z}^a$, and this motion leads to rotation of M^a and N^a . $\mathbf{r}_{,x}^a$ and $\mathbf{r}_{,z}^a$ are analogous to $\mathbf{r}_{,y}^a$. These deformation-related DOFs are examined in the following section to identify the cross-sectional deformation modes under revolute joint constraints.

The standard ANCF formulations are used for the element mass matrix and generalized gravitational force [24,25] and are not repeated here. To define the deformation measures and elastic forces used in the subsequent analysis, the deformation gradient of the beam element is written as:

$$\mathbf{F} = \frac{\partial \mathbf{r}}{\partial \mathbf{r}_0} = \frac{\partial \mathbf{r}}{\partial \boldsymbol{\xi}} \left(\frac{\partial \mathbf{r}_0}{\partial \boldsymbol{\xi}} \right)^{-1} = \mathbf{J} \mathbf{J}_0^{-1} \quad (5)$$

where $\mathbf{r}$ and $\mathbf{r}_0$ are the position vectors in the reference and current configurations, respectively, $\boldsymbol{\xi} = [x \ y \ z]^T$ denotes the material coordinates, and $\mathbf{J}$ and $\mathbf{J}_0^{-1}$ are the corresponding position gradient matrices.

The Green-Lagrange strain tensor is defined as:

$$\mathbf{G} = \frac{1}{2} (\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}) \quad (6)$$

For the isotropic elastic material considered in this study, the second Piola-Kirchhoff stress is

evaluated using the Saint Venant-Kirchhoff constitutive relation:

$$\mathbf{P} = \lambda \text{tr}(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E} \quad (7)$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \mu = \frac{E}{2(1+\nu)} \quad (8)$$

with E and ν denoting Young's modulus and Poisson's ratio, respectively. The element strain energy and the corresponding elastic generalized force are therefore expressed as:

$$U = \frac{1}{2} \int_V \mathbf{P} : \mathbf{G} dV \quad (9)$$

$$\mathbf{Q}_k = -\frac{\partial U}{\partial \mathbf{e}} \quad (10)$$

For comparison with the stress response obtained from the reference simulation, the second Piola-Kirchhoff stress is pushed forward to the Cauchy stress according to:

$$\boldsymbol{\sigma} = \frac{1}{J_F} \mathbf{F} \mathbf{P} \mathbf{F}^T, J_F = \det(\mathbf{F}) \quad (11)$$

The von Mises equivalent stress used in Section 5 is subsequently evaluated from the Cauchy stress tensor.

2.2 Beam element deformation analysis under revolute joint constraints

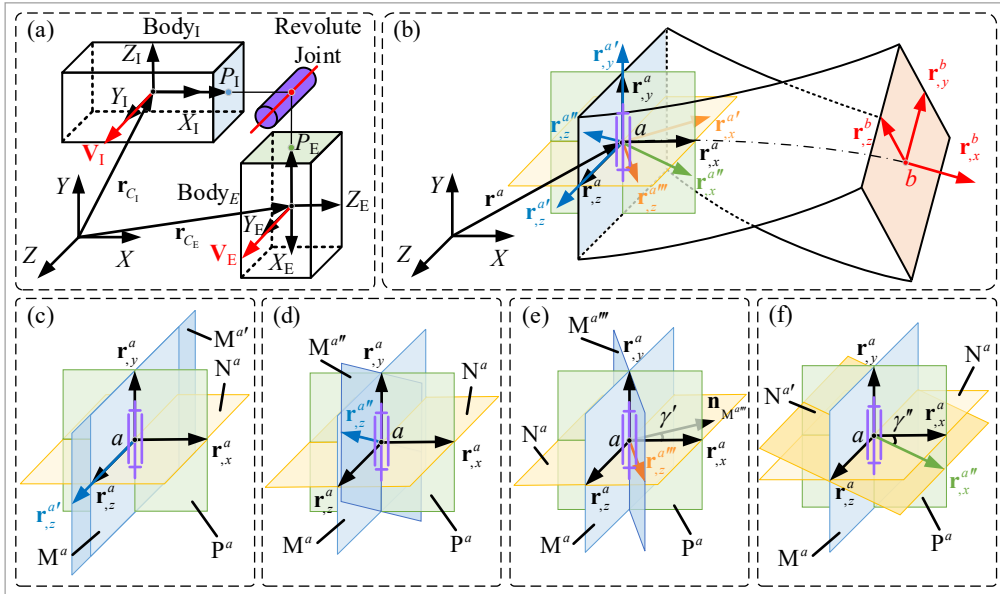


Fig.4 Physical interpretation of the absolute nodal coordinates and kinematic characteristics of the revolute joint constraints:

- (a) Revolute joint model; (b) Nodal coordinates of the ANCF beam subjected to the revolute joint constraints; (c)-(f) Deformation modes at the constrained node

An ideal revolute joint in multibody dynamics is represented by five independent geometric constraints. When the joint connects two rigid components, as illustrated in Fig.4(a), the position coincidence constraints require the two joint points to remain coincident, while the axis parallelism constraints require the two joint axes to remain parallel. These constraints eliminate three relative translation DOFs and two relative rotation DOFs, leaving only the relative rotation about the common joint axis.

For a flexible component connected through a revolute joint, the joint point is located at beam node a . In the reference configuration, the joint axis is physically embedded in the attachment cross-section and aligned with its material y direction. To define the current joint axis direction independently of the selected joint frame, two material points $J^+ : \left(x_a, \frac{d_J}{2}, 0\right)$ and $J^- : \left(x_a, -\frac{d_J}{2}, 0\right)$ are introduced on the initial joint axis, where d_J denotes the initial distance between the two material points.

For the fully parameterized ANCF beam adopted in this study, the position field at the section containing node a is linear with respect to the cross-sectional coordinates and can be expressed as:

$$\mathbf{r}(x_J, y, z) = \mathbf{r}^a + y\mathbf{r}_{,y}^a + z\mathbf{r}_{,z}^a \quad (12)$$

where $\mathbf{r}^a$ is the nodal position vector, and $\mathbf{r}_{,y}^a$ and $\mathbf{r}_{,z}^a$ are the absolute position gradients associated with the two cross-sectional material directions. The current positions of J^+ and J^- are therefore:

$$\mathbf{r}_{J^+} = \mathbf{r}^a + \frac{d_J}{2} \mathbf{r}_{,y}^a \quad (13)$$

$$\mathbf{r}_{J^-} = \mathbf{r}^a - \frac{d_J}{2} \mathbf{r}_{,y}^a \quad (14)$$

Accordingly, the current vector of the material-defined joint axis is:

$$\mathbf{v}^a = \mathbf{r}_{J^+} - \mathbf{r}_{J^-} = d_J \mathbf{r}_{,y}^a \quad (15)$$

and its unit direction is given by:

$$\bar{\mathbf{v}}^a = \frac{\mathbf{r}_{,y}^a}{\|\mathbf{r}_{,y}^a\|} \quad (16)$$

Therefore, the revolute joint axis at node a can be represented by the absolute position gradient $\mathbf{r}_{,y}^a$. Based on the above definition, the geometric constraints of the revolute joint acting on the flexible component are illustrated in Fig.4 (b). The coincidence constraint requires the joint points on the two connected components to remain coincident, thereby restricting the relative translation DOFs of node a . The axis parallelism constraint requires the direction of $\mathbf{r}_{,y}^a$, which represents the joint axis, to remain parallel to the corresponding joint axis of the connected component. Therefore, the two rotation DOFs that change the direction of $\mathbf{r}_{,y}^a$ are constrained, whereas its length variation remains admissible. The remaining gradient vectors can still vary with the deformation of the beam. Consequently, node a retains the deformation-related DOFs associated with $\mathbf{r}_{,x}^a$, the length variation of $\mathbf{r}_{,y}^a$, and the length and directional variations of $\mathbf{r}_{,z}^a$. These remaining DOFs allow the beam element to undergo the deformation modes illustrated in Fig.4(c)-(f).

- (1) In Fig.4 (c), $\mathbf{r}_{,z}^a$ changes to $\mathbf{r}_{,z}^{a'}$, and M^a is correspondingly deformed into $M^{a'}$. An analogous length change can occur in $\mathbf{r}_{,y}^a$. Therefore, the revolute joint constraints do not suppress in-plane extension or distortion of the cross-section.
- (2) In Fig.4(d), $\mathbf{r}_{,z}^a$ rotates to $\mathbf{r}_{,z}^{a''}$, while the constrained joint axis direction represented by $\mathbf{r}_{,y}^a$ remains unchanged. Consequently, M^a is deformed into $M^{a''}$ through a relative change between its two material directions. This mode represents a torsion-related cross-sectional distortion permitted by the fully parameterized beam description.
- (3) In Fig.4(e), $\mathbf{r}_{,z}^a$ rotates to $\mathbf{r}_{,z}^{a'''}$, causing M^a to deform into $M^{a'''}$. The angle γ' between $\mathbf{r}_{,x}^a$ and the normal vector $\mathbf{n}_{M^{a'''}}$ of the deformed plane characterizes one shear deformation mode. An analogous shear mode can be produced by the corresponding variation of $\mathbf{r}_{,x}^a$.
- (4) In Fig.4(f), $\mathbf{r}_{,x}^a$ rotates to $\mathbf{r}_{,x}^{a''}$. The angle γ'' between $\mathbf{r}_{,x}^{a''}$ and the normal direction of M^a characterizes another independent shear deformation mode. Hence, fixing the joint axis does not eliminate the shear deformation of the connected beam section.
- (5) Axial extension, bending, and torsion deformation of the beam element depend on the relative evolution of the nodal coordinates at nodes a and b . Because the revolute joint constraints act only on the joint point and axis direction at node a , these element-level deformation modes are not kinematically eliminated. Their amplitudes are influenced by the joint reactions, element stiffness, and the motion of the adjacent node.

2.3 ANCF-based joint coordinate systems: formulation and comparison

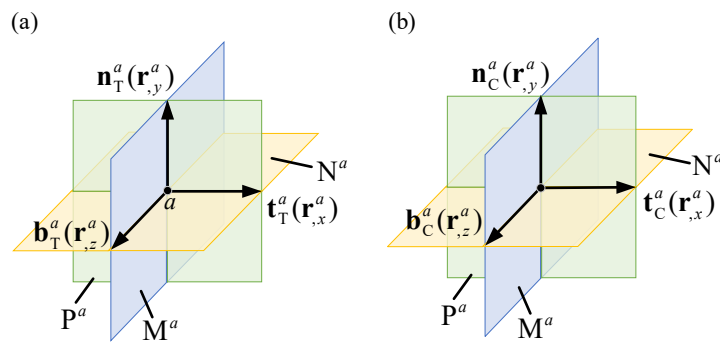


Fig.5 Two types of joint coordinate systems:
(a) Schematic of tangent frame; (b) Cross-section frame

For a flexible component, elastic deformation makes it difficult for a body-fixed coordinate system to represent joint geometric constraints accurately. Therefore, the gradient vectors of the fully parameterized ANCF beam element are employed to construct a joint coordinate system. As

shown in Fig.5, two alternative definitions are considered: the tangent frame and the cross-section frame.

The tangent frame shown in Fig.5(a) is defined as:

$$\mathbf{t}_T^a = \frac{\mathbf{r}_{,x}^a}{|\mathbf{r}_{,x}^a|}, \quad \mathbf{b}_T^a = \mathbf{t}_T^a \times \frac{\mathbf{r}_{,y}^a}{|\mathbf{r}_{,y}^a|}, \quad \mathbf{n}_T^a = \mathbf{b}_T^a \times \mathbf{t}_T^a \quad (17)$$

Its basis matrix is written as:

$$\mathbf{A}_T^a = [\mathbf{t}_T^a \quad \mathbf{n}_T^a \quad \mathbf{b}_T^a] \quad (18)$$

The tangent frame is constructed as an orthogonal joint coordinate system. Its tangent direction $\mathbf{t}_T^a$ is determined by the beam centerline gradient $\mathbf{r}_{,x}^a$, while the other two basis directions are successively generated through cross-product operations. Therefore, the tangent frame maintains a mutually orthogonal set of basis directions and provides a conventional joint coordinate system for formulating the geometric constraints of a revolute joint.

The cross-section frame shown in Fig.5(b) is constructed directly from the gradient vectors as:

$$\mathbf{t}_C^a = \mathbf{r}_{,y}^a \times \mathbf{r}_{,z}^a, \quad \mathbf{b}_C^a = \mathbf{r}_{,z}^a, \quad \mathbf{n}_C^a = \mathbf{r}_{,y}^a \quad (19)$$

Accordingly, the basis matrix of the cross-section frame is expressed as:

$$\mathbf{A}_C^a = [\mathbf{t}_C^a \quad \mathbf{n}_C^a \quad \mathbf{b}_C^a] \quad (20)$$

Here, basis vectors $\mathbf{b}_C^a$ and $\mathbf{n}_C^a$ are directly defined by the gradient vectors $\mathbf{r}_{,z}^a$ and $\mathbf{r}_{,y}^a$, respectively, while $\mathbf{t}_C^a$ is determined by the plane spanned by these two cross-sectional directions. Unlike the rigid-body attitude matrix, $\mathbf{A}_C^a$ is generally non-orthogonal and therefore should be interpreted as a basis matrix rather than a rotation matrix.

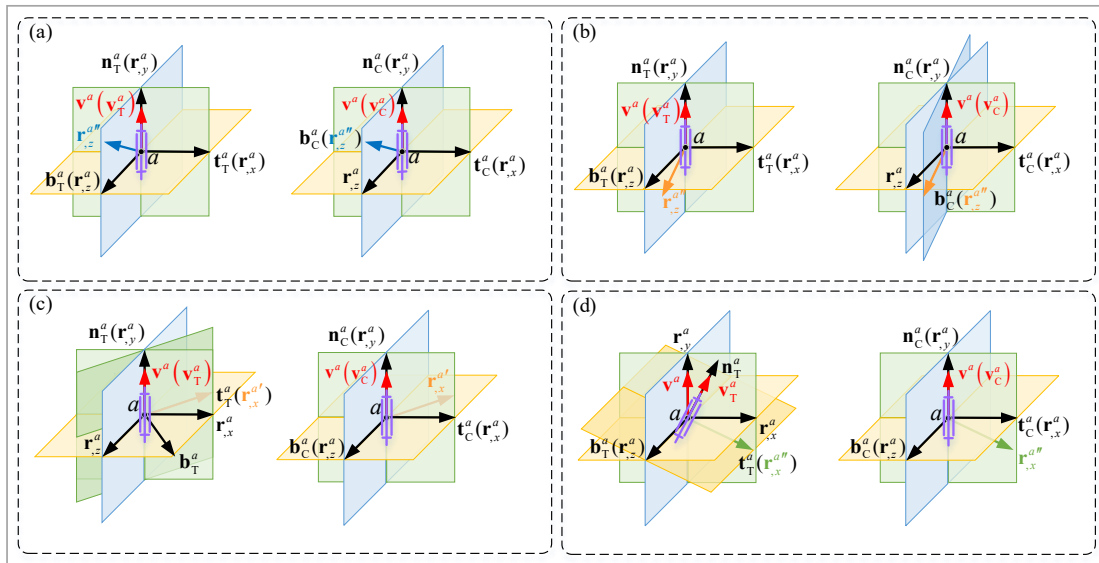


Fig.6 Influence of deformation on the revolute joint axis in the tangent frame and cross-section frame:

(a) $\mathbf{r}_{,z}^a$ rotating about $\mathbf{r}_{,x}^a$; (b) $\mathbf{r}_{,z}^a$ rotating about $\mathbf{r}_{,y}^a$; (c) $\mathbf{r}_{,x}^a$ rotating about $\mathbf{r}_{,y}^a$; (d) $\mathbf{r}_{,x}^a$ rotating about $\mathbf{r}_{,z}^a$

The cross-section frame therefore follows the local deformation of the beam cross-section. In contrast, the tangent frame enforces mutual orthogonality of its basis vectors. Their representations of the revolute joint axis direction under the four deformation modes are compared in Fig.6.

For the deformation modes shown in Fig.6(a)–(c), the material-defined joint axis direction is preserved. Although the associated gradient vectors undergo different local changes, these deformations do not introduce a discrepancy between the joint axis directions represented by the tangent and cross-section frames. Therefore, the two frames provide identical joint axis representations for these three deformation modes.

The essential difference appears in Fig.6(d), where $\mathbf{r}_{,x}^a$ rotates about $\mathbf{r}_{,z}^a$, causing $\mathbf{r}_{,x}^a$ and $\mathbf{r}_{,y}^a$ to become non-orthogonal. The cross-section frame continues to define the joint axis direction directly from $\mathbf{r}_{,y}^a$, whereas the tangent frame reconstructs the corresponding direction to preserve its orthogonal structure. As a result, the joint axis direction represented in the tangent frame deviates from the material-defined joint axis associated with the element deformation.

Accordingly, the discrepancy of direction between the two frames is associated specifically with the xy-shear deformation shown in Fig.6(d). Its geometric origin and magnitude are derived below:

$$\mathbf{p}_x^a = \frac{\mathbf{r}_{,x}^a}{\|\mathbf{r}_{,x}^a\|}, \mathbf{p}_y^a = \frac{\mathbf{r}_{,y}^a}{\|\mathbf{r}_{,y}^a\|} \quad (21)$$

and

$$c_{xy}^a = \mathbf{p}_x^a \cdot \mathbf{p}_y^a = \frac{\mathbf{r}_{,x}^a \cdot \mathbf{r}_{,y}^a}{\|\mathbf{r}_{,x}^a\| \|\mathbf{r}_{,y}^a\|} \quad (22)$$

Because the revolute joint axis direction in the cross-section frame is directly aligned with $\mathbf{r}_{,y}^a$, its unit direction is

$$\bar{\mathbf{v}}_C^a = \frac{\mathbf{r}_{,y}^a}{\|\mathbf{r}_{,y}^a\|} = \mathbf{p}_y^a \quad (23)$$

In contrast, the axis direction generated by the tangent frame is

$$\begin{aligned} \mathbf{v}_T^a &= (\mathbf{p}_x^a \times \mathbf{p}_y^a) \times \mathbf{p}_x^a \\ &= \mathbf{p}_y^a (\mathbf{p}_x^a \cdot \mathbf{p}_x^a) - \mathbf{p}_x^a (\mathbf{p}_y^a \cdot \mathbf{p}_x^a) \\ &= \mathbf{p}_y^a - c_{xy}^a \mathbf{p}_x^a \end{aligned} \quad (24)$$

Eq.(24) demonstrates that the tangent frame does not directly retain $\mathbf{p}_y^a$. Instead, it removes the component of $\mathbf{p}_y^a$ parallel to $\mathbf{p}_x^a$. Equivalently,

$$\mathbf{v}_T^a = (\mathbf{I} - \mathbf{p}_x^a \mathbf{p}_x^{aT}) \mathbf{p}_y^a \quad (25)$$

which shows that $\mathbf{v}_T^a$ is the orthogonal projection of the material-defined joint axis onto the plane perpendicular to $\mathbf{p}_x^a$. Its magnitude is:

$$\|\mathbf{v}_T^a\| = \sqrt{1 - (c_{xy}^a)^2} \quad (26)$$

and the corresponding unit direction is therefore

$$\bar{\mathbf{v}}_T^a = \frac{\mathbf{p}_y^a - c_{xy}^a \mathbf{p}_x^a}{\sqrt{1 - (c_{xy}^a)^2}} \quad (27)$$

Let δ_n^a denote the included angle between the joint axes represented by the two coordinate systems. From Eq.(23) and (27):

$$\begin{aligned} \cos(\delta_n^a) &= \bar{\mathbf{v}}_C^a \cdot \bar{\mathbf{v}}_T^a \\ &= \mathbf{p}_y^a \cdot \frac{\mathbf{p}_y^a - c_{xy}^a \mathbf{p}_x^a}{\sqrt{1 - (c_{xy}^a)^2}} \\ &= \sqrt{1 - (c_{xy}^a)^2} \end{aligned} \quad (28)$$

and hence:

$$\sin(\delta_n^a) = |c_{xy}^a| \quad (29)$$

The included angle between $\mathbf{r}_{,x}^a$ and $\mathbf{r}_{,y}^a$ is:

$$\theta_{xy}^a = \arccos\left(\frac{\mathbf{r}_{,x}^a \cdot \mathbf{r}_{,y}^a}{\|\mathbf{r}_{,x}^a\| \|\mathbf{r}_{,y}^a\|}\right) = \arccos(c_{xy}^a) \quad (30)$$

while the geometric xy-shear angle is defined as:

$$\gamma_{xy}^a = \frac{\pi}{2} - \theta_{xy}^a \quad (31)$$

It follows that:

$$c_{xy}^a = \cos(\theta_{xy}^a) = \sin(\gamma_{xy}^a) \quad (32)$$

Therefore, for the shear deformation range considered here:

$$\delta_n^a = \arcsin(|c_{xy}^a|) = |\gamma_{xy}^a| \quad (33)$$

When no xy-shear deformation exists:

$$\mathbf{r}_{,x}^a \cdot \mathbf{r}_{,y}^a = 0 \quad (34)$$

and consequently:

$$\bar{\mathbf{v}}_C^a = \bar{\mathbf{v}}_T^a \quad (35)$$

Thus, the two coordinate systems provide identical revolute joint axes in the absence of xy-shear. Once xy-shear occurs, however, the tangent frame converts the non-orthogonality between the cross-sectional material directions into a rotation of the axis represented by the orthogonal frame. The magnitude of this axis deviation is equal to the magnitude of the geometric xy-shear angle. By directly retaining the corresponding cross-sectional material direction as the joint axis, the non-orthogonal cross-section frame avoids this orthogonalization-induced deviation and remains consistent with the material-defined joint axis introduced in Section 2.2.

Therefore, the distinction between the two frames is specific to the representation of the revolute joint axis direction under xy-shear deformation rather than to all deformation modes. This relationship provides the geometric basis for the numerical comparison presented in Section 5.

2.4 Revolute joint formulation in the cross-section frame

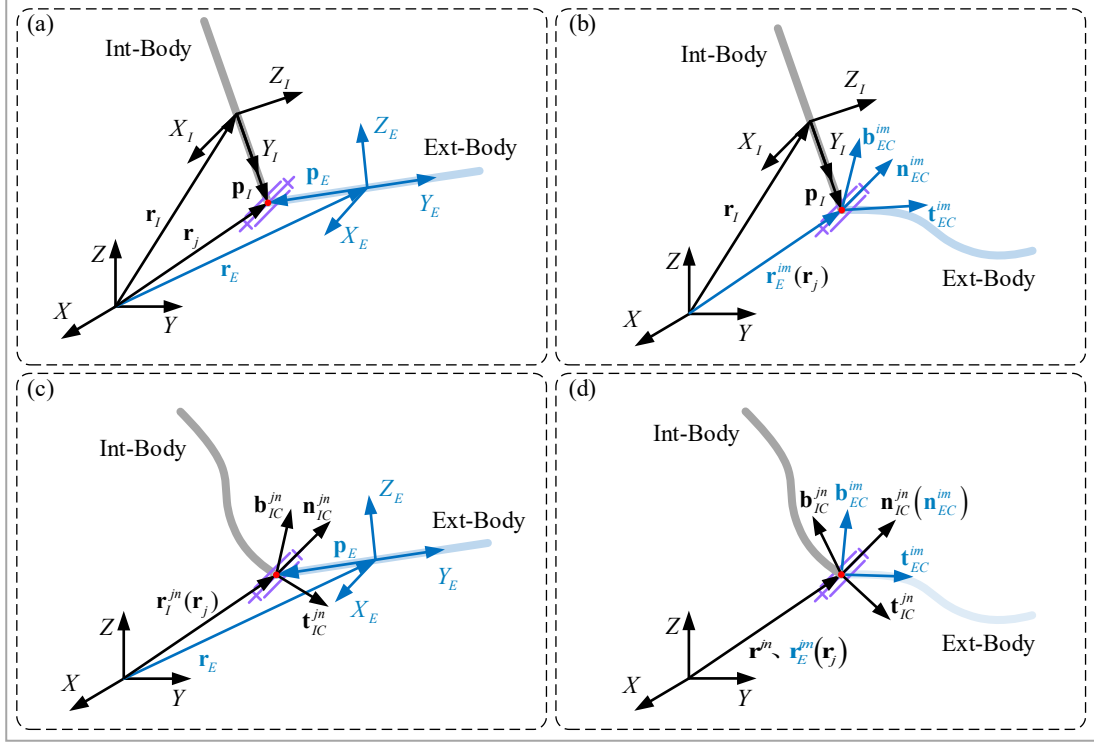


Fig.7 Four cases of revolute joint:

(a) Case I, rigid-rigid; (b) Case II, rigid-flexible; (c) Case III, flexible-rigid; (d) Case IV, flexible-flexible

To formulate the constraint equations for the four joint cases shown in Fig.7, the joint geometry is described in coordinate systems attached to the two connected components. For the Case I joint shown in Fig.7(a), the joint positions on the two components are written as:

$$\mathbf{r}_i^j = \mathbf{r}_i^c + \mathbf{A}_i \mathbf{P}_i \quad (i = I, E) \quad (36)$$

Here, $\mathbf{r}_i^c$ denotes the centroid position vector of component i in the global Cartesian frame; $\mathbf{P}_i$ denotes the local position vector of the joint on component i ; $\mathbf{A}_i$ denotes the attitude transformation matrix of component i , obtained from Eq.(1). The joint axis direction vectors of the two components are expressed as:

$$\mathbf{V}_i^j = \mathbf{A}_i \mathbf{V}_i \quad (i = I, E) \quad (37)$$

The axis parallelism condition is expressed by two orthogonality constraints. Using Eq.(36) and Eq.(37), the constraint equations for the Case I joint are obtained as:

$$\Phi^{RR} = \begin{bmatrix} \mathbf{r}_I^c + \mathbf{A}_I \mathbf{P}_I - \mathbf{r}_E^c - \mathbf{A}_E \mathbf{P}_E & (\mathbf{V}_E^1)^T \mathbf{V}_I & (\mathbf{V}_E^2)^T \mathbf{V}_I \end{bmatrix}^T = \mathbf{0} \quad (38)$$

Here, $\mathbf{V}_E^1$ and $\mathbf{V}_E^2$ are two vectors orthogonal to the joint axis, expressed in the body-fixed frame of

the outer component.

Fig.7(b) shows the Case II joint. The constraints on the rigid component remain those given by Eq.(36) and Eq.(37). The joint position on the flexible component $\mathbf{r}_E^j$ is written as:

$$\mathbf{r}_E^j = \mathbf{S}_E \mathbf{e}_E \quad (39)$$

Here, $\mathbf{S}_E$ denotes the shape function of the element containing the joint; $\mathbf{e}_E$ denotes the absolute nodal coordinates of this element.

The joint axis parallelism constraint is imposed using Eq.(37), with $\mathbf{A}_E$ evaluated from Eq.(20). For the flexible component, the joint axis direction vector $\underline{\mathbf{V}}_E$ and a prescribed reference vector $\underline{\mathbf{V}}_r$, defined in the cross-section frame, are first transformed into the global Cartesian frame through $\mathbf{A}_E$:

$$\mathbf{V}_E = \mathbf{A}_E \underline{\mathbf{V}}_E, \quad \mathbf{V}_r = \mathbf{A}_E \underline{\mathbf{V}}_r \quad (40)$$

Since $\mathbf{A}_E$ is generally non-orthogonal, the two auxiliary vectors perpendicular to the current joint axis are constructed after this transformation. Specifically, they are obtained successively as:

$$\mathbf{V}_E^1 = \mathbf{V}_E \times \mathbf{V}_r, \quad \mathbf{V}_E^2 = \mathbf{V}_E^1 \times \mathbf{V}_E \quad (41)$$

Thus, $\mathbf{V}_E^1$ and $\mathbf{V}_E^2$ provide two independent directions perpendicular to the current joint axis and are used to impose the axis parallelism constraints. The constraint equations for the Case II joint are then written as:

$$\Phi^{RF} = \begin{bmatrix} \mathbf{r}_I + \mathbf{A}_I \underline{\mathbf{P}}_I - \mathbf{S}_E \mathbf{e}_E & (\mathbf{V}_E^1)^T \mathbf{V}_I & (\mathbf{V}_E^2)^T \mathbf{V}_I \end{bmatrix}^T = \mathbf{0} \quad (42)$$

Similarly, the constraint equations for the Case III and Case IV joints (Fig.7(c) and Fig.7(d)) are formulated as:

$$\Phi^{FR} = \begin{bmatrix} \mathbf{S}_I \mathbf{e}_I - \mathbf{r}_E^c - \mathbf{A}_E \underline{\mathbf{P}}_E & (\mathbf{V}_E^1)^T \mathbf{V}_I & (\mathbf{V}_E^2)^T \mathbf{V}_I \end{bmatrix}^T = \mathbf{0} \quad (43)$$

$$\Phi^{FF} = \begin{bmatrix} \mathbf{S}_I \mathbf{e}_I - \mathbf{S}_E \mathbf{e}_E & (\mathbf{V}_E^1)^T \mathbf{V}_I & (\mathbf{V}_E^2)^T \mathbf{V}_I \end{bmatrix}^T = \mathbf{0} \quad (44)$$

The Lagrange equations of the first kind require the Jacobian matrix of the constraint equations. To avoid direct differentiation of the global constraint equations, the Jacobian for each joint type is derived separately and then assembled according to the system topology. The Jacobians of these joint constraints with respect to the generalized coordinates of the inner and outer components are given by Eq.(A-1)-Eq.(A-4) in Appendix A.

2.5 Torsion spring formulation in the cross-section frame

In truss deployable antenna mechanisms, torsion springs are often mounted at revolute joints and apply a torque pair of equal magnitude and opposite direction to the two connected components. Accordingly, the torsion spring torques acting on the inner and outer components are formulated separately for the four joint types.

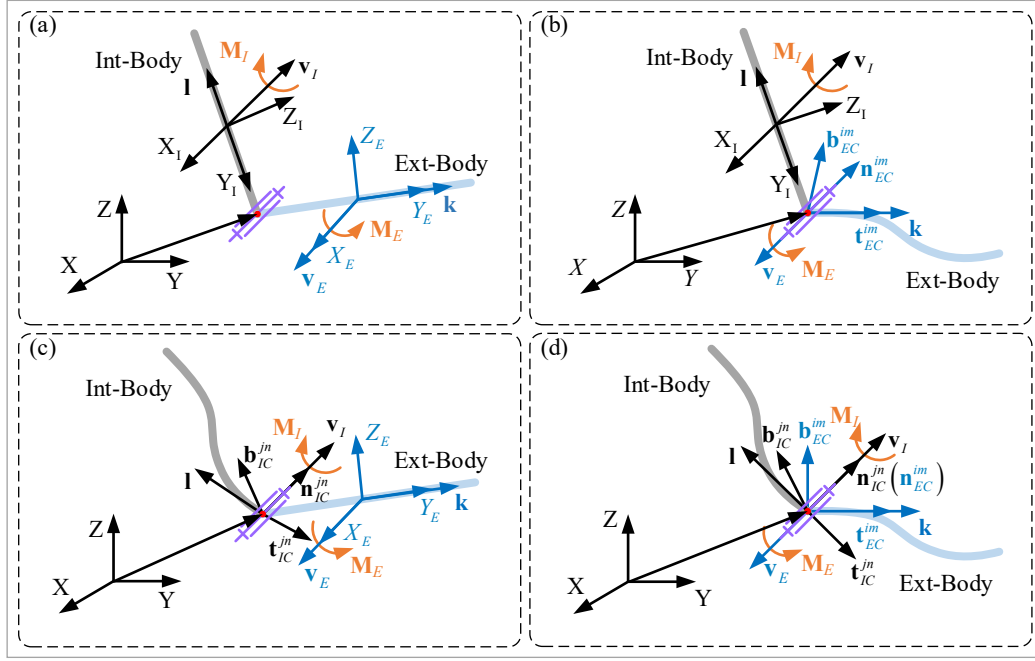


Fig.8 Generalized torques of torsion springs:

(a) Case I, rigid-rigid; (b) Case II, rigid-flexible; (c) Case III, flexible-rigid; (d) Case IV, flexible-flexible

For the Case I joint shown in Fig.8(a), the torques $\mathbf{M}_I^{RR}$ and $\mathbf{M}_E^{RR}$ acting on the inner and outer components are:

$$\mathbf{M}_I^{RR} = k(\varphi - \varphi_0) \mathbf{v}_I \quad (45)$$

$$\mathbf{M}_E^{RR} = k(\varphi - \varphi_0) \mathbf{v}_E \quad (46)$$

Here, k is the torsion spring stiffness, φ_0 denotes the free angle of the torsion spring, and φ denotes the relative angle between the two components, calculated from feature vectors $\mathbf{k}$ and $\mathbf{l}$ as:

$$\varphi = \arccos\left(\frac{\mathbf{k} \cdot \mathbf{l}}{\|\mathbf{k}\| \|\mathbf{l}\|}\right) = \arccos\left(\frac{(\mathbf{A}_I \mathbf{k})^T (\mathbf{A}_E \mathbf{l})}{\sqrt{(\mathbf{A}_I \mathbf{k})^T (\mathbf{A}_I \mathbf{k})} \sqrt{(\mathbf{A}_E \mathbf{l})^T (\mathbf{A}_E \mathbf{l})}}\right) \quad (47)$$

Here, $\mathbf{k}$ and $\mathbf{l}$ are coordinates of $\mathbf{k}$ and $\mathbf{l}$, respectively, in the body-fixed frame. Likewise, torque

directions $\mathbf{v}_I$ and $\mathbf{v}_E$ can be written as $\frac{\mathbf{A}_I \mathbf{v}_I}{\|\mathbf{A}_I \mathbf{v}_I\|}$ and $\frac{\mathbf{A}_E \mathbf{v}_E}{\|\mathbf{A}_E \mathbf{v}_E\|}$.

When a torsion spring is installed at a Case II joint (Fig.8(b)), the cross-section frame is constructed at the joint location on the flexible component as described in Section 2.3. The spring feature vectors $\mathbf{v}_I$, $\mathbf{v}_E$, $\mathbf{k}$ and $\mathbf{l}$ are defined in the body-fixed frame of the rigid component and the cross-section frame of the flexible component. Substituting these vectors into Eq.(45) and Eq.(46) yields the corresponding torques $\mathbf{M}_I^{RF}$ and $\mathbf{M}_E^{RF}$ acting on the flexible component. However, $\mathbf{M}_E^{RF}$ is expressed in the global Cartesian frame, so generalized forces associated with absolute nodal coordinates must be further derived.

The material time derivative of the deformation gradient tensor $\mathbf{F}$ is:

$$\dot{\mathbf{r}} = \frac{D\mathbf{F}}{Dt} = \frac{D}{Dt} \left(\frac{\partial \mathbf{r}}{\partial \mathbf{r}_0} \right) = \frac{D}{\partial \mathbf{r}_0} \left(\frac{\partial \mathbf{r}}{Dt} \right) = \frac{\partial \mathbf{v}}{\partial \mathbf{r}_0} = \frac{\partial \mathbf{v}}{\partial \mathbf{r}} \frac{\partial \mathbf{r}}{\partial \mathbf{r}_0} = \mathbf{L}\mathbf{F} \quad (48)$$

The velocity gradient tensor $\mathbf{L}$ follows as:

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} = \begin{bmatrix} \dot{\mathbf{F}}_{,x_1} & \dot{\mathbf{F}}_{,x_2} & \dot{\mathbf{F}}_{,x_3} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{row_1}^{-1} \\ \mathbf{F}_{row_2}^{-1} \\ \mathbf{F}_{row_3}^{-1} \end{bmatrix} = \sum_{i=1}^3 \left(\dot{\mathbf{F}}_{,x_i} \otimes \mathbf{F}_{row_i}^{-1} \right) \quad (49)$$

$\mathbf{L}$ is further decomposed into the deformation rate tensor $\mathbf{D}$ and spin tensor $\mathbf{W}$:

$$\mathbf{L} = \mathbf{D} + \mathbf{W} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T) + \frac{1}{2}(\mathbf{L} - \mathbf{L}^T) \quad (50)$$

$\mathbf{W}$ characterizes the local rigid-body rotation rate. Since $\mathbf{W}$ is skew-symmetric, its axial vector can be expressed in terms of the absolute nodal coordinates as:

$$\boldsymbol{\omega} = \frac{1}{2} \left(\sum_{i=1}^3 \mathbf{F}_{row_i}^{-1} \times \dot{\mathbf{F}}_{,x_i} \right) = \frac{1}{2} \left(\sum_{i=1}^3 \mathbf{F}_{row_i}^{-1} \times \frac{\partial \mathbf{r}_{,x_i}}{\partial \mathbf{e}} \right) \mathbf{e} = \mathbf{G}(\mathbf{e}) \mathbf{e} \quad (51)$$

The direction of $\boldsymbol{\omega}$ defines the local rotation axis, while its magnitude represents the angular speed. For an infinitesimal virtual rotation $\boldsymbol{\theta}$ of the flexible body about the joint axis, the virtual work principle gives:

$$\delta W = (\mathbf{M}_E^{RF})^T \delta \boldsymbol{\theta} = (\mathbf{M}_E^{RF})^T \boldsymbol{\omega} \delta t = (\mathbf{M}_E^{RF})^T \mathbf{G}(\mathbf{e}_E) \mathbf{e} \delta t = (\mathbf{M}_E^{RF})^T \mathbf{G}(\mathbf{e}_E) \delta \mathbf{e} = \mathbf{Q}_{\mathbf{M}_E^{RF}}^T \delta \mathbf{e} \quad (52)$$

The generalized force associated with the torsion spring torque acting on the flexible component is obtained as:

$$\mathbf{Q}_{\mathbf{M}_E^{RF}}^T = (\mathbf{M}_E^{RF})^T \mathbf{G}(\mathbf{e}_E^{mx_E}) \quad (53)$$

For the Case III joint shown in Fig.8(c), the torque acting on the outer rigid component has the same form as Eq.(46). The torque acting on the inner flexible component and its corresponding generalized force are obtained from Eq.(53) as:

$$\mathbf{Q}_{\mathbf{M}_I^{FR}}^T = (\mathbf{M}_I^{FR})^T \mathbf{G}(\mathbf{e}_I^{nx_I}) \quad (54)$$

For the Case IV joint shown in Fig.8(d), the generalized forces acting on the two flexible components are derived following the same procedure as in Eq.(53) and Eq.(54).

In addition, Jacobian matrices of torsion spring torques and corresponding generalized forces with respect to component generalized coordinates are required for dynamics modeling. For the Case I joint, the Jacobian of the torsion spring torque acting on the inner component is:

$$\begin{aligned} \frac{d\mathbf{M}_I^{RR}}{d(\mathbf{q})^T} &= \left[\frac{\partial \mathbf{M}_I^{RR}}{\partial (\mathbf{q}_I)^T} \mid \frac{\partial \mathbf{M}_I^{RR}}{\partial (\mathbf{q}_E)^T} \right] = \left[\frac{\partial \mathbf{M}_I^{RR}}{\partial (\mathbf{r}_I)^T} \mid \frac{\partial \mathbf{M}_I^{RR}}{\partial (\boldsymbol{\theta}_I)^T} \mid \frac{\partial \mathbf{M}_I^{RR}}{\partial (\mathbf{r}_E)^T} \mid \frac{\partial \mathbf{M}_I^{RR}}{\partial (\boldsymbol{\theta}_E)^T} \right] \\ &= k \left[\mathbf{0} \mid \mathbf{v}_I * \frac{\partial \varphi}{\partial (\boldsymbol{\theta}_I)^T} + (\varphi - \varphi_0) * \frac{\partial \mathbf{v}_I}{\partial (\boldsymbol{\theta}_I)^T} \mid \mathbf{0} \mid \mathbf{v}_I * \frac{\partial \varphi}{\partial (\boldsymbol{\theta}_E)^T} \right] \end{aligned} \quad (55)$$

Differentiating Eq.(47) with respect to attitude coordinates $\boldsymbol{\theta}_I$ and $\boldsymbol{\theta}_E$ of the inner and outer rigid components yields $\partial \varphi / \partial (\boldsymbol{\theta}_I)^T$ and $\partial \varphi / \partial (\boldsymbol{\theta}_E)^T$, respectively.

The Jacobian of the torque $\mathbf{M}_E^{RR}$ acting on the outer component is:

$$\begin{aligned} \frac{\mathbf{M}_E^{RR}}{d(\mathbf{q})^T} &= \left[\frac{\partial \mathbf{M}_E^{RR}}{\partial (\mathbf{q}_I)^T} \middle| \frac{\partial \mathbf{M}_E^{RR}}{\partial (\mathbf{q}_E)^T} \right] = \left[\frac{\partial \mathbf{M}_E^{RR}}{\partial (\mathbf{r}_I)^T} \quad \frac{\partial \mathbf{M}_E^{RR}}{\partial (\boldsymbol{\theta}_I)^T} \middle| \frac{\partial \mathbf{M}_E^{RR}}{\partial (\mathbf{r}_E)^T} \quad \frac{\partial \mathbf{M}_E^{RR}}{\partial (\boldsymbol{\theta}_E)^T} \right] \\ &= k \left[\mathbf{0} \quad \mathbf{v}_E * \frac{\partial \varphi}{\partial (\boldsymbol{\theta}_I)^T} \middle| \mathbf{0} \quad \mathbf{v}_E * \frac{\partial \varphi}{\partial (\boldsymbol{\theta}_E)^T} + (\varphi - \varphi_0) * \frac{\partial \mathbf{v}_E}{\partial (\boldsymbol{\theta}_E)^T} \right] \end{aligned} \quad (56)$$

For the torsion spring at the Case II joint, the Jacobian of the torque $\mathbf{M}_I^{RF}$ acting on the inner component is:

$$\begin{aligned} \frac{d\mathbf{M}_I^{RF}}{d(\mathbf{q})^T} &= \left[\frac{\partial \mathbf{M}_I^{RF}}{\partial (\mathbf{q}_I)^T} \middle| \frac{\partial \mathbf{M}_I^{RF}}{\partial (\mathbf{q}_E)^T} \right] = \left[\frac{\partial \mathbf{M}_I^{RF}}{\partial (\mathbf{r}_I)^T} \quad \frac{\partial \mathbf{M}_I^{RF}}{\partial (\boldsymbol{\theta}_I)^T} \middle| \frac{\partial \mathbf{M}_I^{RF}}{\partial (\mathbf{e}_E^{mx_E})^T} \right] \\ &= k \left[\mathbf{0} \quad \mathbf{v}_I * \frac{\partial \varphi}{\partial (\boldsymbol{\theta}_I)^T} + (\varphi - \varphi_0) * \frac{\partial \mathbf{v}_I}{\partial (\boldsymbol{\theta}_I)^T} \middle| \mathbf{v}_I * \frac{\partial \varphi}{\partial (\mathbf{e}_E^{mx_E})^T} \right] \end{aligned} \quad (57)$$

The derivative of $\partial \varphi / \partial (\mathbf{e}_E^{mx_E})^T$ is obtained in the same manner as that of $\partial \varphi / \partial (\boldsymbol{\theta}_E)^T$. The Jacobian of the generalized force $\mathbf{Q}_{M_E^{RF}}$ corresponding to the torsion spring torque $\mathbf{M}_E^{RF}$ acting on the outer component is:

$$\frac{d\mathbf{Q}_{M_E^{RF}}}{d(\mathbf{q})^T} = \left[\frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\mathbf{q}_I)^T} \middle| \frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\mathbf{q}_E)^T} \right] = \left[\frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\mathbf{r}_I)^T} \quad \frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\boldsymbol{\theta}_I)^T} \middle| \frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\mathbf{e}_E^{mx_E})^T} \right] \quad (58)$$

Where $\frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\mathbf{r}_I)^T} = \mathbf{0}$, $\frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\boldsymbol{\theta}_I)^T} = k \mathbf{G}_E^T \left(\frac{\partial \varphi}{\partial (\boldsymbol{\theta}_I)^T} \mathbf{v}_E \right)$,

$$\frac{\partial \mathbf{Q}_{M_E^{RF}}}{\partial (\mathbf{e}_E^{mx_E})^T} = k (\varphi - \varphi_0) \frac{\partial \mathbf{G}_E^T}{\partial (\mathbf{e}_E^{mx_E})^T} \mathbf{v}_E + k \mathbf{G}_E^T \left(\mathbf{v}_E * \frac{\partial \varphi}{\partial (\mathbf{e}_E^{mx_E})^T} + (\varphi - \varphi_0) \frac{\partial \mathbf{v}_E}{\partial (\mathbf{e}_E^{mx_E})^T} \right)$$

Here, $\mathbf{B}$ is a 24-dimensional row vector with all entries equal to 1. The expression for $\partial \mathbf{G}_E^T / \partial (\mathbf{e}_E^{mx_E})^T$ is:

$$\frac{\partial \mathbf{G}_E^T}{\partial (\mathbf{e}_E^{mx_E})^T} = \frac{\partial}{\partial (\mathbf{e}_E^{mx_E})^T} \left(\frac{1}{2} \sum_{i=1}^3 (\mathbf{J}_E^{-1})_{row_i} \times \left(\frac{\partial (\mathbf{r}_E)_{,x_i}}{\partial (\mathbf{e}_E^{mx_E})^T} \right) \right) = \frac{1}{2} \sum_{i=1}^3 \frac{\partial (\mathbf{F}_E^{-1})_{row_i}}{\partial (\mathbf{e}_E^{mx_E})^T} \times \left(\frac{\partial (\mathbf{r}_E)_{,x_i}}{\partial (\mathbf{e}_E^{mx_E})^T} \right) \quad (59)$$

Following the same procedure, the Jacobians of the torsion spring torques and generalized forces for the Case III joint are obtained as:

$$\frac{d\mathbf{Q}_{M_I^{FR}}}{d(\mathbf{q})^T} = \left[\frac{\partial \mathbf{Q}_{M_I^{FR}}}{\partial (\mathbf{q}_I)^T} \middle| \frac{\partial \mathbf{Q}_{M_I^{FR}}}{\partial (\mathbf{q}_E)^T} \right] = \left[\frac{\partial \mathbf{Q}_{M_I^{FR}}}{\partial (\mathbf{e}_I^{mx_I})^T} \quad \frac{\partial \mathbf{Q}_{M_I^{FR}}}{\partial (\mathbf{r}_E)^T} \quad \frac{\partial \mathbf{Q}_{M_I^{FR}}}{\partial (\boldsymbol{\theta}_E)^T} \right] \quad (60)$$

$$\frac{d\mathbf{M}_E^{FR}}{d(\mathbf{q})^T} = \left[\frac{\partial \mathbf{M}_E^{FR}}{\partial (\mathbf{q}_I)^T} \middle| \frac{\partial \mathbf{M}_E^{FR}}{\partial (\mathbf{q}_E)^T} \right] = \left[\frac{\partial \mathbf{M}_E^{FR}}{\partial (\mathbf{e}_I^{mx_I})^T} \quad \frac{\partial \mathbf{M}_E^{FR}}{\partial (\mathbf{r}_E)^T} \quad \frac{\partial \mathbf{M}_E^{FR}}{\partial (\boldsymbol{\theta}_E)^T} \right] \quad (61)$$

Following the same procedure as in Eq.(58) and Eq.(60), the Jacobians $d\mathbf{Q}_{M_I^{FF}} / d(\mathbf{q})^T$ and

$d\mathbf{Q}_{\mathbf{M}_E^{FF}}/d(\mathbf{q})^T$ for the Case IV joint are obtained with respect to the absolute nodal coordinates of the two flexible components.

Consequently, both the revolute joint constraints and the torsion spring generalized forces are formulated using the cross-section frame. These component-level and joint-level formulations provide the basis for assembling the system-level dynamic equations described in the following section.

3 Assembly of the dynamic equations for the basic module

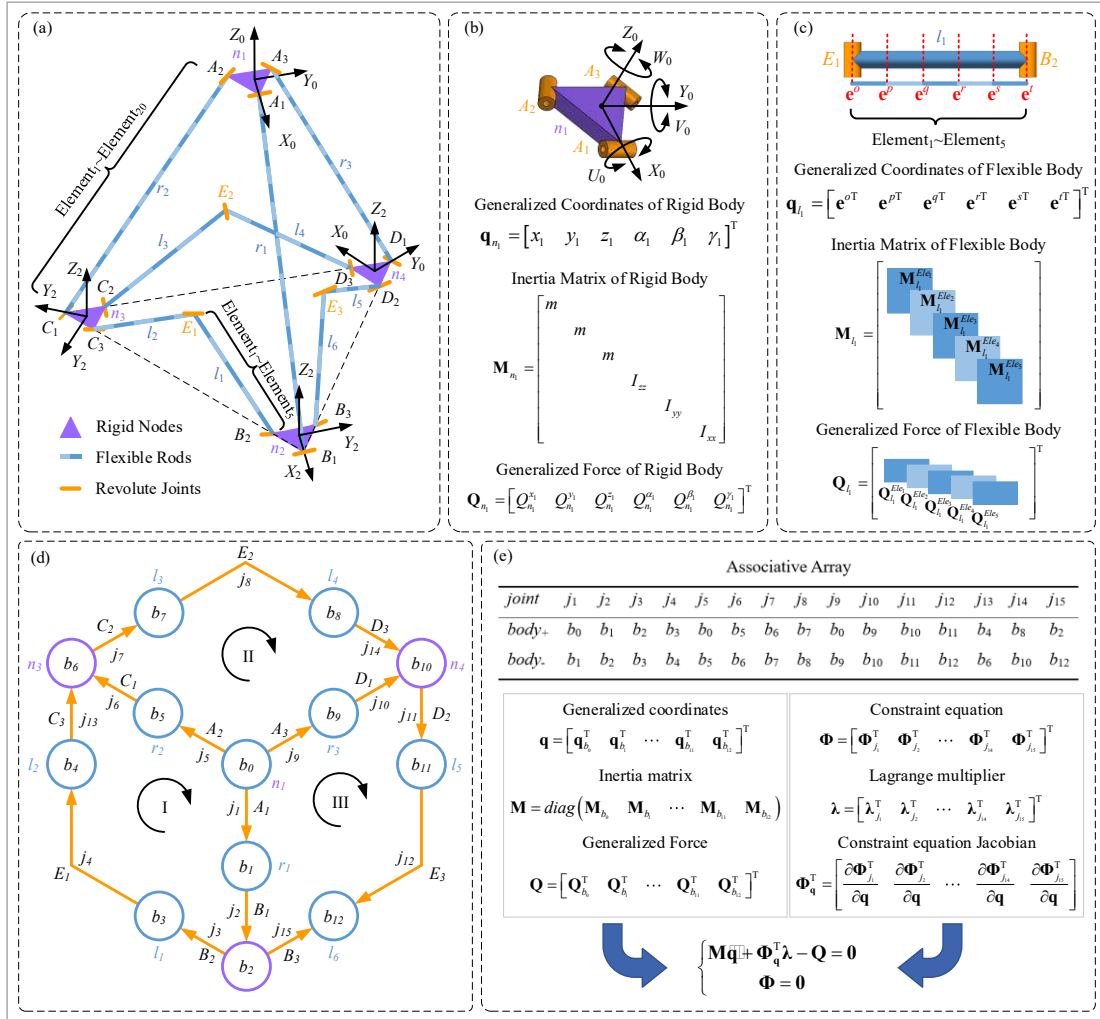


Fig.9 Topology analysis and dynamics equation assembly workflow for basic deployable module:

- (a) Schematic of basic module; (b) Rigid component model; (c) Flexible component model; (d) Topological relationships; (e) Dynamic assembly procedure

Graph theory and associative arrays are used to describe the connectivity among components and joints and to organize the corresponding equations according to the mechanism topology. The truss deployable antenna considered in this study is composed of interconnected basic deployable modules. Therefore, the basic module is adopted as the representative system for illustrating the topology-based equation assembly procedure. Using the same topological description and assembly strategy, multiple basic modules can be further connected to construct the complete deployable

antenna.

The basic deployable module in Fig.1 consists of top node n_1 , bottom nodes n_2 - n_4 , long links r_1 - r_3 , and synchronous links l_1 - l_6 . All components are connected by revolute joints, as sketched in Fig.9(a). During deployment, node components (Fig.9(b)) deform negligibly and are treated as rigid bodies, whereas rod-like components (Fig.9(c)) have large slenderness ratios and exhibit non-negligible elastic deformation under inertial loading, so they are treated as flexible bodies and discretized using ANCF, forming a rigid-flexible multibody system. With n_1 selected as the ground body, the topology of the basic deployable module is shown in Fig.9(d), where the bodies and joints are labeled according to the numbering scheme and structural symmetry.

The physical and geometric parameters of the rigid and flexible components are listed in Table 1 and Table 2:

Table 1 Parameters of rigid components

Body	Density (kg/m ³)	Mass (kg)	I _{xx} (kg · m ²)	I _{yy} (kg · m ²)	I _{zz} (kg · m ²)	Size (m)
n_1	7800	0.13	2.7×10^{-5}	5.3×10^{-5}	5.3×10^{-5}	OA ₁ =0.03
n_2 - n_4		0.12	2.5×10^{-5}	2.5×10^{-5}	4.8×10^{-5}	B ₁ B ₂ =0.05

Table 2 Parameters of flexible components

Body	Length of element (m)	Number of elements	b×h (m)	Density (kg/m ³)	Young's modulus (Pa)	Poisson's ratio
r_1 - r_3	0.02	20	0.01×0.01	7800	2×10^{11}	0.29
l_1 - l_6		5				

Substituting these parameters into the rigid-body and flexible-body formulations established in Section 2.1 yields the inertia matrix $\mathbf{M}_{b_i}$ of each body.

The generalized forces $\mathbf{Q}_{b_i}$ acting on each component in the system include elastic forces $\mathbf{Q}_{b_i}^k$, gravity $\mathbf{Q}_{b_i}^G$, and torsion spring driving torques. Torsion springs are commonly used as deployment actuators in truss deployable antenna mechanisms. The elastic potential energy stored in the preloaded springs during folding is released to drive deployment, thereby simplifying the actuation system. To evaluate the torsion spring model and the assembled dynamic model under different conditions, three torsion spring layouts are considered. The springs are installed at joints A_1 - A_3 , B_1 / C_1 / D_1 , and E_1 - E_3 , respectively. Because of the structural symmetry, the springs at corresponding joints in each layout have the same parameters, as listed in Table 3.

Table 3 Parameters of torsion springs

Parameter	Layout 1	Layout 2	Layout 3
k (N · m/rad)	$6/\pi$	$6/\pi$	$6/\pi$
φ_0 (rad)	π	π	π

Substituting these parameters into the torsion spring formulation in Section 2.5 gives $\mathbf{Q}_{b_i}^M$.

The generalized coordinates $\mathbf{q}_{b_i}$, inertia matrices $\mathbf{M}_{b_i}$, and generalized forces $\mathbf{Q}_{b_i}$ are assembled according to the body indices to form the system-level $\mathbf{q}$, $\mathbf{M}$ and $\mathbf{Q}$. The constraint equations for the different joint types, Eq.(38)-Eq.(44), are assembled according to the joint order defined in the associative array to form the system-level constraint equations. The Lagrange multiplier vector λ contains one multiplier for each scalar constraint and is included among the unknown variables.

Each joint constraint in Eq.(38)-Eq.(44) involves only the generalized coordinates of the two connected components. Therefore, the Jacobian of each joint constraint contains nonzero blocks only in the columns corresponding to the two associated components. Taking Branch $n_1-r_1-n_2-l_1-l_2$ as an example, its constraint Jacobian is written as:

$$\Phi_q' = \begin{bmatrix} \frac{\partial \Phi_{A_1}}{\partial \mathbf{q}_{n_1}^T} & \frac{\partial \Phi_{A_1}}{\partial \mathbf{q}_{r_1}^T} & & & \\ & \frac{\partial \Phi_{B_1}}{\partial \mathbf{q}_{r_1}^T} & \frac{\partial \Phi_{B_1}}{\partial \mathbf{q}_{n_2}^T} & & \\ & & \frac{\partial \Phi_{E_1}}{\partial \mathbf{q}_{l_1}^T} & \frac{\partial \Phi_{E_1}}{\partial \mathbf{q}_{l_2}^T} & \\ & & & \frac{\partial \Phi_{C_3}}{\partial \mathbf{q}_{l_2}^T} & \frac{\partial \Phi_{C_3}}{\partial \mathbf{q}_{n_3}^T} \end{bmatrix} \quad (62)$$

The associative matrix derived from the associative array is an $n \times m$ matrix, where n denotes the number of components and m denotes the number of joints. For Branch I, the transpose of its associative matrix $\mathbf{B}_I$ is given by:

$$\mathbf{B}_I^T = \begin{matrix} & n_1 & r_1 & n_2 & l_1 & l_2 \\ \begin{matrix} A_1 \\ B_1 \\ B_2 \\ E_1 \end{matrix} & \begin{bmatrix} +1 & -1 & & & \\ & +1 & -1 & & \\ & & +1 & -1 & \\ & & & +1 & -1 \end{bmatrix} \end{matrix} \quad (63)$$

The nonzero patterns of the two matrices coincide because each row of $\mathbf{B}_I^T$ and Φ_q corresponds to a joint constraint, and each column of $\mathbf{B}_I^T$ and Φ_q corresponds to a connected body. Based on this, Jacobian matrices of joint constraint equations in Eq.(A-1)-Eq.(A-4) can be assembled directly using the associative array to obtain the global constraint Jacobian of the system, rather than by differentiating constraint equations with respect to generalized coordinates.

Substituting $\mathbf{q}$, $\mathbf{M}$, $\mathbf{Q}$, Φ and Φ_q into the Lagrange equations of the first kind yields the undamped system-level dynamic equations of the basic deployable module:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \Phi_q^T \lambda - \mathbf{Q} = \mathbf{0} \\ \Phi = \mathbf{0} \end{cases} \quad (64)$$

In this system, the total number of generalized coordinates is 1212, while the system contains 15

revolute joints, contributing 75 scalar constraints, together with six grounding constraints applied to n_1 , giving 81 constraints in total.

4 Numerical solution of the dynamic equations

The topology, body, joint, and load parameters of the physical basic deployable module are used to assemble the index-3 dynamic equations following the procedure described in Section 3. For numerical solution, the initial generalized coordinates are determined from the initial configuration, initial velocities are set to zero, and initial accelerations and Lagrange multipliers are obtained by solving a linear system derived from an index-1 formulation (Fig.10(a)).

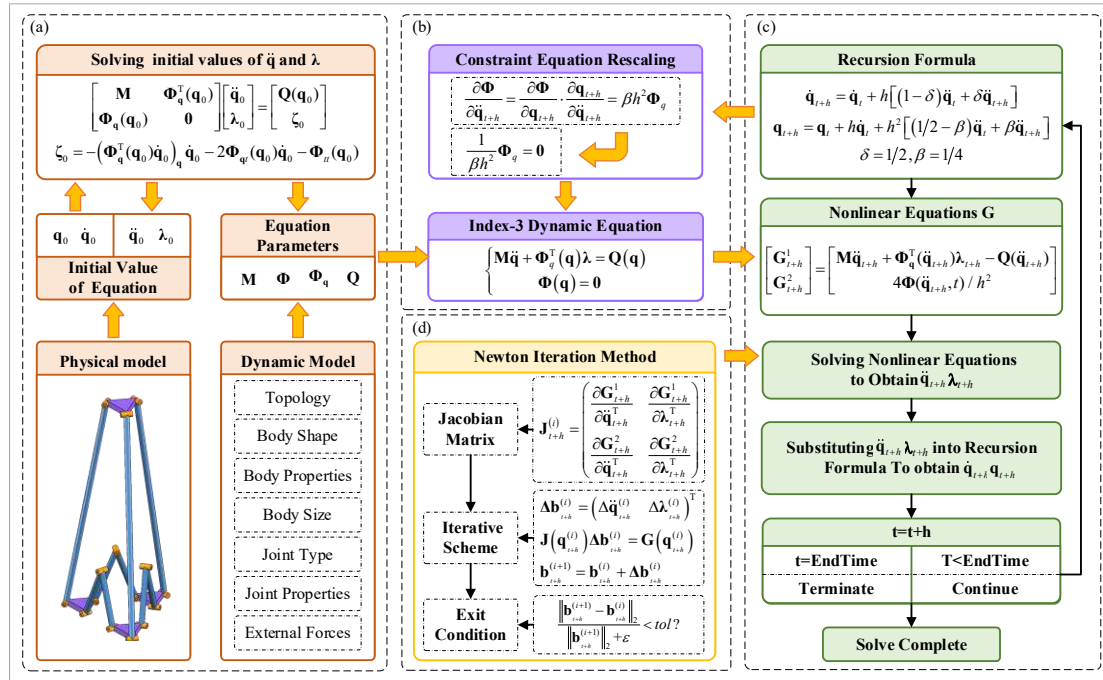


Fig.10 Flowchart for solving the dynamic equations using the Newmark- β method:
(a) Dynamic model formulation; (b) Dynamic equation optimization; (c) Equation solution procedure;
(d) Newton's method flowchart

As shown in Fig.10(b), the nonlinear system is severely ill-conditioned before the constraint equations are rescaled. This ill-conditioning is alleviated by rescaling the coefficients in the constraint equations. Rescaling the constraint equations improves the conditioning of the nonlinear system G_{t+h} .

In Fig.10(c), the Newmark- β method with $\delta=1/2$ and $\beta=1/4$ is adopted, corresponding to the average acceleration scheme without numerical damping. At time t , the displacement q_t , velocity $\dot{q}_t$, acceleration $\ddot{q}_t$, and Lagrange multipliers λ_t are known. The Newmark recurrence relations are then substituted into the dynamic equations to obtain the nonlinear system at $t+h$. The unknown accelerations $\ddot{q}_{t+h}$ and Lagrange multipliers λ_{t+h} are solved iteratively, after which the displacements q_{t+h} and velocities $\dot{q}_{t+h}$ are obtained by back-substitution. The state at $t+h$ is subsequently used as the initial state for the next time step, and the procedure is repeated until the end of the simulation.

The nonlinear system is solved using Newton's method, as shown in Fig.10(d). The Jacobian of $\mathbf{G}_{t+h}$ is given by:

$$\frac{\partial \mathbf{G}_{t+h}^1}{\partial \mathbf{q}_{t+h}^T} = \mathbf{M} + \frac{\partial (\Phi_q^T \lambda_{t+h})}{\partial \mathbf{q}_{t+h}^T} \frac{\partial \mathbf{q}_{t+h}^T}{\partial \mathbf{q}_{t+h}^T} - \frac{\partial \mathbf{Q}}{\partial \mathbf{q}_{t+h}^T} \frac{\partial \mathbf{q}_{t+h}^T}{\partial \mathbf{q}_{t+h}^T}, \quad \frac{\partial \mathbf{G}_{t+h}^1}{\partial \lambda_{t+h}^T} = \Phi_q^T$$

$$\frac{\partial \mathbf{G}_{t+h}^2}{\partial \mathbf{q}_{t+h}^T} = \frac{4}{h^2} \Phi_q \frac{\partial \mathbf{q}_{t+h}^T}{\partial \mathbf{q}_{t+h}^T} = \Phi_q, \quad \frac{\partial \mathbf{G}_{t+h}^2}{\partial \lambda_{t+h}^T} = \mathbf{0}$$

To avoid matrix inversion, a linear system is constructed and solved:

$$\mathbf{J}_{t+h}^{(i)} \Delta \mathbf{b}_{t+h}^{(i)} = \mathbf{G}(\mathbf{q}_{t+h}^{(i)}) \quad (65)$$

Finally, a relative increment convergence criterion based on the Euclidean norm is adopted, with a tolerance of 10^{-5} . This criterion measures the relative change in the generalized accelerations and Lagrange multipliers between two successive iterations. When the relative increment falls below the prescribed tolerance, the changes in the generalized accelerations and Lagrange multipliers are sufficiently small, and the iterative solution at the current time step is considered converged.

5 Numerical verification and dynamic response analysis

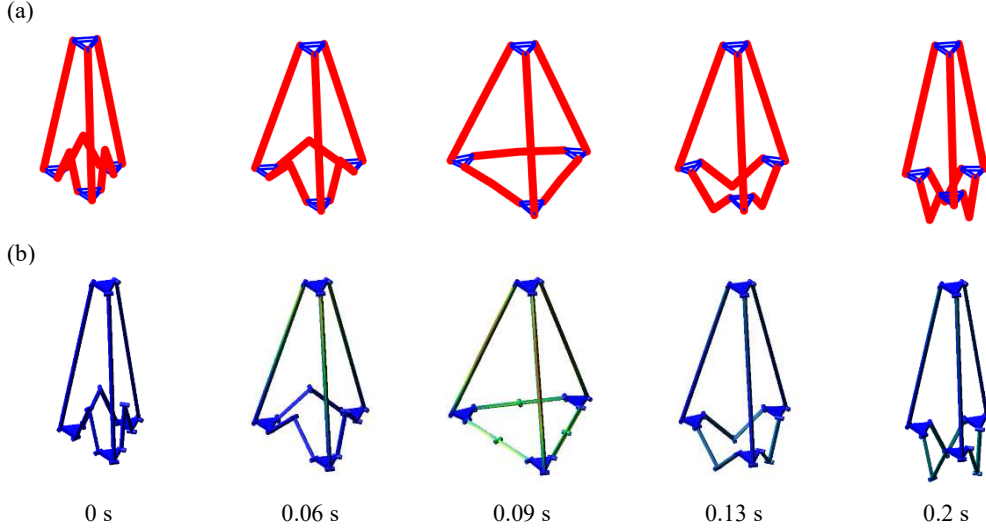


Fig.11 Comparison of the motion predicted by the calculation and the reference simulation for the basic deployable module with torsion springs installed at A₁–A₃ over a deployment–folding cycle from 0 to 0.2 s ($h=10^{-4}$ s):
(a) Calculation; (b) Reference Simulation

The reference model was established in Adams/View, with the flexible links generated using Adams/ViewFlex. First-order tetrahedral solid elements were adopted with target and minimum element sizes of 10 and 5 mm, respectively. Ten normal modes were retained, with modal stress information included for transient stress recovery; the unreduced finite-element models of the long and short links contained 1740 and 996 degrees of freedom, respectively. No modal damping was introduced, and ideal revolute joints and the same gravitational acceleration were used in both models. The Adams/View simulation employed the WSTIFF integrator with the I3 formulation, an error tolerance of 10^{-3} , and initial, minimum, and maximum step sizes of 10^{-4} , 10^{-7} , and 10^{-4} s,

respectively. The Adams/View responses were exported at 2×10^{-4} s intervals, while the proposed model used a constant step of 1×10^{-4} s; comparisons were therefore made directly at common time instants without interpolation. Fig.11 presents representative configurations corresponding to the fully folded (0 s), half-deployed (0.06 s), fully deployed (0.09 s), reverse half-deployed (0.13 s), and reverse fully folded (0.2 s) states. Reverse folding occurs because no locking constraint is imposed on the synchronous links, allowing the mechanism to continue its motion under inertia after reaching the fully deployed configuration. The calculation and the reference simulation predict the same deployment-folding sequence and similar configurations at the selected time instants.

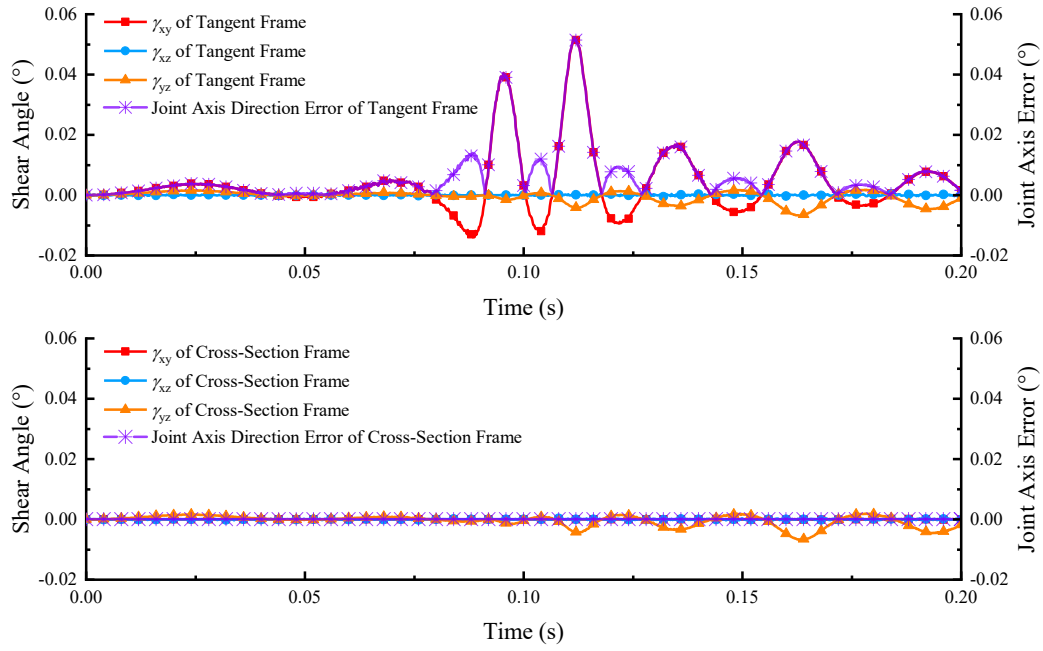


Fig.12 Comparison of shear angles and joint axis direction errors obtained using the tangent and cross-section frames

Fig.12 compares the shear angles and joint axis direction errors associated with the tangent and cross-section frames. As defined in Section 2.2, the material-defined joint axis direction is aligned with the corresponding cross-sectional gradient. In the cross-section frame, the joint axis direction is directly defined by this gradient, and the resulting direction error therefore remains essentially zero. In contrast, the tangent frame reconstructs the joint axis direction through orthogonalization. When xy-shear causes the relevant material directions to become non-orthogonal, the represented axis deviates from the material-defined joint axis direction. As derived in Section 2.3, the magnitude of this deviation is related to the xy-shear angle, whereas the xz- and yz-shear deformations do not directly contribute to the axis deviation. Accordingly, Fig.12 shows that the axis direction error of the tangent frame closely follows the xy-shear angle, while that of the cross-section frame remains essentially zero. The numerical results are consistent with the geometric relationship derived in Section 2.3.

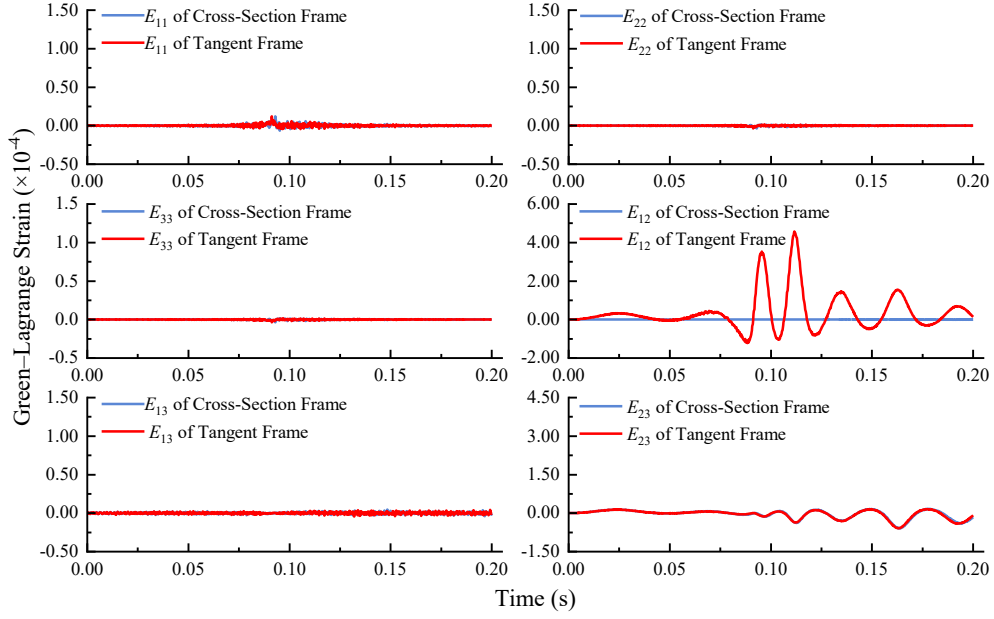


Fig.13 Comparison of Green-Lagrange strain components at joint E_1 obtained using the cross-section frame and tangent frame

To examine how the difference in joint axis direction representation is reflected in the local deformation response, the Green-Lagrange strain components at the revolute joint are compared in Fig.13. The normal strain components E_{11} , E_{22} and E_{33} remain small for both formulations. Among the shear components, E_{13} and E_{23} exhibit nearly identical responses, whereas a pronounced difference is observed in E_{12} . In particular, the tangent frame model produces substantially larger oscillations in E_{12} , while the corresponding response obtained with the cross-section frame remains small. The pronounced difference in E_{12} corresponds closely to the xy-shear response and the joint axis direction error identified in Fig.12, indicating that the frame-dependent axis discrepancy is also reflected in the local xy-shear strain response.

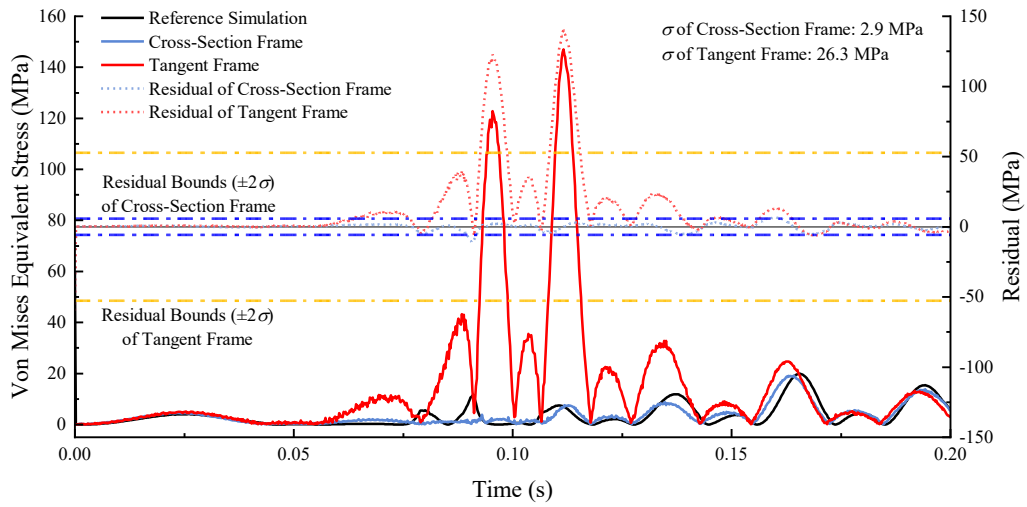


Fig.14 Comparison of von Mises equivalent stress between the reference simulation and the results calculated using the cross-section frame and tangent frame

To evaluate the corresponding stress response, the second Piola-Kirchhoff stress was first pushed forward to the Cauchy stress, and the von Mises equivalent stress was then calculated. Unlike the kinematic comparison in Fig.12, the Adams/View reference simulation in Fig.14 provides an independent reference for assessing the local mechanical responses predicted by the two joint frame formulations. The cross-section frame result closely reproduces the reference response, with a residual standard deviation of 2.9 MPa, compared with 26.3 MPa for the tangent frame, corresponding to an approximately 89.0% reduction. The residual of the cross-section frame remains within the $\pm 2\sigma$ residual bounds throughout the deployment, whereas the tangent frame residual reaches approximately 140 MPa and markedly exceeds the corresponding residual bounds near 0.095 s and 0.112 s. The dominant deviations occur over 0.075-0.125 s and are followed by several smaller peaks after 0.125 s. These dominant stress deviations occur over the same time intervals as the pronounced xy-shear and E_{12} responses identified in Fig.12 and Fig.13, respectively, linking the kinematic axis discrepancy to the local mechanical response. Relative to the Adams/View reference simulation, the cross-section frame provides a substantially more accurate stress prediction at the investigated joint.

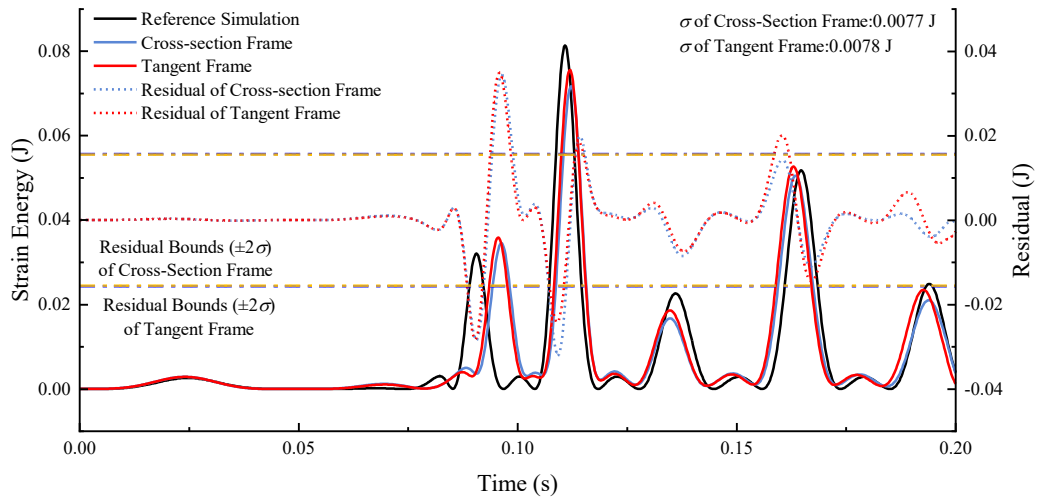


Fig.15 Comparison of strain energy responses between the reference simulation and the results calculated using the cross-section and tangent frames

Fig.15 further compares the strain energy of synchronous link l_1 . Both formulations reproduce the reference strain energy response with similar accuracy, yielding residual standard deviations of 0.0077 J and 0.0078 J for the cross-section and tangent frames, respectively. Their residual histories are also highly similar, while the excursions beyond the $\pm 2\sigma$ residual bounds during approximately 0.07-0.10 s are mainly associated with phase differences relative to the reference response. Unlike the local strain and stress quantities, the strain energy shows little sensitivity to the difference in joint axis direction representation. This indicates that the localized deformation error introduced by the tangent frame has only a limited influence on the overall strain energy response of the investigated link. Although the two frames provide comparable strain energy predictions for link l_1 , the cross-section frame is adopted in the subsequent analysis because of its higher accuracy in joint axis direction representation and the associated local strain and stress responses.

The preceding results demonstrate the local kinematic and mechanical consistency of the proposed joint formulation. The performance of the assembled rigid-flexible multibody dynamics model is further evaluated at the system level using three torsion spring layouts.

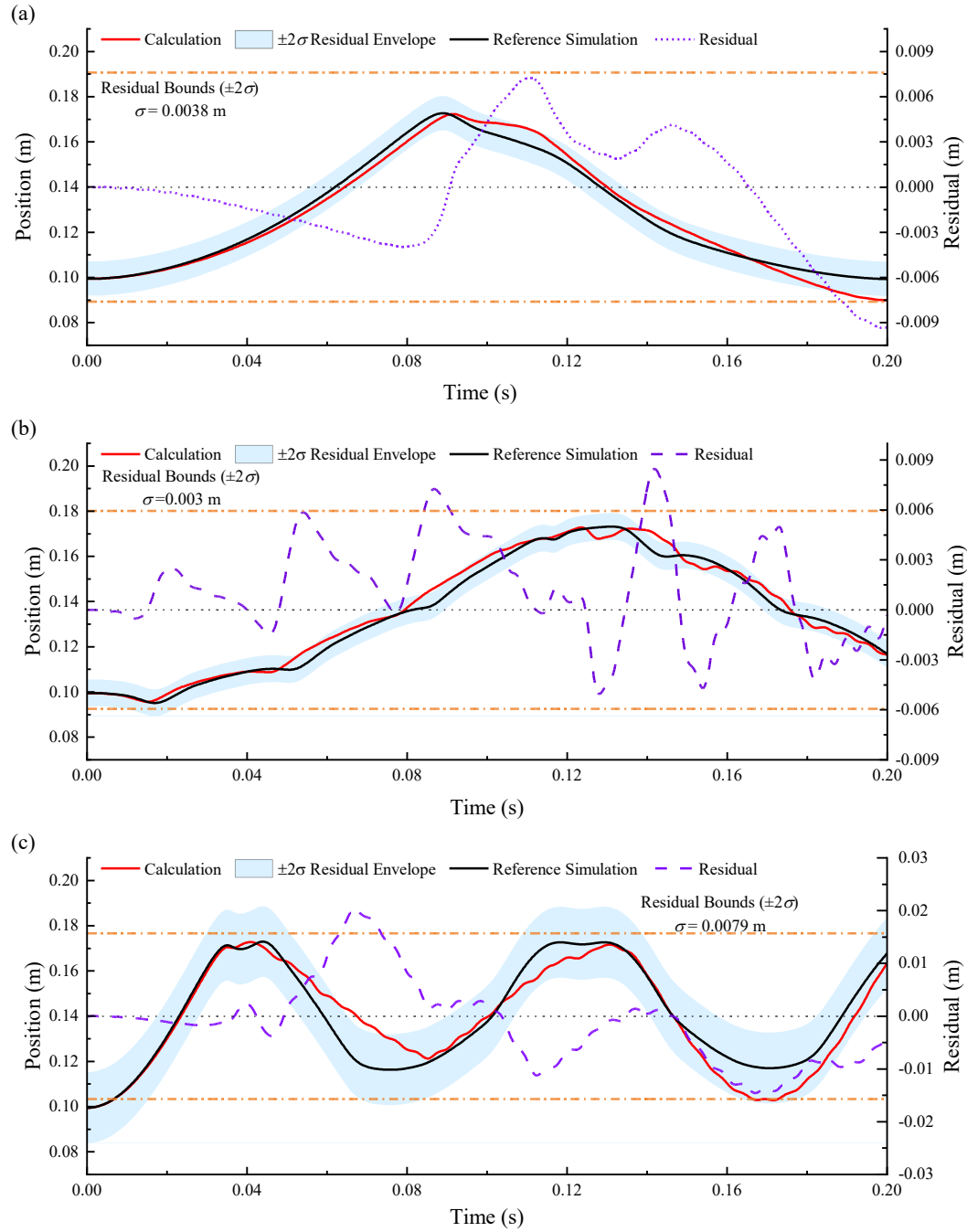


Fig.16 Comparison of the x-component position response of node n_2 predicted by the proposed model and the reference simulation:
(a) Layout 1; (b) Layout 2; (c) Layout 3

Fig.16 compares the x-component position response of node n_2 predicted by the proposed model with that obtained from the reference simulation. For all three layouts, the calculated responses reproduce the principal evolution of the deployment and subsequent folding motion. Layouts 1 and 2 exhibit relatively small residual standard deviations, with $\sigma=0.0038$ and 0.0030 m,

respectively, whereas a larger value of $\sigma=0.0079$ m is observed for Layout 3. The larger residual standard deviation in Layout 3 is likely associated with its faster deployment response under the larger initial torsional preload and is manifested mainly as local phase and amplitude differences. The three torsion spring layouts therefore provide distinct dynamic conditions for evaluating the assembled rigid-flexible multibody dynamics model.

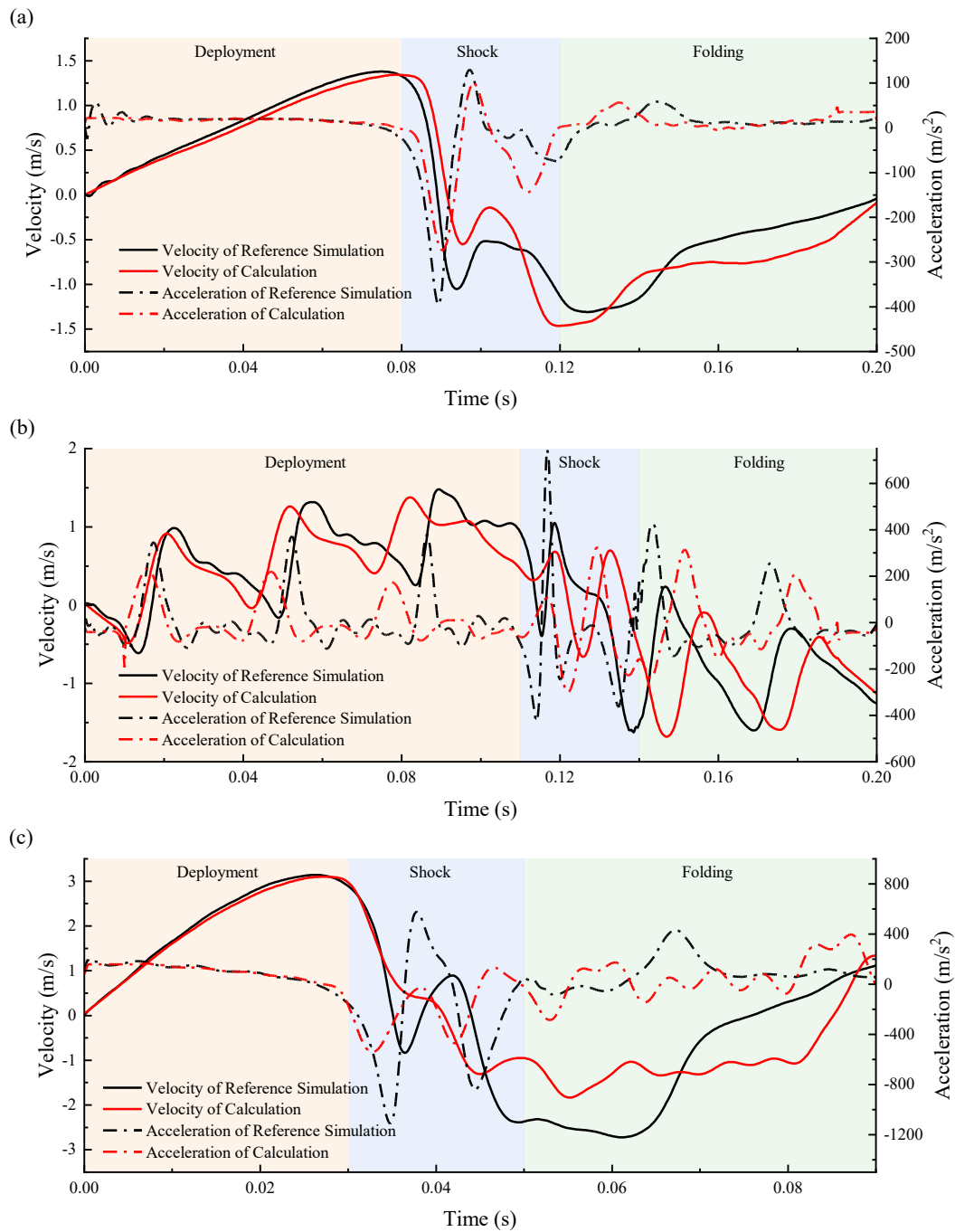


Fig.17 Comparison of the x-component velocity and acceleration responses of node n_2 predicted by the proposed model and the reference simulation:
(a) Layout 1; (b) Layout 2; (c) Layout 3

Velocity and acceleration are more sensitive than position to local phase differences and high-frequency elastic responses. Therefore, Fig.17 compares the x-component velocity and acceleration

of node n_2 . For Layout 3, one complete deployment-folding cycle is selected to provide a response interval comparable with those of Layouts 1 and 2. Across the three layouts, the proposed model reproduces the principal velocity reversals and the timing of the dominant acceleration variations. Layout 1 shows relatively good agreement, whereas stronger oscillations and larger local discrepancies are observed for Layout 2. Layout 3 exhibits the fastest response because of the larger initial torsional preload, resulting in more pronounced transient differences after the fully deployed state.

To quantify the agreement observed in Fig.17, the NRMSE is calculated separately for each stage. For stage k , the NRMSE is defined as:

$$\text{NRMSE}_k = \frac{\sqrt{\frac{1}{N_k} \sum_{i=1}^{N_k} (y_{i,k}^{\text{Sim}} - y_{i,k}^{\text{Cal}})^2}}{\max(y_k^{\text{Sim}}) - \min(y_k^{\text{Sim}})} \quad (66)$$

where $y_{i,k}^{\text{Sim}}$ and $y_{i,k}^{\text{Cal}}$ denote the responses obtained from the reference simulation and the proposed model at the i th data point of stage k , respectively, and N_k is the number of data points in that stage. The reference-response range within each stage is used for normalization. The stage-wise NRMSE values for velocity and acceleration in Layouts 1-3 are listed in Table 4.

Table 4 NRMSE of velocity and acceleration for three layouts

	Deployment	Velocity		Acceleration		
		Shock	Folding	Deployment	Shock	Folding
Layout 1	0.039	0.171	0.187	0.097	0.140	0.189
Layout 2	0.197	0.301	0.399	0.271	0.214	0.389
Layout 3	0.021	0.169	0.269	0.052	0.269	0.379

Except for the acceleration response in Layout 2, the deployment stage yields the lowest NRMSE. For all three layouts, the largest NRMSE values for both velocity and acceleration occur during the folding stage. During the initial deployment, the motion is governed mainly by the response of the spring-driven mechanism, whereas the subsequent shock and folding stages involve stronger elastic vibration and more rapid changes in motion. Consequently, differences in flexible body modeling and time integration methods are more readily manifested as local phase shifts and discrepancies in the high-frequency response. These differences are particularly evident in the acceleration response, which is more sensitive to phase and high-frequency variations than displacement or velocity.

Overall, the local results show that the cross-section frame provides a more accurate representation of the joint axis and the associated strain and stress responses. At the system level, the proposed model reproduces the main deployment responses under all three torsion spring layouts. The remaining discrepancies are concentrated primarily in the shock and folding stages, where stronger elastic vibration and high-frequency responses are accompanied by larger differences in the predicted velocity and acceleration.

6 Conclusions

This study develops a rigid-flexible coupling dynamic modeling framework based on the proposed non-orthogonal cross-section frame. Flexible components are discretized using fully parameterized ANCF beam elements, while the cross-section frame is employed as the joint coordinate system for formulating revolute joint constraints and torsion spring generalized forces. The component, joint, and driving-force formulations are subsequently assembled according to the mechanism topology to obtain the system-level dynamic equations. A spatial multi-closed-loop basic module of a truss deployable antenna is then adopted as a representative validation case to evaluate the proposed modeling framework and its application to flexible deployable mechanisms.

Comparison with the tangent frame shows how the joint coordinate representation affects the local mechanical response. The joint axis direction error of the tangent frame follows the xy-shear deformation, in agreement with the analytical relationship derived in Section 2.3, whereas the cross-section frame remains aligned with the material-defined joint axis. The corresponding difference is also reflected in the E_{12} strain component and the local stress response. Relative to the Adams/View reference simulation, the cross-section frame reduces the standard deviation of the von Mises stress residual from 26.3 to 2.9 MPa, an approximately 89.0% reduction relative to the tangent frame. In contrast, the strain energy responses of the two formulations remain similar, indicating that the localized deformation discrepancy has a limited influence on the strain energy response of the investigated link.

System-level numerical verification is performed under three torsion spring layouts that produce distinct deployment characteristics. The proposed model consistently reproduces the principal position evolution, velocity reversals, acceleration variations, and deployment-shock-folding sequence observed in the Adams/View reference simulations. The stage-wise NRMSE results show that the agreement is generally better during deployment, while larger discrepancies are concentrated in the shock and folding stages, where stronger elastic vibration and high-frequency responses increase the sensitivity of the predicted velocity and acceleration to modeling and numerical differences.

The main contribution of this work is a joint modeling formulation based on a non-orthogonal cross-section frame for fully parameterized ANCF beams. The proposed frame retains the material direction of the deforming cross-section as the revolute joint axis direction, avoiding the orthogonalization-induced axis deviation of the tangent frame under xy-shear deformation. Within the same coordinate representation, the revolute joint constraints and the generalized forces and Jacobians of the driving torsion springs are formulated consistently, enabling both joint kinematics and spring torque transmission to follow the deforming cross-section.

Several limitations remain for future investigation. First, the discrepancies observed during strongly transient stages indicate that the predicted high-frequency response is more sensitive to flexible-body representation and numerical integration; this issue should be examined further through numerical convergence studies and higher-fidelity transient analyses. Second, the present

formulation assumes ideal revolute joints, and the effects of joint friction and clearance are not considered. Third, the current joint model is formulated for revolute joints whose joint axes are defined along the y-direction of the joint coordinate system; extension to arbitrary joint locations and axis orientations would further broaden its applicability to complex deployable multibody systems.

Statements and Declarations

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Competing interests

The authors have no competing interests to declare that are relevant to the content of this article.

Data availability

The data generated and analyzed during the current study are available from the corresponding author upon reasonable request.

Author contributions

Xiaoyu Hu and Yongsheng Zhao: Conceptualization, Methodology, Software, Formal analysis, Investigation, Visualization, Writing-original draft.

Zongke Liu: Software, Validation, Data curation.

Chencheng Zhang: Validation, Data curation, Visualization.

Fei Hu: Investigation, Validation, Writing-review & editing.

Bo Han: Conceptualization, Supervision, Project administration, Funding acquisition, Writing-review & editing.

Jiantao Yao: Supervision, Project administration, Funding acquisition, Writing-review & editing.

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Appendix A

$$\Phi_q^{RR} = \left[\begin{array}{cc|cc} \mathbf{I}_3 & \frac{\partial \mathbf{A}_I \underline{\mathbf{P}}_I}{\partial (\boldsymbol{\theta}_I)^\top} & -\mathbf{I}_3 & -\frac{\partial \mathbf{A}_E \underline{\mathbf{P}}_E}{\partial (\boldsymbol{\theta}_E)^\top} \\ \mathbf{0} & (\mathbf{A}_E \underline{\mathbf{V}}_E^1)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\boldsymbol{\theta}_I)^\top} & \mathbf{0} & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^1}{\partial (\boldsymbol{\theta}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \\ \mathbf{0} & (\mathbf{A}_E \underline{\mathbf{V}}_E^2)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\boldsymbol{\theta}_I)^\top} & \mathbf{0} & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^2}{\partial (\boldsymbol{\theta}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \end{array} \right] \quad (\text{A-1})$$

$$\Phi_q^{RF} = \left[\begin{array}{cc|cc} \mathbf{I}_3 & \frac{\partial \mathbf{A}_I \underline{\mathbf{P}}_I}{\partial (\boldsymbol{\theta}_I)^\top} & & -\mathbf{S}_E \\ \mathbf{0} & (\mathbf{A}_E \underline{\mathbf{V}}_E^1)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\boldsymbol{\theta}_I)^\top} & & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^1}{\partial (\mathbf{e}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \\ \mathbf{0} & (\mathbf{A}_E \underline{\mathbf{V}}_E^2)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\boldsymbol{\theta}_I)^\top} & & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^2}{\partial (\mathbf{e}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \end{array} \right] \quad (\text{A-2})$$

$$\Phi_q^{FR} = \left[\begin{array}{cc|cc} & \mathbf{S}_I & -\mathbf{I}_3 & -\frac{\partial \mathbf{A}_E \underline{\mathbf{P}}_E}{\partial (\boldsymbol{\theta}_E)^\top} \\ (\mathbf{A}_E \underline{\mathbf{V}}_E^1)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\mathbf{e}_I)^\top} & & \mathbf{0} & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^1}{\partial (\boldsymbol{\theta}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \\ (\mathbf{A}_E \underline{\mathbf{V}}_E^2)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\mathbf{e}_I)^\top} & & \mathbf{0} & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^2}{\partial (\boldsymbol{\theta}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \end{array} \right] \quad (\text{A-3})$$

$$\Phi_q^{FF} = \left[\begin{array}{cc|cc} & \mathbf{S}_I & & -\mathbf{S}_E \\ (\mathbf{A}_E \underline{\mathbf{V}}_E^1)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\mathbf{e}_I)^\top} & & & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^1}{\partial (\mathbf{e}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \\ (\mathbf{A}_E \underline{\mathbf{V}}_E^2)^\top \frac{\partial \mathbf{A}_I \underline{\mathbf{V}}_I}{\partial (\mathbf{e}_I)^\top} & & & \left(\left(\frac{\partial \mathbf{A}_E \underline{\mathbf{V}}_E^2}{\partial (\mathbf{e}_E)^\top} \right)^\top \mathbf{A}_I \underline{\mathbf{V}}_I \right)^\top \end{array} \right] \quad (\text{A-4})$$