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A 4D Ultrasound Strain Imaging Framework for In Vivo Identification of Anisotropic Properties in Abdominal Aortic Aneurysms: Preliminary Validation against Mechanical Testing of Porcine and Human Tissue

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Abstract

Abdominal aortic aneurysms (AAAs) carry a substantial risk of rupture, underscoring the need for accurate biomechanical assessment of the individual patient. We present a computationally efficient method to identify the hyperelastic, anisotropic properties of AAA walls in vivo from non-invasive 4D ultrasound strain imaging, using only two load cases. The identification is posed as an inverse finite element problem, in which patient-specific models of the wall and the intraluminal thrombus (ILT) are iteratively optimized until the simulated deformations under blood pressure loading match the measured ones. Because the ILT geometry is resolved as well, the framework is applicable to the majority of clinical cases. As a proof of concept, the method is subjected to a preliminary validation against mechanical testing of both porcine aortic and human AAA wall tissue. The in vitro case comprised a porcine aorta in a controlled inflation–extension experiment; in vivo, the identification was performed in two AAA patients and compared against tensile tests of their excised wall tissue ($n = 6$ specimens). In vitro, the stress–stretch responses derived from ultrasound and from mechanical testing agreed closely ($R^2 = 0.96\text{--}0.99$). In vivo, agreement was good in one patient and moderate in the other. A complementary sensitivity analysis identified wall thickness and the degree of nonlinear stiffening as the parameters with the strongest influence on the identification. These results demonstrate the feasibility of non-invasive, patient-specific identification of AAA wall properties and represent a major step toward personalized biomechanical modeling and improved clinical risk stratification.

Keywords: Abdominal aortic aneurysm, 4D ultrasound strain imaging, Inverse finite element analysis, Material parameter identification, Anisotropic hyperelasticity, Experimental validation

1 Introduction

An abdominal aortic aneurysm (AAA) is a permanent dilation of the abdominal aorta to at least 1.5 times its normal size (Wanhainen et al. 2024). This condition is caused by a multifactorial wall weakening process (Martufi and Gasser 2013; Martufi et al. 2013) and can lead to rupture, an event with mortality rates up to 85% (Humphrey and Taylor 2008; Gasser et al. 2023). A common feature in about 75% of AAAs is the presence of an intraluminal thrombus (ILT) (Tong and Holzapfel 2015), a heterogeneous pseudo-tissue formed from stagnant blood (Wang et al. 2001). The role of the ILT is ambivalent: while it can alter peak wall stress (Inzoli et al. 1993; Mower et al. 1997; Wang et al. 2002), it may also biochemically weaken the underlying wall (Vorp et al. 2001) and is frequently found at rupture sites (Simão da Silva et al. 2000). A reliable risk assessment must therefore account for its presence (Gasser et al. 2023). Characterizing the distinct mechanical behaviors of these components is crucial. Healthy aortas exhibit anisotropic behavior with a pronounced linear elastic regime and subsequent nonlinear stiffening. In contrast, AAA tissue shows stage-dependent alterations, including a reduced linear range, premature stiffening, and a shift toward isotropy (Niestrawska et al. 2019). The ILT itself is nearly isotropic with a slightly nonlinear elastic response (Wang et al. 2001; Vande Geest et al. 2006; Tong et al. 2011).

Experimentally, inflation-extension tests best replicate in vivo conditions (Hayashi 1993), but require intact specimens that are unavailable from AAA repair surgery. Consequently, tensile tests on excised wall specimen are more common, yet they only provide local material behavior. Current surgical criteria still rely on population-based diameter thresholds (Wanhainen et al. 2024), a metric that may lead to overtreatment or missed interventions (Gasser et al. 2023). Biomechanical simulations offer a path toward patient-specific risk assessment (Farotto et al. 2018; Di Martino et al. 1998; Vorp et al. 1998), but their accuracy is

critically dependent on individual boundary conditions, geometry, and, most importantly, material properties, which are often unknown in vivo. Identifying these properties from clinical data, typically limited to two load states (diastole/systole), remains a major challenge.

While several methods have been proposed to identify patient-specific parameters in vitro and in vivo (Avril et al. 2008; Stålhand 2009; Avril et al. 2010; Astrand et al. 2011; Smoljkić et al. 2015; Wittek et al. 2016a; Trabelsi et al. 2016; Liu et al. 2017, 2018; Gade et al. 2019; Liu et al. 2019b,c; Roux and Hild 2020; Narayanan et al. 2021; Karlsson et al. 2025; Guo et al. 2025; Meng et al. 2025), a critical gap remains. To ensure reliability, any in vivo approach must be benchmarked against mechanical tests on harvested tissue (American Society of Mechanical Engineers 2020). Such validations are scarce, particularly for methods that handle patient-specific geometries, anisotropic material models and use AAA tissue samples. To the authors’ knowledge, in vivo approaches benchmarked against mechanical tests on harvested tissue remain scarce and limited in scope. In van Disseldorp et al. (2020), a 4D-US-based global, linearized wall stiffness was verified against biaxial tensile tests on porcine aortas and AAA tissue, but did not identify a full anisotropic constitutive model. Liu et al. (2019b) and Guo et al. (2025) identified anisotropic properties of ascending aortas (ATAA and type-A dissection, respectively), but for tissue lacking an ILT, and both reported only limited agreement between the in vivo and ex vivo material behavior. A validation of fully nonlinear, anisotropic constitutive parameter identification for AAA wall tissue, accounting for the presence of ILT, is still lacking. Our group previously introduced and numerically verified a 4D ultrasound (4D-US) strain imaging approach to identify the hyperelastic and anisotropic behavior of irregularly shaped AAAs (Wittek et al. 2013, 2016a). But the method was computationally expensive, excluded the ILT, and lacked experimental validation.

To address these limitations, this study introduces an extended and computationally efficient

framework for the in vivo identification of patient-specific, anisotropic material properties in AAAs from 4D-US strain imaging data. Extending our previous work, this new framework now incorporates the ILT while reducing computation times sevenfold, thereby enabling its application to a broad spectrum of clinical cases. The framework’s feasibility and accuracy are demonstrated by a preliminary validation in a proof-of-concept study, comprising a controlled experimental in vitro benchmark as well as a direct comparison of in vivo predictions with ex vivo mechanical tests. To present this work clearly, we first outline the core methodologies common to both comparative investigations. The paper then proceeds with two central studies. First, a controlled in vitro benchmark is presented, in which a porcine aorta was subjected to biaxial inflation-extension tests. Wall deformations were captured simultaneously by an optical camera system and our 4D-US device, enabling a direct comparison of the material parameters identified from each data source. Based on this in vitro setup, a sensitivity analysis is then presented to assess the robustness of the parameter identification. Second, an in vivo study on two AAA patients is detailed. Here, material parameters derived from pre-operative in vivo 4D-US data are compared against those obtained from ex vivo uniaxial tensile tests on tissue samples ($n = 6$) collected from the same patients.

2 Materials and methods

2.1 4D ultrasound strain imaging and post-processing

4D-US data were acquired using a real-time 3D echocardiography system (Artida[®], Canon Medical Systems, Japan) with a transthoracic matrix transducer (Toshiba, PST-25SX, 1–4 MHz). Post-processing was performed using the commercial Advanced Cardiology Package (ACP, Canon Medical Systems, Japan). In the ACP, the luminal border was manually segmented in two cross-sections (one section can be seen in Fig. 1), automatically completed, and used as the region of interest (ROI) for subsequent speckle tracking with $10 \times 10 \times 10 \text{ mm}^3$ template volumes across 14–29 time steps per cardiac cycle (Chen et al. 2005; Seo et al. 2009, 2011). All ECG-triggered measurements and

post-processing were performed by a single experienced observer. In terms of continuum mechanics, each template centroid and its measured displacements from speckle-tracking represent a material point, with end-diastole as reference configuration $\Omega_{\text{dia}}^{\text{US}}$ (Karatolios et al. 2013). After excluding the artificial pseudo-apex (cf. Fig. 1), 504–828 material points remained which describe the geometry and local displacements of the luminal vessel wall with resulting vessel lengths of 38.8–78.6 mm. A single manual segmentation, automated completion, and wall displacement estimation over one cardiac cycle is further referred to as one ‘masking’. Since initial segmentation is manual, each masking yields an individual displacements estimate. To reduce random errors, we presented a method to combine multiple maskings into one ‘averaged model’ (Hegner et al. 2023). Discretization of the wall geometry in terms of the finite element (FE) analysis was done by connecting four material points to form rectangular shell elements. As the displacements of these corner points (material points) were known, local in-plane strain tensors could be computed for each element in regions of 2–20 mm². These strains were expressed in local coordinate systems (cf. Fig. 3) aligned with the vessel’s axial ($\mathbf{e}_1$) and circumferential ($\mathbf{e}_2$) directions (Karatolios et al. 2013; Hegner et al. 2023; Derwich et al. 2016, 2020; Wittek et al. 2016b). Local in-plane Biot strain tensors $\boldsymbol{\epsilon}$ were computed element-wise in Matlab R2025a (The MathWorks, Inc., USA) from the right Cauchy-Green tensor $\mathbf{C} = \mathbf{F}^T \mathbf{F}$ and the second-order identity tensor $\mathbf{I}$ using

$$\boldsymbol{\epsilon} = \sqrt{\mathbf{C}} - \mathbf{I} \quad . \quad (1)$$

Deformation gradient $\mathbf{F}$ was derived from US measured displacements, the local coordinate systems and linear shape functions, as described and validated in Hegner et al. (2023). US measured strains $\boldsymbol{\epsilon}^{\text{US}}$ are cyclic strains relative to diastole and are purely kinematic, as they are derived from the known displacement field and therefore independent of any material model (Wittek et al. 2009). Meshes were later used for building the FE models (as seen in Fig. 3 and later described in Sec. 2.6.2 and Sec. 2.7.2).

2.2 Constitutive modeling and model stresses

All vessel walls were modeled using the hyperelastic, anisotropic Gasser-Ogden-Holzapfel (GOH) material model (Gasser et al. 2006), defined by the strain energy density per unit reference volume

$$\psi^{\text{GOH}}(\mathbf{C}, \mathbf{a}_{0\alpha}) = \frac{\mu}{2} (C_1 - 3) + \frac{k_1}{2k_2} \sum_{\alpha=1}^2 \left(e^{k_2 (E_\alpha)^2} - 1 \right) \quad (2)$$

$$\text{with: } E_\alpha = \kappa (C_1 - 3) + (1 - 3\kappa) (C_{4\alpha} - 1) . \quad (3)$$

C_1 is the first invariant of $\mathbf{C}$, $C_{4\alpha}$ is a mixed invariant of $\mathbf{C}$ and the structural tensor $\mathbf{a}_{0\alpha} \otimes \mathbf{a}_{0\alpha}$, capturing anisotropy. If $\mathbf{a}_{01} = (\sin \varphi, \cos \varphi, 0)^T$ and $\mathbf{a}_{02} = (\sin \varphi, -\cos \varphi, 0)^T$ define two symmetric fiber directions in the wall plane where fiber angle φ is measured with respect to $\mathbf{e}_2$, then $C_{4\alpha}$ follows to

$$C_{4\alpha} = \mathbf{C} : \mathbf{a}_{0\alpha} \otimes \mathbf{a}_{0\alpha} = \lambda_1^2 \sin^2 \varphi + \lambda_2^2 \cos^2 \varphi , \quad \alpha = 1, 2 . \quad (4)$$

The incompressible Cauchy stress tensor can be written as (Holzapfel 2000)

$$\boldsymbol{\sigma} = 2\mathbf{F} \frac{\partial \psi}{\partial \mathbf{C}} \mathbf{F}^T - p\mathbf{I} \quad (5)$$

where p is a Lagrange multiplier accounting for the hydrostatic pressure. For plane stress ($\sigma_{33} = 0$) and incompressible, homogeneous deformations according to the principle axes $\mathbf{C} = \lambda_1^2 \mathbf{e}_1 \otimes \mathbf{e}_1 + \lambda_2^2 \mathbf{e}_2 \otimes \mathbf{e}_2 + 1/(\lambda_1^2 \lambda_2^2) \mathbf{e}_3 \otimes \mathbf{e}_3$ where $E_\alpha \geq 0$, Cauchy model stresses in axial σ_{11}^{GOH} and circumferential (circ.) vessel direction σ_{22}^{GOH} are derived to

$$\sigma_{11}^{\text{GOH}} = \mu \left(\lambda_1^2 - \frac{1}{\lambda_1^2 \lambda_2^2} \right) + 4k_1 E e^{k_2 E^2} \times \left((\kappa + (1 - 3\kappa) \sin^2 \varphi) \lambda_1^2 - \frac{\kappa}{\lambda_1^2 \lambda_2^2} \right) , \quad (6)$$

$$\sigma_{22}^{\text{GOH}} = \mu \left(\lambda_2^2 - \frac{1}{\lambda_1^2 \lambda_2^2} \right) + 4k_1 E e^{k_2 E^2}$$

$$\times \left((\kappa + (1 - 3\kappa) \cos^2 \varphi) \lambda_2^2 - \frac{\kappa}{\lambda_1^2 \lambda_2^2} \right) , \quad (7)$$

$$\text{with: } E = \kappa \left(\lambda_1^2 + \lambda_2^2 + \frac{1}{\lambda_1^2 \lambda_2^2} - 3 \right) + (1 - 3\kappa) (\lambda_1^2 \sin^2 \varphi + \lambda_2^2 \cos^2 \varphi - 1) . \quad (8)$$

$\mu > 0$, $k_1 > 0$ are stress-like parameters, $k_2 > 0$ is dimensionless, $0 \leq \kappa \leq 1/3$ and $0^\circ \leq \varphi \leq 90^\circ$ are structure parameters. ILT was modeled using the unconstrained isotropic neo-Hookean strain energy density function (Maas et al. 2024) given by

$$\psi^{\text{nH}} = \frac{1}{2} \mu^{\text{nH}} (C_1 - 3) - \mu^{\text{nH}} \ln J + \frac{1}{2} \lambda^{\text{nH}} (\ln J)^2 , \quad (9)$$

$$\mu^{\text{nH}} = \frac{Y}{2(1 + \nu_{\text{ILT}})} , \quad (10)$$

$$\lambda^{\text{nH}} = \frac{Y \nu_{\text{ILT}}}{(1 + \nu_{\text{ILT}})(1 - 2\nu_{\text{ILT}})} . \quad (11)$$

where J is the determinant of $\mathbf{F}$, Y is the Young's modulus, and ν_{ILT} is the Poisson's ratio which was estimated from the volume ratio $J = V_{\text{sys}}/V_{\text{dia}}$, and axial λ_1 and circ. λ_2 stretches by

$$\nu_{\text{ILT}} = \frac{1 - \frac{V_{\text{sys}}}{V_{\text{dia}}} \frac{1}{\lambda_1 \lambda_2}}{\left(1 - \frac{V_{\text{sys}}}{V_{\text{dia}}} \frac{1}{\lambda_1 \lambda_2} \right) + (\lambda_1 + \lambda_2 - 2)} . \quad (12)$$

2.3 A method for in vivo constitutive parameter optimization

The method starts by building an FE model $\mathbf{X}_{\text{dia}}^{\text{GOH}} = \mathbf{X}_{\text{dia}}^{\text{US}}$ from the 4D-US-measured diastolic configuration $\mathbf{X}_{\text{dia}}^{\text{US}}$, using the GOH material model Eq. (2) for the vessel wall ($\mathbf{X}$ always denotes a matrix containing the coordinates of all material points in the respective configuration). The mesh consists of the material points forming 4-node shell elements (Kroon and Holzapfel 2009), as described in Sec. 2.1. For each parameter vector $\mathbf{p} = (\mu, k_1, k_2, \kappa, \varphi)^T$, the load-free configuration $\mathbf{X}_{\text{LF}}^{\text{GOH}}$ is first iteratively estimated using the Rausch et al. (2017) method, to which later measured diastolic and systolic loadings are

applied, yielding calculated configurations $\mathbf{X}_{\text{dia}}^{\text{GOH}}$ and $\mathbf{X}_{\text{sys}}^{\text{GOH}}$. Strains are then computed for both configurations with respect to the load-free configuration, from where cyclic strains ε^{GOH} are calculated with respect to the diastolic configuration. These cyclic strains are material-dependent and directly comparable to the material-free measured in vivo cyclic strains ε^{US} . GOH parameters are then identified by minimizing the squared error sum Eq. (13) between US-measured strains ε^{US} and FE-computed strains ε^{GOH}

$$f_{\text{err}}^{\text{US}} = \frac{1}{n} \sum_i \sum_{j=1}^n w_i (\varepsilon_{i,j}^{\text{GOH}}(\mu, k_1, k_2, \kappa, \varphi) - \varepsilon_{i,j}^{\text{US}})^2 \quad (13)$$

$$\text{with : } \sum_i w_i = 1 \quad ; \quad i = 11, 22, 12 \quad . \quad (14)$$

Here, n is the number of elements, i denotes the in-plane strain components (11: axial, 22: circ., 12: shear), and w_i are weights. As optimization strategy, we implemented a self-adaptive JADE version with external archive (Zhang and Sanderson 2009) of a Differential Evolution (DE) algorithm in Matlab R2025a. A current-to- k best mutation strategy with $k = 1$ and an initial population of size 35 created using Latin Hypercube Sampling was used. FE solver FEBio (Maas et al. 2012) was used for all simulations. So, for each guess of $\mathbf{p}$:

1. Find load-free configuration $\mathbf{X}_{\text{LF}}^{\text{GOH}}$ using a hybrid technique:
 - 1.1 Apply diastolic loads to the measured diastolic configuration $\mathbf{X}_{\text{dia}}^{\text{GOH}} = \mathbf{X}_{\text{dia}}^{\text{US}}$ and obtain resulting deformation gradients $\mathbf{F}^{\text{GOH}}$.
 - 1.2 $\mathbf{F}^{\text{GOH}}$ are used on $\mathbf{X}_{\text{dia}}^{\text{GOH}}$ as pre-strain for the pre-strain material class (Maas et al. 2016) in FEBio to find an initial guess of $\mathbf{X}_{\text{LF}}^{\text{GOH}}$.
 - 1.3 Iterative search using the augmented version (Rausch et al. 2017) of Sellier's method (Sellier 2011) is performed until the largest difference between identical nodes of the calculated $\mathbf{X}_{\text{dia}}^{\text{GOH}}$ and measured $\mathbf{X}_{\text{dia}}^{\text{US}}$ is < 0.01 mm.
 - 1.4 Obtain final calculated $\mathbf{X}_{\text{LF}}^{\text{GOH}}$ and $\mathbf{X}_{\text{dia}}^{\text{GOH}}$.
2. Apply systolic loads to $\mathbf{X}_{\text{LF}}^{\text{GOH}}$ to obtain calculated $\mathbf{X}_{\text{sys}}^{\text{GOH}}$.

3. Determine in-plane strain tensors for calculated $\mathbf{X}_{\text{dia}}^{\text{GOH}}$ and $\mathbf{X}_{\text{sys}}^{\text{GOH}}$ with respect to $\mathbf{X}_{\text{LF}}^{\text{GOH}}$ using Eq. (1).
4. Convert to calculated cyclic strains ε^{GOH} with respect to diastole and compare with ε^{US} using Eq. (13)

Basic modeling assumptions for the vessel wall remain unchanged and were extensively discussed in Section 2.1 of Wittek et al. (2016a).

2.4 Parameter optimization of mechanical data

To validate the 4D-US method from Sec. 2.3, mechanical tests on tissue samples were performed. GOH parameters $\mu, k_1, k_2, \kappa, \varphi$ were identified by directly fitting model stresses σ_{11}^{GOH} and σ_{22}^{GOH} defined in Eq. (6)–(8) to wall stresses σ_{11}^{exp} and σ_{22}^{exp} obtained from mechanical tests (further detailed in Sec. 2.6.3 and 2.7.3), using the same JADE algorithm. Here, parameters can be estimated by directly comparing model stresses and experimental wall stresses by minimizing the squared error sum

$$f_{\text{err}}^{\text{exp}} = \frac{1}{n} \sum_{i=1}^n w_{11} \left(\sigma_{11,i}^{\text{GOH}}(\mu, k_1, k_2, \kappa, \varphi) - \sigma_{11,i}^{\text{exp}} \right)^2 + \frac{1}{m} \sum_{j=1}^m w_{22} \left(\sigma_{22,j}^{\text{GOH}}(\mu, k_1, k_2, \kappa, \varphi) - \sigma_{22,j}^{\text{exp}} \right)^2, \quad (15)$$

where the weights $w_{11} + w_{22} = 1$, n is the number of axial stresses and m is the number of circ. stresses.

At this point, all underlying methods are introduced here in general form. In the following validation Sec. 2.6 and 2.7, they are specified and adapted to the respective cases.

2.5 Ethics

4D-US imaging, harvesting and use of AAA wall specimen was approved by the Ethics Committee of the Goethe University Hospital, Frankfurt am Main, Germany (Ref.-Num.: 275/17 and S-462/2017). Participants provided informed consent prior to inclusion in the study. Porcine aorta is considered slaughterhouse waste.

2.6 Benchmarking framework: biaxial testing and parameter identification for a porcine aorta

2.6.1 Inflation-extension experiment

Using an inflation-extension setup (Blase et al. 2025), an intact porcine aorta segment was placed in 0.9% saline at 37 °C and physiologically loaded at 0.84 ± 0.01 Hz (cf. Fig. 2). A motor (PR01-52, LinMot, Switzerland) dynamically changed vessel length via an axially movable upper specimen holder, while a fixed lower holder, connected to a second motor (Electroforce LM1, Bose Corp., USA), pumped heated saline into the specimen, inducing dynamic diameter changes. Axial force and transmural pressure were measured at 50 Hz, with deformations recorded simultaneously by two orthogonal CMOS cameras (1.3 Megapixel, Ximea, Germany) at 15 Hz and the clinically used 4D-US device (cf. Sec. 2.1). All components were synchronized in LabVIEW (National Instruments, USA). 17 different deformation paths were generated during the experiment (shown in Fig. 4e), where the specimen experienced axial strains of 5–55%, circ. strains of 0–38%, and transmural pressures of 40–180 mmHg. One deformation path represented an in vivo case (IVC), mimicking the physiological loading of a healthy abdominal aorta and ensuring later applicability to only two load states (diastole/systole) in vivo. IVC exhibited an axial pre-stretch of 1.2, cyclic axial strains of 2.5%, circ. strains of 14.6%, volume change of 29.8%, and transmural pressures of 78–132 mmHg at 0.84 ± 0.01 Hz, aligning well with physiological deformations and loads (Klinke et al. 2010; Wittek et al. 2016b; Horný et al. 2014; Fung 1993). After testing, porcine aorta was cut into six rings. Undeformed wall thickness was measured at 10 evenly spaced points per ring (total of 60 values), yielding a mean of 2.32 ± 0.27 mm (min: 1.72 mm, max: 2.86 mm).

2.6.2 Parameter optimization of porcine aorta from 4D-US

Post-processing of 4D-US data followed Sec. 2.1 while only the IVC was used for optimization to mimic in vivo conditions. Vessel wall was masked five times in the US images (see Fig. 1a) to create one averaged model (cf. Sec. 2.1, Hegner et al.

(2023)). Diastolic $\Omega_{\text{dia}}^{\text{US}}$ and systolic $\Omega_{\text{sys}}^{\text{US}}$ configurations were defined by smallest and largest volume, respectively, from which measured cyclic strains ε^{US} were determined using Eq. (1) for later comparison with calculated cyclic strains ε^{GOH} . Only experimental pressure and force data at these two states were used to match in vivo conditions. Resulting vessel length was approx. 78.6 mm. FE model (see Fig. 3a) was created from the averaged model using 4-node ‘quad4’ shell elements with ‘three-field-shell’ type formulation and the ‘Augmented Lagrangian’ method activated to enforce incompressibility. Anisotropy was modeled using local material axes (cf. Sec. 2.1), as seen in Fig. 3b. Boundary conditions (BCs) of the experiment were applied to the model, measured transmural pressures were applied to the inner surface, and the measured sensor forces were applied to the upper end. GOH parameters were identified by minimizing Eq. (13) as described in Sec. 2.3. During optimization, the measured mean undeformed wall thickness of 2.32 mm (cf. Sec. 2.6.1) was assigned to the current load-free geometry $\mathbf{X}_{\text{LF}}^{\text{GOH}}$.

2.6.3 Parameter optimization of porcine aorta from mechanical data

All 17 deformation paths were used to identify the material behavior based on mechanical data. Deformed vessel radii r_o as well as axial λ_1 and circ. λ_2 stretches were measured from camera images between previously applied markings using ImageJ (<https://ImageJ.net>). Force, pressure, and stretches were averaged for each of the 17 cases across five consecutive load cycles (see Fig. 4a,b), as five maskings were used to create one averaged model. Measured force values were smoothed using the `smoothingspline` option in Matlab R2025a. Only small hystereses were observed, therefore unloading paths were neglected (Holzapfel et al. 2005). For each load path, 25 equally distributed points along each curve were used for calculation of axial $\sigma_{11}^{\text{inf-ext}}$ and circ. $\sigma_{22}^{\text{inf-ext}}$ stresses. For an incompressible, thin-walled cylinder ($\sigma_{33} = 0$) with closed ends and external axial loading, the equilibrium equations

yield the averaged Cauchy stresses

$$\sigma_{11}^{\text{inf-ext}} = \frac{F_{\text{red}} + p_t \pi r_E^2}{\frac{\pi H}{\lambda_1 \lambda_2} \left(2r_o - \frac{H}{\lambda_1 \lambda_2} \right)} \quad , \quad (16)$$

$$\sigma_{22}^{\text{inf-ext}} = p_t \left(\frac{r_o}{H} \lambda_1 \lambda_2 - \frac{1}{2} \right) \quad . \quad (17)$$

Here, reduced axial force F_{red} equals the measured sensor force (Holzapfel et al. 2000), p_t is the measured transmural pressure, r_E is the constant end radius of the upper specimen holder, and H is the undeformed wall thickness measured on the specimen. Averaged measured wall thickness of $H = 2.32$ mm was used. Resulting 3D stress-stretch curves from Eq. (16) and (17) can be seen in Fig. 4c,d. GOH parameters were identified as described in Sec. 2.4 by minimizing Eq. (15), where $\sigma^{\text{exp}} = \sigma^{\text{inf-ext}}$ and $n = m$.

2.7 Patient-specific case study: in vivo AAA data acquisition and ex vivo tissue characterization

2.7.1 In vivo AAA patient data

Data from two AAA patients, one male (AAA-1) and one female (AAA-2), aged 69 and 65 years, and with maximum AAA diameters of 53.1 and 66.2 mm, who presented at the Clinic for Vascular and Endovascular Surgery of the University Hospital Frankfurt am Main, Germany, were evaluated. Pre-operative 4D-US data were acquired according to Sec. 2.1, diastolic and systolic blood pressures were obtained using brachial blood pressure measurements. AAA wall specimens from the same two patients were harvested after 4D-US imaging during open aneurysm repair surgery. Specimens were stored in 0.9% saline solution at 4°C until testing on the same day. Harvested AAA tissues were cut into bone shaped uniaxial tensile specimens, and undeformed wall thickness was measured directly on each. Patient AAA-1 had one longitudinal and one circumferential specimen with wall thickness of 2.20 mm and 1.80 mm. Patient AAA-2 had two longitudinal specimens (3.60 mm, 2.80 mm) and two circumferential specimens (4.20 mm, 3.90 mm).

2.7.2 Parameter optimization of AAA patients from 4D-US

In vivo FE models were generated from US-based segmentations of wall and ILT (Fig. 1b,c). Post-processing of 4D-US data followed Sec. 2.1. Vessel wall and ILT were segmented five times to create one averaged model of each. Ten layers were then created between lumen and wall (Fig. 3e), with the innermost ILT layer matching the lumen segmentation in Fig. 1c and the outermost matching the wall segmentation in Fig. 1b, both measured by US. To reflect physiological BCs, tapered artificial extensions to a healthy diameter of 20 mm were added at both ends (mesh areas outside the red markings in Fig. 3c). The caudal end allowed only radial expansion, while the cranial end could additionally move in the axial direction. In combination with an axial spinal support, the US-measured region could move in a physiological way. Measured blood pressure was applied to the lumen surface, considering an intra-abdominal pressure of 7 mmHg (de Keulenaer et al. 2009). Axial force was set to zero, as AAAs typically lack axial pre-stretch (Horný et al. 2013). The vessel wall was modeled using 8-node 'hex8' solid elements, seen in Fig. 3c and 3e, with a 'three-field-solid' type formulation and activated 'Augmented Lagrangian' method to enforce incompressibility. For undeformed wall thickness, measured thicknesses of the wall specimen (cf. Sec. 2.7.1) were averaged. For patient AAA-1 (2 specimen), mean thickness was 2.00 ± 0.20 mm, and for patient AAA-2 (4 specimen) 3.63 ± 0.52 mm (given as mean $\pm$ std, respectively). During optimization, the measured mean undeformed wall thickness was assigned to the current load-free geometry $\mathbf{X}_{\text{LF}}^{\text{GOH}}$, respectively. Experimentally, the ILT can be roughly divided into three layers (luminal, medial, abluminal) with distinct mechanical properties (Tong et al. 2011). Here, the ILT was modeled as a single layer, with its material properties defined as the average of the three layers. ILT was modeled using 8-node 'HEX8G8' solid elements, seen in Fig. 3e, with an 'elastic-solid' type formulation to allow compressibility. Strain energy was the neo-Hookean model Eq. (9), where parameter $Y = 109.8$ kPa was an average ($n = 43$) of tangential moduli reported in Tong et al. (2011) and Tong and Holzapfel (2015). ν_{ILT} was estimated patient-individually using Eq. (12), where

ILT volume ratio $V_{\text{sys}}/V_{\text{dia}}$ was calculated from the US point clouds using `convexHull` function in Matlab R2025a. Vessel wall was modeled using the GOH model Eq. (2), while anisotropy was modeled using local material axes seen in Fig. 3d. Only regions covered by 4D-US were further evaluated, extensions served solely to ensure physiological boundary conditions. GOH parameters were identified by minimizing Eq. (13) as described in Sec. 2.3.

2.7.3 Parameter optimization of AAA patients from mechanical data

Harvested tissues were cut into separate bone-shaped specimens for the axial and circ. directions of the blood vessel, with sizes of 15×10 mm. There was one axial and one circ. specimen from AAA-1 (AAA-1 S1) and two axial and two circ. specimens from AAA-2 (AAA-2 S1 and S2), where S1/S2 always refers to a combination of one axial and one circ. specimen. Specimens were mounted in a displacement-driven uniaxial testing device using clamps and submerged in 0.9% saline at 37°C . They were tested by applying increasing total displacements until plastic deformation occurred. Displacements were applied quasi-statically at 0.1 mm/s over five cycles to account for viscoelastic effects (Fung et al. 1979), followed by 5 min. of rest. Total displacements ranged from 0.5 to 2.1 mm. During deformation, images were captured using a CMOS camera (1.3 Megapixels, Ximea, Germany). In each case, the last data set in which only elastic deformations occurred was evaluated. Axial λ_1 and circ. λ_2 stretches were measured from camera images between previously applied markings in ImageJ. For each load path, 25 equally distributed points along each curve were used for calculation of axial σ_{11}^{uni} and circ. σ_{22}^{uni} stresses. For an incompressible material, the equilibrium equations yield the Cauchy stresses

$$\sigma_{11}^{\text{uni}} = \frac{F_{\text{axial}}}{A_{\text{axial}}} \lambda_1 \quad , \quad \sigma_{22}^{\text{uni}} = \frac{F_{\text{circ}}}{A_{\text{circ}}} \lambda_2 \quad , \quad (18)$$

where F is the experimentally measured force, and A is the directly on the specimen measured undeformed cross-sectional area, respectively. GOH parameters were identified as described in Sec. 2.4 by minimizing Eq. (15), where $\sigma^{\text{exp}} = \sigma^{\text{uni}}$ and $n = m$.

2.7.4 Sensitivity analysis of the parameter identification process

To assess key influences on in vivo parameter optimization, a sensitivity analysis was performed. We focused on three factors that can only be estimated in vivo, but are either controllable or known during in vitro experiments: wall thickness H (unknown in vivo), reduced axial force F_{red} (population-averaged), and deformation range between diastole and systole, which is restricted by blood pressure. Therefore, H and F_{red} were varied by $\pm 10\%$, and optimizations repeated. During experiment, time-resolved pressure and force data were recorded by the sensors, and 11 different deformation states were captured via 4D-US. But only diastole and systole were used for optimization, as in vivo blood pressure is unknown for intermediate deformation states. The used strain energy formulation Eq. (2) uses a reduced form of invariants, where C_1 and $C_{4\alpha}$ have to be captured over a suitable range of deformations that adequately capture the material's nonlinear response (Holzapfel 2009; Ogden 2009). To analyze the impact of reduced deformation, systole was replaced by earlier frames (with corresponding force/pressure values), keeping the same diastolic reference. True systole represented 100% of deformation range; reduced ranges of 90%, 80%, 70%, 60%, and 50% were tested, and parameters re-identified accordingly. Additionally, the Young's modulus Y is the only unknown or non-measurable material parameter of the ILT. Here, the ILT was modeled as a single layer, with Y calculated as the mean value of mechanical data (cf. Sec. 2.7.2). To assess the influence of the ILT material parametrization and one layer layout, eleven model configurations were compared at a luminal pressure of $p_t = 120$ mmHg. Model M0 served as a reference without ILT. In model M1, the ILT was represented by a single homogeneous layer whose properties were derived from the mean tangent moduli reported by Tong et al. (2011) and Tong and Holzapfel (2015), as later used for the in vivo models described in Sec. 2.7.2. Models M2–M10 employed a three-layered ILT (luminal, medial, abluminal) and systematically varied the layer stiffnesses within ± 1 standard deviation of the corresponding experimental values from the literature. The resulting displacements of the wall nodes were then compared, since the strains to be

minimized in the error functions Eq. (13) and (15) are calculated directly from these displacements. Thus, identical wall displacements would result in identical material behavior.

2.8 Goodness of fit

To evaluate agreement of identified material behavior the coefficient of determination R^2

$$R^2 = 1 - \frac{\sum_i^n (\sigma_i^{\text{Model}} - \sigma_i^{\text{Reference}})^2}{\sum_i^n (\sigma_i^{\text{Reference}} - \hat{\sigma}^{\text{Reference}})^2} \quad , \quad (19)$$

or, if $R^2 < 0$, the Mean Absolute Percentage Error (MAPE)

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^n \left| \frac{\sigma_i^{\text{Model}} - \sigma_i^{\text{Reference}}}{\sigma_i^{\text{Reference}}} \right| \cdot 100\% \quad , \quad (20)$$

were used for both validations. To compare material behavior, stresses σ_i^{Model} and $\sigma_i^{\text{Reference}}$ were computed using the two parameter sets resulting from the two different identification methods (here given as 'Model' and 'Reference' sets), n is the number of stress values and $\hat{\sigma}^{\text{Reference}}$ is the average. To account for anisotropy, goodness of fit was evaluated along three characteristic load paths within the biaxial deformation domain: LP-I (axial uniaxial, $\lambda_2 = \text{const.} = 1$), LP-II (equibiaxial, $\lambda_1 = \lambda_2$), and LP-III (circ. uniaxial, $\lambda_1 = \text{const.} = 1$). The three load paths are shown in Fig. 4e.

For comparison of strain distributions, the Heterogeneity index HI , defined as coefficient of variation (ratio of the standard deviation and mean of the strain distribution) (Karatolios et al. 2013; Derwich et al. 2016; Wittek et al. 2017), was calculated.

3 Results

Validation results in Sec. 3.1 and 3.3 are given as follows. First, the agreement of each identified GOH material response with the underlying data is assessed individually. For the in vivo 4D-US method, a comparison is made between the strains measured by ultrasound and the corresponding strains from the finite element model. In the case where the GOH model was fitted to optical and mechanical data, a comparison is

performed between the experimental wall stresses and the stresses calculated using the optimized model parameters. Subsequently for validation, both identified GOH material responses are compared directly to evaluate their similarity. In both validation results, 'Exp' refers to the experimental data, 'US' denotes the GOH material response identified using the 4D-US approach (Sec. 2.3), and 'OP' denotes the material response identified based on optical and mechanical data (Sec. 2.4).

3.1 Comparison of in vitro material behavior from mechanical testing and 4D-US imaging

Optical/Mechanical: Identified parameter set based on optical and mechanical (OP) data can be found in Tab. 1 in section *Porcine Aorta* OP and illustration of the GOH model fit against experimental data in Fig. 5a,b. Model and experimental data show very good agreement with $R^2 = 0.943$ ($p < 0.001$) for the axial and $R^2 = 0.975$ ($p < 0.001$) for the circ. stresses.

Ultrasound: Identified parameter set based on ultrasound (US) data can be found in Tab. 1 in section *Porcine Aorta* US, comparison of the stretches measured by US and the resulting stretches of the FE model is shown in Fig. 5g. Cyclic axial stretches measured by US showed 1.023 [0.97, 1.10] (median [Q_1 , Q_3]), the FEA model 1.029 [1.02, 1.03], cyclic circ. stretches measured by US showed 1.125 [1.10, 1.16], the FEA model 1.129 [1.11, 1.14] and therefore good agreement between both median values.

Comparison: Stress-stretch comparison between the materials identified based on optical/mechanical and ultrasound deformation measurements can be seen in Fig. 6. Material behavior of the porcine aorta was identified with very good agreement between both methods, which is confirmed by goodness of fit between $R^2 = 0.952$ – 0.990 (all $p < 0.001$) along all load paths.

3.2 Sensitivity analysis

Results of the sensitivity analysis, as described in Sec. 2.7.4, are shown in Fig. 7, Tab. 2, and Tab. 3.

Influence of deformation range: Influences of the deformation range are shown in Fig. 7a,b, with resulting parameters in Tab. 2. Gray area between the vertical dashed lines indicates the true stretch range of the IVC (and thus the approx. physiologically occurring stretch range), with results from the true systole (100%) serving as the reference. Using 70–90% of the deformation range between diastole and systole still yielded sufficiently accurate material identification, with $R_{\text{Long}}^2 = 0.990\text{--}0.995$ and $R_{\text{Circ}}^2 = 0.992\text{--}0.993$. However, using 60% or less resulted in a material that was too soft. For this IVC, where approx. 56.6% of the stretches lie within the non-linear stiffening range, at least 46.9% of this range (or 70% of the healthy pulse pressure range) was required for adequate material identification.

Influence of wall thickness: Reference optimizations were performed using the average wall thickness 2.32 mm. To assess the impact, we varied the thickness by $\pm 10\%$ (2.55 mm and 2.09 mm) and repeated the optimization. Effects on stress-stretch behavior are shown in Fig. 7c,d, with resulting parameters in Tab. 2. Since a 10% change represents about 50% of the range between measured min. and max. wall thickness, its influence is notable. Although $R^2 = 0.958\text{--}0.984$ indicate good agreement, the stresses in the physiological range deviate by 0.0076 ± 0.009 MPa (2.7% of max. occurred stress). The max. deviation was 0.0198 MPa (6.9% of max. occurred stress), which could be significant for rupture stress estimates. Over- or underestimating wall thickness caused nearly identical deviations.

Influence of reduced axial force: The reduced axial force was also varied by $\pm 10\%$, and optimizations were repeated. The effects on stress-stretch behavior are shown in Fig. 7e,f, with parameters in Tab. 2. The influence of axial force fluctuations is small compared to wall thickness. The smallest values of $R^2 = 0.994$ indicate excellent agreement between stress-stretch curves. The resulting stresses deviate by 0.0009 ± 0.0006 MPa (0.31% of max. occurred stress), with a max. deviation of 0.0021 MPa (0.73% of max. occurred stress). Overestimating axial force had a slightly greater impact than underestimating it.

Influence of ILT constitutive parameters:

As summarized in Table 3, introducing the ILT roughly halves the median wall displacement (7.62 mm $\rightarrow$ 3.73 mm), whereas the specific ILT parametrization has a negligible effect: across the nine variants M2–M10 the per-model median displacement varies by only 3.67–3.86 mm, i.e. within about 2.5% of the no-ILT reference. Thus, modeling the ILT as a single layer using parameters averaged from experimental data is sufficient, provided the actual luminal geometry is known.

3.3 In vivo vs. ex vivo results: comparison of patient-specific material behavior

Uniaxial tests: Identified parameter sets based on uniaxial tensile tests on excised tissue samples and optical (OP) measurements can be found in Tab. 1 in sections AAA-1 S1 and AAA-2 OP S1 and OP S2, illustrations of the GOH model against uniaxial test data in Fig. 5c,d for AAA-1 and in Fig. 5e,f for AAA-2. In both cases, the model and experimental data show good agreement. AAA-1 S1 showed $R^2 = 0.9993$ ($p < 0.001$) for the axial and $R^2 = 0.9997$ ($p < 0.001$) for the circ. stresses. Specimen S1 of AAA-2 showed $R^2 = 0.9996$ for the axial and $R^2 = 0.9982$ for the circ. stresses, specimen S2 of AAA-2 showed $R^2 = 0.9975$ for the axial and $R^2 = 0.9790$ for the circ. stresses (all $p < 0.001$).

Ultrasound: Identified parameter sets based on ultrasound (US) data can be found in Tab. 1 in sections AAA-1 US and AAA-2 US, comparison of stretches measured by US and the resulting stretches of the FEA model is shown in Fig. 5h,i. For AAA-1, cyclic axial stretches measured by US showed 1.005 [0.996, 1.014] (median [Q_1 , Q_3]), the FEA model 1.003 [1.02, 1.03], cyclic circ. stretches measured by US showed 1.001 [0.989, 1.014], the FEA model 0.998 [0.997, 0.999] and therefore good agreement between median values. For AAA-2, cyclic axial stretches measured by US showed 1.002 [0.992, 1.015], the FEA model 1.001 [0.999, 1.002], cyclic circ. stretches measured by US showed 0.993 [0.978, 1.029], the FEA model 0.993 [0.998, 1.001] and therefore also good agreement between median values.

Comparison: Stress-stretch comparison between the materials that were identified based on uniaxial test data in vitro and based on 4D-US measurements in vivo for each AAA patient can be seen in Fig. 8a–f for AAA-1 and Fig. 8g–l for AAA-2. For AAA-1, comparison between both methods shows goodness of fit between $R^2 = 0.319\text{--}0.885$ along the three load paths, equi-biaxial load path LP-II shows best agreement with $R_{\text{Axial}}^2 = 0.776$ and $R_{\text{Circ}}^2 = 0.885$. For AAA-2, comparison between both methods shows $\text{MAPE} = 17.83\% - 0.51\%$ along the three load paths. For S1 also equi-biaxial load path LP-II shows best agreement with $\text{MAPE}_{\text{Axial}} = 0.57\%$ and $\text{MAPE}_{\text{Circ}} = 0.51\%$, while for S2 circ. load path LP-III shows best agreement with $\text{MAPE}_{\text{Axial}} = 10.71\%$ and $\text{MAPE}_{\text{Circ}} = 4.44\%$.

4 Discussion

In this study, we performed a preliminary validation of a non-invasive 4D ultrasound strain imaging-based approach to identify the patient-specific constitutive parameters of a hyperelastic, anisotropic material model, using only input variables that are available or assessable in vivo. The validation was performed using inflation-extension tests on a porcine aorta (in vitro) and by comparing in vivo predictions against uniaxial tensile tests of tissues ($n = 6$) harvested intraoperatively from AAA patients (ex vivo). Although six tissue samples were analyzed, the small patient cohort ($N = 2$) limits statistical power and immediate clinical generalization. Consequently, the current in vivo results are explicitly presented as a preliminary proof-of-concept.

The identified parameters from US and OP differ for both AAA patients, yet the qualitative curve characteristics in the stress-stretch diagram along all three LPs are well matched in both cases (cf. Fig. 8). This apparent contradiction, different parameter values but a similar mechanical response, reflects the non-uniqueness of the GOH parameters and is discussed in detail below. Both identified material curves of AAA-1 show an initial linear increase followed by a strong nonlinear stiffening, while all identified material curves of AAA-2 show almost no linear increase, but early nonlinear stiffening. Although the best quantitative agreement for both patients can be found

along the equi-biaxial LP-II, there are still differences. While both material curves of AAA-1 fit well, this is not the case for AAA-2. Here, in vivo curves lie between the curves of the tissue samples, which is not surprising. Deformations of the porcine aorta are significantly more homogeneous compared to both AAAs. This makes it more likely from the outset to identify the correct material behavior for the porcine aorta. However, the greatest influence is due to the nature of the two very different experimental tests. From the inflation-extension tests, the entire geometry is taken into account in an analytical calculation, thereby identifying an averaged material behavior. Material characterization based on the 4D-US approach also identifies an averaged material behavior due to homogeneous material properties (one set of GOH parameters). However, tensile tests of small specimens from the two AAAs each give a local material behavior. Thus, the initial expectation was that the material behavior of the porcine aorta could be determined correctly with both methods (if the deformation range was sufficient). For the AAAs, the expectation was as seen for AAA-2, where the averaged material behavior based on US lies somewhere between the curves from the tensile tests performed on different tissue specimen, so a difference in the curves was therefore to be expected. For AAA-2, we did not calculate an averaged stress-stretch response from the four specimens, but it can be seen from Fig. 8g,l that it would fit quite well with the averaged material behavior identified in vivo. Both AAAs show strong differences in the heterogeneity of their stretch distributions. AAA-1, with $HI_{A1} = 6.8$, shows a heterogeneity index only half as large as AAA-2, with $HI_{A2} = 12.1$ (for reference, porcine aorta showed $HI_P = 0.2$). Calcification in the axial sample of AAA-2 S1 could cause the strong stiffening compared to S2. Both could explain the smaller difference in AAA-1 and the larger difference in AAA-2 between the in vivo approach and excised tissue samples.

A known challenge when fitting the GOH model is the non-uniqueness of the identified parameters when all five parameters are determined solely from mechanical data, as done here. This is exemplified by the coupling between the isotropic (μ) and anisotropic (k_1) stiffness, where similarly good fits can result from either high μ and low k_1 or vice versa, as seen for AAA-1.

The problem is particularly pronounced for stiff tissues such as AAAs, which operate within a narrow deformation range, and is reflected in the differing parameter sets identified from US and OP data. For the porcine aorta, this effect was not observed, likely due to its considerably larger deformation range. Not only the material behavior of the porcine aorta was identified with good agreement based on optical and ultrasound deformation measurements, also the identified parameters of the GOH material model agree very well. Both aneurysms further exhibit high values of k_2 , which is expected since the influence of k_2 on the material response diminishes as its value increases, so that large values are required to reproduce such stiff behavior. To improve the robustness of the identification, the number of free parameters could be reduced, e.g. by fixing the mean fiber angle φ from histological analysis, or by replacing the phenomenological dispersion parameter κ with a micro-structurally motivated fiber distribution computed directly from histology (Niestrawska et al. 2016), leaving only μ , k_1 , and k_2 to be identified. For purely mechanical data, the μ - k_1 coupling could alternatively be addressed by computing a transition point and splitting the identification into two sub-problems (Holzapfel 2006), which is, however, not transferable to the Finite Element approach used here.

The identified parameters also compare well with values reported in the literature for the same constitutive model. For porcine aortic tissue, the GOH parameters identified here show good agreement with those reported by Peña et al. (2022), Fischer et al. (2023), and Díaz et al. (2021). A further group of studies (Peña et al. 2015; Giudici et al. 2021; Ryu et al. 2022) employed slightly modified or extended versions of the model, so that not all parameters are directly comparable; for the comparable ones, however, good agreement is found as well. For AAA tissue, the identified parameters are likewise in good agreement with those reported by Sassani et al. (2015), Haspinger and Holzapfel (2023), and Pierce et al. (2015). The comparatively high values of k_2 obtained here for the two aneurysms are consistent with the similarly high k_2 values reported for AAA tissue by Niestrawska et al. (2019) and Niestrawska et al. (2016). It should be noted, however, that the comparability of parameters depends strongly on whether the structural parameters κ and φ

are measured from histology and fixed during the optimization or, as in the present study, identified freely together with the stiffness parameters. This is well illustrated by Pasta et al. (2016), who reported GOH parameters for the same tissue once with the structural parameters φ and κ obtained from multi-photon imaging analysis and once with κ and φ derived from the macroscopic mechanical response of tensile testing. Comparing both parameter sets, the largest differences occur precisely in the structural parameters, reaching up to 36.4% for φ and 145.5% for κ , with corresponding differences of up to 78% for k_1 and 85.8% for k_2 . Consistent with this, when κ and φ are co-identified rather than fixed, the optimization tends to drive κ more towards isotropy ($\kappa = 1/3$) and the stiffness towards higher k_1 values for healthy tissue.

A comparison of identified material parameters for an anisotropic material equation, taking into account the true patient-specific aortic geometry based on an in vivo approach and harvested tissue samples, was, to the best of the authors' knowledge, long limited to Liu et al. (2019b). Here, an in vivo multi-resolution direct search approach based on patient-specific gated CT images was compared with biaxial tissue samples of ascending thoracic aortic aneurysm for two patients. Material curves showed MAPE between 0.73–9.75%, while our approach showed 0.07–17.83% (but values are calculated in different stress ranges). However, the material curves for the Liu et al. (2019b) approach show qualitative differences between curves, which, as mentioned before, could not be observed here. Also, Liu et al. (2019b) identified nearly identical GOH parameters in vivo for both patients, although the material curves of the tissue samples were markedly different. This, too, differs from our findings, where the parameter sets identified in vivo and their resulting stress-stretch responses are strongly different while matching the respective mechanical data. An approximate comparison is further possible with Cosentino et al. (2019), who used the statically determinate method of Liu et al. (2017) but without patient-individually measured blood pressure, and reported errors of up to MAPE = 77% in the stress predictions for an orthotropic Fung model. As in Liu et al. (2019b), qualitative differences between the in vivo and ex vivo material curves were found. Notably, Cosentino

et al. (2019) rely on the assumption of a statically determinate stress distribution, whereas the strain-based error measure used here is not subject to this assumption. Strains are arguably more suitable than stresses, as they are a direct result of the wall kinematics and thus inherently reflect the mechanical properties of the tissue. A significant difference to Liu et al. (2019b), however, is that ATAA do not show an ILT. More recently, Guo et al. (2025) independently estimated the in vivo anisotropic properties of human ascending aortas (five type-A dissection patients and five donors) from transesophageal echocardiography and CT using an anisotropic Mooney–Rivlin model, validated against ex vivo biaxial tensile tests. They reported a systematic in vivo underestimation of the circumferential stiffness, with a median relative error of -29.4% (maximum relative error: -33.44%), i.e. likewise only moderate in vivo-to-ex vivo agreement. Importantly, none of these studies (Liu et al. 2019b; Cosentino et al. 2019; Guo et al. 2025) share the same input data, imaging modality, or constitutive formulation with the present work, so that they constitute fully independent approaches to the same problem. In our study one of the two patients showed good and the other only moderate agreement, whereas Liu et al. (2019b) reported only moderate agreement for both patients together with nearly identical and partly non-physiological in vivo parameters.

The study by Karlsson et al. (2025) using an in vivo parameter identification approach showed good agreement with the parameters of a healthy abdominal aorta, but it has only been assessed in silico against synthetic data, rather than against mechanical tests on harvested tissue as done here. These results are comparable to the verification performed by our group previously (Wittek et al. 2013, 2016a). The closest study in terms of acquisition is van Disseldorp et al. (2020), who validated a 4D-US-based approach against biaxial tensile tests on porcine aortas and AAA wall tissue, reporting good in vitro and moderate in vivo agreement, similar to the present results. While both approaches share the 4D-US modality, they are fundamentally different: van Disseldorp et al. (2020) calibrated a single, global linearized wall stiffness, whereas the present work identifies the five parameters of a full nonlinear, anisotropic GOH model and additionally accounts for the ILT.

The change from a local search method to a global one prevents the algorithm from getting stuck in local minima. The Differential Evolution version used here, with an external archive (Zhang and Sanderson 2009), even keeps poor candidates and uses them, and the mutation strategy uses the current best, randomly selected from the population and/or bad solutions, which helps keep the population diverse. This additionally prevents from getting stuck in local minima but does not excessively increase optimization times. Prior to this study, verification and performance experiments (not further described here) using numerical data sets with known solutions were generated based on 4D-US and measurement data (healthy and AAA geometries). The optimization routine was started independently five times for each of the two cases to re-find the known solutions. Solutions were found with an average deviation from the known parameters of 0.146% (maximum deviation 0.460%), and (correct) solutions were found in all 10 optimizations. This also identified the optimal population size of 35 for the JADE algorithm, as increasing the population size significantly impacts optimization times. To ensure algorithmic stability against parameter coupling, all optimizations in this study were executed three times independently, consistently converging to the same solution. The initial population was always randomly generated from a very large parameter space using Latin Hypercube Sampling, which guarantees an even use of the entire parameter space. The algorithm required an average of 68 ± 23 iterations, which corresponds to 2380 function or FEA calls (iterations $\times$ population size). The iterations to find the load-free configuration must be added, and switching from the original Sellier method (Sellier 2011) to the augmented method (Rausch et al. 2017) reduced the necessary iterations from ≥ 35 to approx. 7 (80% time saving). The optimization times for the porcine models were 17.3 hours and for AAAs approx. two days, running on an Intel Core i9-10900KF @3.7 GHz with 64 GB RAM. Although optimization times could be reduced by 86% ($7\times$ faster) compared to our previous version, it is probably still too much for clinical use. Using a different mutation strategy than the one employed here, the 35 parameter sets of one population could be evaluated in parallel using the here implemented algorithm. However, this may lead to

slower convergence, which could negate the time advantage and increases the risk of getting stuck in local minima. But this needs to be investigated in the future and was not the main focus in this study. Approaches such as machine learning could save an enormous amount of time, but the necessary large amount of training data is not (yet) available.

The porcine inflation-extension experiment was deliberately performed on a single, comprehensively instrumented specimen. Its purpose here differs from that of the *in vivo* cases: it serves as a controlled ground-truth benchmark against which the accuracy of the identification framework can be isolated from biological and measurement confounders. Unlike the *in vivo* data, it provides a complete and independently measured reference – a full inflation-extension protocol covering a large, near-homogeneous deformation range ($HI_P = 0.2$), a directly measured wall thickness, and known pressures and reduced axial force. Accordingly, the relevant validation quantity is the within-specimen agreement between the two independent modalities (optical and 4D-US), in both the measured deformation fields and the resulting GOH parameters, which is fully assessed on a single specimen. Increasing the number of porcine samples would characterize inter-specimen biological variability rather than the performance of the method itself, which is the objective of this proof-of-concept study; the reliability of the framework is additionally supported by the prior verification against synthetic data sets with known solutions and by the repeated, independent convergence of the optimizer to the same global minimum.

Identification of parameters for a hyperelastic and anisotropic material, described by the nonlinear, multi-objective, and non-convex optimization problem Eq. (13), normally requires measured data of (1) multiple load cases captured over a (2) sufficiently large deformation range. Although ultrasound imaging can record several deformation states, as load cases, only the diastolic and systolic configurations can be used since *in vivo* blood pressure values are only available for these. Problem (1) can be avoided as ultrasound offers a full-field measurement, the benefit of which, when only two load cases can be observed, has been pointed out before (Avril et al. 2008; Roux and Hild 2020; Wittek et al. 2016a). If only the IVC

and not all load paths (see Fig. 4e dark orange squares and orange circles) are used to identify the parameters based on the optical data, a much too stiff material is identified. Therefore, to characterize the material behavior, a sufficiently large area of deformation states within the $\lambda_1 - \lambda_2$ plane must be captured. This is where the power and advantage of full-field measurements, such as 4D-US, come into full effect, as it offers exactly that. This can clearly be seen in Fig. 4e, where the same load cycle of the IVC is shown for the optical data (15 dark orange squares) and for US data (black dots and crosses), even though only 2 of the 15 recorded configurations of this load cycle were used in the US optimization. *In vivo*, it can be assumed that problem (2) is sufficiently regulated by the pulse pressure itself. However, administration of antihypertensive drugs or low temporal 4D-US resolution (which may prevent the systolic configuration from being met precisely) can influence this. Another relevant factor is, that only a reduced form of material model using only invariants C_1 and $C_{4\alpha}$ can be identified using inflation-extension experiments. Parameterization must then cover a deformation range that sufficiently captures the material’s nonlinear behavior, as discussed in Ogden (2009), Holzapfel (2009), and Humphrey (2002). This motivates the estimation of the necessary proportion of nonlinear stiffening during one cardiac cycle *in vivo*.

The sensitivity analysis showed that approx. 50% of the region of non-linear stiffening that occurs during healthy blood pressure 78–132 mmHg is required to identify the material with sufficient accuracy. This corresponds to about 70% of the healthy *in vivo* pulse pressure range. The reduced axial force was known in this study. *In vivo*, axial pre-stretch can be reasonably estimated based on age (Horný et al. 2014), and a corresponding reduced axial force can be computed in a parameter-dependent manner via an additional FE simulation (Wittek et al. 2016a). An error of $\pm 10\%$ in axial force led to a negligible error in the identified material parameters. This seems plausible, as the wall stress contribution from internal pressure was found to be approx. 5.5 times greater than that from axial force, based on the stress calculation from inflation-extension data Eq. (17). Wall thickness has a non-negligible influence on the identification as well, but is very difficult to

assess in vivo, especially due to its strong heterogeneity in AAAs. For the two AAAs studied here, it was known for at least some wall regions (cf. Sec. 2.7.1). For this validation study, using the ex vivo measured wall thickness was intentional. It acted as a ground truth to isolate the algorithm’s performance without the confounding variables of in vivo thickness estimation. However, our sensitivity analysis showed that an error of $\pm 10\%$ resulted in moderate deviations in the identified material parameters. Methods for better in vivo estimation were presented (Joldes et al. 2016), but require imaging procedures that are not clinical standards in AAA care (MRI). A more recent study by Fasen et al. (2025) using intravascular US shows potential, but it is an invasive procedure, and a validated demonstration of locally measured wall thicknesses against a ground truth in vivo is still pending. Therefore, in vivo thickness estimation remains a significant influencing factor for parameter identification. For future non-invasive clinical translation, our framework could be extended to integrate patient-specific thickness maps derived from routine pre-operative CT scans or advanced ultrasound segmentation. Accurate knowledge of the wall thickness nevertheless remains essential for reliable assessment of local wall stresses, such as peak wall stress in rupture analyses. The sensitivity of the results to the ILT is addressed separately together with the ILT modeling below (cf. Sec. 3.2). Superposition of US data with systematic and random errors and its influence on the optimization was estimated and discussed in Wittek et al. (2023). However, since averaged US models (Hegner et al. 2023) were used here which minimize the random error, this will not be discussed further. A phase shift between axial and circ. stretches simulated in the experiment showed no significant influence on the identified material behavior. The extent to which the identified parameters can be used for further simulations, such as high blood pressure or even rupture behavior, will be evaluated in more detail in the future.

The loading rate of 0.84 Hz applied in the inflation-extension experiment cannot be considered quasi-static, so inertial contributions to the normal stresses could, in principle, affect the stress calculation in Eq. (16)–(17). We therefore estimated these effects and found a maximum

deviation of only 0.02% compared to the quasi-static stresses. Adding this worst-case difference to the stresses of all load paths and re-identifying the parameters changed the identified values by only 0.45%, 0.37%, 0%, 0%, and 0% (μ , k_1 , k_2 , κ , φ), corresponding to a negligible change in $R^2 < 0.0005$. Furthermore, a principal stress state was assumed, i.e. shear stresses were neglected. This is justified, as blood-flow-induced shear stresses are known to be four to five orders of magnitude smaller than the hydrostatic transmural pressure (Caro 2012; Vorp 2007). For the uniaxial tensile tests, this is supported by the symmetric fiber angle φ with respect to the circumferential axis, while for the inflation-extension experiment the cyclic strains measured by 4D-US showed median values of $\bar{\varepsilon}_{11} = 2.5\%$, $\bar{\varepsilon}_{22} = 14.6\%$, and $\bar{\varepsilon}_{12} = 0.01\%$. Since $\bar{\varepsilon}_{12}$ is close to zero, neglecting shear stresses seems justified and explains why the agreement of the identified material behavior is nevertheless good. The full in-plane strain tensor including ε_{12} can, however, be measured using 4D-US in future experiments.

Material assumptions regarding the ILT influence the results of the parameter identification. Experimental curves in previous studies have shown that ILT tissue exhibits only weakly non-linear mechanical characteristics (Vande Geest et al. 2006; Tong et al. 2011; Tong and Holzapfel 2015). This observation is consistent with Gasser et al. (2008), who reported an approximately linear load-displacement response for the tissue, and is further corroborated by Di Martino et al. (1998), who found a strongly linear relationship ($r > 0.99$, $n = 21$) across all investigated patients. Morphologically, the ILT consists of three distinct layers where its properties are a function of the distance from the lumen (Gasser et al. 2023). Although our FE models could conceptually accommodate this heterogeneity by assigning layer-specific properties (cf. Fig. 3e), we simplified the ILT as a homogeneous structure. To achieve this, an averaged Young’s modulus was calculated based on the experimentally determined, layer-specific values from Tong et al. (2011) and Tong and Holzapfel (2015) and applied consistently across both patients. This calculated average is in good agreement with the modulus range of 0.05–0.2 MPa ($n = 21$) reported by Di Martino et al. (1998). Since all constituent layers

exhibit isotropic nearly linear elastic behavior, utilizing this averaged macroscopic modulus is a valid approximation. In contrast, the Poisson’s ratio was estimated individually for each patient based on volume ratios and mean strains between diastole and systole derived from ultrasound imaging (cf. Sec. 2.7.2), justified for isotropic linear elastic materials. This is corroborated by our own sensitivity analysis (cf. Sec. 3.2, Tab. 3): introducing the ILT roughly halves the median wall displacement (M0 vs. M1), whereas varying its material parameters across the nine tested layer configurations (M2–M10) changes it by only about 2.5%. Since the error function of the parameter identification is strain-based, and the resulting wall displacements, and hence the underlying strain fields, are nearly identical across all ILT configurations, the identified material behavior is likewise expected to be nearly identical. This is supported by recent findings from Caruso et al. (2025), who demonstrated that while the physical presence of the ILT is essential, the exact material properties of the ILT have only a minor impact on the overall mechanical behavior of the AAA. US data would allow simultaneous material identification for the ILT, but would require experimental data which was not available. The wall strain calculations from Eq. (1) have been experimentally validated (Blase et al. 2025), but it remains unclear how reliable these calculations would be for the ILT regions since no reference is available. Therefore, in this study, we focused only on the clearly identifiable transition from the lumen to the ILT for the geometry reconstruction of the ILT, but did not calculate or use strains in the regions of the ILT. Emerging ultrasound techniques such as bistatic dual-aperture imaging, which has been shown to improve the lumen–wall contrast and to enable strain quantification even within low-contrast ILT regions (van Hal et al. 2025), may help to overcome this limitation in the future.

Parameter identification and recovery of the load-free configuration depend on the BCs, which are not known in vivo. They can only be estimated in vivo, where the aorta is mainly supported by the surrounding tissue, anchoring by branches, diaphragm, and neighboring organs. Thereby, it can move (to a limited extent) in all six degrees of freedom. The considered intra-abdominal pressure and the artificial extensions on both sides of the in vivo FE models allow this type of support

in the examined area (i.e. the area for which US data was available), and therefore, the most physiological approach. Also, it is likely that external supports would not have a significant impact on the load-bearing of the aorta.

Further discrepancies between in vivo and ex vivo material behavior could be attributed to active contractions generated by smooth muscle cells, which are not considered by the constitutive model. While biaxial testing is the preferred method for characterizing anisotropic cardiovascular tissues, it poses significant experimental challenges for excised surgical AAA samples. Using large cruciform specimens is practically precluded by the limited size of the harvested tissue. Alternatively, mounting small square specimens using hooks is highly challenging and risks tissue tearing, particularly due to the frequent presence of calcifications in AAA walls. Consequently, to ensure robust testing and well-defined boundary conditions (zero transverse stress), we opted for orthogonal uniaxial tests (axial and circ.) using dog-bone specimens. Naturally, the uniaxial deformation range in the λ_1 – λ_2 plane is not in the same range as the biaxial deformation range in vivo. This was avoided by combining axial and circ. specimens during optimization (cf. Sec. 2.4). However, uniaxial deformation states are still rather non-physiological. For the GOH model, previous work on porcine specimens has shown the potential to recover biaxial behavior from uniaxial test data (Schroeder et al. 2018). In addition, in unpublished preliminary studies by the authors, GOH parameters of porcine aorta were first identified from inflation-extension tests and subsequently re-identified from uniaxial tensile tests of specimens cut from the same aortas; the comparison of the resulting material behavior yielded $R^2 = 0.941$ – 0.998 , indicating good agreement despite substantial differences between the two parameter sets and loading states. Also, additional sources of heterogeneity such as local variations in wall thickness and elastic properties (calcifications, plaque) were not considered. A further simplification of this approach is that residual stresses are not taken into account, which however is justified by the findings of Labrosse et al. (2009) and Liu et al. (2019a), who showed that residual stress has little influence on identified constitutive parameters and transmural mean stress. Finally, since the in vivo deformations are obtained from

4D-US, the acquisition itself may introduce a confounder: Bracco et al. (2025a) and Bracco et al. (2025b) demonstrated, in silico and in vivo respectively, that the ultrasound probe pressure applied to the abdomen can alter the measured cyclic wall deformation and thereby reduce the apparent aortic stiffness. This effect acts through two mechanisms: a structural flattening of the cross-section, which mainly biases diameter-based stiffness indices, and, for nonlinear wall behavior, a shift of the operating point towards the more compliant toe region of the stress–stretch curve, which can also affect strain-based identification such as the one presented here. Since our approach relies on the full-field cyclic strains rather than on a single diameter change, it is inherently less susceptible to the purely structural component; the nonlinear operating-point effect, however, cannot be excluded and was mitigated by acquiring all data with consistent probe pressure. Controlled light/firm acquisitions or a registration-based correction of the probe-induced deformation are promising routes to quantify and remove this bias in future work.

Homogeneous material properties were assumed here but in vivo they can be highly heterogeneous. Romo et al. (2014) showed that ruptures occur preferentially in areas with extreme strain values, which cannot be mapped by the homogeneous material assumption. This can be seen in Fig. 5h,i from the clearly more homogeneous stretch distributions of the FE models compared to the in vivo stretches measured using US. Thus, a combination of parameter identification to determine the global material behavior, as presented here, in combination with strain-based indices for the local characterization of elastic tissue properties (Witte et al. 2016b; Derwich et al. 2016; Witte et al. 2017; Derwich et al. 2020) could potentially be a suitable means to better classify the current risk potential of the AAA wall in future (clinical) applications.

Beyond such indices, the present approach could itself be extended towards heterogeneous material identification by subdividing the wall into regions, each assigned its own parameter set, as proposed for cerebral aneurysms by Zhao et al. (2011). This would, however, require the time-resolved blood pressure curve rather than only the diastolic and systolic values. A non-invasive

estimate could be obtained from the strong linear relationship between intra-aneurysmal pressure and AAA volume reported by van’t Veer et al. (2008) ($R^2 = 0.93$), by mapping the measured diastolic and systolic pressures onto the time-resolved volume change extracted from the 4D-US geometries; a weakly nonlinear pressure–radius relationship was likewise reported in vivo by Stålhand (2009). The applicability of these relationships needs to be explored in future studies. A regional identification would additionally allow a direct comparison of local in vivo results with the corresponding excised specimens, provided their harvest location is documented.

4.1 Conclusions

In this study, we presented and preliminary validated a significantly extended 4D-US strain imaging framework for the non-invasive, in vivo identification of nonlinear, anisotropic AAA wall properties from only two physiological load cases. The framework now incorporates the ILT and reduces computation time by 86%, allowing the parameters of virtually all clinically relevant AAAs to be identified. In a proof-of-concept validation, the ultrasound-based identification showed excellent agreement with mechanical testing in vitro (porcine aorta) and good-to-moderate agreement in vivo against excised tissue from two AAA patients, with wall thickness and the extent of nonlinear stiffening identified as the main factors governing accuracy. While the in vivo dataset is still small and these findings remain preliminary, the results demonstrate the feasibility of patient-specific, non-invasive characterization of AAA wall mechanics. Future work should enlarge the in vivo cohort, assess the predictive value of the identified parameters for elevated blood pressure and rupture, and extend the framework towards spatially varying wall thickness and heterogeneous constitutive properties.

Declarations

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Conflict of interest. The authors declare that they have no conflict of interest.

Ethical Approval. The study was approved by the local ethics committee at the Goethe University Hospital Frankfurt am Main / Germany.

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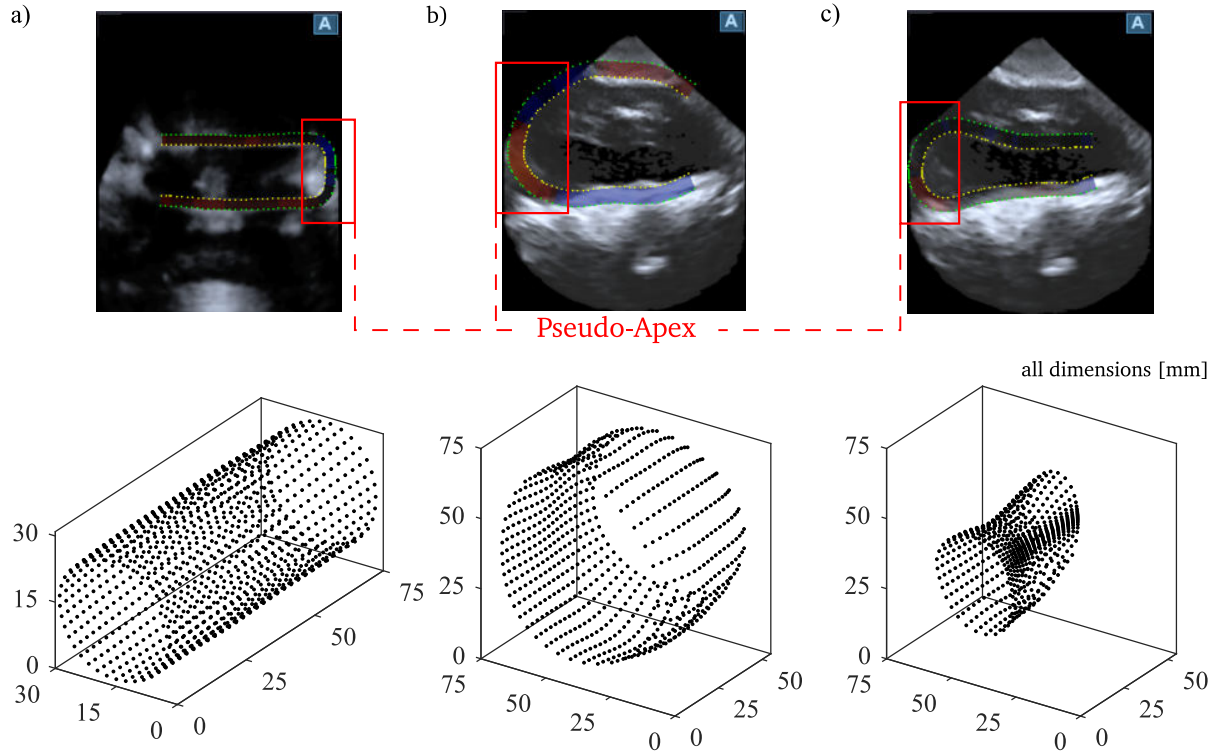
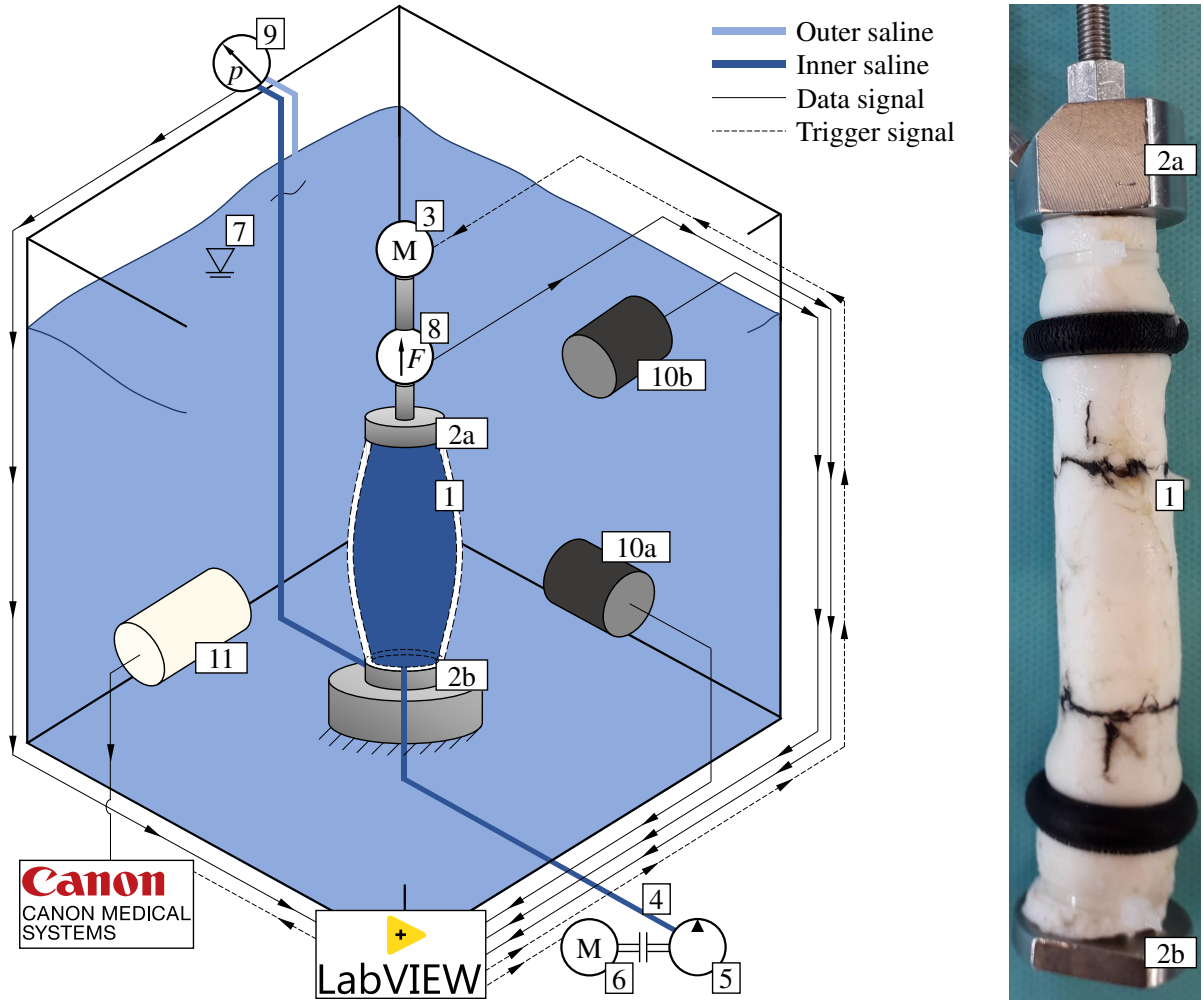


Fig. 1 Ultrasound images (top row) and resulting point clouds after post-processing (bottom row, as described in Sec. 2.1). **a)** Porcine aorta from inflation-extension experiments. **b)** Vessel wall of AAA-1 and in **c)** the corresponding lumen geometry of AAA-1 seen in b). Diastolic configuration is shown in each case, all dimensions of the point clouds are in [mm].



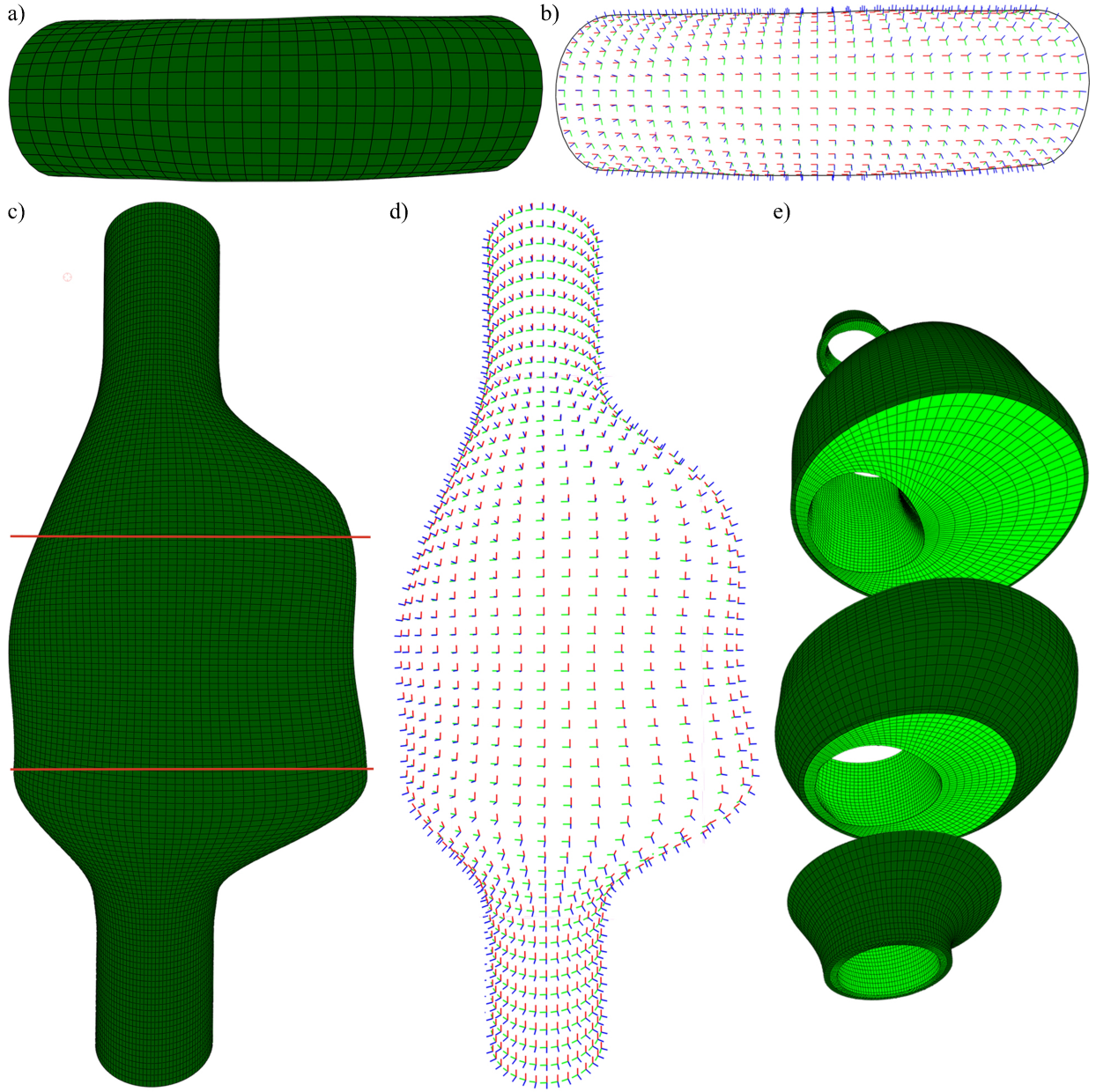


Fig. 3 FEA simulation models of the porcine aorta a)-b) and of AAA-1 c)-e) based on the 4D-US images in Fig. 1. **a)** Meshed diastolic configuration of the porcine aorta based on the point cloud from Fig. 1a. **b)** Local material axes of the porcine aorta from a). The local 1.-direction (red) is oriented in the axial direction of the vessel, the local 2.-direction in the circ. direction and the local 3.-direction (blue) is perpendicular to the respective element. **c)** Geometry of AAA-1 based on the point cloud from Fig. 1b with artificially extended transitions to a healthy vessel diameter. US measurements were only available for the area between the red lines. **d)** Local material axes of AAA-1 from Fig. 3c. The local 1.-direction (red) is oriented in the axial direction of the vessel, the local 2.-direction in the circ. direction and the local 3.-direction (blue) is perpendicular to the respective element (the number of displayed material axes has been reduced for better visualization). **e)** Sliced geometry from c), so that the inner mesh of the ILT (light green) based on the point cloud from Fig. 1c and the wall (dark green) based on the point cloud from Fig. 1b are visible.

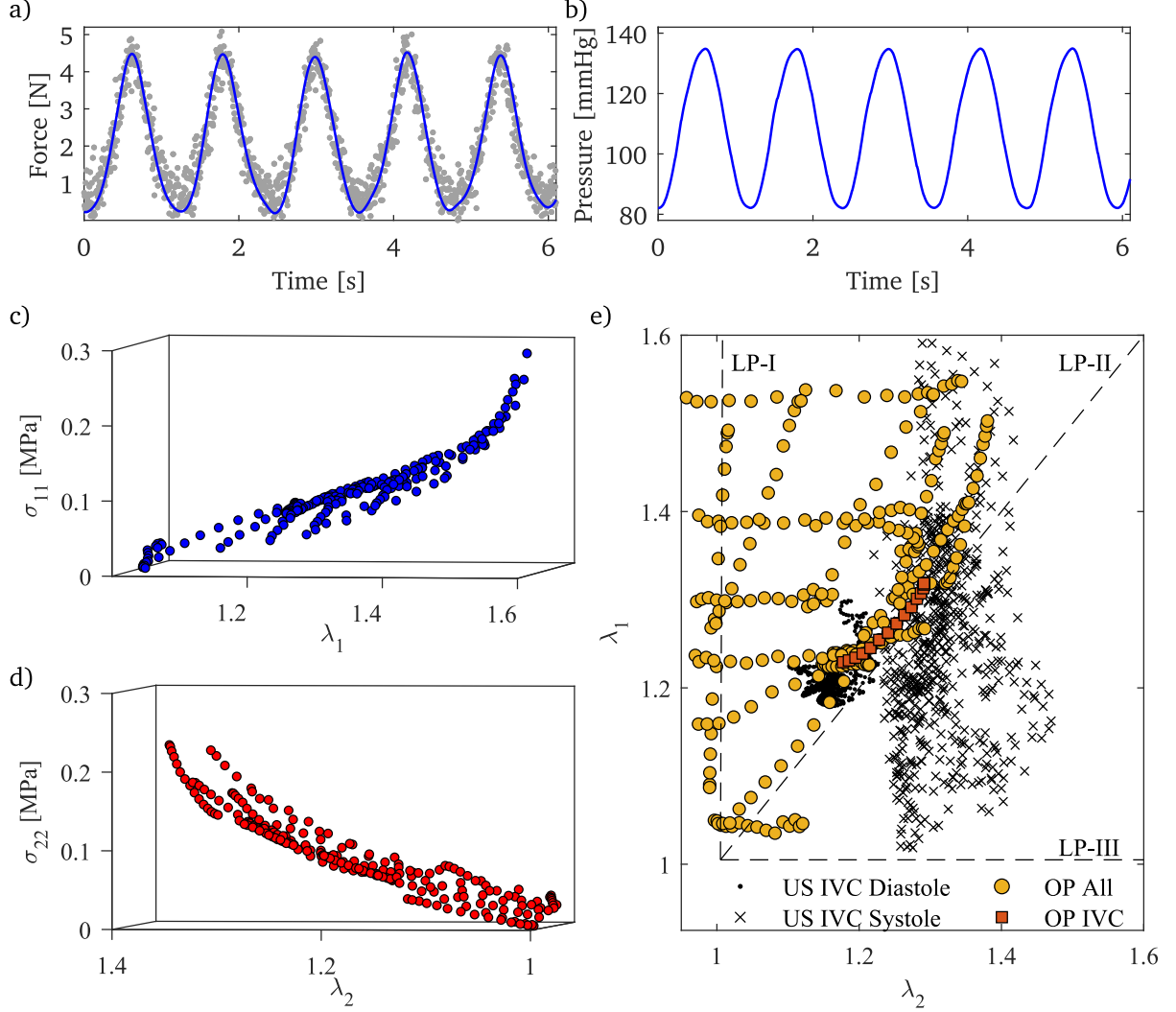


Fig. 4 Measured force/pressure values, calculated 3D stress-stretch curves, and load-paths of the porcine aorta from inflation-extension experiment. **a)** Measured force values for five consecutive load cycles (gray dots) and smoothing (black line). **b)** Measured pressure values for the same five consecutive loading cycles as in a). **c)** 3D axial stresses $\sigma_{11}^{\text{inf-ext}}$ over λ_1 (blue circles) and **d)** 3D circ. stresses $\sigma_{22}^{\text{inf-ext}}$ over λ_2 (red circles) using Eq. (16)–(17). **e)** 2D representation of the 17 different load paths (yellow circles) as well as both US measured stretch states of the diastolic (black dots) $\Omega_{\text{dia}}^{\text{US}}$ and systolic (black crosses) $\Omega_{\text{sys}}^{\text{US}}$ configurations of the IVC shown in the $\lambda_1 - \lambda_2$ plane. The three characteristic load paths LP-I (uniaxial axial), LP-III (uniaxial circ.), and LP-II (equi-biaxial) are shown as dashed lines.

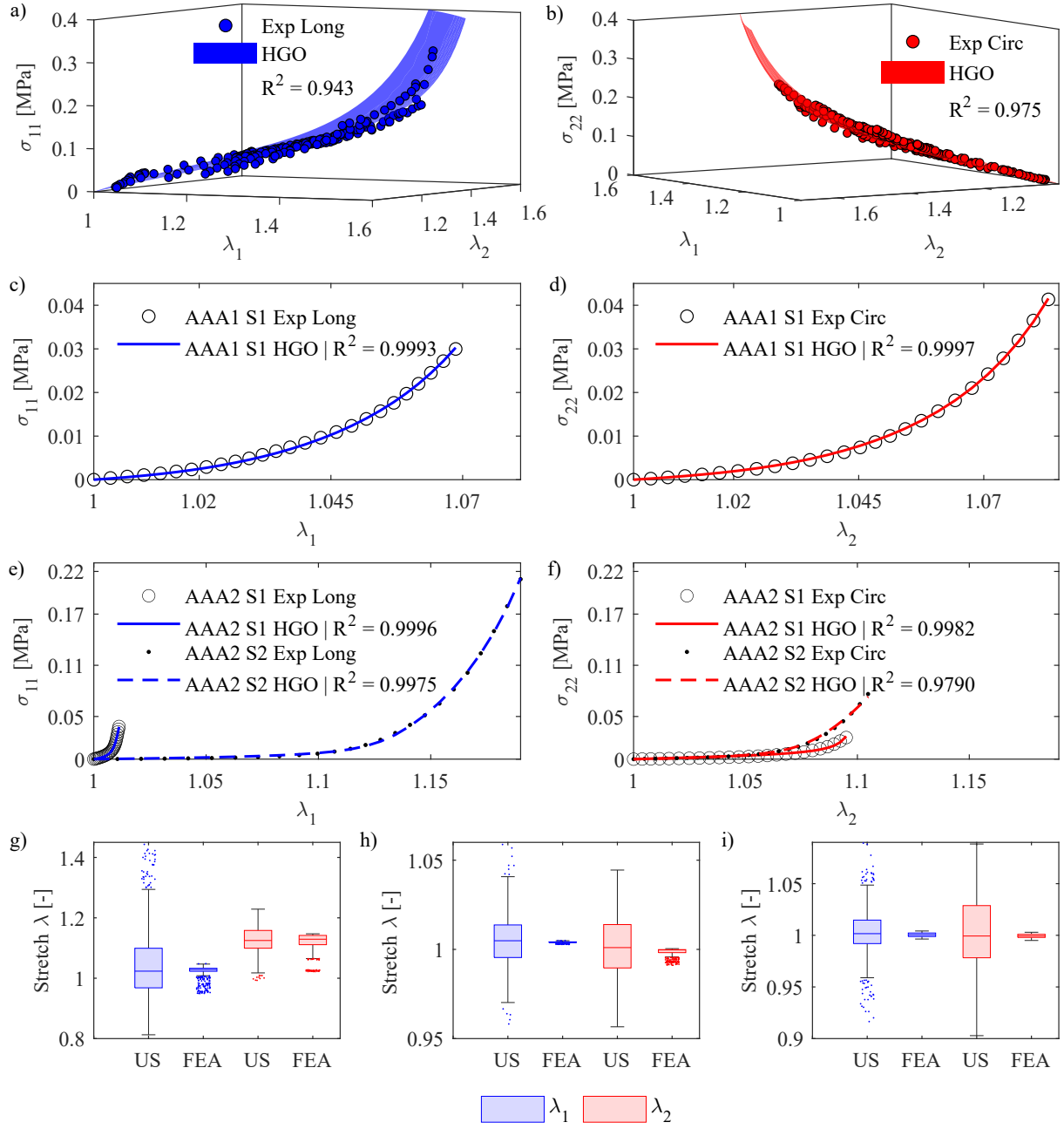


Fig. 5 Results of the fits between the GOH material model and the experimental data. Subfigures **a)** and **b)** show the identified material behavior of the porcine aorta from inflation-extension tests in the axial and circ. directions, respectively. Subfigures **c)** and **d)** display the results for AAA-1 based on uniaxial tensile tests of wall specimen S1 (circles and solid lines) for the axial and circ. directions, respectively. Subfigures **e)** and **f)** show the corresponding results for AAA-2, based on specimens S1 (circles and solid lines) and S2 (dots and dashed lines). All R^2 values indicate the goodness of fit between the GOH model and the experimental data used for parameter identification, all associated $p < 0.001$. Subfigures **g)**, **h)**, and **i)** compare longitudinal (λ_1 , blue) and circumferential (λ_2 , red) stretch distributions obtained from ultrasound (US) measurements and finite element analysis (FEA) using the identified material models, for the porcine aorta (**g**), AAA-1 (**h**), and AAA-2 (**i**).

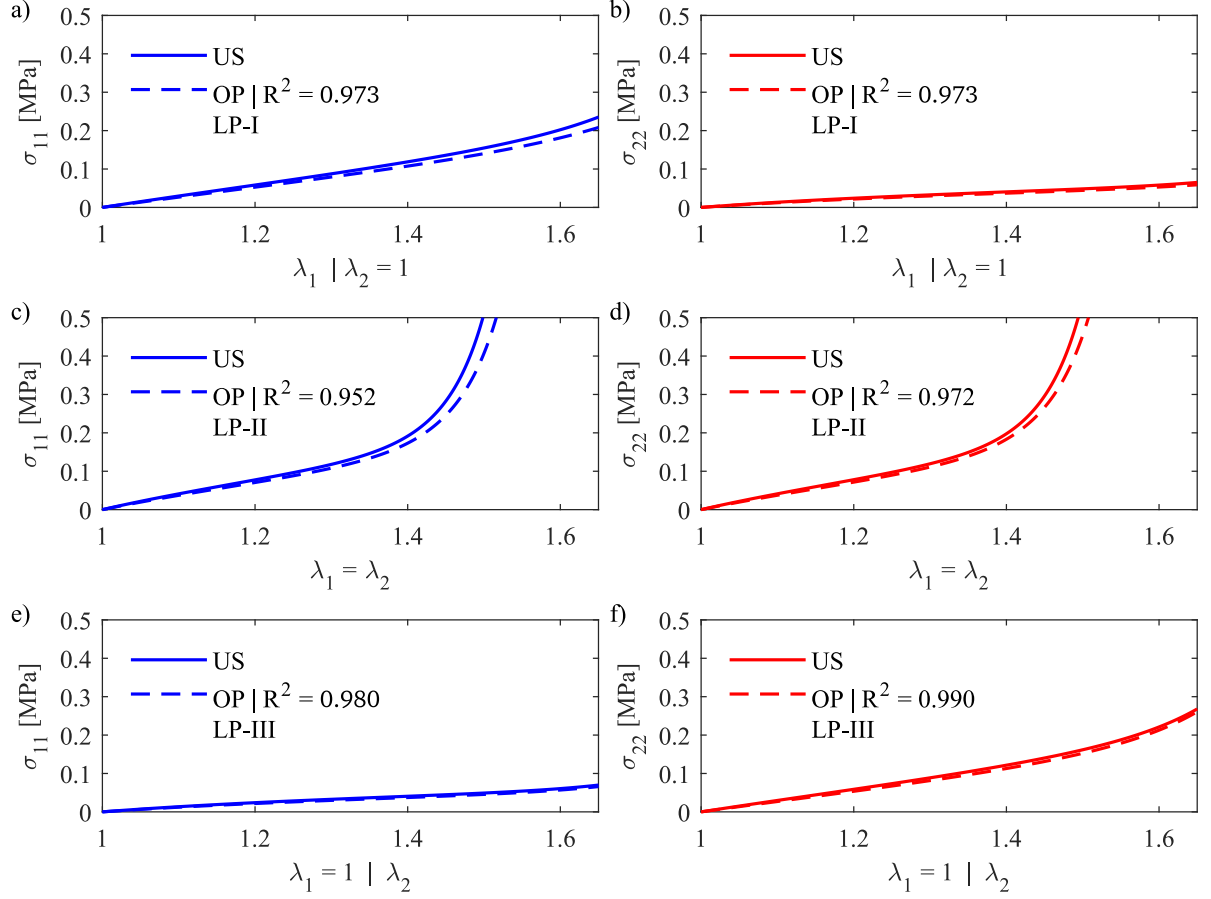


Fig. 6 In vitro validation of the identified GOH material behavior by comparison of model stresses based on optical and mechanical data (OP, dashed lines) and 4D-US (US, continuous lines) data. Subfigures **a)** and **b)** show axial and circ. stress-stretch curves, respectively, for the uniaxial load path LP-I ($\lambda_1 | \lambda_2 = 1$) in axial direction. Subfigures **c)** and **d)** show the same comparison for the equi-biaxial load path LP-II ($\lambda_1 = \lambda_2$). Subfigures **e)** and **f)** present axial and circ. stress-stretch curves for the uniaxial load path LP-III ($\lambda_1 = 1 | \lambda_2$) in the circ. direction. All R^2 values indicate the goodness of fit based on the calculated stresses of the respective GOH material models. All corresponding $p < 0.001$.

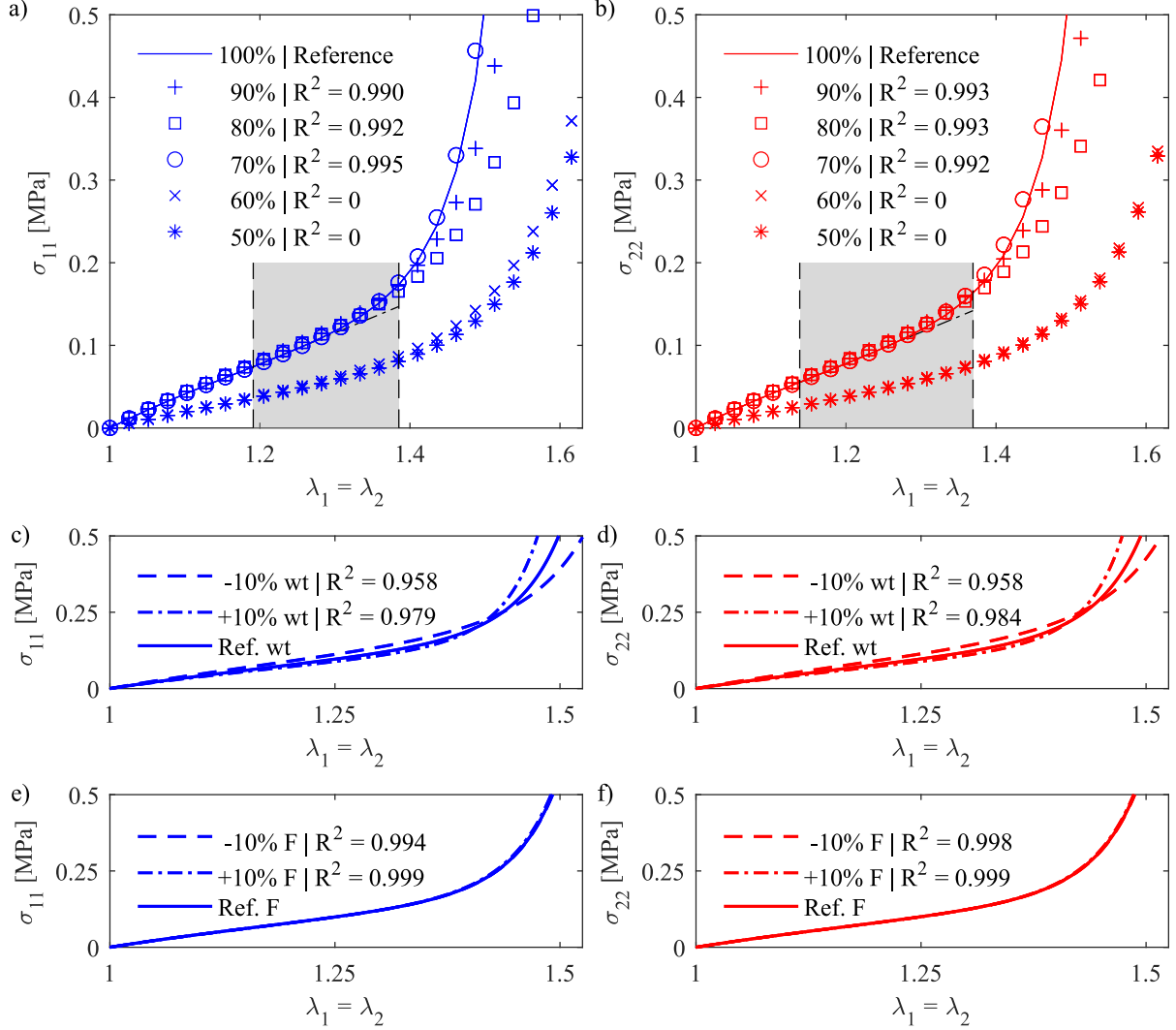


Fig. 7 Results of the sensitivity analysis for the identification of material behavior based on the inflation-extension experiment. Subfigures **a)** and **b)** show the effect of reducing the occurring deformation range from 100% (Reference) to 50%. Subfigures **c)** and **d)** show the effect of varying wall thickness (wt) by +10% (dash-dotted line) and -10% (dashed line) on axial and circ. stress-stretch curves, respectively. Subfigures **e)** and **f)** display the influence of varying axial force by +10% (dash-dotted line) and -10% (dashed line). All results are shown for the equi-biaxial load path LP-II ($\lambda_1 = \lambda_2$), representing the closest approximation to the physiological state. R^2 values indicate deviations from the reference curve (Ref, solid line), which is based on the IVC-identified parameter set.

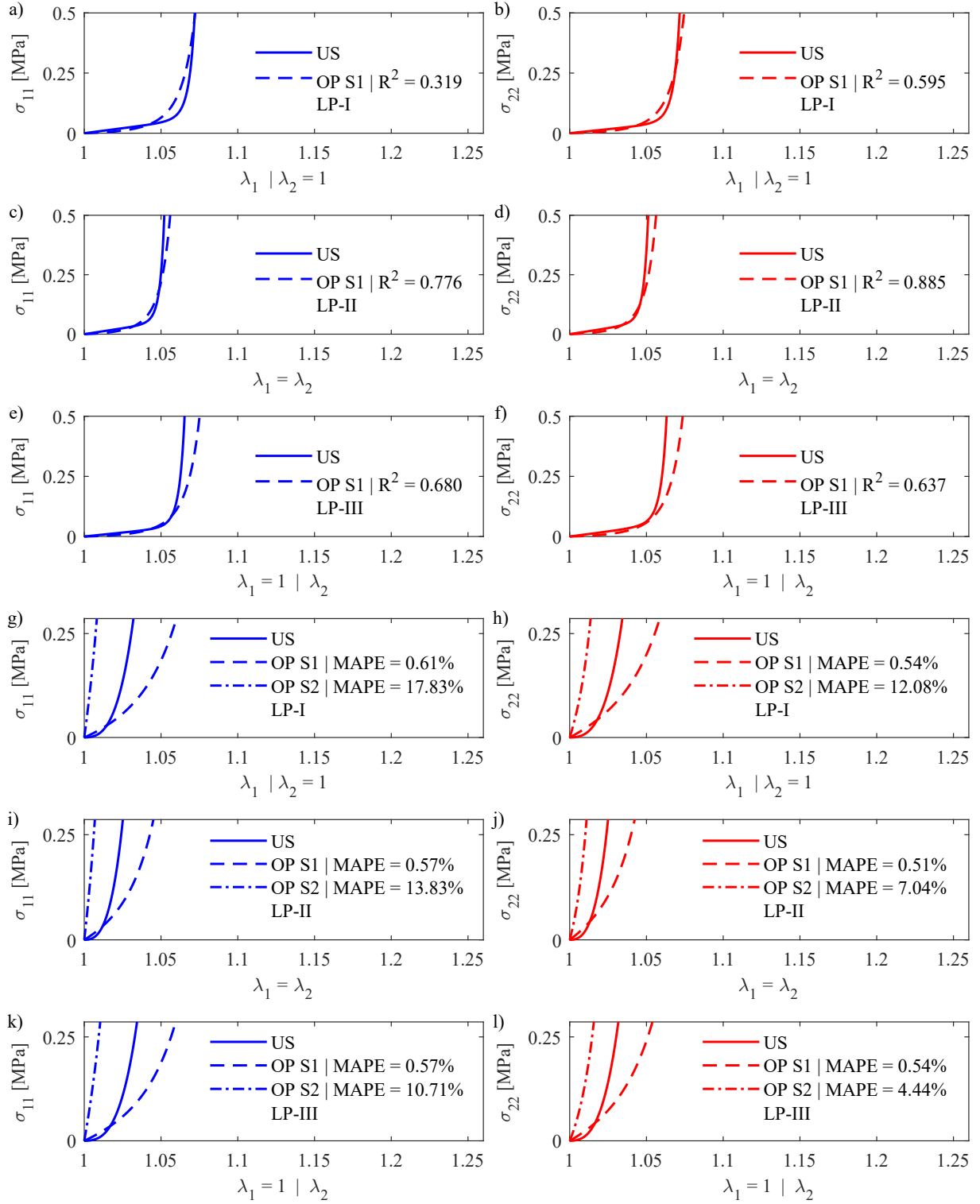


Fig. 8 In vivo validation of the identified GOH material behavior by comparison of model stresses based on optical and mechanical (OP, dashed/dash-dotted lines) and 4D-US (US, solid lines) data. Results of patient AAA-1 are given in a)–f) and for AAA-2 in g)–l). Subfigures a) and b) show axial and circ. stress-stretch curves, respectively, for the uniaxial load path LP-I of AAA-1 and g) and h) for AAA-2. Subfigures c) and d) present the same for the equi-biaxial load path LP-II of AAA-1 and i) and j) for AAA-2, while e) and f) depict results for the uniaxial load path LP-III of AAA-1 and k) and l) for AAA-2. All R^2 values indicate the goodness of fit between the calculated stresses of the respective GOH material models.

Table 1 Identified GOH material parameters. For the *Porcine Aorta*, parameters were identified using both the ultrasound-based approach (US) and optical/mechanical data (OP) from the same inflation-extension experiment. For *AAA-1* and *AAA-2*, parameters were identified using in vivo ultrasound imaging (US) as well as uniaxial tensile tests on harvested tissue samples (OP). *AAA-1* included 2 samples (S1), *AAA-2* included 4 samples (S1 and S2). R^2 and MAPE values compare the material response predicted by US- and OP-based parameter sets. Only worst-to-best values are shown here; full details are provided in Fig. 5, 6 and 8.

	Identified constitutive parameters					R^2	MAPE
	μ [MPa]	k_1 [MPa]	k_2 [-]	κ [-]	φ [°]	[-]	[%]
<i>Porcine Aorta</i>							
US	0.077	0.011	6.3	0.281	37.1		
OP	0.068	0.016	4.4	0.270	36.2	0.952–0.990	0.103–0.056
<i>AAA-1</i>							
US	0.168	0.05	4592.1	0.248	37.0		
OP S1	0.016	9.30	2499.2	0.307	41.6	0.319–0.885	0.757–0.069
<i>AAA-2</i>							
US	0.01	0.78	1551.2	0.100	38.8		
OP S1	0.01	4.63	368.1	0.212	42.1	-	0.615–0.511
OP S2	0.03	32.97	1019.2	0.302	63.8	-	17.827–4.438

Table 2 Identified GOH parameters from the sensitivity analysis of the ultrasound-based optimization. The analysis quantifies the influence of factors that cannot be measured in vivo but are known in the experimental. The *Reference* parameter set (IVC) was identified using the full diastole-to-systole deformation range (100%) and serves as a baseline for all comparisons. Section *Proportion Dia-Sys* shows parameters identified when the usable deformation range was successively reduced to 50%, with diastole fixed and the systolic frame progressively shifted earlier in the experiment. Section *Wall Thickness* shows results when the averaged wall thickness was varied by $\pm 10\%$. Section F_{red} includes results from varying the measured axial force by $\pm 10\%$. All R^2 values are reported with respect to the *Reference* parameter set, the IVC. All associated p -values were $p < 0.001$.

	Identified constitutive parameters					R^2	
	μ [MPa]	k_1 [MPa]	k_2 [-]	κ [-]	φ [°]	Long [-]	Circ [-]
<i>Reference</i>							
IVC	0.077	0.011	6.30	0.281	37.1		
<i>Proportion Dia-Sys</i>							
90%	0.079	0.017	4.01	0.274	41.2	0.990	0.993
80%	0.080	0.017	2.85	0.274	41.2	0.992	0.993
70%	0.075	0.017	5.20	0.277	37.2	0.995	0.992
60%	0.039	0.011	2.10	0.255	50.2	0	0
50%	0.053	0.013	3.99	0.265	40.2	0	0
<i>Wall thickness</i>							
+10% wt	0.069	0.011	7.20	0.278	41.5	0.979	0.984
-10% wt	0.092	0.013	4.10	0.265	36.5	0.958	0.958
F_{red}							
+10% F_{red}	0.080	0.012	6.40	0.281	37.1	0.999	0.999
-10% F_{red}	0.081	0.012	6.43	0.281	37.1	0.994	0.998

Table 3 Wall displacement at $p = 120$ mmHg, reported as median $[Q_1, Q_3]$. For M0 and M1 the statistics are taken over all wall nodes; for the parameter study M2–M10 they describe the model-to-model variability across the nine per-model medians.

Model	Wall displacements (median $[Q_1, Q_3]$)
M0 (no ILT)	7.62 [6.90, 8.05] mm
M1 (one single ILT layer, averaged properties)	3.73 [3.23, 3.93] mm
M2–M10 (three ILT layer models, parameter study)	3.73 [3.67, 3.86] mm