

Effects of multiresolution topography, density representation, and geoid–quasigeoid separation on gravimetric geoid modeling

Sevda Olgun

Kocaeli Universitesi

Cheinway Hwang

cheinway@nycu.edu.tw

National Yang-Ming University: National Chiao Tung University <https://orcid.org/0000-0002-3322-353X>

Rahayu Lestari

National Yang Ming Chiao Tung University

Yu-Shen Hsiao

National Chung Hsing University

Research Article

Keywords: geoid–quasigeoid separation, gravimetric geoid, terrain correction, multiresolution topography, lateral density variation

Posted Date: October 6th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10823626/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

1 **Effects of multiresolution topography, density representation, and geoid–quasigeoid**
2 **separation on gravimetric geoid modeling**

3 Sevda Olgun¹, Cheinway Hwang^{2*}, Rahayu Lestari², and Yu-Shen Hsiao³

4 ¹Department of Geomatics Engineering, Kocaeli University, Türkiye.

5 ²Department of Civil Engineering, National Yang Ming Chiao Tung University, Taiwan.

6 ³Department of Soil and Water Conservation, National Chung Hsing University, 145 Xingda Road,
7 Taichung 402, Taiwan.

8

9 * **Corresponding author:** Cheinway Hwang, cheinway@nycu.edu.tw

10 **Abstract**

11 This study evaluates how topographic resolution, density representation, and geoid–quasigeoid
12 separation (GQS) formulation affect terrain correction, GQS, and the final gravimetric geoid over
13 the rugged terrain of Taiwan. The analysis uses an island-wide 20 m LiDAR DEM, SRTM1, local
14 1 m LiDAR data, and both constant and laterally varying topographic density models. Terrain
15 corrections are computed at the surface and at half the topographic height, and their effects are
16 propagated through a three-term GQS formulation and gravimetric geoid computation. Replacing
17 the 20 m LiDAR DEM with SRTM1 produces local terrain-correction differences of several tens
18 of mGal, GQS differences of up to about 15 cm, and gravimetric-geoid differences reaching the
19 decimeter level in mountainous regions, although the regional RMS geoid difference is about 1
20 cm. Lateral density variations generate smaller but spatially coherent changes in terrain
21 corrections, GQS, and geoid heights that follow regional geological structures; however, they do
22 not systematically reduce the differences between gravimetric and geometric geoid heights relative
23 to the constant-density solution. The three-term GQS formulation produces local geoid-height
24 differences of up to about 30 cm relative to the traditional one-term approximation and yields a
25 modest reduction in the scatter of gravimetric–geometric geoid differences in the denser 1,994-
26 station GNSS/leveling dataset. A local experiment using 1 m LiDAR topography further shows
27 that unresolved near-zone terrain can cause centimeter-to-decimeter changes in GQS and
28 gravimetric geoid heights in areas of extreme relief. These results demonstrate that DEM
29 resolution, density representation, and GQS formulation influence different stages of gravimetric
30 geoid modeling and should be assessed through their complete propagation to the final geoid. The
31 study provides an end-to-end multiresolution assessment of these effects and identifies the

32 conditions under which refined terrain and GQS modeling become important for centimeter-level
33 geoid determination.

34 **Key words:** geoid–quasigeoid separation; gravimetric geoid; terrain correction; multiresolution
35 topography; lateral density variation

36 **Abbreviations**

37 DEM Digital Elevation Model

38 GNSS Global Navigation Satellite System

39 GQS Geoid–Quasigeoid Separation

40 LiDAR Light Detection and Ranging

41 RMS Root Mean Square

42 SRTM Shuttle Radar Topography Mission

43 TC Terrain Correction

1 Introduction

Centimeter-level gravimetric geoid modeling requires accurate representation of several closely related components, including terrain geometry, topographic density, terrain corrections (TCs), and geoid–quasigeoid separation (GQS). These factors are particularly important in mountainous regions, where short-wavelength topographic variations, strong density contrasts, and large gravity gradients can introduce substantial local errors into the gravity reduction and geoid computation processes. Although each of these effects has been investigated individually, their combined propagation through the complete gravimetric geoid modeling chain remains less well quantified.

Terrain correction is strongly controlled by the resolution and quality of the digital elevation model (DEM), assumptions regarding topographic density, and the computational method. Numerous studies have demonstrated that TC is highly sensitive to DEM resolution and the representation of fine-scale terrain morphology (Featherstone and Kirby, 2000; Hwang et al., 2003; Tsoulis et al., 2003; D’Urso and Trotta, 2015; Sampietro et al., 2016; McCubbine et al., 2017; Goyal et al., 2020; Foroughi et al., 2023; Olgun et al., 2023). Errors in TC due to DEM limitations can reach tens of mGal (Olgun and Üstün, 2021; Olgun et al., 2023) and may propagate into geoid errors at the centimeter-to-decimeter level (Olgun et al., 2025). This sensitivity is especially pronounced in rugged terrain, where global DEMs such as SRTM may smooth short-wavelength topographic features that are resolved by high-resolution LiDAR data. Despite this, global DEMs remain widely used because of their broad coverage and accessibility (Tenzer et al., 2015; Nsiah Ababio et al., 2024; Wang et al., 2023; Flury and Rummel, 2009).

Topographic density provides a second source of uncertainty. A constant density of 2.67 g/cm^3 is commonly assumed in gravimetric geoid modeling, but real topographic masses exhibit substantial lateral density variations associated with lithology, geological structure, and tectonic setting. These variations modify the gravitational effect of topography and therefore influence both terrain corrections and the resulting geoid, particularly in regions with strong geological contrasts (Kuhn, 2003; Sjöberg, 2004; Sheng et al., 2019; Tenzer and Nsiah Ababio, 2023; Olgun et al., 2025). The effect of lateral density representation is generally smaller than that of topographic relief, but it may introduce systematic regional patterns that are not represented by constant-density models.

GQS is another important component of the gravimetric geoid modeling process. It represents the separation between the geoid and quasigeoid and is required for transforming between orthometric and normal height systems, relating geoid and quasigeoid models, and unifying vertical datums. Traditionally, GQS has been computed using a simplified formulation based on the simple Bouguer gravity anomaly. However, this approximation can introduce errors ranging from several centimeters to decimeters, particularly in mountainous and rugged regions (Wang et al., 2023; Tenzer et al., 2006; Flury and Rummel, 2009; Tenzer et al., 2016).

More rigorous GQS formulations have therefore been developed for high-accuracy applications. The three-term GQS formulation incorporates the complete Bouguer anomaly, mean topographic gravity, and vertical gravity-gradient contributions (T_1 , T_2 , and T_3 ; see Section 4) and has been shown to improve the representation of GQS in mountainous regions (Flury and Rummel, 2009; Sjöberg, 2010; Wang et al., 2023). Wang et al. (2023), building on the formulation of Sjöberg (2010), derived a mathematically equivalent three-term expression using a Taylor-series expansion of the Bouguer disturbance to account for the nonlinear variation of gravity along the plumb line.

89 Their results in Colorado showed that, although the mean contribution of the higher-order terms is
90 close to zero, with a standard deviation of about 2 mm, local effects can reach up to ± 10 cm.
91 Simplified GQS formulations can therefore introduce errors of 20–30 cm in high-relief regions
92 (Sjöberg, 2010; Wang et al., 2023). Accurate application of the three-term formulation also
93 depends on reliable terrain corrections and estimates of the mean topographic gravity within the
94 topography.

95 Previous studies have demonstrated the importance of terrain geometry, topographic density
96 variations, and higher-order GQS terms in geoid and quasigeoid modeling (Flury and Rummel,
97 2009; Sjöberg, 2010; Foroughi and Tenzer, 2017; Wang et al., 2023). However, these effects have
98 often been investigated separately, using different formulations, DEM resolutions, or
99 computational strategies. Consequently, their relative importance and their propagation from
100 terrain representation through TC and GQS to the final gravimetric geoid remain insufficiently
101 quantified. In particular, few studies have examined, within a common computational framework,
102 the combined influence of meter- to regional-scale topographic representation, lateral density
103 variation, and GQS formulation on terrain corrections evaluated at different heights within the
104 topography and on the resulting gravimetric geoid.

105 This study addresses this gap through a controlled multiresolution analysis over Taiwan,
106 where rugged mountains, steep relief, strong geological contrasts, and abrupt land–sea transitions
107 provide a demanding environment for gravimetric geoid modeling. The analysis systematically
108 compares island-wide 20 m LiDAR topography, local 1 m LiDAR data, SRTM1, and both constant
109 and laterally varying topographic density representations. The effects of DEM resolution and
110 density representation are then propagated through TC and GQS to the final gravimetric geoid. In
111 parallel, the influence of the GQS formulation itself is assessed by comparing one-, two-, three-

term, and traditional formulations. The resulting geoid models are evaluated using two GNSS/leveling datasets. The objective is not to develop a new GQS formulation, but to quantify how topographic resolution, density representation, and GQS formulation jointly affect an end-to-end gravimetric geoid solution in rugged terrain.

2 Study area

The study area covers Taiwan and its surrounding ocean, spanning 21.5°N–25.5°N and 119.5°E–122.5°E. Taiwan is located at the convergent boundary between the Eurasian Plate and the Philippine Sea Plate and is characterized by highly variable and rugged topography. Steep mountain ranges rise abruptly from narrow coastal plains and are adjacent to deep ocean basins, resulting in large elevation contrasts that pose significant challenges for accurate geoid modeling.

The Central Mountain Range dominates the island and reaches elevations up to 3952 m (Hwang et al., 2007). Immediately offshore, the terrain descends rapidly into the Pacific Ocean, where seafloor depths exceed 5,000 m over short horizontal distances. This sharp transition between high mountains and deep ocean basins generates strong spatial variations in the gravity field. In contrast, western Taiwan consists primarily of broad alluvial plains that gently slope toward the shallow Taiwan Strait. Although relief is less pronounced in this region, land–sea transitions and subtle elevation changes still influence the local gravity field.

The combination of high-relief mountain belts, abrupt coastal escarpments, and complex bathymetry makes Taiwan an exceptional natural laboratory for geoid studies. This setting motivates the use of high-resolution topographic data, dense gravity observations, and realistic density information, which are described in Section 3 and form the basis for the methods presented in Section 4.

3 Data

3.1 Digital elevation models

Accurate digital elevation models (DEMs) are essential for reliable TC and GQS. In this study, multiple DEM datasets with different spatial resolutions were employed to represent the land topography of Taiwan. Consistent with the geodetic boundary values required for the gradient solution adopted in this study (Section 4), only land elevations are used in terrain-related computations.

The primary dataset is a 20 m LiDAR-derived DEM provided by the Department of Land Administration, Taiwan (<https://data.gov.tw/en/datasets/169808>). This DEM covers the main island of Taiwan and captures fine-scale topographic features (Fig. 1a). For TC computations, the DEM was extended radially to a distance of 167 km from each gravity station, yielding an effective computational domain spanning 19°–28°N and 116.5°–125°E. Elevations outside the LiDAR coverage were interpolated as described in Section 5.1.

To evaluate the influence of DEM resolution, a second terrain dataset was constructed from SRTM Version 4 at 1" resolution (called SRTM1). Land elevations were prepared on 1" and 15" grids, with the 15" grid obtained by interpolation from the 1" data, and were used in place of the 20 m LiDAR DEM. Fig. 1b shows the elevation differences between the two datasets. Although differences are generally modest in low-relief areas, they become substantially larger in mountainous regions and locally reach several hundred meters. The largest discrepancies are concentrated along steep ridges, deeply incised valleys, and areas of abrupt elevation change, where the coarser SRTM representation smooths short-wavelength terrain features resolved by LiDAR. Additional differences may arise from vegetation-related surface elevations and

inconsistencies in the vertical datums of the global and local datasets. These results demonstrate that DEM selection can strongly affect the representation of rugged terrain and, consequently, the accuracy of terrain corrections.

To further investigate near-zone effects at very high resolution, a 1 m LiDAR DEM was used in selected mountainous areas (Fig. 2). Derived from multi-year airborne LiDAR surveys, this dataset has a vertical accuracy better than 0.2 m and resolves fine-scale terrain features, including sharp ridgelines, river terraces, narrow valleys, and erosional gullies, that are smoothed or omitted in coarser DEMs (Fig. 3a). Such short-wavelength topographic features are particularly important for near-zone terrain corrections, because local slopes and abrupt elevation changes strongly influence the gravitational effect of nearby terrain. Fig. 3b shows the elevation differences between the 1 m and 20 m LiDAR DEMs. The differences range from -520 to 450 m and are concentrated mainly along steep ridges, deeply incised valleys, and abrupt terrain breaks. Their alternating positive and negative patterns indicate that the 20 m DEM smooths and spatially averages sharp terrain features rather than reproducing their local geometry. These differences therefore reflect not only the magnitude of the relief, but also the loss of short-wavelength topographic information caused by spatial-resolution degradation.

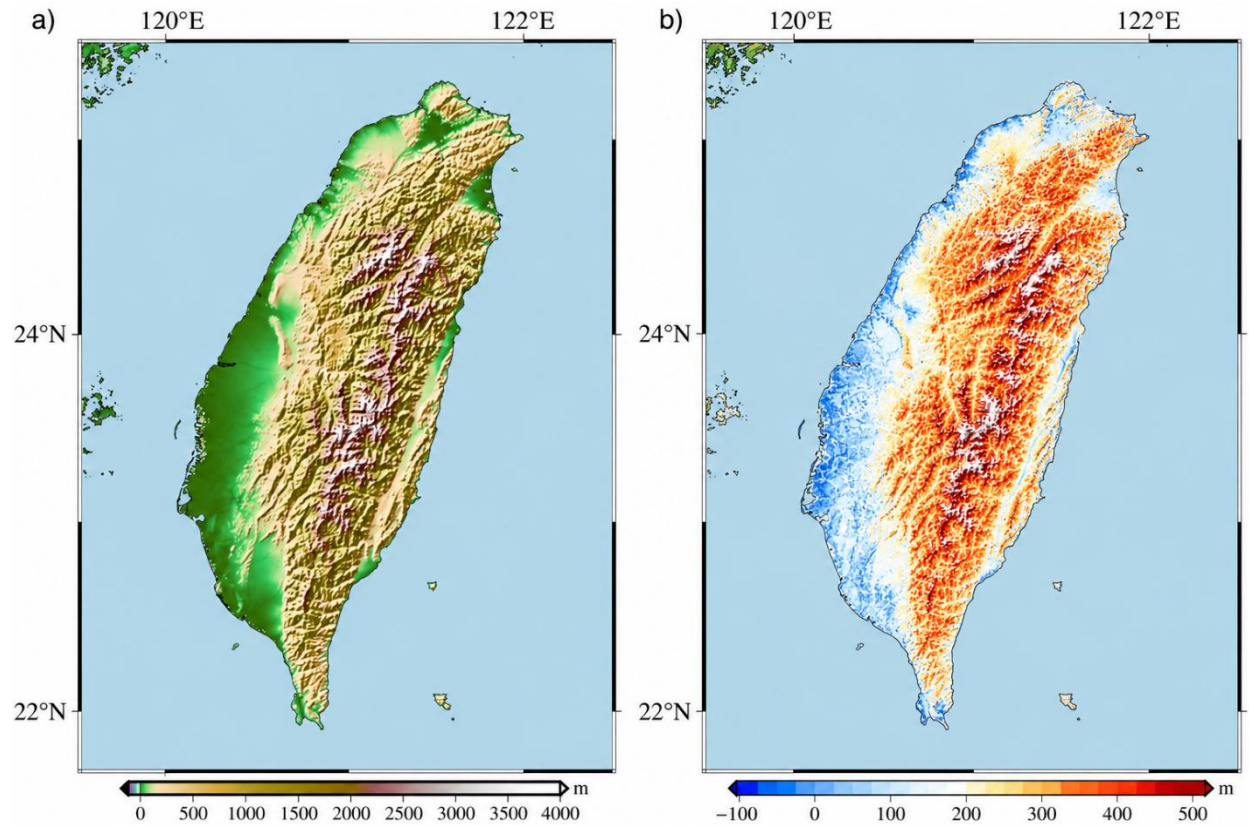


Fig. 1 a Elevation model of Taiwan derived from the 20 m LiDAR DEM, illustrating the island's pronounced relief and steep terrain. **b** Elevation differences between the 20 m LiDAR DEM and SRTM1 heights over land. Although the differences are generally modest, localized deviations become evident in mountainous regions, reflecting the combined effects of terrain steepness, spatial resolution, vegetation penetration, and vertical-datum inconsistencies.

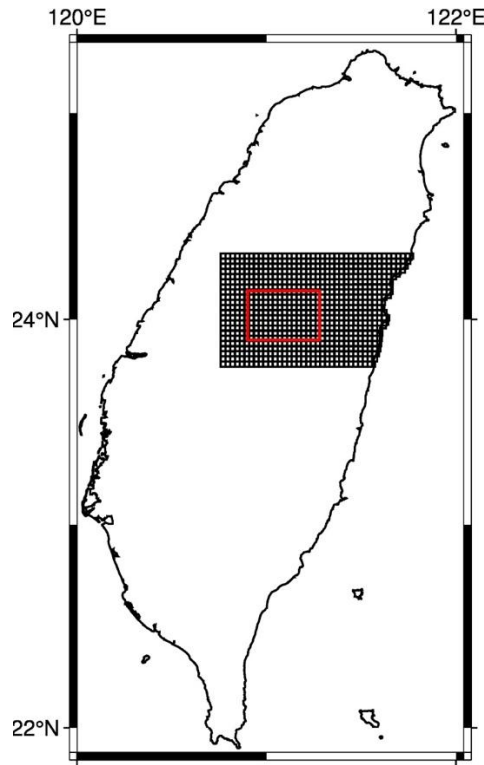


Fig. 2 Coverage of the 1 m LiDAR DEM (black dots) and the geoid modeling area where the 1 m DEM is used for terrain correction (red rectangle).

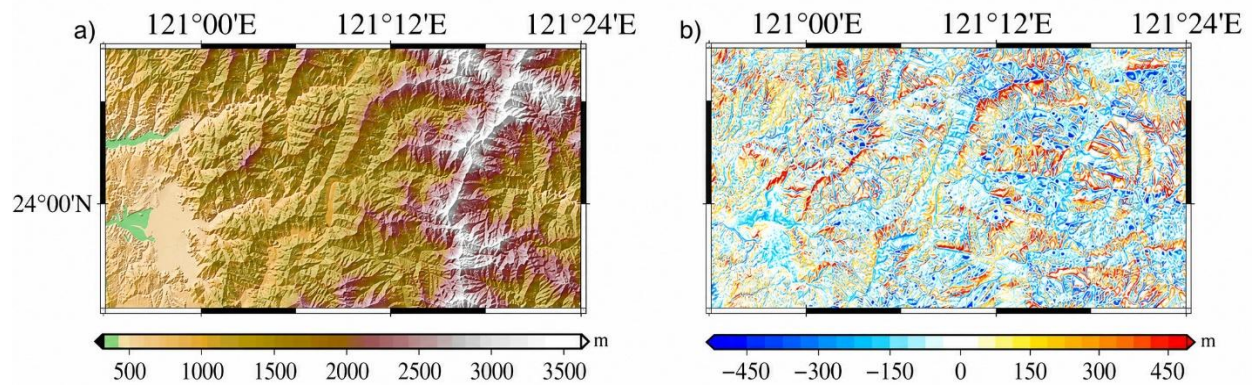


Fig. 3 a High-resolution 1 m LiDAR DEM of the mountainous test area, resolving fine-scale topographic features and steep relief. **b** Elevation differences between the 1 m LiDAR DEM and the 20 m DEM, with localized deviations emphasizing the influence of terrain steepness and spatial-resolution degradation on elevation representation.

3.2 Density model

Incorporating realistic density information into TC and GQS calculations remains a longstanding challenge in physical geodesy. Lateral density variations within the crust modify the gravitational effect of topographic masses and thus influence both geoid and height-anomaly estimates. Although the assumption of a constant topographic density is computationally convenient and widely adopted, it can introduce systematic biases in regions characterized by strong lithological and tectonic contrasts.

To account for these effects, we adopt the lateral density model of Taiwan developed by Hwang and Hsiao (2003). The original model provides density values on a 1' grid and captures the major geological units across the island. The densities range from 1.95 to 3.41 g/cm³, with a mean value of 2.37 g/cm³ (Fig. 4). In areas where local density information is unavailable, a representative constant density of 2.67 g/cm³ is assigned.

To ensure consistency with the DEM datasets, the density model is resampled using nearest-neighbor interpolation to generate grids at 1 m, 20 m, 1", and 15" resolutions. These resampled density fields are then incorporated into the corresponding TC and GQS computations. They form the basis for assessing the influence of lateral density variations on TCs and GQS estimates in the sensitivity analyses presented in Section 5.3.

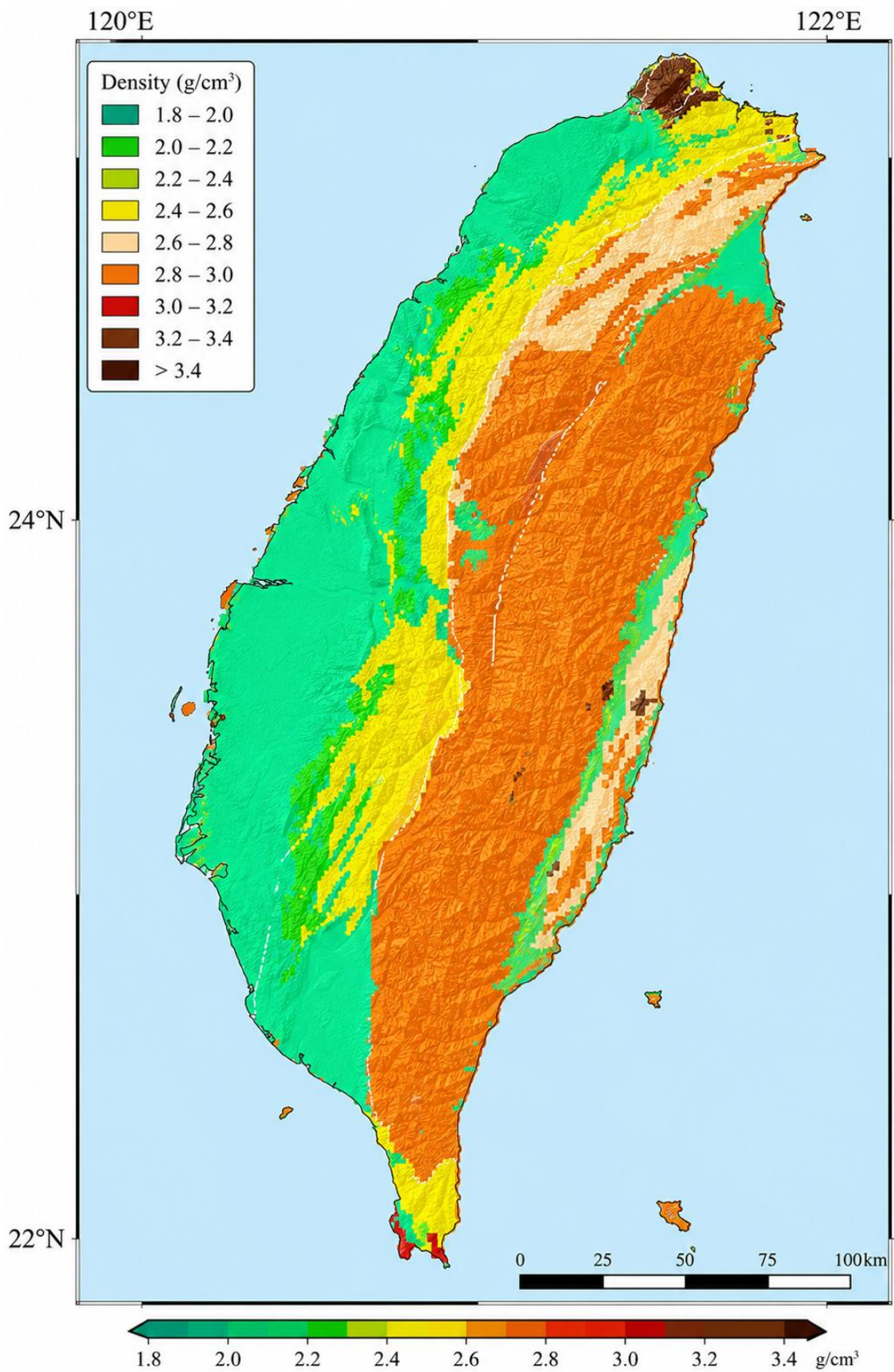


Fig. 4 Lateral topographic density model of Taiwan, resampled to a 15" grid. The model shows pronounced spatial density contrasts associated with the island's major geological and tectonic units and is used to assess the effects of lateral density variations on terrain corrections and geoid–quasigeoid separation.

3.3 Free-air gravity anomalies

The gravity input of the gravimetric quasigeoid computation was a grid of 9"×9" free-air gravity anomalies compiled by Hwang et al. (2014) and subsequently revised and evaluated by Liu (2023). The gravity field is represented by a combination of terrestrial, aerial, shipborne and satellite-altimetry derived gravity observations over Taiwan and the surrounding marine areas (Fig. 5a). The main information source is based on terrestrial observations in accessible land areas, and airborne data improve the coverage of the mountainous areas where surface gravity measurements are not dense. Anomalies from shipborne and altimetry data provide continuity between coastal and offshore regions.

The procedures of collecting, processing, adjusting, and evaluating the accuracy of individual gravity datasets have been described by Hwang et al. (2014; 2020) and are thus not repeated here. In this study the gravity grid is kept fixed in all controlled experiments in order to evaluate the effects of DEM resolution, density representation and GQS formulation independently. The detailed composition of gravity data is relevant for the final geoid accuracy, but its development is not part of the methodological contribution investigated here.

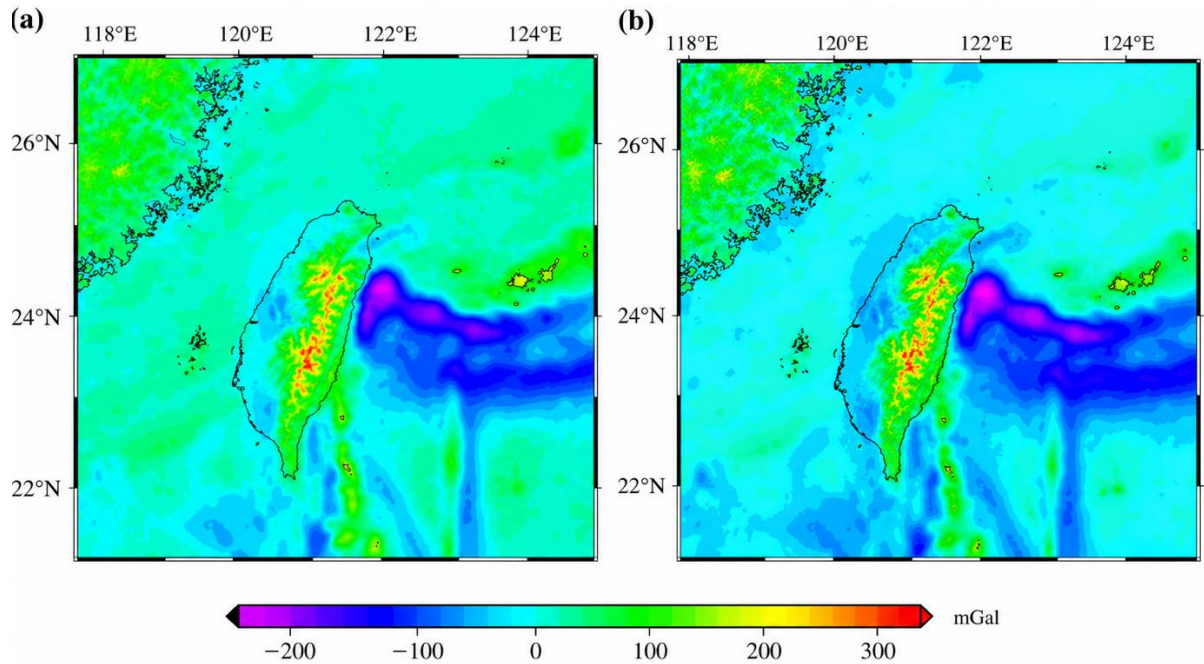


Fig. 5 a Free-air gravity anomalies of Taiwan on a $9'' \times 9''$ grid, compiled from land, airborne, shipborne, and satellite-altimetry observations (Hwang et al. 2014; Liu 2023). This dataset provides the primary gravity input for geoid modeling. **b** The g_1 -corrected ground-level gravity anomalies in Taiwan. The anomalies are obtained from sea-level gravity anomalies by adding the GQS-based correction (see Section 4.3) and terrain corrections computed using the 20 m LiDAR DEM and a constant topographic density.

3.4 GNSS/leveling data for validating geoid model and GQS

Two independent GNSS/leveling datasets are used in this study to validate the gravimetric geoid and GQS results. Each dataset comprises collocated ellipsoidal heights from GNSS observations and orthometric heights from leveling surveys, from which observed geoidal heights are derived. The first dataset is a more recent and denser compilation produced by Liu (2023), which serves as an independent benchmark for assessing the performance of high-resolution geoid

models. The second dataset was used in the geoid modeling and validation described in Hwang et al. (2020). The spatial distributions of the two datasets are shown in Section 5.

The first dataset is the 2023 GNSS/leveling compilation produced by Liu (2023), representing the most recent and spatially dense set of observed geoid heights available for Taiwan. This dataset has ellipsoidal heights derived from 3-hour GNSS sessions taken at first-order leveling benchmarks between 2020 and 2022. The TWD97[2020] reference frame was used to process GNSS observations. This is a new version of the original TWD97 frame (Yang et al. 2001) that includes updated station coordinates as of 2020. After outlier screening based on interpolation residuals and single-station ellipsoidal-height precision, a total of 1,994 stations were retained. The typical standard deviation of ellipsoidal heights ranges from 3 to 5 cm. The orthometric height accuracies are comparable to those of the second dataset below.

The second dataset consists of high-precision orthometric heights derived from leveling surveys on first- and second-order benchmarks with the heights referenced to the Taiwan Vertical Datum 2001 (TWVD2001), together with ellipsoidal heights obtained from static GNSS observations with session lengths ranging from 3 to 24 hours. Leveling accuracies for orthometric heights follow the standard of approximately $2.5\text{--}3.0\text{ mm}\sqrt{k}$, where k is the benchmark-to-benchmark distance in km, and the GNSS heights were rigorously adjusted and corrected for vertical land motion to ensure consistency across observation epochs. A total of 214 stations from this dataset were selected for geoid validation. These GNSS/leveling stations were used in Hwang et al. (2020). In this paper, they are used as the benchmark for assessing the impacts of DEM resolution, terrain corrections, lateral density variations, and GQS formulations, generally attaining sub-decimeter validation accuracy.

258

259 4 Methods

260 4.1 Three-term geoid–quasigeoid separation formulation

261 GQS represents the vertical distance between the geoid and the quasigeoid and is a
262 fundamental parameter for converting between orthometric and normal height systems. The geoid
263 height N is obtained by adding GQS to the height anomaly ζ . The motivation for refining GQS is
264 to improve the classical formulation of Physical Geodesy by Heiskanen and Moritz (1967), which
265 becomes inadequate in regions of complex topography. In areas with large elevation variations,
266 strong gravity gradients arise from both topography and lateral density contrasts, making an
267 accurate representation of GQS essential.

268 In this study, the three-term GQS is evaluated using high-resolution LiDAR DEMs (1 m and
269 20 m) and SRTM1 (~30 m), combined with either a constant density of 2.67 g/cm³ or a lateral
270 density model. The formulation follows the framework of Flury and Rummel (2009), Sjöberg
271 (2010), and Wang et al. (2023), in which GQS is expressed as the sum of multiple physically
272 meaningful components.

273 According to Flury and Rummel (2009), GQS can be written as the sum of two primary terms
274 ($T_1 + T_2$). Sjöberg (2010) and Wang et al (2023) recommended including the second-order
275 expansion term, commonly known as the gravity gradient correction. Accordingly, this study
276 includes this term in GQS, denoted here as T_3 :

$$277 \quad GQS = \frac{\Delta g_{CB}(P)}{\bar{\gamma}} H + \frac{V_{Top}(P_0) - V_{TOP}(P)}{\bar{\gamma}} - \frac{H^2}{2\bar{\gamma}} \frac{\partial \Delta g_{CB}}{\partial H} = T_1 + T_2 + T_3 \quad (1)$$

278 where Δg_{CB} is the complete Bouguer anomaly, H is the orthometric height, $\bar{\gamma}$ is the mean normal
 279 gravity, and V_{Top} is the gravitational potential of the topographic masses between the geoid point
 280 P_0 and the surface point P .

281 The second term in Eq. (1) represents the contribution of the topographic masses to the
 282 potential difference between P_0 and P . Because the product of vertical distance and gravity is
 283 equivalent to the corresponding potential difference, this term can be expressed using the mean
 284 topographic gravity $\bar{g}_{TOP}$,

$$285 \quad V_{Top}(P_0) - V_{TOP}(P) = H \cdot \bar{g}_{TOP} \quad (2)$$

286
 287 Formally, the mean topographic gravity is a height-averaged quantity along the plumb line between
 288 the geoid and the surface. Following Flury and Rummel (2009), it can therefore be related to the
 289 mean terrain effect over the height interval between geoid and surface. A rigorous numerical
 290 evaluation would require integration of the topographic gravity effect along the plumb line or
 291 equivalently, direct computation of the corresponding topographic-potential difference.

292 The mean topographic gravity along the plumb line can be written as the height-averaged
 293 terrain correction (Flury and Rummel 2009),

$$294 \quad \bar{g}_{TOP} = -\frac{1}{H} \int_{P_0}^P g^{TC} dH = -\bar{g}^{TC} \quad (3)$$

295 where g^{TC} denotes the terrain correction and $\bar{g}^{TC}$ is its mean value over the height interval H .

296 The mean normal gravity $\bar{\gamma}$ is approximated by:

$$297 \quad \bar{\gamma} \approx \gamma_P + 0.3086 \frac{H}{2} \quad (4)$$

298 The complete Bouguer anomaly is computed by:

299
$$\Delta g_{CB} = g_P - \gamma_0 + 0.3086H - 2\pi G\rho_i H + \delta g_P^{TC,\rho_i} = \Delta g_{FA} - 2\pi G\rho_i H + \delta g_P^{TC,\rho_i} \quad (5)$$

300 where Δg_{FA} free-air anomaly, ρ_i is topographic density, and $\delta g_P^{TC,\rho_i}$ is the terrain correction
301 evaluated at the surface computation point.

302 In the present implementation, the height-averaged topographic contribution required in the
303 second GQS term is approximated by evaluating the terrain correction at the midpoint height ($H/2$).
304 This midpoint approximation provides a computationally efficient representation of the mean
305 topographic gravity along the plumb line and allows the same terrain-correction framework to be
306 applied consistently at both the surface level (H) and the mid-topographic level ($H/2$). Its effect on
307 the resulting GQS and gravimetric geoid is evaluated in the subsequent sensitivity analyses.

308 4.2 Multiresolution terrain correction at the surface and within the topography

309 In this study, TCs are computed for both GQS determination and the construction of Faye
310 gravity anomalies. All TC computations are at two levels: the surface elevation H and the midpoint
311 height $H/2$. TCs evaluated at the surface level H represent the classical terrain correction and are
312 used in the computation of the complete Bouguer anomaly term (T_1) as well as the Faye anomaly.
313 In the gradient approach to geoid modeling, the ground-level gravity anomaly is analytically
314 continued through the first-order gradient term g_1 . TC represents an approximation of the g_1 term
315 (see the g_1 -corrected gravity anomalies in Fig. 5b) and is added to the ground-level anomaly when
316 computing the height anomaly. Moreover, TC evaluated at $H/2$ approximates the mean
317 gravitational contribution of internal topography (mean topographic gravity, $\bar{g}_{TOP}$, Eq. (3)),
318 required in the three-term GQS formulation.

Following Moritz (1980, Eq. (48-9)) and Hwang et al. (2003), and allowing for lateral density variations, the terrain correction at a computation point P can be expressed as

$$C_p = \frac{GR^2}{2} \iint_{\sigma} \frac{\rho_Q (h_Q - h_P)^2}{l_{PQ}^3} d\sigma_Q \quad (6)$$

Direct numerical evaluation of Eq. (6) is computationally demanding when high-resolution DEMs are used. In this research, TC is calculated utilizing the *tc-cylinder* algorithm (Olgun et al. 2023), which reconfigures the surface integral into a summation of cylindrical mass elements.

In the *tc-cylinder* algorithm, the surrounding topography is divided into a set of concentric rings, each further partitioned into angular sectors. The inner zone (within 10 km of each computation point) is represented using either the 20 m LiDAR DEM or the 1" (~30 m) SRTM1 DEM, while the outer zone (10–167 km) is represented by a coarser 15" (~450 m) DEM. The *tc-cylinder* algorithm is written in C and supports high-resolution DEMs and lateral density models. This makes it possible to accurately measure the effects of terrain in both the near and far zones.

Two main computational templates were used to see how DEM resolution affects TC computations. One uses the 20 m LiDAR DEM for the first 10 km, which works effectively in rough terrain, and the other uses the 1" SRTM1 DEM, which is less detailed but available globally. Both templates use the same parameters based on the 15" DEM for distances greater than 10 km. Because the LiDAR DEM has a higher spatial resolution, it can divide areas into smaller parts and provide more accurate TC estimates in mountainous areas.

For the regional sensitivity analysis, the 20 m LiDAR DEM was replaced by SRTM1 (~30 m), and the resulting changes in terrain correction were propagated through the GQS computation to the final gravimetric geoid (Section 5). A separate high-resolution experiment was conducted

where 1 m LiDAR data were available. In this configuration, the 1 m DEM was used within 1 km of each computation point, with a radial increment of 2 m, to resolve short-wavelength near-zone terrain effects. The 20 m LiDAR DEM was then used from 1 to 10 km, followed by a 15" DEM interpolated from the 20 m model for distances of 10–167 km. This nested multiresolution scheme captures fine-scale terrain effects where they are most influential while retaining computational efficiency at larger distances.

4.3 Determination of ground-level gravity anomalies

In Molodensky theory, the fundamental gravity quantity entering the geodetic boundary-value problem is the ground-level gravity anomaly, defined as the difference between observed gravity at the Earth's physical surface and normal gravity evaluated at the corresponding point on the telluroid (Heiskanen and Moritz, 1967). This quantity provides the appropriate boundary value for determining height anomalies.

In practice, gravity observations are commonly reduced to sea level using the free-air gradient and orthometric heights. The resulting quantity is the sea-level gravity anomaly, which differs from the true ground-level anomaly because orthometric height H is not, in general, equal to the normal height H^* of the telluroid. The difference between these heights is given by

$$H^* - H = N - \zeta = GQS \quad (7)$$

where N is the geoid height, ζ is the height anomaly, and GQS denotes the geoid–quasigeoid separation (Section 4.1).

If the free-air reduction is performed using the orthometric height, a bias is introduced into the ground-level anomaly. This bias equals the product of the vertical gravity gradient

(approximately -0.3086 mGal/m) and the height difference between the telluroid and the physical surface. Consequently, the ground-level gravity anomaly can be obtained from the sea-level anomaly as

$$\Delta g_{ground} = \Delta g_{sea} + 0.3086 GQS \quad (8)$$

where Δg_{sea} is the sea-level gravity anomaly. In Taiwan, the GQS varies between approximately -0.46 and $+0.28$ m (Section 5.2.1), implying that the associated correction to gravity anomalies is small (at sub mGal level). Although the resulting impact on the geoid is typically at the millimeter level, applying this correction ensures internal consistency between gravity anomalies, heights, and the adopted theoretical framework.

In coastal and offshore regions, gravity observations and altimeter-derived gravity anomalies are influenced only by land topography, because the ground-level over the ocean coincides approximately with the sea surface. In the Molodensky sense, the boundary value there is already defined at sea level. Consequently, when computing the terrain correction associated with the first-order gradient term g_1 , only land elevations are included, and ocean-floor topography is excluded. As a result, terrain-correction values decrease smoothly with increasing offshore distance.

Finally, the g_1 -corrected ground-level gravity anomaly used in this study is defined as

$$\Delta g_1 = \Delta g_{sea} + 0.3086 GQS + C \quad (9)$$

where C is the terrain correction (numerically equal to g_1) computed using the *tc-cylinder* algorithm (Olgun et al., 2023) and Δg_1 is the g_1 -corrected ground-level gravity anomaly shown in Fig. 5b.

4.4 Quasigeoid computation and conversion to the gravimetric geoid

The gravimetric geoid determination follows the gradient approach described by Hwang et al. (2020), with a key distinction: all computations are formulated explicitly in terms of ground-level gravity anomalies, obtained from sea-level anomalies through the GQS reductions (Eq. (8)) and g_1 corrections (TCs) described in Section 4.3. In this approach, height anomalies are first determined from the g_1 -corrected ground-level anomalies, and geoid heights are subsequently obtained by adding the GQS and the zero-degree term.

The residual height anomaly at point P is computed using the modified Stokes integral

$$\Delta\zeta_P = \frac{R}{4\pi\gamma_P} \iint_0^{\sigma_0} (\Delta g_{1,Q} - \Delta g_{ref,Q}) S_M(\psi_{PQ}) \quad (10)$$

where R is the mean Earth radius, γ is normal gravity, $\Delta g_{1,Q}$ is the g_1 -corrected ground-level anomaly at running point Q , $\Delta g_{ref,Q}$ is the reference gravity anomaly from a reference field (see below), S_M is the modified Stokes kernel truncated at degree M , ψ_{PQ} is the spherical distance between computation and running points, and σ_0 is the integration cap size.

Eq. (10) relies on the standard assumptions of Molodensky's gradient solution: (i) gravity anomalies vary linearly with height, and (ii) higher-order terms involving $G\rho H^2/\gamma$ are neglected. As shown by Moritz (1980, Eq. (48-29)), this second-order term can reach 5 cm for height $H = 1000$ m, but its effect remains small relative to other error sources and is therefore neglected here, consistent with common geoid practice.

In the gravimetric geoid modeling in this paper, a one-dimensional FFT implementation of the modified Stokes kernel was applied to evaluate the residual height anomalies $\Delta\zeta_P$, with the truncation degree M (for S_M) optimized to minimize the discrepancies against GNSS/leveling

measurements. Table 1 summarizes the parameter settings used for gravimetric geoid determination in this study. The residual terrain model (RTM) approach (Forsberg, 1984) was not adopted because a high-degree reference field from EGM2008 (Pavlis et al. 2012) was employed (for $\Delta g_{ref,Q}$). Although several new high-degree global gravity models have been developed in recent years (see, International Centre for Global Earth Models (ICGEM) at <https://icgem.gfz-potsdam.de/home>), EGM2008 remains the most suitable reference model for the Taiwan region. As this paper focuses on the impacts of high-resolution DEMs and the improved GQS formulation on geoidal accuracy, the detailed procedures of geoid computation are not repeated here and can be found in Hwang et al. (2020).

Indirect effects due to the change of the telluroid were incorporated using the first-order formulation of Moritz (1980) and Sjöberg (2001), which are especially significant in Taiwan's high mountains where they can reach up to 50 cm (Hwang et al. 2020). Following the procedure of Hwang et al. (2020), the gravimetric geoid was generated on a regular $30'' \times 30''$ grid covering Taiwan and its surrounding seas. This resolution provides a continuous geoid surface suitable for integration with GNSS/leveling benchmarks and for subsequent hybrid modeling.

The zero-degree term N_0 can be calculated using the following formula in accordance with the International Height Reference System (IHRs) and the GRS80 reference ellipsoid definitions (Ihde and Sánchez, 2017; Sánchez and Sideris, 2017; Sánchez et al. 2021; Dang et al. 2024):

$$N_0 = \frac{GM - GM_0}{r\gamma} - \frac{W_0 - U_0}{\gamma} \quad (11)$$

where GM and GM_0 are the geocentric gravitational constants defined by the IHRs (Sánchez et al. 2021) and GRS80, respectively; r indicates the geocentric radius, representing the radial distance

423 from the Earth's center to the observation point, including a tide gauge; γ denotes the normal
 424 gravity at the corresponding location on the telluroid; W_0 indicates the conventional geopotential
 425 value of the geoid as defined by the IHRs, which is $62,636,853.4 \text{ m}^2\text{s}^{-2}$ for the epoch 2010.0, and
 426 U_0 refers to the normal gravity potential on the reference ellipsoid. As a result, the gravimetric
 427 geoid height is the sum of height anomaly, GQS and N_0 as

$$428 \quad N = \zeta + GQS + N_0 \quad (12)$$

429 where ζ is the full height anomaly obtained by summing the reference height anomaly from
 430 EGM2008, the residual height anomaly, and the indirect effects.

431 To convert the computed quasigeoid height into the gravimetric geoid height, the GQS terms
 432 T_1 , T_2 , and T_3 (Section 4.1) were applied. For comparison, the conventional GQS approximation
 433 of Heiskanen and Moritz (1967, Section 8-13), approximately corresponding to T_1 , was also used
 434 to generate alternative geoid solutions.

435 **Table 1** Parameters for gravimetric geoid determination in this paper

Parameter	Value
Reference field	EGM 2008 to degree 2190
Integral cap size	0.5°
Max degree in modified Stokes function	140
DEM for TC	LiDAR 20 m DEM + interpolated to 15" DEM from the 20 m DEM for outer zone; SRTM1 1" DEM + interpolated to 15" DEM from the SRTM1 DEM for outer zone; LiDAR 1 m / 20 m DEMs + 15" DEM from the 20 m DEM for outer zone
Density	2.67 g/cm^3 for crust; lateral variations included
TC algorithm	<i>tc-cylinder</i>

5 Results

5.1 Terrain corrections for GQSs and Faye anomalies

5.1.1 Regional terrain corrections: 20 m LiDAR vs. SRTM1 and density effects

This section examines how the modeling choices tested in the TC and GQS experiments propagate into the final gravimetric geoid. In each controlled experiment, only one factor—DEM resolution, lateral density representation, or GQS formulation—is varied relative to the reference configuration, while all other settings are held fixed. The resulting differences are evaluated sequentially at the terrain-correction, GQS, and gravimetric-geoid stages. Their practical significance is then assessed using the two GNSS/leveling validation datasets.

All TC values were computed on a $9'' \times 9''$ grid, resulting in 6,725,201 computation nodes. The resulting TCs at H (Fig. 6a) reflect the rugged topography of Taiwan. TC values are low and approach zero in coastal plains and offshore locations, although they exhibit a significant increase in mountainous areas. As shown in Table 2, the TC values at H using 20 m LiDAR DEM data range from -0.06 to 91.20 mGal, with a mean of 1.32 mGal and a standard deviation of 5.02 mGal.

The effect of DEM resolution is shown in Fig. 6b, which shows the differences between TCs at H derived from the 20 m DEM and those from SRTM1. Differences reach up to 40 mGal (Table 2), particularly in the Central and Eastern Mountain Ranges, where the coarse SRTM1 grid cannot fully capture sharp terrain gradients. Although the mean difference is negligible (-0.04 mGal), the RMS difference of 0.53 mGal confirms that significant local deviations exist. These results show that although SRTM1 may be sufficient for regional studies, the 20 m DEM provides a more detailed representation of terrain effects in regions of steep relief for precise geoid modeling.

The influence of lateral density variations was analyzed by comparing the results obtained using a constant-density model (2.67 g/cm^3) and the Taiwan lateral density model (Fig. 6c). Differences range from -5.1 to $+3.0$ mGal (Table 2). Negative values dominate areas with lower rock densities in sedimentary regions such as western coastal plains and foothills, and fractured formations such as the Longitudinal Valley compared with the constant value, while positive values up to $+3$ mGal occur in northern Taiwan, where crystalline rocks over volcanos such as the Tatun Volcano Group yield higher densities (see Fig. 4). Although the magnitude of density effects is smaller than that of DEM resolution, the differences are systematic and improve the physical realism of terrain corrections in geologically diverse areas.

TCs at $H/2$ height (for subsurface TC) are shown in Fig. 6d-f and Table 2. Using the 20 m LiDAR DEM data the TC values range from -84.19 to 78.85 mGal, with a mean of -0.46 mGal. The influence of lateral density variations on TC values at $H/2$ varies between -17.23 and $+10.66$ mGal. The effect of SRTM1 DEM resolution reaches up to 63 mGal in the mountainous regions.

High-resolution DEMs are necessary for accurately reflecting the steep terrain gradients that characterize TC values, and the inclusion of lateral density variations provides a more physically realistic representation of topographic mass effects. The accurate TCs at H (Fig. 6a-c) are the fundamental input for computing the complete Bouguer anomalies (for T_1 term of GQS) and the midpoint topographic gravity (for T_2 term of GQS) at $H/2$ (Fig. 6d-f) in the three-term GQS formulation. These changes propagate into the GQS and subsequently into the final gravimetric geoid.

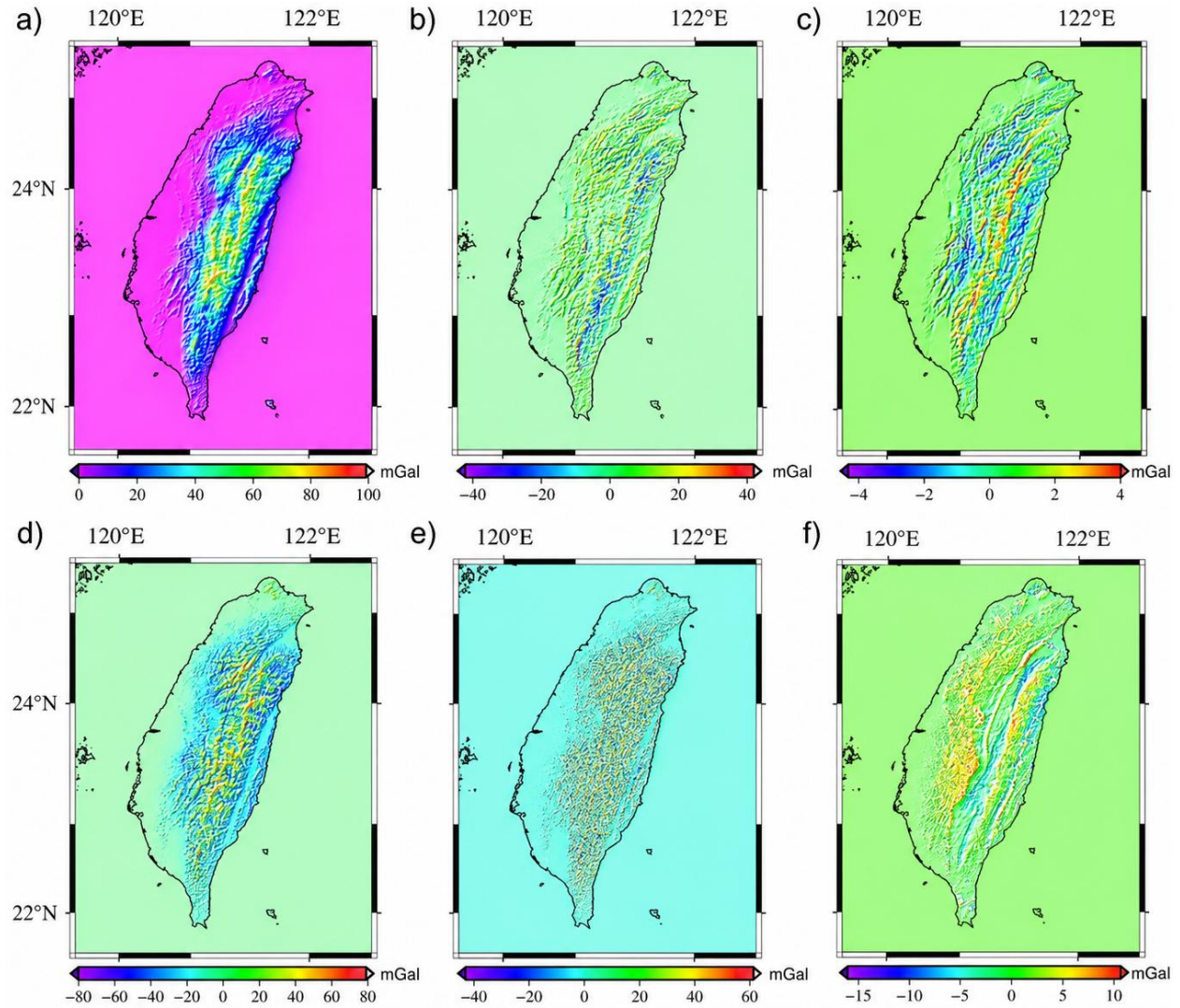


Fig. 6 **a** TC values at H height computed with the 20 m DEM and constant density at gravity stations. **b** Differences (SRTM1- 20m DEM) between TCs at H height derived from SRTM1 and those from the 20 m DEM, showing the effect of DEM resolution. **c** Differences between TCs at H height computed with the 20 m DEM using constant density and those obtained with the lateral density model, showing larger differences over high mountains. **d** TC at $H/2$ height values computed with the 20 m DEM and constant density at gravity stations. **e** Differences between TCs at $H/2$ height derived from SRTM1 and those from the 20 m DEM, showing the effect of DEM resolution. **f** Differences between TCs at $H/2$ height computed with the 20 m DEM using constant

density and those obtained with the lateral density model, again showing larger differences over high mountains.

Table 2 Terrain correction (at H and $H/2$ heights) statistics computed with the 20 m DEM and differences relative to the SRTM1 and density model cases (unit: mGal). Difference is SRTM1 – 20 m DEM

Computation level	DEMs	Density	Result / Difference	Min	Max	Mean	RMS	STD
H	20 m DEM	2.67	Result	-0.06	91.20	1.32	5.20	5.02
H	SRTM1	2.67	Difference	-40.08	32.62	-0.04	0.53	0.53
H	20 m DEM	Model	Difference	-5.06	3.01	-0.03	0.17	0.18
$H/2$	20 m DEM	2.67	Result	-84.19	78.85	-0.46	4.32	4.30
$H/2$	SRTM1	2.67	Difference	-46.71	63.29	0.01	1.71	1.71
$H/2$	20 m DEM	Model	Difference	-17.23	10.66	0.05	0.33	0.33

5.1.2 Near-zone terrain corrections: effects of the 1 m LiDAR DEM

To quantify the impact of DEM resolution on TCs, we conducted experiments using a 1 m LiDAR DEM in central Taiwan, where steep and highly variable topography dominate the near-zone TC signal (Fig. 7). In such environments, meter-scale elevation data are expected to provide a more realistic representation of local mass geometry than coarser DEMs. Because the direct use of a 1 m DEM over large spatial extents is computationally demanding, a multi-resolution strategy was adopted to balance accuracy and efficiency. Specifically, the 1 m LiDAR DEM was applied within a 1 km radius of each computation point to capture detailed near-zone effects. For intermediate distances between 1 and 10 km, the 20 m LiDAR DEM was used, while for the far zone (10–167 km), a 15'' grid interpolated from the 20 m LiDAR dataset was employed to represent long-wavelength mass contributions. This hierarchical approach preserves high-frequency terrain

signals where they matter most while minimizing unnecessary computational cost at larger distances.

Fig. 7a–c show the TCs at height H derived from the 1 m LiDAR DEM, the 20 m LiDAR DEM, and their difference, respectively. The difference map highlights short-wavelength terrain features that are smoothed or omitted in the 20 m DEM, particularly along sharp ridges, steep slopes, and deeply incised valleys. These localized differences correspond to small but systematic gravitational effects associated with abrupt changes in near-zone terrain geometry.

The TC differences between the 1 m and 20 m LiDAR DEMs range from -21.6 to $+29.7$ mGal, with a mean of -1.4 mGal and a standard deviation of 3.1 mGal. The largest differences occur in mountainous areas, confirming that meter-scale topography provides a more detailed representation of near-zone mass geometry where relief is highly variable. For the computation of the midpoint topographic gravity required by the three-term GQS formulation, TCs were also evaluated at $H/2$. These values range from -57.71 to $+52.34$ mGal, with a mean of -5.52 mGal.

Although the use of 1 m DEMs remains computationally intensive and data availability is still limited in many regions, the efficient design of the *tc-cylinder* algorithm substantially reduces the computational burden. Together with the rapid increase in computing power the routine use of meter-scale DEMs for TC computation is expected to become increasingly practical. As high-resolution topography becomes more widely available, such approaches are likely to play a growing role in future accurate geoid modeling.

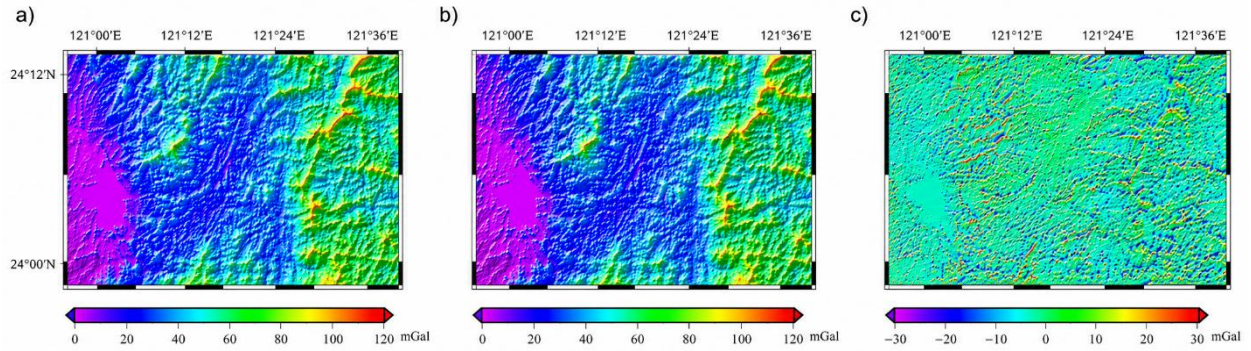


Fig. 7 **a** Terrain correction at H height based on 1 m LiDAR, **b** terrain correction at H height based on 20 m LiDAR and, **c** difference map (1 m minus 20 m), showing the effect of DEM resolution on TCs at H height.

5.2 Evaluation of the three-term geoid–quasigeoid separation

5.2.1 The reference three-term GQS Model

For the reference configuration, GQS values were obtained by summing the three components T_1 , T_2 , and T_3 at each grid node (Fig. 8a–d), using the 20 m LiDAR DEM and a constant topographic density. The statistical characteristics of the individual terms and the total GQS are summarized in Table 3.

The T_1 and T_2 terms exhibit comparable magnitudes, reflecting the combined influence of complete Bouguer anomalies and topographic potential effects. The T_3 term contributes smaller-amplitude variations but displays coherent spatial patterns associated with long-wavelength gravity gradients. The resulting total GQS ranges from -0.459 m to 0.282 m, with a root-mean-square (RMS) value of 0.016 m.

Table 3 Statistics of the geoid-quasigeoid separation terms (T_1 , T_2 , T_3) and the total GQS for grid nodes (unit: m) using the 20 m LiDAR DEM and constant topographic density

Terms	Min	Max	Mean	RMS	STD
T_1	-0.381	0.120	-0.001	0.014	0.014
T_2	-0.311	0.121	-0.001	0.007	0.007
T_3	-0.072	0.253	0.001	0.007	0.007
Total GQS	-0.459	0.282	-0.000	0.016	0.016

5.2.2 Sensitivity of GQS to regional high-resolution topography and density

Replacing the 20 m DEM with SRTM1 introduces differences of up to about 15 cm (Fig. 9a; Table 4). These discrepancies are spatially correlated with regions of steep and complex terrain, demonstrating that DEM resolution has a first-order impact on the GQS.

Using the lateral density model generates differences of approximately ± 5 cm (Fig. 9b; Table 4). Although smaller than the DEM effect, these variations are systematic and follow known geological structures, providing a more physically realistic representation of topographic mass effects, although no systematic improvement is evident in the GNSS/leveling validation.

5.2.3 Near-zone topographic effects on GQS with the 1 m LiDAR Test

A high-resolution test was performed using the 1 m LiDAR DEM in the near-zone (< 1 km), while all other modeling choices followed the reference configuration based on the 20 m LiDAR DEM. The resulting GQS differences between the two solutions range from -13.1 cm to $+14.7$ cm, with a mean of 0.1 cm and a standard deviation of 3.5 cm. The spatial distribution of these

differences (Fig. 10c) shows that the largest deviations are concentrated in steep mountainous regions, whereas low-relief areas exhibit only small and spatially smooth changes.

The influence of DEM resolution on the GQS arises through two pathways. First, the recalculation of TCs modifies the T_1 term by resolving small-scale topographic masses that are represented in the 1 m DEM (Fig. 10a) but are partially smoothed when the 20 m DEM is used (Fig. 10b). This effect is most pronounced in areas of rugged terrain, where short-wavelength height variations contribute significantly to the near-zone gravity field. Second, the use of refined elevations updates the height-dependent quantities in T_2 and T_3 .

Among these contributions, T_3 is controlled primarily by long-wavelength gradients of the disturbing potential and therefore shows only weak sensitivity to DEM resolution. The impact on T_2 is likewise secondary at this scale. In contrast, the enhanced geometric detail provided by the 1 m LiDAR DEM substantially amplifies the short-wavelength component of T_1 , making it the dominant source of GQS differences observed in Fig. 10c.

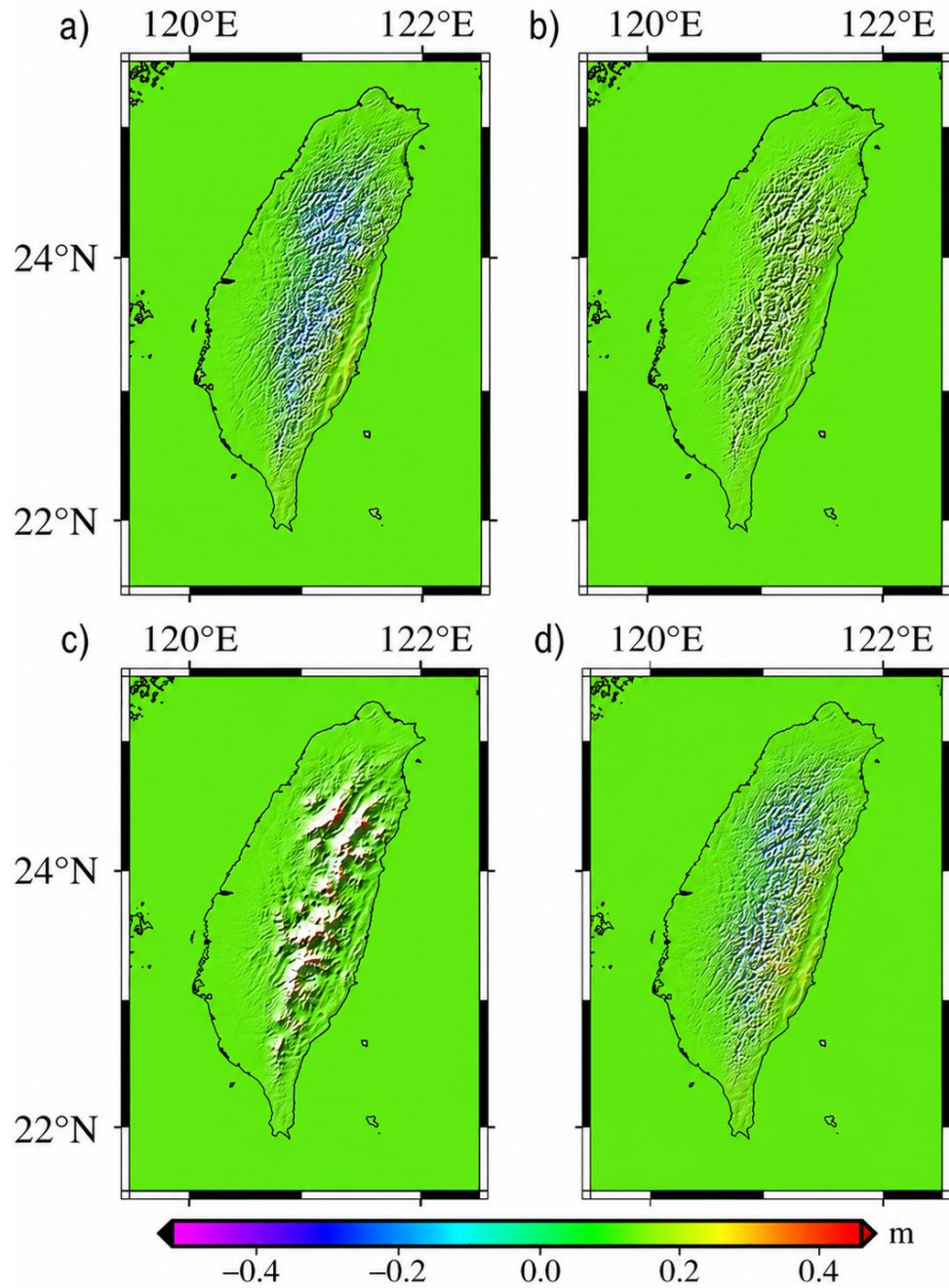
These results indicate that, when cm-level geoid accuracy is sought, the treatment of near-zone terrain becomes the limiting factor in mountainous regions. Ultra-high-resolution DEMs may be relevant for modelling at the centimeter level in areas of extreme local relief, although their practical benefit will depend on numerical treatment, spatial filtering and the resolution of the final geoid solution.

5.2.4 Comparing results of the three-term and traditional GQS formulations

Comparing the three-term formulation with the traditional GQS approach (T_1 only in Eq. (1) and without terrain correction for the Bouguer anomaly) produces the largest differences, reaching

575 –28 cm (Fig. 9c; Table 4). These discrepancies arise primarily from the improved treatment of
576 terrain effects and the explicit representation of vertical gravity gradients.

577 The differences resulting from the different GQS configurations (Fig. 9) represent changes in
578 the resulting gravimetric geoidal heights (Section 4.4) relative to the reference configuration
579 (Section 5.2.1). Taken together, the sensitivity experiments show that GQS estimates in
580 mountainous terrain are most strongly controlled by the treatment of topography, followed by
581 density representation, while methodological simplifications in the GQS formulation have the
582 largest overall impact. Although changes in DEM resolution and density primarily affect the
583 spatial detail and regional patterns of the GQS, the traditional one-term formulation introduces
584 substantially larger deviations by neglecting key terrain and gravity-gradient effects. These
585 findings indicate that the three-term formulation provides a more complete representation of GQS
586 and becomes increasingly important for centimeter-level modeling in areas of complex relief.



587

588 **Fig. 8** Maps showing **a** the first term (T_1), **b** the second term (T_2), **c** the third term (T_3), and **d** the
 589 total geoid-quasigeoid separation computed using the 20 m LiDAR DEM and constant
 590 topographic density.

Table 4 Differences in GQS values relative to the reference case* (unit: m)

DEMs	Density	GQS Terms	Min	Max	Mean	RMS	STD
SRTM1	2.67 g/cm ³	3 Terms	-0.145	0.008	-0.000	0.002	0.002
20 m DEM	Model	3 Terms	-0.022	0.051	0.000	0.002	0.002
20 m DEM	2.67 g/cm ³	Traditional	-0.277	0.063	-0.002	0.014	0.014

* The reference case uses the 20 m DEM, constant density (2.67 g/cm³), and the three-term formulation.

Row 1 shows differences from replacing the 20 m DEM with SRTM1. Row 2 shows differences from using a lateral density model. Row 3 shows differences between the traditional one-term formulation (T_1 only in Eq. (1) without terrain correction) and the reference case.

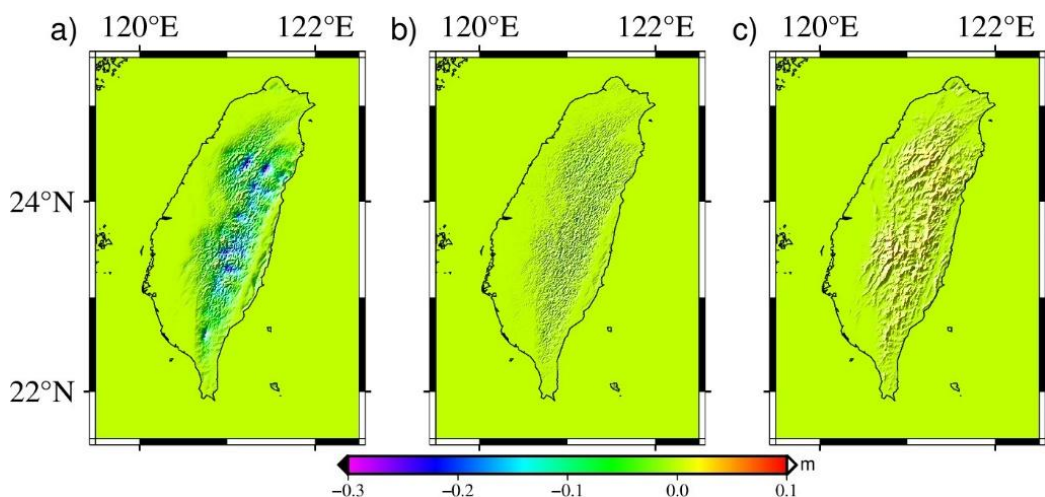


Fig. 9 Comparison of three-term GQS values showing **a** the effect of using SRTM1 DEM, **b** the effect of the lateral density model, and **c** the results from the traditional GQS formulation.

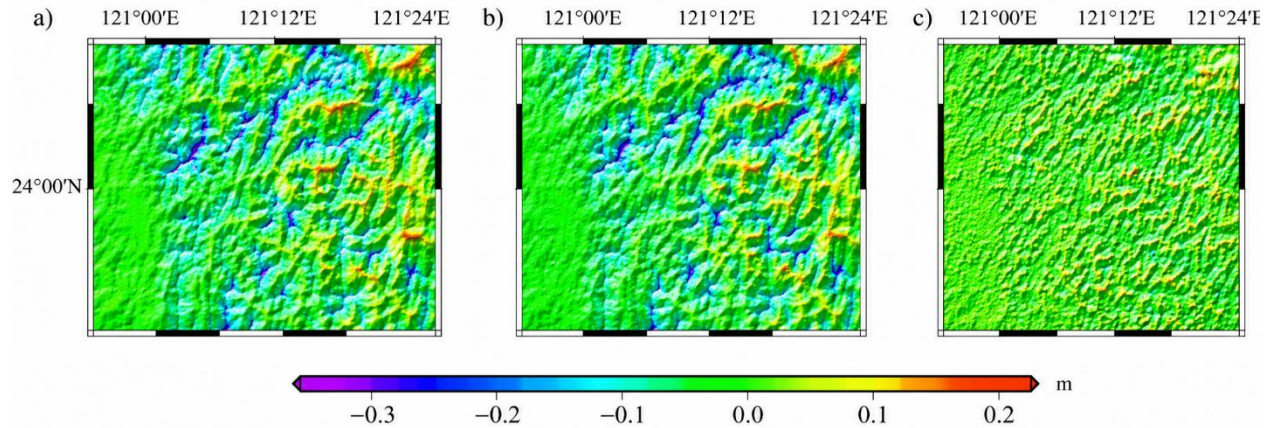


Fig. 10 Geoid–quasigeoid separation maps computed using **a** the 1 m LiDAR DEM and **b** the 20 m LiDAR DEM in the near zone (< 1 km), and **c** their difference (1 m minus 20 m).

5.3 Effects of DEM resolution, density representation, and GQS formulation on the gravimetric geoid

The gravimetric geoid of Taiwan was computed following the method described in Section 4.4, using the g_1 -corrected ground-level gravity anomalies together with the three-term GQS. To isolate and quantify the effects of DEM resolution, lateral density variations, and GQS formulation, a series of gravimetric geoid models was generated using different modeling configurations.

The resulting geoid models, their difference maps, and validation results are presented in Figs. 11-15, while the corresponding statistical summaries are given in Tables 5 and 6. These results show how terrain representation propagates into the final geoid through gravity reduction and GQS.

5.3.1 Propagation of DEM resolution and density effects through GQS to gravimetric geoid heights

The reference configuration is the 20 m LiDAR DEM, constant topographic density of 2.67 g/cm³, and the full three-term GQS formulation. The resulting gravimetric geoid varies from 5.8 to 31.0 m over the study area (Fig. 11a; Table 5). This configuration is used as a common baseline to isolate the effects of DEM resolution and lateral density representation and is not assumed in advance to be the most accurate solution.

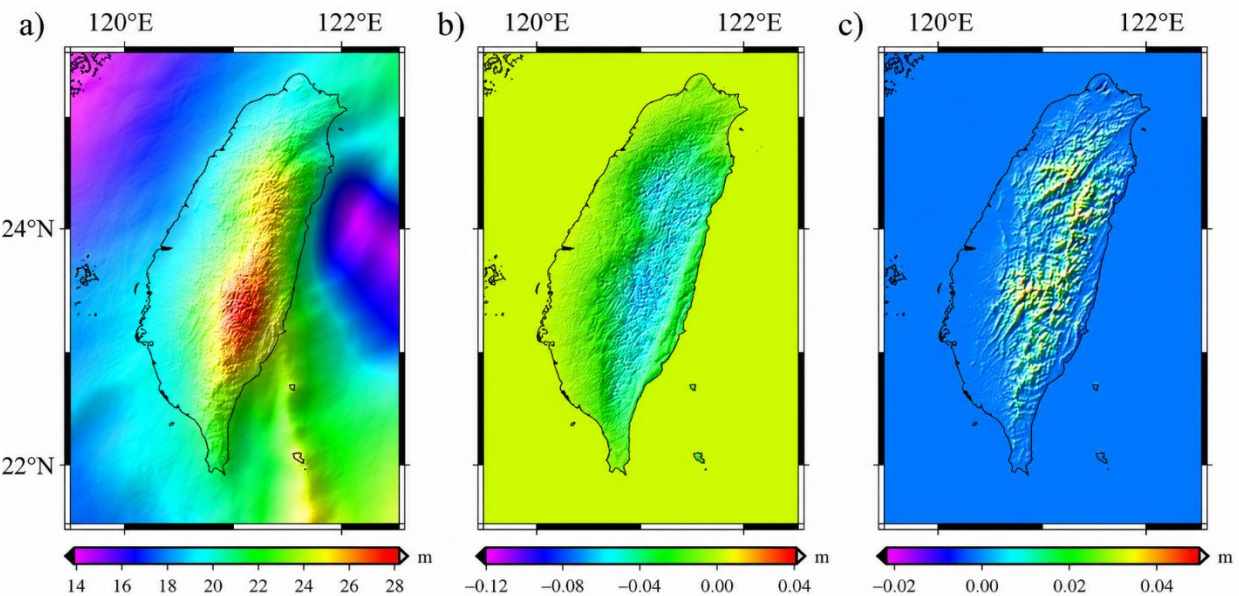


Fig. 11 Maps of gravimetric geoids calculated **a** using 20 m DEM and constant topographic density, along with their differences relative to **b** the SRTM1 DEM and **c** the lateral density model.

Table 5 Gravimetric geoid heights calculated with 20 m DEM at grid nodes, their differences from those calculated from the SRTM1 model and density model at grid nodes (unit: m)

DEMs	Density	Geoid/geoid difference	Min	Max	Mean	RMS	STD
20 m DEM	2.67	Geoid	5.755	31.003	19.791	20.380	4.860
SRTM1	2.67	Difference	-0.117	0.038	-0.001	0.009	0.010

20 m DEM	Model	Difference	-0.023	0.051	0.000	0.002	0.002
----------	-------	------------	--------	-------	-------	-------	-------

Replacing the 20 m LiDAR DEM by SRTM1, the gravimetric geoid changes from -0.117 to 0.038 m, with an RMS difference of 0.009 m (Fig. 11b; Table 5). The largest differences locally occur in areas of steep relief. The regional mean difference is about -1 mm. When the lateral density model is introduced, the changes are smaller, ranging from -0.023 m to +0.051 m, with an RMS value of 0.002 m (Fig. 11c). These variations spatially correlate with major geological units and tectonic boundaries, indicating that realistic density information improves the physical consistency of topographic mass modeling. Although the magnitude of density-induced differences is modest compared with DEM effects, they refine regional geoid patterns in geologically complex areas.

The GQS evaluations (Section 5.2.4) show that replacing the traditional one-term formulation with the three-term GQS formulation can change the gravimetric geoid by up to approximately 30 cm locally. This effect is larger than those associated with DEM resolution and lateral density representation, indicating that the treatment of GQS is a major source of model sensitivity in high-relief regions. Although DEM resolution and density variations also influence the final geoid, their effects are generally smaller and more localized.

5.3.2 Effects of the 1 m LiDAR DEM in mountainous regions on geoids

To further assess the impact of ultra-high-resolution topography on gravimetric geoid accuracy, a targeted experiment was conducted in a steep mountainous region using the 1 m LiDAR DEM in the near zone (≤ 1 km), while all other modeling components followed the reference configuration based on the 20 m LiDAR DEM. The resulting geoid differences between

the 1 m and 20 m DEM solutions (Fig. 12c) range from -28.5 to $+17.6$ cm, with a mean of -3.3 cm and a standard deviation of 6.6 cm. The largest deviations are concentrated in areas of extreme slope and sharp relief, confirming that unresolved micro-topographic features in the 20 m DEM significantly affect near-zone TCs and, consequently, the short-wavelength geoid signal.

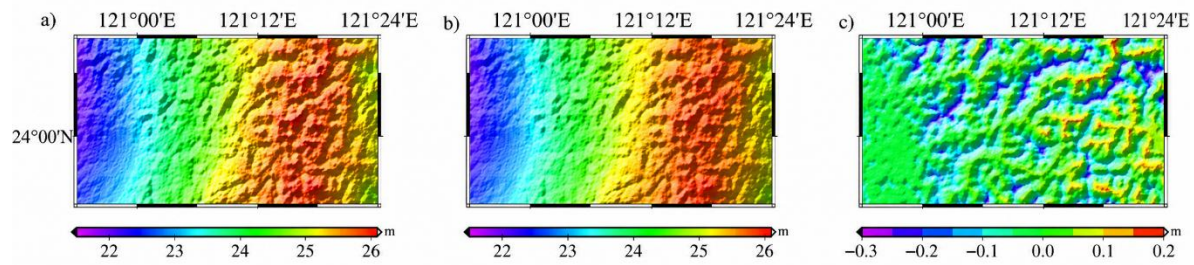


Fig. 12 Gravimetric geoid heights computed using **a** the 1 m LiDAR DEM and **b** the 20 m LiDAR DEM, and **c** their difference (1 m minus 20 m).

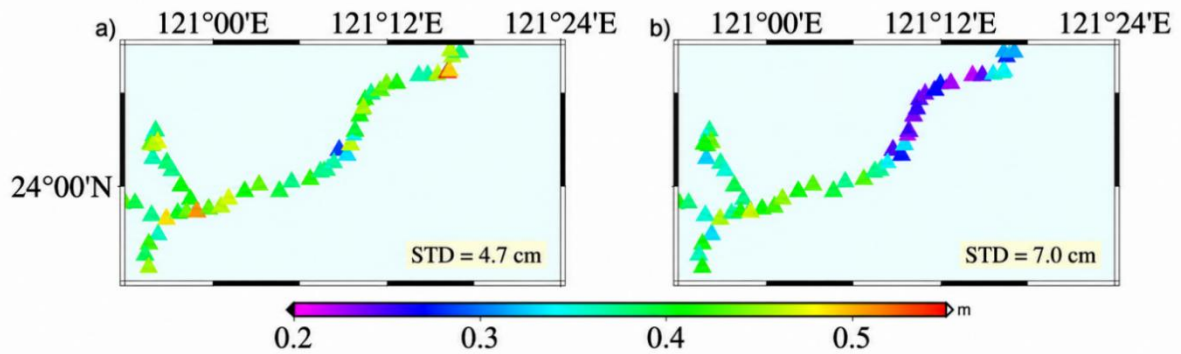


Fig. 13 Differences between gravimetric geoid heights and GNSS/leveling-derived geoid heights for the first validation dataset: **a** geoid solution based on the 1 m LiDAR DEM (STD = 4.7 cm) and **b** geoid solution based on the 20 m LiDAR DEM (STD = 7.0 cm). Colors indicate the magnitude of the geoid-height differences.

659

660

661 Fig. 13 shows the improvements by validation with the first GNSS/leveling dataset. When the
662 gravimetric geoid computed with the 1 m LiDAR DEM is compared with geometric geoidal
663 heights, the standard deviation of the residuals is reduced to 4.7 cm (Fig. 13a), compared with 7.0
664 cm for the corresponding solution based on the 20 m DEM (Fig. 13b). The color distribution along
665 the validation transect further shows that the 1 m DEM suppresses localized mismatches in high-
666 relief segments, leading to a more spatially consistent fit. The influence of the 1 m DEM on the
667 gravimetric geoid arises through three pathways: (i) refined terrain corrections that directly modify
668 the T_1 term, (ii) elevation-dependent quantities entering the Stokes integration, and (iii) secondary
669 effects on the GQS components (T_1 – T_3).

670 Among these contributions, the effect on T_1 is dominant. The enhanced geometric detail
671 provided by the 1 m DEM improves the representation of near-zone topographic masses, which
672 strongly control short-wavelength gravity signals in rugged terrain. In contrast, T_2 and T_3 are
673 governed primarily by longer-wavelength gravity and height-dependent terms and therefore
674 exhibit weaker sensitivity to DEM resolution.

675 In areas with gentle slopes, the differences between the 1 m and 20 m DEM solutions become
676 very small. This shows that ultra-high-resolution DEMs are most useful in mountainous areas
677 where slopes and elevations vary rapidly. The local 1 m experiment results in a smaller residual
678 scattering of GNSS/leveling than the corresponding 20 m solution in the chosen test area, which

is interpreted as evidence for local sensitivity rather than a general improvement for the whole region.

5.4 Island-wide validations of gravimetric geoid solutions with GNSS/leveling data

5.4.1 Primary validation using the 1,994-station dataset

The first island-wide validation was performed using the denser GNSS/leveling dataset of 1,994 stations distributed across Taiwan (Figs. 14 and 15). For the solutions based on the 20 m LiDAR DEM, the dispersion of the gravimetric–geometric geoid residuals decreases progressively as additional GQS terms are included. The standard deviation decreases from 0.062 m for the one-term solution to 0.059 m for the two-term solution and 0.058 m for the three-term solution (Table 6), compared with 0.064 m for the traditional GQS formulation. Although these reductions are modest, they are systematic and consistently favor the more complete treatment of the GQS.

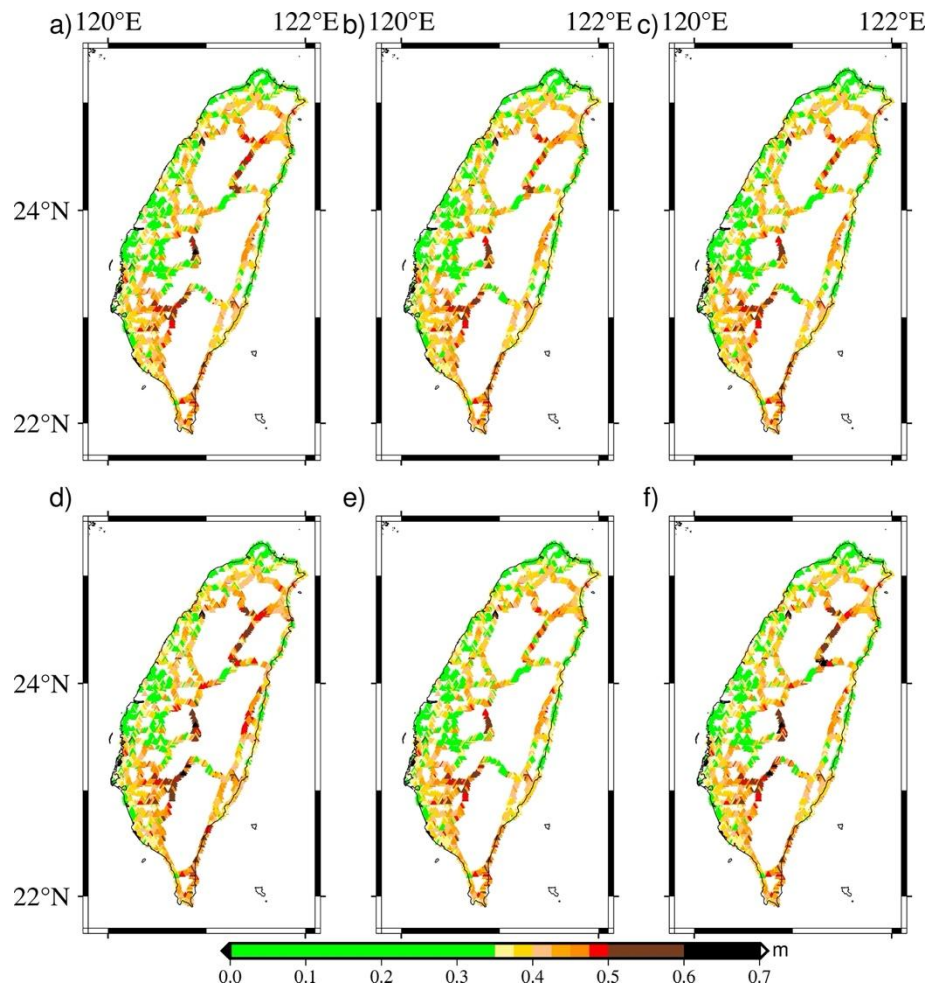
The influence of DEM resolution is also evident, although it is expressed more clearly in the mean residual than in the residual dispersion. The three-term SRTM1 solution has a mean residual of 0.401 m and a standard deviation of 0.062 m, whereas the corresponding 20 m LiDAR solution gives 0.387 m and 0.058 m, respectively. Thus, replacing SRTM1 with the 20 m LiDAR DEM shifts the mean residual by approximately 14 mm while reducing the standard deviation by about 4 mm. This result indicates that improved terrain representation produces not only a modest reduction in residual scatter but also a systematic change in the modeled geoid surface that is relevant to centimeter-level geoid determination.

The relatively small improvement in the island-wide standard deviation should not be interpreted as evidence that DEM resolution or GQS formulation is unimportant. The sensitivity experiments presented in the preceding sections show that their effects are strongly spatially

701 dependent and are largest in areas of steep and complex relief. In contrast, an island-wide statistic
702 combines benchmarks located in mountainous terrain with many stations in plains, valleys, and
703 other low- to moderate-relief areas, where the differences between the 20 m LiDAR and SRTM1
704 representations are much smaller. Consequently, locally significant terrain- and GQS-related
705 corrections can be substantially diluted when summarized by a single network-wide statistic. In
706 addition, the GNSS/leveling residuals contain contributions from other error sources, including
707 uncertainties in GNSS ellipsoidal heights, leveling, gravity observations, datum realization, and
708 the gravimetric geoid computation itself. These errors may be comparable to or larger than the
709 incremental effects of individual model refinements.

710 The constant-density and laterally varying-density three-term solutions both yield a residual
711 standard deviation of 0.058 m. The density model therefore modifies the geoid spatially but does
712 not produce a detectable improvement in the overall residual dispersion for this dataset. Taken
713 together, the results show that the 20 m LiDAR DEM and the three-term GQS provide a
714 measurable but moderate improvement in island-wide GNSS/leveling agreement. Their principal
715 benefit should therefore be interpreted as reducing systematic and terrain-dependent modeling
716 errors, rather than producing a large reduction in a single network-averaged residual statistic.

717



718

719 **Fig. 14** Spatial distribution of gravimetric–geometric geoid differences at the GNSS/leveling
 720 stations of the first dataset (Section 3.4). The gravimetric geoid solutions are based on **a** the one-
 721 term GQS, **b** the two-term GQS, **c** the three-term GQS using the 20 m LiDAR DEM and constant
 722 topographic density, **d** the three-term GQS using SRTM1 and constant topographic density, **e** the
 723 three-term GQS using the 20 m LiDAR DEM and the lateral density model, and **f** the traditional
 724 GQS formulation using the 20 m LiDAR DEM and constant topographic density.

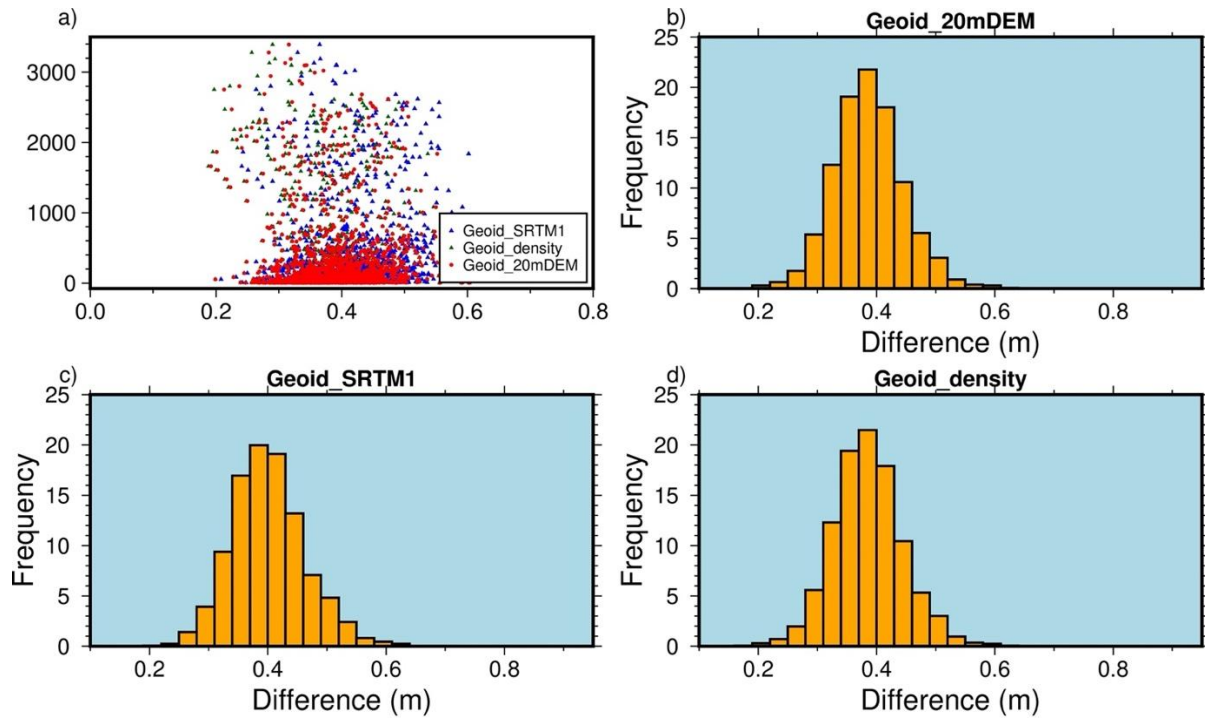


Fig. 15 Comparison of gravimetric–geometric geoid differences at the GNSS/leveling stations of the first dataset. **a** Differences as a function of station elevation (in m) for solutions based on the 20 m LiDAR DEM, SRTM1, and the lateral density model. Histograms show the difference distributions for **b** the 20 m LiDAR DEM with constant topographic density, **c** SRTM1 with constant topographic density, and **d** the 20 m LiDAR DEM with the lateral density model. All solutions use the three-term GQS formulation.

Table 6 Statistics of gravimetric–geometric geoid differences at the 1,994 GNSS/levelling stations for the evaluated DEM, density, and GQS configurations (unit: m)

DEMs	Density	GQS	Min	Max	Mean	STD
20 m DEM	2.67	1 term	0.140	0.622	0.389	0.062
20 m DEM	2.67	2 terms	0.173	0.610	0.388	0.059
20 m DEM	2.67	3 terms	0.192	0.610	0.387	0.058
SRTM1	2.67	3 terms	0.208	0.623	0.401	0.062
20 m DEM	Model	3 terms	0.188	0.610	0.386	0.058

20 m DEM	2.67	traditional	0.178	0.680	0.395	0.064
----------	------	-------------	-------	-------	-------	-------

5.4.2 Validation using the 214-station dataset

The 214-station GNSS/leveling dataset provides an independent assessment of whether the model sensitivities identified using the denser 1,994-station dataset are also detectable in an earlier and more sparsely distributed benchmark network. For the 20 m LiDAR DEM, the two- and three-term GQS solutions both yield a standard deviation of the differences of 0.078 m, compared with 0.081 m for the traditional GQS formulation (Table 7). The corresponding SRTM1-based three-term solution gives a standard deviation of 0.079 m, whereas the solution incorporating the lateral density model produces a larger value of 0.085 m. These differences are small, but the overall tendency is consistent with the primary validation: the more complete GQS treatment gives slightly smaller dispersion of the differences than the traditional formulation.

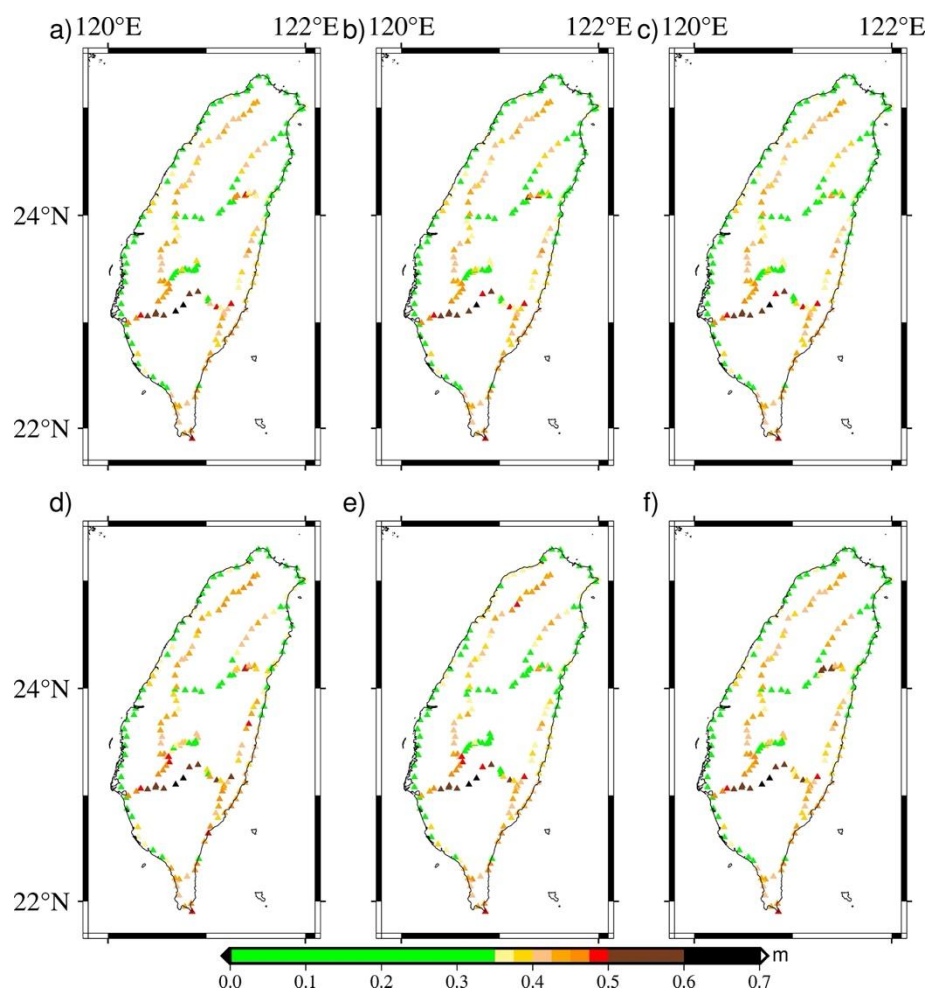
The effect of DEM resolution is more apparent in the mean difference than in the standard deviation. For the three-term solutions, the mean difference decreases from 0.387 m with SRTM1 to 0.368 m with the 20 m LiDAR DEM, corresponding to a systematic shift of approximately 19 mm, whereas the standard deviation changes by only 1 mm. Similarly, the traditional GQS solution has a mean difference of 0.381 m, about 13 mm larger than that of the three-term solution. These results indicate that improvements in terrain representation and GQS formulation can alter the modeled geoid at the centimeter level even when their effect on the overall dispersion of the differences is small.

Fig. 16 provides additional insight into this behavior by showing the spatial distribution of the gravimetric–geometric geoid differences for the different model configurations. The changes are

spatially heterogeneous rather than uniform across Taiwan, consistent with the expectation that DEM resolution and GQS refinement exert their largest influence in areas of steep and complex relief. In contrast, many stations located in plains and other relatively smooth terrain show only small differences among the tested solutions. Consequently, these localized improvements are diluted when all 214 stations are summarized by a single network-wide standard deviation.

The difference distributions shown in Fig. 17 lead to a similar conclusion. The histograms for the principal configurations largely overlap, confirming that the differences in the overall dispersion of the differences are modest. Nevertheless, the distributions shift slightly as the DEM and GQS formulation are changed, consistent with the systematic differences in the mean differences reported in Table 7. Thus, Figs. 16 and 17 together show that the effects of the model refinements are expressed more clearly through changes in the spatial pattern and systematic level of the differences than through a large narrowing of their overall distribution.

The lateral-density model does not improve the validation statistics for this dataset and instead increases the difference standard deviation to 0.085 m. This result does not imply that lateral density variations are physically unimportant; rather, it indicates that the adopted density model does not reduce the combined error measured by this particular GNSS/leveling network. Overall, the 214-station validation supports the conclusions obtained from the denser dataset: DEM resolution and GQS formulation have measurable but moderate effects on island-wide validation statistics, while their more important contribution is the correction of spatially systematic and terrain-dependent geoid errors that may be considerably larger locally than suggested by the network-averaged standard deviation.



776

777 **Fig. 16** Same as Fig. 14, except that the calculations were performed using the second
 778 GNSS/leveling dataset.

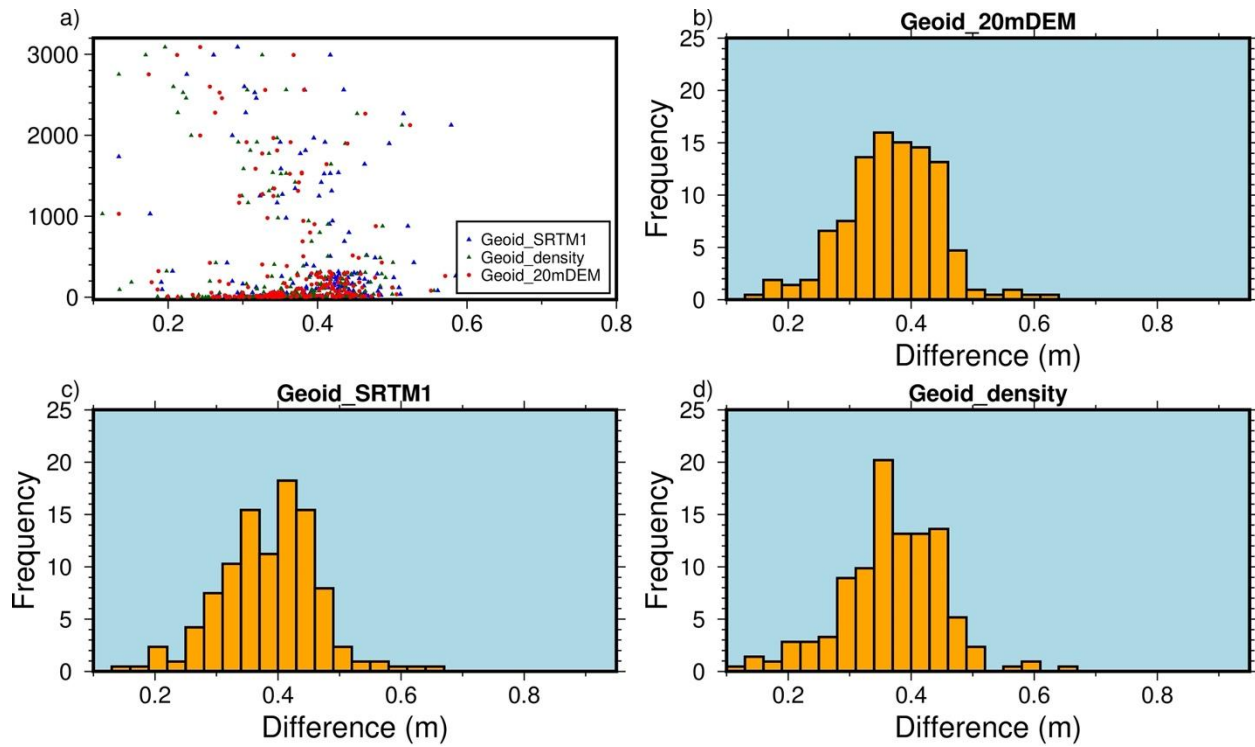


Fig. 17 Same as Fig. 15, except that the calculations were performed using the second GNSS/leveling dataset.

Table 7 Statistics of gravimetric–geometric geoid differences at the 214 GNSS/levelling stations for the evaluated DEM, density, and GQS configurations (unit: m)

DEMs	Density	GQS	Min	Max	Mean	STD
20 m DEM	2.67	1 term	0.039	0.645	0.371	0.081
20 m DEM	2.67	2 terms	0.067	0.628	0.371	0.078
20 m DEM	2.67	3 terms	0.094	0.629	0.368	0.078
SRTM1	2.67	3 terms	0.134	0.667	0.387	0.079
20 m DEM	Model	3 terms	0.071	0.642	0.366	0.085
20 m DEM	2.67	traditional	0.073	0.657	0.381	0.081

6. Discussion and conclusions

786 This study evaluates how terrain resolution, lateral density representation, and GQS
787 formulation propagate through terrain correction, quasigeoid computation, and the final
788 gravimetric geoid over the rugged topography of Taiwan. TCs were computed at both the surface
789 and the mid-topographic level, with the latter used as a midpoint approximation to the height-
790 averaged topographic contribution required by the three-term GQS formulation. These
791 experiments allow the influence of each modeling factor to be traced consistently to the final geoid.

792 Comparison of the 20 m LiDAR DEM with SRTM1 shows that DEM resolution can produce
793 large local differences in TCs, particularly in steep mountainous regions. These differences are
794 substantially attenuated when propagated into GQS and the final gravimetric geoid: regional mean
795 effects are small, although local geoid differences can still reach the decimeter level in areas of
796 extreme relief. This demonstrates that the significance of high-resolution topography should be
797 assessed from its effect on the final geoid rather than from terrain-correction differences alone.

798 The three-term GQS formulation produces locally substantial differences relative to the
799 traditional one-term approximation. Validation with the denser GNSS/leveling dataset shows a
800 modest but consistent reduction in difference scatter as additional GQS terms are included. The
801 improvement is limited at the regional scale but becomes more relevant in high and rugged terrain,
802 where the neglected topographic and gravity-gradient contributions are largest. The practical
803 benefit of the three-term formulation therefore depends on both terrain complexity and the target
804 accuracy of the geoid model.

805 The local 1 m LiDAR experiment further demonstrates that unresolved near-zone topography
806 can produce centimeter-to-decimeter changes in TCs, GQS, and gravimetric geoid heights in areas
807 of extreme relief. Although the experiment is limited to selected mountainous regions and should

not be generalized to the whole study area, it indicates that meter-scale terrain representation may become important when centimeter-level geoid accuracy is required. Additional numerical convergence and scale-consistency tests are needed to establish the broader benefit of meter-scale topography.

The main contribution of this study is an end-to-end multiresolution assessment that propagates terrain and density information from the topographic mass model through TC and three-term GQS to the final gravimetric geoid. Among the factors considered, terrain representation and GQS formulation produce the largest local effects in rugged regions, whereas the influence of lateral density is smaller and less clearly detectable in the GNSS/leveling validation. However, the use of lateral density produces spatially coherent changes in geoid height.

Declarations

Availability of data and materials

The gravity data and high-resolution LiDAR DEMs used in this study are available from the corresponding author (C. Hwang) upon reasonable request and subject to the data providers' access policies. Publicly available DEM products, including SRTM1, can be obtained from their respective data repositories.

Competing interests

The authors declare that they have no competing interests.

827 **Funding**

828 This study was supported by the Ministry of the Interior, Taiwan, through the project “Geoid
829 Model Enhancement and Maintenance Program,” and by the National Science and Technology
830 Council, Taiwan, under projects 112-2221-E-A49-025-MY3 and 114-2811-M-A49-560.

831 **Authors’ contributions**

832 C.H. served as the project principal investigator. S.O. performed the computations and prepared
833 the figures and tables. S.O. and C.H. conducted the analyses and wrote the manuscript. R.L. and
834 Y.-S.H. contributed to data preparation and data editing. All authors reviewed and approved the
835 final manuscript.

836 **Acknowledgements**

837 Not applicable.

838 **References**

- 839 Nsiah Ababio, A., Foroughi, I., Tenzer, R., & Bagherbandi, M. (2024). A conversion of the geoid
840 to the quasigeoid at the Hong Kong territories. *Applied Geomatics*, 16(3), 471-483,
841 <https://doi.org/10.1007/s12518-024-00562-x>
- 842 Dang Y, Jiang T, Guo C, Chen B, Zhang C, Yang Q, Wang Z (2024) Determining the new height
843 of Mount Qomolangma based on the International Height Reference System. *Geo-spatial*
844 *Information Science* 27(4):1182-1191. <https://doi.org/10.1080/10095020.2022.2128901>
- 845 D’Urso MG, Trotta S (2015) Comparative assessment of linear and bilinear prism-based

846 strategies for terrain correction computations. *Journal of Geodesy* 89(3):199–215.
847 <https://doi.org/10.1007/s00190-014-0770-4>

848 Featherstone WE, Kirby JF (2000) The reduction of aliasing in gravity anomalies and geoid
849 heights using digital terrain data. *Geophysical Journal International* 141(1):204–212.
850 <https://doi.org/10.1046/j.1365-246X.2000.00082.x>

851 Flury J, Rummel R (2009) On the geoid–quasigeoid separation in mountain areas. *Journal of*
852 *Geodesy* 83(9):829–847. <https://doi.org/10.1007/s00190-009-0302-9>

853 Foroughi, I, Tenzer R (2017). Comparison of different methods for estimating the geoid-to-quasi-
854 geoid separation. *Geophysical journal international*, 210(2), 1001-1020.

855 Foroughi I, Goli M, Pagiatakis S et al (2023) The uncertainties of the topographical density
856 variations in view of a sub-centimetre geoid. In: *International Association of Geodesy*
857 *Symposia*. Cham: Springer International Publishing 27-35.

858 Forsberg R (1984) A study of terrain reductions, density anomalies and geophysical inversion
859 methods in gravity field modeling. Report No. 355, Department of Geodetic Science, The
860 Ohio State University, Columbus.

861 Forsberg R (1985) Gravity field terrain effect computations by FFT. *Bulletin Géodésique* (1976-
862 1995) 59(4):342-360. <https://doi.org/10.1007/BF02521068>

863 Goyal R, Featherstone WE, Tsoulis D, Dikshit O (2020) Efficient spatial-spectral computation of
864 local planar gravimetric terrain corrections from high-resolution digital elevation models.
865 *Geophysical Journal International* 221(3):1820–1831.

866 <https://doi.org/10.1093/GJI/GGAA107>

867 Heiskanen WA, Moritz H (1967) Physical geodesy. W.H. Freeman and Company, San Francisco

868 Huang W, Cheng CC, Hwang C, Yu D, Chen TW, Chen YT (2021) The 2021 gravimetric and

869 hybrid geoid models for Taiwan and offshore islands: Application to a seamless vertical

870 datum. *Terr Atmos Ocean Sci* 32:873-886. <https://doi.org/10.3319/TAO.2021.10.19.01>

871 Hwang C, Wang CG, Hsiao YS (2003) Terrain correction computation using Gaussian

872 quadrature. *Computers and Geosciences* 29(10):1259–1268.

873 <https://doi.org/10.1016/j.cageo.2003.08.003>

874 Hwang C, Hsiao YS (2003) Orthometric corrections from leveling, gravity, density and elevation

875 data: a case study in Taiwan. *Journal of Geodesy* 77(5):279-291.

876 <https://doi.org/10.1007/s00190-003-0325-6>

877 Hwang C, Hsiao YS, Lu C, Wu WS, Tseng YH (2007) Determination of northeast Asia's highest

878 peak (Mt. Jade) by direct levelling. *Survey Review* 39(303):21-33.

879 <https://doi.org/10.1179/003962607X164961>

880 Hwang C, Hsu HJ, Chang ET, Featherstone W, Tenzer R, Lien T, Hsiao YS, Shih HC, Jai PH

881 (2014) New free-air and Bouguer gravity fields of Taiwan from multiple platforms and

882 sensors. *Tectonophysics* 611:83–93. <https://doi.org/10.1016/j.tecto.2013.11.027>

883 Hwang C, Hsu HJ, Featherstone WE, et al (2020) New gravimetric-only and hybrid geoid

884 models of Taiwan for height modernisation, cross-island datum connection and airborne

885 LiDAR mapping. *Journal of Geodesy* 94(9):1. <https://doi.org/10.1007/s00190-020-01412-5>

886 Ihde J, Sánchez L, Barzaghi R, Drewes H, Foerste C, Gruber T et al (2017) Definition and
887 proposed realization of the International Height Reference System (IHRs). *Surveys in*
888 *Geophysics* 38(3):549-570. <https://doi.org/10.1007/s10712-017-9409-3>

889 Kuhn M (2003) Geoid determination with density hypotheses from isostatic models and
890 geological information. *Journal of Geodesy* 77(1):50–65. [https://doi.org/10.1007/s00190-](https://doi.org/10.1007/s00190-002-0297-y)
891 [002-0297-y](https://doi.org/10.1007/s00190-002-0297-y)

892 Liu WY (2023) Assessing the 2023 Observed Geoidal Heights for Improving Taiwan Geoid
893 Models. Master thesis, National Yang Ming Chiao Tung University

894 McCubbine JC, Featherstone WE, Kirby JF (2017) Fast-Fourier-based error propagation for the
895 gravimetric terrain correction. *Geophysics* 82(4):G71–G76.
896 <https://doi.org/10.1190/geo2016-0627.1>

897 Moritz H (1980) *Advanced physical geodesy*. Wichmann, Karlsruhe

898 Olgun S (2017) Topoğrafik düzensizliklerin ortometrik yüksekliklere etkisinin farklı
899 çözünürlüklü SYM ve silindir şablonlar yardımıyla araştırılması. Master Thesis, Kocaeli
900 University, Kocaeli, 474091.

901 Olgun S, Üstün A (2021) Gravimetrik topoğrafik düzeltmelerin hesaplanmasında yakın bölge
902 katkısı. *Harita Dergisi* 165(1):17-27.

903 Olgun S, Üstün A, Akyılmaz O (2023) tc-cylinder: An optimized algorithm for accurate
904 topography effect from high-resolution digital elevation models. *Computers and*
905 *Geosciences* 170:105264. <https://doi.org/10.1016/j.cageo.2022.105264>

906 Olgun S, Üstün A, Akyılmaz O, Hwang C (2025) Insights on terrain roughness and density
 907 variations for geoid models and orthometric heights: a quantitative comparison in the Konya
 908 Closed Basin, Türkiye and Auvergne, France. *Geophysical Journal International*
 909 243(3):ggaf373. <https://doi.org/10.1093/gji/ggaf373>

910 Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2012) The development and evaluation of the
 911 Earth Gravitational Model 2008 (EGM2008). *Journal of Geophysical Research: Solid Earth*
 912 117(B4):B04406. <https://doi.org/10.1029/2011JB008916>

913 Sánchez L, Sideris MG (2017) Vertical datum unification for the international height reference
 914 system (IHRs). *Geophysical Journal International* 209(2):570-586.
 915 <https://doi.org/10.1093/gji/ggx025>

916 Sánchez L, Ågren J, Huang J, Wang YM, Mäkinen J, Pail R et al (2021) Strategy for the
 917 realisation of the International Height Reference System (IHRs). *Journal of Geodesy*
 918 95(3):1-33. <https://doi.org/10.1007/s00190-021-01481-0>

919 Sampietro D, Capponi M, Triglione D et al (2016) GTE: a new software for gravitational terrain
 920 effect computation: theory and performances. *Pure and Applied Geophysics* 173(7):2435–
 921 2453. <https://doi.org/10.1007/s00024-016-1265-4>

922 Sheng MB, Shaw C, Vaníček P et al (2019) Formulation and validation of a global laterally
 923 varying topographical density model. *Tectonophysics* 762:45–60.
 924 <https://doi.org/10.1016/j.tecto.2019.04.005>

925 Sjöberg LE (2001) The effect of downward continuation of gravity anomaly to sea level in
 926 Stokes' formula. *Journal of Geodesy* 74(11):796-804.

927 <https://doi.org/10.1007/s001900000143>

928 Sjöberg LE (2004) The effect on the geoid of lateral topographic density variations. *Journal of*
929 *Geodesy* 78(1):34–39. <https://doi.org/10.1007/s00190-003-0363-0>

930 Sjöberg LE (2010) A strict formula for geoid-to-quasigeoid separation. *Journal of Geodesy*
931 84(11):699–702. <https://doi.org/10.1007/s00190-010-0407-1>

932 Tenzer, R., Novák, P., Moore, P., Kuhn, M., & Vaníček, P. (2006). Explicit formula for the geoid-
933 quasigeoid separation. *Studia Geophysica et Geodaetica*, 50(4), 607-618.
934 <https://doi.org/10.1007/s11200-006-0038-4>

935 Tenzer, R., Hirt, C., Claessens, S., & Novák, P. (2015). Spatial and spectral representations of the
936 geoid-to-quasigeoid correction. *Surveys in Geophysics*, 36(5), 627-658.
937 <https://doi.org/10.1007/s10712-015-9337-z>

938 Tenzer, R., Hirt, C., Novák, P., Pitoňák, M., & Šprlák, M. (2016). Contribution of mass density
939 heterogeneities to the quasigeoid-to-geoid separation. *Journal of Geodesy*, 90(1), 65-80.
940 <https://doi.org/10.1007/s00190-015-0858-5>

941 Tenzer R, Nsiah Ababio A (2023) On the consistency between a classical definition of the geoid-
942 to-quasigeoid separation and Helmert orthometric heights. *Sensors* 23(11):5185.
943 <https://doi.org/10.3390/s23115185>

944 Tsoulis D, Wziontek H, Petrović S (2003) A bilinear approximation of the surface relief in terrain
945 correction computations. *Journal of Geodesy* 77:338–344. [https://doi.org/10.1007/s00190-](https://doi.org/10.1007/s00190-003-0332-7)
946 003-0332-7

947 Wang YM, Veronneau M, Huang J et al (2023) Accurate computation of geoid-quasigeoid
948 separation in mountainous region - A case study in Colorado with full extension to the
949 experimental geoid region. Journal of Geodetic Science 13(1):20220128.
950 <https://doi.org/10.1515/jogs-2022-0128>

951 Yang M, Tseng CL, Yu JY (2001) Establishment and maintenance of Taiwan geodetic datum
952 1997. Journal of Surveying Engineering 127(4):119-132.
953 [https://doi.org/10.1061/\(ASCE\)0733-9453\(2001\)127:4\(119\)](https://doi.org/10.1061/(ASCE)0733-9453(2001)127:4(119))

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Graphicabstract.png](#)