

Supplementary Materials for

**Broadband polarization-agnostic dispersion flattening with gradient epsilon-
and-mu-near-zero photonic structures**

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Methods

S.1 Molecular beam epitaxy

The InAs nano-layers with spatially varying n-type (Si) doping concentrations were synthesized using a solid-source VEECO Gen-930 molecular beam epitaxy (MBE) system. The chamber is equipped with a valve-controlled arsenic cracker supplying As₂ flux, and a 400 cc SUMO indium effusion cell operated at elevated tip temperature to suppress defect formation associated with indium spitting. All InAs layers were epitaxially grown on epi-ready, semi-insulating GaAs (100) substrates. Single-side polished GaAs wafers were employed to avoid Fabry–Pérot interference effects that can arise in double-side polished substrates. Prior to InAs growth, native oxides were thermally desorbed from the GaAs (100) substrates under an As₂ overpressure of 5×10^{-6} Torr at a substrate temperature of 600 °C. A 50 nm GaAs buffer layer was subsequently deposited at a growth rate of 0.33 ML s⁻¹, during which the substrate temperature was reduced to 580 °C. Following buffer growth, the substrate temperature was raised back to 600 °C and held for 10 minutes under continuous As₂ flux to promote surface smoothing. For the deposition of the InAs nano-layers, the substrate temperature was lowered to 410 °C. The InAs growth rate was fixed at 0.5 ML s⁻¹, while the As/In flux ratio was maintained in the range of 1.2–1.5. Upon completion of the graded InAs film, the As₂ flux was preserved as the substrate was cooled to 325 °C. The arsenic flux was then terminated, and the sample was removed from the growth chamber at approximately 275 °C. Reflection high-energy electron diffraction (RHEED) was employed throughout the growth process to calibrate GaAs and InAs deposition rates and to monitor the As/In flux ratio.

S.2 C_{4v}-symmetric photonic-crystal (PC) slab fabrication

Following the growth of the gradient InAs multilayer stack, a C_{4v}-symmetric photonic crystal (PC) slab was fabricated on top of the structure to enable polarization-neutral coupling to the underlying polaritonic resonances. The photonic crystal geometry was first designed using photonic band-structure calculations to position symmetry-protected bound states in the continuum (BICs near the Γ point) at the targeted wavelength close to the lower edge of the gradient EMNZ resonance band. For fabrication, a photoresist was spin-coated directly onto the surface of the gradient InAs layer. The designed C_{4v}-symmetric photonic crystal pattern was then defined using a direct laser writing lithography system, exposing the lattice geometry with the periodicity and fill factor prescribed by the band-structure optimization. After development of the exposed resist, a

germanium (Ge) layer with a thickness of 1 μm was deposited using electron-beam evaporation. Finally, a lift-off process was carried out to remove the remaining resist and define the final photonic crystal slab geometry atop the gradient InAs stack.

S.3 Effective Bloch parameter retrieval

The effective optical parameters of the photonic crystal were retrieved from the complex specular (0,0) scattering coefficients obtained from RCWA simulations at the Γ point. The specular reflection response of the structure was described by the diagonal scattering parameters S_{11} and S_{22} , corresponding to reflection for illumination from the top and bottom interfaces, respectively. The transmission response was captured by the off-diagonal scattering parameters S_{21} and S_{12} , which were power-wave normalized to account for unequal port impedances arising from air on the top side and the wavelength-dependent substrate refractive index on the bottom side.

The characteristic impedances of the two ports were defined as

$$Z_1 = \frac{\eta_0}{n_{\text{top}}}, Z_2 = \frac{\eta_0}{n_{\text{sub}}},$$

where η_0 is the free-space impedance.

The resulting two-port scattering matrix was converted to the corresponding ABCD transfer matrix (A, B, C, D) using standard relations for unequal port impedances. The complex Bloch wavevector was then extracted from the transfer-matrix elements according to

$$\cos(k_B d) = \frac{A + D}{2},$$

where k_B is the complex Bloch wavevector and d denotes the effective thickness assigned to the homogenized slab. A continuous complex branch was selected to ensure spectral continuity of k_B .

The effective refractive index was calculated as

$$n_{\text{eff}} = \frac{k_B}{k_0}, k_0 = \frac{2\pi}{\lambda}.$$

The Bloch (effective) impedance was obtained from the ABCD matrix as

$$Z_B = \sqrt{\frac{B}{C}},$$

with the sign chosen to enforce physical consistency and continuity.

Finally, the effective permittivity and permeability were computed from the retrieved effective index and impedance as

$$\epsilon_{\text{eff}} = \frac{n_{\text{eff}}}{Z_B/\eta_0}, \mu_{\text{eff}} = n_{\text{eff}} \left(\frac{Z_B}{\eta_0} \right).$$

This retrieval procedure was applied across the full operational bandwidth for both TM- and TE-polarized illumination, yielding the spectrally resolved effective refractive index, permittivity, and permeability shown in Fig. S2.

S.4 Photonic band-structure calculation

Photonic band-structure calculations were performed using a photonic eigenfrequency solver to design and position symmetry-protected bound states in the continuum (BICs) supported by the C_{4v} -symmetric photonic crystal slab. The calculations were carried out on a single primitive unit cell of the photonic crystal, exploiting Bloch's theorem to represent the infinite periodic structure. The in-plane real-space lattice vectors of the square photonic crystal were defined in component form as

$$\mathbf{a}_1 = (a_{1x}, a_{1y}), \mathbf{a}_2 = (a_{2x}, a_{2y}),$$

with

$$a_{1x} = a, a_{1y} = 0, a_{2x} = 0, a_{2y} = a,$$

where a is the lattice constant of the photonic crystal.

The corresponding reciprocal lattice vectors were defined consistently as

$$\mathbf{b}_1 = (b_{1x}, b_{1y}), \mathbf{b}_2 = (b_{2x}, b_{2y}),$$

with

$$b_{1x} = \frac{2\pi a_{2y}}{a_{1x}a_{2y} - a_{1y}a_{2x}}, b_{1y} = -\frac{2\pi a_{2x}}{a_{1x}a_{2y} - a_{1y}a_{2x}},$$

$$b_{2x} = -\frac{2\pi a_{1y}}{a_{1x}a_{2y} - a_{1y}a_{2x}}, b_{2y} = \frac{2\pi a_{1x}}{a_{1x}a_{2y} - a_{1y}a_{2x}},$$

which satisfy the standard reciprocal-lattice condition $\mathbf{a}_i \cdot \mathbf{b}_j = 2\pi\delta_{ij}$.

Bloch-periodic boundary conditions were applied along the in-plane directions according to Bloch's theorem,

$$\mathbf{E}(\mathbf{r} + \mathbf{a}_i) = \mathbf{E}(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{a}_i},$$

where $\mathbf{k} = (k_x, k_y)$ is the in-plane Bloch wavevector.

The Bloch wavevector components were parameterized as fractions of the Brillouin-zone boundary,

$$k_x = k_{x,N} \frac{\pi}{a}, k_y = k_{y,N} \frac{\pi}{a},$$

and swept over a small region around the Γ point ($k_x = k_y = 0$) using a two-dimensional parametric sweep to resolve the near- Γ dispersion.

At each Bloch wavevector, the eigenfrequency solver was configured to compute a fixed number of eigenmodes, yielding the eigenfrequencies $\omega(\mathbf{k})$ of the photonic crystal slab. The eigenfrequency search was tracked continuously as a function of $\mathbf{k}$ to ensure consistent band indexing. The resulting dispersion relation $\omega(\mathbf{k})$ was reconstructed from the eigenfrequency solutions and used to identify symmetry-protected BICs appearing at the Γ point.

S.5 Momentum-space polarization-state and vortex visualization

To characterize the polarization topology of the symmetry-protected bound states in the continuum (BICs), we computed the polarization states of the specular (0,0) diffraction order in momentum space in the vicinity of the Γ point. The analysis was performed using complex reflection or transmission coefficients obtained from RCWA simulations, depending on the polarization channel under consideration.

A two-dimensional parameter sweep was carried out over a discrete set of in-plane wavevectors (k_x, k_y) surrounding the BIC point. The sampling points were constructed in polar coordinates as concentric rings in momentum space, parameterized by an in-plane radial coordinate ρ and an azimuthal angle ϕ , where

$$k_x = \rho \cos \phi, k_y = \rho \sin \phi,$$

with

$$\rho = \frac{a}{\lambda} \sin \theta.$$

Here, a is the photonic crystal lattice constant, λ is the free-space wavelength, and θ is the polar angle of incidence.

At each sampled wavevector, the complex electric-field components of the zeroth-order diffraction mode, E_x and E_y , were extracted and assembled into a Jones vector,

$$\mathbf{J} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}.$$

The Jones vectors were normalized pointwise to remove amplitude variations across momentum space.

From the normalized Jones vectors, the Stokes parameters were calculated as

$$\begin{aligned} S_0 &= |E_x|^2 + |E_y|^2, \\ S_1 &= |E_x|^2 - |E_y|^2, \\ S_2 &= 2 \operatorname{Re}(E_x E_y^*), \\ S_3 &= -2 \operatorname{Im}(E_x E_y^*). \end{aligned}$$

Only points with negligible circular polarization content, identified by the condition $|S_3|/S_0 < \delta$ (with δ a fixed threshold), were retained, ensuring that the polarization director is well defined.

For these points, the polarization director angle θ_p was computed as

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{S_2}{S_1} \right),$$

which is defined modulo π for linear polarization.

Finally, to visualize the polarization state as a continuous momentum-space map, the complex director field $\exp(i2\theta_p)$ was interpolated over the sampled (k_x, k_y) domain. The polarization angle was then reconstructed as

$$\theta_p = \frac{1}{2} \arg[\exp(i2\theta_p)],$$

yielding the momentum-space polarization director-angle maps. The persistence of a polarization vortex across multiple sampling radii provides direct evidence of a symmetry-protected BIC at the Γ point.

S.6 Optical characterization

The optical response of the fabricated structures was characterized using angle-resolved reflectance measurements. All measurements were performed with a Bruker Invenio-R Fourier-transform infrared (FTIR) spectrometer equipped with a Harrick Seagull variable-angle reflection accessory, allowing systematic control of the polar angle of incidence. Polarization-resolved measurements were carried out for both TM- and TE-polarized illumination by inserting a wire-grid polarizer into the optical path and appropriately orienting it with respect to the plane of incidence. For each measurement condition, reflectance spectra were obtained by first acquiring a background signal from a well-calibrated reference mirror, which served as the reflectance standard. All reflectance data were normalized to this reference to account for instrumental response and to enable direct comparison across different angles and polarization states.

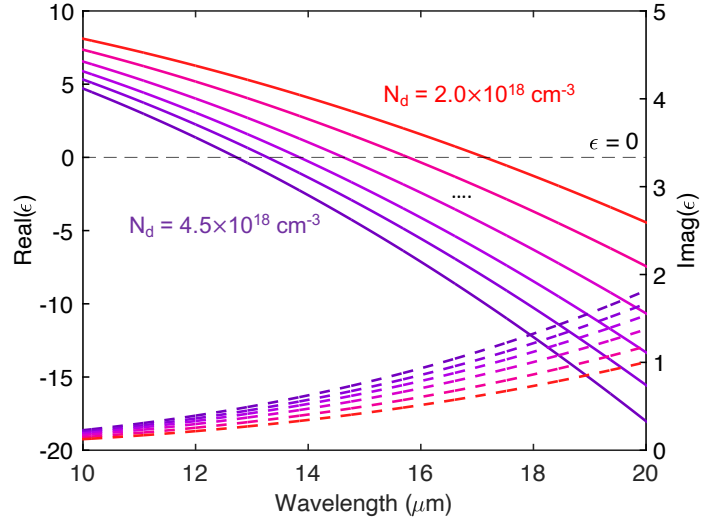


Fig. S1 Doping-dependent permittivity of Si-doped InAs, showing a gradual shift of the epsilon-near-zero (ENZ) crossing in the real part of the permittivity with varying doping concentration, while the imaginary part exhibits only minimal variation across the wavelength range considered.

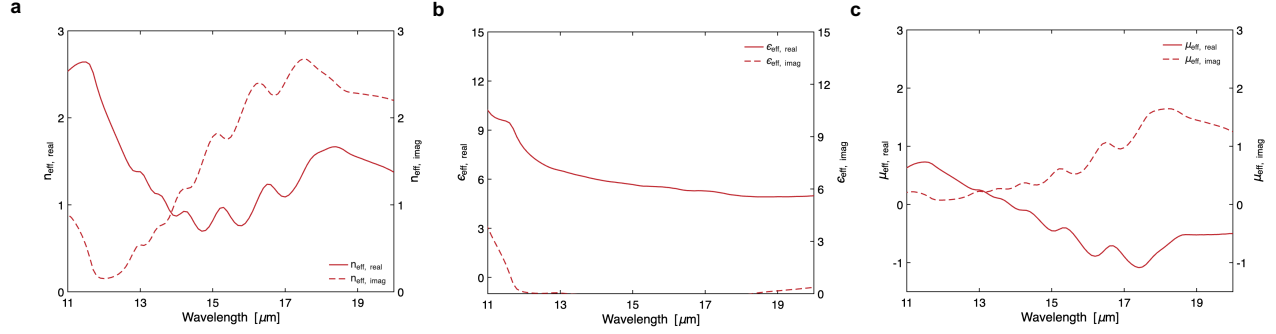


Fig. S2 Bloch parameter retrieval of the gradient EMNZ photonic structure. The retrieved effective refractive index exhibits a spectrally graded near-zero response across the operational bandwidth (a). The effective permittivity shows a dip associated with the gradient ENZ behavior, with inflated values arising from nonlocal effects associated to guided-mode resonances supported by the high-index photonic crystal slab (b). The effective permeability also shows a diminished value consistent with an effective gradient mu-near-zero (MNZ) response, though mediated by a range of non-local effects introduced by the photonic crystal.

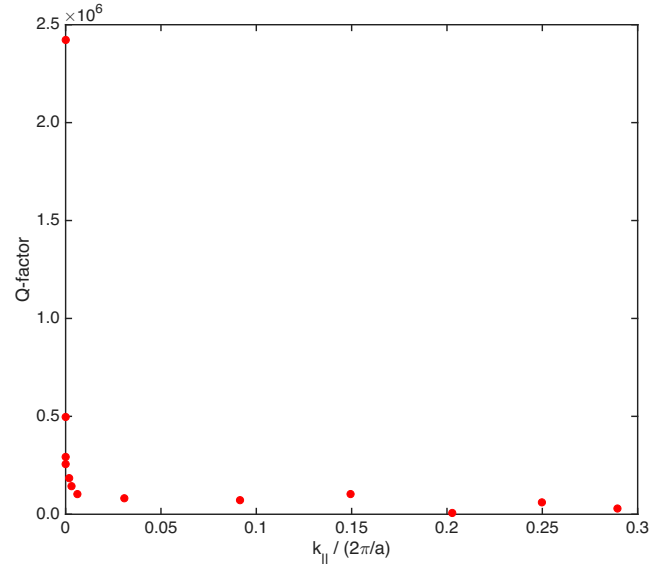


Fig. S3 Q-factor calculations of the designed photonic-crystal (PC) slab diverges to infinity near the Γ -point, a clear signature of a symmetry-protected BIC mode.

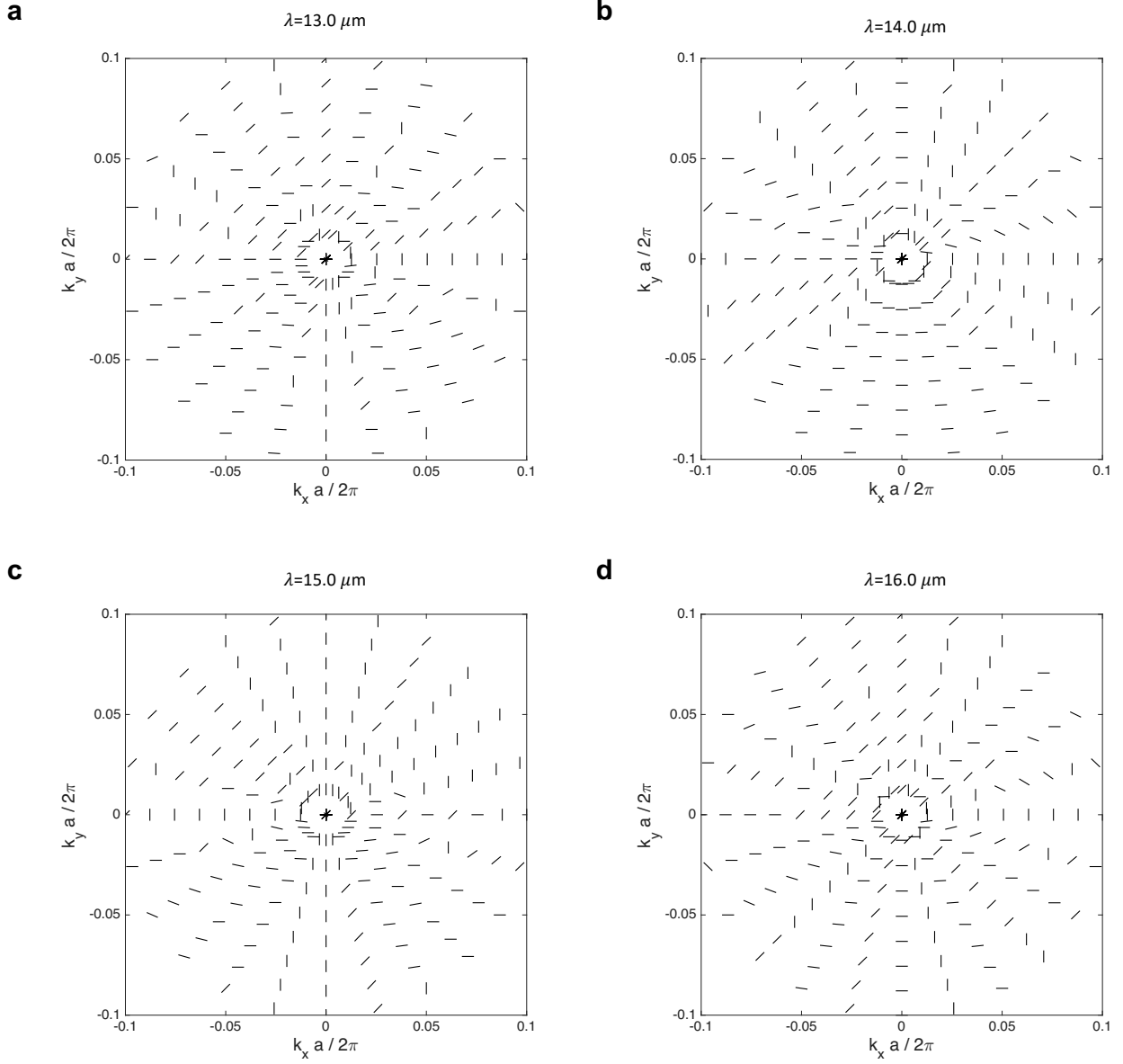


Fig. S4 Momentum-space polarization vortex around the Γ point for the gradient EMNZ photonic crystal at multiple wavelengths across the EMNZ band. The persistence of the polarization singularity confirms symmetry-protected polaritonic BIC behavior and guarantees polarization-agnostic coupling across the broadband EMNZ operational range.

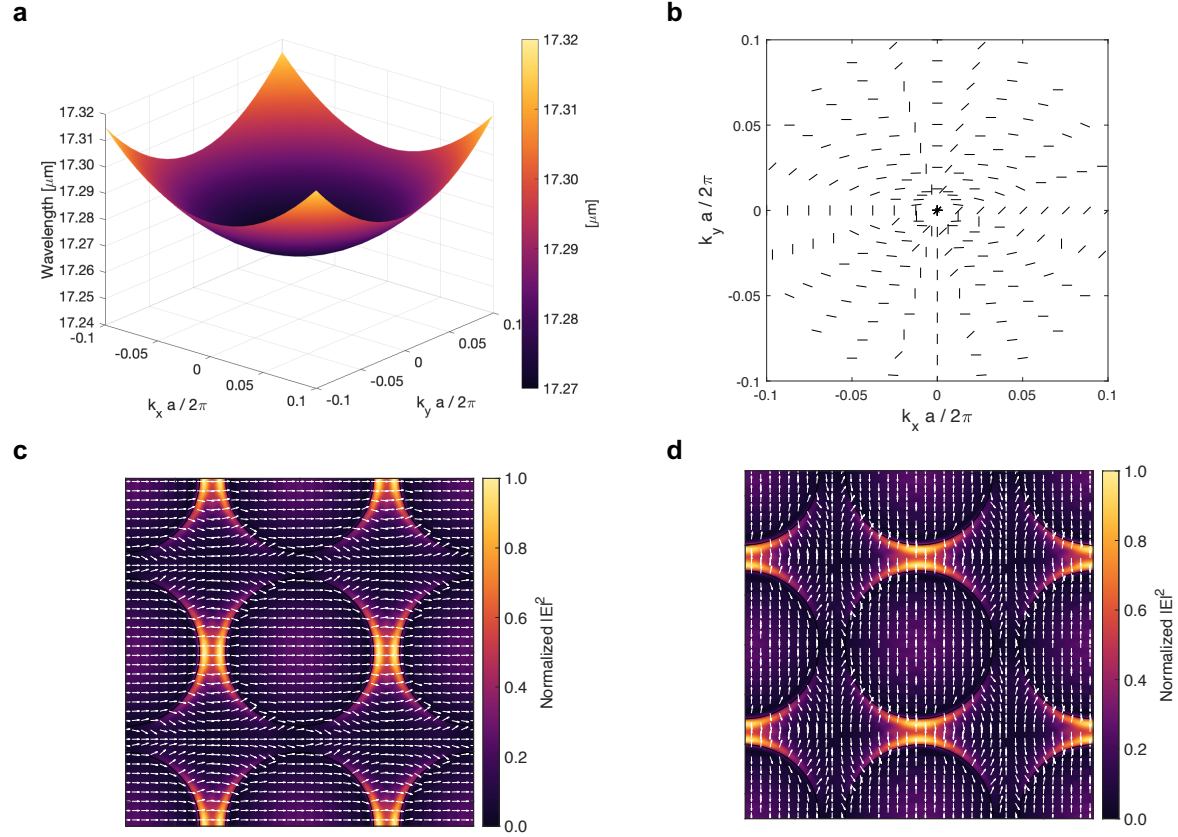


Fig. S5 Photonic crystal design and polaritonic BICs for structure 2. The calculated band structure places a symmetry-protected BIC near 17.4 μm (a), accompanied by a polarization vortex at Γ (b). The corresponding in-plane electric-field intensity distributions for TM and TE polarizations exhibit symmetric profiles (c,d), confirming polarization-degenerate access to the gradient EMNZ polaritonic resonance.

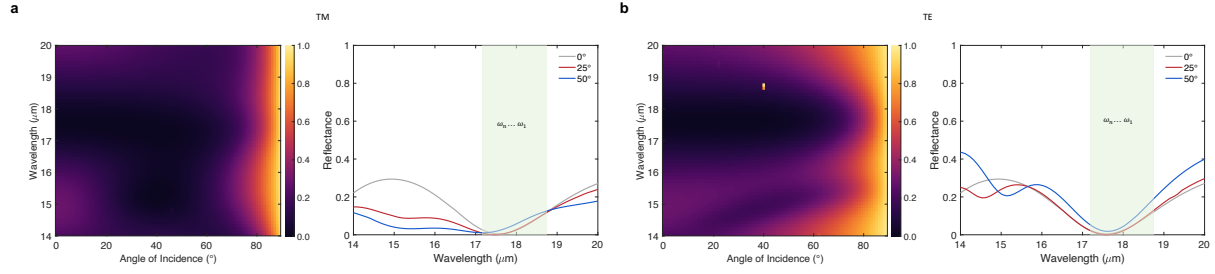


Fig. S6 Experimental polarization-resolved reflectance map for structure 2, demonstrating spectrally complementary (17–19 μm) gradient EMNZ behavior compared to structure 1. (a) Angle–wavelength reflectance map under TM-polarized illumination (left panel) and corresponding spectral line cuts at 0°, 25°, and 50° incidence (right panel). (b) Angle–wavelength reflectance map under TE-polarized illumination (left panel) and corresponding spectral line cuts at 0°, 25°, and 50° incidence (right panel). For both polarizations, pronounced angular invariance is observed within the EMNZ operational band (shaded green region), while spectral features at shorter wavelengths exhibit angular dispersion associated with delocalized photonic crystal modes.

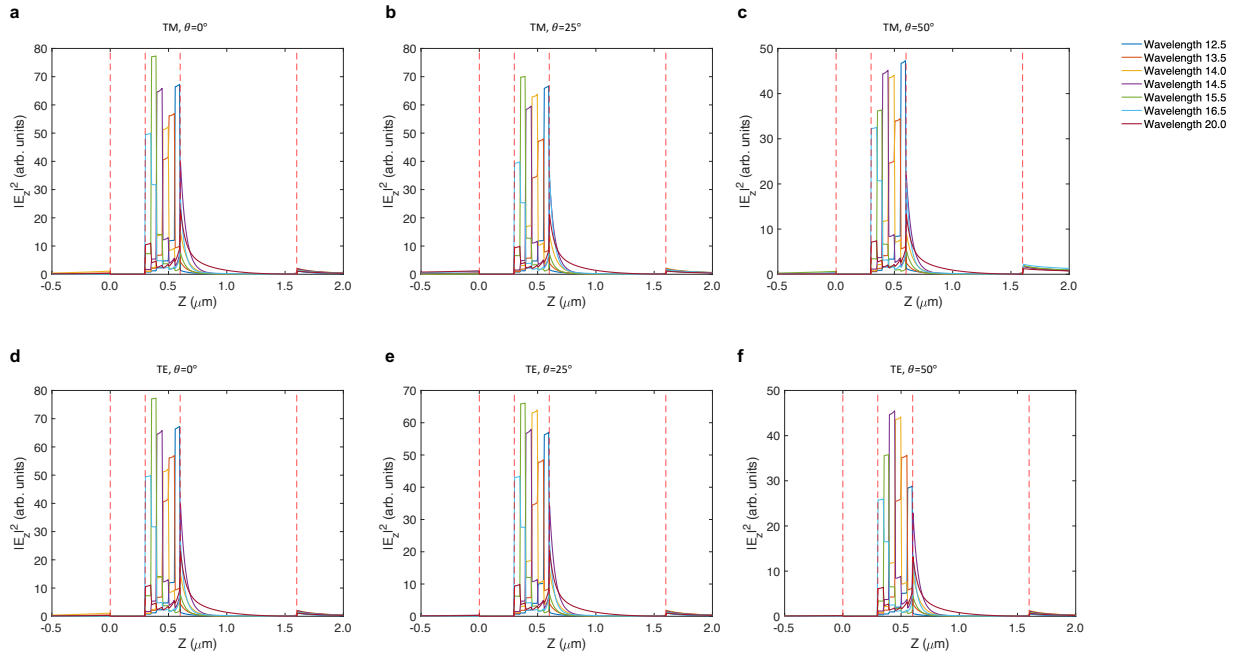


Fig. S7 Layer-resolved electric-field intensity profiles $|E|^2$ within the gradient EMNZ structure across a broad wavelength range, multiple incident angles, and both polarizations. (a–c) $|E|^2$ profiles under TM-polarized illumination at three representative incident angles. (d–f) Corresponding $|E|^2$ profiles under TE-polarized illumination at the same angles. In all cases, the electric field exhibits a ladder-like sequence of strongly localized maxima within individual InAs sublayers, with each peak corresponding to the polaritonic resonance of a nano-layer whose plasma frequency satisfies the near-zero-index condition at a given wavelength. As the wavelength is varied, the dominant localization site shifts systematically along the gradient stack, directly visualizing the spectrally graded EMNZ response. The $|E|^2$ profiles remain nearly invariant with respect to polarization and angle of incidence, confirming broadband, polarization-agnostic, and angle-resilient field confinement.

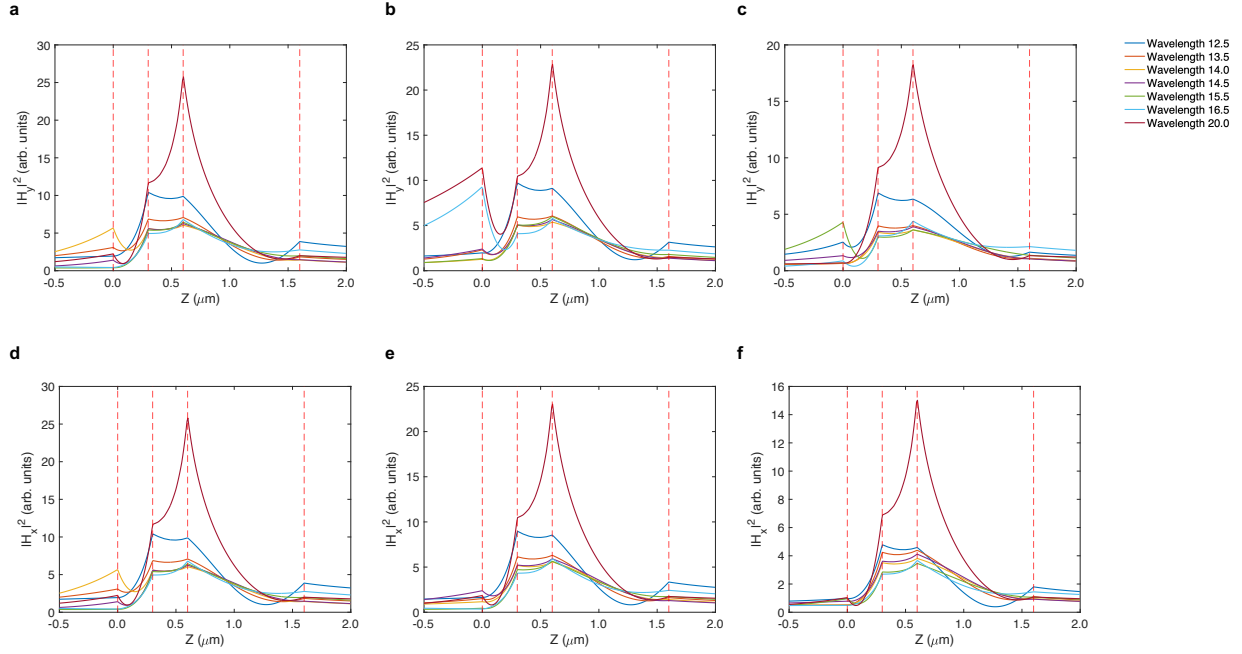


Fig. S8 Layer-resolved magnetic-field intensity profiles $|H|^2$ within the gradient EMNZ structure across the same wavelength range, incident angles, and polarizations as in Fig. S7. (a–c) $|H|^2$ profiles under TM-polarized illumination at three representative incident angles. (d–f) Corresponding $|H|^2$ profiles under TE-polarized illumination. In contrast to the electric field, the magnetic field does not exhibit layer-by-layer localization; instead, $|H|^2$ peaks near the multilayer region and decays monotonically away from the structure, consistent with the behavior expected for ultrathin ENZ and polaritonic modes. The magnetic-field profiles show minimal dependence on wavelength, polarization, and angle of incidence, quantitatively confirming the angular robustness and polarization equivalence of the gradient EMNZ modes.