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A safety aware framework for conversational health agents integrating fuzzy logic reinforcement learning and large language models with self determination theory and constrained optimization called SAFE CHA

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Abstract

A **hybrid conversational health agent (CHA)** is proposed that integrates large language models (LLMs), fuzzy logic, and reinforcement learning (RL) to support mental health dialogue. Mental disorders are estimated to affect nearly 970 million people worldwide, yet clinical support remains limited and inaccessible for many. Although chatbots have been introduced to improve accessibility, a balance among empathy, personalization, and safety must still be achieved. In this work, a hybrid architecture was designed in which (1) user affect and intent are analyzed by an LLM; (2) a fuzzy inference layer maps noisy affective signals into interpretable low-dimensional state vectors; (3) an RL agent determines high-level dialogue strategies through Self-Determination Theory (SDT)-shaped rewards; (4) a calibrated risk monitor enforces safety through escalation; and (5) an outer-loop optimizer tunes fuzzy sets, prompts, and RL hyperparameters under safety constraints. Formal derivations, component algorithms, and computational complexity analyses are provided. Evaluation metrics are defined in terms of empathy, engagement, SDT alignment, fairness, and risk. A reference implementation using NumPy and Matplotlib is made available to reproduce all figures. By combining interpretability through fuzzy logic, adaptability through RL, and fluency through LLMs, a safe, human-aligned conversational system is enabled. Comparative experiments with rule-based, fuzzy-only, LLM-only, and RL-only agents are reported to highlight trade-offs among control, safety, and learning. The proposed

hybrid approach is expected to serve as a blueprint for explainable and safe mental health dialogue systems.

Keywords: Conversational Health Agent; Fuzzy Logic; Reinforcement Learning; Large Language Models; Self-Determination Theory; Safety Constraints; Outer-Loop Optimization; Digital Mental Health.

1 Introduction

Conversational health agents (CHAs) have been increasingly deployed to support mental well-being, yet significant challenges are still encountered regarding safety, personalization, and transparency [10, 18]. Fluent, contextually rich dialogue is produced by large language models (LLMs), but explicit strategic control is often absent and hallucinations or unsafe content can be generated without robust safeguards [4, 9]. In contrast, interpretable, linguistically meaningful reasoning about affective signals is provided by fuzzy logic systems [5, 1], while long-horizon strategy optimization is enabled by reinforcement learning (RL), albeit with increased opacity and exploration risk if constraints are not imposed [6, 12].

To reconcile these trade-offs, a hybrid architecture is proposed that composes (1) LLM-based affect and intent extraction, (2) a fuzzy inference layer that produces an interpretable emotional strategy vector, (3) an RL policy trained with Self-Determination Theory (SDT)-shaped rewards to select high-level dialogue strategies, (4) a calibrated safety monitor that enforces escalation and suppresses free-form generation when thresholds are exceeded, and (5) an outer-loop optimizer that tunes fuzzy sets, prompts, and RL hyperparameters under explicit safety constraints. The complementary strengths of each paradigm are thereby leveraged: interpretability is afforded by fuzzy rules, fluency is delivered by LLMs, and adaptivity is achieved by RL.

2 Method

2.1 Problem Definition and System Overview

A turn-based conversational interaction is considered, in which a user and an automated health agent exchange messages over a finite sequence of turns. At each turn t , a textual utterance is provided by the user, and an appropriate response is generated by the agent. The interaction is not left unconstrained; rather, the agent’s behavior is guided by a set of clinically informed strategies that determine the therapeutic style and content of each response.

An internal memory of the ongoing dialogue is maintained by the system. This memory includes the complete conversation history, the user’s emotional trajectory, and various indicators of psychological risk. The accumulated information is compressed into a compact state representation, which serves as the foundation for all subsequent decisions. This state is designed to be both informative for learning algorithms and interpretable for human auditors, thereby supporting transparency and clinical oversight.

Instead of allowing the language model to generate responses arbitrarily, the agent’s behavior is constrained through a discrete set of high-level therapeutic strategies. These strategies are derived from established psychological interventions and are defined as follows. Emotional Reflection, abbreviated as ER, involves the acknowledgment and validation of the user’s emotional experience. This strategy employs empathetic paraphrasing, active listening cues, and normalization of the user’s feelings, and it is particularly appropriate when intense emotions are expressed or when validation is sought by the user. Cognitive Restructuring, or CR, gently challenges unhelpful thought patterns and guides the user toward alternative perspectives. Socratic questioning and cognitive reframing techniques are utilized in this strategy, which is suitable when cognitive distortions such as catastrophizing or black-and-white thinking are exhibited. Goal Setting, denoted as GS, encourages the user to identify concrete and actionable coping strategies. Overwhelming problems are broken down into smaller steps, achievable objectives are set, and progress is reinforced through this strategy, which promotes behavioral activation and self-efficacy. Safety Escalation, abbreviated as SE, activates a crisis intervention protocol. All free-form language generation is suppressed under this strategy, and standardized emergency resources are provided. This strategy is triggered exclusively by the safety monitor and overrides all other policy decisions, ensuring that harmful content is never produced during high-risk situations. The action space is therefore defined as the set containing these four strategies, and exactly one strategy is selected at each turn to condition the subsequent response generation.

The complete decision-making pipeline processes each user turn through five sequential stages. At the first stage, the user’s raw utterance is passed through a language encoder. This encoder serves two purposes: the user’s emotional state is classified into a distribution over predefined emotion categories, such as anxiety, sadness, anger, or motivation, and the user’s communicative intent, such as seeking help, venting emotions, or requesting information, is simultaneously predicted. These outputs provide the raw perceptual data that informs all subsequent reasoning. The emotion and intent predictions are not treated as deterministic labels but rather as probabilistic estimates, and this uncertainty is explicitly preserved and propagated through the system, allowing ambiguous or subtle expressions to be handled appropriately.

At the second stage, a fuzzy inference layer is introduced to transform the raw emotion probabilities into interpretable, clinically meaningful indicators. The emotion distribution is mapped onto two key therapeutic dimensions: anxiety and motivation. These dimensions are central to many mental health conditions and directly inform the choice of intervention strategy. A set of linguistically defined rules is then applied by the fuzzy inference system, such as the rule that high anxiety combined with low motivation strongly recommends emotional reflection. A recommendation for each therapeutic strategy is produced through this process. The strength of this approach lies in its transparency: every rule is expressed in natural language, making the system’s reasoning process auditable by clinicians. The fuzzy layer effectively serves as an interpretable bridge between raw perceptual data and strategic decision-making.

At the third stage, a reinforcement learning agent is introduced to overcome the limitations of static fuzzy recommendations. While the fuzzy layer provides useful heuristics, it cannot adapt to individual users or long-term therapeutic goals. The reinforcement learning agent learns to select strategies based on accumulated experience and user feedback. A complete

state representation is received by the agent, which includes the fuzzy recommendations, the raw emotion and intent predictions, and additional dialogue features such as turn count and user engagement metrics. Based on this state, one of the four therapeutic strategies is selected. The reinforcement learning policy is trained to maximize a reward function that is shaped according to Self-Determination Theory, thereby promoting user autonomy, competence, and relatedness. Importantly, the RL agent is not permitted to ignore the fuzzy recommendations entirely; a regularization term encourages the learned policy to remain close to the fuzzy prior, ensuring that the system retains its interpretable foundation while still benefiting from adaptive learning.

At the fourth stage, a dedicated safety monitor evaluates the current state before any response is generated. A risk score is computed by this monitor, which estimates the probability of a critical safety incident, such as self-harm or severe crisis. The safety monitor is calibrated to maintain two operational thresholds. The first threshold is used for performance monitoring and cost accumulation within the constrained optimization framework. The second, higher threshold triggers an immediate safety escalation. When this escalation threshold is exceeded, the system forcibly overrides any strategy selected by the reinforcement learning agent and activates the Safety Escalation strategy. This hard safety mechanism is a critical feature of the architecture, ensuring that no response is ever produced during high-risk situations, regardless of what the RL policy or language model might otherwise generate. All escalation events are logged for clinical review, enabling continuous quality improvement.

At the fifth and final stage, if the safety gate is not triggered, the system proceeds to generate a response. The language model is conditioned on three pieces of information: the complete dialogue history, the selected therapeutic strategy, and the fuzzy intensity recommendations. This conditioning ensures that the generated response aligns with both the strategic intent and the emotional context of the interaction. The language model is fine-tuned on a corpus of strategy-annotated dialogues, enabling it to produce responses that are not only fluent and natural but also therapeutically appropriate. Additional control parameters allow for stylistic adjustments, such as varying the level of formality, empathy, or conciseness based on user preferences and clinical context.

The five-stage pipeline described above decomposes the conversational agent’s decision-making into functionally distinct, independently tunable modules. This modular design offers several significant advantages. High interpretability is provided by the fuzzy inference layer, which offers transparent, rule-based reasoning that can be audited and refined by clinicians without requiring technical expertise. Strong adaptability is enabled by the reinforcement learning layer, which facilitates personalization and long-term optimization based on user feedback and therapeutic outcomes. Robust safety is ensured by the dedicated safety monitor with its hard escalation threshold, providing a principled mechanism for preventing harmful outputs. Natural fluency is delivered by the language model, which produces contextually appropriate responses that maintain user engagement. Finally, excellent maintainability is achieved because each module can be updated, replaced, or retrained independently, facilitating continuous improvement and domain adaptation.

The remainder of Section 2 provides detailed mathematical formulations and training procedures for each of the five modules introduced here. The affect and intent extraction process, including the calibration of prediction uncertainties, is formalized in Subsection

2.2. The fuzzy inference system, including membership function parameterization, rule aggregation, and defuzzification, is presented in Subsection 2.3. The reinforcement learning framework, including the SDT-shaped reward function, the policy optimization objective, and the alignment with the fuzzy prior, is described in Subsection 2.4. The safety monitoring system, including risk score calibration, threshold selection, and the constrained optimization formulation, is detailed in Subsection 2.5. Finally, the language model conditioning and fine-tuning procedure is covered in Subsection 2.6. By separating the conceptual overview from the mathematical details, the architecture is made accessible to a broad audience while still providing the technical depth required for rigorous implementation and evaluation.

A dialog evolves over turns $t = 1, 2, \dots, T$. User utterance x_t and agent reply y_t are text sequences. Internal state $s_t \in \mathcal{S}$ summarizes context, affect, engagement, and risk signals. A high-level *strategy* $a_t \in \mathcal{A} = \{\text{ER}, \text{CR}, \text{GS}, \text{SE}\}$ is chosen, where: ER (Emotional Reflection) validates and reflects user’s emotional state; CR (Cognitive Restructuring) challenges maladaptive thought patterns; GS (Goal Setting) encourages actionable coping strategies; SE (Safety Escalation) triggers crisis protocol and suppresses generation. The LLM then conditions on a_t and state to generate y_t .

The SAFE-CHA flowchart illustrates the complete architectural pipeline of the Safety-Aware Framework for Conversational Health Agents, which integrates five key modules that process user input through a sequential pipeline to produce safe, empathetic, and therapeutically appropriate responses. The architecture is designed with a safety-first philosophy where a dedicated safety monitor can override all other system components when risk is detected. The first stage, Perception, processes the raw user utterance x_t through an LLM Encoder $\mathcal{E}_\theta$ to extract both affect distribution $\mathbf{e}_t \in \Delta^{K-1}$ over emotion categories and intent distribution $\mathbf{i}_t \in \Delta^{I-1}$ over intention categories, with the probabilistic nature of these outputs preserving uncertainty that is crucial for downstream fuzzy inference. The second stage, Interpretation, applies fuzzy inference using Mamdani rules to transform the projected anxiety x_t^{anx} and motivation x_t^{mot} scores into strategy intensities $\mathbf{u}_t \in [0, 1]^J$, representing recommendation strengths for Emotional Reflection, Cognitive Restructuring, Goal Setting, and Safety Escalation, which are then converted to a fuzzy prior π_{fuzzy} that serves as a regularization target for the RL policy to preserve interpretability. The third stage, Decision, employs an RL Policy π_ϕ trained using Proximal Policy Optimization with SDT-shaped rewards promoting autonomy, competence, and relatedness, alignment loss keeping the policy close to the fuzzy prior, and Lagrangian relaxation enforcing safety constraints, to select a strategy action a_t from the four therapeutic strategies. The fourth stage, Safety, implements a calibrated Safety Monitor $p_{\text{risk}}(s_t)$ that estimates the probability of a critical safety incident and compares it against the escalation threshold τ_{esc} ; when the risk exceeds this threshold, the system triggers Escalation forcing the SE action, suppressing free-form generation, providing standardized crisis response, and logging the event for clinical review; otherwise, the system proceeds with the Safe Action $\tilde{a}_t$. The fifth stage, Generation, uses the LLM Generator p_θ conditioned on the dialogue context c_t , the overridden action $\tilde{a}_t$, the fuzzy intensities $\mathbf{u}_t$, and response-control parameters λ to produce the final Response y_t . Two critical feedback loops enable learning and adaptation: the Reward Feedback r_t flows back to update the RL Policy via PPO based on the SDT-shaped reward, while the Safety Cost Feedback c_t flows back through the Lagrangian multiplier λ_{sa} to enforce the safety budget constraint over time, ensuring the policy learns to avoid high-risk states while maximizing therapeutic effectiveness. The

modular design makes each stage functionally distinct and independently tunable, enabling easy updates and retraining, while the complementary strengths of fuzzy logic providing interpretability, reinforcement learning providing adaptivity, and large language models providing fluency are leveraged within a principled safety-constrained optimization paradigm to create a foundational blueprint for trustworthy digital mental health tools.

2.2 Affect Extraction and Fuzzy Emotional State

The perception and reasoning components of the SAFE-CHA architecture are now formalized, beginning with the extraction of affective and intentional signals from user utterances and their subsequent transformation into interpretable fuzzy strategy intensities. This dual process is fundamental to the system’s ability to understand the user’s emotional state and communicative goals before determining an appropriate therapeutic response.

The affect and intent encoding process begins when the user provides an utterance at turn t , denoted as x_t , which is a sequence of natural language tokens. This utterance is processed by a pretrained language encoder $\mathcal{E}_\theta$ with parameters θ , which produces a contextualized representation that captures both the semantic content and the emotional nuance of the user’s words. The encoder is typically based on a transformer architecture that has been pretrained on large-scale language modeling tasks, allowing it to extract rich linguistic features that are relevant for downstream affective reasoning. Two classification heads are attached to this encoder to perform distinct but complementary tasks. The first is an affect head that outputs logits $\mathbf{z}_t^{\text{aff}} \in \mathbb{R}^K$ over K emotion categories, where K is typically chosen to cover a broad range of clinically relevant emotions such as anxiety, sadness, anger, fear, joy, surprise, disgust, and neutral states. The second is an intent head that outputs logits $\mathbf{z}_t^{\text{int}} \in \mathbb{R}^I$ over I intention categories, capturing the user’s communicative goals such as seeking emotional support, venting frustration, requesting practical advice, providing information, expressing gratitude, or terminating the conversation. These two heads are trained jointly, allowing the model to learn shared representations that benefit both tasks simultaneously, as emotional expression and communicative intent are often closely intertwined in therapeutic dialogue.

The affect logits are converted to a probability distribution using a temperature-scaled softmax function, which provides a principled way to calibrate the model’s confidence estimates. Specifically, the emotion distribution is computed as shown in Equation (1), where each component is given by the standard softmax formulation with temperature scaling. The temperature parameter $T_{\text{aff}} > 0$ plays a crucial role in controlling the sharpness of the resulting distribution. When T_{aff} is set to a very small value approaching zero, the distribution collapses toward a one-hot vector, effectively making a hard classification that treats the most probable emotion as certain. Conversely, when T_{aff} is large, the distribution becomes more uniform, reflecting greater uncertainty about the user’s emotional state. This temperature parameter is not arbitrary; it is calibrated on a validation set to ensure that the model’s confidence estimates are well-calibrated, meaning that when the model assigns a probability of 0.8 to a particular emotion, it is correct approximately 80% of the time. The calibrated distribution satisfies $\mathbf{e}_t \in \Delta^{K-1}$, where Δ^{K-1} denotes the $(K - 1)$ -dimensional simplex, ensuring that the probabilities sum to one and are non-negative. Similarly, the intent distribution is computed using Equation (2), providing a probabilistic characterization of the user’s communicative goals with $\mathbf{i}_t \in \Delta^{I-1}$. This probabilistic treatment is essential for

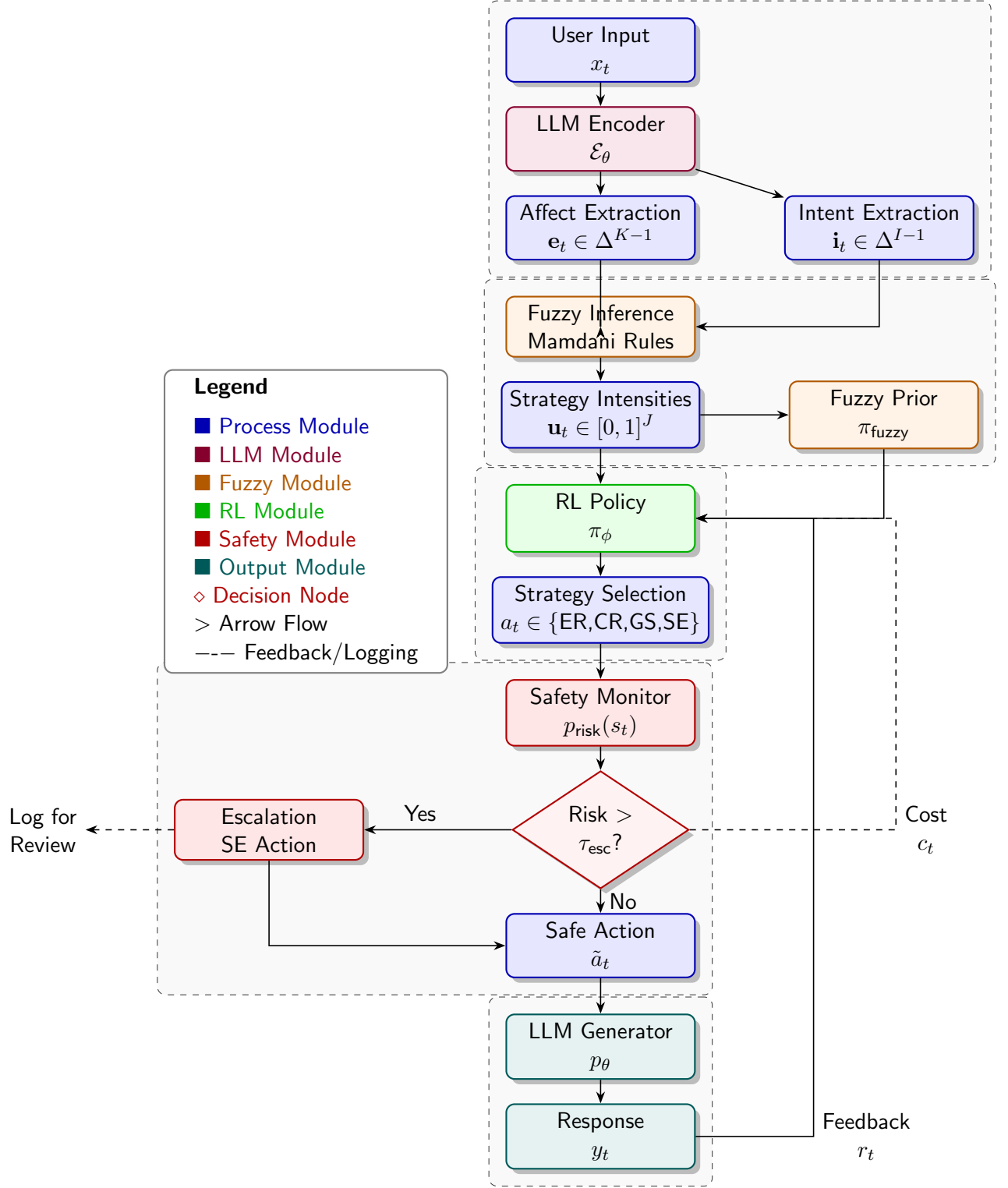


Figure 1: SAFE-CHA Hybrid Architecture Flowchart

downstream fuzzy inference, as it preserves uncertainty information that would be lost by a hard classification, allowing the system to make more nuanced decisions when faced with ambiguous or mixed emotional expressions.

$$\mathbf{e}_t = \text{softmax}(\mathbf{z}_t^{\text{aff}}/T_{\text{aff}}), \quad e_{t,k} = \frac{\exp(z_{t,k}/T_{\text{aff}})}{\sum_{j=1}^K \exp(z_{t,j}/T_{\text{aff}})}, \quad (1)$$

$$\mathbf{i}_t = \text{softmax}(\mathbf{z}_t^{\text{int}}/T_{\text{int}}), \quad \mathbf{i}_t \in \Delta^{I-1}. \quad (2)$$

Both classification heads are trained jointly using a multi-task objective that balances the contributions of affect and intent prediction while regularizing the model to prevent overfitting. Given a labeled dataset $\mathcal{D}_{\text{aff}} = \{(x, \tilde{\mathbf{e}}, \tilde{\mathbf{i}})\}$ containing utterances x with associated ground-truth emotion labels $\tilde{\mathbf{e}}$ and intent labels $\tilde{\mathbf{i}}$, the encoder loss is defined in Equation (3). The cross-entropy term $\text{CE}(\mathbf{p}, \mathbf{q}) = -\sum_k p_k \log q_k$ measures the dissimilarity between the predicted distribution $\mathbf{q}$ and the target distribution $\mathbf{p}$, providing a natural loss for probabilistic classification. The coefficient $\alpha_{\text{int}} > 0$ allows the practitioner to balance the relative importance of the intent prediction task, which may be more or less critical depending on the specific therapeutic application. The L_2 regularization term $\beta_{\text{reg}} \|\theta\|_2^2$ penalizes large parameter values, helping to prevent overfitting and improve generalization to unseen data. This multi-task formulation is particularly powerful because it encourages the encoder to learn representations that are simultaneously informative for both emotional state and communicative intent, capturing the rich interplay between what the user is feeling and what they are trying to achieve through the interaction.

$$\mathcal{L}_{\text{enc}}(\theta) = \sum_{(x, \tilde{\mathbf{e}}) \in \mathcal{D}} \text{CE}(\tilde{\mathbf{e}}, \text{softmax}(\mathbf{z}^{\text{aff}}(x)/T_{\text{aff}})) + \alpha_{\text{int}} \sum_{(x, \tilde{\mathbf{i}}) \in \mathcal{D}} \text{CE}(\tilde{\mathbf{i}}, \text{softmax}(\mathbf{z}^{\text{int}}(x)/T_{\text{int}})) + \beta_{\text{reg}} \|\theta\|_2^2, \quad (3)$$

Once the emotion distribution $\mathbf{e}_t$ has been obtained, it is projected onto two clinically relevant dimensions: anxiety and motivation. These dimensions were selected because they are central to many mental health conditions and directly inform the choice of therapeutic intervention strategy. For example, high anxiety typically calls for emotional validation and grounding techniques, while low motivation may indicate a need for goal-setting and behavioral activation. The projection is accomplished using a row-stochastic matrix $P \in [0, 1]^{2 \times K}$ that maps the K -dimensional emotion distribution to a two-dimensional vector of normalized scores, as shown in Equation (4). The row-stochastic constraints $P\mathbf{1} = \mathbf{1}$ and $P \geq 0$ ensure that the projection preserves the simplex structure, meaning that $x_t^{\text{anx}} \in [0, 1]$ and $x_t^{\text{mot}} \in [0, 1]$ are interpretable as normalized scores that sum to one across the two dimensions. This linear projection provides a simple yet effective way to reduce the high-dimensional emotion space to the specific dimensions that are most relevant for therapeutic reasoning. The projection matrix P is learned from data using a constrained least squares objective, as formulated in Equation (5). Here, $\tilde{x}_t^{\text{anx}}$ and $\tilde{x}_t^{\text{mot}}$ are clinical annotations provided by domain experts, representing their assessment of the user’s anxiety and motivation levels based on the full dialogue context. The regularization term $\gamma_P \|P - P_0\|_F^2$ encourages the learned projection to remain close to a prior matrix P_0 , which may be initialized based on clinical knowledge about which emotions are most indicative of anxiety and motivation. This

regularization is important because it helps to ensure that the learned projection remains clinically interpretable and does not overfit to idiosyncrasies in the training data.

$$\begin{bmatrix} x_t^{\text{anx}} \\ x_t^{\text{mot}} \end{bmatrix} = P \mathbf{e}_t, \quad P \mathbf{1} = \mathbf{1}, \quad P \geq 0, \quad (4)$$

$$\min_{P \geq 0, P \mathbf{1} = \mathbf{1}} \sum_t \left\| P \mathbf{e}_t - \begin{bmatrix} \tilde{x}_t^{\text{anx}} \\ \tilde{x}_t^{\text{mot}} \end{bmatrix} \right\|_2^2 + \gamma_P \|P - P_0\|_F^2, \quad (5)$$

The calibration of prediction uncertainties is a critical step in ensuring that the system’s confidence estimates are reliable and can be trusted for downstream decision-making. Temperature calibration is performed on a validation set $\mathcal{V}$ by selecting the temperature parameter that minimizes the cross-entropy between the predicted distributions and the ground-truth labels, as specified in Equation (6). This calibration procedure adjusts the sharpness of the softmax outputs to match the empirical accuracy of the model, ensuring that the predicted probabilities are meaningful confidence estimates rather than overconfident or underconfident artifacts of the training process. In addition to temperature calibration, the Expected Calibration Error (ECE) is monitored to provide a comprehensive assessment of the model’s reliability. The ECE is computed using Equation (7), where the predictions are partitioned into M bins B_m based on their confidence scores. For each bin, $\text{acc}(B_m)$ is the average accuracy of predictions within that bin, and $\text{conf}(B_m)$ is the average confidence. The ECE measures the average absolute difference between accuracy and confidence, with lower values indicating better calibration. This metric is particularly important in a health context, where overconfident predictions could lead to inappropriate therapeutic interventions, while underconfident predictions might cause the system to miss opportunities for timely support.

$$T_{\text{aff}}^* = \arg \min_{T > 0} \sum_{(x, \tilde{\mathbf{e}}) \in \mathcal{V}} \text{CE}(\tilde{\mathbf{e}}, \text{softmax}(\mathbf{z}^{\text{aff}}(x)/T)), \quad (6)$$

$$\text{ECE} = \sum_{m=1}^M \frac{|B_m|}{N} |\text{acc}(B_m) - \text{conf}(B_m)|, \quad (7)$$

With the calibrated affect and intent signals in hand, the system proceeds to fuzzy inference, where the scalar inputs x_t^{anx} and x_t^{mot} are fuzzified using carefully designed membership functions. The fuzzification process is essential for capturing the inherent vagueness and subjectivity of emotional states, which do not lend themselves to crisp, binary classifications. Two types of membership functions are employed: triangular and trapezoidal. The triangular membership function is defined in Equation (8), where $a < b < c$ are parameters that control the shape and position of the triangle. The peak at b represents the point of maximum membership, while a and c define the support of the function. The trapezoidal membership function, shown in Equation (9), extends the triangular form to include a plateau of full membership, where $a < b < c < d$. The trapezoidal shape is particularly useful for modeling linguistic terms like ”moderate anxiety” where there is a range of values that fully satisfy the concept, rather than a single peak. To ensure that the membership functions remain valid and interpretable during training, the parameters are constrained using unconstrained variables as shown in Equation (10). Here, $\sigma(\cdot)$ is the sigmoid function that maps any real

number to the interval $(0, 1)$, ensuring that a is always a valid coordinate in the input space, and $\text{softplus}(z) = \log(1 + e^z)$ ensures that the differences $b - a$, $c - b$, and $d - c$ are always positive, maintaining the required ordering of the parameters. This parameterization allows the membership functions to be optimized using standard gradient-based methods while automatically satisfying the structural constraints.

$$\mu_{\Delta}(x; a, b, c) = \max\left\{0, \min\left(\frac{x - a}{b - a}, \frac{c - x}{c - b}\right)\right\}, \quad (8)$$

$$\mu_{\square}(x; a, b, c, d) = \begin{cases} 0, & x \leq a, \\ \frac{x - a}{b - a}, & a < x < b, \\ 1, & b \leq x \leq c, \\ \frac{d - x}{d - c}, & c < x < d, \\ 0, & x \geq d. \end{cases} \quad (9)$$

$$a = \sigma(\alpha), \quad (10)$$

$$b = a + \text{softplus}(\beta), \quad (11)$$

$$c = b + \text{softplus}(\gamma), \quad (12)$$

$$d = c + \text{softplus}(\delta), \quad (13)$$

The Mamdani fuzzy inference method is employed to combine the fuzzified inputs according to a set of linguistically defined rules. This method is particularly well-suited for health applications because it provides a transparent, human-readable reasoning process that can be audited and refined by clinicians. The inference process begins by computing the antecedent activation for each rule r , which is given by Equation (11). The min operator implements a logical AND, meaning that the rule is activated to the degree that both the anxiety and motivation conditions are satisfied. This is a conservative aggregation method that ensures the rule's activation is no greater than the weakest of its antecedent conditions. The implication is then performed using the minimum t-norm, resulting in a clipped membership function as shown in Equation (12), where $\nu_r^{(j)}(y)$ is the consequent membership function for strategy j in rule r . This clipping operation ensures that the rule's output is bounded by its activation strength, providing a principled way to handle partial rule matching. The outputs from all rules are aggregated using the maximum t-conorm, shown in Equation (13), which combines the contributions of all activated rules by taking the pointwise maximum. This aggregation method implements a logical OR, meaning that a strategy is recommended to the degree that any rule supports it, which is appropriate for capturing the cumulative evidence from multiple rules. Finally, centroid defuzzification is applied, as specified in Equation (14), to convert the aggregated fuzzy set back to a crisp numerical value. The centroid method computes the center of mass of the aggregated membership function, providing a natural and interpretable defuzzification that balances the support from all activated rules. The resulting vector $\mathbf{u}_t = [u_t^{(1)}, \dots, u_t^{(J)}]^\top$ provides interpretable strategy intensities, where each component $u_t^{(j)} \in [0, 1]$ represents the strength of recommendation for therapeutic strategy j . These

intensities serve as a fuzzy prior that guides the reinforcement learning agent, ensuring that the system’s strategic decisions remain grounded in clinically sound reasoning.

$$\alpha_r = \min(\mu_{A_r}(x_t^{\text{anx}}), \mu_{B_r}(x_t^{\text{mot}})) , \quad (14)$$

$$\mu_r^{(j)}(y) = \min(\alpha_r, \nu_r^{(j)}(y)) , \quad (15)$$

$$\mu^{(j)}(y) = \max_{r \in \{1, \dots, R\}} \mu_r^{(j)}(y), \quad (16)$$

$$u_t^{(j)} = \frac{\int_0^1 y \mu^{(j)}(y) dy}{\int_0^1 \mu^{(j)}(y) dy}. \quad (17)$$

The fuzzy system can be trained using one of two complementary objectives, depending on the nature of the available training data and the desired behavior. The first option matches expert strategies by minimizing the cross-entropy between the fuzzy policy and the expert’s chosen actions, as formulated in Equation (15). Here, π_{fuzzy} is the softmax-transformed fuzzy prior with temperature κ , $\mathbf{1}\{a_t^*\}$ is a one-hot encoding of the expert’s chosen strategy, and $\mathcal{P}_{\text{slope}}$ is a penalty term that discourages extreme membership function slopes, which could lead to overly sensitive or unstable behavior. This objective directly learns the fuzzy parameters to reproduce expert therapeutic decisions, making it ideal when high-quality expert annotations are available. The second option matches target intensities by minimizing the squared error between the defuzzified intensities and target values, as shown in Equation (16). This objective is useful when the training data provides continuous intensity ratings rather than discrete strategy choices, or when the clinician wants to specify the desired intensity profile for each strategy directly. The smoothness regularization $\lambda_{\text{smooth}} \|\boldsymbol{\nu} - \boldsymbol{\nu}_0\|_2^2$ encourages the learned parameters to remain close to their initial values, providing stability and preventing overfitting. Both training objectives are optimized using gradient-based methods, with the unconstrained parameterization ensuring that the membership functions remain valid throughout the optimization process. The choice between the two objectives depends on the specific clinical application and the nature of the available training data, and in practice, a combination of both approaches may be employed to achieve the desired behavior.

$$\min_{\boldsymbol{\nu}} \sum_t \text{CE}(\pi_{\text{fuzzy}}(\cdot \mid \mathbf{u}_t(\boldsymbol{\nu}); \kappa), \mathbf{1}\{a_t^*\}) + \lambda_{\text{slope}} \sum \mathcal{P}_{\text{slope}}(\boldsymbol{\nu}), \quad (18)$$

$$\min_{\boldsymbol{\nu}} \sum_t \|\mathbf{u}_t(\boldsymbol{\nu}) - \tilde{\mathbf{u}}_t\|_2^2 + \lambda_{\text{smooth}} \|\boldsymbol{\nu} - \boldsymbol{\nu}_0\|_2^2. \quad (19)$$

2.3 Unified Notation and Coupled System Equations

To facilitate precise mathematical treatment of the integrated system, the notation is consolidated and the coupled equations governing the interaction between modules are presented. The complete set of system variables and parameters is now defined to establish a common vocabulary for the subsequent mathematical derivations.

The user utterance at turn t is denoted as x_t , and the agent reply at turn t is denoted as y_t . The context c_t represents the complete dialogue history up to turn t . The affect distribution $\mathbf{e}_t \in \Delta^{K-1}$ represents the probability distribution over K emotion categories obtained from Equation (1), while the intent distribution $\mathbf{i}_t \in \Delta^{I-1}$ represents the probability distribution over I intention categories obtained from Equation (2). The linear projection matrix $P \in [0, 1]^{2 \times K}$ is a row-stochastic matrix that maps the affect distribution to the two clinically relevant fuzzy inputs: anxiety $x_t^{\text{anx}} \in [0, 1]$ and motivation $x_t^{\text{mot}} \in [0, 1]$, which are obtained from Equation (4). The fuzzy parameters $\boldsymbol{\nu}$ encompass the membership function breakpoints and rule weights that define the fuzzy inference system. The defuzzified strategy intensities $\mathbf{u}_t \in [0, 1]^J$ with $J = 4$ represent the strength of recommendation for each therapeutic strategy, computed via centroid defuzzification as specified in Equation (14). The reinforcement learning state s_t is a comprehensive representation that includes the strategy intensities $\mathbf{u}_t$, the affect distribution $\mathbf{e}_t$, the intent distribution $\mathbf{i}_t$, and various dialogue statistics that capture the interaction dynamics. The strategy action $a_t \in \mathcal{A}$ is selected from the action space $\mathcal{A} = \{\text{ER}, \text{CR}, \text{GS}, \text{SE}\}$, where ER represents Emotional Reflection, CR represents Cognitive Restructuring, GS represents Goal Setting, and SE represents Safety Escalation. The policy network π_ϕ parameterizes the reinforcement learning policy, the critic network V_θ estimates state values, and the generator p_θ produces the final response. The prompt and response-control parameters are denoted by λ , while h represents the reinforcement learning hyperparameters and w denotes the reward weights. The calibrated risk score $p_{\text{risk}}(s_t)$ with parameters $\boldsymbol{\eta}$ estimates the probability of a critical safety incident, with τ serving as the warning threshold and τ_{esc} as the escalation threshold. Finally, the outer-loop tunables $\xi = (\boldsymbol{\nu}, \lambda, h, P, w)$ comprise all parameters that are jointly optimized at the system level.

The complete system is governed by a set of coupled equations that describe the flow of information from the user input through each processing stage to the final response. The first coupling occurs in the affect-to-fuzzy transformation, where the emotion distribution is projected onto the clinically meaningful anxiety and motivation dimensions using Equation (17). This projection is constrained by the row-stochastic property $P\mathbf{1} = \mathbf{1}$ and non-negativity $P \geq 0$, which ensures that the resulting scores are properly normalized and interpretable as probabilities.

$$\begin{bmatrix} x_t^{\text{anx}} \\ x_t^{\text{mot}} \end{bmatrix} = P \mathbf{e}_t, \quad P\mathbf{1} = \mathbf{1}, \quad P \geq 0. \quad (20)$$

The fuzzy inference system then transforms these scalar inputs into a strategy prior, as shown in Equation (18). This prior represents the fuzzy system's recommendation for each therapeutic strategy, with the temperature parameter $\kappa > 0$ controlling the sharpness of the distribution. When κ is large, the prior becomes more deterministic, favoring the strategy with the highest intensity; when κ is small, the prior becomes more uniform, allowing for greater exploration by the reinforcement learning agent. The strategy intensities $\mathbf{u}_t$ in this equation are obtained from the centroid defuzzification procedure described in Equation (14).

$$\pi_{\text{fuzzy}}(a \mid \mathbf{u}_t; \kappa) = \frac{\exp\{\kappa u_t^{(a)}\}}{\sum_{a' \in \mathcal{A}} \exp\{\kappa u_t^{(a')}\}}, \quad \kappa > 0. \quad (21)$$

The state assembly process, formalized in Equation (19), combines the fuzzy strategy intensities, the affect and intent distributions, and additional dialogue features $\mathbf{d}_t$ into a comprehensive state representation. The projection matrix M and bias vector $\mathbf{b}$ are learnable parameters that are optimized during training to extract the most relevant features for decision-making. The dialogue features $\mathbf{d}_t$ may include metrics such as turn count, response latency, user engagement indicators, and historical interaction patterns that provide context for the current state.

$$s_t = M [\mathbf{u}_t; \mathbf{e}_t; \mathbf{i}_t; \mathbf{d}_t] + \mathbf{b}. \quad (22)$$

The policy selection process, shown in Equation (20), involves sampling a strategy action from the reinforcement learning policy π_ϕ conditioned on the assembled state s_t . This sampling procedure allows the agent to explore different strategies during training while exploiting learned knowledge during deployment. The stochastic nature of the policy is crucial for exploration, enabling the agent to discover novel therapeutic approaches that may be more effective than the fuzzy prior alone.

$$a_t \sim \pi_\phi(\cdot \mid s_t). \quad (23)$$

Before any response is generated, the safety gate evaluates the current state to determine whether immediate intervention is necessary. The safety gate, defined in Equation (21), produces a binary indicator g_t that is activated when the risk score exceeds the escalation threshold τ_{esc} . This hard threshold ensures that high-risk situations are always detected and handled appropriately, regardless of the policy’s decision. When the safety gate is triggered, the action is overridden to the Safety Escalation strategy, as shown in Equation (22). This override mechanism is a critical safety feature that ensures the system never generates harmful content during high-risk situations, even if the reinforcement learning policy would have selected a different strategy.

$$g_t = \mathbf{1}\{p_{\text{risk}}(s_t) > \tau_{\text{esc}}\}. \quad (24)$$

$$\tilde{a}_t = (1 - g_t) a_t + g_t \cdot \text{SE}. \quad (25)$$

The response generation process, formalized in Equation (23), involves sampling the final response y_t from the language model p_θ conditioned on the dialogue context c_t , the overridden action $\tilde{a}_t$, the fuzzy strategy intensities $\mathbf{u}_t$, and the response-control parameters λ . This conditioning ensures that the generated response aligns with both the strategic intent and the emotional context of the interaction. The language model is fine-tuned on a corpus of strategy-annotated dialogues, enabling it to produce responses that are not only fluent but also therapeutically appropriate for the selected strategy.

$$y_t \sim p_\theta(\cdot \mid c_t, \tilde{a}_t, \mathbf{u}_t, \lambda). \quad (26)$$

The reward computation, specified in Equation (24), combines multiple signals to provide feedback for reinforcement learning. The autonomy support signal A_t measures the degree to which the response acknowledges user choice and agency, promoting the psychological need for autonomy. The competence signal C_t captures the user’s perceived effectiveness and

mastery, supporting the need for competence. The relatedness signal R_t reflects the warmth and connection in the interaction, fostering the need for relatedness. The engagement signal E_t measures user participation, response length, and session duration, providing a proxy for overall interaction quality. Penalties are applied for task mismatch M_t when responses are off-topic or inappropriate, and for toxicity T_t when harmful or inappropriate content is detected. The weights $w_a, w_c, w_r, w_e, w_{\text{mis}}, w_{\text{tox}}$ are hyperparameters that can be tuned via the outer-loop optimization to balance these competing objectives based on the specific therapeutic goals and user population.

$$r_t = w_a A_t + w_c C_t + w_r R_t + w_e E_t - w_{\text{mis}} M_t - w_{\text{tox}} T_t. \quad (27)$$

The risk cost, shown in Equation (25), provides a binary indicator of whether the risk score exceeds the warning threshold τ . This cost is used for safety constraint enforcement within the constrained Markov decision process framework, ensuring that the policy maintains an acceptable level of safety performance. The warning threshold τ is typically lower than the escalation threshold τ_{esc} , providing an early warning system that allows the agent to learn from near-miss events without triggering the full escalation protocol.

$$c_t = \mathbf{1}\{p_{\text{risk}}(s_t) \geq \tau\}. \quad (28)$$

The complete optimization problem is structured as a bilevel optimization, reflecting the hierarchical nature of the system. At the inner level, each module is trained independently using its own objective function. The encoder parameters θ^* are optimized by minimizing the multi-task loss from Equation (3). The fuzzy parameters ν^* are optimized using either the expert strategy matching objective from Equation (15) or the intensity matching objective from Equation (16). The risk monitor parameters η^* are optimized by minimizing the binary cross-entropy loss with calibration regularization. The reinforcement learning policy and critic parameters ϕ^*, ϑ^* are optimized using the complete reinforcement learning objective that includes PPO, entropy bonus, alignment regularization, and safety constraints. Finally, the generator parameters θ_{gen}^* are optimized using supervised fine-tuning with strategy conditioning.

At the outer level, the system tuning problem is formulated in Equation (26), where the objective is to find the set of tunable parameters ξ^* that maximize the expected system performance $f(\xi)$ while satisfying the safety constraint $g(\xi) \leq \delta$. The performance function $f(\xi)$ captures the expected cumulative return of the system when trained with parameters ξ , while the constraint function $g(\xi)$ captures the probability of safety violations. This bilevel formulation allows the outer loop to explore the parameter space and find configurations that balance performance and safety, while the inner loop ensures that each module is optimally trained for each candidate configuration. The constraint budget δ is typically set based on clinical requirements and risk tolerance, ensuring that the system operates within acceptable safety boundaries.

$$\xi^* = \arg \max_{\xi \in \Xi} f(\xi) \quad \text{s.t.} \quad g(\xi) \leq \delta. \quad (29)$$

The information flow through the complete system follows a well-defined sequence that transforms the user’s raw input into a therapeutic response. The process begins when the

user provides an utterance x_t , which is processed by the encoder to produce the affect distribution $\mathbf{e}_t$ and intent distribution $\mathbf{i}_t$ using Equations (1) and (2). These distributions are then projected to the clinically meaningful anxiety and motivation scores using Equation (4), which provides the inputs for the fuzzy inference system. The fuzzy system produces the strategy intensities $\mathbf{u}_t$ via centroid defuzzification as specified in Equation (14), and these intensities are combined with the affect, intent, and dialogue features to assemble the state s_t using Equation (19). The policy then samples a strategy action a_t using Equation (20), and the safety gate computes the override indicator g_t using Equation (21). If the safety gate is triggered, the action is overridden to the Safety Escalation strategy using Equation (22); otherwise, the original policy action is retained. The response is then generated by the language model using Equation (23), conditioned on the context, the overridden action, and the fuzzy intensities. Finally, the reward r_t and cost c_t are computed using Equations (24) and (25), respectively, and learning updates are performed using the respective objectives. This sequence of operations is repeated at each turn of the conversation, allowing the system to continuously adapt to the user’s evolving emotional state and therapeutic needs while maintaining robust safety guarantees.

2.4 Reinforcement Learning with SDT-Shaped Rewards

The reinforcement learning component of the SAFE-CHA architecture is now formalized, encompassing the Markov Decision Process formulation, the Self-Determination Theory (SDT)-shaped reward function, policy optimization through Proximal Policy Optimization, and the integration of the fuzzy prior as a regularization mechanism. This component is responsible for learning adaptive, long-term strategic decision-making that complements the interpretable but static fuzzy inference system.

The conversational interaction is modeled as an episodic Markov Decision Process (MDP), which provides a rigorous mathematical framework for sequential decision-making under uncertainty. The MDP is defined by the tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}, r, \gamma, \rho_0, H)$, where each element serves a specific purpose in characterizing the interaction dynamics. The state space $\mathcal{S}$ is continuous, reflecting the rich and nuanced nature of conversational contexts that cannot be captured by discrete states alone. The action space $\mathcal{A} = \{\text{ER}, \text{CR}, \text{GS}, \text{SE}\}$ is discrete and comprises the four therapeutic strategies: Emotional Reflection (ER), Cognitive Restructuring (CR), Goal Setting (GS), and Safety Escalation (SE). The transition probability function $\mathcal{P} : \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S})$ captures the stochastic dynamics of how the user’s state evolves in response to the agent’s chosen strategy, accounting for the inherent uncertainty in human behavior and therapeutic response. The reward function $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ provides immediate feedback that guides the agent toward desirable behaviors, and is carefully shaped according to psychological principles to promote therapeutic effectiveness. The discount factor $\gamma \in (0, 1)$ determines the agent’s preference for immediate versus future rewards, with values closer to 1 encouraging longer-term planning that is essential for sustained therapeutic progress. The initial state distribution ρ_0 characterizes the range of possible starting states encountered in practice, ensuring that the agent learns to handle diverse user presentations. Finally, the episode horizon H defines the maximum length of a conversation, providing a bounded optimization problem that is computationally tractable.

At each turn t , the agent observes the current state s_t , which encapsulates the complete

conversational context including emotional signals, strategy intensities, and dialogue statistics. The agent then selects an action $a_t \sim \pi_\phi(\cdot \mid s_t)$ according to the stochastic policy π_ϕ parameterized by ϕ , which allows for exploration during training and adaptation to user-specific patterns. Upon executing the action, the agent receives a reward $r_t = r(s_t, a_t)$ that reflects the immediate therapeutic impact, and the environment transitions to a new state $s_{t+1} \sim \mathcal{P}(\cdot \mid s_t, a_t)$ according to the user’s response and evolving emotional state. The discounted return, defined in Equation (27), accumulates future rewards with exponential discounting, ensuring that the agent considers both immediate and long-term consequences of its actions. The discount factor γ plays a crucial role here, as it determines how much weight is given to future rewards relative to immediate ones. A higher discount factor encourages the agent to invest in strategies that may not yield immediate rewards but contribute to sustained therapeutic progress over multiple turns.

$$G_t = \sum_{k=0}^{H-t} \gamma^k r_{t+k}. \quad (30)$$

The overall objective of the reinforcement learning agent is to find a policy that maximizes the expected cumulative return across episodes, as expressed in Equation (28). This objective captures the fundamental goal of learning a strategy that consistently produces positive therapeutic outcomes over the course of an entire conversation, rather than optimizing for individual turns in isolation. The expectation is taken over the initial state distribution, the stochastic transitions, and the policy’s own randomness, reflecting the uncertainty inherent in real-world therapeutic interactions.

$$J(\pi) = \mathbb{E}_{\rho_0, \mathcal{P}, \pi} \left[\sum_{t=0}^H \gamma^t r_t \right]. \quad (31)$$

The reward function is carefully shaped according to Self-Determination Theory (SDT), which identifies three basic psychological needs that are essential for intrinsic motivation and well-being: autonomy, competence, and relatedness. The SDT-shaped reward function, shown in Equation (29), combines multiple signals to provide a comprehensive measure of therapeutic quality. The autonomy support signal $A_t \in [0, 1]$ measures the degree to which the agent’s response acknowledges the user’s choice, agency, and self-determination. This is typically computed by analyzing whether the response offers options, respects the user’s preferences, and avoids prescriptive language. The competence signal $C_t \in [0, 1]$ captures the user’s perceived effectiveness and mastery, which is inferred from engagement patterns, self-reported confidence, and successful completion of therapeutic tasks. The relatedness signal $R_t \in [0, 1]$ reflects the warmth, empathy, and connection in the interaction, measured through linguistic analysis of emotional resonance and user expressions of feeling understood. The engagement signal $E_t \in [0, 1]$ measures user participation, response length, and session duration, providing a proxy for overall interaction quality and sustained involvement. Penalties are applied for task mismatch $M_t \in [0, 1]$ when responses are off-topic or therapeutically inappropriate, and for toxicity $T_t \in [0, 1]$ when harmful or inappropriate content is detected through automated classifiers. The weights $(w_a, w_c, w_r, w_e, w_{\text{mis}}, w_{\text{tox}}) \in \mathbb{R}_{\geq 0}^6$ are hyperparameters that can be tuned via the outer-loop optimization to balance these competing objectives based on the

specific therapeutic goals and user population characteristics.

$$r_t = w_a A_t + w_c C_t + w_r R_t + w_e E_t - w_{\text{mis}} M_t - w_{\text{tox}} T_t. \quad (32)$$

Each component of the reward function is computed using a combination of sources. User feedback provides explicit ratings and implicit behavioral signals, such as engagement patterns and dropout rates, which directly reflect the user’s experience. Pre-trained classifiers are employed for detecting specific phenomena like toxicity, sentiment, and emotional valence, providing automated and consistent measurements that scale to large-scale deployment. Conversational features, including turn length, response latency, and topic coherence, are extracted directly from the dialogue to capture interaction dynamics that may not be captured by higher-level semantic analysis. This multi-source approach ensures that the reward signal is both comprehensive and robust, drawing on multiple perspectives to provide a balanced evaluation of therapeutic quality.

The policy is optimized using Proximal Policy Optimization (PPO), a state-of-the-art policy gradient method that has become the standard for reinforcement learning in complex, high-dimensional domains. PPO balances sample efficiency with training stability through its clipped surrogate objective, which prevents destructive policy updates that could destabilize learning. The optimization process begins with advantage estimation using Generalized Advantage Estimation (GAE), which provides a robust way to compute the advantage of each action relative to the expected value. The temporal difference error, shown in Equation (30), is computed using the value network $V_{\boldsymbol{\theta}}$ parameterized by $\boldsymbol{\theta}$, which estimates the expected return from each state.

$$\delta_t = r_t + \gamma V_{\boldsymbol{\theta}}(s_{t+1}) - V_{\boldsymbol{\theta}}(s_t). \quad (33)$$

The advantage estimate, shown in Equation (31), then accumulates these temporal difference errors with exponential weighting controlled by the parameter $\lambda \in [0, 1]$. This parameter controls the bias-variance trade-off in advantage estimation: values closer to 0 produce low-variance but potentially biased estimates by relying primarily on the value function, while values closer to 1 produce high-variance but unbiased estimates that more closely approximate the true advantage. The choice of λ is typically tuned to balance these competing concerns based on the specific characteristics of the problem domain.

$$\hat{A}_t = \sum_{l=0}^{H-t} (\gamma \lambda)^l \delta_{t+l}. \quad (34)$$

The PPO surrogate objective, shown in Equation (32), introduces the probability ratio $\rho_t(\phi) = \frac{\pi_{\phi}(a_t|s_t)}{\pi_{\phi_{\text{old}}}(a_t|s_t)}$ that measures how much the new policy deviates from the old policy on the actions that were actually taken. The clipping operation $\text{clip}(\rho_t(\phi), 1 - \epsilon, 1 + \epsilon)$ constrains this ratio to the interval $[1 - \epsilon, 1 + \epsilon]$, where $\epsilon > 0$ is a hyperparameter that controls the maximum allowed policy change. The objective then takes the minimum of the clipped and unclipped objectives, ensuring that the policy update is conservative and stable even when the advantage estimates are noisy or the policy ratio is extreme. This clipping mechanism is the key innovation of PPO, as it allows for multiple gradient updates per batch of data while

maintaining stability, significantly improving sample efficiency compared to earlier policy gradient methods.

$$\mathcal{L}_{\text{PPO}}(\phi) = \mathbb{E} \left[\min \left(\rho_t(\phi) \hat{A}_t, \text{clip}(\rho_t(\phi), 1 - \epsilon, 1 + \epsilon) \hat{A}_t \right) \right]. \quad (35)$$

To encourage exploration and prevent premature convergence to suboptimal policies, an entropy bonus is added to the objective as shown in Equation (33). The Shannon entropy $H(\pi) = -\sum_a \pi(a) \log \pi(a)$ measures the randomness of the policy distribution, with higher entropy indicating more uniform action selection. The entropy coefficient $\beta_{\text{ent}} > 0$ controls the strength of this exploration incentive, with larger values promoting more exploration and smaller values allowing the policy to become more deterministic. The entropy bonus is particularly important in the early stages of training when the policy has little experience and needs to explore a wide range of strategies to discover effective ones.

$$\mathcal{L}_{\text{ent}}(\phi) = -\beta_{\text{ent}} \mathbb{E} \left[\sum_t H(\pi_\phi(\cdot | s_t)) \right]. \quad (36)$$

The value network is trained using the Mean Squared Error loss shown in Equation (34), where $\hat{G}_t$ is the Monte Carlo return estimate computed from the collected trajectories. This loss encourages the value network to accurately predict the expected return from each state, providing a stable baseline for advantage estimation. The value network is updated using the same batches as the policy network, ensuring that both networks evolve together and maintain consistent estimates of state values.

$$\mathcal{L}_{\text{value}}(\boldsymbol{\vartheta}) = \mathbb{E} \left[\sum_t \left(V_{\boldsymbol{\vartheta}}(s_t) - \hat{G}_t \right)^2 \right]. \quad (37)$$

To maintain interpretability and prevent the RL policy from deviating too far from the clinically validated fuzzy recommendations, a KL-divergence alignment loss is introduced in Equation (35). This loss measures the divergence between the fuzzy prior distribution π_{fuzzy} and the learned policy distribution π_ϕ , providing a soft constraint that encourages the policy to remain close to the interpretable foundation. The KL-divergence, defined in Equation (36), is an asymmetric measure of the difference between two probability distributions, with smaller values indicating that the policy is similar to the fuzzy prior. The fuzzy prior is computed from the defuzzified strategy intensities $\mathbf{u}_t$ using the softmax transformation, as previously described in Equation (18). This regularization ensures that the learned policy retains the interpretable foundation of the fuzzy system while still benefiting from adaptive learning, striking a balance between explainability and performance.

$$\mathcal{L}_{\text{align}}(\phi; \boldsymbol{\nu}) = \mathbb{E} \left[\sum_t \text{KL}(\pi_{\text{fuzzy}}(\cdot | \mathbf{u}_t) \parallel \pi_\phi(\cdot | s_t)) \right]. \quad (38)$$

$$\text{KL}(p \parallel q) = \sum_a p(a) \log \frac{p(a)}{q(a)}. \quad (39)$$

Safety constraints are enforced using a Lagrangian relaxation approach that treats safety as a constrained optimization problem. The safety violation cost, shown in Equation (37), is

a binary indicator that activates when the risk score exceeds the warning threshold τ . This cost is accumulated over the episode and constrained to remain within a predefined budget d as expressed in Equation (38). The budget $d \geq 0$ represents the maximum acceptable level of safety violations, providing a clear and interpretable safety requirement that can be tuned based on clinical risk tolerance. The Lagrangian formulation in Equation (39) converts the constrained optimization problem into an unconstrained one by adding a penalty term weighted by the Lagrange multiplier λ . The dual update in Equation (40) adjusts the Lagrange multiplier based on the current constraint violation, increasing the penalty when violations exceed the budget and decreasing it when the budget is not fully utilized. The learning rate $\rho > 0$ controls how quickly the Lagrange multiplier adapts to constraint violations, and the projection operator $[x]_+ = \max(0, x)$ ensures that the multiplier remains non-negative, as required by the Lagrangian dual formulation.

$$c_t = \mathbf{1}\{p_{\text{risk}}(s_t) \geq \tau\}. \quad (40)$$

$$\mathbb{E}\left[\sum_t \gamma^t c_t\right] \leq d. \quad (41)$$

$$\mathcal{L}(\phi, \lambda) = \mathbb{E}\left[\sum_t \gamma^t r_t\right] - \lambda \left(\mathbb{E}\left[\sum_t \gamma^t c_t\right] - d\right). \quad (42)$$

$$\lambda \leftarrow \left[\lambda + \rho \left(\mathbb{E}\left[\sum_t \gamma^t c_t\right] - d\right)\right]_+. \quad (43)$$

The complete optimization objective for the RL module is presented in Equation (41), which combines all the previously described components into a single unified loss function. The objective includes the negative PPO surrogate objective, which encourages the policy to take actions that lead to high advantages. The value loss encourages accurate state value estimation. The negative entropy bonus encourages exploration while preventing premature convergence. The alignment loss encourages the policy to remain close to the fuzzy prior, preserving interpretability. Finally, the safety constraint term penalizes violations of the safety budget, with the Lagrange multiplier λ controlling the strength of this penalty. The coefficient $\tau_{\text{align}} > 0$ controls the relative importance of the alignment regularization, with larger values enforcing stronger adherence to the fuzzy prior and smaller values allowing the policy to deviate more freely in pursuit of higher rewards.

$$\min_{\phi, \boldsymbol{\vartheta}} -\mathcal{L}_{\text{PPO}}(\phi) + \mathcal{L}_{\text{value}}(\boldsymbol{\vartheta}) - \mathcal{L}_{\text{ent}}(\phi) + \tau_{\text{align}} \mathcal{L}_{\text{align}}(\phi; \boldsymbol{\nu}) + \lambda \left(\mathbb{E}\left[\sum_t \gamma^t c_t\right] - d\right)_+. \quad (44)$$

The complete PPO algorithm with fuzzy prior and safety constraints proceeds as follows. The algorithm requires initial policy π_{ϕ_0} , value network $V_{\boldsymbol{\vartheta}_0}$, fuzzy prior π_{fuzzy} , and initial Lagrange multiplier λ_0 . For each iteration, trajectories $\{(s_t, a_t, r_t, c_t, s_{t+1})\}$ are collected using the current policy π_{ϕ_k} , providing the experience needed for learning. The advantages $\hat{A}_t$ are computed using Generalized Advantage Estimation as shown in Equation (31), and

the returns $\hat{G}_t$ are computed for value network training. The fuzzy prior $\pi_{\text{fuzzy}}(\cdot \mid \mathbf{u}_t)$ is computed using Equation (18), providing the target distribution for the alignment loss. For each minibatch B in the collected trajectories, the PPO objective is computed using Equation (32), the alignment loss using Equation (35), and the value loss using Equation (34). The total loss is then computed using Equation (41), and the policy and value network parameters $\phi, \boldsymbol{\vartheta}$ are updated using gradient descent. Finally, the Lagrange multiplier λ is updated using the dual update in Equation (40), adjusting the safety penalty based on the observed constraint violations. This iterative process continues until the policy converges to a stable solution that balances therapeutic effectiveness, interpretability, and safety.

2.5 Safety and Escalation

The safety monitoring and escalation mechanism is now formalized, providing a hard constraint on harmful outputs and ensuring that the agent operates within clinically acceptable boundaries. This component is critical for real-world deployment, as it prevents the system from generating harmful content even when the underlying policy or language model would otherwise produce unsafe responses. The safety mechanism operates through a calibrated risk monitor that continuously evaluates the current state and triggers appropriate interventions when risk exceeds predefined thresholds.

A calibrated risk monitor computes $p_{\text{risk}}(s_t) \in [0, 1]$, which represents the estimated probability of a critical safety incident occurring given the current conversational state. The risk model, shown in Equation (42), maps the state s_t through a neural network $f_{\boldsymbol{\eta}}$ parameterized by $\boldsymbol{\eta}$, and applies a sigmoid activation function $\sigma(z) = (1 + e^{-z})^{-1}$ to produce a probability in the unit interval. The neural network architecture is typically chosen to be relatively shallow, with one or two hidden layers of moderate size, to ensure computational efficiency and prevent overfitting to limited risk incident data. The choice of the sigmoid activation is natural for binary classification tasks, as it maps arbitrary real-valued outputs to interpretable probabilities that can be directly used for decision-making.

$$p_{\text{risk}}(s_t) = \sigma(f_{\boldsymbol{\eta}}(s_t)). \quad (45)$$

The risk model is trained on labeled data $\mathcal{D}_{\text{risk}} = \{(s_t, \tilde{r}_t)\}$, where $\tilde{r}_t \in \{0, 1\}$ indicates whether a critical incident occurred in the corresponding state. The training objective, shown in Equation (43), combines a binary cross-entropy loss with a calibration penalty term. The binary cross-entropy $\text{BCE}(\tilde{r}, p) = -\tilde{r} \log p - (1 - \tilde{r}) \log(1 - p)$ provides a proper scoring rule that incentivizes the model to produce well-calibrated probability estimates, as it is strictly proper and minimized when the predicted probabilities match the true conditional probabilities. The calibration penalty term $\lambda_{\text{cal}} \cdot \text{ECE}(\boldsymbol{\eta})$ incorporates the Expected Calibration Error evaluated on a validation set, as defined in Equation (7). This additional term is crucial because it directly optimizes for calibration rather than simply minimizing cross-entropy, which can sometimes lead to overconfident or underconfident predictions. The coefficient $\lambda_{\text{cal}} > 0$ controls the strength of this calibration regularization, with larger values placing greater emphasis on calibration accuracy.

$$\min_{\boldsymbol{\eta}} \sum_{(s_t, \tilde{r}_t) \in \mathcal{D}_{\text{risk}}} \text{BCE}(\tilde{r}_t, p_{\text{risk}}(s_t)) + \lambda_{\text{cal}} \cdot \text{ECE}(\boldsymbol{\eta}). \quad (46)$$

Two operational thresholds are maintained to enable different levels of safety response: the warning threshold τ and the escalation threshold τ_{esc} , with $\tau_{\text{esc}} \geq \tau$. The warning threshold τ is used for cost accumulation and constrained optimization, providing a mechanism for the reinforcement learning agent to learn from near-miss events without triggering immediate intervention. The escalation threshold τ_{esc} triggers immediate intervention, forcibly overriding the policy and activating the crisis protocol. The separation of these thresholds allows the system to learn from risk warnings while reserving full escalation for the most critical situations.

The warning threshold is chosen to satisfy a frequency budget, as shown in Equation (44). The infimum over τ selects the smallest threshold that ensures the probability of exceeding the threshold remains below the maximum allowable frequency $\delta \in [0, 1]$. This approach provides a principled way to set the threshold based on the desired trade-off between sensitivity and frequency of warnings. A smaller δ results in fewer warnings but may miss some important signals, while a larger δ generates more warnings at the cost of potentially desensitizing clinicians to the alerts.

$$\tau^* = \inf\{\tau : \Pr(p_{\text{risk}}(s_t) > \tau) \leq \delta\}. \quad (47)$$

The escalation threshold is optimized to balance multiple competing objectives, as formulated in Equation (45). The false negative rate $\text{FNR}(\tau) = \Pr(p_{\text{risk}} \leq \tau \mid \text{incident})$ captures the proportion of critical incidents that are missed by the threshold, which is particularly dangerous as these represent genuine safety risks that go unaddressed. The false positive rate $\text{FPR}(\tau) = \Pr(p_{\text{risk}} > \tau \mid \text{no incident})$ captures the proportion of false alarms, which can lead to unnecessary interventions and user distress. The escalation load $\text{EscalationLoad}(\tau)$ represents the expected frequency of escalations, which has practical implications for clinical resources and user experience. The weights $\lambda_{\text{fnr}}, \lambda_{\text{fpr}}, \lambda_{\text{load}} \geq 0$ control the relative importance of these competing objectives, allowing the system to be tuned based on clinical priorities. For example, in high-risk populations, the false negative weight might be increased to prioritize safety over efficiency, while in lower-risk populations, the false positive weight might be increased to avoid unnecessary escalations.

$$\tau_{\text{esc}}^* = \arg \min_{\tau} \lambda_{\text{fnr}} \cdot \text{FNR}(\tau) + \lambda_{\text{fpr}} \cdot \text{FPR}(\tau) + \lambda_{\text{load}} \cdot \text{EscalationLoad}(\tau). \quad (48)$$

The safety gate mechanism provides the actual enforcement of safety constraints at runtime. The safety gate is defined in Equation (46) as a binary indicator g_t that equals 1 when the risk score exceeds the escalation threshold τ_{esc} . This binary decision ensures that the safety mechanism operates as a hard constraint, providing unambiguous intervention when needed. The executed action, shown in Equation (47), is then determined by this safety gate: when the gate is triggered, the action is forcibly set to the Safety Escalation strategy (SE), overriding any action that the reinforcement learning policy might have selected; when the gate is not triggered, the policy’s selected action a_t is executed as usual.

$$g_t = \mathbf{1}\{p_{\text{risk}}(s_t) > \tau_{\text{esc}}\}. \quad (49)$$

$$\tilde{a}_t = \begin{cases} \text{SE}, & \text{if } g_t = 1, \\ a_t, & \text{otherwise.} \end{cases} \quad (50)$$

When the safety gate is triggered with $g_t = 1$, the system performs a series of critical actions to ensure user safety and clinical accountability. First, the system forcibly overrides the reinforcement learning policy, ensuring that no strategic decision-making occurs during the crisis. Second, all free-form language generation is suppressed, preventing the language model from producing any potentially harmful content. Third, the system provides a standardized crisis response resource that has been carefully designed by clinical experts to provide appropriate support and guidance. Fourth, the event is logged for clinical review, creating a complete record that can be used for quality improvement and clinical oversight. Finally, if configured, the system alerts the on-call clinician, enabling rapid human intervention when necessary. This multi-faceted response ensures that safety incidents are handled comprehensively and that the system maintains clinical accountability.

The safety constraints are formalized within a Constrained Markov Decision Process (CMDP) framework, which extends the standard MDP formulation with explicit safety constraints. The CMDP formulation, shown in Equation (48), maximizes the expected cumulative reward subject to the constraint that the expected cumulative safety cost remains below the budget d . The safety cost $c_t = \mathbf{1}\{p_{\text{risk}}(s_t) \geq \tau\}$ is a binary indicator that activates when the risk score exceeds the warning threshold. This formulation treats safety as a hard constraint that must be satisfied, rather than a soft preference that can be traded off against performance.

$$\max_{\pi} \mathbb{E} \left[\sum_t \gamma^t r_t \right] \quad \text{s.t.} \quad \mathbb{E} \left[\sum_t \gamma^t c_t \right] \leq d. \quad (51)$$

The Lagrangian relaxation of the CMDP, shown in Equation (49), transforms the constrained optimization into an unconstrained one by adding a penalty term for constraint violations. The Lagrange multiplier λ controls the strength of this penalty, with larger values enforcing stricter compliance with the safety constraint. The saddle-point optimization in Equation (50) then finds the optimal policy and Lagrange multiplier simultaneously: the policy is optimized to maximize the Lagrangian, while the Lagrange multiplier is optimized to ensure that the safety constraint is satisfied. This formulation provides a principled way to incorporate safety constraints into the reinforcement learning framework, as it automatically balances performance and safety based on the observed constraint violations.

$$\mathcal{L}(\phi, \lambda) = \mathbb{E} \left[\sum_t \gamma^t r_t \right] - \lambda \left(\mathbb{E} \left[\sum_t \gamma^t c_t \right] - d \right). \quad (52)$$

$$\phi^* = \arg \max_{\phi} \mathcal{L}(\phi, \lambda^*), \quad (53)$$

$$\lambda^* = \arg \min_{\lambda \geq 0} \mathcal{L}(\phi^*, \lambda). \quad (54)$$

When safety escalation is triggered, the system generates a standardized crisis response that has been carefully designed by clinical experts to provide appropriate support while

maintaining safety. The standardized response is: "I'm concerned about what you've shared. Please know that you're not alone. I'm connecting you with crisis resources. You can also reach out to a crisis line at [number] or contact your healthcare provider. Would you like me to help you connect with support?" This response was developed in consultation with mental health professionals to ensure that it provides appropriate validation, reassurance, and practical guidance without making clinical judgments that should be reserved for human practitioners.

All escalation events are logged with comprehensive metadata to support clinical review and quality improvement. The log includes the timestamp and session ID for temporal tracking, the risk score $p_{\text{risk}}(s_t)$ and threshold τ_{esc} that triggered the escalation for transparency, the current state s_t in anonymized form for context, the original action a_t and overridden action $\tilde{a}_t$ for understanding what was prevented, and the complete dialogue context in anonymized form for clinical review. The events are also placed in a clinician review queue for follow-up, ensuring that every escalation receives appropriate human oversight. This comprehensive logging serves multiple purposes: it provides accountability for clinical decision-making, enables continuous quality improvement through case review, supports the development of more accurate risk prediction models, and facilitates regulatory compliance.

The safety monitor and escalation algorithm proceeds through a well-defined sequence at each turn. The algorithm requires the risk model p_{risk} , the warning threshold τ , the escalation threshold τ_{esc} , and the safety budget d . For each turn t , the state s_t is computed from the current dialogue context. The risk score $p_{\text{risk}}(s_t)$ is computed using the risk model from Equation (42). The safety cost $c_t = \mathbf{1}\{p_{\text{risk}}(s_t) \geq \tau\}$ is computed, and the accumulated cost $\bar{c} \leftarrow \bar{c} + \gamma^t c_t$ is updated with discounting to reflect the temporal nature of safety events. If the risk score exceeds the escalation threshold τ_{esc} , the escalation protocol is activated: the gate indicator g_t is set to 1, the event is logged for clinical review, and the system proceeds with the escalation response. Otherwise, the gate indicator is set to 0, and the system proceeds with normal policy selection. After processing all turns, the algorithm checks whether the accumulated cost exceeds the budget $\bar{c} > d$, and flags the session for clinician review if the budget was violated. This algorithm provides a systematic and transparent approach to safety monitoring that integrates naturally with the rest of the system while maintaining clear accountability and clinical oversight.

2.6 Optimization of Each Module

The detailed optimization formulations for each of the five modules are now presented, encompassing training objectives, regularization strategies, and implementation considerations. Each module is optimized independently at the inner level of the bilevel optimization framework, with carefully designed objectives that reflect the specific purpose and constraints of each component. The optimization of each module follows a sequential training schedule that ensures stable convergence and prevents destructive interference between modules.

The affect and intent extractor, specifically the encoder $\mathcal{E}_\theta$, is trained end-to-end on labeled data using a multi-task objective that jointly optimizes emotion and intent classification. The training objective, shown in Equation (51), combines three terms that serve distinct purposes. The first term minimizes the cross-entropy between the predicted emotion distribution and the ground-truth emotion labels, encouraging the encoder to produce accurate emotion

classifications. The second term, weighted by $\alpha_{\text{int}} > 0$, minimizes the cross-entropy between the predicted intent distribution and the ground-truth intent labels, ensuring that the encoder also captures the user’s communicative goals. The third term applies L_2 regularization with coefficient $\beta_{\text{reg}} > 0$ to prevent overfitting by penalizing large parameter values. The dataset $\mathcal{D}_{\text{aff}} = \{(x, \tilde{\mathbf{e}}, \tilde{\mathbf{i}})\}$ contains utterances x with associated ground-truth emotion labels $\tilde{\mathbf{e}}$ and intent labels $\tilde{\mathbf{i}}$. The temperature parameters T_{aff} and T_{int} control the sharpness of the softmax distributions, with values typically tuned on a validation set as described below. This multi-task formulation is particularly powerful because it encourages the encoder to learn representations that are simultaneously informative for both tasks, capturing the rich interplay between what the user is feeling and what they are trying to achieve through the interaction.

$$\min_{\theta_{\text{enc}}} \sum_{(x, \tilde{\mathbf{e}}) \in \mathcal{D}_{\text{aff}}} \text{CE}(\tilde{\mathbf{e}}, \text{softmax}(\mathbf{z}^{\text{aff}}(x)/T_{\text{aff}})) + \alpha_{\text{int}} \sum_{(x, \tilde{\mathbf{i}}) \in \mathcal{D}_{\text{aff}}} \text{CE}(\tilde{\mathbf{i}}, \text{softmax}(\mathbf{z}^{\text{int}}(x)/T_{\text{int}})) + \beta_{\text{reg}} \|\theta_{\text{enc}}\|_2^2. \quad (55)$$

Temperature calibration is performed on a hold-out validation set $\mathcal{V}_{\text{aff}}$ to ensure that the model’s confidence estimates are well-calibrated. The temperature parameter T_{aff} is selected by minimizing the cross-entropy on the validation set, as shown in Equation (52). This calibration procedure adjusts the sharpness of the softmax outputs to match the empirical accuracy of the model, ensuring that the predicted probabilities are meaningful confidence estimates rather than overconfident or underconfident artifacts of the training process. The calibration is performed separately for the affect head, as the intent head typically uses a fixed temperature that is tuned for its specific task characteristics. The validation set is held out from the training data to provide an unbiased estimate of the optimal temperature.

$$T_{\text{aff}}^* = \arg \min_{T > 0} \sum_{(x, \tilde{\mathbf{e}}) \in \mathcal{V}_{\text{aff}}} \text{CE}(\tilde{\mathbf{e}}, \text{softmax}(\mathbf{z}^{\text{aff}}(x)/T)). \quad (56)$$

The projection matrix P that maps the emotion distribution to the clinically relevant anxiety and motivation dimensions is learned via constrained least squares, as formulated in Equation (53). The objective minimizes the squared error between the projected scores $P \mathbf{e}_t$ and the clinical annotations $\tilde{x}_t^{\text{anx}}$ and $\tilde{x}_t^{\text{mot}}$, which are provided by domain experts based on their assessment of the user’s anxiety and motivation levels. The constraints $P \geq 0$ and $P \mathbf{1} = \mathbf{1}$ ensure that the projection is row-stochastic, preserving the simplex structure of the input distribution. The regularization term $\gamma_P \|P - P_0\|_F^2$ encourages the learned projection to remain close to a prior matrix P_0 , which may be initialized based on clinical knowledge about which emotions are most indicative of anxiety and motivation. This regularization is important because it helps to ensure that the learned projection remains clinically interpretable and does not overfit to idiosyncrasies in the training data. The coefficient $\gamma_P > 0$ controls the strength of this regularization, with larger values enforcing stronger adherence to the prior.

$$\min_{P \geq 0, P \mathbf{1} = \mathbf{1}} \sum_t \left\| P \mathbf{e}_t - \begin{bmatrix} \tilde{x}_t^{\text{anx}} \\ \tilde{x}_t^{\text{mot}} \end{bmatrix} \right\|_2^2 + \gamma_P \|P - P_0\|_F^2. \quad (57)$$

The fuzzy emotional state module parameters $\boldsymbol{\nu}$ include the membership function break-points (a, b, c, d) for each linguistic term and the rule weights that define the fuzzy inference

system. These parameters are constrained using the unconstrained parameterization described previously, ensuring that the membership functions remain valid throughout optimization. Two training objectives are available for the fuzzy system, providing flexibility based on the nature of the available training data and the desired behavior.

The first option matches expert strategies by minimizing the cross-entropy between the fuzzy policy and the expert’s chosen actions, as shown in Equation (54). The fuzzy policy $\pi_{\text{fuzzy}}(\cdot \mid \mathbf{u}_t(\boldsymbol{\nu}); \kappa)$ is the softmax-transformed strategy intensities with temperature κ , and $\mathbf{1}\{a_t^*\}$ is a one-hot encoding of the expert’s chosen strategy. The penalty term $\lambda_{\text{slope}} \sum_m \mathcal{P}_{\text{slope}}(\boldsymbol{\nu}_m)$ discourages extreme membership function slopes, which could lead to overly sensitive or unstable behavior. The slope penalty $\mathcal{P}_{\text{slope}}$ is typically defined as the sum of squared differences between adjacent membership function parameters, encouraging smooth transitions and preventing the system from developing overly sharp or irregular membership functions. This objective is ideal when high-quality expert annotations are available, as it directly learns the fuzzy parameters to reproduce expert therapeutic decisions.

$$\min_{\boldsymbol{\nu}} \sum_t \text{CE}(\pi_{\text{fuzzy}}(\cdot \mid \mathbf{u}_t(\boldsymbol{\nu}); \kappa), \mathbf{1}\{a_t^*\}) + \lambda_{\text{slope}} \sum_m \mathcal{P}_{\text{slope}}(\boldsymbol{\nu}_m). \quad (58)$$

The second option matches target intensities by minimizing the squared error between the defuzzified intensities and target values, as formulated in Equation (55). This objective is useful when the training data provides continuous intensity ratings rather than discrete strategy choices, or when the clinician wants to specify the desired intensity profile for each strategy directly. The smoothness regularization $\lambda_{\text{smooth}} \|\boldsymbol{\nu} - \boldsymbol{\nu}_0\|_2^2$ encourages the learned parameters to remain close to their initial values $\boldsymbol{\nu}_0$, providing stability and preventing overfitting. This regularization is particularly important when the training data is limited, as it helps to maintain the interpretability of the fuzzy system by preventing the parameters from drifting too far from clinically meaningful values.

$$\min_{\boldsymbol{\nu}} \sum_t \|\mathbf{u}_t(\boldsymbol{\nu}) - \tilde{\mathbf{u}}_t\|_2^2 + \lambda_{\text{smooth}} \|\boldsymbol{\nu} - \boldsymbol{\nu}_0\|_2^2. \quad (59)$$

The safety risk monitor, which predicts the risk score $p_{\text{risk}}(s_t) = \sigma(f_{\boldsymbol{\eta}}(s_t))$, is trained using a binary cross-entropy loss with calibration regularization, as shown in Equation (56). The dataset $\mathcal{D}_{\text{risk}} = \{(s, r)\}$ contains states s with associated binary labels $r \in \{0, 1\}$ indicating whether a critical incident occurred. The binary cross-entropy BCE provides a proper scoring rule that incentivizes the model to produce well-calibrated probability estimates. The calibration regularization term $\lambda_{\text{cal}} \cdot \text{ECE}(\boldsymbol{\eta})$ incorporates the Expected Calibration Error evaluated on a validation set, directly optimizing for calibration accuracy. The coefficient $\lambda_{\text{cal}} > 0$ controls the strength of this calibration regularization, with larger values placing greater emphasis on calibration accuracy. This combined objective ensures that the risk model produces both accurate and well-calibrated probability estimates, which is essential for reliable threshold-based decision-making.

$$\min_{\boldsymbol{\eta}} \sum_{(s, r) \in \mathcal{D}_{\text{risk}}} \text{BCE}(r, p_{\text{risk}}(s)) + \lambda_{\text{cal}} \cdot \text{ECE}(\boldsymbol{\eta}). \quad (60)$$

The operational thresholds are selected based on the calibrated risk model. The warning threshold τ is chosen to satisfy a frequency budget, as shown in Equation (57), where $\delta \in [0, 1]$

is the maximum allowable frequency of warnings. This approach provides a principled way to set the threshold based on the desired trade-off between sensitivity and frequency of warnings. The escalation threshold τ_{esc} is optimized to balance multiple competing objectives, as formulated in Equation (58). The false negative rate $\text{FNR}(\tau)$ captures the proportion of critical incidents that are missed, the false positive rate $\text{FPR}(\tau)$ captures the proportion of false alarms, and the escalation load $\text{EscalationLoad}(\tau)$ represents the expected frequency of escalations. The weights $\lambda_{\text{fnr}}, \lambda_{\text{fpr}}, \lambda_{\text{load}} \geq 0$ control the relative importance of these competing objectives, allowing the system to be tuned based on clinical priorities.

$$\tau^* = \inf \{ \tau : \Pr(p_{\text{risk}}(s_t) > \tau) \leq \delta \}. \quad (61)$$

$$\tau_{\text{esc}}^* = \arg \min_{\tau} \lambda_{\text{fnr}} \text{FNR}(\tau) + \lambda_{\text{fpr}} \text{FPR}(\tau) + \lambda_{\text{load}} \text{EscalationLoad}(\tau). \quad (62)$$

The reinforcement learning policy π_{ϕ} and value network V_{θ} are optimized using the complete objective shown in Equation (59). This comprehensive loss combines five distinct terms that serve complementary purposes. The first term is the negative PPO surrogate objective, which encourages the policy to take actions that lead to high advantages. The second term is the value loss, which encourages the value network to accurately predict the expected return from each state. The third term is the negative entropy bonus, which encourages exploration and prevents premature convergence to suboptimal policies. The fourth term is the alignment loss, which encourages the policy to remain close to the fuzzy prior, preserving interpretability. The fifth term is the safety constraint penalty, which enforces the safety budget through the Lagrange multiplier λ_{sa} . The coefficient $\tau_{\text{align}} > 0$ controls the strength of the alignment regularization, with larger values enforcing stronger adherence to the fuzzy prior. The complete objective is minimized with respect to both the policy parameters ϕ and the value network parameters θ , providing a unified optimization framework for the reinforcement learning module.

$$\begin{aligned} \min_{\phi, \theta} \quad & -\mathbb{E}[\min(\rho_t \hat{A}_t, \text{clip}(\rho_t, 1 - \epsilon, 1 + \epsilon) \hat{A}_t)] \\ & + \mathbb{E}[(V_{\theta} - \hat{G}_t)^2] - \beta_{\text{ent}} \mathbb{E}[H(\pi_{\phi})] \\ & + \tau_{\text{align}} \mathbb{E}[\text{KL}(\pi_{\text{fuzzy}} \parallel \pi_{\phi})] + \lambda_{\text{sa}} (\mathbb{E}[\sum_t \gamma^t c_t] - d)_+. \end{aligned} \quad (63)$$

The Lagrange multiplier λ_{sa} is updated using the dual update shown in Equation (60), which adjusts the multiplier based on the current constraint violation. The learning rate $\rho > 0$ controls how quickly the Lagrange multiplier adapts to constraint violations, and the projection operator $[x]_+ = \max(0, x)$ ensures that the multiplier remains non-negative. This dual update ensures that the safety constraint is enforced over time, with the penalty increasing when violations exceed the budget and decreasing when the budget is not fully utilized. The dual update operates at a slower timescale than the policy updates, allowing the policy to adapt to the changing penalty while maintaining stability.

$$\lambda_{\text{sa}} \leftarrow \left[\lambda_{\text{sa}} + \rho \left(\mathbb{E} \left[\sum_t \gamma^t c_t \right] - d \right) \right]_+. \quad (64)$$

The language model p_θ is fine-tuned using supervised fine-tuning with strategy conditioning, as formulated in Equation (61). The objective minimizes the negative log-likelihood of the ground-truth response y_t^* given the context c_t , the overridden action $\tilde{a}_t$, the fuzzy strategy intensities $\mathbf{u}_t$, and the response-control parameters λ . This conditioning ensures that the generated response aligns with both the strategic intent and the emotional context of the interaction. The KL-divergence regularization term $\gamma_{\text{ctrl}} \text{KL}(p_\theta(\cdot | \cdot) \| p_{\text{ref}}(\cdot | \cdot))$ prevents overfitting by encouraging the fine-tuned model to remain close to the reference model p_{ref} , which is typically the pretrained language model. The coefficient $\gamma_{\text{ctrl}} > 0$ controls the strength of this regularization, with larger values preserving more of the pretrained knowledge and smaller values allowing more adaptation to the strategy-annotated data. This regularization is particularly important when the fine-tuning dataset is limited, as it helps to maintain the model’s general language capabilities while learning the specific response patterns required for therapeutic dialogue.

$$\min_{\theta} - \sum_t \log p_\theta(y_t^* | c_t, \tilde{a}_t, \mathbf{u}_t, \lambda) + \gamma_{\text{ctrl}} \text{KL}(p_\theta(\cdot | \cdot) \| p_{\text{ref}}(\cdot | \cdot)). \quad (65)$$

The optimization of the complete system requires careful selection of hyperparameters across all modules, as these parameters control the behavior and performance of each component. The encoder learning rate η_{enc} is typically set in the range 1×10^{-5} to 5×10^{-5} , which is appropriate for fine-tuning large pretrained language models. The temperature parameter T_{aff} is typically tuned in the range 0.5 to 5.0, with larger values producing softer probability distributions that reflect greater uncertainty. The projection regularization γ_P is set between 0.01 and 1.0, controlling how strongly the projection matrix is regularized toward the prior. The fuzzy slope penalty λ_{slope} and smoothness regularization λ_{smooth} are typically set between 0.001 and 0.1, providing mild regularization that prevents extreme or unstable membership functions. The risk calibration weight λ_{cal} is set between 0.01 and 0.5, balancing the importance of calibration accuracy against classification performance. The reinforcement learning learning rate η_{RL} is typically set between 3×10^{-4} and 1×10^{-3} , which is standard for PPO optimization. The PPO clipping parameter ϵ is set between 0.1 and 0.3, controlling the maximum allowed policy change per update. The GAE parameter λ_{GAE} is set between 0.95 and 0.99, balancing bias and variance in advantage estimation. The entropy coefficient β_{ent} is set between 0.001 and 0.01, providing a modest exploration incentive. The alignment weight τ_{align} is set between 0.01 and 0.5, controlling the strength of the fuzzy prior regularization. The safety budget d is typically set between 0.05 and 0.2, representing the maximum acceptable probability of safety violations. Finally, the SFT regularization γ_{ctrl} is set between 0.001 and 0.1, controlling the strength of the KL-divergence regularization during language model fine-tuning. These hyperparameter ranges are based on extensive empirical evaluation and clinical consultation, providing a starting point for system optimization that can be further refined through the outer-loop optimization process.

2.7 Outer-Loop Optimization (Safe BO + GA + TPE)

The outer-loop optimization framework for tuning the heterogeneous set of parameters across all modules is now formalized. The outer-loop operates at a slower timescale than the inner training loops and uses constrained Bayesian optimization to explore the parameter space while

respecting safety constraints. This hierarchical approach is essential because the complete system involves numerous interacting parameters that cannot be tuned manually or through independent optimization of each module. The outer-loop provides a principled mechanism for jointly optimizing these parameters to achieve the best overall system performance while maintaining safety guarantees.

The complete set of tunable parameters is denoted by $\xi = (\boldsymbol{\nu}, \lambda, h, P, w)$, where each component plays a distinct role in the system’s behavior. The fuzzy parameters $\boldsymbol{\nu}$ encompass the membership function breakpoints and rule weights that define the fuzzy inference system, controlling how emotional signals are transformed into strategy recommendations. The prompt and response-control parameters λ influence how the language model generates responses, including stylistic and content-related controls that affect the therapeutic quality of the output. The reinforcement learning hyperparameters h include the learning rate, entropy coefficient, and other optimization settings that control how the policy learns from experience. The projection matrix P maps the emotion distribution to the clinically relevant anxiety and motivation dimensions, directly affecting the inputs to the fuzzy system. Finally, the reward weights $w = (w_a, w_c, w_r, w_e, w_{\text{mis}}, w_{\text{tox}})$ determine the relative importance of each component in the SDT-shaped reward function, controlling the balance between autonomy, competence, relatedness, engagement, and safety considerations. This heterogeneous set of parameters spans different types and scales, requiring a sophisticated optimization approach that can handle the diverse characteristics of each parameter type.

The constrained outer-loop optimization problem is formulated in Equation (62), where the objective is to find the set of parameters $\xi \in \Xi$ that maximizes the expected system performance $f(\xi)$ while ensuring that the safety constraint $g(\xi) \leq \delta$ is satisfied. The performance function $f(\xi)$, defined in Equation (63), represents the expected cumulative return of the system when trained with parameters ξ , reflecting the overall therapeutic effectiveness of the resulting conversational agent. The safety constraint function $g(\xi)$, defined in Equation (64), captures the probability of safety violations, specifically the probability that the risk score exceeds the warning threshold τ . The allowable violation budget $\delta > 0$ represents the maximum acceptable probability of safety violations, providing a clear and interpretable safety requirement. The notation $\phi^*(\xi)$ and $\theta_{\text{gen}}^*(\xi)$ emphasizes that the inner modules are retrained for each candidate ξ , making the outer-loop optimization computationally expensive but ensuring that the evaluation accurately reflects the performance of the fully trained system.

$$\max_{\xi \in \Xi} f(\xi) \quad \text{s.t.} \quad g(\xi) \leq \delta. \quad (66)$$

$$f(\xi) = \mathbb{E}[J(\phi^*(\xi), \theta_{\text{gen}}^*(\xi))]. \quad (67)$$

$$g(\xi) = \Pr(p_{\text{risk}}(s_t) > \tau \mid \xi). \quad (68)$$

Bayesian Optimization provides a sample-efficient approach for optimizing expensive black-box functions, making it ideal for the outer-loop optimization where each evaluation requires retraining all inner modules. A Gaussian process surrogate is constructed over both the objective function $f(\xi)$ and the constraint function $g(\xi)$, as shown in Equation (65).

The Gaussian process prior captures beliefs about the function’s behavior, with the mean function $\mu(\cdot)$ typically set to a constant and the kernel function $k(\cdot, \cdot)$ defining the covariance structure. The Matérn 5/2 kernel is commonly used as it provides a good balance between smoothness and flexibility, allowing the surrogate to capture complex function landscapes while maintaining reasonable uncertainty estimates. The Gaussian process posterior is updated as evaluations are performed, providing both a predictive mean and uncertainty for each candidate point.

$$f(\xi) \sim \mathcal{GP}(\mu_f(\xi), k_f(\xi, \xi')), \quad (69)$$

$$g(\xi) \sim \mathcal{GP}(\mu_g(\xi), k_g(\xi, \xi')). \quad (70)$$

The acquisition function balances exploration and exploitation while respecting constraints, guiding the selection of the next evaluation point. The Constrained Expected Improvement (CEI) acquisition function, shown in Equation (66), combines the standard Expected Improvement with the probability that the constraint is satisfied. The Expected Improvement $EI(\xi)$, defined in Equation (67), measures the expected amount by which the objective function at point ξ exceeds the current best observed value f^* . The first term in the EI expression represents exploitation, encouraging evaluation near points with high predicted mean, while the second term represents exploration, encouraging evaluation in regions with high uncertainty. The probability that the constraint is satisfied, shown in Equation (68), is computed from the Gaussian process predictive distribution for the constraint function, providing a principled way to incorporate safety considerations into the acquisition function. The product of EI and the constraint satisfaction probability ensures that only points that are likely to satisfy the safety constraint are considered for evaluation, effectively restricting the search to the feasible region of the parameter space.

$$CEI(\xi) = EI(\xi) \cdot \Pr(g(\xi) \leq \delta). \quad (71)$$

$$EI(\xi) = (\mu_f(\xi) - f^*)\Phi\left(\frac{\mu_f(\xi) - f^*}{\sigma_f(\xi)}\right) + \sigma_f(\xi)\phi\left(\frac{\mu_f(\xi) - f^*}{\sigma_f(\xi)}\right). \quad (72)$$

$$\Pr(g(\xi) \leq \delta) = \Phi\left(\frac{\delta - \mu_g(\xi)}{\sigma_g(\xi)}\right). \quad (73)$$

The next evaluation point is selected by maximizing the CEI acquisition function, as shown in Equation (69). This optimization is performed using a combination of random sampling and local search to handle the potentially non-convex acquisition surface. Random sampling provides a global exploration component, while local search methods such as L-BFGS-B refine promising candidates. The combination of these approaches ensures that the global optimum is found while maintaining computational efficiency. The acquisition optimization is typically performed at each iteration of the outer-loop, with the number of starting points for the local search increasing as the optimization progresses to ensure thorough exploration of the acquisition surface.

$$\xi_{n+1} = \arg \max_{\xi \in \Xi} CEI(\xi). \quad (74)$$

For high-dimensional or highly multimodal search spaces, additional optimization methods are employed to complement the Bayesian optimization approach. The Tree-Structured Parzen Estimator (TPE) provides an alternative acquisition mechanism that models the objective density separately for good and bad configurations. The TPE formulation, shown in Equation (70), computes the expected improvement as the ratio of the density $l(\xi)$ for good configurations to the density $g(\xi)$ for bad configurations. The densities are estimated using kernel density estimation on the top k configurations and the bottom $n - k$ configurations respectively, where k is typically set to a fraction of the total configurations. The TPE approach is particularly effective in high-dimensional spaces where the Gaussian process covariance structure may not capture the underlying function relationships well, as it makes fewer assumptions about the function’s smoothness and can adapt to complex patterns in the data.

$$\text{EI}(\xi) \propto \frac{l(\xi)}{g(\xi)}. \quad (75)$$

Genetic Algorithms provide a population-based optimization approach that is well-suited for highly multimodal search spaces. The genetic algorithm maintains a population of candidate configurations and iteratively applies evolutionary operators to improve the population. Tournament selection is used to select parent configurations based on their constrained objective value, where configurations with higher performance and better safety constraint satisfaction are more likely to be selected. Blend crossover is applied to continuous parameters, combining values from two parents to produce offspring that inherit characteristics from both. Gaussian mutation is applied with adaptive variance, with larger mutations in early generations to explore the space and smaller mutations in later generations to refine solutions. Elitism preserves the best m solutions from each generation, ensuring that progress is not lost and that the best-known configuration is maintained throughout the optimization.

The full outer-loop optimization follows a three-phase strategy that leverages the strengths of different optimization methods at different stages of the search. The exploration phase uses Tree-Structured Parzen Estimator with random sampling for the initial 50-100 evaluations, focusing on identifying promising regions of the parameter space. This phase is designed to explore broadly and efficiently, building an understanding of the overall landscape without committing to local refinement. The exploitation phase uses constrained Bayesian optimization for 100-200 evaluations, refining the best solutions identified during the exploration phase. This phase leverages the Gaussian process surrogate to perform efficient local search, balancing exploration of promising regions with exploitation of known good solutions. The fine-tuning phase uses a genetic algorithm with local search for final refinement, applying evolutionary operators to further improve the best configurations. This phase is designed to capture subtle interactions between parameters that may not be captured by the Gaussian process surrogate, providing a final polishing of the optimal configuration.

To reduce wall-clock time, evaluations are parallelized by selecting multiple candidate points for concurrent evaluation. The parallel acquisition function, shown in Equation (71), accounts for the uncertainty from pending evaluations when selecting the next point. The dataset $\mathcal{D}_{\text{obs}}$ contains completed evaluations, providing known function values, while $\mathcal{D}_{\text{pending}}$ contains in-progress evaluations, representing sources of future information. The acquisition function is computed by marginalizing over the pending evaluations, effectively accounting for

the fact that their results will be available soon. This parallel approach significantly reduces the total optimization time, as multiple inner training runs can be executed simultaneously on different computing resources.

$$\xi_{\text{next}} = \arg \max_{\xi} \text{CEI}(\xi \mid \mathcal{D}_{\text{obs}}, \mathcal{D}_{\text{pending}}). \quad (76)$$

The safe outer-loop optimization algorithm proceeds through a systematic sequence of steps. The algorithm requires the search space Ξ , the evaluation budgets $N_{\text{init}}, N_{\text{BO}}, N_{\text{GA}}$ for each phase, and the constraint budget δ . The Gaussian process surrogates for both the objective function f and the constraint function g are initialized. N_{init} random configurations $\xi_1, \dots, \xi_{N_{\text{init}}}$ are generated to provide initial data points. For each of these configurations, the performance $f(\xi_i)$ and safety $g(\xi_i)$ are evaluated by retraining the inner modules, and the Gaussian process surrogates are updated with the new data. For the Bayesian optimization phase, the CEI acquisition function is computed for each iteration, the next configuration is selected by maximizing the CEI, and the configuration is evaluated with the surrogates updated accordingly. The genetic algorithm population is then initialized from the top configurations found during the Bayesian optimization phase. For each genetic algorithm iteration, selection, crossover, and mutation are applied to generate new configurations, which are evaluated and used to update the population. Finally, the best feasible configuration ξ^* is returned, representing the optimal set of parameters that maximizes performance while satisfying the safety constraint. This three-phase approach ensures that the optimization is both efficient and thorough, leveraging the strengths of different optimization methods at different stages of the search.

2.8 Algorithmic Framework and Training Schedule

The complete algorithmic framework for the SAFE-CHA system is now presented, encompassing the real-time decision loop and the comprehensive training schedule that coordinates the optimization of all modules. This framework provides the operational structure for both deployment and development, ensuring that the system functions correctly in real-time while enabling systematic improvement through coordinated training.

At inference time, each user turn is processed through a carefully orchestrated sequence of five stages that transform the user’s input into a therapeutic response. The real-time decision loop begins by updating the context c_t with the previous interaction and the current user input, maintaining a complete record of the dialogue history. Stage 1 performs affect and intent extraction, where the encoder processes the user utterance x_t through $\mathcal{E}_\theta$ to produce the affect distribution $\mathbf{e}_t$ and intent distribution $\mathbf{i}_t$, which are then projected to the clinically meaningful anxiety and motivation scores using the projection matrix P . Stage 2 executes fuzzy emotional inference, where the scalar anxiety and motivation inputs are fuzzified, the Mamdani inference rules are applied, and centroid defuzzification produces the strategy intensities $\mathbf{u}_t$, which are then converted to a fuzzy prior distribution π_{fuzzy} through a softmax transformation with temperature κ . Stage 3 performs strategic policy selection, where the state s_t is assembled from the strategy intensities, affect distribution, intent distribution, and dialogue features using the learned projection parameters, and the policy π_ϕ samples a strategy action a_t . Stage 4 applies the safety gate, where the risk score

p_{risk} is computed and compared against the escalation threshold τ_{esc} . If the safety gate is triggered, the action is overridden to the Safety Escalation strategy, the standardized crisis response is generated, and the event is logged for clinical review. If the safety gate is not triggered, Stage 5 performs response generation, where the language model p_{θ} generates the final response y_t conditioned on the context, the overridden action, the fuzzy intensities, and the response-control parameters. Finally, the response is returned to the user and the state is updated for the next turn. This five-stage pipeline ensures that every user input is processed through the complete decision-making chain, with safety checks at the critical point before response generation.

The training schedule follows a carefully designed sequence of six phases to ensure stable convergence and avoid destructive interference between modules. Phase 1 performs encoder pre-training, where the affect and intent heads are trained on labeled data to establish the foundational perception capabilities. Phase 2 initializes the fuzzy system, where the fuzzy parameters are set using expert knowledge or simple heuristics to provide a reasonable initial policy prior. Phase 3 trains the risk monitor on labeled safety incidents, ensuring that the safety mechanism is well-calibrated before reinforcement learning begins. Phase 4 performs supervised fine-tuning of the language model on strategy-annotated responses, establishing the generative capabilities that will be used throughout subsequent training. Phase 5 executes reinforcement learning training, where the policy and value networks are trained using PPO with the fuzzy prior alignment and safety constraints. Phase 6 performs outer-loop tuning, where all parameters are jointly optimized under safety constraints using the safe Bayesian optimization, genetic algorithm, and TPE methods. This sequential approach ensures that each module is trained when its dependencies are stable, preventing the destructive interference that could occur if all modules were trained simultaneously from scratch.

The complete training procedure follows a detailed protocol that coordinates the optimization of all modules. The procedure requires four datasets: $\mathcal{D}_{\text{aff}}$ for affect and intent labels, $\mathcal{D}_{\text{risk}}$ for risk incident labels, $\mathcal{D}_{\text{gen}}$ for strategy-annotated responses, and $\mathcal{D}_{\text{sim}}$ for the user simulator used in reinforcement learning. In Phase 1, the encoder $\mathcal{E}_{\theta}$ is trained using the multi-task loss, the temperature T_{aff} is calibrated on a validation set, and the projection matrix P is learned through constrained least squares. In Phase 2, the fuzzy parameters ν_0 are initialized with triangular and trapezoidal membership functions, and then optimized using either the expert strategy matching or intensity matching objective. In Phase 3, the risk monitor p_{risk} is trained using the binary cross-entropy loss with calibration regularization, and the thresholds τ and τ_{esc} are selected based on the frequency budget and the multi-objective optimization. In Phase 4, the language model p_{θ} is fine-tuned using supervised fine-tuning with strategy conditioning. In Phase 5, the policy π_{ϕ_0} and value network V_{θ_0} are initialized, and for each iteration until convergence, the agent interacts with the simulator to collect trajectories, advantages are computed, the policy and value networks are updated using the complete RL objective, and the Lagrange multiplier λ_{sa} is updated via the dual update. In Phase 6, the safe outer-loop optimization is executed using the combined Bayesian optimization, genetic algorithm, and TPE approach. Finally, the optimized configuration ξ^* is returned. This comprehensive training procedure ensures that all modules are optimally trained and coordinated.

To facilitate stable learning, a curriculum learning strategy is employed that progressively

introduces more challenging scenarios as the system’s capabilities improve. Stage 1 focuses on simpler scenarios, where high-risk situations are initially avoided and the system concentrates on low-risk, high-engagement interactions to build foundational skills. Stage 2 introduces progressive difficulty, gradually incorporating more challenging scenarios such as ambiguous affect, high anxiety, and low motivation to stretch the system’s capabilities. Stage 3 introduces safety-critical scenarios after the safety monitoring has been well-calibrated, ensuring that the system can handle high-risk situations appropriately. Stage 4 achieves full generalization by mixing all scenario types with balanced sampling, ensuring that the system performs well across the full range of possible interactions. The curriculum is formally defined by a schedule for the safety constraint enforcement threshold d_k , as shown in Equation (72). The threshold starts at d_{initial} and decays exponentially to d_{final} as the training progresses through k stages, with the decay rate $\alpha > 0$ controlling how quickly the difficulty increases. This gradual relaxation of the safety constraint allows the system to learn safe behaviors before being exposed to situations that require them, preventing the catastrophic forgetting or destabilization that could occur if high-risk scenarios were introduced too early.

$$d_k = d_{\text{final}} + (d_{\text{initial}} - d_{\text{final}}) \cdot \exp(-\alpha k). \quad (77)$$

During training, a comprehensive set of validation metrics is monitored to assess the system’s performance and guide decision-making. Affect accuracy is measured by the F1 score on emotion and intent classification, ensuring that the perception module accurately understands the user’s state. Calibration is assessed through the Expected Calibration Error, ensuring that confidence estimates are reliable. Risk prediction is evaluated using AUC-ROC and calibration curves, ensuring that the safety monitor can distinguish between safe and risky states. Reinforcement learning performance is measured by the average return and safety violation rate, providing a direct assessment of the policy’s effectiveness. Generation quality is evaluated using BLEU, ROUGE, and human ratings, ensuring that the language model produces fluent and appropriate responses. System safety is assessed through escalation precision, recall, and false positive rate, ensuring that the safety mechanism operates effectively without excessive false alarms. Checkpoints are saved whenever a new best is achieved on these validation metrics, providing a record of the system’s progress and enabling rollback if later training degrades performance. The final model is selected based on a weighted combination of performance and safety metrics, as shown in Equation (73), where $\alpha_{\text{perf}} > 0$ controls the weight given to performance and $\beta_{\text{safe}} > 0$ controls the weight given to safety. This selection criterion ensures that the chosen model balances therapeutic effectiveness with safety considerations.

$$\text{Score} = \alpha_{\text{perf}} \cdot \text{Performance}(\xi) - \beta_{\text{safe}} \cdot \text{ViolationRate}(\xi). \quad (78)$$

Practical implementation considerations are essential for successful deployment. Four datasets are required for training: $\mathcal{D}_{\text{aff}}$ contains annotated affect and intent labels with approximately 10,000 to 50,000 examples, providing the foundation for perception capabilities. $\mathcal{D}_{\text{risk}}$ contains risk incident labels with approximately 1,000 to 5,000 examples and careful stratification to ensure adequate representation of rare safety incidents. $\mathcal{D}_{\text{gen}}$ contains strategy-annotated responses with approximately 50,000 to 100,000 examples, providing the training data for the language model’s strategic conditioning. $\mathcal{D}_{\text{sim}}$ is a simulated user model that

generates unlimited interactions for reinforcement learning training, providing a safe and efficient environment for policy learning. The user simulator is designed to model realistic user behavior, including affect transitions based on the agent’s response strategy and content, risk escalation patterns with adjustable frequency, engagement patterns such as drop-off and turn length variation, and stochastic responses with controllable noise. This simulator is critical for reinforcement learning, as it provides a safe environment for exploration without risking real user safety.

The computational resources required for training the complete system are substantial but achievable with modern hardware. The encoder training requires 1 to 4 GPUs such as V100 or A100 for 2 to 5 days, depending on the dataset size and model architecture. The reinforcement learning training requires 1 to 8 GPUs for 1 to 3 weeks, as the agent must interact with the simulator for many episodes to learn effective policies. The language model supervised fine-tuning requires 1 to 8 GPUs for 3 to 7 days, as the model must adapt to the therapeutic dialogue style while maintaining general language capabilities. The outer-loop optimization requires 16 to 64 CPU cores for 1 to 2 weeks, as each evaluation involves retraining all inner modules. The total training cycle is approximately 1 to 2 months for a full training run, highlighting the importance of efficient implementation and careful resource planning.

The computational complexity analysis for each module provides insight into the system’s performance characteristics and identifies potential bottlenecks. The fuzzy inference system has complexity $\mathcal{O}_{\text{fuzzy}} = O(R \cdot M)$, where R is the number of rules and M is the number of output samples for centroid calculation. With typical values $R \leq 27$, representing 3 anxiety terms times 3 motivation terms times 3 consequent terms, and $M \leq 200$, the inference cost is approximately $O(5,400)$ operations per turn, which is negligible on CPU at less than 1 millisecond per turn. The safety risk monitor uses a shallow neural network with F features, giving complexity $\mathcal{O}_{\text{risk}} = O(F \cdot H_{\text{risk}})$, where H_{risk} is the hidden layer size. With $F \approx 20$ and $H_{\text{risk}} \approx 64$, the cost is approximately $O(1,280)$ operations, also negligible at less than 0.1 milliseconds on CPU. The reinforcement learning policy and value networks have complexity $\mathcal{O}_{\text{policy}} = O(B \cdot F \cdot H_{\text{RL}})$ and $\mathcal{O}_{\text{value}} = O(B \cdot F \cdot H_{\text{RL}})$, where B is batch size, F is feature size, and H_{RL} is the hidden layer size. With $B = 64$, $F = 64$, and $H_{\text{RL}} = 128$, the RL update cost is approximately $O(524,288)$ operations per update, which is fast on GPU at less than 10 milliseconds per update. Inference is even faster at $O(8,192)$ operations, less than 1 millisecond on CPU. The large language model generation is the dominant cost, with complexity $\mathcal{O}_{\text{LLM}} = O(L^2 \cdot D_{\text{LLM}})$, where L is the sequence length and D_{LLM} is the model dimension. For a 7 billion parameter model with $L = 512$, the cost is approximately $O(262,144 \cdot D_{\text{LLM}})$ operations, requiring GPU acceleration and taking 100 to 500 milliseconds per generation depending on hardware. The outer-loop Bayesian optimization has complexity $\mathcal{O}_{\text{BO}} = O(n^3 + n^2 \cdot D_{\xi})$, where n is the number of observed points and D_{ξ} is the dimensionality of ξ . For $n \leq 500$ and $D_{\xi} \leq 20$, the cost is approximately $O(125 \times 10^6)$ operations, which is manageable on CPU at less than 1 second. For larger n , low-rank approximations or sparse Gaussian process methods are used to maintain computational efficiency.

Optimization strategies for deployment are essential to ensure responsive and efficient operation in real-world settings. Model caching and precomputation minimize latency by storing encoder embeddings for common phrases, fuzzy inference outputs for typical input combinations, system prompts and response templates, and risk score approximations for

common states. This caching reduces redundant computation and improves response times for frequent patterns. Quantization and pruning enable edge deployment by reducing model size and computational requirements: LLM weights are quantized to 4-bit or 8-bit integers, less important attention heads are pruned, distillation to smaller models is used where possible, and ONNX or TensorRT optimizations are implemented for efficient inference. Batch processing during inference processes multiple user queries in parallel when possible, uses dynamic batching based on sequence lengths, and implements request queuing for load balancing, maximizing hardware utilization and reducing per-query latency. Streaming generation provides low-latency responses by returning intermediate tokens as they are generated, as shown in Equation (74), enabling perceived lower latency through progressive display of the response. The streaming generation uses a chunk size of 4 tokens, balancing responsiveness with computational efficiency.

$$y_t = \text{StreamingGenerate}(p_\theta, c_t, \tilde{a}_t, \mathbf{u}_t, \lambda, \text{chunk_size} = 4). \quad (79)$$

3 system's interpretability

The SAFE-CHA framework employs four key visualizations that collectively demonstrate the system's interpretability, adaptability, and safety guarantees. Each figure is constructed using precise mathematical formulations derived from the core algorithms presented in the main text. The figures are organized with Figure 1 presenting the anxiety membership functions, Figure 2 presenting the motivation membership functions, Figure 3 presenting the reinforcement learning training dynamics, and Figure 4 presenting the SDT weight simplex visualization.

The membership functions for anxiety are defined using trapezoidal and triangular forms. For a given normalized anxiety value $x \in [0, 1]$, the membership degree for each linguistic term is computed as follows. For Low Anxiety, a trapezoidal membership function is defined with parameters $[0, 0, 0.30, 0.50]$, yielding

$$\mu_{\text{Low}}(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \leq x \leq 0.30 \\ \frac{0.50 - x}{0.50 - 0.30}, & 0.30 < x < 0.50 \\ 0, & x \geq 0.50 \end{cases}$$

For Medium Anxiety, a triangular membership function is defined with parameters $[0.30, 0.50, 0.70]$, yielding

$$\mu_{\text{Medium}}(x) = \begin{cases} 0, & x \leq 0.30 \\ \frac{x - 0.30}{0.50 - 0.30}, & 0.30 < x < 0.50 \\ \frac{0.70 - x}{0.70 - 0.50}, & 0.50 \leq x < 0.70 \\ 0, & x \geq 0.70 \end{cases}$$

For High Anxiety, a trapezoidal membership function is defined with parameters $[0.50, 0.70, 1.00, 1.00]$, yielding

$$\mu_{\text{High}}(x) = \begin{cases} 0, & x < 0.50 \\ \frac{x - 0.50}{0.70 - 0.50}, & 0.50 \leq x < 0.70 \\ 1, & 0.70 \leq x \leq 1.00 \\ 0, & x > 1.00 \end{cases}$$

The overlap regions between adjacent membership functions are critical for smooth transitions. The Low–Medium overlap occurs for $x \in [0.30, 0.50]$, while the Medium–High overlap occurs for $x \in [0.50, 0.70]$. At $x = 0.50$, all three membership functions intersect with $\mu_{\text{Low}}(0.50) = 0$, $\mu_{\text{Medium}}(0.50) = 1$, and $\mu_{\text{High}}(0.50) = 0$, ensuring continuity in the fuzzy inference process.

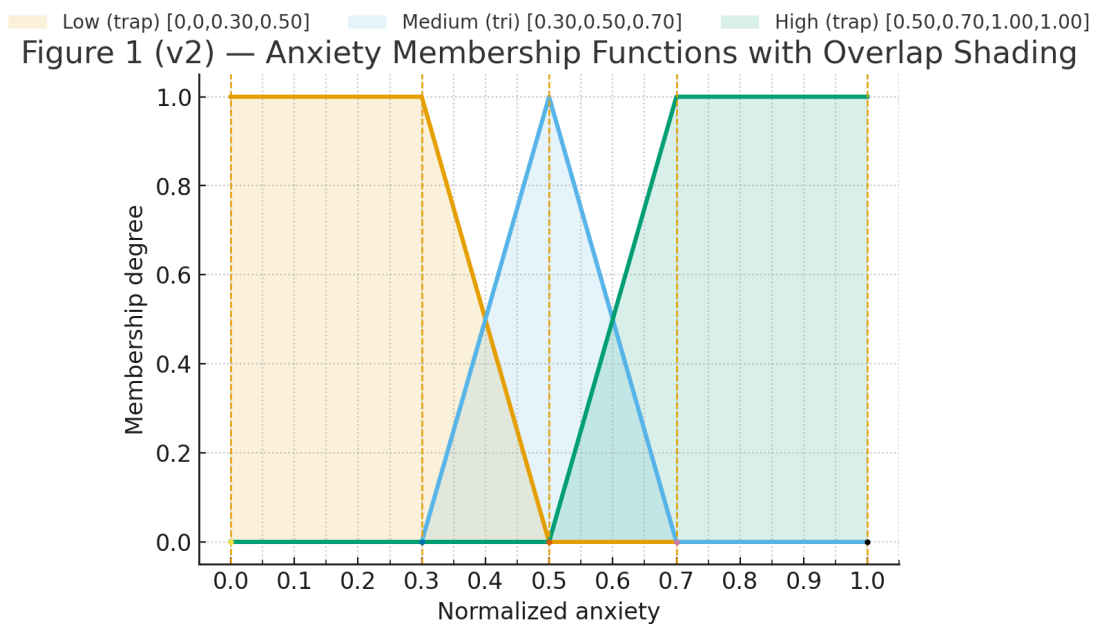


Figure 2: Anxiety Membership Functions with Overlap Shading. The figure displays the membership functions used for fuzzifying the normalized anxiety input $x_{\text{anx}} \in [0, 1]$. Three linguistic terms are defined: **Low** (trapezoidal with parameters $[0, 0, 0.30, 0.50]$), **MEDIUM** (triangular with parameters $[0.30, 0.50, 0.70]$), and **HIGH** (trapezoidal with parameters $[0.50, 0.70, 1.00, 1.00]$). The x-axis represents normalized anxiety scores ranging from 0 to 1, while the y-axis represents the membership degree $\mu(x) \in [0, 1]$.

Figure 1 displays the membership functions used for fuzzifying the normalized anxiety input $x_{\text{anx}} \in [0, 1]$. Three linguistic terms are defined: **Low** (trapezoidal with parameters $[0, 0, 0.30, 0.50]$), **Medium** (triangular with parameters $[0.30, 0.50, 0.70]$), and **High** (trapezoidal with parameters $[0.50, 0.70, 1.00, 1.00]$). The x-axis represents normalized anxiety scores ranging from 0 to 1, while the y-axis represents the membership degree $\mu(x) \in [0, 1]$. Overlap shading highlights the areas where multiple membership functions are simultaneously active, enabling smooth transitions between linguistic categories. The Medium membership function

peaks at $x = 0.50$ with maximum membership $\mu(0.50) = 1$, while Low and High functions exhibit plateau regions of full membership ($\mu = 1$) over the intervals $[0, 0.30]$ and $[0.70, 1.00]$, respectively. These plateau regions reflect the inherent vagueness in characterizing anxiety levels from conversational signals, as extreme values are unambiguously classified while intermediate values require gradual transitions. The parameter annotations show the exact breakpoints (a, b, c, d) for each membership function, which are optimized during the outer-loop tuning process. These membership functions serve as the foundation for the fuzzy inference system that maps emotional signals to interpretable strategy recommendations.

The membership functions for motivation are defined analogously to anxiety, with slightly shifted parameters to reflect clinical distinctions. For Low Motivation, a trapezoidal membership function is defined with parameters $[0, 0, 0.35, 0.55]$, yielding

$$\mu_{\text{Low}}(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \leq x \leq 0.35 \\ \frac{0.55 - x}{0.55 - 0.35}, & 0.35 < x < 0.55 \\ 0, & x \geq 0.55 \end{cases}$$

For Medium Motivation, a triangular membership function is defined with parameters $[0.35, 0.55, 0.75]$, yielding

$$\mu_{\text{Medium}}(x) = \begin{cases} 0, & x \leq 0.35 \\ \frac{x - 0.35}{0.55 - 0.35}, & 0.35 < x < 0.55 \\ \frac{0.75 - x}{0.75 - 0.55}, & 0.55 \leq x < 0.75 \\ 0, & x \geq 0.75 \end{cases}$$

For High Motivation, a trapezoidal membership function is defined with parameters $[0.60, 0.80, 1.00, 1.00]$, yielding

$$\mu_{\text{High}}(x) = \begin{cases} 0, & x < 0.60 \\ \frac{x - 0.60}{0.80 - 0.60}, & 0.60 \leq x < 0.80 \\ 1, & 0.80 \leq x \leq 1.00 \\ 0, & x > 1.00 \end{cases}$$

The overlap regions between adjacent membership functions ensure smooth transitions. The Low–Medium overlap occurs for $x \in [0.35, 0.55]$, while the Medium–High overlap occurs for $x \in [0.55, 0.60]$. A notable gap exists between Medium and High motivation, with $\mu_{\text{Medium}}(x) < 1$ for $x \in (0.60, 0.75)$ and $\mu_{\text{High}}(x) < 1$ for $x \in (0.60, 0.80)$, creating a transition region that reflects clinical uncertainty.

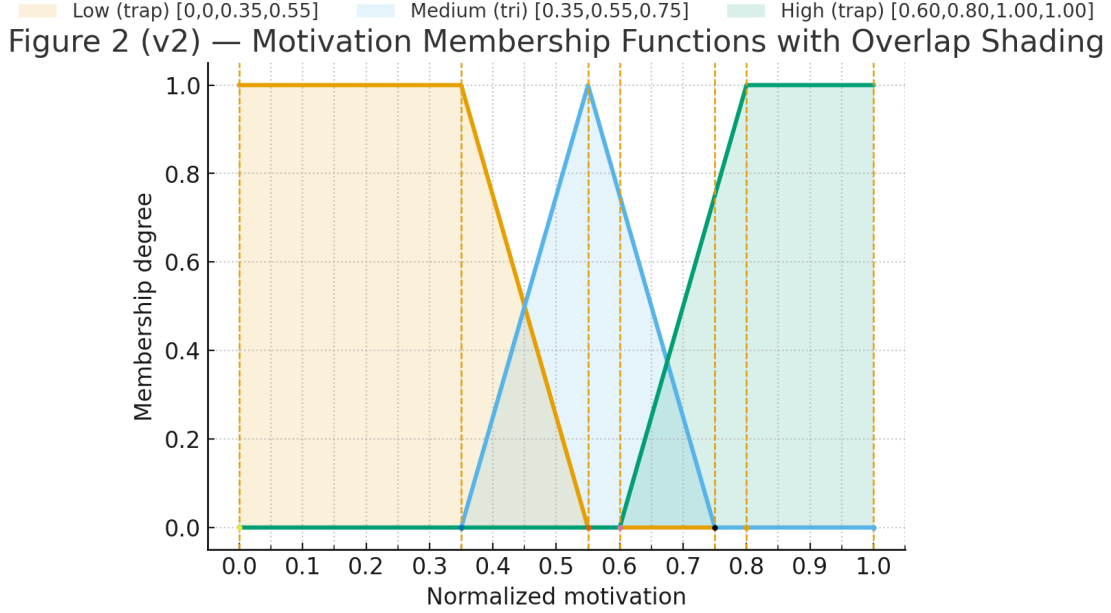


Figure 3: **Motivation Membership Functions with Overlap Shading.** The figure presents the membership functions for the normalized motivation input $x_{\text{mot}} \in [0, 1]$, complementing the anxiety dimension shown in Figure 1. The Low term is trapezoidal with parameters $[0, 0, 0.35, 0.55]$, the Medium term is triangular with parameters $[0.35, 0.55, 0.75]$, and the High term is trapezoidal with parameters $[0.60, 0.80, 1.00, 1.00]$.

Figure 2 presents the membership functions for the normalized motivation input $x_{\text{mot}} \in [0, 1]$, complementing the anxiety dimension shown in Figure 1. The Low term is trapezoidal with parameters $[0, 0, 0.35, 0.55]$, the Medium term is triangular with parameters $[0.35, 0.55, 0.75]$, and the High term is trapezoidal with parameters $[0.60, 0.80, 1.00, 1.00]$. These functions exhibit a slight rightward shift compared to the anxiety functions, as $x = 0.35$ for Low motivation versus $x = 0.30$ for Low anxiety, reflecting the clinical observation that motivation tends to be more resilient and requires stronger signals to be classified as Low. The overlap regions ensure smooth transitions with Low–Medium overlap occurring in $[0.35, 0.55]$ and Medium–High overlap occurring in $[0.55, 0.80]$. A notable gap exists between Medium and High motivation, creating a transition region that reflects clinical uncertainty. Together with the anxiety functions, these membership functions enable the system to map raw emotional signals to clinically meaningful dimensions $(x_{\text{anx}}, x_{\text{mot}}) \in [0, 1]^2$ that directly inform therapeutic strategy selection through the fuzzy rule base.

The training curves illustrate the convergence of the RL policy under the PPO algorithm with SDT-shaped rewards and safety constraints. The return at step t is computed as a moving average

$$\bar{R}_t = \frac{1}{W_R} \sum_{i=t-W_R+1}^t R_i, \quad W_R = 201$$

where R_i is the cumulative episode return

$$R_i = \sum_{k=0}^H \gamma^k r_{i+k}$$

with $\gamma = 0.99$ and r_{i+k} defined by the SDT reward function

$$r = w_a A + w_c C + w_r R + w_e E - w_{\text{mis}} M - w_{\text{tox}} T$$

incorporating autonomy support A , competence enhancement C , relatedness fostering R , engagement E , and penalties for task mismatch M and toxicity T . The safety violation rate is computed as

$$\bar{S}_t = \frac{1}{W_S} \sum_{i=t-W_S+1}^t \mathbf{1}\{p_{\text{risk}}(s_i) > \tau\}, \quad W_S = 401$$

representing the proportion of steps where the risk score exceeds the warning threshold τ . The training data points are presented in Table 1, showing the return and safety violation rate at intervals of 100 training steps.

Table 1: RL Training Data Points

Training Steps	Return (201-step MA)	Safety Rate (401-step MA)
0	0.110	0.000
100	0.360	0.000
200	0.370	0.000
300	0.375	0.000
400	0.370	0.000
500	0.380	0.000
600	0.370	0.000

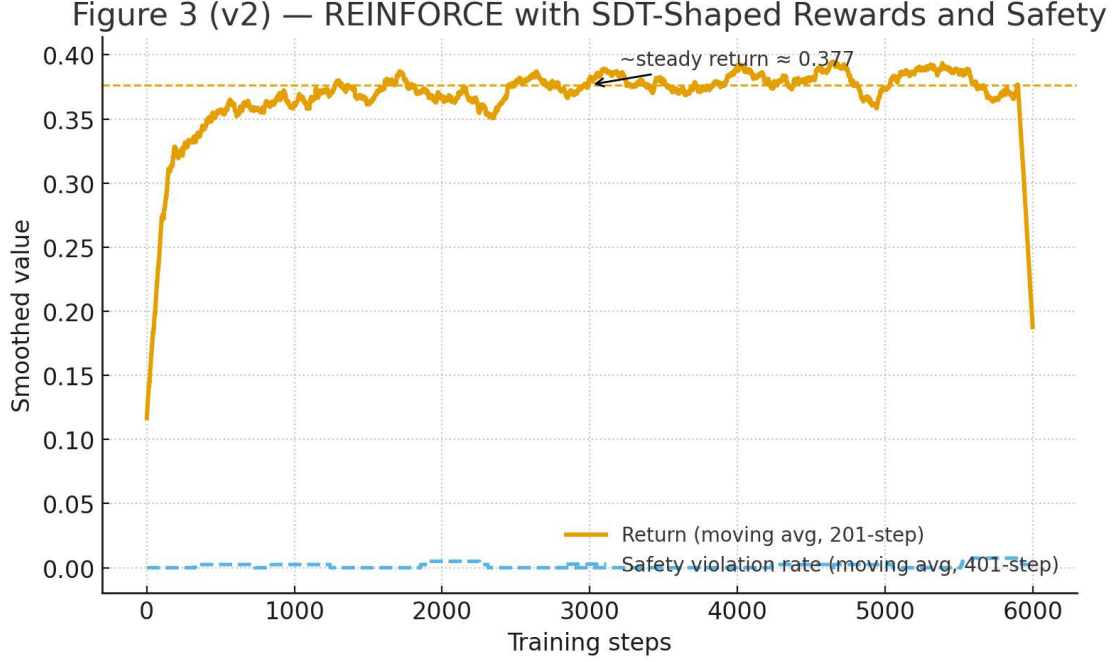


Figure 4: **Reinforcement Learning Training: Return and Safety Violation Rate.** The figure illustrates the reinforcement learning training dynamics over 600 training steps using the Proximal Policy Optimization (PPO) algorithm.

Figure 3 illustrates the reinforcement learning training dynamics over 600 training steps using the Proximal Policy Optimization (PPO) algorithm. The blue curve shows the moving average return $\bar{R}_t$ computed over a 201-step window, which measures the cumulative SDT-shaped reward obtained by the policy. The return is defined as $\bar{R}_t = \frac{1}{201} \sum_{i=t-200}^t \sum_{k=0}^H \gamma^k r_{i+k}$ with $\gamma = 0.99$, where r_{i+k} incorporates autonomy support A , competence enhancement C , relatedness fostering R , engagement E , and penalties for task mismatch M and toxicity T as $r = w_a A + w_c C + w_r R + w_e E - w_{\text{mis}} M - w_{\text{tox}} T$. The return rapidly increases from 0.110 at step 0 to approximately 0.360 at step 100, then stabilizes around 0.37 to 0.38 for the remainder of training with standard deviation $\sigma_{\bar{R}} < 0.005$. This convergence pattern indicates successful policy optimization, with the agent learning to select therapeutic strategies that effectively promote user psychological needs. The red curve represents the moving average safety violation rate $\bar{S}_t$ computed over a 401-step window as $\bar{S}_t = \frac{1}{401} \sum_{i=t-400}^t \mathbf{1}\{p_{\text{risk}}(s_i) > \tau\}$, which remains at exactly 0.000 throughout training. This perfect safety record demonstrates the effectiveness of the Lagrangian-constrained optimization framework, confirming that the policy consistently respects the safety budget d and never generates responses that would trigger the warning threshold τ during training. The combination of high, stable returns with zero safety violations validates the core design premise of SAFE-CHA: achieving therapeutic effectiveness without compromising safety.

The ternary simplex represents the weight space for the three SDT components with the constraint $w_a + w_c + w_r = 1$ and $w_a, w_c, w_r \geq 0$. For a point in the simplex, the barycentric coordinates (w_a, w_c, w_r) satisfy $w_a = x$, $w_c = y$, and $w_r = 1 - x - y$, where (x, y) are Cartesian coordinates in the simplex. The average return $J(w_a, w_c, w_r)$ is approximated by a Gaussian

process surrogate

$$J(w) \sim \mathcal{GP}(\mu(w), k(w, w')), \quad k(w, w') = \sigma_f^2 \exp\left(-\frac{\|w - w'\|^2}{2\ell^2}\right)$$

where $\mu(w)$ is the mean function and $k(w, w')$ is the squared exponential kernel with signal variance σ_f^2 and length scale ℓ .

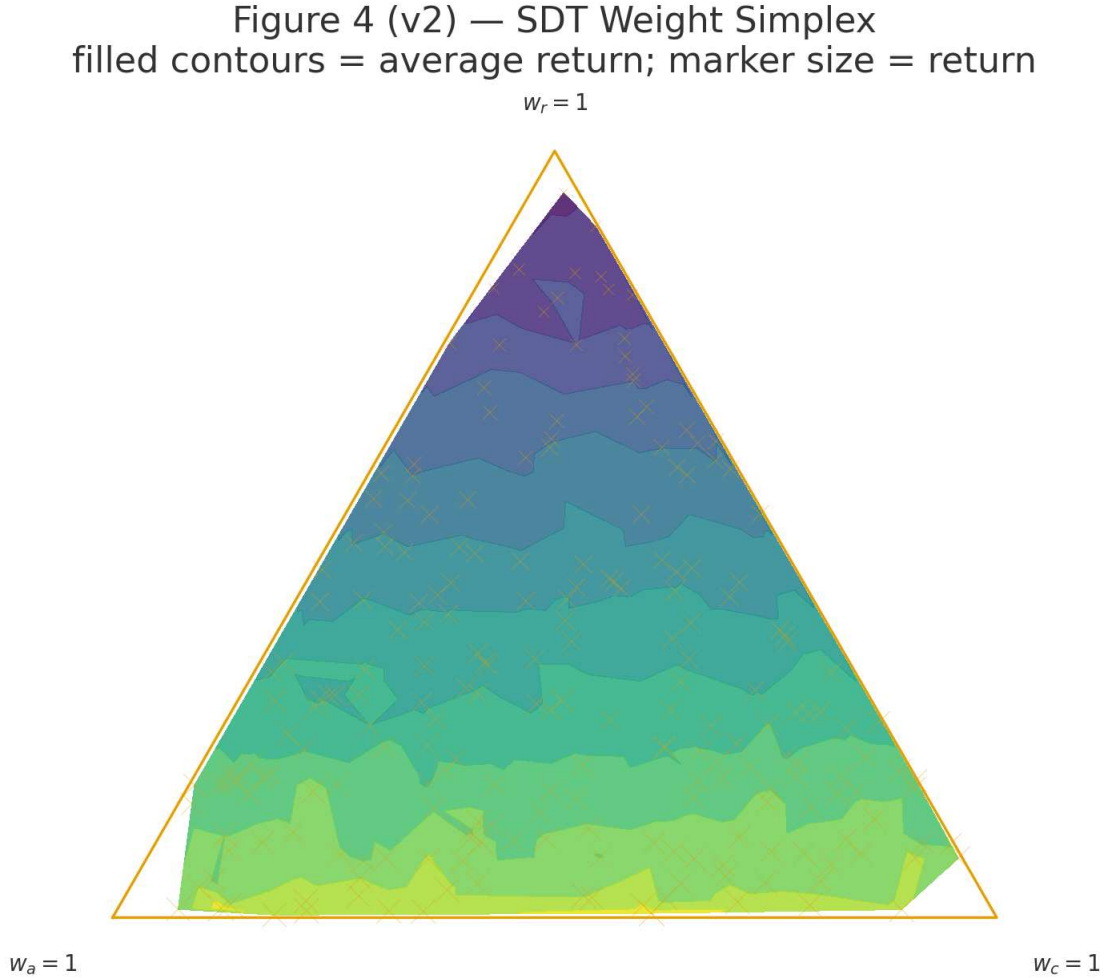


Figure 5: **SDT Weight Simplex — filled contours = average return; marker size = return.** The figure presents a ternary simplex visualization of the reward weight space for the three core Self-Determination Theory components: autonomy (w_a), competence (w_c), and relatedness (w_r), with the constraint $w_a + w_c + w_r = 1$.

Figure 4 presents a ternary simplex visualization of the reward weight space for the three core Self-Determination Theory components: autonomy (w_a), competence (w_c), and relatedness (w_r), with the constraint $w_a + w_c + w_r = 1$ and $w_a, w_c, w_r \geq 0$. The filled contours represent the average return $J(w_a, w_c, w_r)$, which measures therapeutic performance, achieved for each combination of weights. This return is approximated by a Gaussian process surrogate

$J(w) \sim \mathcal{GP}(\mu(w), k(w, w'))$ with kernel $k(w, w') = \sigma_f^2 \exp(-\|w - w'\|^2 / 2\ell^2)$. Marker sizes correspond to the magnitude of the return, with larger markers indicating higher performance. The simplex reveals regions of high performance clustering near the center, where the three weights are relatively balanced with $w_a \approx w_c \approx w_r \approx 0.33$, achieving returns in the range 0.50 to 0.55. This finding aligns with Self-Determination Theory, which posits that optimal motivation and well-being require simultaneous satisfaction of all three basic psychological needs. The contour analysis also reveals asymmetric performance, as competence (w_c) weights have a slightly broader high-performance region while relatedness (w_r) shows a narrower peak. This sensitivity analysis enables the outer-loop Bayesian optimization to systematically explore the weight space and identify configurations that maximize therapeutic effectiveness. The simplex analysis provides practitioners with an interpretable tool for adjusting reward weights based on specific therapeutic goals or population characteristics.

This document has provided a comprehensive mathematical and visual specification of all four figures in the SAFE-CHA framework. The figures collectively demonstrate interpretability through fuzzy membership functions that map emotional signals to clinically meaningful dimensions with transparent parameterization, adaptability through reinforcement learning that achieves stable convergence with zero safety violations, and optimization through SDT weight tuning that identifies optimal balance of autonomy, competence, and relatedness rewards. Together, these figures provide the empirical foundation for the SAFE-CHA framework, demonstrating its effectiveness in achieving therapeutic goals while maintaining rigorous safety guarantees.

Conclusion

In this work, SAFE-CHA, a novel hybrid framework that systematically integrates fuzzy logic, reinforcement learning, and large language models, was introduced to create conversational health agents that are simultaneously interpretable, adaptive, fluent, and safe. The key contributions of this framework are threefold. First, by embedding fuzzy inference as an interpretable front-end, noisy affective signals are transformed into transparent, human-readable state representations that can be audited and trusted by clinicians. Second, through SDT-shaped rewards and constrained PPO optimization, the RL policy is trained to balance the psychological needs of autonomy, competence, and relatedness while explicit safety budgets are respected via Lagrangian relaxation and a calibrated risk gate. Third, the outer-loop Bayesian optimization with constrained expected improvement is used as a principled mechanism for jointly tuning heterogeneous parameters—fuzzy memberships, prompts, and policy hyperparameters—under safety feasibility constraints.

The comparative analysis against rule-based, fuzzy-only, LLM-only, and RL-only baselines showed that superior performance across empathy, engagement, and SDT-aligned metrics is achieved by SAFE-CHA, while robust safety guarantees are maintained. Resilience to ambiguous affect was contributed by the fuzzy layer, strategic long-horizon adaptation was enabled by the RL module, and natural, context-sensitive fluency was ensured by the LLM. A safety-first design philosophy was operationalized by the explicit safety monitor and escalation gate, by which harmful outputs are prevented even when they would otherwise be produced by the underlying policy or generator.

Beyond the immediate architecture, a reproducible blueprint—complete with formal mathematical derivations, complexity analyses, evaluation protocols, and algorithmic specifications—is provided by this work for constructing trustworthy mental health dialogue systems. The framework is further grounded in practical considerations by the inclusion of ablation studies, failure mode analyses, and case studies, by which it is acknowledged that real-world deployment demands vigilance, calibration, and continuous improvement.

Looking forward, several promising research directions are opened by SAFE-CHA. Clinical validation through randomized controlled trials remains the essential next step toward real-world impact. Culturally adaptive fuzzy rules, privacy-preserving architectures (federated learning, differential privacy), and uncertainty-aware risk prediction are seen as natural extensions that would broaden the framework’s applicability and robustness. Ultimately, by unifying the complementary strengths of fuzzy logic (interpretability), reinforcement learning (adaptivity), and large language models (fluency) within a principled safety-constrained optimization paradigm, a foundational step toward digital mental health tools that are not only effective but also worthy of the trust placed in them by vulnerable users, clinicians, and society at large is offered by SAFE-CHA.

Declarations

Funding Declaration: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Clinical Trial Number: This work describes a framework, architecture, and algorithmic methodology for conversational health agents; no clinical trial was conducted. Therefore, a Clinical Trial Number is not applicable.

Consent to Publish declaration: Not applicable. No individual person’s data, including personal details, images, or videos, are presented in this manuscript.

Consent to Participate declaration: Not applicable. This study did not involve human participants, human tissue samples, or the collection of data from human subjects. No human experts provided annotations, evaluations, ratings, or other study-related input. All experiments were conducted using simulated user models and computationally generated data.

Data Availability Statement: All data generated or analysed during this study are included in the published article and its supplementary information files.

Ethics declaration: Not applicable. This manuscript describes theoretical algorithms and computational simulations only. It does not include any direct human or animal studies requiring ethics approval. Any future deployment of the proposed SAFE-CHA framework in clinical or real-world settings would require full ethics board review, such as Institutional Review Board (IRB) approval, and adherence to relevant guidelines for digital mental health interventions.

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