

ALTYN Inversion: A MATLAB-Based 2D/2.5D Finite-Strike Gravity Inversion Framework with Box-Constrained IRLS Regularization

Ahmad Ali Muckharom

ahmadalimuckharom@gmail.com

Universitas Diponegoro Fakultas Sains dan Matematika

Research Article

Keywords: gravity inversion, finite-strike modeling, density contrast, IRLS regularization, bound-constrained inversion, geothermal geophysics

Posted Date: September 24th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10764937/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

ALTYN Inversion: A MATLAB-Based 2D/2.5D Finite-Strike Gravity Inversion Framework with Box-Constrained IRLS Regularization

Ahmad Ali Muckharom^{a,*}

^aDepartment of Physics, Faculty of Science and Mathematics, Universitas Diponegoro, Semarang 50275, Indonesia

*Corresponding Author email: ahmadalimuckharom@gmail.com

Abstract

Gravity inversion is intrinsically non-unique: the recovered density distribution depends not only on the observations, but also on forward geometry, data uncertainty, regularization, and prior constraints. This paper presents ALTYN Inversion, a MATLAB-based 2D/2.5D profile framework that combines finite-strike rectangular-prism forward modeling, topography-consistent X–Z discretization, uncertainty-weighted data fitting, explicit density bounds, optional regional nuisance estimation, L2 and iteratively reweighted least-squares (IRLS) regularization, and a projected Newton/active-set preconditioned conjugate-gradient solver. The framework is examined using three synthetic baseline experiments of increasing structural complexity and a 171-station gravity profile, approximately 9.23 km long, from the Dieng geothermal area in Indonesia. The synthetic tests recover the dominant anomaly polarity and approximate lateral position of simple targets, but depth smearing and loss of structural fidelity increase as the source geometry becomes more complex. In the most demanding benchmark, an uncertainty-consistent WRMS is obtained despite substantial model-space error, showing that a good gravity fit does not guarantee correct source geometry. For the field profile, both L2 and IRLS satisfy the adopted discrepancy criterion but select noticeably different density textures. L2 produces smoother, more distributed contrasts, whereas the chosen IRLS configuration sharpens vertical transitions. Features shared by both solutions are therefore more defensible than detailed boundaries, density extrema, or deep continuations. Quantitative robustness to observational-noise level, strike-length choice, and mesh resolution is not claimed from the present runs. ALTYN is best viewed as a transparent framework for constructing and comparing geophysically admissible density models under explicit assumptions rather than as a unique reconstruction of subsurface geology.

Keywords: gravity inversion; finite-strike modeling; density contrast; IRLS regularization; bound-constrained inversion; geothermal geophysics.

1. Introduction

The gravity method is capable of detecting subsurface mass heterogeneity and is therefore widely used to investigate density variations associated with sedimentary basins, lithological contacts, intrusive bodies, mineral systems, volcanic structures, geothermal environments, and crustal architecture. However, the interpretation of gravity measurements remains subject to several challenges, one of the most important being the modeling stage. The inverse problem is fundamentally ill-posed because substantially different density distributions can produce identical or nearly identical gravity fields at the observation surface (Al-Chalabi, 1971; Boulanger & Chouteau, 2001; Li & Oldenburg, 1998; Uieda & Barbosa, 2016). Consequently, an inversion model should not be interpreted as a unique representation of the actual subsurface, but rather as one geophysically admissible model that is consistent with the data and with the information explicitly incorporated into the inversion. Stable gravity inversion requires appropriate model parameterization, regularization, data-error weighting, geological or petrophysical constraints, and numerical strategies capable of controlling the trade-off between data fit and model complexity.

Various inversion formulations have been developed to address these challenges, and the resulting density images can depend strongly on the selected parameterization and stabilizing functional. Continuous cell-based approaches generally estimate density contrasts by minimizing a weighted data-misfit term together with a quadratic minimum-structure penalty, producing stable but inherently smooth models. In contrast, compactness and focusing formulations seek to recover more spatially concentrated anomalous bodies through minimum-volume, minimum-support, or related stabilizers (Barbosa & Silva, 1994; Guillen & Menichetti, 1984; Last & Kubik, 1983; Portniaguine & Zhdanov, 1999). Non-smooth and piecewise-constant models can be obtained using total variation or iterative reweighting formulations, which are more appropriate when the true geology is characterized by sharp physical-property contrasts (Bertete-Aguirre et al., 2002; Farquharson, 2008; Vatankhah et al., 2018a). Other formulations employ discrete or binary density states, sensitivity-dependent weighting, and compactness constraints to reduce ambiguity and near-surface bias (Ghalehnoee et al., 2017; Krahenbuhl & Li, 2006), whereas randomized matrix decompositions have been used to improve the computational efficiency of focused inversion in large model spaces (Vatankhah et al., 2018b). Stochastic and population-based optimization methods provide alternatives to deterministic local optimization, particularly for nonlinear geometry estimation and uncertainty assessment (Pallero et al., 2015; Song et al., 2021). More recent formulations have also explored Lp-norm penalties and structure-based constraints to

enhance compactness or boundary recovery (Liu et al., 2022; Peng & Liu, 2021). These developments emphasize that data fit alone is insufficient for evaluating an inversion; smoothness, compactness, model boundaries, depth sensitivity, numerical convergence, and geological plausibility collectively determine the characteristics and interpretability of the recovered density distribution.

Another practical issue arises when gravity observations are acquired along a profile crossing geological structures that can be approximated in cross-section but have a finite, rather than infinite, extent perpendicular to the profile. Conventional two-dimensional modeling generally assumes invariance along strike, whereas full three-dimensional inversion explicitly represents lateral variations at the expense of substantially greater model dimensionality and computational cost. Analytical gravity expressions for finite-length sources and rectangular prisms demonstrate that source extent contributes directly to the forward response and should therefore be represented when the infinite-strike approximation is inappropriate (Cady, 1980; Nagy et al., 2000). This consideration motivates an intermediate 2.5D formulation, in which the inversion remains parameterized in the X–Z cross-section while the forward operator accounts for finite strike length. In this context, ALTYN was developed not to introduce a new standalone regularizer, but to integrate finite-strike prism physics, topography-consistent active cells, observation-uncertainty weighting, a fixed reference model, explicit density bounds, directional smallness and smoothness regularization, L2/IRLS schemes, an optional regional nuisance component, constrained optimization, and weighted-residual diagnostics within a transparent and reproducible profile-inversion framework.

An important reference for cell-based gravity inversion is the GROWTH software. Its initial implementation determines anomalous-body geometry through progressive exploration, in which cells belonging to a three-dimensional prism partition are progressively assigned predefined positive or negative density contrasts. Candidate additions are evaluated using a criterion that balances the fit to the gravity data against model size or anomalous mass, while a regional gravity trend can be adjusted simultaneously (Camacho et al., 2002). The development of GROWTH2.0 introduced a graphical user interface, increasingly automated parameter selection, robust treatment of outliers, options for depth-dependent density variation, and improved handling of the balance between model fit and smoothness (Camacho et al., 2011). GROWTH3.0 subsequently simplified the operational package and incorporated automatic determination of density contrast, subhorizontal layered structures, a background medium with density increasing downward, and updates to both graphical and computational

procedures (Camacho et al., 2021). GROWTH is therefore a relevant tool for constrained gravity inversion; however, its model-growth formulation differs fundamentally from the ALTYN approach. In GROWTH, predefined density contrasts are assigned to progressively selected cells to construct anomalous-body geometry, whereas ALTYN estimates continuous density contrasts on a fixed X–Z discretization through explicitly regularized and bounded numerical optimization.

This study presents ALTYN Inversion as a MATLAB-based 2D/2.5D gravity inversion framework for recovering continuous density-contrast distributions beneath a gravity profile using finite-strike rectangular-prism forward modeling (Figure 1). The framework discretizes the subsurface using a topography-following X–Z mesh and incorporates uncertainty-normalized data fitting, explicit density bounds, a fixed reference density, directional model regularization, and optional constant or linear regional components. Two regularization strategies are implemented: conventional L2 regularization as a smooth minimum-structure model and non-quadratic IRLS to investigate solutions exhibiting different degrees of spatial segmentation. Model updates are obtained using a projected Newton/active-set preconditioned conjugate-gradient strategy, allowing density bounds to be enforced within the solution process rather than imposed afterward. The primary objective is to identify first-order density features that can be consistently recovered while maintaining transparent control over the assumptions underlying the inversion. No claim is made regarding quantitative robustness to perturbations for which dedicated tests have not yet been performed.

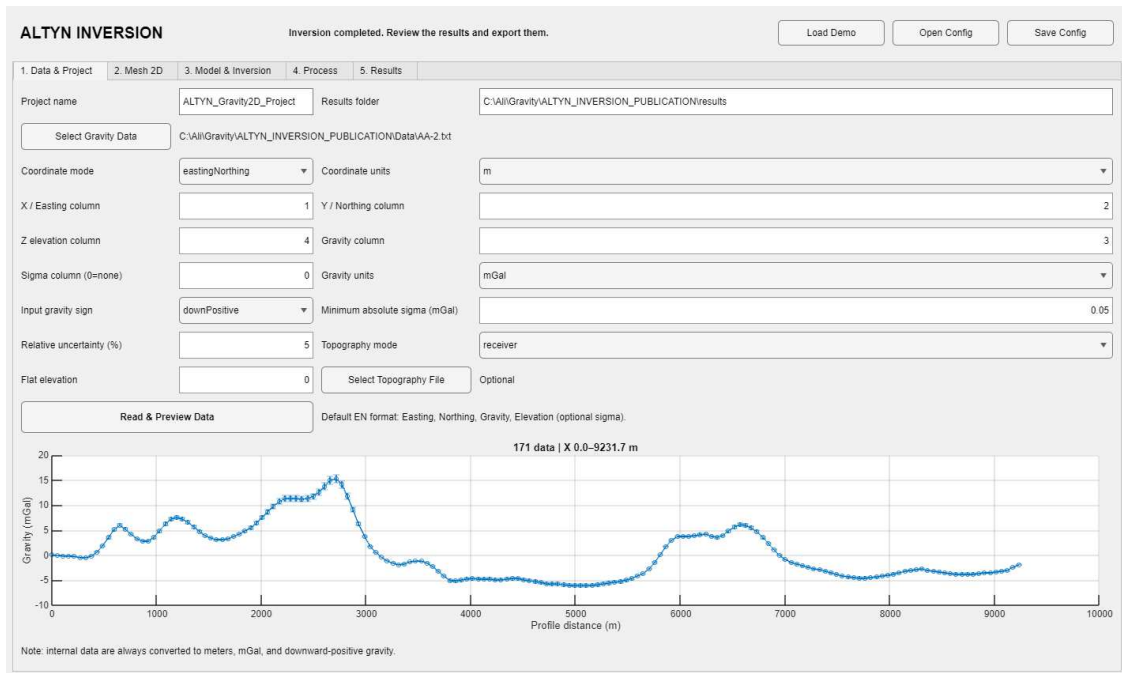


Figure 1. ALTYN Inversion graphical user interface (GUI) used for gravity-data input and preliminary profile visualization.

2. ALTYN Inversion Methodology

2.1 Forward gravity formulation

ALTYN Inversion represents the subsurface as a two-dimensional distribution of density contrast in the vertical X – Z plane, while evaluating the gravity response by accounting for a finite extent perpendicular to the profile. Let N denote the number of gravity observations and M the number of active subsurface cells. The observed gravity-anomaly vector is denoted by $\mathbf{d}_{\text{obs}} \in \mathbb{R}^N$, whereas the unknown density-contrast model is represented by $\mathbf{m} \in \mathbb{R}^M$. For a prescribed model geometry, the density contribution to the gravity response is linear with respect to $\mathbf{m}$. ALTYN also allows an optional regional component to be estimated simultaneously with the density distribution; therefore, the predicted gravity anomaly is expressed as shown in Equation (1).

$$\mathbf{d}^{\text{pred}} = \mathbf{J}\mathbf{m} + \mathbf{B}\mathbf{c} \quad (1)$$

where $\mathbf{J} \in \mathbb{R}^{N \times M}$ is the finite-strike gravity sensitivity matrix, $\mathbf{B}$ is the basis matrix for the optional regional component, and $\mathbf{c}$ contains its nuisance coefficients. The observation model can then be written as Equation (2).

$$\mathbf{d}^{\text{obs}} = \mathbf{J}\mathbf{m} + \mathbf{B}\mathbf{c} + \boldsymbol{\varepsilon} \quad (2)$$

where $\boldsymbol{\varepsilon}$ represents observational errors and gravity components not captured by the adopted model. Internally, ALTYN uses meters for horizontal and vertical coordinates, g/cm^3 for density contrast, and mGal for the gravity response. The vertical coordinate is defined as positive upward, whereas the modeled gravity response is taken as positive downward.

This formulation follows a continuous cell-based gravity inversion approach, in which density contrast is treated as the inversion parameter on a fixed mesh, as commonly adopted in minimum-structure gravity inversion (Li & Oldenburg, 1998). This approach should be distinguished from the GROWTH family of methods. GROWTH progressively assigns predefined positive or negative density contrasts to selected cells and thereby constructs the geometry of anomalous bodies through model exploration, whereas ALTYN continuously estimates the density contrasts of all active cells under explicit regularization and bound constraints (Camacho et al., 2002). Although both approaches employ cell-based representations, their inversion parameterizations and optimization strategies are fundamentally different.

2.2 2.5D finite-strike prism response

The principal physical approach adopted in ALTYN is to represent the geological cross-section using an X – Z inversion mesh, in which each cell has a prescribed finite extent perpendicular to the profile. Each active cell is treated as a three-dimensional rectangular

prism formed by symmetrically extruding the X - Z cross-section from $\frac{-L_y}{2}$ to $\frac{+L_y}{2}$, where L_y denotes the finite strike length. Density may vary from cell to cell within the inversion cross-section but is assumed to remain constant along the strike extent of each prism. This results in a 2.5D formulation: the unknown model remains two-dimensional, while the gravity forward operator explicitly accounts for the three-dimensional geometry of finite-length sources. This formulation avoids the infinite-strike assumption inherent in conventional 2D models without introducing a fully three-dimensional density parameterization.

For receiver i and active cell j , the sensitivity coefficient is calculated from the analytical vertical gravitational attraction of a rectangular prism. Under the ALTYN coordinate convention, this coefficient can be expressed as Equation (3).

$$J_{ij} = G 10^8 \sum_{p=1}^2 \sum_{q=1}^2 \sum_{r=1}^2 (-1)^{p+q+r} F(x_p, y_q, z_r), \quad (3)$$

with

$$F(x, y, z) = x \ln(y + R) + y \ln(x + R) - z \tan^{-1} \left(\frac{xy}{zR} \right), \quad (4)$$

and

$$R = \sqrt{(x^2 + y^2 + z^2)} \quad (5)$$

Here x_p and z_r are the prism-boundary coordinates relative to the gravity receiver, while $y_1 = \frac{-L_y}{2}$, $y_2 = \frac{+L_y}{2}$. This formulation is consistent with the analytical gravitational attraction equation for a rectangular prism, as summarized by Nagy et al. (2000). In the implementation, logarithmic and arctangent evaluations are numerically safeguarded for limiting geometric configurations to avoid singular floating-point evaluations. The factor in Equation (3) arises from $10^8 = 10^3 \times 10^5$, where 10^3 converts the input density unit from g/cm^3 to kg/m^3 , and 10^5 converts gravitational acceleration from m/s^2 to mGal . Accordingly, each column of $\mathbf{J}$ represents the gravity response in mGal generated by a unit density contrast within a single finite-strike cell. The relationship between the X - Z inversion mesh and the finite-strike extrusion used in the forward operator is illustrated in Figure 2.

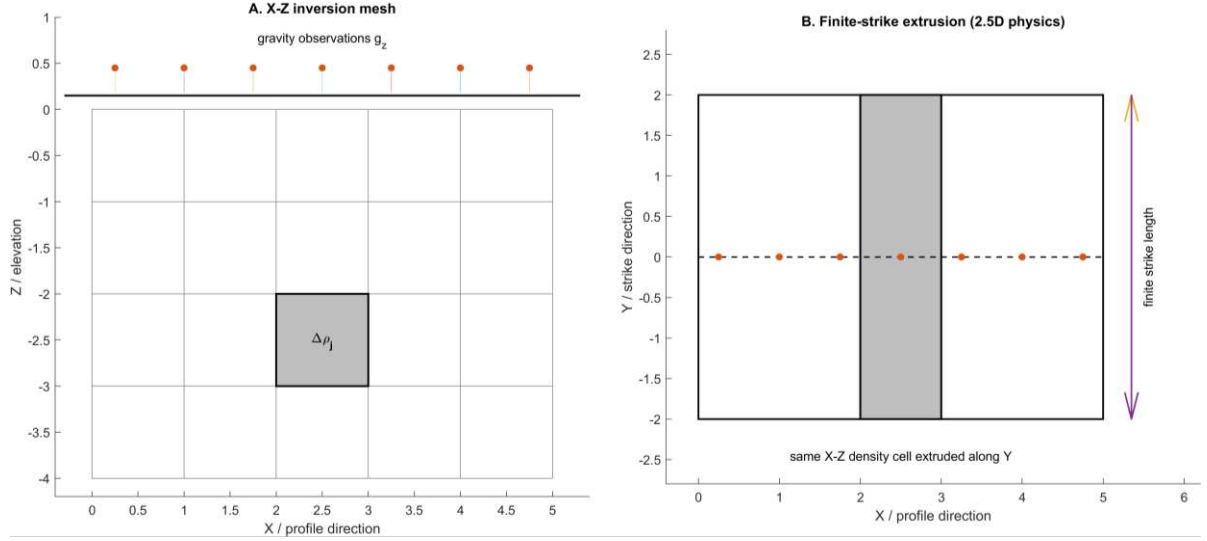


Figure 2. Geometry of the ALTYN 2.5D finite-strike gravity formulation. The inversion parameters are defined in the X - Z section, whereas each density cell $\Delta\rho_j$ is extruded over the prescribed finite strike length L_y for computation of its gravity response.

Finite strike length is an assumed physical parameter rather than a quantity uniquely determined from profile gravity data. Its specification should therefore reflect the expected geological extent of the target, and uncertainty in this parameter forms part of the model-assumption uncertainty.

2.3 X - Z model discretization and topographic constraint

ALTYN discretizes the model domain using a uniform rectangular X - Z mesh characterized by the horizontal cell width, dx , the nominal vertical cell height, dz , the maximum investigation depth, and lateral padding beyond the observation interval. The density model is expressed as Equation (6).

$$\mathbf{m} = [m_1, m_2, \dots, m_M]^T, \quad (6)$$

where M includes only cells classified as active cells beneath the surface. The lateral boundaries are extended beyond the first and last observation points by a prescribed padding distance so that near-surface density structures at the ends of the profile remain represented beneath the outermost receivers.

Topography is incorporated directly into the physical model geometry. For each horizontal mesh column, ALTYN evaluates the topographic elevation over the entire column interval. Under the adopted *topography-safe* convention, the minimum surface elevation intersecting that interval is used to determine whether a nominal cell lies entirely or partially beneath the surface. Cells located above the surface are excluded from the active model. For the uppermost active cell, the nominal upper boundary is truncated to conform to the local

surface elevation. Consequently, the actual height of the uppermost cell may differ from the nominal dz , and cells with a negligible residual height are removed.

This operation is important because topography is not applied merely as a graphical mask after the inversion. The truncated cell boundaries are passed directly to the finite-prism forward calculation in Equation (3), such that topographic relief modifies the geometry represented by $\mathbf{J}$. The cell centers, widths, and heights subsequently used in the regularization terms are also calculated from the final active-cell geometry. This treatment prevents the prisms used in the forward calculation from extending above the modeled ground surface and ensures geometric consistency among the topography, forward kernel, and inversion model.

2.4 Data uncertainty and weighted data misfit

ALTYN explicitly incorporates the uncertainty associated with each gravity observation. If positive and finite standard deviations are available in the input data, these values are retained. If such values are unavailable or invalid, a combined absolute–relative uncertainty model is constructed, as expressed in Equation (7).

$$\sigma_i = \sqrt{\sigma_0^2 + (r|d_i^{obs}|)^2}, \quad (7)$$

where σ_0 is an absolute error floor in mGal and r is a fractional relative uncertainty. The quadrature combination ensures that the uncertainty remains finite near zero anomaly while allowing the assumed error magnitude to increase with anomaly amplitude. The resulting data-weighting matrix is:

$$\mathbf{W}_d = \text{diag}(\sigma_1^{-1}, \sigma_2^{-1}, \dots, \sigma_N^{-1}), \quad (8)$$

and the statistically normalized data-misfit functional is:

$$\phi_d = \|\mathbf{W}_d[\mathbf{d}^{obs} - \mathbf{J}\mathbf{m} - \mathbf{B}\mathbf{c}]\|_2^2, \quad (9)$$

Equivalently, the normalized squared residual can be written as:

$$X^2 = \sum_{i=1}^N \left[\frac{d_i^{obs} - d_i^{pred}}{\sigma_i} \right]^2, \quad (10)$$

and ALTYN reports the weighted root-mean-square residual as:

$$\text{WRMS} = \sqrt{\frac{X^2}{N}} \quad (11)$$

This normalization permits a direct discrepancy-based interpretation of the final residuals. If the observations are statistically independent and the assigned σ_i values are representative of their standard deviations, a WRMS near unity indicates that the magnitude of the residual is broadly compatible with the assumed data uncertainty. It does not, however, demonstrate that the recovered density model is unique or geologically correct; the usual ambiguity of

gravity inversion remains, and the recovered solution continues to depend on parameterization, regularization, bounds, and the finite-strike assumption.

2.5 Regional gravity component

Long-wavelength or datum-related gravity components may otherwise be incorrectly represented as subsurface density structure. ALTYN therefore permits a low-order regional term to be solved simultaneously with the density model. Three modes are implemented: no regional term, a constant offset, or a profile-parallel linear trend. For a constant regional component, the formulation is:

$$g_{reg}(x_i) = c_0, \quad (12)$$

whereas the linear mode is parameterized as:

$$g_{reg}(x_i) = c_0 + c_1 \frac{x_i - \bar{x}}{1000}, \quad (13)$$

where c_0 is the gravity offset in mGal, c_1 is the slope in mGal/km, x_i is profile distance in meters, and $\bar{x}$ is its mean. Centering the distance coordinate reduces correlation between offset and slope.

The associated coefficients are appended to the density parameters and estimated as part of the same data-fitting problem. However, they are intentionally neither density bounded nor included in the density-model regularization. Before the main inversion loop, their starting values are estimated by uncertainty-weighted least squares against the residual remaining after the starting density model. This initialization reduces the tendency for a simple gravity datum offset to be mapped immediately into density structure.

2.6 Model regularization and depth weighting

Because the number of active cells can be substantially greater than the number of observations, stable inversion requires explicit model regularization. Minimum-structure formulations and depth weighting are well established in cell-based gravity inversion, particularly for controlling non-uniqueness and compensating for the strong decay of gravity sensitivity with depth (Li & Oldenburg, 1998). ALTYN minimizes the combined objective function expressed in Equation (14).

$$\Phi(\mathbf{m}, \mathbf{c}) = \gamma^2 \varphi_d + \beta^2 \varphi_m \quad (14)$$

where γ scales the data term and β controls the balance between data agreement and model regularization. The use of β^2 , rather than β , is retained consistently in the implemented normal equations. The model objective is:

$$\phi_m = \alpha_s \phi_s + \alpha_x \phi_x + \alpha_z \phi_z, \quad (15)$$

where α_s , α_x , and α_z independently control model smallness, horizontal smoothness, and vertical smoothness. For a fixed reference model $\mathbf{m}_{ref}$, the three components are:

$$\phi_s = \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{\text{ref}})\|_2^2, \quad (16)$$

$$\phi_x = \|\mathbf{W}_x \mathbf{D}_x(\mathbf{m} - \mathbf{m}_{\text{ref}})\|_2^2 \quad (17)$$

260 and

$$\phi_z = \|\mathbf{W}_z \mathbf{D}_z(\mathbf{m} - \mathbf{m}_{\text{ref}})\|_2^2 \quad (18)$$

261 $\mathbf{D}_x$ and $\mathbf{D}_z$ are first-order difference operators connecting horizontally and vertically
 262 adjacent active cells. The reference model is fixed throughout the inversion rather than
 263 replaced by the previous iteration. Regularization geometry accounts for the dimensions of the
 264 active rectangular cells. For each neighboring pair, the magnitude of the discrete difference
 265 operator is proportional to:

$$\left(\frac{\text{shared -- face dimension}}{\text{distance between cell centers}} \right)^{1/2} \quad (19)$$

266 The horizontal operator therefore uses the common vertical face dimension, whereas the
 267 vertical operator uses the common horizontal face dimension. Additional face weights are
 268 derived from the adjacent cell weights, providing a continuous transition between cell-based
 269 and derivative-based regularization even where topographic clipping alters upper-cell
 270 dimensions. ALTYN further introduces a cell-area scaling:

$$w_j^{(A)} = \sqrt{\frac{A_j}{\text{median}(A)}}, \quad (20)$$

271 where A_j is the actual X - Z area of cell j . Depth is measured downward from the locally
 272 interpolated topographic surface to the cell center rather than from a flat global datum. When
 273 depth weighting is enabled, the depth-dependent factor is:

$$w_j^{(d)} = C_h [\max(d_j, d_{\min,j})]^{-Pd/2}, \quad (21)$$

274 where d_j is the local depth, Pd is the configured depth exponent, C_h is the depth-weighting
 275 coefficient, and $d_{\min,j}$ prevents singular amplification close to the surface. In the implemented
 276 geometry, $d_{\min,j}$ is tied to one-half of the cell height. An optional sensitivity weight may
 277 additionally be formed from the uncertainty-normalized kernel:

$$s_j = \left[\sum_{i=1}^N \left(\frac{j_{ij}}{\sigma_i} \right)^2 \right]^{1/2}, \quad (22)$$

278 followed by normalization, flooring, and exponentiation.

$$w_j^{(s)} = \left[\max \left(\frac{s_j}{\max_t s_t}, s_{\min} \right) \right]^\eta \quad (23)$$

279 When sensitivity weighting is disabled, $w_j^{(s)} = 1$. The base cell weight used in
 280 regularization can therefore be represented conceptually as:

$$w_j^{\text{base}} = w_j^{(A)} w_j^{(d)} w_j^{(s)}. \quad (24)$$

Depth and sensitivity weighting alter the prior metric of the inverse problem and thus influence the admissible recovered structure; they should not be interpreted as data-derived corrections independent of prior assumptions.

2.7 L2 and IRLS formulations

ALTYN supports both quadratic L2 regularization and an iteratively reweighted least-squares formulation for nonquadratic L_p -type stabilization. In L2 mode, the configured norms satisfy:

$$P_s = P_x = P_z = 2, \quad (25)$$

and all IRLS multipliers remain unity. The resulting objective is therefore quadratic for each fixed β , favoring comparatively smooth, distributed, minimum-structure-type density variations. Such solutions are useful when strong discontinuities are not assumed a priori and provide an important reference against which focused solutions can be evaluated.

For IRLS mode, ALTYN repeatedly solves weighted quadratic subproblems whose weights are updated from the current model. This strategy is widely used to approximate nonquadratic stabilization and to generate more focused or piecewise-structured potential-field models (Farquharson, 2008; Portniaguine & Zhdanov, 1999; Vatankhah et al., 2017, 2018b). For outer iteration k , the smallness multiplier is:

$$w_{s,j}^{(k)} = \left[\left(m_j^{(k)} - m_{ref,j} \right)^2 + \epsilon_s \right]^{(P_s-2)/4}, \quad (26)$$

while the horizontal-gradient multiplier is:

$$w_{x,l}^{(k)} = \left[\left(D_x [m^{(k)} - m_{ref}] \right)_l^2 + \epsilon_x \right]^{(P_x-2)/4}, \quad (27)$$

and the corresponding vertical multiplier is:

$$w_{z,l}^{(k)} = \left[\left(D_z [m^{(k)} - m_{ref}] \right)_l^2 + \epsilon_z \right]^{(P_z-2)/4} \quad (28)$$

The implemented norms satisfy:

$$0 \leq P_s, P_x, P_z \leq 2 \quad (29)$$

The exponent $(p - 2)/4$ arises because each IRLS multiplier is subsequently used within a squared weighted Euclidean norm; the resulting squared weight produces the corresponding $(p - 2)/2$ dependence required by the quadratic IRLS surrogate. The positive quantities ϵ_s , ϵ_x and ϵ_z stabilize the weights near zero model or gradient differences and prevent singular amplification.

A reproducibility-relevant feature of ALTYN is that all IRLS differences are evaluated relative to the fixed $\mathbf{m}_{ref}$. The previous model is therefore used to calculate the current IRLS

weights but does not become a new reference model. In the first outer iteration, the IRLS multipliers are initialized to unity; subsequent weights are updated after each accepted density solution.

2.8 Density bounds and constrained numerical solution

Physical prior information is incorporated through explicit bounds on every active density cell.

$$m_{\min} \leq m_j \leq m_{\max}, j = 1, \dots, M \quad (30)$$

These limits define the feasible model space and are enforced during optimization rather than by clipping an unconstrained solution afterward. Regional nuisance coefficients remain unbounded. Defining the augmented parameter vector:

$$\mathbf{x} = \begin{bmatrix} \mathbf{m} \\ \mathbf{c} \end{bmatrix} \quad (31)$$

and the augmented forward matrix:

$$\mathbf{H} = [\mathbf{J} \ \mathbf{B}] \quad (32)$$

the fixed-weight quadratic problem at outer iteration k can be expressed through the normal operator:

$$\mathbf{A}_k = \gamma^2 \mathbf{H}^T \mathbf{C}_d^{-1} \mathbf{H} + \beta_k^2 \tilde{\mathbf{R}}_k \quad (33)$$

where

$$\mathbf{C}_d^{-1} = \text{diag}(\sigma_1^{-2}, \dots, \sigma_N^{-2}), \quad (34)$$

and

$$\tilde{\mathbf{R}}_k = \begin{bmatrix} \mathbf{R}_k & 0 \\ 0 & 0 \end{bmatrix} \quad (35)$$

The zero regional block in Equation (35) expresses explicitly that nuisance coefficients are not subject to density-model regularization. If

$$\mathbf{R}_k = \mathbf{R}_{s,k} + \mathbf{R}_{x,k} + \mathbf{R}_{z,k},$$

the implemented right-hand side is

$$\mathbf{b}_k = \gamma^2 \mathbf{H}^T \mathbf{C}_d^{-1} \mathbf{d}^{\text{obs}} + \beta_k^2 \begin{bmatrix} \mathbf{R}_{s,k} \mathbf{m}_{\text{ref}} \\ 0 \end{bmatrix} \quad (36)$$

Only the smallness reference contribution appears explicitly because the ALTYN reference density is spatially constant, and its first-order directional derivatives therefore vanish. The corresponding constrained quadratic objective is:

$$\min_{\mathbf{x}} \left[\frac{1}{2} \mathbf{x}^T \mathbf{A}_k \mathbf{x} - \mathbf{b}_k^T \mathbf{x} \right] \quad (37)$$

subject to the density bounds in equation (30). ALTYN solves this subproblem using a projected Newton/active-set procedure in which cells satisfying the appropriate lower- or upper-bound optimality condition are temporarily fixed, and the Newton system is solved

only over the remaining free variables. The reduced linear system is solved by preconditioned conjugate gradients (PCG). When diagonal preconditioning is enabled, the preconditioner is assembled from the diagonal of the uncertainty-weighted data normal operator together with the regularization contribution.

For a Newton direction $\mathbf{p}$, ALTYN evaluates the quadratic optimal step

$$\alpha_N = -\frac{\mathbf{g}^T \mathbf{p}}{\mathbf{p}^T \mathbf{A}_k \mathbf{p}} \quad (38)$$

where $\mathbf{g} = \mathbf{A}_k \mathbf{x} - \mathbf{b}_k$, and restricts it to the maximum feasible step before any density parameter crosses its bound. If the PCG/Newton direction is non-finite, non-descent, or degenerate, the algorithm falls back to a diagonally scaled projected-gradient direction. Trial density parameters are then projected onto the feasible interval and the active set is re-evaluated.

Optimality is monitored through the projected gradient associated with the Karush–Kuhn–Tucker conditions. In the implementation, gradient entries are removed from the projected residual when a variable lies at a lower bound with an outward gradient or at an upper bound with the opposite outward gradient. The reported normalized projected KKT residual is:

$$\text{rKKT} = \frac{\|\mathbf{g}_{\text{proj}}\|_{\infty}}{\max(1, \|\mathbf{b}_k\|_{\infty})}, \quad (39)$$

Thus, density bounds influence the inversion throughout the optimization. They represent prior physical constraints and consequently restrict the family of admissible models rather than serving solely as a numerical safeguard.

2.9 Regularization-parameter cooling and convergence

For both L2 and IRLS modes, ALTYN uses an outer-loop cooling schedule for the regularization trade-off parameter. Starting from $\beta_0 > 0$, the schedule is:

$$\beta_{k+1} = q\beta_k, \quad 0 < q \leq 1, \quad (40)$$

where q is the prescribed reduction factor. Large initial β values emphasize stabilization, whereas progressive cooling gives increasing influence to the data-misfit term. This differs from selecting one fixed regularization parameter independently of the inversion trajectory, and it permits the solution to move gradually toward the data-consistent region of model space.

The principal ALTYN stopping rule is based on the normalized discrepancy criterion. If the requested target weighted RMS is WRMS_target , the inversion terminates when:

$$\chi^2 \leq N(\text{WRMS}_{\text{target}})^2 \quad (41)$$

For the common choice:

$$WRMS_{target} = 1$$

equation (41) reduces to:

$$\chi^2 \leq N \quad (42)$$

Use of χ^2 -type principles for regularization and discrepancy assessment has precedent in focused gravity inversion, but its statistical interpretation depends directly on the reliability of the assigned data uncertainties. A WRMS near one therefore indicates consistency with the adopted uncertainty model rather than proof of a unique subsurface reconstruction.

A secondary stopping condition may be applied to the relative change in the density model,

$$\delta_m^{(k)} = \frac{\|\mathbf{m}^{(k)} - \mathbf{m}^{(k-1)}\|_2}{\max(\|\mathbf{m}^{(k-1)}\|_2, 1)}, \quad (43)$$

falls below a user-specified threshold. If this criterion is disabled, cooling continues until the discrepancy target or the maximum number of outer iterations is reached.

For diagnostic transparency, ALTYN stores substantially more information than is required for stopping. The recorded quantities include unweighted RMS in mGal, WRMS, χ^2 , model norm, smallness contribution, horizontal- and vertical-smoothness contributions, total model objective, combined objective, relative model change, fraction of cells located at density bounds, projected KKT residual, PCG iteration count, active-set iteration count, and elapsed computational time. These quantities are diagnostic outputs; they should not all be interpreted as independent convergence conditions. The ALTYN inversion workflow is illustrated in Figure 3.

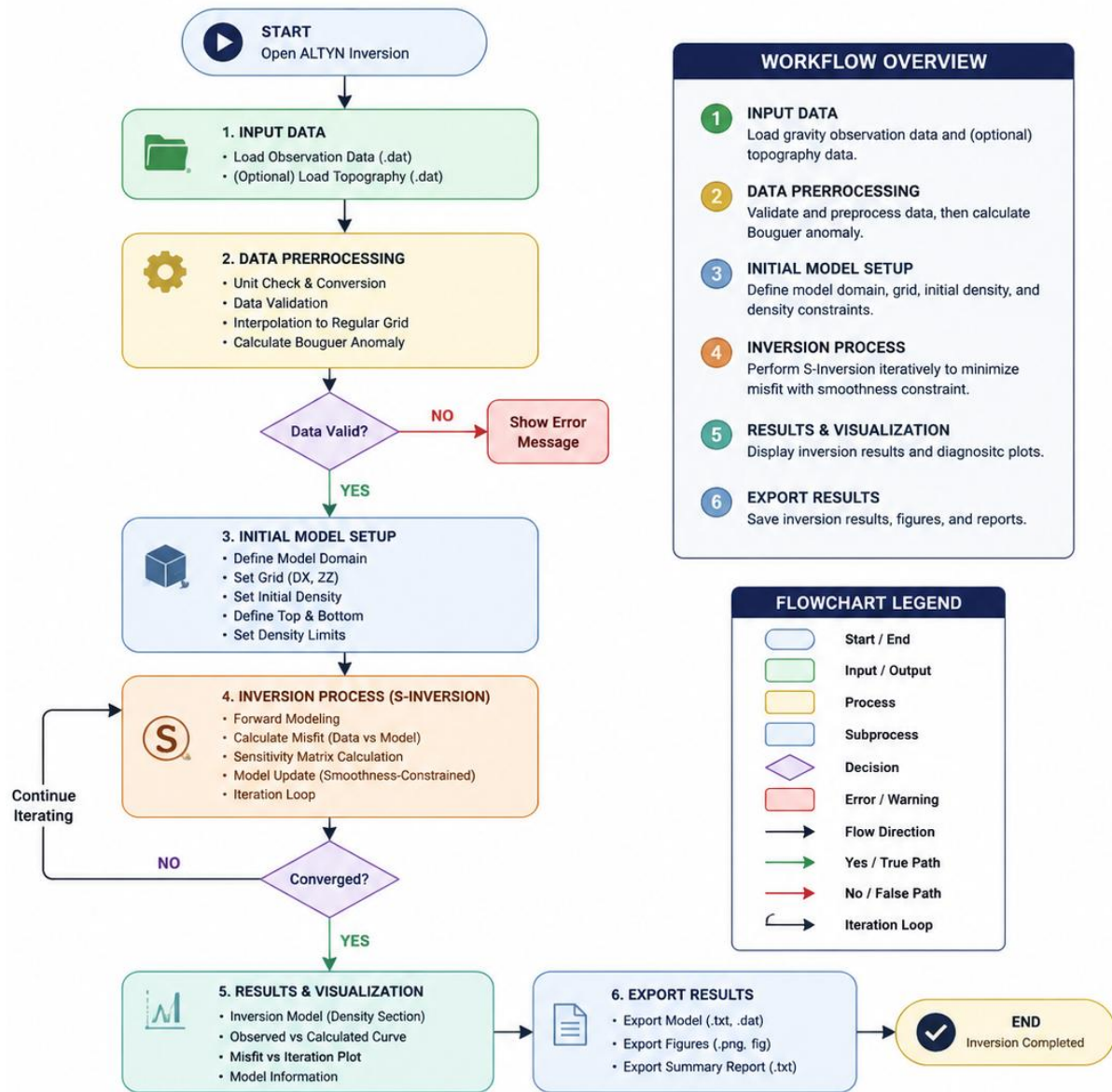


Figure 3. Computational workflow of the ALTYN Inversion framework.

3. Synthetic Baseline Validation

3.1 Synthetic benchmark design

Synthetic experiments were used to evaluate ALTYN under conditions in which the prescribed density distribution is known independently of the inversion. Such tests are important because close agreement between observed and predicted gravity does not, by itself, demonstrate that the source distribution has been reconstructed correctly; markedly different density models may produce similar surface responses because of the inherent non-uniqueness and depth-dependent sensitivity of gravity inversion (Li & Oldenburg, 1998). The three experiments therefore assess both data-space recovery, expressed through gravity

residuals, and model-space recovery through direct comparison of the prescribed and recovered density distributions.

All three benchmarks used 81 gravity observations distributed from 0 to 6 km at a uniform interval of 75 m. The synthetic forward models and inverse models employed 150-m nominal cell dimensions in both the horizontal and vertical directions, a model depth of 3 km, 600 m of lateral padding, and a finite strike length of 4 km. Density contrasts were constrained between -0.60 and $+0.60$ g/cm³, with zero starting and reference models. Synthetic noise was deterministic, zero-mean, and scaled to the prescribed standard deviation, allowing the experiments to be repeated without variation caused by random-number generation. Benchmark 1 used $\sigma = 0.03$ mGal, whereas Benchmarks 2 and 3 used $\sigma = 0.05$ mGal. The target discrepancy was WRMS = 1.05. The three topographic configurations resulted in 1008, 1035, and 1079 active subsurface cells, respectively (see Table 1).

Table 1. Principal parameters used in the three synthetic ALTYN benchmarks

Parameter	Benchmark 1	Benchmark 2	Benchmark 3
Number of observations	81	81	81
Station spacing	75 m	75 m	75 m
Horizontal cell size, dx	150 m	150 m	150 m
Vertical cell size, dz	150 m	150 m	150 m
Maximum model depth	3000 m	3000 m	3000 m
Lateral padding	600 m	600 m	600 m
True strike length	4000 m	4000 m	4000 m
Inversion strike length	4000 m	4000 m	4000 m
Assigned σ	0.03 mGal	0.05 mGal	0.05 mGal
Density bounds	$-0.60, +0.60$ g/cm ³	$-0.60, +0.60$ g/cm ³	$-0.60, +0.60$ g/cm ³
Regional component	None	Constant	Linear
WRMS target	1.05	1.05	1.05
Active cells	1008	1035	1079

3.2 Benchmark 1: Compact positive-density anomaly

Benchmark 1 represents the simplest recovery problem. The prescribed model consists of a single positive-density elliptical body centered at approximately 3.0 km along the profile and at an elevation of -0.85 km. The defining semi-axes are approximately 0.75 km horizontally and 0.50 km vertically, and the prescribed density contrast is $+0.35$ g/cm³. After discretization, the nonzero target occupies the central portion of the section between approximately 2.33 and 3.68 km along the profile and elevations of about -0.38 to -1.28 km.

The prescribed and recovered density distributions are compared in Figure 4. ALTYN correctly identifies the positive polarity and lateral center of the source. The maximum recovered positive density is concentrated close to the true body position, indicating that the profile location of the causative structure is resolved. The geometry, however, is not reproduced literally. Instead of the compact prescribed body, the recovered positive anomaly extends vertically through a much greater fraction of the model domain and gradually broadens with depth. Lower-amplitude negative contrasts also occur on both sides of the central positive structure. Because these negative regions are absent from the prescribed model, they should be interpreted as compensating components of the regularized inverse solution rather than as recovered target structures. This result illustrates an important distinction between source localization and geometrical reconstruction. The inversion identifies the principal lateral position and correct density sign, but gravitational depth ambiguity permits the required mass effect to be distributed over a considerably larger vertical interval. The imposed smoothness also spreads the abrupt boundaries of the compact target into gradual transitions, a behavior characteristic of minimum-structure-type potential-field inversions (Portniaguine & Zhdanov, 1999). Despite this model-space discrepancy, the predicted gravity reproduces the synthetic observations with an RMS residual of 0.02602 mGal and a WRMS of 0.867, while no model cells reach the imposed density bounds. Thus, the excellent data-space agreement coexists with appreciable geometrical smearing.

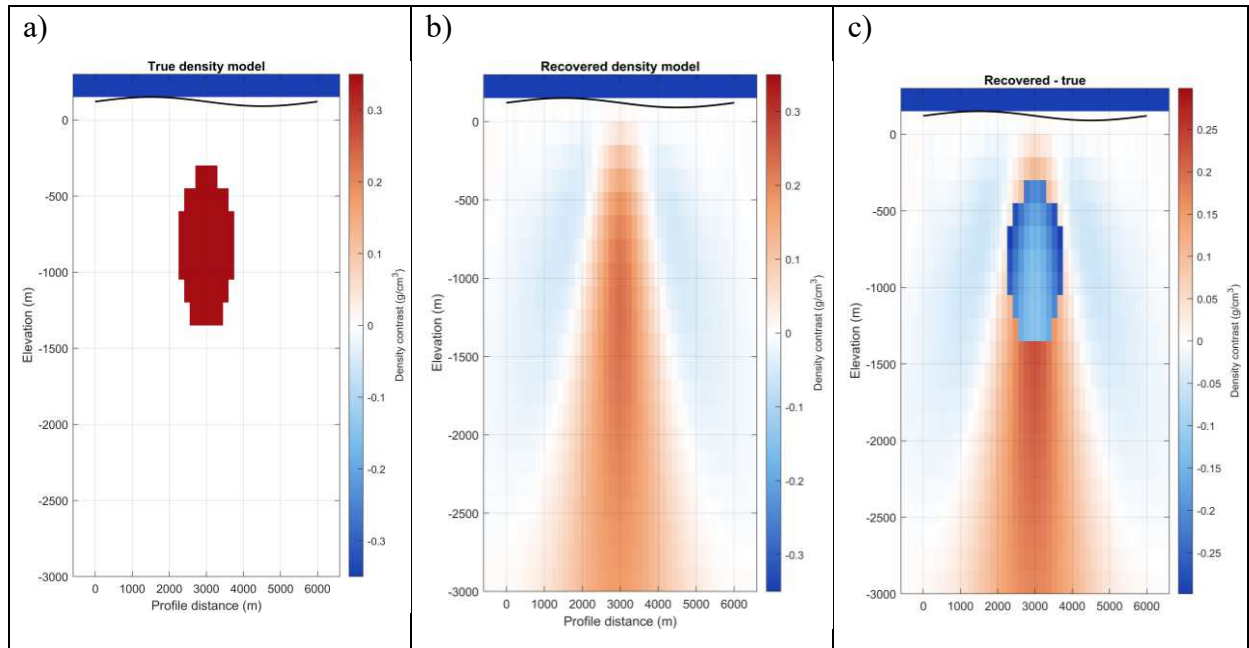


Figure 4. Synthetic Benchmark 1. (a) Prescribed compact positive density-contrast model used to generate the synthetic observations. (b) Density distribution recovered by ALTYN. (c) Cell-wise difference between the recovered and prescribed models. Identical density limits are used for the true and recovered models to permit direct comparison.

3.3 Benchmark 2: Positive and negative density anomalies

Benchmark 2 increases the complexity by introducing two spatially separated bodies with opposite density contrasts. The positive body is centered at approximately 1.9 km and -0.95 km elevation, with horizontal and vertical semi-axes of about 0.65 and 0.55 km and a prescribed contrast of $+0.32$ g/cm³. A second body centered near 4.2 km and -1.35 km elevation has semi-axes of approximately 0.70 and 0.65 km and a contrast of -0.26 g/cm³. A constant regional offset of 0.25 mGal is superimposed on the density response before noise is added. This benchmark therefore tests not only dual-polarity recovery but also the partitioning of the total gravity field between local density structure and a nuisance datum component. Figure 5 shows that the inversion correctly recovers the broad lateral separation and polarity of the two principal anomalies. A positive-density zone develops beneath the western target and a negative-density zone beneath the eastern target, demonstrating that the inversion is not restricted to a single density sign. As in Benchmark 1, however, both recovered anomalies are considerably smoother and extend substantially deeper than the prescribed bodies. Their amplitudes are also attenuated relative to the true contrasts, indicating that the required gravitational moment is distributed over a larger model volume rather than concentrated within the exact target geometry.

The recovered-minus-true panel makes this behavior particularly clear. Within the prescribed positive body, negative differences indicate under-recovery of its imposed amplitude, whereas positive differences beneath the body reflect density assigned below the true target. The converse pattern occurs around the negative body: the true negative contrast is incompletely recovered within its prescribed geometry, while additional negative density is distributed at greater depth. The regional-density decomposition also demonstrates that separation of a broad regional signal from long-wavelength density effects is itself non-unique. The nuisance component accounts for a substantial part of the long-wavelength field, but some trade-off remains between regional offset and distributed density. Accordingly, satisfactory recovery of the total gravity field should not be interpreted as unique recovery of either component individually.

The gravity observations are nevertheless reproduced to within their assigned uncertainty, with an RMS of 0.04624 mGal and WRMS of 0.925. The density-bound fraction remains 0.0%, showing that the solution is not being forced against the $\pm 0.60 \text{ g/cm}^3$ limits. The principal success of this experiment is therefore recovery of anomaly polarity and approximate lateral position, rather than exact body boundaries or density amplitudes.

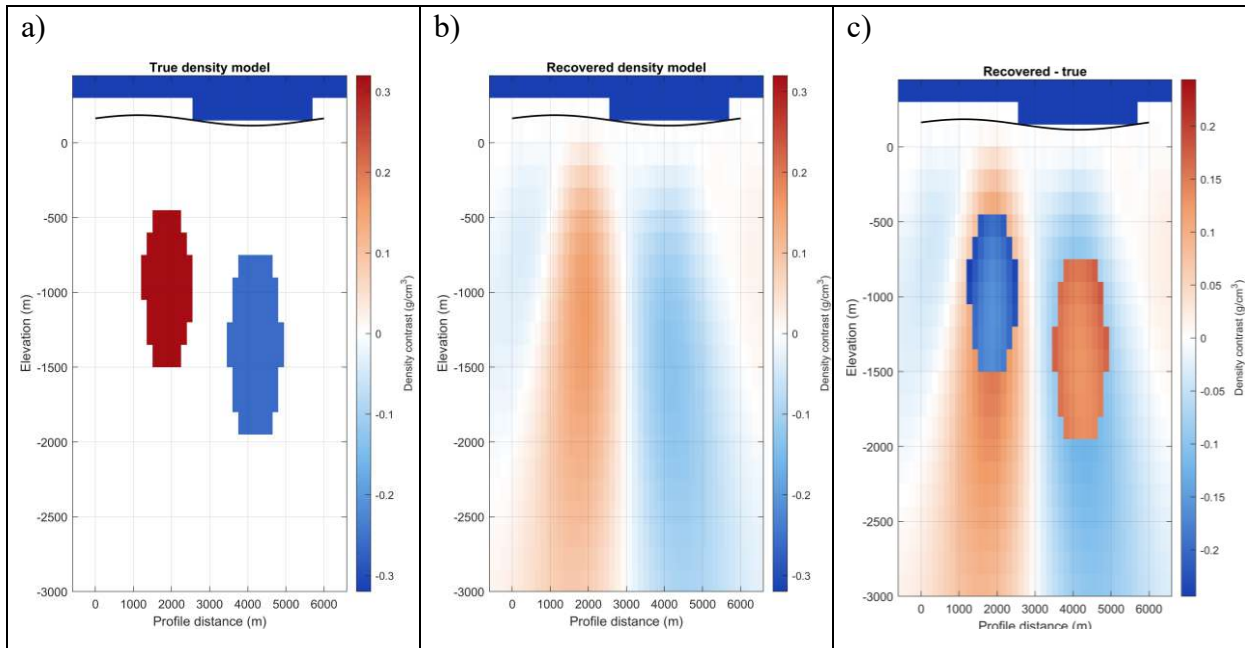


Figure 5. Synthetic Benchmark 2. (a) Prescribed model containing spatially separated positive and negative density bodies. (b) Recovered density distribution. (c) Recovered minus prescribed density difference. The inversion reproduces the principal polarity and lateral position of both targets but distributes their density contrasts over broader depth intervals.

3.4 Benchmark 3: Structurally complex density model

Benchmark 3 provides the most demanding baseline experiment. The topography contains a pronounced central edifice superimposed on lower-amplitude relief. Beneath this surface, the synthetic model contains a shallow low-density zone centered near 2.9 km and -0.35 km elevation, with approximate semi-axes of 1.15 and 0.52 km and a density contrast of -0.18 g/cm³. A deeper positive intrusion is centered near 3.3 km and -1.75 km, with semi-axes of approximately 0.70 and 0.85 km and a prescribed contrast of $+0.34$ g/cm³. These bodies are supplemented by a narrow inclined positive-density feeder with a contrast of $+0.22$ g/cm³.

The increased difficulty of recovering vertically and laterally complex density structure is illustrated in Figure 6. The shallow negative-density component is recognizable in the recovered model: its lateral position beneath the central topographic high and its negative polarity are reproduced. However, the negative anomaly extends far deeper than its prescribed geometry. More importantly, the deeper positive intrusion is not distinctly recovered at its true location. Instead, the central deep region remains dominated by the vertically extended negative solution, while lower-amplitude positive density is displaced principally toward the flanks, especially to the east. The narrow positive feeder is likewise not resolved as a discrete structure.

The difference model emphasizes that this benchmark produces a substantially larger model-space discrepancy than the simpler experiments. Positive differences within much of the prescribed shallow negative body indicate that the magnitude of the negative contrast is underestimated locally, whereas strong negative differences over the location of the prescribed deep positive intrusion indicate failure to recover its positive amplitude there. Positive differences outside the true intrusive body represent compensating positive density assigned elsewhere in the section. Thus, although the inversion captures part of the shallow polarity pattern, it does not uniquely separate the shallow low-density body, feeder, and deeper positive source under the adopted baseline constraints.

Nevertheless, the total gravity response remains closely matched: RMS is 0.04579 mGal, WRMS is 0.916, and the density-bound fraction is 0.0%. This result is particularly instructive because it demonstrates that a statistically satisfactory gravity fit can coexist with substantial disagreement in model space. The benchmark therefore illustrates the central limitation of interpreting gravity inversions solely through residual magnitude. It also suggests stronger trade-offs among depth distribution, regional trend, and density amplitude when several vertically superposed sources contribute overlapping long-wavelength signals.

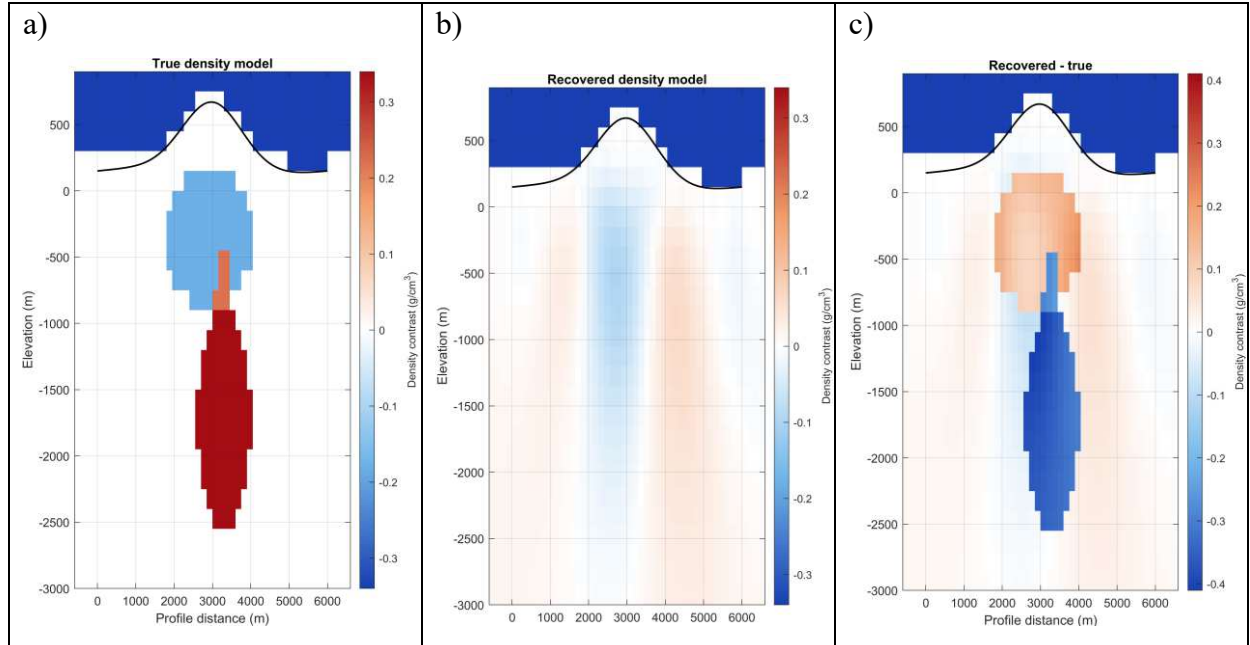


Figure 6. Synthetic Benchmark 3. (a) Prescribed structurally complex model containing a shallow negative-density zone, deeper positive intrusion, and narrow feeder beneath variable topography. (b) ALTYN recovered density model. (c) Recovered-minus-prescribed density difference. A linear regional gravity component is included in the synthetic observations.

3.5 Quantitative baseline recovery assessment

The verified data-space metrics for the three experiments are summarized in Table 2. All cases reach WRMS values below the adopted target of 1.05, and none requires density values at the prescribed model bounds. The lowest residual is obtained for Benchmark 1, whereas Benchmarks 2 and 3 produce slightly larger residuals consistent with their larger assigned data uncertainty. These differences should not be interpreted as a ranking of model-space accuracy. In particular, Benchmark 3 attains $WRMS = 0.916$, comparable with Benchmark 2, despite a substantially poorer reconstruction of the deeper prescribed density structure.

Table 2. Verified quantitative inversion diagnostics for the three synthetic benchmarks.

Metric	Benchmark 1	Benchmark 2	Benchmark 3
Gravity RMS (mGal)	0.02602	0.04624	0.04579
WRMS	0.867	0.925	0.916
Bound fraction (%)	0.0	0.0	0.0

The comparison therefore demonstrates progressively different levels of recovery. Benchmark 1 establishes that an isolated anomaly can be localized laterally and assigned the correct polarity, although depth smearing remains pronounced. Benchmark 2 shows that

positive and negative sources can be discriminated simultaneously, but their amplitudes and vertical extents remain regularization dependent and trade off with the regional component. Benchmark 3 provides the clearest demonstration of non-uniqueness: the shallow negative anomaly is identified, but the deeper positive intrusion and feeder are not independently reconstructed despite a satisfactory gravity fit.

4. L2 and IRLS Regularization Analysis

4.1 Comparative inversion design

Because gravity inversion is intrinsically non-unique, the spatial distribution of recovered density is controlled not only by the observations but also by the regularization imposed on the model. Two solutions may reproduce essentially the same gravity field while differing substantially in anomaly amplitude, smoothness, continuity, and depth extent (Li & Oldenburg, 1998; Portniaguine & Zhdanov, 1999). The L2 and IRLS inversions were therefore compared to evaluate the sensitivity of the ALTYN solution to the adopted model norm. The supplied runs use the same 171-point gravity profile, receiver-based topography, and uncertainty prescription consisting of a 0.05 mGal absolute floor combined with a 5% relative component. Their sensitivity and depth-weighting distributions are also identical, indicating that the data geometry, active model discretization, and fixed sensitivity treatment were held constant. Both inversions simultaneously estimate a constant regional nuisance component. Thus, the principal methodological difference between the two calculations is the regularization metric rather than the gravity data or forward geometry.

In the L2 calculation, the smallness and directional terms remain quadratic, with $P_s = P_x = P_z = 2$. The IRLS configuration retains quadratic smallness and horizontal smoothness, $P_s = P_x = 2$, but applies the nonquadratic exponent $P_z = 0.5$ to the vertical-gradient term. Consequently, the IRLS case should not be interpreted as a uniformly sparse or compact inversion. Its nonquadratic behavior acts specifically on vertical density gradients, encouraging a smaller number of comparatively sharp vertical transitions while horizontal smoothness and model smallness retain their L2 character. This distinction is important because focused inversion behavior depends on which component of the model objective is subjected to nonquadratic weighting (Farquharson, 2008; Vatankhah et al., 2018a).

Both calculations use the same regularization-cooling strategy. The diagnostic histories indicate an initial β of 500, a reduction factor of 0.8 per outer iteration, and a target WRMS of 1.0. The L2 solution reaches the discrepancy target after 36 outer iterations, whereas the IRLS calculation requires 39 iterations. Because IRLS weights are updated after each non-terminal

outer solution, the latter run undergoes repeated nonquadratic reweighting and can therefore be considered a developed IRLS solution rather than a calculation that terminated before the reweighting mechanism became active.

Table 3. Principal controlled parameters in the L2 and IRLS comparison.

Parameter	L2	IRLS
Number of gravity observations	171	171
Absolute uncertainty floor	0.05 mGal	0.05 mGal
Relative uncertainty	5%	5%
Topographic representation	Receiver elevations	Receiver elevations
Regional formulation	Constant	Constant
P_s	2.0	2.0
P_x	2.0	2.0
P_z	2.0	0.5
IRLS stabilizers	Not active	10^{-8}
Initial β	500	500
Beta reduction factor	0.8	0.8
Target WRMS	1.0	1.0
Outer iterations to target	36	39

The common field-model settings were a 100 m $\times$ 100 m X–Z discretization, a 5000 m model depth, 500 m left/right lateral padding, zero starting and reference density, and density bounds of -0.50 to $+0.55$ g/cm³. The regularization coefficients were $\alpha_s = 20$, $\alpha_x = 100$, and $\alpha_z = 1$. Depth weighting was enabled with exponent 2, whereas additional sensitivity weighting was disabled. The strike-length input was set to 0, which in ALTYN activates the automatic option of 10 times the profile length; for the 9231.7 m A–A' profile this corresponds to an effective finite strike length of approximately 92,317 m. These geometry, bound, weighting, and regional settings were held common between the L2 and IRLS calculations, so the comparison isolates the effect of the regularization norm described above.

4.2 L2-regularized solution

The L2 result is characterized by comparatively smooth density variations and gradual transitions between positive and negative regions. The recovered density distribution, data fit, and convergence behavior are summarized in Figure 7. The most prominent positive-density feature occurs between approximately 2 and 3 km profile distance beneath the highest part of

the topography. Its highest amplitudes are concentrated at shallow elevations, but a lower-amplitude positive continuation extends downward over a broad depth interval. A second substantial positive-density region occurs approximately between 5.8 and 7.0 km. Smaller positive features are also resolved in the western part of the profile.

Negative density contrasts occupy broad regions surrounding the positive anomalies, particularly beneath the central portion of the section and toward the eastern half of the profile. These negative variations are comparatively diffuse and generally decay in amplitude with depth. This smooth spatial distribution is consistent with the quadratic penalty imposed on both model amplitude and directional gradients: abrupt changes are penalized strongly, so the inversion tends to accommodate the gravity data by distributing density contrasts among neighboring cells rather than concentrating them into sharply bounded regions. The observed and predicted gravity responses agree closely over most of the profile. The L2 model yields an RMS residual of 0.5751 mGal, a WRMS of 0.972, and a final density-bound fraction of 1.9%.

The standardized-residual diagnostic gives $\chi^2 \approx 161$ for 171 observations, thereby satisfying the discrepancy criterion. The largest residuals are spatially localized near the high-amplitude gravity peak at approximately 2.6–3.0 km, while much of the remainder of the profile is reproduced with comparatively small residuals. The nonzero bound fraction indicates that a small subset of cells reaches an imposed density limit, but the solution as a whole is not dominated by bound-saturated cells.

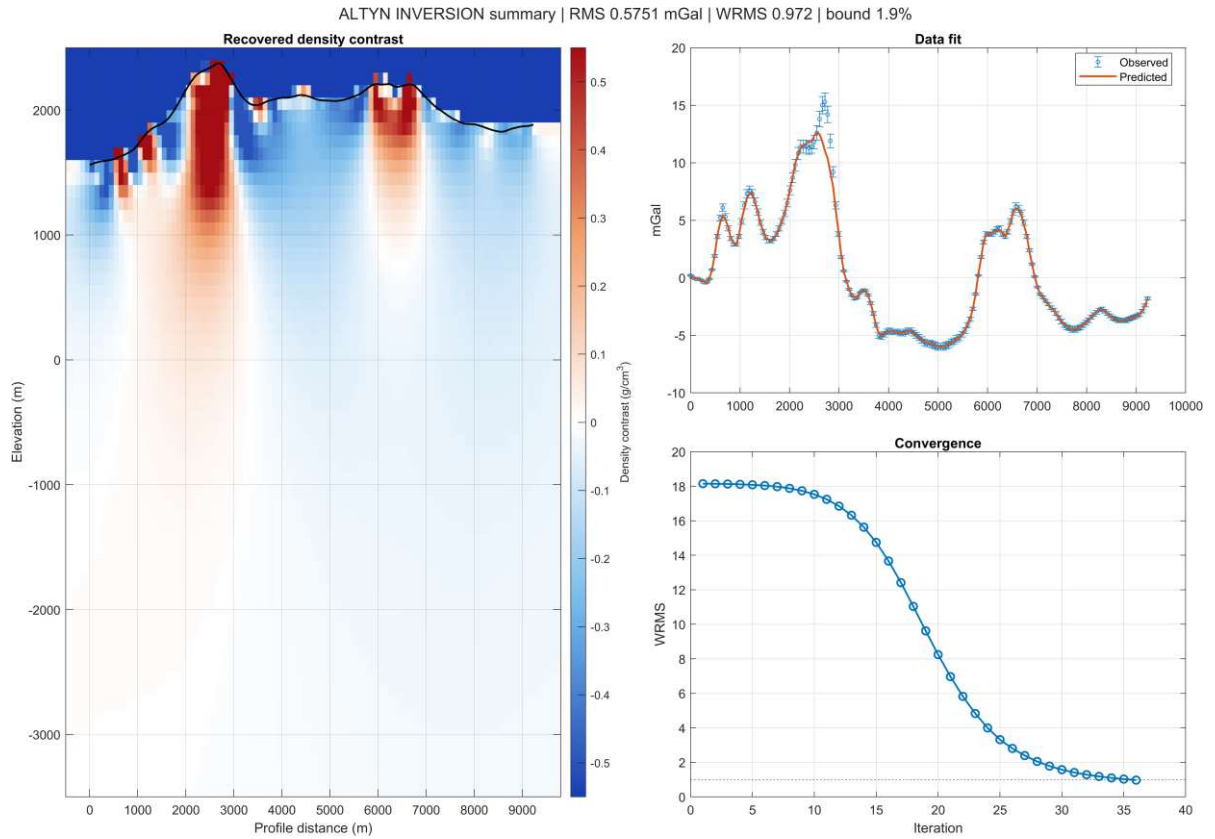


Figure 7. L2-regularized ALTYN inversion. Recovered density-contrast distribution, observed and predicted gravity responses, and WRMS convergence history for the quadratic L2 regularization. The recovered model exhibits comparatively smooth and spatially distributed density variations.

4.3 IRLS-regularized solution

The corresponding IRLS result is shown in Figure 8. The large-scale pattern remains recognizable: the strong positive-density feature beneath approximately 2–3 km persists, as does the positive structure around 6–7 km and the principal negative-density regions separating and surrounding them. The persistence of these major anomaly locations despite the change in regularization indicates that their broad lateral positions are comparatively stable with respect to this particular model-norm perturbation.

The detailed model character, however, differs substantially from the L2 solution. Positive anomalies in the IRLS model display sharper lateral and vertical transitions and more internally coherent high-amplitude cores. The principal positive feature around 2–3 km is more columnar, whereas the positive-density structure around 6–7 km is divided into more localized high-amplitude regions. Negative contrasts are also more vertically coherent and persist more strongly into the deeper model. This behavior is consistent with the actual IRLS configuration: because $P_z = 0.5$ is applied only to the vertical-gradient term, the method

preferentially permits a limited number of stronger changes in the vertical direction rather than enforcing quadratic smoothing at every depth transition. The result is therefore better described as more vertically segmented and structurally focused than as uniformly more compact.

The IRLS model produces an RMS residual of 0.5488 mGal, WRMS of 0.950, and density-bound fraction of 2.2%. The corresponding standardized-residual diagnostic gives $\chi^2 = 154$, again below the nominal target of 171. Thus, IRLS reduces the RMS by only about 4.6% relative to L2 and the WRMS by about 2.3%. These differences are modest because both solutions already reproduce the observations at approximately the level of the assumed measurement uncertainty. They therefore do not constitute evidence that the IRLS density model is geologically more accurate.

Another notable difference occurs in the decomposition between density response and the simultaneously fitted regional term. Both inversions employ a constant regional model, but the fitted regional level is visibly greater in the IRLS calculation. The regularization change therefore influences not only the detailed density distribution but also the trade-off between long-wavelength density structure and the nuisance regional component. This illustrates that nuisance parameters and model regularization cannot necessarily be interpreted independently when they are estimated simultaneously.

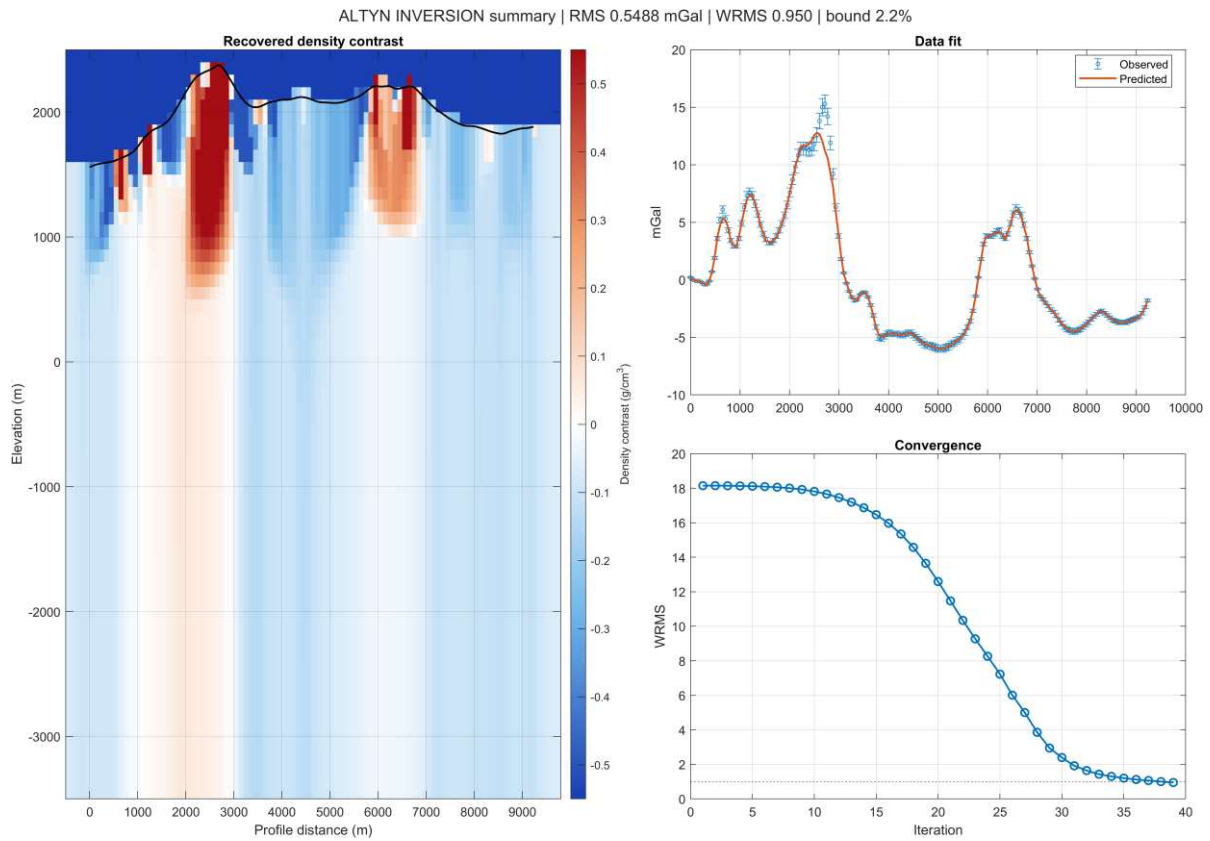


Figure 8. IRLS-regularized ALTYN inversion. Recovered density-contrast distribution, gravity-data fit, and convergence history obtained using iterative reweighting. The nonquadratic vertical-gradient regularization produces sharper vertical transitions and a more segmented density structure than the corresponding L2 solution.

4.4 Direct L2–IRLS model comparison

A same-scale comparison of the two recovered models is presented in Figure 9. Both panels should use identical horizontal and vertical limits, topographic representation, density limits, and diverging color scale centered at zero. Under these conditions, the major similarities and differences can be attributed to the inversion results rather than graphical normalization.

The strongest evidence of regularization stability is the persistence of the main positive-density zones beneath approximately 2–3 km and 6–7 km profile distance. Their exact amplitudes and geometries vary, but their lateral positions and density signs remain broadly unchanged. Smaller shallow positive anomalies toward the western part of the profile are likewise present in both solutions. These features are therefore comparatively insensitive to the transition from quadratic L2 regularization to the adopted IRLS formulation, although such persistence does not itself constitute independent geological validation.

The main differences lie in model texture and vertical distribution. L2 produces broader transitions between anomaly cores and surrounding cells, together with a more gradual decrease in density amplitude with depth. IRLS produces sharper vertical contrasts and stronger columnar continuity, especially beneath the main positive bodies and within the intervening negative-density zones. This behavior is consistent with the fact that only the vertical-gradient term is nonquadratic. The comparison should therefore not be reduced to a simple “smooth L2 versus compact IRLS” distinction. In this configuration, IRLS redistributes vertical gradients and changes the depth continuity of density anomalies while horizontal smoothing remains quadratic.

IRLS also places slightly more cells at their density limits—2.2% compared with 1.9% for L2. The difference is small, but it indicates that the more strongly structured IRLS solution makes somewhat greater use of the admissible density extremes. The density bounds therefore remain part of the model prior in both cases and must be considered when interpreting local high-amplitude regions.

The field comparison is based on density sections displayed on a common scale together with scalar inversion diagnostics. A formal cell-by-cell IRLS-minus-L2 difference map is not used because the native recovered-density vectors needed for a numerical subtraction are not available in the archived comparison package. Figure 9 therefore shows only density sections that can be compared directly; subtracting rendered image pixels would not be an acceptable substitute for a model-space difference.

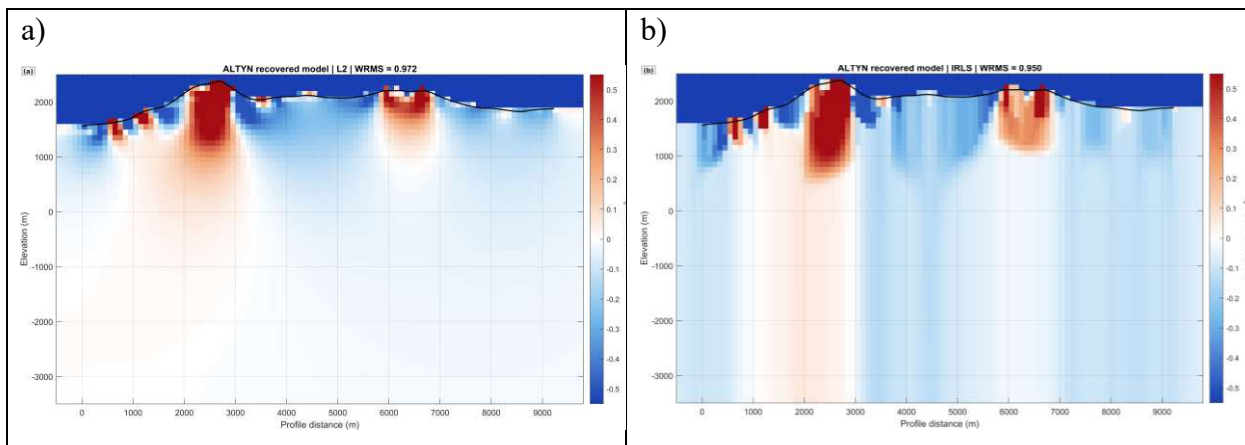


Figure 9. Direct comparison of L2 and IRLS density models. (a) L2 recovered density distribution. (b) IRLS recovered density distribution. Both panels use identical profile-distance, elevation, topography, and density-color limits to expose regularization-dependent differences directly.

4.5 Comparative data fit, convergence, and model characteristics

The principal diagnostics are summarized in Table 4. Both inversions terminate by satisfying the WRMS discrepancy criterion rather than merely reaching the final available iteration. L2 attains the target after approximately 36 outer iterations, corresponding to $\beta \approx 0.203$, whereas IRLS reaches it after approximately 39 iterations at $\beta \approx 0.104$. The later convergence of IRLS reflects the different evolution of the reweighted model objective rather than a failure to fit the observations.

Table 4. Comparative diagnostics for the L2 and IRLS inversions

Metric	L2	IRLS
RMS (mGal)	0.5751	0.5488
WRMS	0.972	0.950
χ^2	≈ 161	≈ 154
Bound fraction (%)	1.9	2.2
Outer iterations	36	39
Final β	≈ 0.203	≈ 0.104
Stopping condition	WRMS target	WRMS target

The convergence histories show a broadly similar progression in both cases: WRMS remains high while regularization dominates at large β , then decreases rapidly as β is cooled. The L2 curve begins its rapid decline somewhat earlier, whereas the IRLS curve remains at larger WRMS for more iterations before converging toward the same discrepancy level. This difference is consistent with repeated IRLS modification of the vertical-gradient metric during the outer sequence.

The constrained-solver diagnostics provide an additional distinction that should not be obscured by the satisfactory WRMS values. In the L2 calculation, the projected KKT residual decreases strongly during intermediate iterations but rises again during the final low- β iterations, accompanied by an increase in inner PCG effort. The IRLS calculation maintains smaller KKT residuals through most of its trajectory, although its final iteration also shows some increase. Thus, the data-discrepancy criterion and the constrained-optimality diagnostic do not convey identical information. Model selection should consider both data consistency and optimization diagnostics rather than minimizing the gravity residual alone.

5. Application to Real Gravity Data

5.1 Study area and gravity dataset

Following the synthetic validation and the regularization comparison, ALTYN was applied to the A-A' gravity profile from the Dieng dataset used in this study (Figure 10). The Dieng geothermal field is located within the Dieng Volcanic Complex, Central Java, Indonesia, a high-temperature volcanic geothermal system characterized by numerous Quaternary volcanic centers, ridges, cones, explosion craters, and active thermal manifestations. Previous geological and geothermal studies describe a heterogeneous volcanic succession dominated by basaltic to andesitic lava and pyroclastic units, locally intruded by denser igneous bodies and extensively modified by faulting and hydrothermal alteration (Harijoko et al., 2016; Shalihin et al., 2020, 2022). Earlier integrated interpretations of gravity, magnetotelluric, and well data also emphasized strong structural compartmentalization within Dieng (Boedihardi et al., 1991; Layman et al., 2002), while a more recent 3D gravity study inferred multiple fault-controlled structures and spatially separated low- to moderate-density reservoir candidates (Rosid & Sibarani, 2021). This geological complexity provides a plausible basis for substantial lateral and vertical density contrasts; however, gravity anomalies alone cannot uniquely distinguish among lithological, structural, porosity-related, and alteration-related causes.

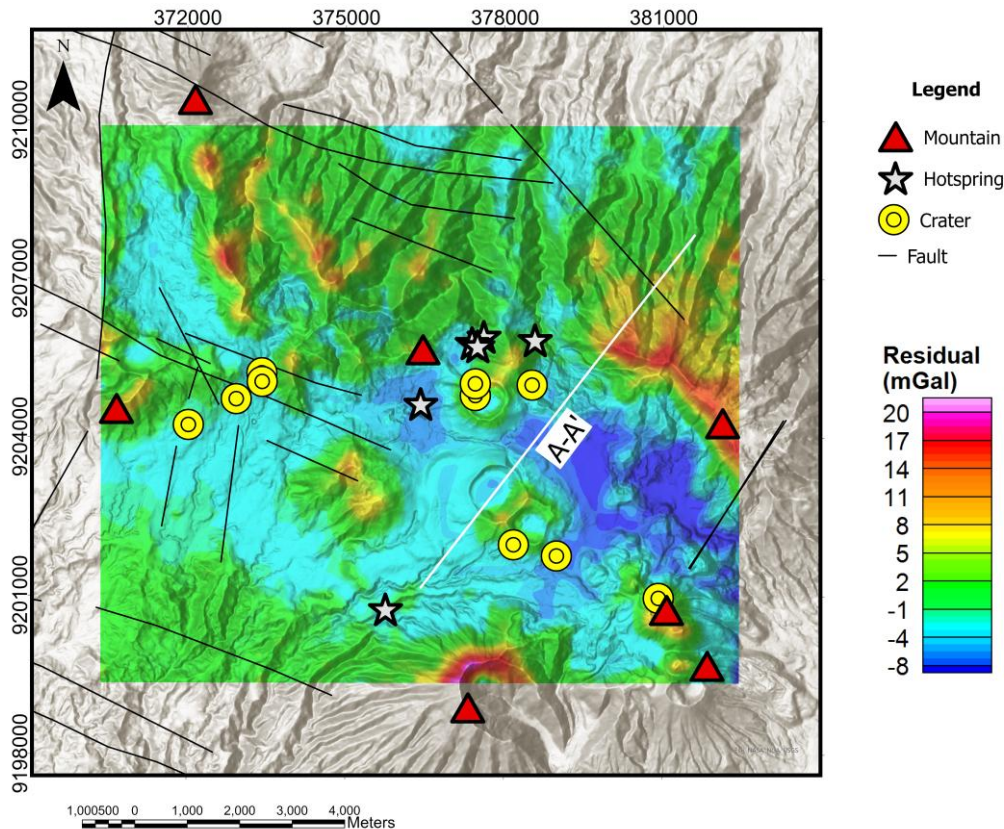


Figure 10. Study area and A-A' gravity profile in the Dieng geothermal field. The map provides the complete Bouguer anomaly context together with mapped faults, volcanic centers, craters, and thermal manifestations used for geological reference.

Structural complexity is a defining characteristic of the field. NW-SE-trending faults have been inferred from geological mapping and from the alignment of thermal manifestations, while crater-related structures occur around several volcanic centers. Subsurface investigations further indicate widespread hydrothermal alteration involving mineral replacement, dissolution, and secondary mineral deposition, with pyroclastic units locally showing more intense alteration than lava units (Shalihin et al., 2020). These observations support the use of gravity inversion to investigate first-order density architecture, but they do not establish a one-to-one correspondence between a recovered density anomaly and a particular geological unit or geothermal component.

The field inversion uses the A-A' traverse, identified by the ALTYN input interface as dataset A-A'. The input coordinates are Easting and Northing in meters, gravity is expressed in mGal, and receiver elevations define the topographic surface. The profile contains 171 observations over a total length of 9231.7 m. Inspection of the numerical arrays indicates a relatively uniform station spacing of approximately 54.3 m, receiver elevations of about 1560–2375 m, and gravity values ranging from -8.0 to 20.0 mGal.

5.2 Field-data preparation and inversion configuration

The input consists of projected Easting-Northing coordinates, gravity values, and elevations. Because no explicit standard-deviation column is provided, ALTYN constructs observation uncertainties using the adopted absolute-relative quadrature model with a minimum absolute floor of 0.05 mGal and a relative component of 5%. Receiver elevations are used to construct the topographic surface, and the internal calculations use meters, mGal, and the positive-downward gravity convention. The field mesh uses $dx = dz = 100$ m, a maximum model depth of 5000 m, and 500 m of lateral padding on both sides. The resulting mesh contains 103×60 cells (6180 total), of which 5682 are active subsurface cells. A constant regional nuisance term is estimated simultaneously so that a profile-wide datum contribution is not forced entirely into the recovered subsurface density distribution.

The same field profile, topography, uncertainty model, forward geometry, density bounds, and regional formulation are used for the L2 and IRLS calculations. Both inversions start from a zero-density model and use a zero reference model, density bounds of -0.50 to +0.55 g/cm³, $\alpha_s = 20$, $\alpha_x = 100$, $\alpha_z = 1$, depth weighting with exponent 2, no additional sensitivity

weighting, $\gamma = 1$, an initial β of 500, a β reduction factor of 0.8, and a target WRMS of 1.0. The L2 model uses $p_s = p_x = p_z = 2$, whereas the IRLS model retains $p_s = p_x = 2$ and applies $p_z = 0.5$ with $\epsilon_s = \epsilon_x = \epsilon_z = 10^{-8}$. The two density sections are therefore interpreted jointly, with Figures 7–9 providing the principal model-space comparison.

The projected PCG/KKT tolerance is 10^{-4} , with a maximum of 1000 PCG iterations for each reduced linear solve. These numerical controls complement the projected active-set strategy and are reported to improve reproducibility of the field calculations.

The strike-length setting requires particular care in interpretation. A GUI input of zero does not represent zero physical strike extent; instead, it activates the ALTYN automatic option, $L_y = 10 \times \text{profile length}$. For A-A', this corresponds to an effective finite strike length of approximately 92.317 km, roughly ten times the 9.23-km profile length. The field run should therefore be regarded as a long finite-strike approximation rather than as a cross-profile extent independently constrained by geology. Strike length remains a physical geometric prior that cannot be determined uniquely from a single gravity profile. Accordingly, the A-A' application demonstrates the integrated inversion workflow but does not establish the superiority or robustness of this particular strike-length choice; such a claim would require dedicated perturbation experiments that were not performed in the present study.

5.3 Gravity response and residual characteristics

The A-A' profile exhibits pronounced variations in both wavelength and amplitude. The western part of the line contains several local gravity highs, followed by the strongest positive anomaly between approximately 2 and 3 km. The signal then decreases rapidly into a broad negative interval across much of the central profile before rising again toward a secondary positive anomaly at approximately 6–7 km; the easternmost part is again dominated by negative values. Both the L2 and IRLS solutions reproduce these first-order variations well, as illustrated in Figures 7 and 8. The final RMS-WRMS pairs are 0.5751 mGal and 0.972 for L2, and 0.5488 mGal and 0.950 for IRLS.

Both WRMS values satisfy the adopted uncertainty-normalized discrepancy criterion. The modest reduction in residual level obtained by IRLS should therefore not be interpreted as evidence that its subsurface density model is uniquely superior. In a non-unique gravity inverse problem, closely comparable data fits can correspond to materially different admissible density distributions (Al-Chalabi, 1971; Li & Oldenburg, 1998; Portniaguine & Zhdanov, 1999).

The spatial residual pattern is more informative than the aggregate WRMS alone. The clearest concentration of misfit occurs at approximately 2.6–3.0 km profile distance, close to the steepest and highest-amplitude part of the observed anomaly. In the L2 result, several standardized residuals in this interval exceed $+4\sigma$, with the largest approaching $+5.5\sigma$; a negative standardized residual is also present near 3 km. Nevertheless, the total normalized misfit is $\chi^2 = 161$, below the nominal target of 171.

The IRLS solution displays a very similar spatial residual pattern and retains several standardized residuals above $+4\sigma$ in the same interval, while the global value decreases to $\chi^2 = 154$.

This residual concentration is important because WRMS close to one does not imply that every observation is fitted within one assigned standard deviation. Much of the central and eastern profile has comparatively small standardized residuals, whereas the localized cluster around the strongest anomaly contributes disproportionately to the global objective function. Such coherent residual structure can arise from inadequacy of the assumed finite-strike geometry, unresolved three-dimensional source variation, an overly simple regional component, limitations of the discretization, locally underestimated observational uncertainty, or some combination of these effects. The residual pattern by itself cannot discriminate among these possibilities, but it identifies the interval in which the adopted parameterization is least satisfactory.

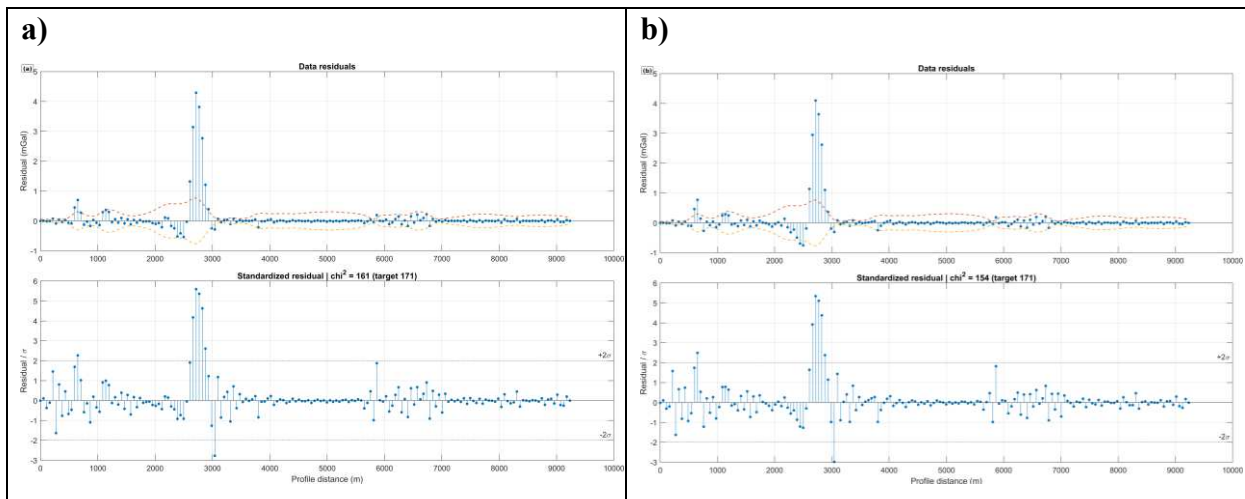


Figure 11. Residual and uncertainty-normalized fit diagnostics for the A-A' gravity inversion.

Residual and standardized-residual distributions for the L2 and IRLS solutions. Both inversions satisfy the global WRMS discrepancy criterion, but localized standardized residuals remain concentrated near the steep, high-amplitude gravity anomaly at approximately 2.6–3.0 km profile distance.

5.4 Recovered density structure

The L2 and IRLS density models in Figures 7–9 share several first-order features. Both recover a strong positive-density anomaly beneath approximately 2–3 km profile distance, directly beneath the highest-topography sector and the strongest positive gravity anomaly; in the study-area map this sector is associated with Mount Prau. A second positive-density domain occurs at approximately 6–7 km, while smaller shallow positive features appear farther west. Broad negative-density zones separate and flank these positive domains, particularly across the central part of the profile and east of the second positive anomaly.

These recurrent features are more defensible than detailed model boundaries because their polarity and approximate lateral position persist under both regularization schemes. The principal 2–3 km positive anomaly occurs at essentially the same lateral position in the L2 and IRLS models, and the broad negative domains remain recognizable despite differences in texture. Their persistence provides evidence of relative stability with respect to the specific L2-versus-IRLS perturbation tested here, although it does not constitute independent geological validation.

Geometry within these broad domains is considerably less stable. L2 distributes density changes smoothly among neighboring cells and produces gradual transitions with depth. The selected IRLS formulation produces sharper vertical changes and more columnar density patterns because the nonquadratic term acts on the vertical gradient while smallness and horizontal smoothness remain quadratic. This behavior is consistent with the expected influence of nonquadratic or focusing regularization, which can sharpen selected model components without resolving the intrinsic non-uniqueness of potential-field inversion (Farquharson, 2008; Portniaguine & Zhdanov, 1999; Vatankhah et al., 2018a, 2018b).

The sensitivity diagnostics reinforce this distinction. Kernel sensitivity is greatest in the shallow part of the section and decreases strongly with depth, even though depth weighting modifies the regularization metric to reduce excessive near-surface concentration. Depth weighting therefore changes the prior metric rather than adding independent information to the observations (Li & Oldenburg, 1998). Deep vertical continuations and density extrema should consequently be assigned lower spatial confidence than the broad polarity and lateral position of the principal anomalies.

5.5 Geophysical interpretation and limitations

Geologically, the recovered models should first be interpreted as maps of relative density contrast rather than as direct lithological cross-sections. The Dieng subsurface comprises basaltic to andesitic lava, pyroclastic and volcanoclastic sequences, younger volcanic products,

hydrothermally modified units, and locally intrusive rocks (Harijoko et al., 2016; Shalihin et al., 2020, 2022). Shalihin et al. (2020) reported dioritic material in wells at depths of roughly 2 km in parts of the field and documented widespread hydrothermal alteration, with pyroclastic rocks locally more intensely altered than lava. Earlier geothermal studies also emphasized the structurally compartmentalized character of Dieng and the close association of the resource with NW-trending volcanic and structural elements (Boedihardi et al., 1991; Layman et al., 2002).

On this basis, prominent positive-density zones recovered by ALTYN may be compatible with relatively dense and coherent volcanic or intrusive material, whereas negative-density contrasts may reflect lower-density pyroclastic or volcanoclastic sequences, increased fracture or pore volume, hydrothermal leaching or alteration, or combinations of these processes. These interpretations remain hypotheses. A positive anomaly cannot be identified automatically as a dioritic intrusion merely because diorite is known elsewhere in the field, and a negative anomaly cannot be equated uniquely with a geothermal reservoir, clay cap, or alteration zone. Comparable ambiguity is evident in other gravity inversions in which model focusing, bounds, and depth weighting strongly influence the recovered geometry (Boulanger & Chouteau, 2001; Ghalehnoee et al., 2017; Krahenbuhl & Li, 2006).

Previous work confirms substantial structural and geothermal complexity at Dieng. NW-SE faulting and crater-related structures have been linked to permeability and thermal manifestations (Shalihin et al., 2020, 2022), and previous 3D gravity analysis inferred multiple normal faults together with separated low- to moderate-density zones interpreted as reservoir candidates (Rosid & Sibarani, 2021). Earlier integrated gravity, magnetotelluric, and well interpretations likewise separated the field into distinct sectors and associated some gravity highs with relatively low-permeability barriers (Boedihardi et al., 1991). These studies provide important regional context, but without verified spatial registration of A-A' against their specific structures, wells, and 3D models, it would be inappropriate to identify an individual ALTYN density transition as a named fault, reservoir boundary, or lithological contact.

The field results therefore support a deliberately limited interpretation. The broad polarity and lateral position of the principal density contrasts are better constrained than their absolute density amplitude, detailed geometry, or maximum depth. The localized residual concentration at 2.6–3.0 km further suggests that the strongest anomaly is not represented perfectly by the adopted 2.5D parameterization despite the satisfactory global WRMS. Locally more complex three-dimensional source geometry is one plausible explanation,

although observational uncertainty, preprocessing effects, regional-field simplification, and discretization remain viable alternatives.

Several limitations remain fundamental. Gravity inversion is intrinsically non-unique; finite strike length imposes an assumed cross-profile geometry; a 2.5D formulation cannot represent arbitrary out-of-profile heterogeneity; sensitivity decreases with source depth; L2 and IRLS impose different spatial priors; density bounds restrict the admissible model family; and the simultaneously estimated regional term can trade off against long-wavelength density structure (Al-Chalabi, 1971; Boulanger & Chouteau, 2001; Li & Oldenburg, 1998). Out-of-profile three-dimensional geology is particularly relevant in a volcanic complex containing numerous eruption centers and structurally segmented geothermal sectors (Harijoko et al., 2016). The A-A' result should therefore be treated as a geophysically admissible density-contrast distribution consistent with the processed gravity profile and the stated uncertainty, finite-strike, bound, regional, and regularization assumptions, rather than as a unique reconstruction of the Dieng subsurface.

6. Discussion

6.1 Performance of the finite-strike inversion framework

The baseline synthetic tests and the field application demonstrate that ALTYN can execute a consistent profile-gravity inversion workflow while accounting explicitly for topography, observation uncertainty, model regularization, density bounds, a regional nuisance component, and prescribed finite strike extent. The three synthetic benchmarks become progressively more demanding. Benchmark 1 localizes the isolated positive source laterally but smears its depth extent; Benchmark 2 recovers separated positive and negative polarities while retaining amplitude and depth ambiguity; and Benchmark 3 shows that an uncertainty-consistent gravity fit can coexist with failure to reconstruct the deeper positive source at its prescribed location. The results therefore support first-order recovery of anomaly polarity and approximate lateral position more strongly than exact body boundaries, density amplitude, or depth geometry.

This progression is consistent with the fundamental behavior of gravity inversion rather than with a simple transition from successful to failed optimization. Gravity observations are integrated responses to subsurface mass contrast, and the inverse problem admits many density distributions capable of generating nearly indistinguishable surface fields (Al-Chalabi, 1971; Boulanger & Chouteau, 2001; Li & Oldenburg, 1998). Sensitivity also decays strongly with depth, so increasingly complex combinations of shallow and deep sources enlarge the range of model-space trade-offs available to the inversion (Li & Oldenburg, 1998;

Portniaguine & Zhdanov, 1999). The synthetic tests are therefore an important internal control: ALTYN can reproduce the dominant characteristics of relatively simple sources, but it does not artificially remove the intrinsic non-uniqueness of the gravity inverse problem.

The finite-strike formulation occupies an intermediate geometric position between conventional 2D infinite-strike modeling and a fully variable 3D inversion (Nagy et al., 2000). For a given X–Z density distribution, changing the strike extent changes the predicted gravitational attraction and can be compensated by changes in density amplitude, source volume, or depth. Finite-strike flexibility therefore introduces an additional geometric prior rather than an independently resolved parameter. This is particularly important for the present field case: the synthetic benchmarks use an explicit 4-km strike length, whereas A-A' uses the automatic long-strike value of approximately 92.3 km. The field result should consequently be interpreted as a demonstration of the integrated profile workflow, not as validation of a geologically determined cross-profile strike extent.

ALTYN does not define a fundamentally new class of gravity inverse problem. Its contribution is the computational integration of established ideas within a single constrained profile formulation. Minimum-structure inversion and depth weighting are well-established strategies for stabilizing cell-based gravity inversion (Li & Oldenburg, 1998); compact and focusing stabilizers have long been used to favor spatially concentrated anomalies (Guillen & Menichetti, 1984; Last & Kubik, 1983; Portniaguine & Zhdanov, 1999); IRLS and related nonquadratic approaches permit piecewise or focused models (Farquharson, 2008; Vatankhah et al., 2017, 2018b); and physical bounds or discrete constraints can substantially reduce the admissible model space (Boulianger & Chouteau, 2001; Ghalehnoee et al., 2017; Krahenbuhl & Li, 2006). Within that established methodological landscape, ALTYN combines topography-consistent active-cell geometry, finite-prism physics, uncertainty-normalized fitting, optional simultaneous regional estimation, explicit box constraints, directional L2/IRLS regularization, and a projected Newton/active-set PCG strategy. Its principal scientific value is therefore transparency: the assumptions that select one data-compatible model can be stated, compared, and diagnosed without changing the underlying forward-data convention.

The A-A' application shows that the same numerical framework can be transferred from controlled synthetic experiments to a real profile with substantial topographic relief. Both regularization choices reproduce the principal field-gravity variations at approximately the level expected from the adopted uncertainty model. This consistency supports the numerical applicability of the forward and constrained-inverse formulation, but it does not establish

uniqueness or geological correctness. That distinction is especially important in Dieng, where previous studies document multiple volcanic centers, fault-controlled geothermal sectors, intrusive activity, and spatially variable hydrothermal alteration (Boedihardi et al., 1991; Harijoko et al., 2016; Layman et al., 2002; Shalihin et al., 2020, 2022).

6.2 Data-space fit versus model-space resolution

One of the clearest outcomes of this study is the distinction between satisfactory data-space fit and reliable model-space recovery. This distinction appears in both the synthetic and field calculations and is most obvious in Benchmark 3. The synthetic gravity response for that experiment is reproduced with WRMS close to the adopted discrepancy level, yet the deeper prescribed positive-density structure is not distinctly reconstructed at its correct position. Instead, the recovered model redistributes density over a broader volume while retaining a gravity response compatible with the synthetic observations. A small residual can therefore coexist with substantial source-geometry error.

This behavior is a direct consequence of non-uniqueness. Data-space quantities such as RMS, WRMS, standardized residuals, and χ^2 measure consistency between predicted and observed gravity, but they do not determine whether the corresponding density distribution is close to the true source (Al-Chalabi, 1971; Li & Oldenburg, 1998; Uieda & Barbosa, 2016). In synthetic tests, model-space fidelity can be quantified because the prescribed density distribution is known; recovered-minus-true differences, model RMSE, relative error, and correlation therefore provide information unavailable for field data. For real observations there is no equivalent true model, and reliability must instead be evaluated indirectly through regularization comparisons, sensitivity diagnostics, geological or petrophysical constraints, and consistency with independent observations. Stochastic and population-based gravity inversions make the same broader point from another direction: uncertainty in geometry and density should be explored rather than inferred from a single best-fitting model (Pallero et al., 2015; Song et al., 2021).

The interpretation of WRMS is consequently critical. A value near unity indicates that the residual magnitude is broadly compatible with the assigned observational standard deviations; it does not imply that density amplitude is correct, that source depth has been uniquely resolved, or that the solution reproduces the actual geology. The field results illustrate this directly: L2 and IRLS both obtain acceptable WRMS values but produce noticeably different density textures and vertical continuities. Likewise, all three baseline synthetic benchmarks satisfy the discrepancy criterion even though the discrepancy between prescribed and

recovered geometries increases with structural complexity. The discrepancy principle is therefore a data-consistency criterion, not a model-validation criterion.

Depth resolution is a major component of this separation between data space and model space. Near-surface cells exert a stronger gravitational influence at the receivers than equivalent cells at depth, whereas deeper sources generally produce smoother, longer-wavelength responses. The inversion can therefore trade source depth against density amplitude and volume. The recovered synthetic models display exactly this behavior: compact prescribed bodies become broadened or vertically extended, especially as structural complexity increases, and the deeper positive component of Benchmark 3 is much less distinguishable than the shallower negative component. Comparable depth-amplitude trade-offs are central to classical minimum-structure and focused gravity inversion (Li & Oldenburg, 1998; Portniaguine & Zhdanov, 1999).

Depth weighting can reduce the tendency of minimum-structure inversion to concentrate density immediately beneath the observation surface, but it should not be interpreted as restoring missing resolution. It modifies the regularization prior to counteract spatial variation in sensitivity; it does not add independent information to the data (Ghalehnoee et al., 2017; Li & Oldenburg, 1998). Deep density structures recovered by ALTYN, as in other potential-field inversions, should therefore be interpreted with lower spatial confidence than shallower lateral anomaly positions.

The same distinction applies to numerical convergence. Satisfying the WRMS target establishes a data-discrepancy condition, whereas the projected KKT residual assesses how adequately the box-constrained numerical subproblem has been solved. These diagnostics answer different questions. A statistically acceptable model can still show a less completely resolved final constrained subproblem, particularly at low regularization strength. Numerical optimality diagnostics therefore complement rather than replace statistical misfit assessment.

6.3 Influence of regularization and prior assumptions

The L2-IRLS comparison demonstrates how different regularization priors select different members of the family of gravity-compatible density models. In the L2 formulation, density amplitudes and directional gradients are penalized quadratically, favoring broad and smoothly varying structure. This behavior is consistent with minimum-structure approaches that stabilize ill-posed potential-field inversion by suppressing model complexity, but can smear abrupt geological boundaries (Li & Oldenburg, 1998; Portniaguine & Zhdanov, 1999). The A-L2 model accordingly exhibits gradual transitions between positive and negative density regions and distributes anomaly amplitude over relatively large volumes.

The adopted IRLS configuration is more specific than a generic smooth-versus-focused comparison. Smallness and horizontal smoothness remain quadratic, whereas the vertical-gradient term uses $p_z < 2$; the nonquadratic weighting therefore acts preferentially on vertical variation. The resulting model contains stronger vertical segmentation, more pronounced columnar continuity, and sharper transitions among some density domains. This response is consistent with the behavior of IRLS, Lp, total-variation, and focusing approaches reported in previous gravity studies, in which changing the stabilizing functional can sharpen or compact selected features (Farquharson, 2008; Peng & Liu, 2021; Vatankhah et al., 2017, 2018b, 2018a). It should not, however, be interpreted as independent evidence that the IRLS geometry is geologically more accurate. The difference in field-data fit is modest, and the slightly lower IRLS RMS/WRMS does not establish improved geological fidelity.

Conversely, several first-order features persist under both regularization schemes. The principal positive-density domains occur at nearly the same lateral positions, and the broad negative-density regions remain recognizable. Their persistence indicates relative stability with respect to the specific regularization perturbation that was actually tested. More interpretive confidence can therefore be assigned to the existence and approximate lateral location of these common anomalies than to small features or detailed boundaries that change between L2 and IRLS. Nevertheless, stability under two regularizers is not equivalent to independent geological validation, a caution also emphasized by studies comparing smooth, compact, binary, and focused potential-field formulations (Ghalehnoee et al., 2017; Krahenbuhl & Li, 2006; Last & Kubik, 1983).

Density bounds constitute another prior assumption. Explicit limits are useful because they prevent the inversion from fitting the data through unrealistically large density contrasts, but they also restrict the admissible model space and can alter the trade-off among anomaly amplitude, volume, and depth (Boulanger & Chouteau, 2001; Krahenbuhl & Li, 2006). The small bound-active fraction in the baseline synthetic experiments indicates that those solutions are not dominated by widespread saturation at the imposed limits, and the field inversions likewise show limited bound activity. Even so, the bounds remain part of the prior model definition even when only a small number of cells lie exactly at their limiting values.

The regional component introduces a further trade-off. Simultaneously estimating a constant or linear nuisance term with the density model reduces the tendency for a datum offset or broad trend to be mapped entirely into subsurface density. Related simultaneous regional treatment has been used in other constrained gravity-inversion frameworks, including the GROWTH family (Camacho et al., 2002, 2011, 2021). The decomposition is not unique,

however, because broad or deep density structure can generate long-wavelength gravity responses that overlap a low-order regional term. The estimated regional coefficient should therefore be treated as part of the inversion parameterization rather than as an independently resolved geological field.

Finite strike length and model discretization introduce similar prior dependence. An inaccurate strike extent can be compensated by density amplitude, source depth, or volume, while a finer mesh adds degrees of freedom without automatically adding independent information to the data. Fine discretization can represent sharper or smaller anomalies, but features that appear only at the finest scale may reflect parameterization rather than true resolving power; conversely, an excessively coarse mesh can suppress meaningful structural variation. These issues are common to cell-based gravity inversion and become particularly important when geological complexity approaches or exceeds the dimensionality assumed by the forward model (Cady, 1980; Li & Oldenburg, 1998; Nagy et al., 2000).

Among the principal assumptions examined here, dependence on regularization is tested directly through the completed L2-versus-IRLS comparison. Quantitative invariance to observational noise, finite strike length, and mesh spacing has not been demonstrated by repeated runs and is therefore not claimed. Similar WRMS values obtained under different assumptions do not by themselves establish model-space robustness. The present evidence supports greater confidence in first-order anomaly polarity and lateral position than in density extrema, exact boundaries, or deep continuation.

6.4 Implications for interpreting the Dieng gravity profile

The A-A' field inversion reinforces the conclusions drawn from the synthetic experiments: the most defensible result is the first-order density architecture rather than the detailed geometry of individual cells. L2 and IRLS both identify the main positive-density zones at similar profile positions and reproduce broad negative-density domains between and around them, whereas local amplitude, vertical continuity, and detailed boundaries vary more strongly. This indicates that the presence and approximate lateral position of the principal density contrasts are more stable than their exact subsurface extent.

The geological setting of Dieng provides several plausible mechanisms for these density variations. Positive relative contrasts may be associated with coherent lava packages, denser volcanic units, intrusive material, or locally shallower dense basement, whereas negative contrasts may reflect volcanoclastic or pyroclastic sequences, stronger fracturing, higher porosity, hydrothermal alteration or leaching, or combinations of these effects. The regional literature supports such heterogeneity: Harijoko et al. (2016) documented a complex multi-

stage volcanic and magmatic evolution; Shalihin et al. (2022, 2020) described extensive subsurface hydrothermal alteration, varied volcanic lithologies, and structurally controlled geothermal circulation; and Layman et al. (2002) emphasized the association of the geothermal resource with a NW-trending volcanic-structural zone. These studies make density heterogeneity geologically plausible, but they do not provide a unique conversion from density contrast to lithology.

Previous geophysical interpretations also provide an important comparison. Boedihardi et al. (1991) integrated gravity, magnetotelluric, and well information and interpreted the Dieng field as several geothermal sectors separated by comparatively low-permeability zones, some of which corresponded to gravity highs and conductivity lows. Rosid and Sibarani (2021), using derivative analysis and 3D gravity inversion, identified multiple predominantly normal faults and two low- to moderate-density graben-like zones interpreted as separate reservoir candidates, with comparison to geological and magnetotelluric information. The present ALTYN result is qualitatively consistent with the broader conclusion that Dieng is strongly density-heterogeneous and structurally segmented. However, because A-A' is a single 2.5D profile and is not registered here in a way that permits direct cell-by-cell comparison with those earlier 3D interpretations, the agreement should be regarded as regional consistency rather than direct validation of any named reservoir, fault, or lithological body.

For the same reason, the field density section should not be translated directly into a geological cross-section without independent constraints. A positive density anomaly should not automatically be labeled as an intrusion, and a negative-density region cannot uniquely be assigned to a geothermal reservoir, clay cap, fractured zone, or hydrothermally altered unit. Such interpretations would become substantially stronger if supported by borehole lithology and density measurements, mapped structures, magnetotelluric resistivity, seismic velocity, magnetic data, or other independent observations. This requirement is consistent with the broader gravity-inversion literature, which emphasizes that geological plausibility and external constraints are essential for discriminating among non-unique density models (Boulanger & Chouteau, 2001; Pallero et al., 2015).

The residual distribution provides an additional constraint on interpretation. Although the global WRMS is statistically acceptable, both field inversions retain localized standardized residuals around the strongest gravity-gradient zone at 2.6–3.0 km. Coherent misfit coincident with a steep, high-amplitude anomaly indicates that this interval may be inadequately represented by the adopted parameterization. Possible causes include local three-dimensional source geometry, spatially variable strike extent, unresolved small-scale sources, a simplified

regional model, topographic or discretization effects, or underestimated local observational uncertainty. The residual pattern cannot distinguish among these causes, but it marks the part of the profile where geological interpretation should be most cautious.

Taken together, the real-data application supports a hierarchical interpretation. Broad anomaly polarity and lateral position receive the highest confidence because they persist under both regularization schemes and are required to reproduce the principal gravity variations. Exact density extrema, narrow secondary anomalies, vertical boundaries, and deep continuations receive progressively lower confidence because they are more strongly influenced by regularization, depth sensitivity, and geometric assumptions. This hierarchy is consistent with both the synthetic recovery results presented here and the longstanding recognition that potential-field inversions resolve some aspects of source location more reliably than unique three-dimensional source geometry (Al-Chalabi, 1971; Li & Oldenburg, 1998; Portniaguine & Zhdanov, 1999).

7. Conclusions

ALTYN provides a 2D/2.5D finite-strike gravity-inversion framework that integrates finite-prism forward modeling, topography-consistent discretization, uncertainty-weighted data fitting, explicit density bounds, optional regional nuisance estimation, L2/IRLS regularization, and constrained iterative optimization. The baseline synthetic tests show that the framework can recover the principal polarity and approximate lateral position of relatively simple density anomalies, whereas increasingly complex and deeper structures are less well resolved. Satisfactory RMS or WRMS values do not guarantee accurate recovery of source geometry; Benchmark 3 demonstrates that an acceptable data-space fit can coexist with substantial model-space error. The L2-IRLS comparison likewise shows that both schemes can reproduce the observations at approximately the adopted uncertainty level while selecting different admissible density distributions. L2 favors smoother and more distributed structure, whereas the adopted IRLS configuration produces stronger vertical segmentation and sharper density transitions. Large-scale anomaly locations are therefore more defensible than detailed boundaries, density extrema, and deep continuations.

Application to the real A-A' gravity profile demonstrates that ALTYN can accommodate variable topography and recover coherent first-order positive- and negative-density domains while maintaining a fit consistent with the assumed uncertainty of the processed gravity anomaly. The field calculation uses an automatic long finite-strike prior of approximately 92.3 km and therefore should not be interpreted as validation of a uniquely constrained geological strike extent. Localized residual concentration and the differences between the L2

and IRLS solutions further show that the field model is not a unique geological cross-section. Quantitative robustness to observational-noise level, strike-length choice, and mesh resolution has not been established through repeated perturbation experiments and is not claimed here. ALTYN is therefore best regarded as a transparent framework for constructing and comparing geophysically admissible density models under explicitly stated forward, uncertainty, bound, regional, and regularization assumptions.

Supplementary Material

Supplementary materials are available online at: https://bit.ly/Altyn_Inversion.

Author Contribution Statements

Ahmad Ali Muckharom: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Visualization, Writing – original draft, Writing – review & editing.

Conflict of Interest Statement

The author declares that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

Acknowledgments

The author would like to express sincere gratitude to Universitas Diponegoro for the institutional support and academic environment that contributed to the completion of this research. In addition, Ahmad Ali Muckharom wants to thank, in particular, the patience, care and support from Seftyana Indica Berlyansyah over the passed years.

Data Availability

The data and supplementary materials supporting the findings of this study, including the resources associated with the ALTYN Inversion framework, are available at https://bit.ly/Altyn_Inversion. Additional information related to the data and implementation may be obtained from the corresponding author upon reasonable request.

References

Al-Chalabi, M. (1971). Some studies relating to nonuniqueness in gravity and magnetic inverse problems. *Geophysics*, 36(5), 835–855. <https://doi.org/10.1190/1.1440219>

- 1189 Barbosa, V. C. F., & Silva, J. B. C. (1994). Generalized compact gravity inversion.
1190 *Geophysics*, 59(1), 57–68. <https://doi.org/10.1190/1.1443534>
- 1191 Bertete-Aguirre, H., Cherkaev, E., & Oristaglio, M. (2002). Non-smooth gravity problem with
1192 total variation penalization functional. *Geophysical Journal International*, 149(2), 499–
1193 507. <https://doi.org/10.1046/j.1365-246X.2002.01664.x>
- 1194 Boedihardi, M., Suranto, & Sudarman, S. (1991). Evaluation of the Dieng Geothermal Field,
1195 review of development strategy. *20th Annual Convention Proceedings*, 347–361.
1196 <https://doi.org/10.29118/ipa.1483.347.361>
- 1197 Boulanger, O., & Chouteau, M. (2001). Constraints in 3D gravity inversion. *Geophysical*
1198 *Prospecting*, 49(2), 265–280. <https://doi.org/10.1046/j.1365-2478.2001.00254.x>
- 1199 Cady, J. W. (1980). Calculation of gravity and magnetic anomalies of finite-length right
1200 polygonal prisms. *Geophysics*, 45(10), 1507–1512. <https://doi.org/10.1190/1.1441045>
- 1201 Camacho, A. G., Fernández, J., & Gottsmann, J. (2011). The 3-D gravity inversion package
1202 GROWTH2.0 and its application to Tenerife Island, Spain. *Computers and Geosciences*,
1203 37(4), 621–633. <https://doi.org/10.1016/j.cageo.2010.12.003>
- 1204 Camacho, A. G., Montesinos, F. G., & Vieira, R. (2002). A 3-D gravity inversion tool based
1205 on exploration of model possibilities q. *Computers & Geosciences*, 28(2), 191–204.
1206 [https://doi.org/10.1016/S0098-3004\(01\)00039-5](https://doi.org/10.1016/S0098-3004(01)00039-5)
- 1207 Camacho, A. G., Prieto, J. F., Aparicio, A., Ancochea, E., & Fernández, J. (2021). Upgraded
1208 GROWTH 3.0 software for structural gravity inversion and application to El Hierro
1209 (Canary Islands). *Computers and Geosciences*, 150.
1210 <https://doi.org/10.1016/j.cageo.2021.104720>
- 1211 Farquharson, C. G. (2008). Constructing piecewise-constant models in multidimensional
1212 minimum-structure inversions. *Geophysics*, 73(1), K1–K9.
1213 <https://doi.org/10.1190/1.2816650>
- 1214 Ghalehnoee, M. H., Ansari, A., & Ghorbani, A. (2017). Improving compact gravity inversion
1215 using new weighting functions. *Geophysical Journal International*, 208(1), 546–560.
1216 <https://doi.org/10.1093/gji/ggw413>
- 1217 Guillen, A., & Menichetti, V. (1984). Gravity and magnetic inversion with minimization of a
1218 specific functional. *Geophysics*, 49(8), 1354–1360. <https://doi.org/10.1190/1.1441761>

- 1219 Harijoko, A., Uruma, R., Wibowo, H. E., Setijadji, L. D., Imai, A., Yonezu, K., & Watanabe,
1220 K. (2016). Geochronology and magmatic evolution of the Dieng Volcanic Complex,
1221 Central Java, Indonesia and their relationships to geothermal resources. *Journal of*
1222 *Volcanology and Geothermal Research*, 310, 209–224.
1223 <https://doi.org/10.1016/j.jvolgeores.2015.12.010>
- 1224 Krahenbuhl, R. A., & Li, Y. (2006). Inversion of gravity data using a binary formulation.
1225 *Geophysical Journal International*, 167(2), 543–556. [https://doi.org/10.1111/j.1365-](https://doi.org/10.1111/j.1365-246X.2006.03179.x)
1226 [246X.2006.03179.x](https://doi.org/10.1111/j.1365-246X.2006.03179.x)
- 1227 Last, B. J., & Kubik, K. (1983). Compact gravity inversion. *Geophysics*, 48(6), 713–721.
1228 <https://doi.org/10.1190/1.1441501>
- 1229 Layman, E. B., Agus, L., & Warsa, S. (2002). The Dieng Geothermal Resource, Central Java,
1230 Indonesia. *Geothermal Resources Council Transactions*, 26, 60.
- 1231 Li, Y., & Oldenburg, D. W. (1998). 3-D inversion of gravity data. *Geophysics*, 63(1), 109–
1232 119. <https://doi.org/10.1190/1.1444302>
- 1233 Liu, S., Jin, S., & Chen, Q. (2022). Three-dimensional gravity inversion based on
1234 optimization processing from edge detection. *Geodesy and Geodynamics*, 13(5), 503–
1235 524. <https://doi.org/10.1016/j.geog.2022.03.005>
- 1236 Nagy, D., Papp, G., & Benedek, J. (2000). The gravitational potential and its derivatives for
1237 the prism. *Journal of Geodesy*, 74(7–8), 552–560.
1238 <https://doi.org/10.1007/s001900000116>
- 1239 Pallero, J. L. G., Fernández-Martínez, J. L., Bonvalot, S., & Fudym, O. (2015). Gravity
1240 inversion and uncertainty assessment of basement relief via Particle Swarm
1241 Optimization. *Journal of Applied Geophysics*, 116, 180–191.
1242 <https://doi.org/10.1016/j.jappgeo.2015.03.008>
- 1243 Peng, G., & Liu, Z. (2021). 3D inversion of gravity data using reformulated Lp-norm model
1244 regularization. *Journal of Applied Geophysics*, 191.
1245 <https://doi.org/10.1016/j.jappgeo.2021.104378>
- 1246 Portniaguine, O., & Zhdanov, M. S. (1999). Focusing geophysical inversion images.
1247 *Geophysics*, 64(3). <https://doi.org/10.1190/1.1444596>

- Rosid, M. S., & Sibarani, C. (2021). Reservoir identification at Dieng geothermal field using 3D inversion modeling of gravity data. *Journal of Physics: Conference Series*, 1816(1). <https://doi.org/10.1088/1742-6596/1816/1/012083>
- Shalihin, Darmawan, D., Abi Tiwana, R., & Chandra, V. R. (2022). The Geology and Geothermal System of the Dieng Geothermal Field, Central Java, Indonesia. *PROCEEDINGS, 47th Workshop on Geothermal Reservoir Engineering*.
- Shalihin, Utami, P., & Nurpratama, M. I. (2020). The Subsurface Geology and Hydrothermal Alteration of the Dieng Geothermal Field, Central Java: A Progress Report. *IOP Conference Series: Earth and Environmental Science*, 417(1). <https://doi.org/10.1088/1755-1315/417/1/012010>
- Song, T., Hu, X., Du, W., Cheng, L., Xiao, T., & Li, Q. (2021). Lp-norm inversion of gravity data using adaptive differential evolution. *Applied Sciences (Switzerland)*, 11(14). <https://doi.org/10.3390/app11146485>
- Uieda, L., & Barbosa, V. C. F. (2016). Fast non-linear gravity inversion in spherical coordinates with application to the South American Moho. *Geophysical Journal International*, 208(1), 162–176. <https://doi.org/10.1093/gji/ggw390>
- Vatankhah, S., Renaut, R. A., & Ardestani, V. E. (2017). 3-D Projected L1 inversion of gravity data using truncated unbiased predictive risk estimator for regularization parameter estimation. *Geophysical Journal International*, 210(3), 1872–1887. <https://doi.org/10.1093/gji/ggx274>
- Vatankhah, S., Renaut, R. A., & Ardestani, V. E. (2018a). A fast algorithm for regularized focused 3D inversion of gravity data using randomized singular-value decomposition. *Geophysics*, 83(4), G25–G34. <https://doi.org/10.1190/geo2017-0386.1>
- Vatankhah, S., Renaut, R. A., & Ardestani, V. E. (2018b). Total variation regularization of the 3-D gravity inverse problem using a randomized generalized singular value decomposition. *Geophysical Journal International*, 213(1), 695–705. <https://doi.org/10.1093/gji/ggy014>