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Article

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H_∞ Leader–Following Control for Heterogeneous Nonlinear Multi-Agent Systems via Alternating LMIs

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Abstract

This paper investigates the distributed $\boldsymbol{H_\infty}$ leader-following control for heterogeneous nonlinear multi-agent systems MASs subject to external disturbances. A compact tracking-error model is formulated by introducing a lumped nonlinear mismatch term, which converts the error dynamics into a closed-loop representation. Sufficient conditions for stability and prescribed $\boldsymbol{H_\infty}$ attenuation are derived via Lyapunov stability theory and a quadratic constraint on the mismatch. For controller synthesis, a nonconvex BMI condition is established, and a verification-based alternating LMI procedure is proposed to address it. The framework naturally reduces to the linear heterogeneous case when the mismatch term vanishes. Numerical examples, including a heterogeneous nonlinear MAS and a leader-following UGV network, demonstrate the effectiveness.

Keywords: Heterogeneous multi-agent systems, leader–following control, $\boldsymbol{H_\infty}$ control, Nonlinear systems, Alternating LMI.

1 Introduction

Multi-agent systems (MASs) have been widely studied due to their ability to achieve distributed coordination through local information exchange among agents. Compared

with centralized architectures, MASs provide scalability and flexibility, which have enabled their applications in cooperative robotics, autonomous vehicles, and networked systems[1–6]. Among various coordination problems, consensus and leader-following control have been extensively investigated, where agents achieve agreement or track reference trajectories through distributed protocols.

In the context of consensus control, identical agent dynamics are commonly assumed in existing studies, which simplifies system modeling and facilitates controller design. Under this assumption, many elegant results have been obtained for homogeneous MASs. However, in practical applications, different agents may exhibit heterogeneous dynamics due to variations in physical parameters, hardware configurations, and operating environments. Therefore, heterogeneous multi-agent systems (HMASs), where agent-dependent dynamics are explicitly considered, have attracted increasing research interest. For example, distributed consensus problems of nonlinear MASs with different agent dynamics were investigated in [7]. Consensus of heterogeneous MASs composed of linear and nonlinear agents was studied in [8]. Moreover, robust containment and adaptive consensus problems for heterogeneous MASs have been investigated under various communication structures and system uncertainties [9–11]. Although these results have significantly promoted the development of heterogeneous consensus theory, most existing studies mainly focus on disturbance-free consensus or nominal convergence performance.

In practical networked systems, external disturbances and modeling uncertainties may degrade tracking performance. The H_∞ framework characterizes disturbance attenuation by constraining the induced $\mathcal{L}_2$-gain from external disturbances to regulated outputs [12]. Based on this framework, robust consensus and leader-following control problems have been investigated for linear and heterogeneous MASs under various uncertainties and communication constraints [13–15].

Nevertheless, existing studies usually focus on these issues individually. When heterogeneous dynamics, nonlinear characteristics, external disturbances, and distributed controller synthesis are considered simultaneously, the resulting leader–following control problem becomes more involved. In particular, heterogeneous agent dynamics combined with nonlinear increments complicate the construction of a compact error system, and most existing heterogeneous consensus results are developed for linear agents or specific nonlinear structures [15].

To describe nonlinear dynamics beyond conventional Lipschitz conditions, incremental quadratic constraints have recently been introduced to characterize a broader class of nonlinearities and derive tractable stability conditions using Lyapunov analysis and LMIs [16, 17]. However, integrating such nonlinear descriptions into distributed H_∞ leader–following controller synthesis remains nontrivial. Moreover, the simultaneous design of Lyapunov matrices and distributed feedback gains generally leads to bilinear coupling in the synthesis conditions. Consequently, the controller design problem is formulated as a bilinear matrix inequality (BMI), which cannot be directly handled by standard LMI-based optimization methods [18, 19].

Recent studies have further investigated multi-agent control problems by incorporating heterogeneous dynamics, nonlinear characteristics, and external disturbances. For heterogeneous nonlinear MASs, distributed consensus and tracking problems

have been studied under more practical conditions, including switched dynamics, sampled-data interactions, and uncertain environments [20–22]. Meanwhile, robust control strategies have been developed to improve disturbance rejection performance in networked systems with complex communication constraints and uncertain factors [23, 24]. Nevertheless, most existing studies still focus on partial aspects of the overall problem, while the simultaneous consideration of heterogeneous nonlinear dynamics, prescribed H_∞ disturbance attenuation, and BMI-based distributed controller synthesis remains insufficiently investigated.

This paper aims to address the distributed H_∞ leader–following control problem of heterogeneous nonlinear MASs subject to finite-energy external disturbances. The contributions of this paper can be summarized as follows. First, a compact leader–following error system is formulated while preserving the heterogeneous follower dynamics and directed communication topology. Second, the nonlinear increment is characterized by an incremental quadratic constraint, based on which Lyapunov-based sufficient conditions are derived to guarantee asymptotic stability and prescribed H_∞ disturbance attenuation performance. Third, a verification-based alternating LMI synthesis procedure is developed to handle the BMI arising from the coupled design of the Lyapunov matrix and distributed feedback gains. The resulting controller is further verified through the original H_∞ matrix inequality.

The main features of representative studies and this work are compared in Table 1.

Table 1 Comparison of related works and the proposed method

Reference	Heterogeneous	Nonlinear	H_∞	BMI handling
Zheng et al. [25]	✓	×	×	✓
Miao et al. [26]	✓	×	✓	✓
Nguyen [27]	✓	✓	×	✓
Yao et al. [28]	×	✓	✓	✓
This work	✓	✓	✓	✓

It can be observed that existing methods usually address heterogeneous dynamics, nonlinear behaviors, or H_∞ disturbance attenuation separately. In contrast, this work considers heterogeneous nonlinear multi-agent systems with external disturbances and develops an alternating LMI-based synthesis approach to handle the resulting BMI controller design problem.

The main contributions of this paper are summarized as follows.

1. A distributed H_∞ leader–following control framework is established for heterogeneous nonlinear MASs with external disturbances. Different from homogeneous consensus formulations, the proposed framework explicitly considers heterogeneous follower dynamics and directed leader–following communication structures.
2. An incremental quadratic constraint based analysis framework is developed to characterize the nonlinear and heterogeneity-induced terms without requiring explicit nonlinear function forms.

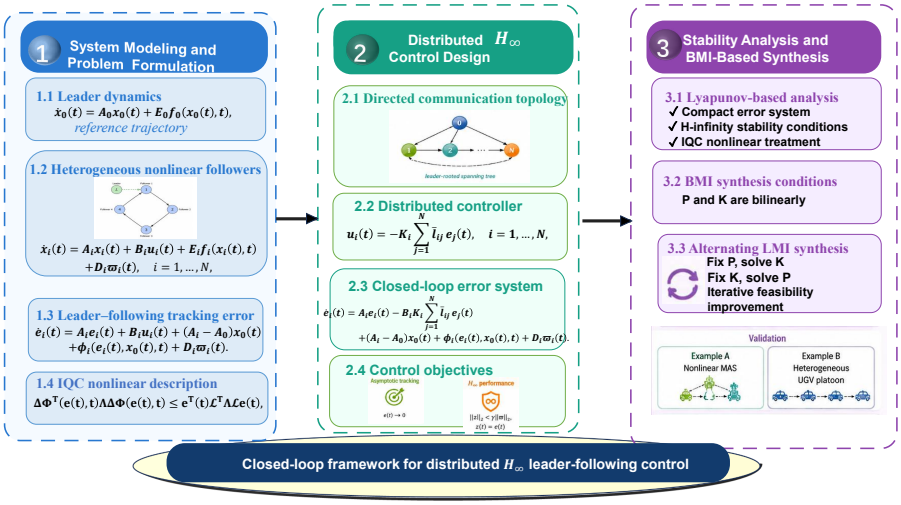


Fig. 1 Framework of the proposed distributed H_∞ leader–following control scheme for heterogeneous nonlinear multi-agent systems.

3. The controller synthesis problem is formulated by considering the bilinear coupling between the Lyapunov matrix and heterogeneous distributed feedback gains. A verification-based alternating LMI synthesis procedure is developed to solve the BMI controller design problem through two convex optimization steps.

The remainder of this paper is organized as follows. Section II presents the graph-theoretic preliminaries, heterogeneous nonlinear MAS model, distributed leader–following protocol, and closed-loop error formulation. Section III establishes the H_∞ disturbance attenuation condition and develops the certified alternating LMI synthesis procedure. Section IV provides simulation examples. Finally, Section V concludes this paper. As illustrated in Fig. 1, the proposed method consists of three sequential components: system modeling, distributed H_∞ controller synthesis, and stability verification.

2 Preliminaries and Problem Formulation

2.1 Graph Theory and Notation

The information exchange among the follower agents is described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of nodes representing the followers, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges, and $\mathcal{W} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the weighted adjacency matrix. An edge $(j, i) \in \mathcal{E}$ indicates that follower i can receive information from follower j, with $a_{ij} > 0$, and $a_{ij} = 0$ otherwise. Self-loops are excluded, i.e., $a_{ii} = 0$ for all i.

The neighbor set of follower i is defined as

$$\mathcal{N}_i = \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}\}. \tag{1}$$

The corresponding in-degree matrix is given by

$$D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}, \tag{2}$$

where

$$d_i = \sum_{j \in \mathcal{N}_i} a_{ij}. \tag{3}$$

The Laplacian matrix associated with $\mathcal{G}$ is then defined as

$$L = D - \mathcal{W}. \tag{4}$$

For leader-following MASs, a diagonal pinning matrix

$$G = \mathrm{diag}\{g_1, g_2, \ldots, g_N\} \tag{5}$$

is introduced to characterize the coupling between the leader and the followers, where $g_i > 0$ if follower i has access to the leader information, and $g_i = 0$ otherwise. Accordingly, the augmented Laplacian matrix is defined as

$$\bar{L} = L + G, \tag{6}$$

which incorporates both the follower-follower and leader-follower pinning links.

Throughout this paper, $\|\cdot\|$ denotes the Euclidean norm, $\otimes$ denotes the Kronecker product, and $\mathbb{R}$ denotes the set of real numbers. For a collection of vectors or matrices $\{Y_i\}_{i=1}^N$, $\mathrm{diag}\{Y_1, \ldots, Y_N\}$ denotes the block diagonal matrix with Y_i as its diagonal blocks, and $\mathrm{col}\{Y_1, \ldots, Y_N\}$ denotes the column vector formed by stacking $Y_1, \ldots, Y_N$. For a square matrix X, denote

$$\mathrm{He}(X) = X + X^T. \tag{7}$$

Assumption 1. *The communication topology among the agents contains a directed spanning tree rooted at the leader.*

2.2 Heterogeneous Nonlinear MAS Model

Consider a leader-following heterogeneous nonlinear MAS consisting of one leader and N followers. The dynamics of the leader are given by

$$\dot{x}_0(t) = A_0 x_0(t) + E_0 f_0(x_0(t), t), \tag{8}$$

where $x_0(t) \in \mathbb{R}^n$ denotes the leader state, A_0 and E_0 are known constant matrices with compatible dimensions, and $f_0(x_0(t), t)$ represents the nonlinear dynamics of the leader.

The dynamics of follower i are described by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + E_i f_i(x_i(t), t) + D_i \varpi_i(t), \quad i = 1, \ldots, N, \tag{9}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ denote the state vector and control input of follower i, respectively. The matrices A_i, B_i, E_i, and D_i are known system matrices. The nonlinear function $f_i(x_i(t), t)$ characterizes the agent-dependent nonlinear dynamics, and $\varpi_i(t)$ denotes the external disturbance.

In contrast to homogeneous MASs, the followers considered in this paper are allowed to possess different local dynamics. That is, the system matrices of different followers are not necessarily identical, namely,

$$(A_i, B_i, E_i, D_i) \neq (A_j, B_j, E_j, D_j), \quad i \neq j. \tag{10}$$

This heterogeneity makes the leader-following control problem more involved, since the same distributed interaction signal may act through different local dynamics.

Define the tracking error between follower i and the leader as

$$e_i(t) = x_i(t) - x_0(t). \tag{11}$$

Combining (8) and (9), one obtains

$$\begin{aligned} \dot{e}_i(t) =& A_i x_i(t) + B_i u_i(t) + E_i f_i(x_i(t), t) + D_i \varpi_i(t) \\ & - A_0 x_0(t) - E_0 f_0(x_0(t), t). \end{aligned} \tag{12}$$

Since $x_i(t) = e_i(t) + x_0(t)$, the above equation can be rewritten as

$$\begin{aligned} \dot{e}_i(t) =& A_i e_i(t) + B_i u_i(t) + (A_i - A_0) x_0(t) \\ & + E_i f_i(e_i(t) + x_0(t), t) - E_0 f_0(x_0(t), t) + D_i \varpi_i(t). \end{aligned} \tag{13}$$

For notational convenience, define

$$\phi_i(e_i(t), x_0(t), t) = E_i f_i(e_i(t) + x_0(t), t) - E_0 f_0(x_0(t), t). \tag{14}$$

Then, the tracking error dynamics of follower i become

$$\dot{e}_i(t) = A_i e_i(t) + B_i u_i(t) + (A_i - A_0) x_0(t) + \phi_i(e_i(t), x_0(t), t) + D_i \varpi_i(t). \tag{15}$$

2.3 Distributed leader–following Protocol

To achieve leader-following consensus, the following distributed control protocol is adopted:

$$u_i(t) = K_i \left[\sum_{j=1}^{N} a_{ij} \big(x_j(t) - x_i(t) \big) + g_i \big(x_0(t) - x_i(t) \big) \right], \quad i = 1, \ldots, N, \tag{16}$$

where K_i denotes the distributed feedback gain to be designed.

By using the tracking error $e_i(t) = x_i(t) - x_0(t)$, the protocol (16) can be equivalently expressed as

$$u_i(t) = -K_i \sum_{j=1}^{N} \bar{l}_{ij} e_j(t), \quad i = 1, \ldots, N, \tag{17}$$

where $\bar{L} = [\bar{l}_{ij}]$ is the augmented leader-following Laplacian matrix.

Substituting (17) into (15) yields

$$\begin{aligned} \dot{e}_i(t) =& A_i e_i(t) - B_i K_i \sum_{j=1}^{N} \bar{l}_{ij} e_j(t) \\ &+ (A_i - A_0) x_0(t) + \phi_i(e_i(t), x_0(t), t) + D_i \varpi_i(t). \end{aligned} \tag{18}$$

The objective of this paper is to design the distributed feedback gains K_i, $i = 1, \ldots, N$, such that all followers track the leader trajectory while achieving a prescribed H_∞ disturbance attenuation performance in the presence of heterogeneous dynamics, nonlinear increments, and external disturbances.

2.4 Compact Error System

For the convenience of subsequent stability analysis and controller synthesis, the individual error dynamics (18) are transformed into a compact form.

Define the stacked tracking error vector as

$$e(t) = \mathrm{col}\{e_1(t), e_2(t), \ldots, e_N(t)\} \in \mathbb{R}^{Nn}. \tag{19}$$

Let

$$\mathcal{A} = \mathrm{diag}\{A_1, A_2, \ldots, A_N\}, \tag{20}$$

$$\mathcal{B}_0 = \mathrm{diag}\{B_1, B_2, \ldots, B_N\}, \tag{21}$$

and

$$\mathcal{K} = \mathrm{diag}\{K_1, K_2, \ldots, K_N\}. \tag{22}$$

Moreover, define

$$\mathcal{D} = \mathrm{diag}\{D_1, D_2, \ldots, D_N\}, \tag{23}$$

and

$$\varpi(t) = \mathrm{col}\{\varpi_1(t), \varpi_2(t), \ldots, \varpi_N(t)\}. \tag{24}$$

The nonlinear and heterogeneity-induced terms are collected into

$$\Delta\Phi(e(t), t) = \mathrm{col}\left\{\Delta\phi_1(e_1(t), x_0(t), t), \ldots, \Delta\phi_N(e_N(t), x_0(t), t)\right\}, \tag{25}$$

where

$$\Delta\phi_i(e_i(t), x_0(t), t) = (A_i - A_0) x_0(t) + \phi_i(e_i(t), x_0(t), t). \tag{26}$$

Using the Kronecker product property and the augmented Laplacian matrix $\bar{L}$, the closed-loop error dynamics can be compactly written as

$$\dot{e}(t) = \mathcal{A}_K e(t) + \Delta\Phi(e(t), t) + \mathcal{D}\varpi(t), \tag{27}$$

where

$$\mathcal{A}_K = \mathcal{A} - \mathcal{B}_0\mathcal{K}(\bar{L} \otimes I_n). \tag{28}$$

The controlled output is defined as

$$z(t) = C_e e(t), \tag{29}$$

where $z(t)$ characterizes the leader-following tracking performance and C_e is a given weighting matrix.

2.5 H_∞ Consensus Definition and Structured Nonlinear Constraint

In order to quantify the disturbance attenuation capability of the closed-loop error system, the following H_∞ consensus notion is introduced.

Definition 1 The closed-loop error system (27) is said to achieve leader-following H_∞ consensus with attenuation level $\gamma > 0$ if, under the zero initial condition $e(0) = 0$, for any nonzero finite-energy disturbance $\varpi \in \mathcal{L}_2[0, \infty)$, the controlled output satisfies

$$\|z\|_2 < \gamma\|\varpi\|_2. \tag{30}$$

Define the set of positive diagonal multiplier matrices as

$$\mathbb{D}^{Nn} = \{\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{Nn}\} \mid \lambda_k > 0,\ k = 1, \ldots, Nn\}. \tag{31}$$

The following assumption is imposed to characterize the nonlinear and heterogeneity-induced term in a structured quadratic form.

Assumption 2. *To characterize the nonlinear increment and the mismatch caused by heterogeneous dynamics in a unified manner, the following structured weighted quadratic constraint is imposed:*

$$\Delta\Phi^T(e(t), t)\Lambda\Delta\Phi(e(t), t) \le e^T(t)\mathcal{L}^T\Lambda\mathcal{L}e(t), \tag{32}$$

for every $\Lambda \in \mathbb{D}^{Nn}$, where $\mathcal{L}$ is a known matrix with compatible dimension.

Remark 1 The proposed structured quadratic constraint offers a flexible description of nonlinear and heterogeneity-induced terms via a diagonal multiplier matrix. Compared with the conventional scalar counterpart, the introduction of such a diagonal structure brings extra degrees of freedom into the stability analysis, which is beneficial for reducing conservatism in the derived H_∞ condition. This advantage becomes particularly evident when the system

exhibits strong nonlinear couplings, where a scalar multiplier may be insufficient to capture the underlying dynamics.

Remark 2 Assumption 2 encompasses a class of nonlinear systems satisfying the usual Lipschitz condition. Specifically, if the nonlinear increment satisfies $\|\Delta\Phi(e,t)\| \leq \rho\|e\|$, then Assumption 2 can be recovered by simply setting $L = \rho I$. Therefore, the present framework is general enough to include many nonlinear MASs that have been considered in previous consensus studies. In this sense, our results extend those existing ones while retaining the ability to handle more complex nonlinearities.

3 Main Results

In this section, the H_∞ performance analysis of the nonlinear closed-loop error system is first developed. Then, the distributed controller synthesis problem is formulated as a BMI condition. Finally, a certified alternating LMI procedure is proposed to obtain feasible distributed controller gains.

3.1 H_∞ Performance Analysis

Based on the compact error system (27) and the structured weighted quadratic constraint (32), the following theorem provides a sufficient condition for the internal stability and prescribed H_∞ disturbance attenuation performance.

Theorem 1 *Consider the nonlinear error system (27) with the controlled output (29). Suppose that Assumption 2 holds. For a prescribed disturbance attenuation level $\gamma > 0$ and given feedback gains K_i, $i = 1, \ldots, N$, if there exist a symmetric matrix $P = P^T > 0$ and a diagonal multiplier matrix $\Lambda \in \mathbb{D}^{Nn}$ such that*

$$\Xi_\Lambda = \begin{bmatrix} \Xi_{11}^\Lambda & P\mathcal{D} & P \\ * & -\gamma^2 I_\varpi & 0 \\ * & * & -\Lambda \end{bmatrix} < 0, \tag{33}$$

where

$$\Xi_{11}^\Lambda = \mathrm{He}(P\mathcal{A}_K) + C_e^T C_e + \mathcal{L}^T \Lambda \mathcal{L}, \tag{34}$$

I_ϖ is the identity matrix with compatible dimension, and $\mathcal{A}_K$ is defined in (28), then the disturbance-free closed-loop error system is internally asymptotically stable. Moreover, under the zero initial condition $e(0) = 0$, the controlled output satisfies

$$\|z\|_2 < \gamma\|\varpi\|_2 \tag{35}$$

for every nonzero $\varpi \in \mathcal{L}_2[0,\infty)$. Therefore, the closed-loop error system achieves the prescribed H_∞ disturbance attenuation level γ.

Proof Choose the quadratic Lyapunov function

$$V(e(t)) = e^T(t)Pe(t), \tag{36}$$

where $P = P^T > 0$. Along the trajectories of (27), its time derivative is given by

$$\dot{V}(e(t)) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t)$$

$$
\begin{aligned}
=& e^T(t)\,\mathrm{He}(P\mathcal{A}_K)e(t) + 2e^T(t)P\Delta\Phi(e(t),t) \\
&+ 2e^T(t)P\mathcal{D}\varpi(t). \qquad (37)
\end{aligned}
$$

Consider the instantaneous dissipation expression

$$
\mathcal{J}(t) = \dot{V}(e(t)) + z^T(t)z(t) - \gamma^2\varpi^T(t)\varpi(t). \qquad (38)
$$

Using $z(t) = C_e e(t)$, one obtains

$$
\begin{aligned}
\mathcal{J}(t) =& e^T(t)\left[\mathrm{He}(P\mathcal{A}_K) + C_e^T C_e\right] e(t) \\
&+ 2e^T(t)P\Delta\Phi(e(t),t) + 2e^T(t)P\mathcal{D}\varpi(t) \\
&- \gamma^2\varpi^T(t)\varpi(t). \qquad (39)
\end{aligned}
$$

From Assumption 2, for every $\Lambda \in \mathbb{D}^{Nn}$,

$$
e^T(t)\mathcal{L}^T\Lambda\mathcal{L}e(t) - \Delta\Phi^T(e(t),t)\Lambda\Delta\Phi(e(t),t) \geq 0. \qquad (40)
$$

Adding the nonnegative term in (40) to the right-hand side of (39) yields the following upper bound:

$$
\begin{aligned}
\mathcal{J}(t) \leq& e^T(t)\left[\mathrm{He}(P\mathcal{A}_K) + C_e^T C_e + \mathcal{L}^T\Lambda\mathcal{L}\right] e(t) \\
&+ 2e^T(t)P\Delta\Phi(e(t),t) - \Delta\Phi^T(e(t),t)\Lambda\Delta\Phi(e(t),t) \\
&+ 2e^T(t)P\mathcal{D}\varpi(t) - \gamma^2\varpi^T(t)\varpi(t). \qquad (41)
\end{aligned}
$$

Define

$$
\xi(t) = \mathrm{col}\left\{e(t),\, \varpi(t),\, \Delta\Phi(e(t),t)\right\}. \qquad (42)
$$

Then (41) can be written as

$$
\mathcal{J}(t) \leq \xi^T(t)\Xi_\Lambda\xi(t). \qquad (43)
$$

If (33) holds, there exists a scalar $\alpha > 0$ such that

$$
\xi^T(t)\Xi_\Lambda\xi(t) \leq -\alpha\|\xi(t)\|^2. \qquad (44)
$$

Thus,

$$
\dot{V}(e(t)) + z^T(t)z(t) - \gamma^2\varpi^T(t)\varpi(t) < 0. \qquad (45)
$$

First, consider the disturbance-free case. Let $\varpi(t) = 0$. It follows from (43) and the strict negativity of Ξ_Λ that

$$
\dot{V}(e(t)) \leq -\alpha\|e(t)\|^2, \quad e(t) \neq 0. \qquad (46)
$$

Therefore, the disturbance-free closed-loop error system is internally asymptotically stable.

Next, consider the case with nonzero disturbance. Integrating (45) over $[0, T]$ yields

$$
\begin{aligned}
&V(e(T)) - V(e(0)) + \int_0^T z^T(t)z(t)\,dt \\
&- \gamma^2\int_0^T \varpi^T(t)\varpi(t)\,dt < 0. \qquad (47)
\end{aligned}
$$

Under the zero initial condition $e(0) = 0$, one has $V(e(0)) = 0$. Moreover, $P > 0$ implies $V(e(T)) \geq 0$. Thus,

$$
\int_0^T z^T(t)z(t)\,dt < \gamma^2\int_0^T \varpi^T(t)\varpi(t)\,dt. \qquad (48)
$$

Letting $T \to \infty$ gives

$$
\|z\|_2^2 < \gamma^2\|\varpi\|_2^2. \qquad (49)
$$

For every nonzero $\varpi \in \mathcal{L}_2[0,\infty)$, this is equivalent to

$$
\|z\|_2 < \gamma\|\varpi\|_2. \qquad (50)
$$

Hence, the closed-loop error system achieves the prescribed H_∞ disturbance attenuation level γ. This completes the proof. □

Remark 3 Theorem 1 provides an analysis condition for given distributed feedback gains. Compared with the scalar multiplier case, the diagonal multiplier matrix Λ introduces additional degrees of freedom into the quadratic constraint and may reduce conservatism. For fixed K_i, $i = 1, \ldots, N$, the prescribed H_∞ performance can be verified by solving an LMI feasibility problem with respect to P and Λ.

3.2 BMI-Based Distributed Controller Synthesis

Theorem 1 provides a verification condition for given controller gains. However, when the feedback gains are treated as decision variables, the condition in (33) becomes nonconvex. The following theorem formulates the corresponding distributed controller synthesis condition.

Theorem 2 *Consider the nonlinear heterogeneous error system (27) with the controlled output (29). Suppose that Assumption 2 holds. For a prescribed disturbance attenuation level $\gamma > 0$, if there exist a symmetric matrix $P = P^T > 0$, a diagonal multiplier matrix $\Lambda \in \mathbb{D}_{++}^{Nn}$, and a distributed gain matrix*

$$\mathcal{K} = \mathrm{diag}\{K_1, K_2, \ldots, K_N\} \tag{51}$$

such that

$$\mathcal{M}(P, \mathcal{K}, \Lambda) < 0, \tag{52}$$

where

$$\mathcal{M}(P, \mathcal{K}, \Lambda) = \begin{bmatrix} \Theta(P, \mathcal{K}, \Lambda) & P\mathcal{D} & P \\ * & -\gamma^2 I_{\varpi} & 0 \\ * & * & -\Lambda \end{bmatrix}, \tag{53}$$

with

$$\begin{aligned} \Theta(P, \mathcal{K}, \Lambda) = {} & \mathrm{He}\left[P\left(\mathcal{A} - \mathcal{B}_0 \mathcal{K}(\bar{L} \otimes I_n)\right)\right] \\ & + C_e^T C_e + \mathcal{L}^T \Lambda \mathcal{L}, \end{aligned} \tag{54}$$

then the distributed controller

$$u_i(t) = -K_i \sum_{j=1}^{N} \bar{l}_{ij} e_j(t), \quad i = 1, \ldots, N, \tag{55}$$

guarantees the internal asymptotic stability of the disturbance-free closed-loop error system and achieves the prescribed H_∞ disturbance attenuation level γ.

Proof Under the controller (55), the closed-loop matrix is given by

$$\mathcal{A}_K = \mathcal{A} - \mathcal{B}_0 \mathcal{K}(\bar{L} \otimes I_n). \tag{56}$$

Substituting this expression into (33) gives exactly the matrix inequality (52). Hence, if (52) holds, all conditions of Theorem 1 are satisfied. Therefore, the closed-loop error system is internally asymptotically stable for $\varpi(t) = 0$ and satisfies the prescribed H_∞ disturbance attenuation performance. This completes the proof. □

Remark 4 The synthesis condition (52) is a BMI because $\Theta(P, \mathcal{K}, \Lambda)$ contains the bilinear term

$$P\mathcal{B}_0\mathcal{K}(\bar{L} \otimes I_n), \tag{57}$$

which couples the Lyapunov matrix P and the distributed gain matrix $\mathcal{K}$. The diagonal multiplier matrix Λ enters $\mathcal{M}(P, \mathcal{K}, \Lambda)$ affinely through $\mathcal{L}^T\Lambda\mathcal{L}$ and $-\Lambda$. Therefore, the nonconvexity of the synthesis condition is induced by the coupling between P and $\mathcal{K}$.

3.3 Verification-Based Alternating LMI Synthesis

To solve the BMI condition in Theorem 2, a certified alternating LMI procedure is proposed. The main idea is to alternately fix one group of decision variables and solve for the other group. The final controller is accepted only when the original strict BMI condition is directly satisfied.

Theorem 3 *Consider the BMI synthesis condition (52). For a fixed distributed gain matrix $\mathcal{K} = \mathcal{K}^{(r)}$, the feasibility problem*

$$\mathcal{M}(P, \mathcal{K}^{(r)}, \Lambda) \preceq -\eta I, \quad P = P^T > 0, \quad \Lambda \in \mathbb{D}_{++}^{Nn}, \quad \eta > 0, \tag{58}$$

is an LMI feasibility problem with respect to the decision variables P, Λ, and η.

Moreover, for a fixed Lyapunov matrix $P = P^{(r)}$, the feasibility problem

$$\mathcal{M}(P^{(r)}, \mathcal{K}, \Lambda) \preceq -\eta I, \quad \Lambda \in \mathbb{D}^{Nn}, \quad \eta > 0, \tag{59}$$

is an LMI feasibility problem with respect to the decision variables $\mathcal{K}$, Λ, and η.

Proof For a fixed distributed gain matrix $\mathcal{K} = \mathcal{K}^{(r)}$, the closed-loop matrix

$$\mathcal{A}_K^{(r)} = \mathcal{A} - \mathcal{B}_0\mathcal{K}^{(r)}(\bar{L} \otimes I_n) \tag{60}$$

is known. Hence, the term

$$\mathrm{He}\left(P\mathcal{A}_K^{(r)}\right) \tag{61}$$

is affine in the decision variable P. Since $\mathcal{L}^T\Lambda\mathcal{L}$ and $-\Lambda$ are affine in Λ, the inequality (58) is an LMI with respect to P, Λ, and η.

Next, consider the case where $P = P^{(r)}$ is fixed. Then

$$\begin{aligned} &\mathrm{He}\left[P^{(r)}\left(\mathcal{A} - \mathcal{B}_0\mathcal{K}(\bar{L} \otimes I_n)\right)\right] \\ =&\,\mathrm{He}\left(P^{(r)}\mathcal{A}\right) - \mathrm{He}\left[P^{(r)}\mathcal{B}_0\mathcal{K}(\bar{L} \otimes I_n)\right]. \end{aligned} \tag{62}$$

Since $P^{(r)}$ is fixed, the first term on the right-hand side of (62) is constant, while the second term is affine in the entries of $\mathcal{K}$. Moreover, $\mathcal{L}^T\Lambda\mathcal{L}$ and $-\Lambda$ are affine in Λ. Therefore, $\mathcal{M}(P^{(r)}, \mathcal{K}, \Lambda)$ is affine in $\mathcal{K}$ and Λ. It follows that (59) is an LMI feasibility problem with respect to $\mathcal{K}$, Λ, and η. This completes the proof. □

Theorem 4 *Consider the nonlinear heterogeneous error system (27) with the controlled output (29). Suppose that Assumption 2 holds. For a prescribed disturbance attenuation level $\gamma > 0$, consider the BMI condition (52).*

Starting from an initial distributed feedback gain

$$\mathcal{K}^{(0)} = \mathrm{diag}\{K_1^{(0)}, K_2^{(0)}, \ldots, K_N^{(0)}\}, \tag{63}$$

the following alternating LMI procedure is performed.

At the rth iteration, with $\mathcal{K} = \mathcal{K}^{(r)}$ fixed, solve for $P_A^{(r)} = (P_A^{(r)})^T > 0$, $\Lambda_A^{(r)} \in \mathbb{D}_{++}^{Nn}$, and $\eta_A^{(r)} > 0$ such that

$$\mathcal{M}(P_A^{(r)}, \mathcal{K}^{(r)}, \Lambda_A^{(r)}) \preceq -\eta_A^{(r)} I. \tag{64}$$

Then, with $P = P_A^{(r)}$ fixed, solve for a distributed gain matrix

$$\mathcal{K}_B^{(r)} = \mathrm{diag}\{K_{1,B}^{(r)}, K_{2,B}^{(r)}, \ldots, K_{N,B}^{(r)}\}, \tag{65}$$

together with $\Lambda_B^{(r)} \in \mathbb{D}_{++}^{Nn}$ and $\eta_B^{(r)} > 0$ such that

$$\mathcal{M}(P_A^{(r)}, \mathcal{K}_B^{(r)}, \Lambda_B^{(r)}) \preceq -\eta_B^{(r)} I. \tag{66}$$

After this step, set

$$P^{(r+1)} = P_A^{(r)}, \qquad \mathcal{K}^{(r+1)} = \mathcal{K}_B^{(r)}. \tag{67}$$

If the alternating procedure returns a pair $(P^\star, \mathcal{K}^\star)$ for which there exists $\Lambda^\star \in \mathbb{D}^{Nn}$ satisfying the strict certificate

$$P^\star > 0, \qquad \Lambda^\star > 0, \qquad \lambda_{\max}\left[\mathcal{M}(P^\star, \mathcal{K}^\star, \Lambda^\star)\right] < 0, \tag{68}$$

then the distributed controller

$$u_i(t) = -K_i^\star \sum_{j=1}^{N} \bar{l}_{ij} e_j(t), \quad i = 1, \ldots, N, \tag{69}$$

guarantees the internal asymptotic stability of the disturbance-free closed-loop error system and achieves the prescribed H_∞ disturbance attenuation level γ.

Proof For fixed $\mathcal{K}^{(r)}$, the closed-loop matrix

$$\mathcal{A} - \mathcal{B}_0 \mathcal{K}^{(r)} (\bar{L} \otimes I_n) \tag{70}$$

is known. Hence,

$$\mathrm{He}\left[P\left(\mathcal{A} - \mathcal{B}_0 \mathcal{K}^{(r)} (\bar{L} \otimes I_n)\right)\right] \tag{71}$$

is affine in the decision variable P. Since the remaining terms in $\mathcal{M}(P, \mathcal{K}^{(r)}, \Lambda)$ are affine in P and Λ, the fixed-controller condition (64) is an LMI with respect to $P_A^{(r)}$, $\Lambda_A^{(r)}$, and $\eta_A^{(r)}$.

Next, for fixed $P = P_A^{(r)}$, one has

$$\begin{aligned} &\mathrm{He}\left[P_A^{(r)}\left(\mathcal{A} - \mathcal{B}_0 \mathcal{K} (\bar{L} \otimes I_n)\right)\right] \\ &= \mathrm{He}(P_A^{(r)} \mathcal{A}) - \mathrm{He}\left[P_A^{(r)} \mathcal{B}_0 \mathcal{K} (\bar{L} \otimes I_n)\right]. \end{aligned} \tag{72}$$

Since $P_A^{(r)}$ is fixed, the right-hand side of (72) is affine in the entries of the distributed gain matrix $\mathcal{K}$. Moreover, $\mathcal{L}^T \Lambda \mathcal{L}$ and $-\Lambda$ are affine in Λ. Therefore, the fixed-Lyapunov condition (66) is an LMI with respect to $\mathcal{K}_B^{(r)}$, $\Lambda_B^{(r)}$, and $\eta_B^{(r)}$.

The alternating procedure therefore solves two convex LMI feasibility problems at each iteration. However, since the original synthesis condition (52) is a BMI, the alternating procedure itself only provides a local feasibility search and does not constitute a stability certificate by numerical convergence alone.

If the final certificate (68) holds, then

$$\mathcal{M}(P^\star, \mathcal{K}^\star, \Lambda^\star) < 0. \tag{73}$$

Thus, the BMI condition in Theorem 2 is satisfied. It follows from Theorem 2 that the controller (69) guarantees the internal asymptotic stability of the disturbance-free closed-loop error system and achieves the prescribed H_∞ disturbance attenuation performance. This completes the proof. □

Remark 5 Theorem 4 does not claim global convergence of the alternating LMI procedure. The role of the alternating steps is to search for a feasible solution of the BMI condition by solving two convex subproblems successively. The final controller is accepted only when the strict certificate (68) is satisfied. Therefore, the stability and H_∞ performance guarantee is not based on the convergence of the iteration, but on the direct verification of the original BMI condition (52).

4 Simulation Examples

In this section, both numerical and application-oriented simulations are presented to illustrate the effectiveness of the proposed distributed H_∞ leader-following control method. Example A considers a heterogeneous nonlinear multi-agent system and is used to illustrate the certified alternating LMI synthesis and the resulting closed-loop performance. Example B further applies the proposed control method to a leader-following coordination problem of heterogeneous unmanned ground vehicles (UGVs).

4.1 Example A

Consider a leader-following heterogeneous nonlinear MAS consisting of one leader and four followers. The leader dynamics are described by

$$\dot{x}_0(t) = A_0 x_0(t) + E_0 f_0(x_0(t), t),$$

where

$$A_0 = \begin{bmatrix} 0 & 1 \\ -2.10 & -1.20 \end{bmatrix}, \qquad E_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad f_0(x_0(t), t) = \sin(x_{01}(t)).$$

The dynamics of follower i are given by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + E_i f_i(x_i(t), t) + D_i \varpi_i(t), \qquad i = 1, \ldots, 4,$$

where

$$B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad D_i = I_2, \qquad i = 1, \ldots, 4.$$

The heterogeneous system matrices are selected as

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2.00 & -1.10 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 \\ -2.25 & -1.25 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ -1.85 & -1.35 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 0 & 1 \\ -2.40 & -1.20 \end{bmatrix}.$$

The nonlinear channel matrices are chosen as

$$E_1 = \begin{bmatrix} 0 \\ 0.55 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 \\ 0.60 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 \\ 0.50 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0 \\ 0.65 \end{bmatrix}.$$

To make the simulation consistent with the theoretical tracking-error model, the nonlinearities are selected in the matched form described previously. Accordingly, the structured nonlinear bound is constructed as

$$\mathcal{L} = \operatorname{diag}\{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4\},$$

where

$$\mathcal{L}_1 = \begin{bmatrix} 0 & 0 \\ 1.1000 & 0.1000 \end{bmatrix}, \qquad \mathcal{L}_2 = \begin{bmatrix} 0 & 0 \\ 1.1500 & 0.0500 \end{bmatrix},$$
$$\mathcal{L}_3 = \begin{bmatrix} 0 & 0 \\ 1.2500 & 0.1500 \end{bmatrix}, \qquad \mathcal{L}_4 = \begin{bmatrix} 0 & 0 \\ 1.3000 & 0 \end{bmatrix}.$$

These matrices are selected according to the upper bounds of the nonlinear increments to satisfy Assumption 2.

The directed communication graph consists of 4 followers, with follower 1 pinned to the leader, as shown in Fig. 2.

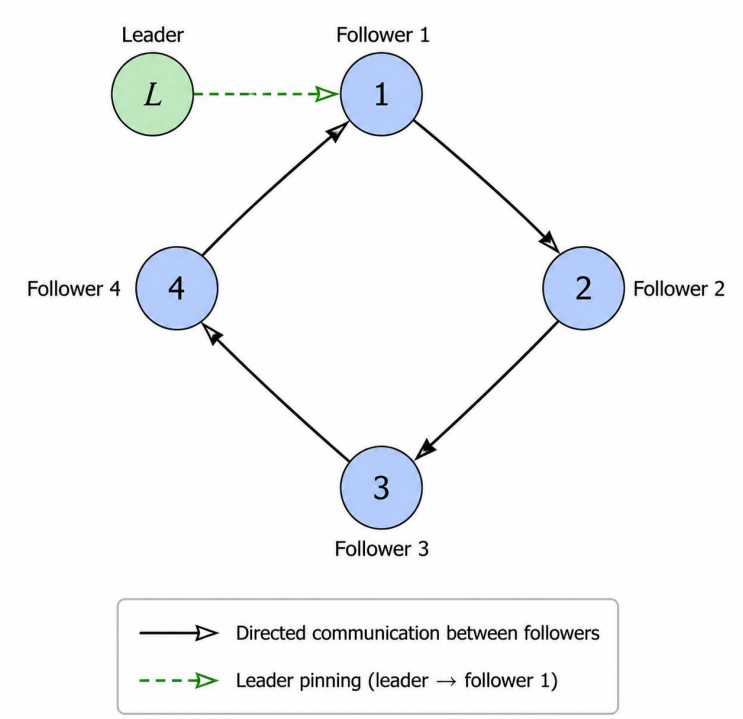


Fig. 2 Directed communication topology of the leader-follower network.

The corresponding augmented Laplacian matrix is

$$\bar{L} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

with eigenvalues

$$\sigma(\bar{L}) = \{2.3803,\ 1.2194 \pm 0.9145\mathrm{j},\ 0.1808\}.$$

Since all eigenvalues have positive real parts, the graph contains a directed spanning tree rooted at the leader.

The prescribed disturbance attenuation level is chosen as

$$\gamma = 5.$$

The certified alternating H_∞ synthesis procedure is then implemented. The alternating LMI procedure terminates after 34 iterations, and the resulting certified controller gains are

$$K_1 = \begin{bmatrix} 304.0496 & 307.8110 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 304.0162 & 307.8770 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 253.0248 & 255.8396 \end{bmatrix}, \qquad K_4 = \begin{bmatrix} 226.0942 & 228.8029 \end{bmatrix}.$$

The corresponding numerical certificate satisfies the strict H_∞ condition, with

$$\lambda_{\max}(\mathcal{M}) = -1.0224 \times 10^{-7} < 0,$$

and

$$\max \operatorname{Re} \lambda(A_{\mathrm{cl}}) = -9.9066 \times 10^{-1} < 0.$$

Hence, the certified controller satisfies the prescribed disturbance attenuation condition and the homogeneous closed-loop tracking-error system is exponentially stable.

For the subsequent time-domain simulation, the initial distributed gain

$$K_1^{(0)} = \begin{bmatrix} 6.4932 & 6.0162 \end{bmatrix}, \qquad K_2^{(0)} = \begin{bmatrix} 6.1399 & 5.6427 \end{bmatrix},$$
$$K_3^{(0)} = \begin{bmatrix} 6.5119 & 5.6808 \end{bmatrix}, \qquad K_4^{(0)} = \begin{bmatrix} 6.0682 & 5.6508 \end{bmatrix}$$

is used. The distributed controller is implemented as

$$u_i(t) = -K_i \sum_{j=1}^{4} \bar{l}_{ij} e_j(t), \qquad i = 1, \dots, 4.$$

The disturbance acting on follower i is selected as

$$\varpi_i(t) = (0.05 + 0.01i) e^{-0.12t} \begin{bmatrix} \sin(0.70t + 0.30i) \\ 0.8 \cos(0.45t - 0.20i) \end{bmatrix}, \qquad i = 1, \dots, 4.$$

The initial states of the leader and followers are chosen as

$$x_0(0) = \begin{bmatrix} 0.8 \\ -0.5 \end{bmatrix}, \qquad x_1(0) = \begin{bmatrix} 1.2 \\ -0.7 \end{bmatrix},$$
$$x_2(0) = \begin{bmatrix} -0.9 \\ 1.0 \end{bmatrix}, \qquad x_3(0) = \begin{bmatrix} 0.7 \\ 1.2 \end{bmatrix},$$
$$x_4(0) = \begin{bmatrix} -1.3 \\ -0.8 \end{bmatrix}.$$

The closed-loop nonlinear system is simulated over $[0, 60]$ s using the MATLAB `ode45` solver.

Fig. 3 illustrates the state trajectories of the leader and four heterogeneous followers. Here, $x_{ij}(t)$, $i = 0, 1, \dots, 4$, $j = 1, 2$, denotes the j-th component of the state vector of agent i. It can be observed that all follower states progressively approach the leader state despite heterogeneous local dynamics, nonlinear terms, different initial conditions, and external disturbances.

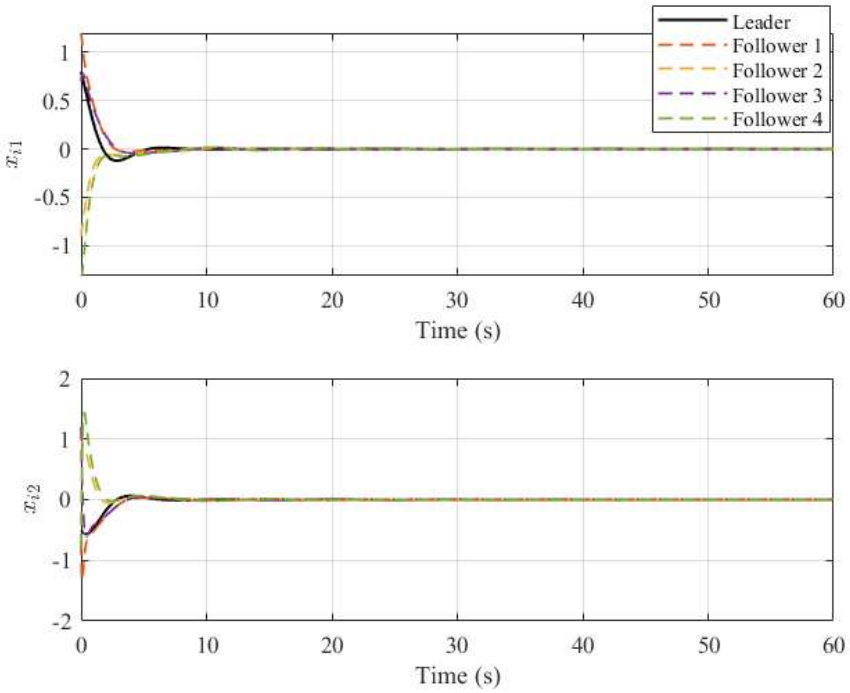


Fig. 3 State trajectories of the leader and heterogeneous followers in Example A.

Fig. 4 depicts the tracking error components of each follower with respect to the leader. All error components exhibit a clear decay during the transient stage. The remaining small deviations are caused by the external disturbances, which gradually vanish with time.

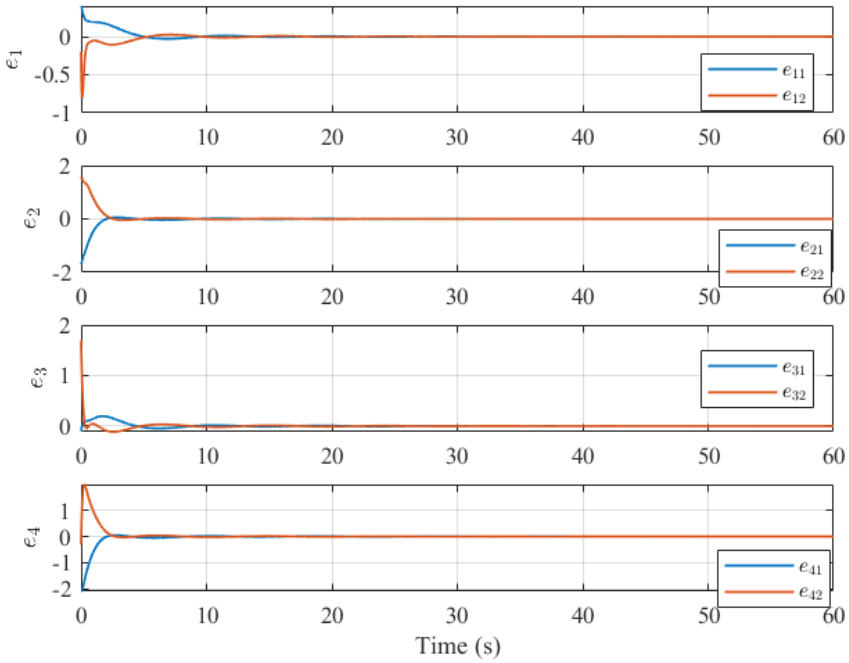


Fig. 4 Tracking errors of the four followers with respect to the leader in Example A.

To further characterize the convergence process, define the global tracking error energy as

$$|e(t)|_2^2 = \sum_{i=1}^{4} |e_i(t)|^2.$$

Fig. 5 illustrates the evolution of $|e(t)|_2^2$. The global error energy decreases from 1.2740×10^{1} to 1.6230×10^{-8} at $t = 60$ s. The corresponding terminal tracking error is

$$|e(60)|_2 = 1.2740 \times 10^{-4}.$$

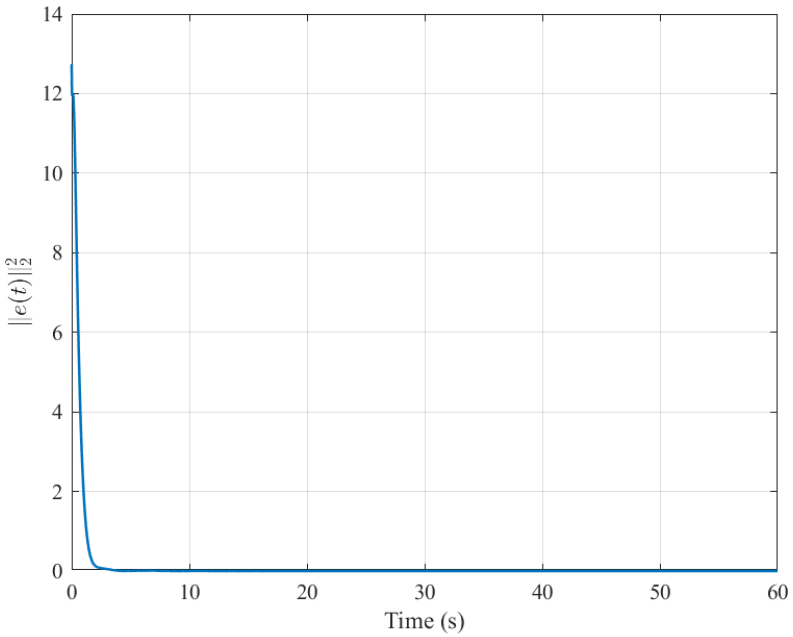


Fig. 5 Evolution of the global tracking-error energy $|e(t)|_2^2$ in Example A.

Fig. 6 presents the distributed control inputs of the four followers. Relatively large control actions are generated at the beginning of the simulation to compensate for the initial tracking mismatch. As the tracking errors decrease, the control inputs remain bounded and gradually decrease.

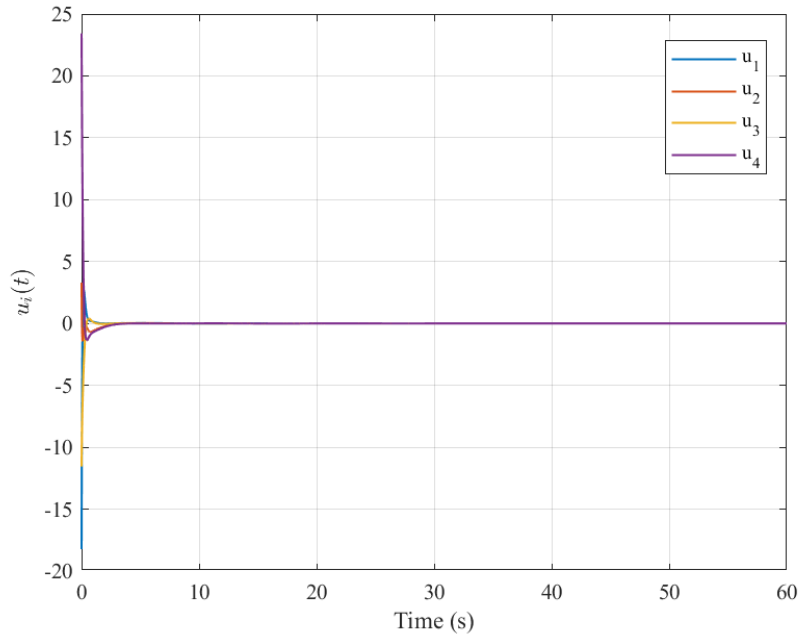


Fig. 6 Distributed control inputs of the four followers in Example A.

Finally, a zero-state finite-horizon test is performed to evaluate the H_∞ disturbance attenuation performance. All leader and follower states are initialized at zero, and the controlled output is selected as

$$z(t) = e(t).$$

The numerical results give

$$|z|_2 = 3.3420 \times 10^{-1}, \qquad |\varpi|_2 = 3.0360 \times 10^{-1},$$

and hence

$$\frac{|z|_2}{|\varpi|_2} = 1.1008 < 5.$$

Therefore, the finite-horizon simulation is consistent with the prescribed disturbance attenuation level.

4.2 Example B

We further apply the proposed distributed control method to a platoon of unmanned ground vehicles (UGVs) consisting of one leader and four followers. This example is intended to verify the leader-following coordination performance under heterogeneous vehicle dynamics, where different UGVs are characterized by different mechanical masses and translational friction coefficients.

Let

$$x_i(t) = \begin{bmatrix} p_i(t) \\ v_i(t) \end{bmatrix}, \qquad i = 0, 1, \ldots, 4,$$

where $p_i(t)$ and $v_i(t)$ denote the position and linear velocity of UGV i, respectively. The dynamics of each follower UGV are described by

$$\begin{bmatrix} \dot{p}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -F_i/M_i \end{bmatrix} \begin{bmatrix} p_i(t) \\ v_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_i \end{bmatrix} u_i(t), \qquad i = 1, \ldots, 4.$$

The leader evolves according to

$$\dot{x}_0(t) = A_0 x_0(t).$$

In this example, the vehicle parameters are selected as

$$(M_i, F_i) = \begin{cases} (0.8, 1.0), & i = 0, \\ (0.7, 0.9), & i = 1, \\ (0.9, 1.2), & i = 2, \\ (0.6, 1.0), & i = 3, \\ (1.0, 0.9), & i = 4. \end{cases}$$

It is seen that both the state matrices and input matrices of the followers are different, which reflects the heterogeneous nature of the considered UGV platoon.

Define the tracking error of follower i with respect to the leader as

$$e_i(t) = x_i(t) - x_0(t), \qquad i = 1, \ldots, 4.$$

Then, the corresponding tracking-error dynamics can be written as

$$\dot{e}_i(t) = A_i e_i(t) + B_i u_i(t) + (A_i - A_0) x_0(t) + D_i \varpi_i(t).$$

Compared with the matched nonlinear case in Example A, an additional term

$$\eta_i(t) = (A_i - A_0) x_0(t)$$

is introduced here due to the heterogeneity among the vehicle dynamics. Since the leader velocity satisfies

$$\dot{v}_0(t) = -\frac{F_0}{M_0} v_0(t),$$

the above term is a vanishing finite-energy signal. Hence, this example provides an application-oriented verification of the proposed method for UGV systems with physically heterogeneous parameters.

The same directed communication topology as in Example A is adopted. The augmented Laplacian matrix is chosen as

$$\bar{L} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

The eigenvalues of $\bar{L}$ are obtained as

$$\sigma(\bar{L}) = \{2.3803,\ 1.2194 + 0.9145\mathrm{j},\ 1.2194 - 0.9145\mathrm{j},\ 0.1808\}.$$

Obviously, all eigenvalues have positive real parts, which is consistent with the existence of a directed spanning tree rooted at the leader.

For the UGV platoon, the distributed controller gains are selected as

$$K_1 = \begin{bmatrix} 4.5516 & 3.3075 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 5.7333 & 4.3651 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 4.9428 & 3.8932 \end{bmatrix}, \qquad K_4 = \begin{bmatrix} 5.6339 & 4.6528 \end{bmatrix}.$$

Accordingly, the distributed control law is implemented as

$$u_i(t) = -K_i \sum_{j=1}^{4} \bar{l}_{ij} e_j(t), \qquad i = 1, \ldots, 4.$$

For the homogeneous part of the closed-loop tracking-error system, the maximum real part of the eigenvalues is calculated as

$$\max \mathrm{Re}\left\{\lambda(A_{\mathrm{cl}})\right\} = -0.9749 < 0.$$

This implies that the unforced homogeneous error dynamics are exponentially stable.

Choose the initial states of the leader and followers as

$$x_0(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \qquad x_1(0) = \begin{bmatrix} -4 \\ 1.5 \end{bmatrix},$$
$$x_2(0) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \qquad x_3(0) = \begin{bmatrix} 6.5 \\ -1.5 \end{bmatrix},$$
$$x_4(0) = \begin{bmatrix} -4 \\ -7 \end{bmatrix}.$$

For the tracking simulation, the external disturbance is set to zero, namely $\varpi_i(t) \equiv 0$, $i = 1, \ldots, 4$. The closed-loop system is simulated over $[0, 60]$ s.

Fig. 7 shows the position and velocity trajectories of the leader and follower UGVs. Here, $x_{ij}(t)$, $i = 0, 1, \ldots, 4$, $j = 1, 2$, denotes the j-th component of the state vector of UGV i. It can be observed that all follower states progressively synchronize their

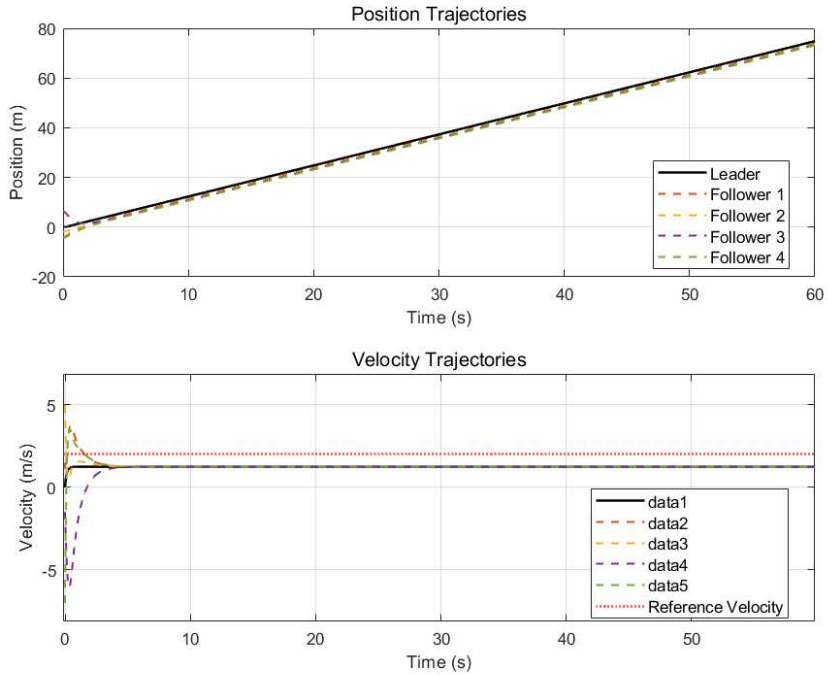


Fig. 7 Position and velocity trajectories of the leader and follower UGVs in Example B.

motion with the leader despite heterogeneous vehicle parameters and distinct initial conditions.

Fig. 8 further gives the state-space trajectories of $(p_i(t), v_i(t))$ for the leader and follower UGVs. The follower trajectories start from different initial positions in the state space and gradually approach the leader trajectory, which provides a geometric interpretation of the leader-following convergence process.

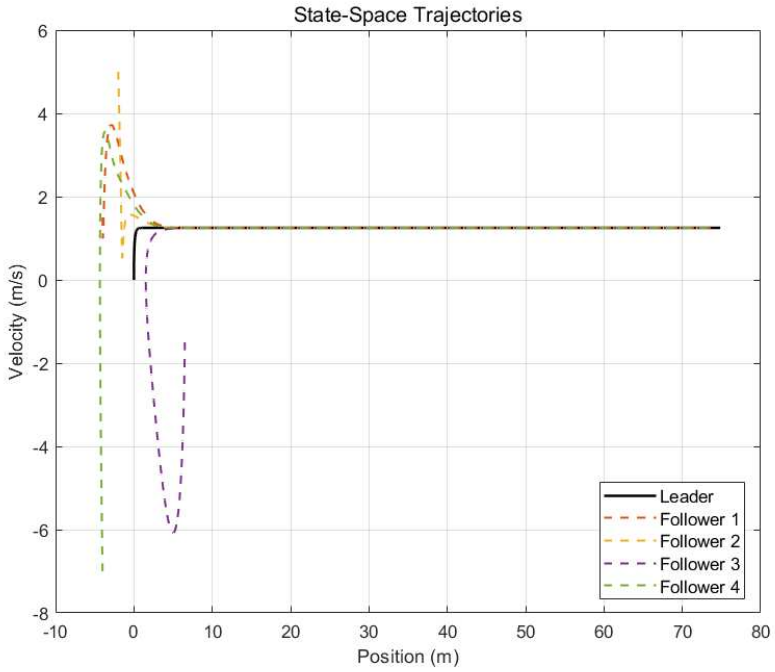


Fig. 8 State-space trajectories of the leader and follower UGVs in Example B.

In addition, Fig. 9 depicts the position and velocity tracking error components of each follower UGV with respect to the leader. It can be seen that all tracking error components converge to a neighborhood of zero and finally vanish numerically. Specifically, the tracking error satisfies

$$|e(60)|_2 = 1.0087 \times 10^{-14},$$

while the global error energy decreases from

$$|e(0)|_2^2 = 2.1375 \times 10^2$$

to

$$|e(60)|_2^2 = 1.0174 \times 10^{-28}.$$

These results demonstrate the effectiveness of the proposed distributed controller in achieving leader-following coordination for heterogeneous UGVs.

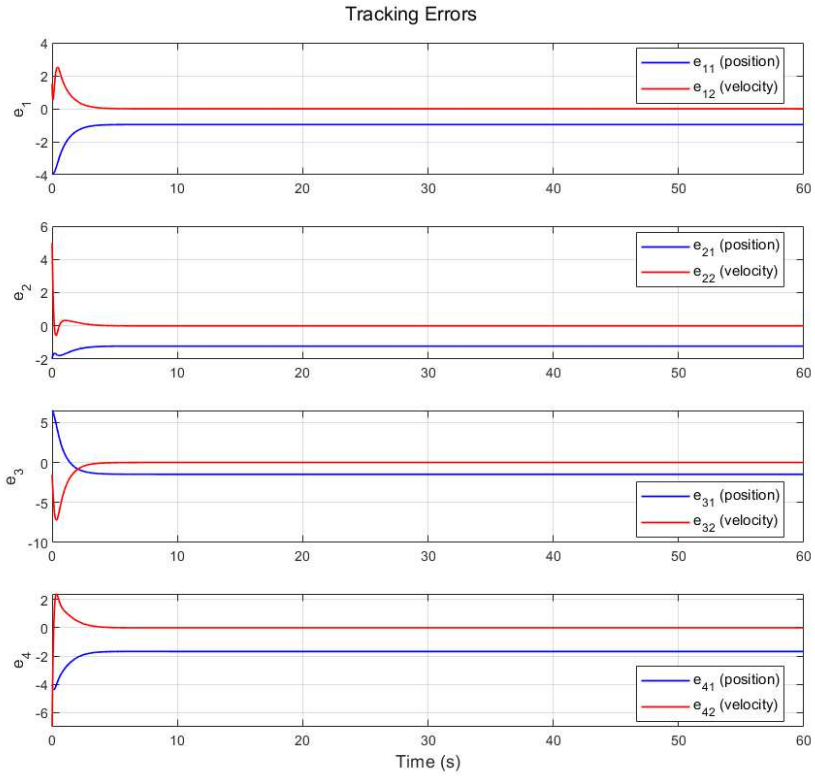


Fig. 9 Position and velocity tracking errors of the follower UGVs with respect to the leader in Example B.

Fig. 10 shows the distributed control inputs of the four follower UGVs. Relatively large control actions appear during the initial transient stage due to the substantial disagreement among the initial states. As the tracking errors decrease, the control inputs remain bounded and gradually attenuate.

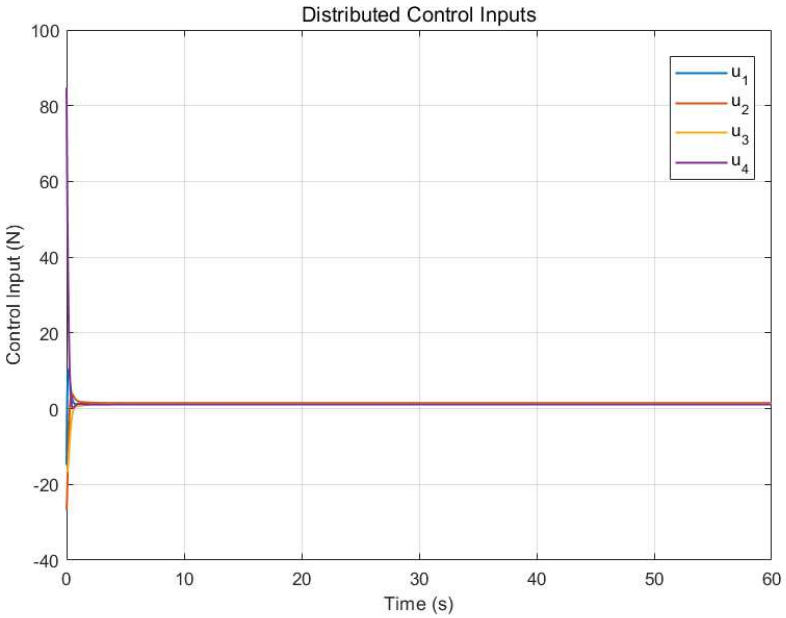


Fig. 10 Distributed control inputs of the follower UGVs in Example B.

Finally, a zero-state finite-horizon test is carried out to evaluate the disturbance attenuation performance of the UGV network. The leader and follower states are initialized at zero, and the disturbance acting on follower i is selected as

$$\varpi_i(t) = (0.03 + 0.005i)e^{-0.10t} \begin{bmatrix} \sin(0.40t + 0.25i) \\ 0.7\cos(0.55t - 0.15i) \end{bmatrix}, \qquad i = 1, \ldots, 4.$$

The controlled output is chosen as

$$z(t) = e(t).$$

The numerical results give

$$|z|_2 = 3.1326 \times 10^{-1}, \qquad |\varpi|_2 = 1.7941 \times 10^{-1},$$

and hence

$$\frac{|z|_2}{|\varpi|_2} = 1.7461 < 5.$$

Consequently, the simulation results verify that the proposed distributed control scheme can ensure favorable leader-following coordination and disturbance attenuation performance for heterogeneous UGV systems.

5 Conclusion

In this paper, we have studied the distributed H_∞ leader–following control problem for heterogeneous nonlinear MASs subject to external disturbances. A unified error system has been established to capture the effects of heterogeneous dynamics and nonlinear terms. Based on the incremental quadratic constraint and Lyapunov method, sufficient conditions have been derived to guarantee asymptotic stability and prescribed H_∞ disturbance attenuation performance. Moreover, a verification-based alternating LMI procedure has been developed to design the distributed feedback gains. Future work will focus on extending the proposed framework to switching topologies and reducing the required control effort.

Supplementary information

No supplementary information is available for this paper.

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Declarations

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Conflict of interest

The authors declare that they have no competing interests.

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Data availability

All data generated or analyzed during this study are included in this published article.

Code availability

The codes used for numerical simulations are available from the corresponding author upon reasonable request.

Author contribution

Xin Zhang was responsible for the theoretical analysis and controller design. Liangzhi Xu conducted the numerical simulations. Hao Zhang and Ye Fei participated in manuscript drafting and revision. All authors reviewed and gave final approval to the submitted version.

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