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Non-local strain gradient isogeometric analysis of thermally loaded FG-TPMS sandwich nanoplates on elastic foundations

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Abstract

Functionally graded triply periodic minimal surface (FG-TPMS) sandwich nanoplates offer attractive combinations of lightweight architecture and tunable mechanical performance, yet their vibration behavior under coupled size-dependent, thermal, and foundation effects remains insufficiently understood. This study develops a high-continuity isogeometric framework for their free-vibration analysis. The sandwich structure comprises temperature-dependent Ti-6Al-4V/ZrO₂ functionally graded face sheets and a density-graded sheet-based TPMS core with Primitive (P), Gyroid (G), or I-graph and wrapped package-graph (IWP) topology. Non-local strain gradient theory is combined with a five-variable higher-order shear deformation theory to capture non-local softening and strain-gradient hardening. The nonlinear through-thickness temperature field is obtained from steady heat conduction with spatially varying conductivity, while thermal initial stresses and Winkler–Pasternak foundation effects are incorporated consistently. High-order NURBS basis functions provide the continuity required by the strain-gradient formulation. The model is verified against available benchmark solutions. Parametric results reveal the competing effects of non-local softening and strain-gradient hardening and show that thermal loading consistently reduces the fundamental frequency. Increasing the TPMS-core density does not

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necessarily improve vibration performance because of the competing stiffness and inertia effects, while an intermediate core-thickness ratio provides a favorable response. The P-type core generally yields the highest fundamental frequency among the considered architectures.

Keywords: Triply periodic minimal surface (TPMS); Sandwich nanoplate; Non-local strain gradient theory; Isogeometric analysis; Thermal vibration; Elastic foundation.

1. Introduction

Triply periodic minimal surface (TPMS) architectures have emerged as a promising class of cellular materials for lightweight structural applications owing to their continuous geometries, high surface-to-volume ratios, and tunable mechanical and transport characteristics. Their effective behavior can be tailored through both topology and relative-density distribution, offering substantial flexibility for the development of architected structures with improved stiffness-to-weight performance. Fu et al. [1] demonstrated the pronounced influence of TPMS topology on the mechanical response of hollow cellular structures, while Chawla and Kiran [2] investigated the thermoelastic characteristics of TPMS-based composites. Ejeh et al. [3] further examined functionally graded additively manufactured TPMS beams, and Hassan et al. [4] addressed the design, fabrication, and mechanical evaluation of graded TPMS structures. These advances have progressively extended TPMS concepts from conventional cellular solids toward functionally graded structural components, making functionally graded TPMS (FG-TPMS) architectures particularly attractive for plates and sandwich systems operating under coupled mechanical and thermal environments.

In parallel, considerable effort has been devoted to developing refined computational models for FG-TPMS plates. Nguyen-Xuan et al. [5] established effective-property models for P-, G-, and IWP-type FG-TPMS plates based on relative-density-dependent constitutive relations. Nguyen et al. [6] extended this concept to size-dependent FG-TPMS microplates, whereas Nguyen et al. [7] developed a refined quasi-three-dimensional isogeometric formulation for their nonlinear analysis. Tang et al. [8] introduced three-dimensional density design variables for the vibration optimization of FG-TPMS plates, while Thai et al. [9] proposed a meshfree computational framework. More recently, Tran et al. [10,11] investigated crystalline rotations and graphene-

platelet reinforcement in FG-TPMS plate systems. Despite these developments, most existing formulations consider either single-layer FG-TPMS plates or structures in which the TPMS medium constitutes the plate itself. Comparatively less attention has been paid to sandwich nanoplates in which the TPMS architecture serves as a graded lightweight core bounded by independently graded face sheets. Such a configuration introduces an additional interaction among face-sheet gradation, core topology, through-thickness density distribution, inertia, and bending stiffness that cannot be inferred directly from conventional single-layer FG-TPMS models.

At the nanoscale, the mechanical response is further complicated by size-dependent effects that cannot generally be represented within classical continuum elasticity. Non-local elasticity captures the stiffness-softening behavior associated with long-range interatomic interactions, whereas strain-gradient theories introduce intrinsic length scales capable of describing size-dependent hardening. The non-local strain gradient theory (NSGT) combines these two mechanisms within a unified generalized-continuum framework. Alghanmi et al. [12] employed NSGT to investigate bending of porous functionally graded nanoplates, while Esen and Özmen [13] examined thermal vibration and buckling of multifunctional nanoplates. Thai et al. [14] subsequently developed an isogeometric NSGT formulation for functionally graded nanoplates. For architected cellular materials, Phung-Van et al. [15] studied porosity-dependent FG-TPMS nanoplates within NSGT, and Nguyen et al. [16] proposed an isogeometric NSGT model for graphene-platelet-reinforced FG-TPMS nanoplates. Hung et al. [17] also investigated FG-TPMS microplates using modified strain-gradient theory and isogeometric analysis. These studies have clearly demonstrated the importance of generalized continuum theories for small-scale TPMS structures; however, the simultaneous treatment of independently graded face sheets and a density-graded TPMS core within a three-layer nanoplate configuration remains insufficiently explored.

The higher-order spatial operators introduced by strain-gradient theories also impose additional requirements on numerical discretization. Isogeometric analysis (IGA), originally introduced by Hughes et al. [18], provides a particularly suitable framework because non-uniform rational B-spline (NURBS) basis functions can be employed consistently for both geometry representation and field approximation while offering controllable high-order continuity. Cottrell et al. [19] demonstrated the favorable spectral properties of IGA for structural vibration, and Kiendl et al. [20] exploited spline continuity in rotation-free Kirchhoff–Love shell formulations. More directly relevant to generalized continua, Hosseini and Niiranen [21] showed that IGA provides a

natural numerical setting for strain-gradient elasticity, whose weak forms involve higher-order spatial derivatives. This feature is especially important in the present NSGT–HSDT formulation, where gradients of higher-order plate strains require derivatives beyond those available in standard (C^0) finite element approximations. High-continuity NURBS spaces therefore permit these derivatives to be evaluated directly, without introducing additional independent fields or mixed variables. The advantages of IGA have also been increasingly exploited in TPMS mechanics. Do et al. [22] developed an isogeometric dynamic model for graphene-platelet-reinforced FG-TPMS plates supported by a Pasternak foundation, Nguyen et al. [23] proposed a NURBS-based reconstruction strategy for TPMS structures, and Chu et al. [24] employed IGA in multi-objective FG-TPMS optimization. These contributions further motivate the use of high-continuity spline discretizations for the present small-scale TPMS sandwich problem.

Thermal loading and elastic-foundation support introduce additional physical mechanisms that can substantially modify the vibration characteristics of nanoplates. Temperature changes affect not only the constituent material properties but also the prestressed state of the structure through thermally induced membrane stresses. At the same time, the stiffness of the surrounding medium can alter the balance between structural flexibility and external restraint. Kumar et al. [25] investigated porous functionally graded nanoplates supported by elastic foundations, while Rai and Gupta [26] examined non-local vibration characteristics of porous FG nanoplates on elastic media and later extended their work to thermally loaded sandwich FG nanoplates [27]. Mottaghi et al. [28] studied hygrothermal FG porous nanoplates using NSGT, and Koç et al. [29] investigated the thermomechanical vibration response of auxetic-core sandwich nanoplates. Foundation effects were further considered by Baghdali et al. [30], whereas Saffari et al. [31] analyzed thermally exposed sandwich nanoplates within an NSGT framework. Collectively, these studies indicate that size effects, temperature-dependent constitutive behavior, thermally induced initial stress, and foundation stiffness may interact strongly and should therefore be incorporated within a consistent vibration formulation.

Recent investigations have also begun to extend TPMS architectures toward sandwich structural systems. Demirhan et al. [32] analyzed sandwich plates incorporating FG-TPMS cores, while Trai et al. [33] developed a triangular-element formulation for FG-TPMS plates integrated with magneto-electric layers. Dao et al. [34] examined sandwich plates with auxetic cores resting on multi-parameter foundations, and Do et al. [35] investigated the dynamic behavior of FG-TPMS

plates supported by viscoelastic foundations. Buğday and Esen [36] further studied the thermomechanical vibration of curved sandwich nanostructures containing TPMS-based cores. These studies demonstrate the growing interest in combining architected cellular cores with multilayer structural concepts. Nevertheless, the principal ingredients relevant to the present problem have largely been treated separately. In particular, a unified high-continuity formulation for a three-layer nanoplate composed of temperature-dependent FG Ti-6Al-4V/ZrO₂ face sheets and a density-graded sheet-based TPMS core, while simultaneously accounting for non-local softening, strain-gradient hardening, nonlinear through-thickness temperature variation, thermally induced initial stress, and Winkler-Pasternak foundation effects, has not yet been systematically established.

Motivated by these observations, the present study develops a high-continuity non-local strain gradient isogeometric framework for the free-vibration analysis of FG-TPMS sandwich nanoplates subjected to nonlinear through-thickness thermal fields and elastic-foundation effects. A five-variable higher-order shear deformation theory (HSDT) is combined with NSGT to describe the nanoplate kinematics and competing small-scale softening and hardening mechanisms. The two Ti-6Al-4V/ZrO₂ face sheets are functionally graded independently of a sheet-based TPMS core with P, G, or IWP topology, whose local effective properties vary according to prescribed mean-preserving relative-density distributions. The through-thickness temperature field is obtained from the steady heat-conduction problem using spatially varying reference-state conductivity, after which the local temperature is employed to update the constitutive properties and thermal initial stresses. A two-parameter Winkler-Pasternak foundation is incorporated into the variational formulation, while cubic NURBS basis functions with sufficient continuity are employed to evaluate directly the higher-order derivatives required by the strain-gradient operators.

The resulting formulation is verified against benchmark solutions covering size-dependent, foundation, and thermal effects. It is then used to investigate the individual and coupled influences of the non-local and strain-gradient parameters, TPMS topology, through-thickness core-density pattern, average relative density, face-sheet material gradation, core-thickness ratio, thermal loading, and elastic-foundation stiffness. Particular attention is devoted to identifying the competing stiffness–inertia mechanisms governing the vibration response and to clarifying how TPMS architecture and sandwich configuration modify the sensitivity of nanoplates to small-scale

and thermal effects.

2. Theoretical framework

2.1. The FG-TPMS sandwich nanoplate

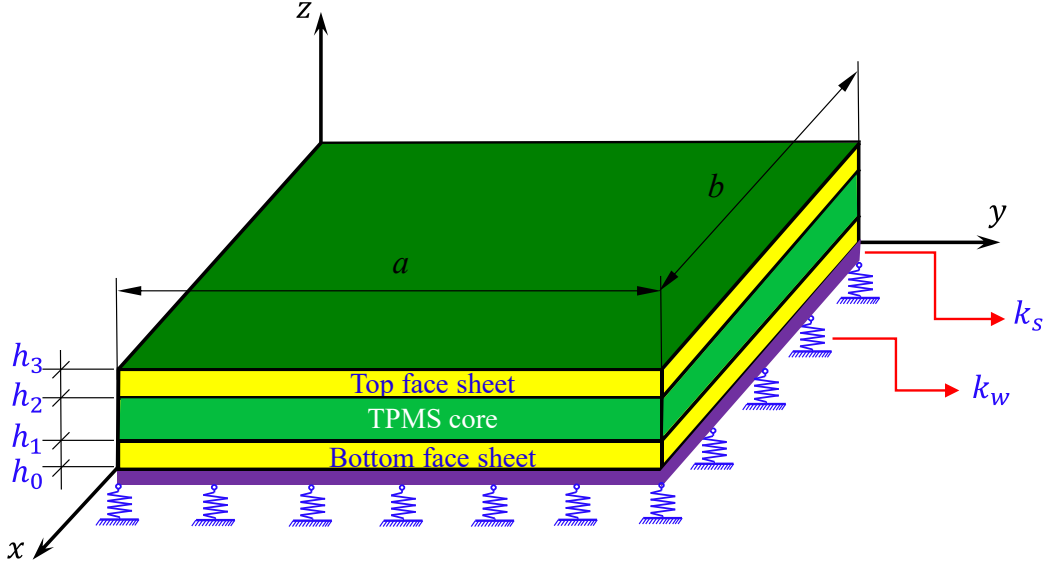


Fig. 1. Geometry and material configuration of the three-layer FG-TPMS sandwich nanoplate resting on a two-parameter Winkler-Pasternak elastic foundation.

Consider a rectangular three-layer sandwich nanoplate of length a , width b , and total thickness h , as illustrated in Fig. 1. A Cartesian coordinate system (x, y, z) is introduced with the x - and y -axes lying on the mid-surface plate and the z -axis directed through the thickness. The lower and upper surfaces of the nanoplate are located at

$$h_0 = -\frac{h}{2}, \quad h_3 = \frac{h}{2}, \quad (1)$$

while h_1 and h_2 denote the interfaces separating the two face sheets from the TPMS core. Accordingly, the three constituent regions are defined as

$$h_0 \leq z \leq h_1, \quad h_1 \leq z \leq h_2, \quad h_2 \leq z \leq h_3. \quad (2)$$

The lower and upper face sheets are composed of functionally graded Ti-6Al-4V/ZrO₂, whereas the intermediate layer is a sheet-based TPMS core fabricated from Ti-6Al-4V. Three

representative TPMS architectures are considered, namely Primitive (P), Gyroid (G), and I-graph and wrapped package-graph (IWP), as shown in Fig. 2. In contrast to a conventional FGM sandwich configuration with a homogeneous core, the present nanoplate combines continuously graded face sheets with an architected porous core whose effective mechanical and thermal properties vary locally through the thickness according to the prescribed relative-density distribution. This configuration therefore allows the effects of both material gradation in the face sheets and topology-dependent density gradation in the core to be incorporated within a unified sandwich model.

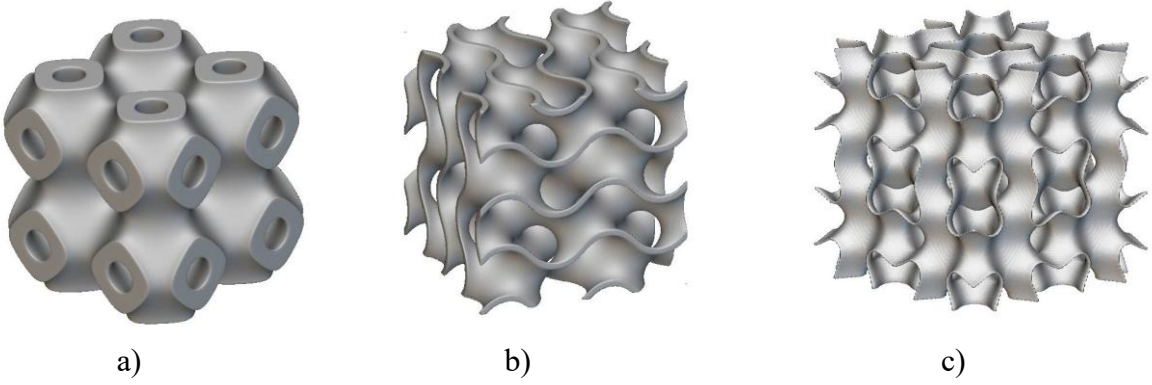


Fig. 2. Sheet-based TPMS architectures: (a) Primitive (P), (b) Gyroid (G), and (c) IWP.

For the two face sheets, the ceramic volume fraction is assumed to vary continuously through the thickness according to a power-law distribution. Specifically, the ZrO_2 volume fraction is defined as [37]

$$V_c(z) = \begin{cases} \left(\frac{h_1 - z}{h_1 - h_0} \right)^p, & h_0 \leq z \leq h_1, \\ \left(\frac{z - h_2}{h_3 - h_2} \right)^p, & h_2 \leq z \leq h_3, \end{cases} \quad (3)$$

where $p \geq 0$ is the material gradient index controlling the through-thickness variation of the ceramic phase. The corresponding Ti-6Al-4V volume fraction follows from the local mixture constraint,

$$V_m(z) = 1 - V_c(z). \quad (4)$$

For $p > 0$, the two outer surfaces are fully ceramic-rich, whereas the material composition gradually changes toward Ti-6Al-4V when approaching the interfaces with the TPMS core. Hence,

$$V_c(h_0) = V_c(h_3) = 1, \quad V_c(h_1) = V_c(h_2) = 0. \quad (5)$$

This symmetric grading scheme provides ceramic-rich exterior surfaces together with metal-rich face/core interfaces, thereby preserving a smooth compositional transition toward the Ti-6Al-4V core. In the limiting case $p = 0$, the ceramic volume fraction becomes

$$V_c(z) = 1, \quad (6)$$

and both face sheets reduce to homogeneous ZrO_2 layers.

For a generic material property $\mathcal{P}$, the effective property of the functionally graded face sheets is evaluated using the rule of mixtures as [37]

$$\mathcal{P}_f(z, T) = \mathcal{P}_m(T) + [\mathcal{P}_c(T) - \mathcal{P}_m(T)]V_c(z) \quad (7)$$

where $\mathcal{P}_m(T)$ and $\mathcal{P}_c(T)$ denote the temperature-dependent properties of Ti-6Al-4V and ZrO_2 , respectively, while $T = T(z)$ represents the local absolute temperature at the through-thickness coordinate. Depending on the subsequent formulation, $\mathcal{P}$ may represent Young's modulus E , Poisson's ratio ν , mass density ρ , coefficient of thermal expansion α , or thermal conductivity K . The specific temperature dependence of these constituent properties and the determination of $T(z)$ are introduced in Section 2.2.

Within the core region, the sheet-based TPMS architecture is modeled as an equivalent continuum whose local effective properties are governed by the relative density. The three TPMS topologies considered in this study are illustrated in Fig. 2. To conveniently describe the through-thickness variation of the core density, a normalized coordinate is introduced as

$$\eta_c = \frac{z - h_1}{h_2 - h_1}, \quad 0 \leq \eta_c \leq 1, \quad (8)$$

such that $\eta_c = 0$ and 1 correspond to the lower and upper face/core interfaces, respectively. The local relative density of the TPMS core is then expressed as a function of this normalized

coordinate,

$$\rho_{\text{rel}} = \rho_{\text{rel}}(\eta_c). \quad (9)$$

To systematically investigate the influence of through-thickness density gradation independently of the total amount of core material, two families of density distributions are introduced. Their basic shape functions are defined as [5]

$$\Phi_A(\eta_c) = \eta_c^{k_z}, \quad (10)$$

and

$$\Phi_B(\eta_c) = \left[1 - \cos\left(\pi\left(\eta_c - \frac{1}{2}\right)\right) \right]^{k_z}, \quad (11)$$

where $k_z > 0$ is the density-gradient exponent controlling the degree of through-thickness variation. The A-type function generates a monotonic distribution from the lower to the upper core interface, whereas the B-type function produces a distribution symmetric with respect to the core mid-surface. Increasing k_z progressively modifies the concentration of material through the core thickness and therefore provides a convenient means of controlling the spatial density contrast.

To separate the influence of material redistribution from that of the average core density, the local relative-density field is constructed in the mean-preserving form

$$\rho_{\text{rel}}(\eta_c) = \rho_{\text{avg}} + A_r[\Phi_r(\eta_c) - \overline{\Phi}_r], \quad r \in \{A, B\} \quad (12)$$

where

$$\overline{\Phi}_r = \int_0^1 \Phi_r(\eta_c) d\eta_c, \quad (13)$$

is the thickness-average value of the corresponding shape function, ρ_{avg} is the prescribed average relative density of the TPMS core, and A_r controls the amplitude of the density variation. By construction,

$$\int_0^1 \rho_{\text{rel}}(\eta_c) d\eta_c = \rho_{\text{avg}}. \quad (14)$$

Thus, all density profiles generated from Eq. (12) contain the same average amount of core

material for a prescribed ρ_{avg} , allowing the influence of through-thickness redistribution to be examined independently. For the normalized A- and B-type shape functions adopted here,

$$\Phi_{r,\min} = 0, \quad \Phi_{r,\max} = 1, \quad (15)$$

and hence the density amplitude is simply

$$A_r = \rho_{\text{rel},\max} - \rho_{\text{rel},\min}. \quad (16)$$

In the present study, six representative density patterns, denoted by $A1, A2, A3, B1, B2$, and $B3$, are considered. The corresponding values of $\rho_{\text{rel},\min}$, $\rho_{\text{rel},\max}$, and k_z for $\rho_{\text{avg}} = 0.35$ are summarized in Table 1. The A-type patterns represent monotonic density variations, whereas the B-type patterns provide their symmetric counterparts; within each family, the selected parameters produce progressively stronger through-thickness density redistribution. This construction provides a unified description of the graded TPMS core and enables the local relative density $\rho_{\text{rel}}(z)$ to be subsequently mapped to the topology-dependent equivalent mechanical properties of the P-, G-, and IWP-type architectures.

Table 1. Parameters of the six relative-density patterns at $\rho_{\text{avg}} = 0.35$.

Pattern	$\rho_{\text{rel},\min}$	$\rho_{\text{rel},\max}$	k_z
$A1$	0.2	0.5	1
$A2$	0.2	0.8	3
$A3$	0.25	1	6.5
$B1$	0.2	0.5	0.561
$B2$	0.2	0.8	1.757
$B3$	0.25	1	3.943

At each through-thickness position within the core, the local relative density $\rho_{\text{rel}}(z)$ determined from Eqs. (8)-(15) is used to evaluate the effective properties of the selected TPMS architecture. For a given topology $\mathcal{T} \in \{\text{P}, \text{G}, \text{IWP}\}$, the equivalent Young's modulus, shear modulus, and Poisson's ratio are expressed in the normalized form [5]

$$E_{\text{TPMS}}(z, T) = E_s(T) \hat{E}_{\mathcal{T}}(\rho_{\text{rel}}(z)), \quad (17)$$

$$G_{\text{TPMS}}(z, T) = G_s(T) \hat{G}_{\mathcal{T}}(\rho_{\text{rel}}(z)), \quad (18)$$

$$\nu_{\text{TPMS}}(z) = \hat{\nu}_{\mathcal{T}}(\rho_{\text{rel}}(z)), \quad (19)$$

where $E_s(T)$ and $G_s(T)$ denote, respectively, the temperature-dependent Young's and shear moduli of the fully dense Ti-6Al-4V parent material, while $\hat{E}_{\mathcal{T}}$, $\hat{G}_{\mathcal{T}}$, and $\hat{\nu}_{\mathcal{T}}$ are the topology-dependent normalized effective-property functions. In this representation, the effects of the parent material and the TPMS architecture are separated explicitly: temperature dependence is introduced through the solid constituent properties, whereas the influence of topology and local porosity is described through the normalized fitted relations.

The effective mass density of the TPMS core is evaluated directly from the local relative density as [5]

$$\rho_{\text{TPMS}}(z, T) = \rho_s(T) \rho_{\text{rel}}(z), \quad (20)$$

where $\rho_s(T)$ denotes the temperature-dependent mass density of fully dense Ti-6Al-4V. The normalized effective-property functions adopted for the three sheet-based TPMS architectures are given below [5].

- P-type core

$$\hat{E}_{\text{P}}(\rho_{\text{rel}}) = \begin{cases} 0.317 \rho_{\text{rel}}^{1.264}, & \rho_{\text{rel}} \leq 0.25, \\ 1.007 \rho_{\text{rel}}^{2.006} - 0.007, & \rho_{\text{rel}} > 0.25, \end{cases} \quad (21)$$

$$\hat{G}_{\text{P}}(\rho_{\text{rel}}) = \begin{cases} 0.705 \rho_{\text{rel}}^{1.189}, & \rho_{\text{rel}} \leq 0.25, \\ 0.953 \rho_{\text{rel}}^{1.715} + 0.047, & \rho_{\text{rel}} > 0.25, \end{cases} \quad (22)$$

$$\hat{\nu}_{\text{P}}(\rho_{\text{rel}}) = \begin{cases} 0.314 e^{-1.004 \rho_{\text{rel}}} + 0.119, & \rho_{\text{rel}} \leq 0.55, \\ 0.152 \rho_{\text{rel}}^2 - 0.235 \rho_{\text{rel}} + 0.383, & \rho_{\text{rel}} > 0.55. \end{cases} \quad (23)$$

- G-type core

$$\hat{E}_{\text{G}}(\rho_{\text{rel}}) = \begin{cases} 0.596 \rho_{\text{rel}}^{1.467}, & \rho_{\text{rel}} \leq 0.45, \\ 0.962 \rho_{\text{rel}}^{2.351} + 0.038, & \rho_{\text{rel}} > 0.45, \end{cases} \quad (24)$$

$$\hat{G}_G(\rho_{\text{rel}}) = \begin{cases} 0.777\rho_{\text{rel}}^{1.544}, & \rho_{\text{rel}} \leq 0.45, \\ 0.973\rho_{\text{rel}}^{1.982} + 0.027, & \rho_{\text{rel}} > 0.45, \end{cases} \quad (25)$$

$$\hat{\nu}_G(\rho_{\text{rel}}) = \begin{cases} 0.192e^{-1.349\rho_{\text{rel}}} + 0.202, & \rho_{\text{rel}} \leq 0.50, \\ 0.402\rho_{\text{rel}}^2 - 0.603\rho_{\text{rel}} + 0.501, & \rho_{\text{rel}} > 0.50. \end{cases} \quad (26)$$

- **IWP-type core**

$$\hat{E}_{\text{IWP}}(\rho_{\text{rel}}) = \begin{cases} 0.597\rho_{\text{rel}}^{1.225}, & \rho_{\text{rel}} \leq 0.35, \\ 0.987\rho_{\text{rel}}^{1.782} + 0.013, & \rho_{\text{rel}} > 0.35, \end{cases} \quad (27)$$

$$\hat{G}_{\text{IWP}}(\rho_{\text{rel}}) = \begin{cases} 0.529\rho_{\text{rel}}^{1.287}, & \rho_{\text{rel}} \leq 0.35, \\ 0.960\rho_{\text{rel}}^{2.188} + 0.040, & \rho_{\text{rel}} > 0.35, \end{cases} \quad (28)$$

$$\hat{\nu}_{\text{IWP}}(\rho_{\text{rel}}) = \begin{cases} 2.597e^{-0.157\rho_{\text{rel}}} - 2.244, & \rho_{\text{rel}} \leq 0.13, \\ 0.201\rho_{\text{rel}}^2 - 0.227\rho_{\text{rel}} + 0.326, & \rho_{\text{rel}} > 0.13. \end{cases} \quad (29)$$

A notable feature of the adopted equivalent-continuum model is that the effective shear modulus is determined independently from its own topology-dependent fitting relation rather than being reconstructed from E_{TPMS} and ν_{TPMS} through the isotropic relation. This treatment preserves the distinct shear response associated with each TPMS architecture and is therefore retained in the subsequent higher-order plate formulation.

Combining the functionally graded face sheets and the density-graded TPMS core, the complete through-thickness material description of the sandwich nanoplate can be written compactly as

$$\mathcal{P}(z, T) = \begin{cases} \mathcal{P}_f(z, T), & h_0 \leq z \leq h_1, \\ \mathcal{P}_{\text{TPMS}}(z, T), & h_1 \leq z \leq h_2, \\ \mathcal{P}_f(z, T), & h_2 \leq z \leq h_3. \end{cases} \quad (30)$$

Here, $\mathcal{P}_f(z, T)$ denotes the temperature- and position-dependent effective property of the Ti-6Al-4V/ZrO₂ functionally graded face sheets defined by Eq. (7), whereas $\mathcal{P}_{\text{TPMS}}(z, T)$ denotes the corresponding equivalent property of the selected P-, G-, or IWP-type TPMS core. Therefore, Eq. (30) provides a unified layer-wise material representation that will be employed in the subsequent thermal and mechanical formulations.

2.2. Nonlinear temperature field through the thickness

The FG-TPMS sandwich nanoplate is assumed to be subjected to a thermal field that is uniform over the in-plane coordinates and varies only through the thickness. The temperatures prescribed at the bottom and top surfaces are denoted by T_b and T_t , respectively. To account for the temperature dependence of the constituent properties, a generic material property $\mathcal{P}$ is expressed, following the temperature-dependent material model adopted by Daikh et al. [37], as

$$\mathcal{P}(T) = \mathcal{P}_0(\mathcal{P}_{-1}T^{-1} + 1 + \mathcal{P}_1T + \mathcal{P}_2T^2 + \mathcal{P}_3T^3), \quad (31)$$

where $\mathcal{P}_0$ is the reference value of the material property, while $\mathcal{P}_{-1}$, $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ are the corresponding temperature coefficients, and T denotes the local absolute temperature. The absolute local temperature is decomposed into the reference temperature T_0 and the corresponding temperature rise ΔT as

$$T = T_0 + \Delta T, \quad T_0 = 300\text{K}. \quad (32)$$

For the three-layer sandwich configuration defined in Section 2.1, the local thermal conductivity is piecewise defined through the thickness as

$$K(z) = \begin{cases} K^{(1)}(z), & h_0 \leq z \leq h_1, \\ K^{(2)}(z), & h_1 \leq z \leq h_2, \\ K^{(3)}(z), & h_2 \leq z \leq h_3. \end{cases} \quad (33)$$

Within the lower and upper FG face sheets, the effective thermal conductivity is evaluated using the same rule of mixtures adopted for the other graded material properties [37],

$$K^{(n)}(z) = K_m(T) + [K_c(T) - K_m(T)]V_c^{(n)}(z), \quad n = 1, 3, \quad (34)$$

where $K_m(T)$ and $K_c(T)$ denote the thermal conductivities of Ti-6Al-4V and ZrO₂, respectively, and $V_c^{(n)}(z)$ is the local ceramic volume fraction in the corresponding face sheet.

For the sheet-based TPMS core, a topology-independent density-scaling approximation is adopted for the effective thermal conductivity

$$K^{(2)}(z) = K_{\text{TPMS}}(z) = \rho_{\text{rel}}(z)K_m(T), \quad (35)$$

where $\rho_{\text{rel}}(z)$ is the through-thickness relative-density distribution introduced in Section 2.1.

Accordingly, the local conductivity varies continuously within the core according to the prescribed density pattern. Unlike the equivalent elastic properties introduced in Eqs. (17)-(29), the thermal-conductivity model in Eq. (35) does not distinguish among the P-, G-, and IWP-type architectures; their thermal response is approximated solely through the local relative density.

In the present formulation, the conductivity distributions entering the heat-conduction problem are evaluated at the reference temperature T_0 and subsequently kept fixed during the solution of the steady thermal problem. Thus, Eqs. (34) and (35) are evaluated using $K_m(T_0)$ and $K_c(T_0)$ when constructing $K(z)$. This assumption decouples the determination of the through-thickness temperature field from the subsequent temperature-dependent constitutive update. Accordingly, the nonlinearity of the resulting temperature profile with respect to z originates from the spatial variation of the reference-state conductivity across the FG face sheets and the density-graded TPMS core, rather than from an iterative dependence of K on the unknown local temperature.

In the absence of internal heat generation, the steady one-dimensional heat-conduction problem is therefore governed by [37]

$$\frac{d}{dz} \left[K(z) \frac{dT(z)}{dz} \right] = 0. \quad (36)$$

subject to the prescribed surface temperatures

$$T(h_0) = T_b, \quad T(h_3) = T_t. \quad (37)$$

Perfect thermal contact is assumed at the two face/core interfaces. Accordingly, both the temperature and the through-thickness heat flux remain continuous at $z = h_1$ and $z = h_2$:

$$\begin{aligned} T^{(1)}(h_1) &= T^{(2)}(h_1), \\ T^{(2)}(h_2) &= T^{(3)}(h_2), \\ K^{(1)}(h_1) \frac{dT^{(1)}}{dz} \Big|_{z=h_1} &= K^{(2)}(h_1) \frac{dT^{(2)}}{dz} \Big|_{z=h_1}, \\ K^{(2)}(h_2) \frac{dT^{(2)}}{dz} \Big|_{z=h_2} &= K^{(3)}(h_2) \frac{dT^{(3)}}{dz} \Big|_{z=h_2}. \end{aligned} \quad (38)$$

Because the steady heat flux is constant through the thickness, the solution of Eq. (36) can be

expressed conveniently in terms of the cumulative thermal resistance. The temperature in the n -th constituent region is written as [37]

$$T^{(n)}(z) = T_b + (T_t - T_b)\chi^{(n)}(z), \quad n = 1, 2, 3. \quad (39)$$

where $\chi^{(n)}(z)$ is a dimensionless layer-wise temperature function. Introducing the total through-thickness thermal resistance

$$R_T = \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} \frac{1}{K^{(n)}(\zeta)} d\zeta, \quad (40)$$

the corresponding layer-wise functions are obtained as

$$\begin{aligned} \chi^{(1)}(z) &= \frac{\int_{h_0}^z \frac{1}{K^{(1)}(\zeta)} d\zeta}{R_T}, \quad h_0 \leq z \leq h_1, \\ \chi^{(2)}(z) &= \frac{\int_{h_0}^{h_1} \frac{1}{K^{(1)}(\zeta)} d\zeta + \int_{h_1}^z \frac{1}{K^{(2)}(\zeta)} d\zeta}{R_T}, \quad h_1 \leq z \leq h_2, \\ \chi^{(3)}(z) &= \frac{\int_{h_0}^{h_1} \frac{1}{K^{(1)}(\zeta)} d\zeta + \int_{h_1}^{h_2} \frac{1}{K^{(2)}(\zeta)} d\zeta + \int_{h_2}^z \frac{1}{K^{(3)}(\zeta)} d\zeta}{R_T}, \quad h_2 \leq z \leq h_3. \end{aligned} \quad (41)$$

The temperatures at the two face/core interfaces follow directly from Eq. (41) as

$$T_1 = T^{(1)}(h_1) = T^{(2)}(h_1), \quad T_2 = T^{(2)}(h_2) = T^{(3)}(h_2). \quad (42)$$

Finally, the local temperature rise relative to the reference state is defined in each layer by

$$\Delta T^{(n)}(z) = T^{(n)}(z) - T_0, \quad n = 1, 2, 3. \quad (43)$$

The resulting through-thickness temperature field is subsequently employed to evaluate the local temperature-dependent constitutive properties and the thermally induced initial stresses entering the vibration formulation. Because the reference-state thermal conductivity varies spatially within both the graded face sheets and the density-graded TPMS core, the temperature distribution is generally nonlinear through thickness even though the steady conduction problem itself is solved using fixed conductivity fields.

2.3. Non-local strain gradient theory

At nanometric length scales, the mechanical response may exhibit both stiffness softening associated with non-local interactions and stiffness hardening induced by strain-gradient effects. To incorporate these competing size-dependent mechanisms within a unified continuum description, the non-local strain gradient theory (NSGT) is adopted. The equation of motion is written as [15,38]

$$\tau_{ij,j} + b_i = \rho \ddot{u}_i \text{ in } \Omega. \quad (44)$$

where τ_{ij} denotes the total non-local stress, b_i is the body-force component, ρ is the mass density, $\ddot{u}_i$ is the acceleration component, and Ω denotes the material domain.

Within the NSGT framework, total non-local stress is decomposed into a conventional non-local contribution and a higher-order contribution associated with the strain gradient [15],

$$\tau_{ij} = \tau_{ij}^{(0)} - \nabla \cdot \tau_{ij}^{(1)}. \quad (45)$$

The corresponding non-local stress measures are defined through spatial averaging over the material domain as [15]

$$\tau_{ij}^{(0)}(\mathbf{x}) = \mathcal{A}_\mu[\sigma_{ij}](\mathbf{x}), \quad \tau_{ij}^{(1)}(\mathbf{x}) = l^2 \mathcal{A}_\mu[\nabla \sigma_{ij}](\mathbf{x}). \quad (46)$$

where $\mathcal{A}_\mu[\cdot]$ denotes the non-local averaging operator associated with the attenuation kernel φ_μ , μ is the non-local parameter, l is the material length-scale parameter, and $\mathbf{x}$ denotes the position vector. The first relation accounts for long-range non-local interactions, whereas the second introduces an energetic penalty associated with stress gradients.

In Eq. (46), the local Cauchy stress is related to the infinitesimal strain through

$$\sigma_{ij} = C_{ijkl}(z, T) \varepsilon_{kl}, \quad \varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}). \quad (47)$$

The elastic coefficients $C_{ijkl}(z, T)$ vary through the thickness according to the FG face-sheet and TPMS-core properties established in Section 2.1 and are evaluated using the local temperature field $T(z)$ determined in Section 2.2. The thermally induced initial stress is treated separately in the subsequent variational formulation so that the temperature-dependent elastic response and the

thermal prestress contribution are not counted twice.

Using a Helmholtz-type attenuation kernel, the integral non-local relations in Eq. (46) can be recast in differential form as [15,38]

$$(1 - \mu \nabla^2) \tau_{ij}^{(0)} = \sigma_{ij}, \quad (1 - \mu \nabla^2) \tau_{ij}^{(1)} = \lambda \nabla \sigma_{ij}, \quad (48)$$

where

$$\lambda = l^2, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \quad (49)$$

In the present plate formulation, the non-local and strain-gradient operators are taken with respect to the in-plane coordinates x and y . Accordingly, μ and λ are squared intrinsic length-scale parameters, whereas l has the dimension of length. With these definitions, substitution of Eqs. (45) and (48) into the equation of motion yields

$$(1 - \lambda \nabla^2) \sigma_{ij,j} + (1 - \mu \nabla^2) b_i = (1 - \mu \nabla^2) \rho \ddot{u}_i. \quad (50)$$

For the free-vibration problem considered here, the body force is neglected, $b_i = 0$. Multiplying Eq. (50) by an admissible virtual displacement, integrating over Ω , and applying integration by parts while omitting the prescribed boundary-traction contributions lead to the weak form [38]

$$\int_{\Omega} \delta \varepsilon_{ij} \sigma_{ij} \, d\Omega + \lambda \int_{\Omega} \nabla \delta \varepsilon_{ij} \cdot \nabla \sigma_{ij} \, d\Omega + \int_{\Omega} \rho \delta u_i \ddot{u}_i \, d\Omega + \mu \int_{\Omega} \rho \nabla \delta u_i \cdot \nabla \ddot{u}_i \, d\Omega = 0 \quad (51)$$

The four terms in Eq. (51) are identified, respectively, as the classical elastic contribution, the strain-gradient contribution, the conventional inertia contribution, and the non-local inertia contribution. For subsequent reduction to the plate midsurface, they are denoted by

$$\begin{aligned}
\delta W_e &= \int_{\Omega} \delta \varepsilon_{ij} \sigma_{ij} d\Omega, \\
\delta W_{sg} &= \lambda \int_{\Omega} \nabla \delta \varepsilon_{ij} \cdot \nabla \sigma_{ij} d\Omega, \\
\delta W_I &= \int_{\Omega} \rho \delta u_i \ddot{u}_i d\Omega, \\
\delta W_{nl} &= \mu \int_{\Omega} \rho \nabla \delta u_i \cdot \nabla \ddot{u}_i d\Omega.
\end{aligned} \tag{52}$$

These definitions preserve the sign convention of the dynamic residual in Eq. (51). Under harmonic motion, the acceleration terms introduce the conventional and non-local inertia operators with the expected negative ω^2 contribution in the resulting eigenvalue problem. The parameter μ governs the non-local softening mechanism, whereas l , or equivalently $\lambda = l^2$, controls the strain-gradient hardening contribution. The relative magnitudes of these two intrinsic length scales therefore determine the competition between stiffness reduction and stiffness enhancement at the nanoscale. The adopted formulation recovers the principal limiting continuum models as

$$\begin{cases} \lambda = 0, & \text{pure non-local elasticity model,} \\ \mu = 0, & \text{pure strain-gradient model,} \\ \mu = \lambda = 0, & \text{classical elasticity model.} \end{cases} \tag{53}$$

The weak form in Eq. (51) involves gradients of the strain field and therefore requires spatial derivatives of higher order than those encountered in classical elasticity. This regularity requirement provides a direct motivation for the high-continuity isogeometric discretization introduced in Section 2.7, where NURBS basis functions are employed to evaluate the required higher-order derivatives without introducing auxiliary field variables.

2.4. Five-variable HSDT kinematics and thickness-integrated coefficients

A five-variable higher-order shear deformation theory (HSDT) is adopted to describe the kinematics of the FG-TPMS sandwich nanoplate. The transverse displacement is assumed to be independent of the thickness coordinate, and thickness stretching is neglected. For an arbitrary material point (x, y, z) , the displacement field is expressed as [39]

$$\begin{aligned}
u(x, y, z, t) &= u_0(x, y, t) - z w_{0,x}(x, y, t) + f(z)\theta_x(x, y, t), \\
v(x, y, z, t) &= v_0(x, y, t) - z w_{0,y}(x, y, t) + f(z)\theta_y(x, y, t), \\
w(x, y, z, t) &= w_0(x, y, t).
\end{aligned} \tag{54}$$

where where t denotes time, and the generalized displacement variables are collected in the vector

$$\mathbf{q} = \{u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y\}^T. \tag{55}$$

In Eq. (54), the higher-order thickness function is selected as [39]

$$\begin{aligned}
f(z) &= z \left(1 - \frac{3z^2}{2h^2} + \frac{2z^4}{5h^4} \right), \\
f'(z) &= 1 - \frac{9z^2}{2h^2} + \frac{2z^4}{h^4}, \quad f' \left(\pm \frac{h}{2} \right) = 0.
\end{aligned} \tag{56}$$

Note that, the condition $f'(\pm h/2) = 0$ ensures that the transverse shear strains vanish automatically at the upper and lower surfaces. Consequently, no shear correction factor is required.

Under the infinitesimal-strain assumption, the nonzero in-plane strain components become

$$\begin{aligned}
\varepsilon_{xx} &= u_{0,x} - z w_{0,xx} + f(z)\theta_{x,x}, \\
\varepsilon_{yy} &= v_{0,y} - z w_{0,yy} + f(z)\theta_{y,y}, \\
\gamma_{xy} &= u_{0,y} + v_{0,x} - 2z w_{0,xy} + f(z)(\theta_{x,y} + \theta_{y,x}).
\end{aligned} \tag{57}$$

These strains can be decomposed into membrane, bending, and higher-order contributions as

$$\boldsymbol{\varepsilon}_p = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \boldsymbol{\varepsilon}^0 + z\boldsymbol{\kappa}^b + f(z)\boldsymbol{\kappa}^h, \tag{58}$$

where

$$\boldsymbol{\varepsilon}^0 = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}, \tag{59}$$

$$\boldsymbol{\kappa}^b = - \begin{Bmatrix} w_{0,xx} \\ w_{0,yy} \\ 2w_{0,xy} \end{Bmatrix}, \tag{60}$$

and

$$\boldsymbol{\kappa}^h = \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{Bmatrix}. \quad (61)$$

The transverse shear strains are

$$\boldsymbol{\gamma}_s = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = f'(z) \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix}. \quad (62)$$

For subsequent use, the generalized in-plane strain vector is introduced as

$$\bar{\boldsymbol{\varepsilon}} = \{(\boldsymbol{\varepsilon}^0)^T, (\boldsymbol{\kappa}^b)^T, (\boldsymbol{\kappa}^h)^T\}^T. \quad (63)$$

The local constitutive coefficients vary through thickness because of both material gradation and temperature dependence. In the incremental vibration formulation, the mechanical stresses induced by the displacement perturbation are treated separately from the thermally induced initial stresses. The local constitutive relations are therefore written as [39]

$$\boldsymbol{\sigma}_p^m = \mathbf{Q}_b(z, T) \boldsymbol{\varepsilon}_p, \quad \boldsymbol{\tau}_s = \mathbf{Q}_s(z, T) \boldsymbol{\gamma}_s, \quad (64)$$

where the corresponding stress vectors are

$$\boldsymbol{\sigma}_p^m = \{\sigma_{xx}^m \quad \sigma_{yy}^m \quad \tau_{xy}^m\}^T, \quad \boldsymbol{\tau}_s = \{\tau_{xz} \quad \tau_{yz}\}^T. \quad (65)$$

In Eq. (64), the reduced in-plane and transverse shear stiffness matrices are

$$\mathbf{Q}_b = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}, \quad \mathbf{Q}_s = \begin{bmatrix} Q_{55} & 0 \\ 0 & Q_{44} \end{bmatrix}. \quad (66)$$

with

$$\begin{aligned} Q_{11} = Q_{22} &= \frac{E(z, T)}{1 - \nu^2(z, T)}, \\ Q_{12} &= \frac{\nu(z, T)E(z, T)}{1 - \nu^2(z, T)}, \\ Q_{66} = Q_{44} = Q_{55} &= G(z, T). \end{aligned} \quad (67)$$

Within the FG face sheets, the shear modulus is evaluated from, $G(z, T) =$

$E(z, T)/[2(1 + \nu(z, T))]$. In contrast, within the TPMS core, $G(z, T)$ is obtained directly from the topology-dependent fitted relation introduced in Section 2.1 rather than reconstructed from E and ν . This treatment preserves the independent effective shear response of the selected TPMS topology. Integration of the in-plane stresses through the thickness gives the membrane, bending, and higher-order stress resultants,

$$\mathbf{N} = \int_{-h/2}^{h/2} \boldsymbol{\sigma}_p^m dz, \quad \mathbf{M} = \int_{-h/2}^{h/2} z \boldsymbol{\sigma}_p^m dz, \quad \mathbf{P} = \int_{-h/2}^{h/2} f(z) \boldsymbol{\sigma}_p^m dz. \quad (68)$$

These generalized stress resultants are related to the generalized strains through

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{P} \end{Bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \\ \mathbf{B} & \mathbf{D} & \mathbf{F} \\ \mathbf{C} & \mathbf{F} & \mathbf{H} \end{bmatrix}}_{\mathbf{C}_b} \begin{Bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa}^b \\ \boldsymbol{\kappa}^h \end{Bmatrix}. \quad (69)$$

where $\mathbf{C}_b$ generalized in-plane constitutive matrix which is computed based on the thickness-integrated stiffness matrices:

$$\begin{aligned} \mathbf{A} &= \int_{-h/2}^{h/2} \mathbf{Q}_b dz, & \mathbf{B} &= \int_{-h/2}^{h/2} z \mathbf{Q}_b dz, & \mathbf{C} &= \int_{-h/2}^{h/2} f(z) \mathbf{Q}_b dz, \\ \mathbf{D} &= \int_{-h/2}^{h/2} z^2 \mathbf{Q}_b dz, & \mathbf{F} &= \int_{-h/2}^{h/2} z f(z) \mathbf{Q}_b dz, & \mathbf{H} &= \int_{-h/2}^{h/2} f^2(z) \mathbf{Q}_b dz. \end{aligned} \quad (70)$$

The transverse-shear stiffness matrix is obtained as

$$\mathbf{J}_s = \int_{-h/2}^{h/2} \mathbf{Q}_s(z, T) [f'(z)]^2 dz, \quad \mathbf{R}_s = \mathbf{J}_s \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix}. \quad (71)$$

To account consistently for the kinetic energy associated with the HSDT displacement field, six through-thickness inertia moments are introduced [39]:

$$\{I_1, I_2, I_3, I_4, I_5, I_6\} = \int_{-h/2}^{h/2} \rho(z, T) \{1, z, z^2, f(z), zf(z), f^2(z)\} dz. \quad (72)$$

These quantities are assembled into the inertia-moment matrix

$$\mathbf{I}_m = \begin{bmatrix} I_1 & I_2 & I_4 \\ I_2 & I_3 & I_5 \\ I_4 & I_5 & I_6 \end{bmatrix}. \quad (73)$$

When acting on the three physical displacement components, the corresponding block inertia operator is written as

$$\hat{\mathbf{I}}_m = \mathbf{I}_m \otimes \mathbf{1}_3, \quad (74)$$

where $\mathbf{1}_3$ is the 3×3 identity matrix.

The temperature field established in Section 2.2 is next used to construct the thermally induced initial-stress coefficients. The local temperature rise is

$$\Delta T(z) = T(z) - T_0. \quad (75)$$

For isotropic thermal expansion in the plate plane, the thermal strain vector is [37]

$$\boldsymbol{\varepsilon}^{th}(z) = \alpha \Delta T(z) \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}. \quad (76)$$

where $\alpha = \alpha(z, T)$ is the thermal expansion. The corresponding thermal stress measure is [37]

$$\boldsymbol{\sigma}^{th}(z) = \mathbf{Q}_b(z, T) \boldsymbol{\varepsilon}^{th}(z) = \{\sigma_{xx}^{th}(z) \quad \sigma_{yy}^{th}(z) \quad 0\}^T. \quad (77)$$

To retain the same HSDT thickness representation used for the mechanical and inertia terms, six thermal normal-stress moments are introduced for $\beta \in \{x, y\}$:

$$\{T_\beta^{(1)}, T_\beta^{(z)}, T_\beta^{(z^2)}, T_\beta^{(f)}, T_\beta^{(zf)}, T_\beta^{(f^2)}\} = \int_{-h/2}^{h/2} \sigma_{\beta\beta}^{th}(z) \{1, z, z^2, f(z), zf(z), f^2(z)\} dz. \quad (78)$$

These coefficients are arranged into the thermal moment matrix

$$\mathbf{T}_\beta^{(m)} = \begin{bmatrix} T_\beta^{(1)} & T_\beta^{(z)} & T_\beta^{(f)} \\ T_\beta^{(z)} & T_\beta^{(z^2)} & T_\beta^{(zf)} \\ T_\beta^{(f)} & T_\beta^{(zf)} & T_\beta^{(f^2)} \end{bmatrix}, \quad (79)$$

and the corresponding block operator

$$\hat{\mathbf{T}}_\beta = \mathbf{T}_\beta^{(m)} \otimes \mathbf{1}_3. \quad (80)$$

The thermal stress field in Eq. (77) acts as an initial in-plane stress in the subsequent vibration analysis. For small-amplitude oscillations about the thermally prestressed equilibrium configuration, the incremental elastic strains remain linear in the displacement perturbation, whereas the geometric contribution generated by the initial stress arises from the quadratic part of the Green-Lagrange strain. Because the thermal shear stress is zero, the corresponding positive virtual-work measure is

$$\delta W_{th} = \sum_{\beta \in \{x,y\}} \int_{\Omega} \sigma_{\beta\beta}^{th}(z) \delta \mathbf{u}_{,\beta}^T \mathbf{u}_{,\beta} d\Omega, \quad \text{with} \quad \mathbf{u} = [u, v, w]^T. \quad (81)$$

In Eq. (77), σ_x^{th} and σ_y^{th} are introduced as positive thermal-stress magnitudes. Under restrained heating, the actual prestress is compressive; hence, the thermal geometric contribution enters the total virtual-work statement with a negative sign. After substitution of the HSDT displacement field, the through-thickness reduction of this contribution is represented by the thermal moment operators defined in Eq. (79).

Because the sandwich nanoplate consists of two FG face sheets and a TPMS core with distinct through-thickness material distributions, all thickness integrations are performed layer by layer. For a generic scalar integrand $\Psi(z)$,

$$\int_{-h/2}^{h/2} \Psi(z) dz = \int_{h_0}^{h_1} \Psi(z) dz + \int_{h_1}^{h_2} \Psi(z) dz + \int_{h_2}^{h_3} \Psi(z) dz. \quad (82)$$

At each through-thickness integration point, the local temperature and the corresponding material properties are evaluated according to Sections 2.1 and 2.2. The same layer-wise integration procedure is consistently employed to construct the elastic stiffness, transverse-shear stiffness, inertia, and thermal initial-stress coefficients used in the subsequent variational formulation.

2.5. Elastic foundation model

The interaction between the FG-TPMS sandwich nanoplate and the supporting medium is modeled using the two-parameter Winkler-Pasternak foundation. In this representation, the Winkler parameter k_w characterizes the local normal stiffness of the foundation, whereas the

Pasternak parameter k_s accounts for the shear interaction between neighboring foundation points. Since the transverse displacement defined in Eq. (54) is independent of the thickness coordinate z , the displacement transmitted to the plate-foundation interface is directly represented by $w_0(x, y, t)$.

Accordingly, the transverse foundation reaction is expressed as [40]

$$q_f = -k_w w_0 + k_s \nabla^2 w_0, \quad \nabla^2 w_0 = \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}. \quad (83)$$

The first term represents the local Winkler restoring force, whereas the second term introduces the shear-layer coupling associated with the Pasternak model. The corresponding elastic potential energy stored in the foundation is

$$U_f = \frac{1}{2} \int_{\bar{\Omega}} [k_w w_0^2 + k_s (w_{0,x}^2 + w_{0,y}^2)] d\bar{\Omega}. \quad (84)$$

where $\bar{\Omega}$ denotes the midsurface domain of the nanoplate.

Taking the first variation of Eq. (84) gives the foundation contribution to the variational formulation,

$$\delta W_f \equiv \delta U_f = \int_{\bar{\Omega}} [k_w w_0 \delta w_0 + k_s (w_{0,x} \delta w_{0,x} + w_{0,y} \delta w_{0,y})] d\bar{\Omega}. \quad (85)$$

Eq. (85) represents the positive elastic contribution of the supporting medium and will be incorporated directly into the combined plate-level virtual-work statement developed in the next section. Hereafter,

2.6. Weak form based on the principle of virtual displacements

The governing weak form of the FG-TPMS sandwich nanoplate is obtained by combining the elastic, strain-gradient, thermal initial-stress, foundation, conventional inertia, and non-local inertia contributions introduced in Sections 2.3-2.5. Accordingly, the principle of virtual displacements gives

$$\delta W_e + \delta W_{sg} - \delta W_{th} + \delta W_f + \delta W_I + \delta W_{nll} = 0. \quad (86)$$

The negative sign preceding δW_{th} accounts for the compressive character of the thermally induced prestress under restrained heating, consistent with the sign convention introduced in Section 2.4. Eq. (86) therefore represents the complete dynamic variational statement prior to reduction through the plate thickness.

To perform this reduction systematically, the physical displacement field in Eq. (54) is reorganized according to the three thickness functions 1, z , and $f(z)$. The displacement vector can be written as

$$\mathbf{u}(z) = \mathbf{u}^{(0)} + z\mathbf{u}^{(1)} + f(z)\mathbf{u}^{(f)}, \quad (87)$$

with

$$\mathbf{u}^{(0)} = \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix}, \quad \mathbf{u}^{(1)} = \begin{Bmatrix} -w_{0,x} \\ -w_{0,y} \\ 0 \end{Bmatrix}, \quad \mathbf{u}^{(f)} = \begin{Bmatrix} \theta_x \\ \theta_y \\ 0 \end{Bmatrix}. \quad (88)$$

For compactness, these three vector fields are collected into the generalized kinematic vector

$$\mathbf{U} = \begin{Bmatrix} \mathbf{u}^{(0)} \\ \mathbf{u}^{(1)} \\ \mathbf{u}^{(f)} \end{Bmatrix}. \quad (89)$$

Because the thickness functions depend only on z , differentiation with respect to an in-plane coordinate $\beta \in \{x, y\}$ gives

$$\mathbf{u}_{,\beta}(z) = \mathbf{Z}(z)\mathbf{U}_{,\beta}, \quad (90)$$

where

$$\mathbf{Z}(z) = [\mathbf{1}_3 \quad z\mathbf{1}_3 \quad f(z)\mathbf{1}_3]. \quad (91)$$

This representation is particularly convenient because the same thickness basis $\{1, z, f(z)\}$ is used consistently in the elastic, inertia, and thermal initial-stress terms derived in Section 2.4.

Substituting the HSDT kinematics into Eq. (86), introducing the thickness-integrated stiffness, inertia, and thermal stress-moment coefficients defined in Section 2.4, and carrying out

the layer-wise integrations through the lower FG face sheet, TPMS core, and upper FG face sheet reduce the variational statement to the plate midsurface $\bar{\Omega}$. The resulting two-dimensional weak form is

$$\begin{aligned}
0 = & \int_{\bar{\Omega}} (\delta \bar{\boldsymbol{\varepsilon}}^T \mathbf{C}_b \bar{\boldsymbol{\varepsilon}} + \delta \boldsymbol{\theta}^T \mathbf{J}_s \boldsymbol{\theta}) d\bar{\Omega} \\
& + \lambda \int_{\bar{\Omega}} \sum_{\vartheta \in \{x,y\}} (\delta \bar{\boldsymbol{\varepsilon}}_{,\vartheta}^T \mathbf{C}_b \bar{\boldsymbol{\varepsilon}}_{,\vartheta} + \delta \boldsymbol{\theta}_{,\vartheta}^T \mathbf{J}_s \boldsymbol{\theta}_{,\vartheta}) d\bar{\Omega} \\
& - \sum_{\beta \in \{x,y\}} \int_{\bar{\Omega}} \delta \mathbf{U}_{,\beta}^T \hat{\mathbf{T}}_{\beta} \mathbf{U}_{,\beta} d\bar{\Omega} \\
& + \int_{\bar{\Omega}} [k_w \delta w_0 w_0 + k_s (\delta w_{0,x} w_{0,x} + \delta w_{0,y} w_{0,y})] d\bar{\Omega} \\
& + \int_{\bar{\Omega}} \delta \mathbf{U}^T \hat{\mathbf{I}}_m \ddot{\mathbf{U}} d\bar{\Omega} \\
& + \mu \int_{\bar{\Omega}} \sum_{\vartheta \in \{x,y\}} \delta \mathbf{U}_{,\vartheta}^T \hat{\mathbf{I}}_m \ddot{\mathbf{U}}_{,\vartheta} d\bar{\Omega}.
\end{aligned} \tag{92}$$

The six groups in Eq. (92) have direct physical interpretations. The first represents the classical HSDT elastic contribution, including both in-plane and transverse-shear deformation. The second introduces the strain-gradient stiffness governed by λ . The third corresponds to the geometric contribution of the thermally induced initial stress, whereas the fourth represents the elastic restraint supplied by the Winkler-Pasternak foundation. The final two terms describe the conventional and non-local inertia contributions, respectively.

Eq. (92) therefore provides the complete plate-level weak form governing the free vibration of the thermally prestressed FG-TPMS sandwich nanoplate. Importantly, each term can be traced directly to its corresponding contribution in Eq. (86), while all through-thickness heterogeneity associated with the FG face sheets, density-graded TPMS core, nonlinear temperature distribution, and thermal prestress is embedded in the thickness-integrated coefficients. This form provides the starting point for the high-continuity isogeometric discretization developed in the following section.

2.7. Isogeometric discretization

2.7.1. NURBS basis functions

The plate geometry and the generalized displacement fields are discretized within the isogeometric framework using the same NURBS basis functions. This unified geometric and field representation is particularly advantageous for the present NSGT-HSDT formulation because the strain-gradient terms involve higher-order spatial derivatives and therefore require basis functions with sufficient inter-element continuity.

Consider two open knot vectors associated with the parametric coordinates ξ and η [18],

$$\Xi^\xi = \{\xi_1, \xi_2, \dots, \xi_{n+p_\xi+1}\}, \quad \Xi^\eta = \{\eta_1, \eta_2, \dots, \eta_{m+p_\eta+1}\}, \quad (93)$$

where p_ξ and p_η denote the polynomial degrees in the ξ - and η -directions, respectively, while n and m are the corresponding numbers of univariate basis functions. The zeroth-degree B-spline basis functions in the ξ -direction are defined by [18]

$$N_{i,0}(\xi) = \begin{cases} 1, & \xi_i \leq \xi < \xi_{i+1}, \\ 0, & \text{otherwise,} \end{cases} \quad (94)$$

and the higher-degree functions follow from the Cox-de Boor recursion [18],

$$N_{i,p_\xi}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p_\xi} - \xi_i} N_{i,p_\xi-1}(\xi) + \frac{\xi_{i+p_\xi+1} - \xi}{\xi_{i+p_\xi+1} - \xi_{i+1}} N_{i+1,p_\xi-1}(\xi), \quad (95)$$

with zero contribution whenever a denominator vanishes. The basis functions $M_{j,p_\eta}(\eta)$ are obtained analogously. The tensor-product B-spline basis is

$$B_{ij}^{p_\xi, p_\eta}(\xi, \eta) = N_{i,p_\xi}(\xi) M_{j,p_\eta}(\eta). \quad (96)$$

Introducing the positive control-point weights w_{ij} , the bivariate NURBS basis functions are defined as [18]

$$R_{ij}^{p_\xi, p_\eta}(\xi, \eta) = \frac{N_{i,p_\xi}(\xi) M_{j,p_\eta}(\eta) w_{ij}}{\sum_{i=1}^n \sum_{j=1}^m N_{i,p_\xi}(\xi) M_{j,p_\eta}(\eta) w_{ij}}. \quad (97)$$

For convenience, the pair of indices (i, j) is subsequently replaced by a single global control-point

index A . The physical geometry is therefore represented as [18]

$$\mathbf{x}^h(\xi, \eta) = \sum_{A=1}^{n_{cp}} R_A(\xi, \eta) \mathbf{P}_A, \quad (98)$$

where $\mathbf{P}_A$ denotes the physical-coordinate vector of control point A and n_{cp} is the total number of control points.

The derivatives of the NURBS basis functions with respect to the physical coordinates are obtained through the geometry mapping. Introducing the Jacobian matrix [18]

$$\mathbf{J} = \frac{\partial(x, y)}{\partial(\xi, \eta)}, \quad (99)$$

the first-order derivatives satisfy [18]

$$\begin{Bmatrix} R_{A,x} \\ R_{A,y} \end{Bmatrix} = \mathbf{J}^{-T} \begin{Bmatrix} R_{A,\xi} \\ R_{A,\eta} \end{Bmatrix}. \quad (100)$$

Higher-order derivatives are obtained by successive application of the chain rule to the mapping defined in Eq. (98). This step is essential in the present formulation because the bending strains contain second derivatives of w_0 , while the strain-gradient contribution requires their in-plane gradients and therefore introduces third-order derivatives of w_0 .

Accordingly, cubic NURBS basis functions are employed in both parametric directions. With simple interior knots, a cubic spline basis possesses C^2 inter-element continuity, which provides the regularity required for direct evaluation of the third-order derivatives appearing in the strain-gradient operators. Consequently, the higher-order NSGT terms can be discretized without introducing auxiliary variables or mixed-field approximations, providing a natural connection between the high-continuity NURBS approximation and the weak form established in Section 2.6.

2.7.2. Discrete NSGT formulation

Using the NURBS basis functions introduced in Section 2.7.1, the five generalized displacement variables are approximated over the plate midsurface as [21]

$$\mathbf{q}^h(\xi, \eta, t) = \sum_{A=1}^{n_{cp}} R_A(\xi, \eta) \mathbf{q}_A(t), \quad \text{with} \quad \mathbf{q}_A = \{u_{0A} \quad v_{0A} \quad w_{0A} \quad \theta_{xA} \quad \theta_{yA}\}^T. \quad (101)$$

Substitution of Eq. (101) into the generalized strain measures defined in Section 2.4 gives

$$\bar{\boldsymbol{\varepsilon}}^h = \sum_{A=1}^{n_{cp}} \mathbf{B}_A^b \mathbf{q}_A, \quad \boldsymbol{\theta}^h = \sum_{A=1}^{n_{cp}} \mathbf{B}_A^s \mathbf{q}_A. \quad (102)$$

where

$$\mathbf{B}_A^b = \begin{bmatrix} \mathbf{B}_A^0 \\ \mathbf{B}_A^\kappa \\ \mathbf{B}_A^h \end{bmatrix} \quad \text{and} \quad \mathbf{B}_A^s = \begin{bmatrix} 0 & 0 & 0 & R_A & 0 \\ 0 & 0 & 0 & 0 & R_A \end{bmatrix}, \quad (103)$$

with

$$\mathbf{B}_A^0 = \begin{bmatrix} R_{A,x} & 0 & 0 & 0 & 0 \\ 0 & R_{A,y} & 0 & 0 & 0 \\ R_{A,y} & R_{A,x} & 0 & 0 & 0 \end{bmatrix}, \quad (104)$$

$$\mathbf{B}_A^\kappa = \begin{bmatrix} 0 & 0 & -R_{A,xx} & 0 & 0 \\ 0 & 0 & -R_{A,yy} & 0 & 0 \\ 0 & 0 & -2R_{A,xy} & 0 & 0 \end{bmatrix}, \quad (105)$$

$$\mathbf{B}_A^h = \begin{bmatrix} 0 & 0 & 0 & R_{A,x} & 0 \\ 0 & 0 & 0 & 0 & R_{A,y} \\ 0 & 0 & 0 & R_{A,y} & R_{A,x} \end{bmatrix}. \quad (106)$$

The strain-gradient contribution in Eq. (92) requires the in-plane derivatives of the generalized strain fields. These are evaluated directly from the differentiated NURBS basis functions as

$$\nabla \bar{\boldsymbol{\varepsilon}}^h = \sum_{A=1}^{n_{cp}} \mathbf{B}_A^{b,\nabla} \mathbf{q}_A \quad \text{and} \quad \nabla \boldsymbol{\theta}^h = \sum_{A=1}^{n_{cp}} \mathbf{B}_A^{s,\nabla} \mathbf{q}_A, \quad (107)$$

with

$$\mathbf{B}_A^{(\cdot),\nabla} = \begin{bmatrix} \mathbf{B}_{A,x}^{(\cdot)} \\ \mathbf{B}_{A,y}^{(\cdot)} \end{bmatrix}. \quad (108)$$

To discretize the inertia and thermal initial-stress contributions, the grouped physical displacement vector introduced in Eq. (91) is interpolated in terms of the same control variables:

$$\mathbf{U}^h = \sum_{A=1}^{n_{cp}} \mathbf{N}_A^U \mathbf{q}_A, \quad \text{with} \quad \mathbf{N}_A^U = \begin{bmatrix} \mathbf{N}_A^0 \\ \mathbf{N}_A^z \\ \mathbf{N}_A^f \end{bmatrix}, \quad (109)$$

where

$$\mathbf{N}_A^0 = \begin{bmatrix} R_A & 0 & 0 & 0 & 0 \\ 0 & R_A & 0 & 0 & 0 \\ 0 & 0 & R_A & 0 & 0 \end{bmatrix}, \quad (110)$$

$$\mathbf{N}_A^z = \begin{bmatrix} 0 & 0 & -R_{A,x} & 0 & 0 \\ 0 & 0 & -R_{A,y} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (111)$$

$$\mathbf{N}_A^f = \begin{bmatrix} 0 & 0 & 0 & R_A & 0 \\ 0 & 0 & 0 & 0 & R_A \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (112)$$

Consequently, the in-plane derivatives of the grouped displacement field and the transverse displacement are expressed as

$$\mathbf{G}_\beta^h = \sum_{A=1}^{n_{cp}} \mathbf{N}_{A,\beta}^U \mathbf{q}_A \quad \text{and} \quad w_0^h = \sum_{A=1}^{n_{cp}} \mathbf{N}_A^w \mathbf{q}_A, \quad (113)$$

with

$$\mathbf{N}_A^w = [0 \quad 0 \quad R_A \quad 0 \quad 0]. \quad (114)$$

Substitution of the above approximations into the plate-level weak form in Eq. (92) yields the discrete operators governing the vibration problem. The classical elastic stiffness matrix is

$$\mathbf{K}_e = \int_{\bar{\Omega}} [(\mathbf{B}^b)^T \mathbf{C}_b \mathbf{B}^b + (\mathbf{B}^s)^T \mathbf{J}_s \mathbf{B}^s] d\bar{\Omega}, \quad (115)$$

whereas the strain-gradient stiffness matrix is

$$\mathbf{K}_{sg} = \lambda \int_{\bar{\Omega}} \sum_{\vartheta \in \{x,y\}} \left[(\mathbf{B}_{,\vartheta}^b)^T \mathbf{c}_b \mathbf{B}_{,\vartheta}^b + (\mathbf{B}_{,\vartheta}^s)^T \mathbf{J}_s \mathbf{B}_{,\vartheta}^s \right] d\bar{\Omega}. \quad (116)$$

The first matrix represents the conventional HSDT elastic stiffness, while the second accounts for the additional stiffness associated with the strain-gradient mechanism.

Using the thermal stress-moment operators defined in Eq. (80), the geometric stiffness associated with the thermally induced initial stress is obtained as

$$\mathbf{K}_T = \int_{\bar{\Omega}} \sum_{\beta \in \{x,y\}} (\mathbf{N}_{,\beta}^U)^T \hat{\mathbf{T}}_{\beta} \mathbf{N}_{,\beta}^U d\bar{\Omega}. \quad (117)$$

The contribution of the Winkler-Pasternak foundation is discretized as

$$\mathbf{K}_f = \int_{\bar{\Omega}} \left[k_w (\mathbf{N}^w)^T \mathbf{N}^w + k_s \left((\mathbf{N}_{,x}^w)^T \mathbf{N}_{,x}^w + (\mathbf{N}_{,y}^w)^T \mathbf{N}_{,y}^w \right) \right] d\bar{\Omega}. \quad (118)$$

Similarly, the conventional and non-local inertia terms lead to mass matrices

$$\mathbf{M}_0 = \int_{\bar{\Omega}} (\mathbf{N}^U)^T \hat{\mathbf{I}}_m \mathbf{N}^U d\bar{\Omega}, \quad \mathbf{M}_{nl} = \mu \int_{\bar{\Omega}} \sum_{\vartheta \in \{x,y\}} (\mathbf{N}_{,\vartheta}^U)^T \hat{\mathbf{I}}_m \mathbf{N}_{,\vartheta}^U d\bar{\Omega}. \quad (119)$$

The resulting global stiffness and mass matrices are therefore assembled as

$$\mathbf{K} = \mathbf{K}_e + \mathbf{K}_{sg} + \mathbf{K}_f - \mathbf{K}_T, \quad \mathbf{M} = \mathbf{M}_0 + \mathbf{M}_{nl}. \quad (120)$$

The negative sign of $\mathbf{K}_T$ follows directly from the thermal-stress convention introduced in Section 2.4: restrained heating produces a compressive initial stress and consequently reduces the effective tangent stiffness governing small-amplitude vibration.

After assembly over all knot-span elements and imposition of the prescribed essential boundary conditions, the semi-discrete equation of motion becomes

$$\mathbf{M} \ddot{\mathbf{d}} + \mathbf{K} \mathbf{d} = \mathbf{0}, \quad (121)$$

where $\mathbf{d}$ is the global vector of control variables. For free harmonic vibration, the displacement vector is assumed in the form

$$\mathbf{d}(t) = \mathbf{\Phi} e^{i\omega t}, \quad (122)$$

where ω is the circular natural frequency and $\mathbf{\Phi}$ is the corresponding mode-shape vector. Substitution of Eq. (122) into Eq. (121) gives the generalized eigenvalue problem,

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{\Phi} = \mathbf{0}. \quad (123)$$

Nontrivial solutions of Eq. (123) exist for the eigenfrequencies ω satisfying the characteristic equation. The associated eigenvectors define the vibration modes of the FG-TPMS sandwich nanoplate. Through $\mathbf{K}$ and $\mathbf{M}$, the resulting eigenproblem incorporates simultaneously the effects of material and density gradation, TPMS topology, non-local softening, strain-gradient hardening, thermal prestress, temperature-dependent constitutive properties, and Winkler-Pasternak foundation support.

3. Computational results and discussion

3.1. Verification studies

The accuracy and consistency of the proposed formulation are assessed through a sequence of benchmark problems designed to verify the principal components of the model separately. Specifically, the first benchmark examines the non-local and strain-gradient size effects, the second evaluates the Winkler-Pasternak foundation contribution, and the last two verify the thermal response of functionally graded sandwich nanoplates under uniform and nonlinear through-thickness temperature fields. In all cases, the present results are compared with previously published solutions obtained for equivalent material, geometric, and loading conditions.

Table 2. Material properties of the constituents used in the validation examples

Material properties	SUS304	Si ₃ N ₄	Al	Al ₂ O ₃
E (GPa)	201.04	348.43	70	380
ν	0.3	0.3	0.3	0.3
ρ (kg/m ³)	8166	2370	2707	3800

Table 3. Nondimensional fundamental frequency of a simply supported SUS304/Si₃N₄ functionally graded square nanoplate for different non-local and strain-gradient parameters.

n	l	Method	μ			
			0	1	2	4
1	0	Present	0.0541	0.0494	0.0458	0.0404
		Thai et al. [38]	0.0542	0.0495	0.0459	0.0405
	1	Present	0.0592	0.0541	0.0501	0.0442
		Thai et al. [38]	0.0593	0.0542	0.0502	0.0443
	2	Present	0.0724	0.0661	0.0613	0.0541
		Thai et al. [38]	0.0723	0.0660	0.0612	0.054
	4	Present	0.1103	0.1008	0.0934	0.0825
		Thai et al. [38]	0.1094	0.0999	0.0925	0.0816
5	0	Present	0.0437	0.0399	0.037	0.0327
		Thai et al. [38]	0.0438	0.0400	0.0371	0.0327
	1	Present	0.0478	0.0437	0.0405	0.0357
		Thai et al. [38]	0.0479	0.0438	0.0406	0.0358
	2	Present	0.0585	0.0534	0.0495	0.0437
		Thai et al. [38]	0.0584	0.0533	0.0494	0.0436
	4	Present	0.0891	0.0814	0.0754	0.0666
		Thai et al. [38]	0.0884	0.0807	0.0747	0.0659
10	0	Present	0.0415	0.0379	0.0351	0.031
		Thai et al. [38]	0.0416	0.038	0.0352	0.0311
	1	Present	0.0454	0.0415	0.0384	0.0339
		Thai et al. [38]	0.0455	0.0415	0.0385	0.034
	2	Present	0.0555	0.0507	0.047	0.0415
		Thai et al. [38]	0.0554	0.0506	0.0469	0.0414
	4	Present	0.0846	0.0773	0.0716	0.0632
		Thai et al. [38]	0.0839	0.0766	0.0709	0.0626

The first benchmark concerns the fundamental frequency of a simply supported SUS304/Si₃N₄ functionally graded square nanoplate and is used to verify the implementation of the non-local strain gradient theory. The constituent material properties are summarized in Table 2. The plate has side length $a = 10$ and a length-to-thickness ratio $a/h = 10$. The effective properties vary continuously through thickness according to a power-law distribution with material gradient indices $n = 1, 5$, and 10 . The non-local parameter is taken as $\mu = 0, 1, 2$, and 4 , while the strain-gradient length-scale parameter is selected as $l = 0, 1, 2$, and 4 . The fundamental frequency is reported in the nondimensional form [38]

$$\bar{\omega} = \omega h \sqrt{\frac{\rho_c}{G_c}}, \quad (124)$$

with

$$G_c = \frac{E_c}{2(1 + \nu_c)}, \quad (125)$$

where E_c , ν_c , ρ_c , and G_c denote the elastic modulus, Poisson's ratio, mass density, and shear modulus of the ceramic constituent, respectively. The dimensional convention adopted in Section 2.3 is retained: μ has units of length squared, whereas l has units of length. The results are compared with those reported by Thai et al. [38] in Table 3. Excellent agreement is obtained over the entire range of material gradation and small-scale parameters, confirming the correct implementation of both the non-local and strain-gradient contributions. For a fixed l , increasing μ consistently reduces the fundamental frequency, reflecting the stiffness-softening effect associated with non-local interactions. For example, at $n = 1$ and $l = 0$, $\bar{\omega}$ decreases from 0.0541 to 0.0404 as μ increases from 0 to 4. Conversely, for fixed μ , increasing l produces a pronounced increase in frequency owing to the strain-gradient hardening mechanism; at $n = 1$ and $\mu = 0$, $\bar{\omega}$ rises from 0.0541 at $l = 0$ to 0.1103 at $l = 4$. The frequency also decreases with increasing n , as the through-thickness composition becomes progressively richer in the metallic constituent and the effective stiffness is reduced. These trends are fully consistent with the reference solution and verify the size-dependent part of the present formulation.

The second benchmark evaluates the treatment of elastic-foundation effects. A simply supported Al/Al₂O₃ functionally graded square nanoplate resting on a two-parameter Winkler-Pasternak foundation is considered. The constituent properties are $E_m = 70$ GPa, $\rho_m = 2702$ kg/m³, and $\nu_m = 0.3$ for aluminum, and $E_c = 380$ GPa, $\rho_c = 3800$ kg/m³, and $\nu_c = 0.3$ for alumina, as summarized in Table 2. The material properties vary through the thickness according to [30,40]

$$\mathcal{P}(z) = \mathcal{P}_m + (\mathcal{P}_c - \mathcal{P}_m)(z/h + 1/2)^p, \quad (126)$$

where p is the material gradient index. Four thickness ratios, $h/a = 0.05, 0.10, 0.15$, and 0.20 , and four values of $p = 0, 0.5, 1$, and 2 are examined. The local continuum limit is adopted for this

benchmark. The nondimensional frequency is defined as [30,40]

$$\bar{\omega} = \omega h \sqrt{\rho_m / E_m}. \quad (127)$$

The dimensionless foundation parameters are $\bar{k}_w = k_w a^4 / D_m$ and $\bar{k}_s = k_s a^2 / D_m$, where $D_m = E_m h^3 / [12(1 - \nu_m^2)]$. Two support conditions are examined, namely $(\bar{k}_w, \bar{k}_s) = (100, 0)$ and $(100, 100)$, corresponding to a Winkler foundation and a combined Winkler-Pasternak foundation, respectively. The present results are compared with those of Baghdali et al. [30] and Sobhy [40] in Table 4. Very good agreement is obtained for all considered thickness ratios, material gradient indices, and foundation parameters. The addition of the Pasternak shear-layer contribution systematically increases the fundamental frequency because of the enhanced coupling and transverse restraint supplied by the supporting medium. This benchmark therefore confirms the correct implementation of both the Winkler and Pasternak terms in the present isogeometric formulation.

Table 4. Nondimensional fundamental frequency of a simply supported Al/Al₂O₃ functionally graded square nanoplate on a Winkler–Pasternak elastic foundation.

$\bar{k}_w$	$\bar{k}_s$	h/a	p	Present	Baghdali et al. [30]	Sobhy [40]
100	0	0.05	0	0.0298	0.0298	0.0298
			0.5	0.0256	0.0258	0.0256
			1	0.0233	0.0238	0.0233
			2	0.0214	0.0221	0.0214
		0.1	0	0.1170	0.1162	0.1162
			0.5	0.1006	0.1012	0.1000
			1	0.0917	0.0933	0.0911
			2	0.0842	0.0867	0.0837
		0.15	0	0.2552	0.2519	0.2518
			0.5	0.2203	0.2204	0.2174
			1	0.2008	0.2036	0.1982
			2	0.1841	0.1889	0.1818
		0.20	0	0.4349	0.4273	0.4272
			0.5	0.3772	0.3758	0.3704
			1	0.3441	0.3476	0.3382
			2	0.3147	0.3219	0.3097
100	100	0.05	0	0.0411	0.0411	0.0411
			0.5	0.0393	0.0395	0.0392
			1	0.0385	0.0388	0.0384

	2	0.0381	0.0386	0.0381
0.1	0	0.1630	0.1619	0.1619
	0.5	0.1560	0.1563	0.1550
	1	0.1529	0.1542	0.1520
	2	0.1515	0.1535	0.1505
0.15	0	0.3608	0.3560	0.3560
	0.5	0.3468	0.3460	0.3423
	1	0.3403	0.3422	0.3361
	2	0.3370	0.3412	0.3329
0.20	0	0.6272	0.6162	0.6161
	0.5	0.6062	0.6026	0.5954
	1	0.5956	0.5978	0.5855
	2	0.5894	0.5970	0.5802

Table 5. Temperature-dependent coefficients of elastic modulus $E(\text{Pa})$, coefficient of thermal expansion $\alpha(\text{K}^{-1})$, mass density $\rho(\text{kg/m}^3)$, thermal conductivity $K(\text{W/mK})$, and Poisson's ratio ν .

	$\mathcal{P}_0$	$\mathcal{P}_{-1}$	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$
ZrO ₂					
E	244.27×10^9	0	-1.371×10^{-3}	1.214×10^{-6}	-3.681×10^{-10}
α	12.766×10^{-6}	0	-1.491×10^{-3}	1.006×10^{-5}	-6.778×10^{-11}
ρ	3000	0	1.133×10^{-4}	0	0
k	1.8	0	0	0	0
ν	0.3	0	0	0	0
Ti-6Al-4V					
E	122.56×10^9	0	4.586×10^{-4}	0	0
α	7.5788×10^{-6}	0	6.638×10^{-4}	-3.147×10^{-6}	0
ρ	4429	0	0	0	0
k	7.82	0	0	0	0
ν	0.3	0	0	0	0

The third benchmark is introduced to assess the response of the formulation for functionally graded sandwich nanoplates with temperature-dependent material properties under a uniform thermal field. A simply supported Ti-6Al-4V/ZrO₂ sandwich square nanoplate is considered, with the constituent properties evaluated at $T_b = T_t = 400\text{K}$ using the temperature-dependent coefficients listed in Table 5. Three sandwich configurations, 1–1–1, 1–2–1, and 2–2–1, are examined. The power-law index varies over $p = 0, 0.5, 1, 1.5$, and 2, while the non-local parameter

is taken as $\mu = 0, 0.5, 1, 1.5$, and 2 . The geometry is defined by $a = b = 10, h = 1$, and $a/h = 10$. The nondimensional frequency is expressed as [37]

$$\bar{\omega} = \omega \frac{a^2}{h} \sqrt{\frac{(1 - \nu_0^2)\rho_0}{E_0}}, \quad (128)$$

where E_0, ρ_0 , and ν_0 denote the Ti-6Al-4V reference properties evaluated at $T_0 = 300K$. The present solutions are compared in Table 6 with those reported by Daikh et al. [37] for the same geometric and thermal conditions. The comparison shows good agreement for all considered values of p and μ and for all three sandwich schemes. The small remaining differences are modest and remain systematic over the parameter range. The frequency decreases monotonically with increasing μ , consistently reflecting the non-local softening effect. A similar reduction occurs as p increases because the sandwich becomes relatively richer in Ti-6Al-4V and therefore less stiff. The layer-thickness arrangement also influences the vibration response: for $p > 0$, the 1-2-1 configuration generally produces the highest frequency, followed by the 2-2-1 and 1-1-1 schemes. For $p = 0$, all three schemes reduce to the same fully ceramic face-sheet configuration and consequently yield identical frequencies. Overall, this benchmark verifies the temperature-dependent sandwich formulation under a spatially uniform thermal state.

Table 6. Nondimensional fundamental frequency of a simply supported Ti-6Al-4V/ZrO₂ functionally graded sandwich square nanoplate in a uniform thermal field.

p	μ	1-1-1		1-2-1		2-2-1	
		Present	Daikh et al. [37]	Present	Daikh et al. [37]	Present	Daikh et al. [37]
0	0	7.8437	8.0194	7.8437	8.0194	7.8437	8.0194
	0.5	7.4831	7.6507	7.4831	7.6507	7.4831	7.6507
	1	7.1681	7.3286	7.1681	7.3286	7.1681	7.3286
	1.5	6.8898	7.0441	6.8898	7.0441	6.8898	7.0441
	2	6.6415	6.7903	6.6415	6.7903	6.6415	6.7903
0.5	0	6.9336	7.0416	7.1090	7.2308	7.0148	7.1292
	0.5	6.6149	6.7179	6.7822	6.8984	6.6923	6.8015
	1	6.3364	6.4351	6.4967	6.6079	6.4106	6.5151
	1.5	6.0904	6.1852	6.2444	6.3514	6.1617	6.2621

	2	5.8709	5.9624	6.0194	6.1225	5.9397	6.0365
	0	6.5394	6.6215	6.7775	6.8773	6.6510	6.7418
	0.5	6.2387	6.3171	6.4659	6.5611	6.3453	6.4319
1	1	5.9761	6.0512	6.1937	6.2849	6.0781	6.1611
	1.5	5.7441	5.8162	5.9532	6.0409	5.8421	5.9219
	2	5.5371	5.6067	5.7387	5.8232	5.6316	5.7085
	0	6.1980	6.2580	6.4783	6.5584	6.3321	6.4030
	0.5	5.9131	5.9703	6.1805	6.2569	6.0410	6.1086
1.5	1	5.6641	5.7189	5.9203	5.9934	5.7867	5.8515
	1.5	5.4442	5.4969	5.6904	5.7607	5.5620	5.6243
	2	5.2480	5.2988	5.4854	5.5532	5.3616	5.4216
	0	5.9248	5.9635	6.2226	6.2833	6.0703	6.1241
	0.5	5.6525	5.6893	5.9366	5.9944	5.7913	5.8426
2	1	5.4145	5.4498	5.6867	5.7421	5.5475	5.5966
	1.5	5.2043	5.2382	5.4658	5.5191	5.3321	5.3793
	2	5.0168	5.0495	5.2689	5.3202	5.1400	5.1855

The final benchmark extends the thermal verification to a nonlinear through-thickness temperature distribution. A simply supported FGM-A Ti-6Al-4V/ZrO₂ sandwich square nanoplate is considered with the bottom-surface temperature fixed at $T_b = 300\text{K}$ and the top-surface temperature prescribed as $T_t = 600\text{K}$. The same three sandwich schemes, 1–1–1, 1–2–1, and 2–2–1, are investigated. The material gradient index varies from $p = 0$ to 2, and the non-local parameter is taken as $\mu = 0, 0.5, 1, 1.5$, and 2. The nondimensional frequency is defined using the same normalization as in the preceding benchmark, see Eq. (127), with the reference Ti-6Al-4V properties evaluated at $T_0 = 300\text{ K}$. The corresponding results are compared with those of Daikh et al. [37] in Table 7. Good agreement is again obtained over the complete parameter range, demonstrating that the present formulation reproduces the vibration characteristics of FGM sandwich nanoplates subjected to a nonuniform thermal field. Increasing p leads to a progressive decrease in the fundamental frequency because the face sheets contain a larger proportion of the metallic constituent over the thickness and hence exhibit lower effective stiffness. The three sandwich configurations also produce slightly different frequencies, reflecting the influence of layer arrangement on the through-thickness stiffness distribution. The consistent agreement with the reference solutions confirms the capability of the present formulation to account

simultaneously for material gradation, non-local size effects, temperature-dependent properties, and nonlinear through-thickness thermal variation.

Table 7. Nondimensional fundamental frequency of a simply supported Ti-6Al-4V/ZrO₂ functionally graded sandwich square nanoplate under a nonlinear through-thickness temperature field.

p	μ	1-1-1		1-2-1		2-2-1	
		Present	Daikh et al. [37]	Present	Daikh et al. [37]	Present	Daikh et al. [37]
0	0	5.7948	5.9306	5.7948	5.9306	5.7948	5.9306
	0.5	5.3165	5.4443	5.3165	5.4443	5.3165	5.4443
	1	4.8813	5.0020	4.8813	5.0020	4.8813	5.0020
	1.5	4.4794	4.5939	4.4794	4.5939	4.4794	4.5939
	2	4.1031	4.2121	4.1031	4.2121	4.1031	4.2121
0.5	0	5.3518	5.4450	5.3984	5.4986	5.4175	5.5064
	0.5	4.9478	5.0364	4.9770	5.0717	5.0080	5.0917
	1	4.5831	4.6678	4.5955	4.6854	4.6383	4.7174
	1.5	4.2495	4.3309	4.2454	4.3311	4.3001	4.3752
	2	3.9407	4.0191	3.9199	4.0019	3.9869	4.0584
1	0	5.1775	5.2504	5.2283	5.3123	5.2253	5.2980
	0.5	4.8067	4.8759	4.8333	4.9126	4.8439	4.9126
	1	4.4734	4.5396	4.4768	4.5520	4.5008	4.5659
	1.5	4.1702	4.2335	4.1507	4.2223	4.1880	4.2499
	2	3.8910	3.9520	3.8488	3.9171	3.8994	3.9585
1.5	0	5.0471	5.1030	5.0853	5.1547	5.0698	5.1287
	0.5	4.7061	4.7591	4.7150	4.7803	4.7139	4.7697
	1	4.4010	4.4516	4.3817	4.4435	4.3947	4.4477
	1.5	4.1248	4.1732	4.0779	4.1366	4.1047	4.1553
	2	3.8720	3.9185	3.7978	3.8537	3.8383	3.8867
2	0	4.9747	5.0168	4.9815	5.0388	4.9711	5.0173
	0.5	4.6593	4.6994	4.6336	4.6874	4.6377	4.6815
	1	4.3785	4.4169	4.3215	4.3724	4.3397	4.3814
	1.5	4.1255	4.1625	4.0382	4.0864	4.0701	4.1098
	2	3.8953	3.9309	3.7780	3.8238	3.8234	3.8615

Taken together, the four benchmark studies verify the principal ingredients of the proposed model in a modular manner: the first establishes the accuracy of the NSGT size-dependent operators, the second validates the elastic-foundation contribution, and the third and fourth confirm the temperature-dependent sandwich formulation under uniform and nonuniform thermal environments, respectively. These comparisons provide the numerical basis for the subsequent

parametric investigation of FG-TPMS sandwich nanoplates.

3.2. Parametric studies and physical discussion

Following the verification studies, the proposed formulation is employed to investigate the combined influences of small-scale parameters, TPMS topology, core-density distribution, face-sheet gradation, thermal loading, core-thickness ratio, and elastic-foundation stiffness on the fundamental vibration response of FG-TPMS sandwich nanoplates. Unless otherwise specified, the nanoplate is simply supported, the constituent material properties are those listed in Table 5, and the frequency is normalized with respect to the Ti-6Al-4V reference properties.

Table 8. Nondimensional fundamental frequency of a simply supported FG-TPMS sandwich square nanoplate for different sandwich schemes, TPMS topologies, and small-scale parameters (A1 core-density pattern).

Sandwich layer	TPMS type	l	μ			
			0	1	2	4
1-1-1	P	0	0.1264	0.1155	0.107	0.0945
		1	0.1383	0.1264	0.1171	0.1034
		2	0.1691	0.1545	0.1432	0.1264
		4	0.2578	0.2356	0.2183	0.1927
	G	0	0.126	0.1152	0.1067	0.0942
		1	0.1379	0.126	0.1168	0.1031
		2	0.1686	0.1541	0.1427	0.126
		4	0.257	0.2348	0.2176	0.1921
	IWP	0	0.1259	0.1151	0.1066	0.0942
		1	0.1378	0.1259	0.1167	0.103
		2	0.1685	0.154	0.1427	0.1259
		4	0.2568	0.2347	0.2175	0.192
1-2-1	P	0	0.1259	0.115	0.1066	0.0941
		1	0.1377	0.1259	0.1166	0.103
		2	0.1684	0.1539	0.1426	0.1259
		4	0.2567	0.2346	0.2174	0.1919
	G	0	0.1248	0.114	0.1057	0.0933
		1	0.1366	0.1248	0.1156	0.1021
		2	0.1669	0.1526	0.1414	0.1248
		4	0.2545	0.2326	0.2155	0.1902
	IWP	0	0.1246	0.1139	0.1055	0.0931
		1	0.1364	0.1246	0.1155	0.1019
		2	0.1667	0.1523	0.1411	0.1246
		4	0.2541	0.2322	0.2152	0.1899

2-2-1	P	0	0.1237	0.1131	0.1048	0.0925
		1	0.1354	0.1237	0.1146	0.1012
		2	0.1655	0.1513	0.1402	0.1237
		4	0.2523	0.2306	0.2136	0.1886
	G	0	0.1231	0.1125	0.1043	0.092
		1	0.1347	0.1231	0.1141	0.1007
		2	0.1647	0.1505	0.1395	0.1231
		4	0.2511	0.2295	0.2126	0.1877
	IWP	0	0.1232	0.1126	0.1043	0.0921
		1	0.1348	0.1232	0.1142	0.1008
		2	0.1648	0.1506	0.1396	0.1232
		4	0.2513	0.2296	0.2128	0.1878

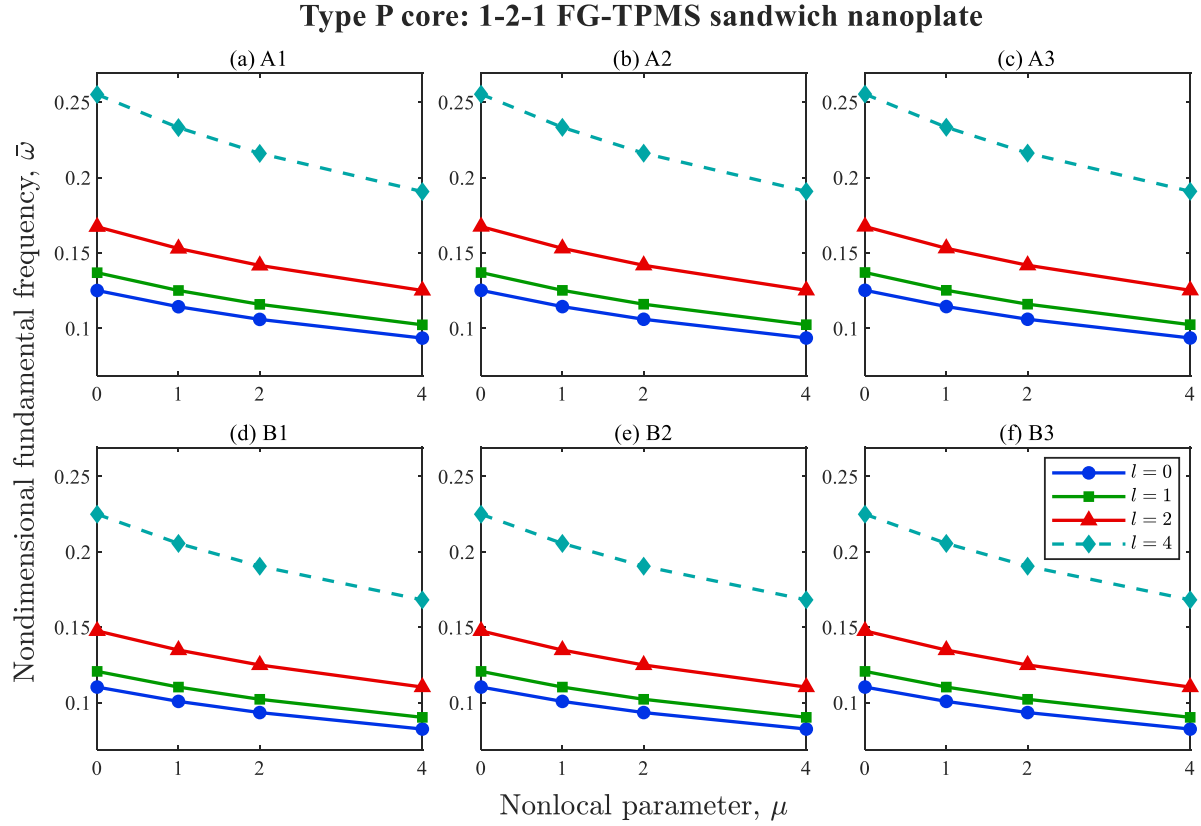


Fig. 3. Effect of the non-local parameter on the nondimensional fundamental frequency of the 1-2-1 FG-TPMS sandwich nanoplate with a P-type core for different density patterns and strain-gradient parameters.

Table 8 first examines the effects of the non-local and strain-gradient parameters together with the TPMS topology and sandwich thickness arrangement. A square nanoplate with $a = b =$

10, $h = 1$, and $\frac{a}{h} = 10$ is considered. The two face sheets are composed of FG Ti-6Al-4V/ZrO₂ with the power-law index $p = 1$, while the TPMS core follows the *A1* density pattern. For this pattern, ρ_{rel} varies from 0.20 to 0.50 through the core thickness with $k_z = 1$, giving an average relative density $\rho_{\text{avg}} = 0.35$. The material properties are evaluated at $T = 300\text{K}$. Three sandwich configurations, 1–1–1, 1–2–1, and 2–2–1, and three sheet-based TPMS topologies, P, G, and IWP, are considered. The nondimensional fundamental frequency is defined as

$$\bar{\omega} = \omega h \sqrt{\frac{\rho_{\text{Ti}}}{G_{\text{Ti}}}} \quad \text{with} \quad G_{\text{Ti}} = \frac{E_{\text{Ti}}}{2(1 + \nu_{\text{Ti}})}, \quad (129)$$

where $E_{\text{Ti}} = 105.698\text{GPa}$, $\nu_{\text{Ti}} = 0.3$, $\rho_{\text{Ti}} = 4429\text{kg/m}^3$. The strain-gradient length-scale parameter l and the non-local parameter μ are varied independently over 0, 1, 2, and 4, following the dimensional convention introduced in Section 2.3.

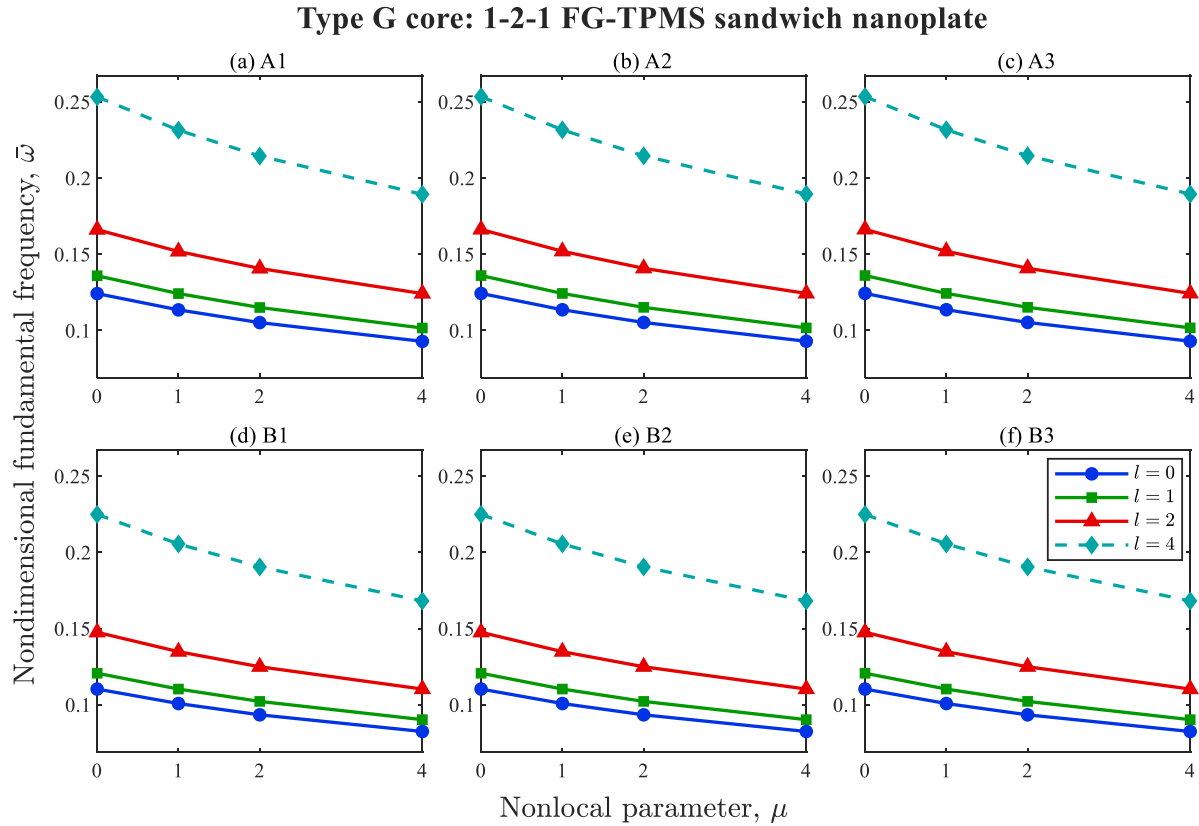


Fig. 4. Effect of the non-local parameter on the nondimensional fundamental frequency of the 1-2-1 FG-TPMS sandwich nanoplate with a G-type core for different density patterns and strain-gradient parameters.

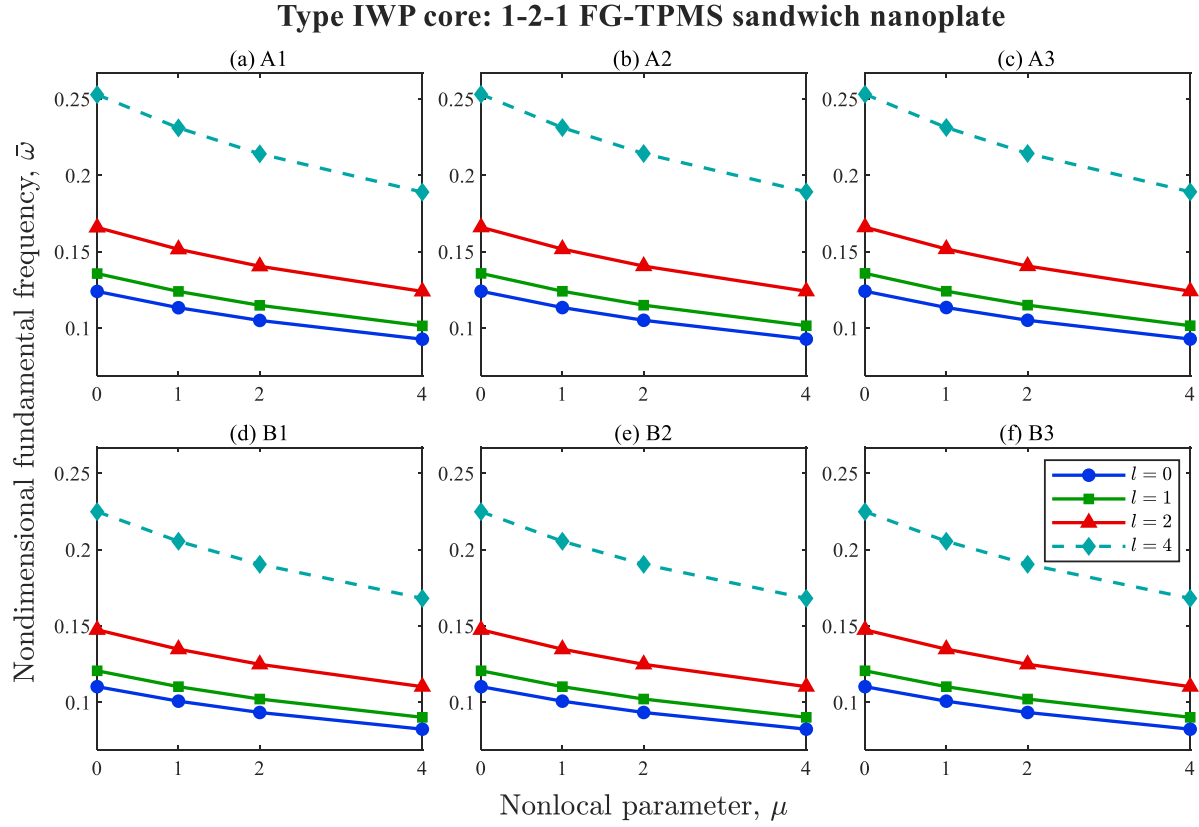


Fig. 5. Effect of the non-local parameter on the nondimensional fundamental frequency of the 1-2-1 FG-TPMS sandwich nanoplate with an IWP-type core for different density patterns and strain-gradient parameters.

Consistent competition between the two small-scale mechanisms is observed throughout Table 8. For a fixed sandwich configuration and TPMS topology, increasing l raises the fundamental frequency, whereas increasing μ reduces it. The former trend arises from the additional stiffness introduced by the strain-gradient contribution, while the latter reflects the softening effect associated with non-local interactions. The two mechanisms therefore act in opposite directions, and the resulting frequency is governed by their relative magnitudes. Among the three TPMS architectures, the P-type core generally produces the highest frequencies, whereas the G- and IWP-type cores give slightly lower and closely spaced values. The influence of the sandwich thickness arrangement is also evident, although less pronounced: the 1-1-1 configuration generally yields a slightly higher frequency than the 1-2-1 and 2-2-1 configurations. These results indicate that, within the investigated range, the vibration response is

dominated by the small-scale parameters, while TPMS topology and layer-thickness distribution provide secondary but systematic modifications to the overall stiffness-to-inertia balance.

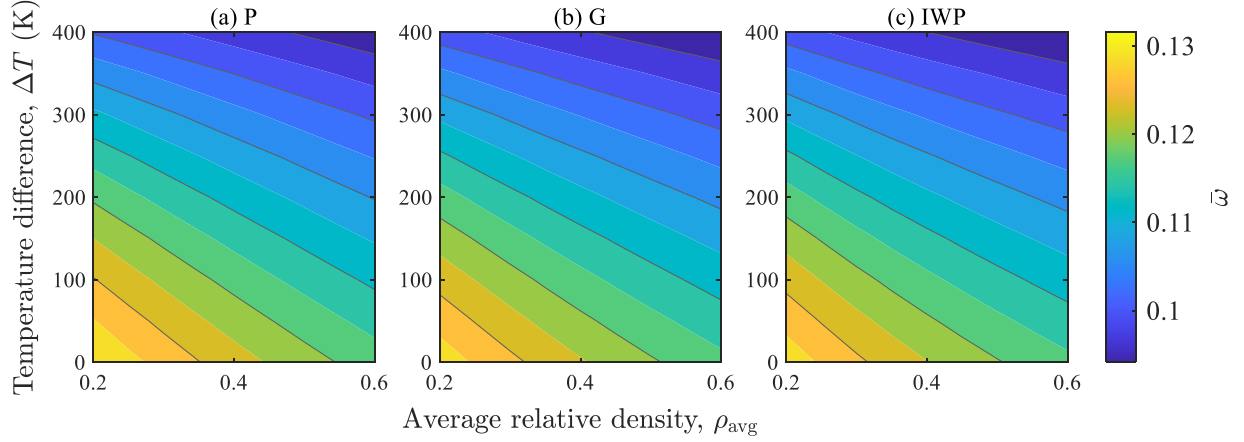


Fig. 6. Coupled effects of average core relative density and temperature difference on the nondimensional fundamental frequency of the 1-2-1 FG-TPMS sandwich nanoplate with pattern A1: (a) P-type core, (b) G-type core, and (c) IWP-type core.

The influence of the through-thickness core-density distribution is examined next in Fig. 3 to Fig. 5. To isolate this effect, the sandwich configuration is fixed as 1–2–1, with $a = b = 10$, $a/h = 10$, and $h = 1$, and $T = 300\text{K}$. The average relative density is maintained at $\rho_{\text{avg}} = 0.35$, while six mean-preserving density patterns, A1, A2, A3, B1, B2, and B3, are considered. Thus, differences among the patterns arise from redistribution of the same average amount of core material rather than from variations in the total core density. Fig. 3 to Fig. 5 correspond to the P-, G-, and IWP-type cores, respectively. For all three TPMS architectures and all six density patterns, increasing μ reduces $\bar{\omega}$, whereas increasing l increases it, confirming that the qualitative small-scale behavior is insensitive to the particular form of density gradation. More importantly, the A-type distributions consistently yield slightly higher frequencies than their B-type counterparts. Because the average relative density is identical for all cases, this difference is attributable to the through-thickness location of the core material rather than to its total amount. The relatively small differences among A1–A3 and among B1–B3 further indicate that, for the present geometry and material system, the vibration response is more sensitive to the general character of the density distribution (monotonic or symmetric) than to moderate changes in the intensity of gradation.

within each family. The similarity of the trends among P, G, and IWP also suggests that the effects of density redistribution and TPMS topology are largely separable within the investigated parameter range.

The interaction between the average core density and thermal loading is subsequently explored in Fig. 6. A simply supported 1–2–1 FG-TPMS sandwich nanoplate with $a = b = 10$, $h = 1$, and $a/h = 10$ is considered. The face-sheet gradient index is fixed at $p = 1$, and the TPMS core follows the $A1$ density pattern. The bottom-surface temperature is maintained at $T_b = 300\text{K}$, while the temperature difference ΔT is increased from 0 to 400K. The average relative density varies from $\rho_{\text{avg}} = 0.20$ to 0.60, and the non-local parameter is set to $\mu = 0$ so that the coupled density-temperature effect can be examined without an additional non-local softening contribution. The nonlinear through-thickness temperature field is obtained from the steady conduction model of Section 2.2, and the corresponding thermal geometric stiffness is included in the vibration analysis. The frequency is normalized as

$$\bar{\omega} = \omega h \sqrt{\frac{\rho_{Ti}(300)}{G_{Ti}(300)}}. \quad (130)$$

A progressive reduction in $\bar{\omega}$ is observed with increasing ΔT for all three TPMS topologies. This reduction results from the combined influence of temperature-dependent constitutive degradation and thermally induced compressive prestress. A less intuitive result is that $\bar{\omega}$ also decreases as ρ_{avg} increases over the range considered. Increasing the core density simultaneously increases both stiffness and inertia; the observed decrease in frequency indicates that, for the present FG-TPMS sandwich configurations, the associated increase in inertia dominates the gain in effective core stiffness. Consequently, increasing the amount of core material does not necessarily improve the vibration performance. This result highlights the importance of considering the stiffness-to-mass ratio rather than stiffness alone when selecting the relative density of a TPMS core. The P-type architecture again provides the highest frequency level, followed closely by the G- and IWP-type cores. Nevertheless, the contour patterns are nearly parallel among the three topologies, indicating that their sensitivities to average density and thermal loading are qualitatively similar. Thus, within the present parameter range, the TPMS topology mainly shifts the overall frequency level, whereas the dominant variation is governed by the coupled effects of thermal loading and the core stiffness-to-inertia balance.

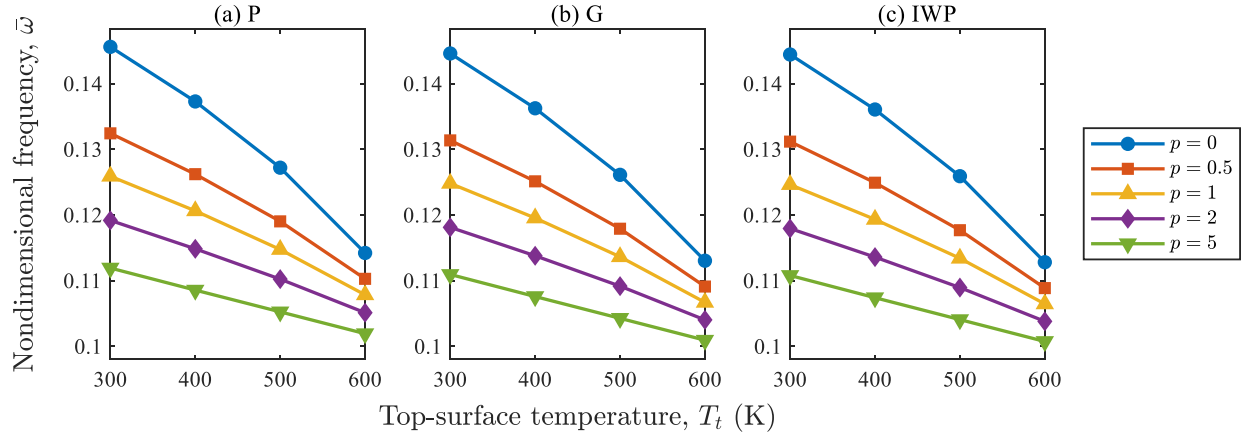


Fig. 7. Coupled effects of the face-sheet gradient index and top-surface temperature on the nondimensional fundamental frequency of the 1-2-1 FG-TPMS sandwich nanoplate with pattern A1: (a) P-type core, (b) G-type core, and (c) IWP-type core.

Fig. 7 further examines the interaction between face-sheet material gradation and temperature. The geometry, sandwich configuration, core-density pattern, and frequency normalization are retained from Fig. 6. The face-sheet power-law index is varied as $p = 0, 0.5, 1, 2$, and 5 , while the top-surface temperature increases from 300K to 600K. For each TPMS topology, the fundamental frequency decreases monotonically as p increases. According to the grading law introduced in Section 2.1, $p = 0$ corresponds to fully ceramic ZrO_2 face sheets, whereas increasing p enlarges the fraction of Ti-6Al-4V over the face-sheet thickness. The resulting reduction in effective face-sheet stiffness lowers the bending rigidity and hence the fundamental frequency. Increasing T_t produces an additional frequency reduction for every value of p because of the combined constitutive softening and compressive thermal-prestress effects. An additional feature evident from Fig. 7 is that the frequency differences among the various gradient indices become smaller as the temperature increases. Thermal loading therefore tends to attenuate the relative influence of face-sheet gradation, although the ordering of the curves remains unchanged throughout the investigated temperature range. This behavior indicates that the thermal contribution becomes increasingly dominant at elevated temperatures, reducing (but not eliminating) the relative sensitivity of the vibration response to material gradation. The P-type core

maintains a slightly higher frequency than the G- and IWP-type cores, while all three architectures exhibit essentially the same coupled p - T_t trend.

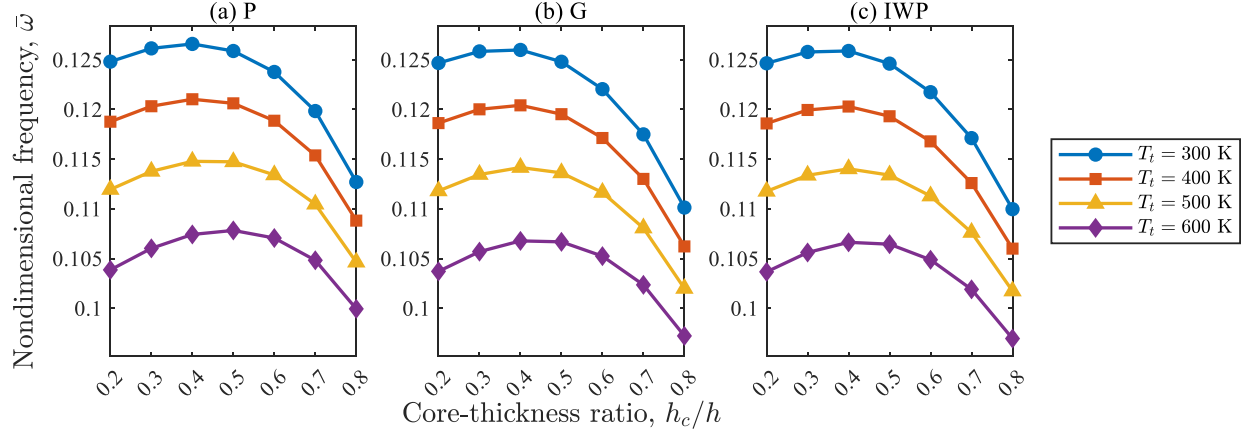


Fig. 8. Coupled effects of the core-thickness ratio and top-surface temperature on the nondimensional fundamental frequency of the FG-TPMS sandwich nanoplate with pattern $A1$: (a) P-type core, (b) G-type core, and (c) IWP-type core.

The combined influence of the core-thickness ratio and temperature is investigated in Fig. 8. The core-thickness ratio h_c/h is varied from 0.2 to 0.8, whereas the top-surface temperature is taken as $T_t = 300, 400, 500$, and 600 K. The remaining parameters are unchanged from the preceding thermal studies. Unlike the monotonic trends observed for p and ΔT , the fundamental frequency exhibits a distinctly non-monotonic dependence on h_c/h . For all three TPMS topologies, $\bar{\omega}$ initially increases as the core becomes thicker, reaches a maximum at an intermediate ratio of approximately $h_c/h = 0.4$, and subsequently decreases as the core occupies a larger fraction of the total thickness. This response reflects the competition between two opposing sandwich mechanisms. At relatively small h_c/h , increasing the core thickness increases the separation between the face sheets and improves the bending efficiency of the sandwich section. Beyond the intermediate range, however, further thickening of the relatively compliant TPMS core necessarily reduces the relative contribution of the stiffer FG face sheets. The resulting deterioration of the effective stiffness-to-inertia ratio eventually outweighs the benefit of increased face-sheet separation, producing the observed reduction in frequency. Increasing T_t shifts the entire family of curves downward but causes only a small change in the location of the maximum. The

favorable intermediate core-thickness ratio is therefore comparatively insensitive to temperature within the investigated range, even though the absolute frequency is substantially affected by thermal loading. The three TPMS architectures again display nearly identical qualitative behavior, with the P-type core producing slightly higher frequencies. These results demonstrate that neither an excessively thin nor an excessively thick TPMS core is optimal from the viewpoint of vibration performance; instead, an intermediate core-thickness ratio provides the most favorable stiffness-to-inertia balance for the present material system and parameter range.

Table 9. Effects of Winkler-Pasternak foundation stiffness, temperature, and small-scale parameters on the nondimensional fundamental frequency of the simply supported 1-2-1 FG-TPMS sandwich nanoplate with a P-type core (*A1* pattern).

$\bar{k}_w$	$\bar{k}_s$	$T_t(K)$	l	μ			
				0	1	2	4
0	0	300	0	0.1259	0.115	0.1066	0.0941
			1	0.1377	0.1259	0.1166	0.103
			2	0.1684	0.1539	0.1426	0.1259
			4	0.2567	0.2346	0.2174	0.1919
		400	0	0.1206	0.1102	0.1021	0.0902
			1	0.132	0.1206	0.1118	0.0987
			2	0.1614	0.1475	0.1366	0.1206
			4	0.246	0.2248	0.2083	0.1839
		500	0	0.1147	0.1048	0.0971	0.0858
			1	0.1255	0.1147	0.1063	0.0938
			2	0.1535	0.1403	0.13	0.1147
			4	0.234	0.2138	0.1981	0.1749
		600	0	0.1078	0.0985	0.0913	0.0806
			1	0.118	0.1078	0.0999	0.0882
			2	0.1442	0.1318	0.1221	0.1078
			4	0.2199	0.2009	0.1862	0.1644
	0	300	0	0.1405	0.1284	0.119	0.1051
			1	0.1538	0.1405	0.1302	0.115
			2	0.188	0.1718	0.1592	0.1405
			4	0.2866	0.2619	0.2427	0.2142
		400	0	0.1358	0.1241	0.115	0.1015
			1	0.1486	0.1358	0.1258	0.1111
			2	0.1817	0.1661	0.1539	0.1358
			4	0.277	0.2531	0.2345	0.207
		500	0	0.1306	0.1194	0.1106	0.0976
			1	0.1429	0.1306	0.121	0.1068

100	100	600	2	0.1747	0.1597	0.1479	0.1306
			4	0.2663	0.2434	0.2255	0.1991
			0	0.1246	0.1138	0.1055	0.0931
			1	0.1363	0.1246	0.1154	0.1019
			2	0.1666	0.1523	0.1411	0.1246
			4	0.254	0.2321	0.2151	0.1899
		300	0	0.3112	0.2844	0.2635	0.2327
			1	0.3406	0.3112	0.2884	0.2546
			2	0.4164	0.3805	0.3525	0.3112
			4	0.6347	0.58	0.5374	0.4744
		400	0	0.3089	0.2823	0.2616	0.2309
			1	0.338	0.3089	0.2862	0.2527
			2	0.4133	0.3777	0.3499	0.3089
			4	0.63	0.5757	0.5334	0.4709
		500	0	0.3064	0.28	0.2595	0.2291
			1	0.3353	0.3064	0.2839	0.2507
			2	0.4099	0.3746	0.3471	0.3064
			4	0.6249	0.5711	0.5291	0.4671
		600	0	0.3037	0.2775	0.2571	0.227
			1	0.3323	0.3037	0.2814	0.2484
			2	0.4063	0.3713	0.344	0.3037
			4	0.6193	0.5659	0.5244	0.4629

Finally, Table 9-Table 11 investigate the combined effects of elastic-foundation support, thermal loading, and small-scale parameters for P-, G-, and IWP-type cores, respectively. A simply supported square nanoplate with $a = b = 10$ and $a/h = 10$ is considered. The sandwich configuration is fixed as 1–2–1, the face-sheet gradient index is $p = 1$, and the TPMS core follows the A1 pattern with $\rho_{\text{avg}} = 0.35$. The bottom-surface temperature is maintained at $T_b = 300\text{K}$, while the top-surface temperature varies as $T_t = 300, 400, 500$, and 600K . The small-scale parameters are selected as $l = 0, 1, 2$, and 4 and $\mu = 0, 1, 2$, and 4 . Three foundation conditions are considered: $(\bar{k}_w, \bar{k}_s) = (0,0)$, $(100,0)$, and $(100,100)$, corresponding to the unsupported, Winkler, and Winkler–Pasternak cases, respectively. The nondimensional foundation parameters are defined by

$$\bar{k}_w = k_w a^4 / D_{\text{ref}} \quad \text{and} \quad \bar{k}_s = k_s a^2 / D_{\text{ref}}, \quad (131)$$

with $D_{\text{ref}} = E_{\text{Ti}} h^3 / (12[1 - \nu_{\text{Ti}}^2])$. The nondimensional frequency is again evaluated as

$$\bar{\omega} = \omega h \sqrt{\rho_{\text{Ti}} / G_{\text{Ti}}}. \quad (132)$$

The results confirm that the opposing effects of l and μ persist irrespective of thermal loading, foundation condition, and TPMS topology. For example, for the P-type core without foundation at $T_t = 300K$ and $\mu = 0$, increasing l from 0 to 4 raises $\bar{\omega}$ from 0.1259 to 0.2567. Conversely, at $l = 0$, increasing μ from 0 to 4 reduces $\bar{\omega}$ from 0.1259 to 0.0941. These variations demonstrate that the competing strain-gradient hardening and non-local softening mechanisms remain robust even when additional thermal and foundation effects are present. The foundation stiffness has a pronounced stabilizing influence on the vibration response. For the local P-type case at $T_t = 300K$, $\bar{\omega}$ increases from 0.1259 for the unsupported nanoplate to 0.1405 with the Winkler foundation and to 0.3112 with the combined Winkler-Pasternak foundation. The markedly larger increase produced by the latter case results from the additional shear-layer restraint represented by k_s . Thermal loading lowers the frequency under all support conditions, but its relative influence becomes substantially weaker as the foundation stiffness increases. For the same local P-type configuration, increasing T_t from 300K to 600K reduces $\bar{\omega}$ from 0.1259 to 0.1078 without foundation, from 0.1405 to 0.1246 with Winkler support, and from 0.3112 to 0.3037 with Winkler-Pasternak support. These changes correspond to frequency reductions of approximately 14.4%, 11.3%, and 2.4%, respectively. Thus, the Pasternak shear-layer contribution not only raises the absolute frequency but also substantially decreases the relative sensitivity of the vibration response to thermal loading. The P-, G-, and IWP-type cores exhibit the same overall dependence on l , μ , temperature, and foundation stiffness, with the P-type architecture generally maintaining a slightly higher frequency. The comparatively small differences among the three topologies, relative to the much stronger variations produced by the small-scale, thermal, and foundation parameters, indicate that the supporting medium and generalized-continuum effects dominate the vibration response in this particular parameter regime.

Taken together, the parametric results reveal several competing mechanisms governing the vibration behavior of FG-TPMS sandwich nanoplates. Non-locality and strain-gradient elasticity produce opposite size-dependent effects; through-thickness redistribution of a fixed amount of TPMS material modifies the response without changing the average core density; increasing the average density may reduce rather than increase the frequency when the inertia penalty exceeds the stiffness benefit; thermal loading consistently decreases the vibration frequency; and an intermediate core-thickness ratio provides a favorable balance between sandwich action and face-sheet stiffness. Elastic-foundation support, particularly the Pasternak shear-layer contribution, can

substantially increase the frequency and reduce the relative thermal sensitivity. These observations demonstrate that the vibration performance of FG-TPMS sandwich nanoplates is governed by coupled stiffness, inertia, thermal-prestress, small-scale, and support mechanisms rather than by any single design parameter.

Table 10. Effects of Winkler-Pasternak foundation stiffness, temperature, and small-scale parameters on the nondimensional fundamental frequency of the simply supported 1-2-1 FG-TPMS sandwich nanoplate with a G-type core (*A1* pattern).

$\bar{k}_w$	$\bar{k}_s$	$T_t(K)$	l	μ			
				0	1	2	4
0	0	300	0	0.1248	0.114	0.1057	0.0933
			1	0.1366	0.1248	0.1156	0.1021
			2	0.1669	0.1526	0.1414	0.1248
			4	0.2545	0.2326	0.2155	0.1902
		400	0	0.1195	0.1092	0.1012	0.0893
			1	0.1308	0.1195	0.1107	0.0978
			2	0.1599	0.1461	0.1354	0.1195
			4	0.2437	0.2227	0.2064	0.1822
		500	0	0.1136	0.1038	0.0962	0.0849
			1	0.1243	0.1136	0.1053	0.0929
			2	0.152	0.1389	0.1287	0.1136
			4	0.2317	0.2117	0.1962	0.1732
		600	0	0.1067	0.0975	0.0903	0.0797
			1	0.1167	0.1067	0.0988	0.0873
			2	0.1427	0.1304	0.1208	0.1067
			4	0.2175	0.1988	0.1842	0.1626
100	0	300	0	0.1396	0.1276	0.1182	0.1043
			1	0.1527	0.1396	0.1293	0.1142
			2	0.1867	0.1706	0.1581	0.1396
			4	0.2846	0.2601	0.241	0.2128
		400	0	0.1349	0.1232	0.1142	0.1008
			1	0.1476	0.1349	0.125	0.1103
			2	0.1804	0.1649	0.1528	0.1349
			4	0.275	0.2513	0.2329	0.2056
		500	0	0.1296	0.1185	0.1098	0.0969
			1	0.1419	0.1296	0.1201	0.106
			2	0.1734	0.1585	0.1468	0.1296
			4	0.2644	0.2416	0.2238	0.1976
		600	0	0.1236	0.1129	0.1046	0.0924
			1	0.1352	0.1236	0.1145	0.1011
			2	0.1653	0.1511	0.14	0.1236
			4	0.252	0.2303	0.2134	0.1884

100	100	300	0	0.3109	0.2841	0.2632	0.2324
			1	0.3402	0.3109	0.288	0.2543
			2	0.4159	0.38	0.3521	0.3109
			4	0.6339	0.5793	0.5368	0.4739
		400	0	0.3086	0.282	0.2613	0.2307
			1	0.3376	0.3086	0.2859	0.2524
			2	0.4128	0.3772	0.3495	0.3086
			4	0.6292	0.575	0.5328	0.4704
		500	0	0.3061	0.2797	0.2592	0.2288
			1	0.3349	0.3061	0.2836	0.2504
			2	0.4095	0.3742	0.3467	0.3061
			4	0.6242	0.5704	0.5285	0.4666
		600	0	0.3034	0.2772	0.2569	0.2268
			1	0.3319	0.3034	0.2811	0.2481
			2	0.4058	0.3709	0.3436	0.3034
			4	0.6186	0.5653	0.5238	0.4624

Table 11. Effects of Winkler-Pasternak foundation stiffness, temperature, and small-scale parameters on the nondimensional fundamental frequency of the simply supported 1-2-1 FG-TPMS sandwich nanoplate with an IWP-type core (*A1* pattern).

$\bar{k}_w$	$\bar{k}_s$	$T_t(K)$	l	μ			
				0	1	2	4
0	0	300	0	0.1246	0.1139	0.1055	0.0931
			1	0.1364	0.1246	0.1155	0.1019
			2	0.1667	0.1523	0.1411	0.1246
			4	0.2541	0.2322	0.2152	0.1899
		400	0	0.1193	0.109	0.101	0.0892
			1	0.1306	0.1193	0.1105	0.0976
			2	0.1596	0.1459	0.1351	0.1193
			4	0.2433	0.2223	0.206	0.1819
		500	0	0.1134	0.1036	0.096	0.0848
			1	0.1241	0.1134	0.1051	0.0928
			2	0.1517	0.1386	0.1284	0.1134
			4	0.2312	0.2113	0.1958	0.1728
		600	0	0.1064	0.0973	0.0901	0.0796
			1	0.1165	0.1064	0.0986	0.0871
			2	0.1424	0.1301	0.1206	0.1064
			4	0.2171	0.1984	0.1838	0.1623
100	0	300	0	0.1394	0.1274	0.1181	0.1042
			1	0.1526	0.1394	0.1292	0.114
			2	0.1865	0.1704	0.1579	0.1394
			4	0.2843	0.2598	0.2407	0.2125
		400	0	0.1347	0.1231	0.114	0.1007

100	100	500	1	0.1474	0.1347	0.1248	0.1102
			2	0.1802	0.1647	0.1526	0.1347
			4	0.2746	0.251	0.2325	0.2053
			0	0.1294	0.1183	0.1096	0.0968
			1	0.1416	0.1294	0.1199	0.1059
			2	0.1732	0.1582	0.1466	0.1294
			4	0.264	0.2412	0.2235	0.1973
		600	0	0.1234	0.1127	0.1045	0.0922
			1	0.135	0.1234	0.1143	0.1009
			2	0.165	0.1508	0.1397	0.1234
			4	0.2516	0.2299	0.213	0.1881
		300	0	0.3108	0.284	0.2632	0.2323
			1	0.3401	0.3108	0.288	0.2542
			2	0.4158	0.38	0.3521	0.3108
			4	0.6338	0.5792	0.5367	0.4738
		400	0	0.3085	0.2819	0.2612	0.2306
			1	0.3376	0.3085	0.2858	0.2523
			2	0.4127	0.3771	0.3494	0.3085
			4	0.6291	0.5749	0.5327	0.4703
		500	0	0.306	0.2797	0.2591	0.2288
			1	0.3349	0.306	0.2835	0.2503
			2	0.4094	0.3741	0.3466	0.306
			4	0.624	0.5703	0.5284	0.4665
		600	0	0.3033	0.2772	0.2568	0.2267
			1	0.3319	0.3033	0.281	0.2481
			2	0.4057	0.3708	0.3435	0.3033
			4	0.6185	0.5652	0.5237	0.4623

4. Conclusions

This study developed a high-continuity isogeometric framework for the free-vibration analysis of FG-TPMS sandwich nanoplates under thermal loading and elastic-foundation effects. The formulation combines non-local strain gradient theory with a five-variable higher-order shear deformation theory and incorporates temperature-dependent material properties, nonlinear through-thickness temperature fields, thermal initial stresses, and Winkler-Pasternak foundation support. Its accuracy was verified against available benchmark solutions. The main findings are summarized as follows:

- The non-local and strain-gradient parameters produce competing size effects: non-locality reduces the fundamental frequency through softening, whereas the strain-gradient effect increases it through hardening.

- Through-thickness redistribution of the TPMS-core density influences the vibration response even at a fixed average relative density, with the monotonic A-type patterns generally yielding slightly higher frequencies than the symmetric B-type patterns.
- Increasing the average core density does not necessarily improve vibration performance, as the associated inertia increase may outweigh the gain in effective stiffness.
- Thermal loading consistently reduces the fundamental frequency through material degradation and compressive thermal prestress. The core-thickness ratio exhibits a non-monotonic effect, with a favorable intermediate value of approximately for the investigated configurations.
- The P-type core generally provides the highest fundamental frequency, while the G- and IWP-type cores exhibit slightly lower responses. Elastic-foundation support increases the frequency and reduces its thermal sensitivity, particularly when the Pasternak shear-layer effect is included.

Overall, the vibration response of FG-TPMS sandwich nanoplates is governed by the coupled interplay among size effects, TPMS architecture and density distribution, sandwich geometry, thermal loading, and foundation restraint. The proposed framework provides an effective computational tool for understanding and tailoring these interactions in advanced sandwich nanostructures.

Conflicts of Interest

The authors declare no conflict of interest related to this work.

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