# Supplementary Information

## Online Resource 1

# Formal Derivations, Numerical Audit Definitions, and Claim-Evidence Mapping for a Discrete Space-Cell Framework

*A Discrete Information-Theoretic Model of Physics on a Space-Cell Substrate: Local Update Dynamics, Event-Based Time, Continuum Limits, and Candidate Geometric Realizations*

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**Purpose.** This Online Resource supplies compact derivations and audit definitions for formal and computational claims summarized in the main manuscript. It does not add a claim that physical space is cellular, that q or χ are known physical fields, that the provisional gauge layer is electromagnetism, or that a continuum stable particle has been obtained.

**Evidence rule.** Exact statements below are conditional on the specified update/action. Numerical statements refer to finite toy-model audits. A passed toy audit establishes a property of that model under the stated test; failed predeclared criteria and mapped failure boundaries are retained explicitly.

## S1. Scope, notation, and evidence classes

The main article deliberately separates several evidence classes. This supplement uses the same hierarchy so that formal deductions are not conflated with continuum approximations or finite simulation outcomes. Internal series labels (CI35-CI169 and V17-QF0-QF3) are experiment identifiers in the author's retained research archive; they are not independent physical evidence.

| Evidence class | What it establishes | What it does not establish |
|---|---|---|
| Exact/conditional | A theorem, identity, variational equation, or conservation statement for a specified local model. | That nature uses the model or that its variables equal measured physical fields. |
| Continuum/controlled approximation | A long-wavelength or low-momentum effective equation under stated scaling and branch assumptions. | Exact finite-lattice Lorentz symmetry or uniqueness of the microscopic realization. |
| Finite toy-model checkpoint | A local mechanism survives defined controls over tested grids, parameters, dimensions, and durations. | Asymptotic stability, continuum universality, or empirical validation unless separately shown. |
| Failure-boundary result | A tempting inference fails a stronger control, scale, duration, or adversarial test. | Failure of every discrete/informational model family. |

## S2. Representative reciprocal q-χ action family

The formal field/action track uses a dynamics-first characteristic recurrence as a representative model family. With $u=(q, \chi)^T$, one retained uniform one-dimensional realization is

$$\mathbf{u}_j^{n+1} + \mathbf{u}_j^{n-1} = A\left(\mathbf{u}_{j+1}^n + \mathbf{u}_{j-1}^n\right) - \Delta t^2 G\mathbf{u}_j^n \tag{S1}$$

$$A = \mathrm{diag}(\gamma_q, \gamma_\chi), \quad \gamma_q = 1 - \tfrac{1}{2}m_q^2\Delta t^2, \quad \gamma_\chi = 1 - \tfrac{1}{2}\mu_\chi^2\Delta t^2, \quad G_{11} = G_{22} = 0, \quad G_{12} = G_{21} = g \tag{S2}$$

The nearest-neighbor support advances by at most one lattice cell per update. The reciprocal off-diagonal coupling is symmetric. One-way and wrong-sign couplings are therefore deliberate non-action controls rather than alternative members of the same real-symmetric family. The continuum χ/Klein-Gordon-like equation is treated as a low-energy target of this update, not as the primitive equation placed on a grid.

## S2.1 Equivalent local discrete action and variation

For the retained symmetric recurrence, one convenient equivalent action representation is

$$S = \Delta t\, \Delta x \sum_{n,j} \mathcal{L}_j^n \tag{S3a}$$

$$\mathcal{L}_j^n = -\left(\mathbf{u}_j^{n+1}\right)^T \mathbf{u}_j^n + \left(\mathbf{u}_j^n\right)^T A\mathbf{u}_{j+1}^n - \frac{\Delta t^2}{2}\left(\mathbf{u}_j^n\right)^T G\mathbf{u}_j^n \tag{S3b}$$

where A and G are symmetric and boundary terms are handled according to the chosen periodic or open lattice. Varying u_j^n receives contributions from the forward and backward temporal bonds, the two spatial bonds, and the on-site bilinear term. The discrete Euler-Lagrange equation obtained from Eqs. (S3a)-(S3b) is exactly Eq. (S1). This displayed action is an equivalent representative form written here to make the retained CI40/CI49 recurrence transparent; it is not asserted to be a unique microscopic action.

For the symmetric continuum/static mass matrix, positivity of the coupled quadratic sector requires a nonnegative determinant. This yields the family-specific coupling condition

$$g^2 \le m_q^2 \mu_\chi^2 \tag{S4}$$

This is not a universal constant of nature. Finite-step spectral stability imposes additional band conditions, and a localized defect can create a separate localized instability even when open-channel scattering is unitary. The latter distinction became a standing audit rule after CI49-CI51.

## S2.2 Local defect, scattering channels, and conserved flux

A one-site interaction is introduced only through a local quadratic action term

$$\Delta S_{\mathrm{def}} = -\tfrac{1}{2}\Delta t\, \Delta x\; \mathbf{u}_0^T V \mathbf{u}_0 \tag{S5}$$

so the recurrence at the defect acquires -Δt²V u_0. For a uniform lead at fixed dimensionless quasifrequency Ω, the open-channel polarizations satisfy

$$[2\cos k\, A - \Delta t^2 G - 2\cos\Omega\, I]\mathbf{v} = 0 \tag{S6}$$

For a right-moving lead channel, the spatial bilinear/action flux used for normalization is

$$J = 2\sin k\; \mathbf{v}^\dagger A \mathbf{v} \tag{S7}$$

Reflection, transmission, and conversion fractions are normalized by this channel flux. The decisive controls are no defect, no reciprocal q-χ coupling, one-way coupling, wrong-sign antisymmetric backreaction, overcritical coupling, localized-mode stability, and exact causal support.

| Audit | Selected result in retained model family | Interpretation / limit |
|---|---|---|
| CI49 analytic one-site scattering | Flux closure, reciprocity, and S-matrix unitarity at roundoff scale; maximum total mode conversion ≈0.419 in the tested sweep. | Exact scattering result for the specified linear action family; not physical particle scattering. |

| Audit | Selected result in retained model family | Interpretation / limit |
|---|---|---|
| CI50 localized-mode audit | A one-dimensional q-defect instability threshold was located near Vq,crit=3.56772 and was essentially grid independent for N=128-512. | Open-channel unitarity is not sufficient; localized spectral modes require a separate stability audit. |
| CI51 held-out packets | Mean channel RMSE ≈3.4×10^-4; maximum channel error ≈0.00525 below the localized-instability threshold. | Direct local propagation reproduced the action-derived scattering prediction in held-out stable cases. |
| CI61-CI64 3D lift | 3D optical identity ≈4×10^-16; reciprocity near roundoff; held-out branch-conversion error ≤0.00777; invariant drift ≈4×10^-15. | Demonstrates dimensional lift of the formal scattering/resolvent family; thresholds are dimension dependent. |

## S3. Continuum hierarchy: effective Hamiltonian, Dirac branch, and Schrödinger envelope

For a translationally invariant unitary walk U(k), a branch-dependent effective Hamiltonian is defined by

$$H_{\mathrm{eff}}(k) = \tfrac{i\hbar}{\Delta t}\,\mathrm{Log}\,U(k) \tag{S8}$$

The matrix logarithm is multivalued. Therefore a continuum statement requires an explicit quasienergy branch and a low-momentum region that does not cross a branch discontinuity. For a two-component spinor walk, take

$$a = c\Delta t, \qquad \theta = \frac{mc^2\Delta t}{\hbar}, \qquad H_{\mathrm{eff}} = cp\,\sigma_z + mc^2\sigma_y + O\big((ka)^2, \theta^2, ka\theta\big) \tag{S9}$$

with |ka|≪1 and |θ|≪1. To leading order this gives a 1+1 dimensional Dirac-like Hamiltonian up to representation choice. The lattice remains a preferred microscopic discretization; the Lorentz-like equation is an effective long-wavelength result, not exact microscopic Lorentz invariance.

### S3.1 Nonrelativistic envelope

After selecting the positive-energy branch, factoring out the rest-energy phase, and assuming |p|≪mc and |V|≪mc², the small spinor component can be eliminated to leading order. The large component obeys

$$i\hbar\,\partial_t\phi = \left(\frac{p^2}{2m} + V\right)\phi + O\!\left(\frac{p^4}{m^3c^2}\right) \tag{S10}$$

The Schrödinger equation is therefore secondary in the hierarchy: local discrete update → branch-selected Dirac-like continuum → low-momentum positive-energy envelope. Higher-order corrections include the usual relativistic p^4 term and, in richer dimensions/couplings, spin and Darwin-type terms. The supplement does not claim a derivation of the Hilbert-space axioms, Born rule, or quantum field theory from non-quantum primitives [1-3,5].

## S4. Provisional gauge/charge checkpoint

The gauge result summarized in the main manuscript is deliberately narrow. Starting from a complex matter field built from the internal two-component sector, local phase freedom requires matter and link variables to transform together:

$$\psi_x \mapsto e^{i\alpha_x}\psi_x, \qquad U_{xy} \mapsto e^{i\alpha_x}U_{xy}e^{-i\alpha_y} \tag{S11}$$

The covariant neighbor value at x is U_xy ψ_y. Gauge-invariant curvature information is represented by closed-loop plaquette holonomy

$$W_p = \prod_{(xy)\in\partial p} U_{xy} = e^{i\Phi_p} \tag{S12}$$

and the variational construction uses a temporal potential before temporal gauge fixing so that a Gauss-like constraint arises from variation rather than being inserted as a post-hoc repair:

$$\sum_i (E_{x,i} - E_{x-\hat{i},i}) = \rho_x \tag{S13}$$

The CI71-CI85 sequence retraced the construction in 1D, 2D, and 3D; added plaquette curvature and Wilson holonomy; coupled matter current reciprocally to electric-link and curvature variables; and compared true gauge-equivalent states with matter-only, link-only, and flux/holonomy spoofs. Wrong-sign or one-sided backreaction produced visible Gauss/closure failures, while the reciprocal action family closed near roundoff in the tested cases.

**Non-inference.** This checkpoint does not derive electromagnetism, measured electric charge, charge quantization, a physical photon, color charge, weak isospin, or Standard Model U(1). Its narrower role is to show that a local phase/link/Gauss/holonomy structure can be embedded inside the same causality/conservation audit architecture.

## S5. Numerical audit definitions and retained failure boundaries

The computational program uses controls to distinguish a mechanism from a success-specific repair rule. Table S2 summarizes the principal audit obligations relevant to claims made in the main manuscript. A missing or failed obligation narrows the claim rather than being silently absorbed by parameter retuning.

| Claim area | Primary positive audit | Mandatory adverse controls / scaling checks | Current defensible status |
|---|---|---|---|
| Causal local dynamics | Zero support outside the graph-update cone. | Nonlocal/global solver control; boundary/source audit. | Exact for specified nearest-neighbor/finite-depth updates. |
| Reciprocal q-χ interaction | Local/global ledger closure; real symmetric action. | No coupling; one-way coupling; wrong sign; overcritical g. | Formal checkpoint in specified family. |
| Scattering | Flux-complete R/T/conversion and held-out packet agreement. | No defect; no coupling; localized spectral-mode stability; reciprocity. | Action-derived finite-model checkpoint. |
| Protected identity | True/template match with local ledger and false leakage below threshold. | No carrier/template; wrong/swapped/spoofed carrier; deletion; no phase lock; bridge/tail stress. | Toy reconstruction mechanism; not a fundamental particle law. |
| Internal symmetry identity | Conserved internal state/charge under symmetric dynamics. | Explicit symmetry breaking; near-parallel false states. | Exact charge/continuity within specified action; $F=\cos^2\theta$ sets a resolution boundary. |
| Topology/charge-current response | Sector preservation, connected support, conserved charge, systematic Q/winding response. | Topology-only; charge-only; current-only; weak binding; long duration; fixed-h volume; held-out orientation; recentering. | Finite-lattice response/failure-boundary checkpoint; no equilibrium or stable soliton. |
| Continuum/universality | Held-out k/angle; multiple stencils/families; resolvent/time reconstruction. | Finite-size and branch-choice audit; blind surfaces; 2D/3D comparison. | Strong numerical/formal evidence; not an RG universality theorem. |
| χ / geometry | Source-specific causal response and path bias. | False/shuffled sources; moving source; calibration/refinement; deformation comparator. | Model-selection evidence; full GR not derived. |

### S5.1 Predeclared and long-duration topology/charge-current corrections

The late topology branch is included because it illustrates the project's failure-boundary rule. Winding and conserved charge/current produced systematic finite-lattice response, but stronger audits invalidated the interpretation of a fitted zero or short-window crossing as equilibrium evidence.

| Series | Test fixed before or applied to stronger inference | Result | Publication consequence |
|---|---|---|---|
| CI157 | Eight held-out orientations; fixed parameters; N=24 and N=36; criterion ≥7/8 sign-bracketed at every grid plus fine-grid spread limits. | Overall FAIL: N=24 gave 6/8; N=36 gave 8/8. Fine-grid IQR/median ≈0.0039. | Preserve the coarse-grid failure; do not call fine-grid clustering continuum orientation independence. |
| CI158 | Long-duration t=40 drift screen for principal n=2 candidates. | Structure, winding, Q and energy accounting persisted, but the pre-specified final-quarter \|dR/dt\|≤0.02 screen failed. | Long-lived connected oscillation ≠ boundedness proof. |
| CI165-CI166 | Fixed-h volume comparison of fitted contraction/expansion crossings. | Finite-time crossings appeared but shifted strongly with volume. | A fitted crossing is a finite-time response statistic, not equilibrium. |
| CI168 | Pre-specified t=40 extension of larger-volume t=16 positive brackets. | Upper positive brackets reversed; all four t=40 second-half fitted slopes were negative. | No persistent long-duration sign bracket. |
| CI169 | Equal-duration windows plus periodic-center/recentered-radius audit. | Every case changed fitted sign across physical time; recentering preserved all 16 signs. Maximum center displacement ≈0.0372 lattice units; maximum radius correction ≈5.94×10^-4. | The fitted linear coefficient is time-local and not a translation artifact; never use fitted zero alone as a balance claim. |

## S6. QMM/FID-χ bridge audit

The main manuscript notes strong architectural convergence with the Quantum Memory Matrix / finite-information-deviation literature, while avoiding an identification of the QMM entropy field with χ. A bounded V17 bridge audit tested that distinction directly.

In a normalized one-excitation lattice sector, a one-cell reduced state has eigenvalues $p_x$ and $1\text{-}p_x$, so the local von Neumann entropy is

$$S_x = -\mathrm{Tr}(\rho_x \ln \rho_x) = H_2(p_x) = -p_x \ln p_x - (1-p_x)\ln(1-p_x) \tag{S14}$$

Thus $S_x$ is a deterministic scalar reparameterization of occupancy in this sector. Same-density states with different phase/current structure therefore have identical local entropy, even though their relational currents differ. This gives a direct reason not to set χ=S as a general identity.

| Audit | Selected diagnostic | Interpretation |
|---|---|---|
| QF0 formal crosswalk | 6/16 features were explicit/core in both programs; 13/16 were explicit-or-adjacent. | Organizational comparison only; the counts are not scientific similarity statistics. |
| QF1 capacity/coarse-graining | Errors decreased with resolution/capacity, but fitted density exponents differed between smooth and rough states (≈-1.06 vs ≈-0.88; maximum exponent difference ≈0.178). | Finite-capacity suppression is generic enough that the exponent is not, by itself, a distinctive signature. |

| Audit | Selected diagnostic | Interpretation |
|---|---|---|
| QF2 entropy specificity | Maximum entropy-field separation across same-density phase variants ≈1.95×10^-16, while current separations were 0.084-0.230 in the tested normalization; shifted-density entropy separation ≈0.567. | Local entropy retains scalar density geometry but is blind to same-density phase/current/provenance. |
| QF3 entropy-sourced χ | Local response had zero detected amplitude outside its update cone; an instantaneous global control placed 22.8% of response norm outside the initial source support. Occupancy- vs entropy-sourced final responses correlated 0.999754 with ≈2.1% best-scaled relative shape error. | S can be a restricted local scalar source candidate, but χ propagation is extra dynamical structure and entropy does not carry the full relational state. |

**Bridge status.** The current defensible relation is not χ=S. A possible hierarchy is full local relational state → reduced local state → scalar entropy diagnostic → candidate source for an independently local χ dynamics. Even this hierarchy remains a model hypothesis until a reciprocal action, physical mapping, and cross-model controls are derived.

## S7. Spectral, correlation, and representation checks

The action family was also audited in representations that expose poles, residues, resolvents, correlations, and equivalent reconstructions. These are not new physical degrees of freedom; they are alternative mathematical views of the same retained dynamics.

| Series | Result | Limit |
|---|---|---|
| CI41-CI42 | Two stable linear branches; direct impulse reconstructed from poles/residues to roundoff; thermodynamic spectral measure and resolvent-discontinuity signature. | Linear family only. |
| CI52 | Closed q-projected resolvent agreed with Brillouin-zone quadrature at ≈3.5×10^-15; physical nonanalyticities are square-root branch points at real band edges. | Formal result for specified 1D linear action. |
| CI53-CI60 | Gaussian critical baselines ν≈1/2 and γ≈1; 2D logarithmic and 3D 1/r correlations; connected four-point at roundoff; cross-stencil/family convergence. | Ordinary Gaussian/critical behavior; no LCFT inference. |
| CI55 / CI64 | Equivalent representations preserved the same response while coefficient counts differed substantially; 3D spread ≈12.7-fold. | Representation efficiency is not a physical parameter. |
| CI63 | 3D spectral-edge density ρ∝√ε with cumulative weight ∝ε^(3/2). | Dimension-specific; do not transfer the 1D inverse-square-root edge unchanged. |

## S8. Claim-evidence map for the main manuscript

| Main-manuscript topic | Primary support in this Online Resource | External/archival support | Claim ceiling |
|---|---|---|---|
| Local continuity / reciprocal action | S2, S5 | Reardon 2026f [5] | Conditional model-family derivation. |
| Action-derived scattering | S2.2, S5 | Reardon 2026h [6] plus retained CI49-CI64 code/audits | Finite-model scattering law; no identification with particle physics. |
| Dirac / Schrödinger hierarchy | S3 | Arrighi [1]; Di Molfetta & Arrighi [2]; Reardon 2026f [5] | Controlled continuum/envelope limits under branch/scaling assumptions. |
| Gauge/charge layer | S4, S5 | Reviewer-only CI49-CI169 checkpoint | Provisional local U(1)-like structure; not electromagnetism/SM. |
| Protected/relational identity | S5 | Reardon 2026h [6] | Toy mechanism and symmetry constraints; no ontic hidden tag claim. |
| Topology/charge-current | S5.1 | Reviewer-only CI49-CI169 checkpoint | Systematic finite-lattice response and failure boundaries; no equilibrium/stable soliton. |
| Weak-field geometry/model selection | S5 audit obligations | Reardon 2026g [7] | Candidate geometry comparison; full GR not derived. |
| QMM/FID convergence and χ bridge | S6 | Neukart & Marx [8] | Architectural convergence and restricted sourcing hypothesis; no theory equivalence. |

## S9. Reproducibility, archive, and reviewer context

The main manuscript is a foundations/status synthesis rather than a scenario-level simulation report. The published technical reports cited below provide the formal and branch-level background. The later CI identifiers refer to retained deterministic Python scripts and compact contact-sheet/checkpoint artifacts. For review, a current CI49-CI169 checkpoint is supplied separately as a reviewer-only related file so that positive findings can be read together with the negative criteria that narrowed them.

- Reardon (2026f) supplies the deeper local-unitary, Dirac/Schrödinger, causal-cone, and no-signaling formal background.
- Reardon (2026g) supplies the weak-field deformation-versus-χ model-selection framework and its source-specificity / false-source obligations.
- Reardon (2026h) supplies the protected-identity, local-ledger, and scattering-reconstruction branch background.
- The reviewer-only CI49-CI169 checkpoint records the action/scattering, internal-symmetry, provisional gauge, topology, charge-current, and failure-boundary sequence, including failed predeclared criteria.
- Research data and large branch artifacts are intended for repository deposition where practical; SI is not being used as a substitute for a reproducibility repository, consistent with Springer guidance.

## References


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**Online Resource caption for the main article.** Online Resource 1: Formal derivations, numerical audit definitions, and claim-evidence mapping for the representative q-χ action family, continuum hierarchy, provisional gauge/charge checkpoint, topology failure-boundary audits, and QMM/FID-χ bridge.