

Supplementary Material

When Does Persistent Shared State Matter?

The supplement reports manuscript-level statistics and key seed-level contrasts supporting the main text; public reproducibility materials are cited at the end.

Supplement A - Primary scope x temporal-availability qualification

Each episode contains a two-bit selector and two later evidence vectors. The selector determines whether each target bit is taken from the first or second evidence vector. Scope (consumer-local versus cross-scope) and temporal availability (CURRENT versus PAST-ONLY) are varied independently, while the target rule, architecture roles, and optimization recipe remain fixed. A common ownership firewall after the initial communication opportunity prevents same-stage copying from erasing the scope manipulation.

Condition	Persistent	Reset	Local	NoComm
consumer-local + CURRENT	1.0000 / 1.0000	1.0000 / 1.0000	1.0000 / 1.0000	0.2500 / 0.2500
consumer-local + PAST-ONLY	1.0000 / 1.0000	1.0000 / 1.0000	1.0000 / 1.0000	0.2500 / 0.2500
cross-scope + CURRENT	1.0000 / 1.0000	1.0000 / 1.0000	1.0000 / 1.0000	0.2500 / 0.2500
cross-scope + PAST-ONLY	1.0000 / 1.0000	0.5811 / 0.5701	0.5825 / 0.5725	0.2500 / 0.2500

Critical contrast	seed 1	seed 2	mean
Persistent - Reset	+0.4189	+0.4299	+0.4244
Persistent - Local	+0.4175	+0.4275	+0.4225

Supplement B - Final-clean solution-accessibility characterization

The accessibility study uses a separate final-clean DSTAR_TEMPORAL host and is not another primary factorial cell. The final runtime-reduced characterization contains 288 primary units: 192 in the complete 64-seed Adam incidence arm and 48 units in each of two matched 24-seed alternative-recipe arms.

Architecture	PASS	BASIN	OTHER FAIL	Basin proportion	Wilson 95% CI
Local	63	1	0	0.0156	[0.0028, 0.0833]
Reset	60	4	0	0.0625	[0.0246, 0.1500]
Persistent	58	6	0	0.0938	[0.0437, 0.1898]

Architecture	Adam matched 24	Frozen full recipe	SGD+momentum
Persistent	23 PASS + 1 BASIN	24 OTHER FAIL	24 OTHER FAIL
Reset	23 PASS + 1 BASIN	24 OTHER FAIL	24 OTHER FAIL

Architecture	n	both recovered	constant only	warm only	neither by 7.5k
Persistent	6	2	1	1	2
Reset	4	3	0	0	1

Across the ten selected basin checkpoints, 7/10 recovered under at least one frozen continuation by 7.5k. The alternative-recipe controls demonstrate that disappearance of the named basin is not equivalent to improved accessibility.

Supplement C - Causal decomposition

Cross-scope + CURRENT Reset	ALL	INTERMEDIATE ONLY	FINAL ONLY	NONE
Joint accuracy	1.0000	1.0000	0.2500	0.2500

Cross-scope + PAST-ONLY Persistent	ALL	INTERMEDIATE ONLY	FINAL ONLY	NONE
Joint accuracy	1.0000	1.0000	1.0000	0.2500

Carry-removal condition	seed 1 drop	seed 2 drop	mean drop
consumer-local + PAST-ONLY	0.5100	0.5334	0.5217
cross-scope + CURRENT	0.2981	0.4399	0.3690
cross-scope + PAST-ONLY	0.6284	0.4263	0.5273

Frozen causal interpretation: current-context broadcast causal = true; past-context broadcast causal = true; persistence causal main effect = true; persistence scope-specific = false.

Supplement D - Consumer and host validation

Consumer binding	seed 1 P-Reset	seed 2 P-Reset	mean
map 1	0.4314	0.4365	0.4340
map 2	0.4248	0.4399	0.4324
map 3	0.4268	0.4309	0.4288

Memory-access geometry	seed 1 P-Reset	seed 2 P-Reset	mean
canonical	0.4216	0.4290	0.4253
state-only	0.4309	0.4316	0.4313

The state-only geometry removes the previous-hidden-state contribution to input-attention selection while preserving ordinary recurrent h/c carry.

Supplement E - Same-construct rule generalization

Rule	seed 1 P-Reset	seed 2 P-Reset	mean
original	0.4314	0.4297	0.4305
altered same-construct rule	0.4358	0.4338	0.4348

Before training, both rules retained oracle accuracy 1.0, selector-hidden Bayes joint accuracy 0.5625, direct consumer-token chance accuracy 0.25, and matched information geometry.

Supplement F - Source-faithful second-family transportability

Condition	Persistent	Reset	Local/direct	NoComm
consumer-local + PAST-ONLY	1.0000	1.0000	1.0000	0.2500
cross-scope + PAST-ONLY	1.0000	0.5637	0.5651	0.2500

Seed	Persistent - Reset	Persistent - Local/direct
20276201	+0.4343	+0.4338
20276202	+0.4382	+0.4360
mean	+0.4363	+0.4349

All predeclared cell-by-seed gates passed across 16 scientific units with zero engineering failures. The result supports qualitative transportability under the common frozen host/bridge, not numerical or bridge-independent universality.

Supplement G - Prospective theory-boundary tests

Endpoint	Persistent	Reset	Local	NoComm	Decision
remote/past Persistent-favoring	1.0000	0.5625	0.5625	0.2500	Persistent - Local +0.4375 on both seeds
proposed Local-favoring	1.0000	1.0000	1.0000	0.2500	Local - Persistent 0.0000; gate failed

A subsequent finite four-candidate Local-side mechanism screen produced Persistent = Local = 1.0000 for every candidate and both paired seeds. Across 8 paired groups / 16 scientific units, Local - Persistent was 0.0000 throughout, positive_candidates was empty, and no candidate was selected. The route was closed by the frozen stop rule rather than extended with a fifth post-exposure candidate.

Supported theory: local/communication sufficiency -> Persistent shared-state advantage under joint cross-scope and temporal-unavailability demand. A finite negative screen does not prove Local-dominant regimes impossible.

Reproducibility and data availability

Publication-facing code, frozen protocols, sanitized result summaries, source-artifact identities, and reproducibility metadata are available at <https://github.com/Utopia-YangLin/persistent-shared-state-boundaries> (frozen branch submission-v0.1, commit f58e6f1287afc965612f71cf444c86e0460bb8a1). Supplements A-G provide the reader-facing statistics and validation results; machine-local and private orchestration paths are not republished.