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A Mathematical Modeling Method of the EHT Synthesized Beam for Radio Synthesis Imaging

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Abstract

The synthesized beam, or point spread function (PSF), is a critical factor influencing the quality of black hole imaging with the Event Horizon Telescope (EHT); its continuous functional representation is essential for black hole image restoration and error propagation analysis. The synthesized beam often exhibits radial symmetry; to construct a high-precision mathematical representation of the EHT synthesized beam, this paper proposes a high-precision modeling method based on Linear radial basis functions (LiRBM: Linear RBF-based Beam Modeling). This method constructs a model using a set of optimal basis functions to achieve a high-precision representation of the synthesized beam, where the symmetry of the radial basis functions enables an effective representation of the beam's symmetric morphological structure. In synthesized beam modeling experiments using real EHT observations, LiRBM achieved the lowest root-mean-square error (RMS) among the three methods (0.015405), more than 88.92% lower than those of Clark CLEAN and Asp-Clean2016; it also achieved the highest structural similarity index (SSIM) (0.992112), more than 69.41% higher than those of the other two methods. These results demonstrate that LiRBM can establish a higher-precision mathematical model for synthesized beams and support subsequent quantitative analysis, degradation simulation, and astronomical image restoration.

Keywords: Black hole imaging; Astronomical imaging; Synthesized beam; Modeling; Linear radial basis functions;

1 Introduction

Imaging at the event horizon scale of black holes is important for studying strong gravity, accretion processes, and relativistic jets. The Event Horizon Telescope (EHT) utilizes an earth-scale very-long-baseline interferometric array to achieve high angular resolution observations of the event horizon scale of black holes [1]. The EHT imaging studies of the central supermassive black hole in M87* have demonstrated the need to recover event-horizon-scale structures under sparse interferometric sampling conditions [2]. However, under finite spatial frequency sampling constraints, the imaging process is influenced by the structure of the synthesized beam's mainlobe and sidelobes. Classical image restoration research indicates that an observed image can be expressed as the convolution of the true image with the synthesized beam; consequently, the accuracy of the Point Spread Function (PSF) representation and the sampling conditions directly impact the quality of the image restoration [3–5].

For PSFs with different spatial variations, many modeling approaches have been proposed in previous studies. Fetick employed blind deconvolution combined with parameter estimation of the PSF in adaptive optics astronomical imaging, offering a methodology for constructing PSF models using limited observational data [6]. Jia et al. utilized deep neural networks and calibration data to establish a PSF model for

wide-field small-aperture telescopes; Shajkofci and Liebling, on the other hand, employed CNNs to describe PSFs that vary with spatial position in optical microscopy imaging [7–8]. Liu et al. studied PSF variations in space-based survey weak lensing measurements, further highlighting the need for accurately characterizing PSF responses under different observational conditions [9]. Principal component analysis and low-order aberration models describe PSF variations within the field of view from statistical and physical perspectives, respectively, and provide different mathematical formulations for continuous-response modeling [10–11]. Although these methods employ different model forms, their common objective is to model the PSF, thereby providing a variety of mathematical models for subsequent quantitative analysis and image processing.

Regarding continuous PSF modeling, numerous studies have explored this topic from various observational perspectives. Valenzuela et al. used edge response estimation to estimate the PSF of satellite imagers; Liaudat et al. systematically summarized the mechanisms by which the PSF of astronomical telescopes is influenced by the optical system, detector, and observational conditions [12–13]. Jarvis et al., working with dark energy survey data, employed the PIFF method to model the PSF at different field-of-view positions, thereby obtaining a continuous PSF response field that varies with spatial position [14]. Methods such as atmospheric PSF interpolation in short-exposure weak gravitational lensing imaging, as well as PSFEx, MCCD, and WaveDiff, demonstrate that the actual PSF can typically only be obtained at a finite set of star positions and may vary depending on the field-of-view position, observation wavelength, and system state; consequently, it is necessary to employ spatial interpolation, combined focal-plane modeling, or differentiable optical forward models to recover a continuous PSF response field [15–18]. The results indicate that, compared with common methods such as polynomial fitting, B-splines, and inverse distance weighting, the RBF method can effectively describe the spatial variation characteristics of the PSF and exhibits superior overall performance [19].

Radial Basis Functions (RBFs) represent a class of high-precision modeling methods featuring multiple optional basis functions, which have been widely applied across numerous fields. Principal component analysis and low-order aberration models describe PSF variations within the field of view from statistical and physical perspectives, respec. The studies by Hardy on multiquadric interpolation, Franke on discrete-data interpolation, and Broomhead and Lowe on RBF networks collectively demonstrate that RBFs can be effectively used for spatial representation and spatial-function fitting of discrete data [20–22]. Additionally, Patrikar further elaborated on the model approximation relationship between RBF networks and multi-layer perceptrons, providing valuable insights for understanding the functional expressiveness of RBF models [23]. Buhmann, Wendland, and Fornberg, among others, provided a systematic review of RBF theory, approximation of irregularly distributed data, and its applications in scientific computing, thereby laying the theoretical foundation for this paper [24–26]. Furthermore, studies on shape parameters reveal that basis functions that explicitly contain ε are affected by parameter selection and numerical stability. Fasshauer and Zhang discussed how the choice of shape parameters in RBF approximations affects the error, while Fasshauer and McCourt analyzed the stable evaluation problem under small shape parameter conditions using Gaussian RBFs as a case study [27–28]. RBFs are a class of modeling methods that construct continuous functions from the distance relationships between sample points, making them particularly suitable for spatial-data

interpolation and continuous spatial-response fitting.

In radio interferometric imaging, the modeling accuracy of the Point Spread Function (PSF) significantly impacts the quality of image reconstruction. The Högbom CLEAN algorithm reconstructs images by repeatedly locating residual peak values and subtracting their corresponding dirty beam responses, thereby representing the observed image as a combination of point-source components [29]. The Shapelets method models images by describing their structure through a local complete basis function expansion, with the resulting model being subsequently applicable to image analysis and simulation [30–31]. Building upon Högbom CLEAN, Clark introduced a framework where component search and precise residual calculation are divided into minor cycles and major cycles; this approach utilizes finite PSF patches to rapidly select components, followed by a unified correction of the cumulative model using the full PSF, thereby reducing the computational cost associated with repeated convolutions [32]. Zhang et al. further extended these methods for extended source imaging by approximating the mainlobe of the PSF as a single Gaussian; they then fit the convolved Gaussian components in the residual map and recover the scale and amplitude through analytical deconvolution, effectively reducing the computational cost of adaptive-scale methods while maintaining comparable imaging performance [33]. The modeling accuracy of the PSF plays a crucial role in numerous related studies, such as those addressing noisy compact emission recovery [34], random-scale parameterized reconstruction models [35], adaptive-scale model reconstruction [36], and hybrid-scale parameterized reconstruction approaches [37]. However, the accuracy of PSF modeling is highly dependent on the complexity of the PSF characteristics; for PSFs exhibiting strong and complex properties—such as those encountered in the Event Horizon Telescope (EHT)—existing methods often struggle to achieve high-precision modeling.

To address this issue, this paper proposes a modeling method based on radial basis functions (RBFs), namely LiRBM, for achieving high-precision modeling of complex PSFs, with validation performed using EHT PSFs. Section 2 reviews relevant prior work; Section 3 describes the model and methodology; Section 4 presents the experimental setup; Section 5 analyzes the results; and Section 6 summarizes the paper and outlines future research directions.

2 Related Work

2.1 PSF Estimation and Modeling

Early studies in image restoration demonstrated that the Point Spread Function (PSF) is a crucial function for describing the imaging degradation process. In the classical image restoration model, Richardson assumed that a degraded image can be represented as the convolution of the original image with a synthesized beam [3]; to maintain consistency with the subsequent notation used in this paper for radio imaging and RBF modeling, the Richardson–Lucy approach is reformulated here using modern continuous convolution notation.

$$I^{\text{dirty}}(x,y) = B(x,y) * I^{\text{true}}(x,y) + I^{\text{noise}}(x,y) \quad (1)$$

Here, I^{dirty} denotes the observed dirty or degraded image; I^{true} represents the true sky brightness distribution; B is the PSF or synthesized beam; I^{noise} is the noise; and $*$ indicates two-dimensional convolution.

Lucy further proposed a method for handling the observed distribution using conditional probability and iterative correction [4], as shown in the following equation:

$$I^{(t+1)}(x,y)=I^{(t)}(x,y)[B^* \frac{I^{dirty}}{B^* I^{(t)}}](x,y), \quad (2)$$

where $B^*(x,y)=B(-x,-y)$

The MCS deconvolution method emphasizes the impact of sampling conditions on the deconvolution results and the control of artifacts [5]. These studies collectively demonstrate that the spatial morphology of the PSF is not merely an ordinary image texture, but rather a functional structure that connects imaging degradation, sampling conditions, and the restoration process.

These studies show that, in both image-degradation analysis and iterative solution, PSF modeling serves as a fundamental link connecting system response, sampling conditions, and observational data. In practical observations, the PSF is often difficult to obtain directly; consequently, subsequent research has shifted focus from image restoration under known PSF conditions to the acquisition and estimation of the PSF itself. Addressing the challenge of obtaining the PSF directly in adaptive optics astronomical imaging, Fetick et al. proposed estimating the PSF from the scientific images themselves using blind deconvolution or parametric marginalization methods, which can then be applied for subsequent restoration [6]. Meanwhile, PSF modeling has gradually been expanded to various imaging systems, showing strong applicability. Jia et al. utilized deep neural networks and calibration data to estimate the PSF for wide-field small-aperture telescopes; Liu et al. discussed the issue of chromatic aberration PSF in weak lensing measurements conducted by the Chinese Space Station Survey Telescope; while Shajkofci and Liebling highlighted the applicability of PSF modeling across different imaging systems from the perspective of spatially varying CNN-based PSF estimation in optical microscopy [7–9].

In practical wide-field astronomical observations, the Point Spread Function (PSF) varies with the field-of-view position, observation wavelength, and system state. Methods such as PIFF, PSFEx, MCCD, WaveDiff, principal component analysis, and low-order aberration models have been developed to model this spatial variation continuously from both data-driven and physical mechanism perspectives [10–11,14,16–18]. Since the PSF is typically only measurable at a finite set of star positions, it is necessary to recover a continuous response field at arbitrary locations through spatial interpolation. Relevant studies have demonstrated that the Radial Basis Function (RBF) shows good overall performance in interpolating PSF anisotropy, providing a basis for using RBF to construct a continuous PSF response in this study [15,19].

2.2 The RBF Method for Spatial Data Fitting

The radial basis function (RBF) method is well-suited for handling irregularly sampled data with spatial positional relationships and spatial fitting problems. The Multiquadric method proposed by Hardy was an early application of RBFs to terrain and irregular-data interpolation [20]. Franke systematically tested various discrete data interpolation methods, comparing their performance across criteria such as computational time, memory usage, accuracy, visual quality of interpolation results, and implementation complexity, thereby establishing a reference standard for subsequent evaluations of methods in spatial interpolation and numerical computation [21]. Broomhead and Lowe further integrated the RBF concept with multivariate function interpolation and adaptive networks, making RBFs an important tool in machine learning and nonlinear approximation [22]. Patrikar discussed the connection

between RBF networks and multi-layer perceptrons from the perspective of model approximation, providing a complementary perspective for understanding the expressive power of RBF networks [23]. Buhmann, Wendland, Fornberg, and others have systematically summarized RBF theory, the approximation of data with irregular distributions, and meshless approximation methods, laying a theoretical foundation for the application of RBFs in engineering and scientific computing [24–26]. Building upon these theoretical foundations, this paper focuses on EHT synthesized beam images and introduces two-dimensional spatial coordinates and response value data into the RBF modeling framework. This section mainly describes the theoretical basis and applicability of RBFs for spatial fitting; the two-dimensional continuous RBF model, distance definition, and weight-calculation method are presented in Section 3.

The RBF method is characterized by a model response governed primarily by the distances between sample points rather than by fixed-order polynomial terms. This characteristic makes it well-suited for capturing the spatial distribution in PSF images, which features a prominent central peak followed by outward attenuation.

2.3 PSF Processing and Parametric Modeling in Radio Interferometric Imaging

In PSF modeling comparisons, Högbom CLEAN can be regarded as a local representation method based on discrete components and a known dirty beam response; however, it does not directly output a continuous PSF function model [29]. The algorithmic procedure for Högbom CLEAN is as follows:

First, define the current residual peak position:

$$(x_k, y_k) = \arg \max_{x, y} |R_k(x, y)| \quad (3)$$

Current residual peak value is:

$$p_k = R_k(x_k, y_k) \quad (4)$$

The residual update relation is:

$$R_{k+1}(x, y) = R_k(x, y) - \gamma p_k B(x - x_k, y - y_k) \quad (5)$$

Here, γ denotes the loop gain, p_k represents the current residual peak, B denotes the PSF, and R_k denotes the residual map at the k -th iteration.

The Shapelet method models images by linearly decomposing the image target into a set of local, complete, orthogonal basis functions; it has been applied to image analysis and image simulation studies involving PSFs, noise, and pixelation effects [30–31]. The specific formulation of this method is as follows:

$$f(x) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} f_{n_1, n_2} B_{n_1, n_2}(x; \beta) \quad (6)$$

Here, f_{n_1, n_2} denotes the Shapelet expansion coefficients, $B_{n_1, n_2}(x; \beta)$ represents the two-dimensional Shapelet basis function, and β is the scale parameter of the basis function.

From the perspective of modeling, CLEAN mainly performs residual updates under known PSF conditions, whereas Shapelets represent image structures using expansions based on finite-order basis functions; however, neither approach is intended to directly construct a continuous response model based on PSF samples.

Building upon the Högbom CLEAN algorithm, Clark divides the component search and precise residual update process into a minor cycle and a major cycle: the minor cycle utilizes a finite PSF patch to rapidly select multiple components, while the

major cycle then leverages the full PSF to uniformly correct the cumulative model [32]. Based on Clark's approach, in which the full PSF is used in the major cycle to correct the cumulative model uniformly, the method can be expressed as follows:

$$R_{\text{new}} = I^{\text{dirty}} - B * M_{\text{new}} \quad (7)$$

Here, I^{dirty} denotes the dirty image, B represents the complete PSF, M_{new} indicates the cumulative model updated during the major cycle, and R_{new} denotes the exact residual recalculated using the complete PSF. This relationship illustrates that most components first respond rapidly under a local approximation, and then undergo unified correction during the major cycle using the complete PSF.

In their work on Asp-Clean, Zhang et al. approximate the mainlobe of the PSF for extended source imaging as a single Gaussian, directly fit the convolved Gaussian components to the residual map, and then recover the true component scale and amplitude via analytical deconvolution [33]. Their fitting process determines the component parameters by minimizing the discrepancy between the residual map and the Gaussian model; the objective function is as follows:

$$\chi^2 = \|I_i^{\text{residual}} - I_{ic}^{\text{gauss}}(a_{ic}, x_{ic}, y_{ic}, w_{ic})\|_2^2 \quad (8)$$

In this formula, I_i^{residual} denotes the residual map for the i -th iteration; I_{ic}^{gauss} represents the convolved Gaussian component fitted to the residual map; while a_{ic} , (x_{ic}, y_{ic}) and w_{ic} denote the amplitude, center coordinates, and width of this Gaussian component, respectively.

After obtaining the parameters of the Gaussian components from the convolution operation, Zhang et al. leveraged the additive property of the variance resulting from Gaussian function convolution to analytically deconvolve the effect of the PSF mainlobe. The width of the model component after deconvolution is given by:

$$\omega_i = \sqrt{\omega_{ic}^2 - \omega_b^2} \quad (9)$$

In this formula, ω_{ic} denotes the width of the fitted convolved Gaussian component obtained through fitting, ω_b denotes the Gaussian width used to approximate the mainlobe of the PSF, and ω_i denotes the width of the true model component after deconvolution. According to this scale-restoration relationship, the amplitude of the model components after deconvolution can be expressed as:

$$\alpha_i = \frac{\alpha_{ic} \omega_{ic}^2}{2\pi \alpha_b \omega_b^2 \omega_i^2} \quad (10)$$

Here, α_{ic} denotes the amplitude of the Gaussian component after convolution, α_b denotes the amplitude of the Gaussian mainlobe of the PSF, and α_i denotes the amplitude of the model component after deconvolution.

2.4 Motivation

The modeling of synthesized beams is crucial for applications such as image restoration and PSF parameter estimation; existing research primarily employs various modeling approaches, including parametric estimation [6], deep learning-based prediction [7–9], CLEAN-type component reconstruction [29,32], Shapelet-based basis function expansion [30–31], and Gaussian component fitting [33]. The accuracy of PSF modeling often serves as a key factor influencing image reconstruction and error analysis.

Among these methods, Zhang et al. in Asp-Clean employ a Gaussian function to approximate the mainlobe of the Point Spread Function (PSF), thereby reducing the computational cost of model inversion and achieving a good balance between reconstruction accuracy and computational overhead—making it particularly suitable for imaging with medium-to-dense telescope arrays. This advancement enabled the adaptive scale method to achieve the first successful validation using real observational data in the field of radio interferometry [33]. The Clark CLEAN algorithm constructs beam patches utilizing both the mainlobe and its adjacent first sidelobe, then applies the full PSF during the major cycle to perform correction; image reconstruction is accomplished by repeatedly locating residual peaks and subtracting their corresponding PSF responses [29,32]. The approximation used in the first method is mainly applicable when sidelobe levels are relatively low, whereas the finite beam patch in the second method approximates only local responses within the minor cycle and cannot independently form a continuous model covering far-sidelobe regions. Additionally, the Shapelets method models the PSF using expansions based on local orthogonal basis functions, whose modeling accuracy is influenced by the scale parameter and the maximum expansion order [30–31]. The precision of all these methods is constrained by their local approximation schemes and expansion orders, making them inadequate for complex PSF modeling or data processing scenarios requiring high accuracy. To address these limitations, this paper proposes LiRBM fitting, a global PSF fitting approach based on radial basis functions, to achieve high-precision fitting and a functional representation of the EHT synthesized beam.

3 Methods

3.1 Radial Basis Function (RBF) Fitting Model

This paper explores a high-precision modeling method for PSFs based on RBFs. The core idea of this approach is to use the sampled image pixels as RBF centers, employ the Euclidean distance between the target estimation position and each sampling center as the distance response, and obtain the model response value at that position by performing a weighted superposition of these distance responses using corresponding weight coefficients. Unlike methods that construct fixed-degree polynomials from coordinates x and y , the RBF model does not assume a global polynomial form for the PSF response; instead, it obtains the response values through weighting based on the distance relationships among the sample points. Consequently, the RBF method is well suited to fitting discrete data with irregular spatial distributions or spatial structures that cannot be adequately described by simple polynomials. The fitting function proposed in this paper is denoted as $f(x, y)$, and its general form is as follows:

$$f(x,y)=\sum_{i=1}^N w_i \phi(r_i) \quad (11)$$

Here, $r_i=\sqrt{(x-x_i)^2+(y-y_i)^2}$ denotes the Euclidean distance between the query position $p=(x,y)$ and the i -th sample point $p_i=(x_i, y_i)$; w_i is the weight coefficient associated with the i -th RBF center, and $\phi(r_i)$ denotes the radial basis function.

The local response is influenced by sampling points near the central mainlobe, while the sidelobes and edge regions collectively affect the overall PSF response through distance terms involving multiple center points. Consequently, this

distance-based modeling approach exhibits excellent compatibility with the spatial structure of the EHT synthesized beam. Leveraging this compatibility, the RBF modeling method can reconstruct discrete samples into a continuous spatial response. In the program, a two-dimensional coordinate grid is first constructed from the image dimensions, and the response values obtained from image-intensity mapping are used as z_i , thereby forming the fitting samples (x_i, y_i, z_i) .

3.1.1 Basis Function Types

In the RBF model, the basis function $\phi(r)$ determines how distance r is converted into a spatial response; therefore, the basis-function form directly affects the smoothness, local-response behavior, and parameter sensitivity of the LiRBM fitting result. Different basis functions correspond to different distance-response rules: some basis functions emphasize local smoothness, some amplify the influence of long distances, while others have their distance-response scale controlled by the shape parameter ϵ .

The Linear basis function is defined as $\phi(r) = r$, where Linear indicates that the basis function responds linearly to the distance variable r ; this does not mean that LiRBM fitting produces a flat response, nor is it equivalent to a first-order polynomial fit. Since the final model is obtained by weighting and superimposing the distance responses from N sampling centers, the relatively simple individual basis functions can still combine to produce a continuous PSF response that closely matches the spatial structure of the EHT synthesized beam.

Compared with basis functions that explicitly contain ϵ , such as Gaussian, Multiquadric, and Inverse, the Linear basis function does not require adjustment of the distance response scale via shape parameters, thereby reducing the issue of spatial-structure mismatch caused by improper parameter selection. Existing studies on RBF shape parameters have shown that the choice of ϵ affects both approximation accuracy and numerical stability; in particular, highly smooth basis functions like the Gaussian may encounter stability issues during evaluation under small shape parameter conditions [27–28]. According to the experimental results presented in this paper, the Linear basis function exhibits a relatively stable response to changes in distance; while preserving the central mainlobe and sidelobe structures, it avoids the local oscillations or heightened sensitivity to higher-order distance terms observed in the results for Cubic and Thin Plate basis functions. Based on these characteristics, this paper adopts the Linear basis function as the primary model in the main body, while the other basis functions are primarily used for comparative validation and are provided in the Supplementary Information.

Table 1: Common basis function types and their expressions

Basis Function Type	Expression
Linear	$\phi(r)=r$
Thin Plate	$\phi(r)=r^2\ln(r)$
Gaussian	$\phi(r)=\exp[-(r/\epsilon)^2]$
Cubic	$\phi(r)=r^3$
Inverse	$\phi(r)=1/\sqrt{1+(r/\epsilon)^2}$
Multiquadric	$\phi(r)=\sqrt{1+(r/\epsilon)^2}$

3.1.2 Shape Parameters

Different basis functions exhibit varying degrees of dependence on the shape parameter ε . In this paper, we uniformly set the value of ε in the code to 2.0; this parameter affects basis functions that explicitly include ε , such as the Gaussian, Multiquadric, and Inverse basis functions. The Linear basis function $\phi(r) = r$ does not explicitly include ε ; thus, its output is primarily determined by the positions of the sampling points, the distance matrix, and the weight calculation process. For basis functions that explicitly include ε , changes in ε will affect the distance response scale.

3.2 LiRBM Fitting Procedure

To align the model construction process described in Section 3 with the program implementation workflow, this paper presents the RBF-based PSF fitting steps in pseudocode, as shown in Algorithm 1. This workflow also outputs: (1) a comprehensive error comparison across all six basis functions; (2) 4,096 sets of center coordinates and weight coefficients for the optimal Linear continuous function; and (3) the LiRBM fitting results for both the mainlobe and the region of the mainlobe and the first sidelobe. The final mathematical model is jointly defined by the fitting results of each basis function, the corresponding error metrics, the residual distribution, the center coordinates, and the weight coefficients.

Algorithm 1: LiRBM Fitting Procedure

Input: PSF response matrix (Z), overall sampling interval (s), and local sampling interval (p).

Output: LiRBM fitting results, residuals, RMS, SSIM, RBF center coordinates, and weight coefficients.

1. Construct the two-dimensional coordinate grid and generate the overall training samples from (Z) using the prescribed sampling interval.
2. Use the sampled coordinates as RBF centers, construct the LiRBM model, and solve for the corresponding weight coefficients.
3. Evaluate LiRBM fitting results
4. Calculate residual, RMS, SSIM, and residual statistics, and record the RBF center coordinates and weight coefficients.
5. Locate the mainlobe center and define the mainlobe region (D_m) and the combined mainlobe and first sidelobe region (D_s).
6. For each local region ($D_k \in D_m, D_s$):
 - 6.1 Select training nodes according to the local sampling interval while retaining the boundary nodes.
 - 6.2 Construct the local LiRBM model and solve for the corresponding weight coefficients.
 - 6.3 Evaluate the fitting results over the complete local region.
 - 6.4 Calculate the local residual, RMS, SSIM, and residual statistics.
 - 6.5 Record the local RBF center coordinates, weight coefficients, and fitting results.
7. Output the overall LiRBM result, the mainlobe fitting result, and the fitting result for the region of the mainlobe and the first sidelobe .

4 Data Processing and Experimental Protocol

4.1 Data Sources and Numerical Processing

To validate the effectiveness of the method proposed in this paper, the synthesized beam is derived from the 2017 1.3 mm very long baseline interferometry observations conducted by the Event Horizon Telescope (EHT) on M87* and the resulting synthesized beam represents the instrumental response associated with the first-ever black hole image of M87*. This array consists of eight stations—ALMA, APEX, JCMT, SMA, SMT, LMT, IRAM 30-m telescope, and the South Pole Telescope—and spans six geographic locations; notably, phased ALMA was incorporated into the global array for the first time as a high-sensitivity VLBI station. The imaging data for M87* were primarily obtained during four observation nights: April 5, 6, 10, and 11, 2017 [1–2]. The EHT operates within the framework of typical VLBI; its sparse

distribution results in an exceptionally complex synthesized beam, requiring the fitting process not only to describe the central peak but also to characterize the mainlobe width, zero-crossing positions, the polarity of the first sidelobe, and their spatial oscillation relationships.

In this experiment, we used a 128×128 pixel PSF. As shown in Figure 1, the EHT's PSF exhibits strong and complex sidelobes, which poses significant challenges for data processing.

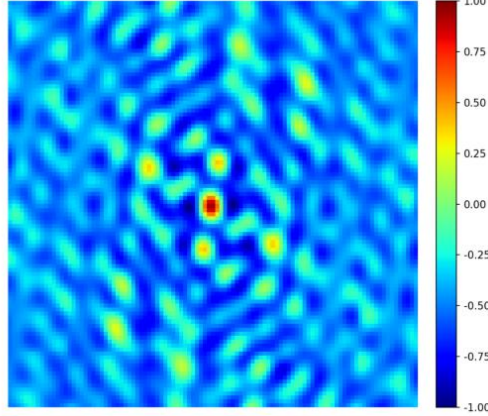


Figure 1: Original Image

4.2 Parameter Settings

During the image digitization stage, the PSF response range is set to $[-1, 1]$. The program performs uniform sampling of 2,048 points across the image, using these points as sample points. Subsequently, the cKDTree algorithm is employed to perform nearest-intensity matching for each pixel, with the results being constrained within the range $[-1, 1]$.

The numerically processed PSF contains many pixels; in this paper, every two pixels, one sampling point is selected as an RBF center, resulting in 4096 center points in total. The coordinate calculation still employs the original pixel spacing, with the corresponding coordinate scaling factor set to 1.0. The LiRBM fitting results are further refined on a grid with twice the original resolution; this grid is used for three-dimensional visualization and overall-error calculation. The original modeling data are mapped to the high-density grid using Linear interpolation; any NaN values generated during interpolation are filled using the nearest neighbor method. For the 3D plot, the aspect ratio parameter Z_{aspect} is set to 0.60; this parameter only controls the image aspect ratio and does not participate in the RBF weight computation or the error calculation.

The choice of basis functions includes six forms: Linear, Thin Plate, Multiquadric, Inverse, Cubic, and Gaussian; for all models, the interface parameter ε is uniformly set to 2.0. Here, ε influences the distance response scale for the Gaussian, Multiquadric, and Inverse basis functions. Since the functional expressions for the Linear, Thin Plate, and Cubic basis functions do not explicitly contain ε , they are unaffected by this parameter. The Gaussian smoothing parameter smooth is set to 0.0, and no additional smoothing term is incorporated for the other basis functions. For this experiment, the final model is determined solely based on the uniform condition $\varepsilon=2.0$.

This paper also employs local evaluation to provide a more comprehensive assessment of LiRBM's performance. In this experiment, prior to locating the mainlobe center, a Gaussian filter with $\sigma=1.0$ is applied to suppress isolated noise; subsequently, the point with the highest contrast relative to the background is identified within the central region spanning 20%–80% of both the horizontal and vertical dimensions of the image. Using the detected mainlobe center as a reference point, a window of 7 pixels is defined outward from this center; when this window expands to 16 pixels, it can cover both the mainlobe and its adjacent first sidelobe. Therefore, these two regions are utilized for evaluating the mainlobe and the combination of the mainlobe and the first sidelobe, respectively. During the local fitting process, training nodes are selected every two pixels, while sample points along the region boundaries are retained.

4.3 Evaluation

This paper evaluates the model from two perspectives: overall fitting accuracy and local structural preservation capability.

The overall evaluation employs the mean absolute residual (MAR), maximum σ_e absolute residual (MaxAR), residual standard deviation (σ_e), root mean square error (RMS), and structural similarity index (SSIM). These metrics are used to compare the overall fitting errors of six RBF basis functions.

Building upon the overall evaluation, this paper utilizes RMS and SSIM to further assess the numerical errors and structural consistency of the final LiRBM model across the full PSF range. Local evaluations are conducted for both the mainlobe region and the region of the mainlobe and the first sidelobe, with RMS and SSIM values calculated separately to analyze the model's ability to preserve the morphology of the central mainlobe, the positions of the sidelobes, and their oscillatory structure.

By combining overall evaluation with local evaluation, the modeling performance of LiRBM for the EHT synthesized beam can be assessed from two perspectives: numerical error and spatial structure.

According to the residual calculation method in the procedure, let $z_{\text{fit},i}$ denote the predicted response value at the i -th predicted grid point, $z_{\text{ori},i}$ denote the original response value after resampling, and N denote the total number of valid sample points; then, the residual for the i -th point is defined as:

$$e_i = z_{\text{fit},i} - z_{\text{ori},i} \quad (12)$$

In matrix form, this can be written as:

$$\text{Residual} = Z_{\text{fit}} - Z_{\text{original_resampled}} \quad (13)$$

The mean absolute residual is used to measure the overall error level, and its calculation formula is:

$$\text{MAR} = \frac{1}{N} \sum_{i=1}^N |e_i| \quad (14)$$

The maximum absolute residual is used to reflect the local maximum deviation; its calculation formula is:

$$\text{MaxAR} = \max(|e_i|), \quad i = 1, 2, \dots, N \quad (15)$$

The residual standard deviation is used to describe the dispersion of the residual distribution. Let $\bar{e}$ denote the mean residual; the residual standard deviation is

calculated as follows:

$$\sigma_e = \sqrt{\frac{1}{N} \sum_{i=1}^N (e_i - \bar{e})^2} \quad (16)$$

The aforementioned overall residual metrics correspond to the functions `np.nanmean(np.abs(residual))`, `np.nanmax(np.abs(residual))`, and `np.nanstd(residual)` in the program. During computation, NaN values are automatically ignored; therefore, the N in the formula represents the number of valid grid points. For `np.nanstd(residual)`, the default value `ddof=0` is used, so the standard deviation is calculated for the full population; a smaller value indicates a more concentrated residual distribution and better fitting stability.

To evaluate the fitting performance of key local regions, let a local region contain M valid pixels; let the original response value and the predicted response value be Z_j and $\hat{Z}_j$, respectively; then:

$$\text{RMS} = \sqrt{\frac{1}{M} \sum_{j=1}^M (Z_j - \hat{Z}_j)^2} \quad (17)$$

The SSIM is computed using `skimage.metrics.structural_similarity`, where `data_range` is set to the difference between the maximum and minimum values of the local raw response.

Building upon the aforementioned quantitative evaluation, this paper further employs the LiRBM fitting result, the central profile, and residual distribution to analyze the spatial fitting performance qualitatively. The quantitative evaluation provides objective metrics for both model error and structural similarity, while the image-based structural analysis is used to verify the model's ability to preserve spatial morphological characteristics; together, these two approaches complement each other and serve as the basis for comparing the performance of different basis functions and for determining that LiRBM is the primary modeling approach.

5 Fitting Results and Discussion

5.1 Basis Function Fitting Results

According to Table 2, the Linear basis function achieves the smallest values across all three metrics—mean absolute residual, maximum absolute residual, and residual standard deviation; therefore, it is selected as the final model for continuous PSF modeling. In this section, we analyze the modeling performance of LiRBM from four perspectives: numerical error, fitting result, central profile, and residual distribution, using the other five basis functions for comparison.

5.1.1 Linear Basis Function Fitting Results

The Linear basis function uses the spatial distance r as the response term, enabling the establishment of a functional relationship between the sample center and the position to be estimated without introducing additional shape parameters. For the current EHT synthesized beam samples, this characteristic helps reduce the impact of parameter

selection on the fitting results while preserving the overall continuous trend, thereby facilitating the construction of a high-precision synthesized beam mathematical model.

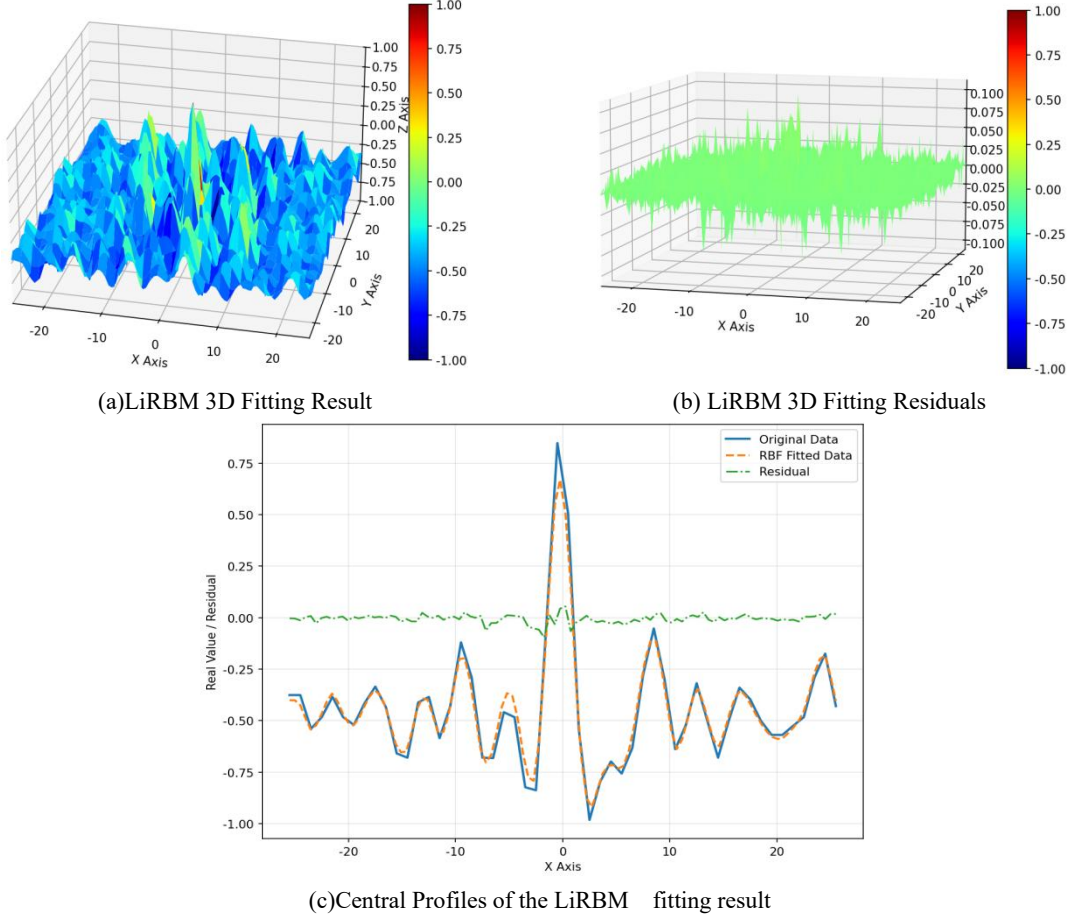


Figure 2: Fitting result using the LiRBM with $\varepsilon = 2.0$

Figure 2(a) shows the LiRBM fitting result, while Figure 2(b) shows the corresponding three-dimensional residual distribution. The LiRBM fitting result in Figure 2(a) preserves the positions of the main peaks and valleys, the overall fluctuation trend, and the edge-attenuation trend; meanwhile, the residuals in Figure 2(b) are distributed around zero, with the largest errors concentrated near local peaks and valleys where the response changes more rapidly.

An analysis of the residual distribution reveals that most regional errors fluctuate around zero with relatively uniform ranges, without any concentrated high-error regions. This indicates that LiRBM exhibits good error control capability across the entire cross-section. In regions with more pronounced variations, the LiRBM fitting curve shows a certain degree of smoothness, suggesting that this basis function prioritizes overall structural continuity over overfitting local details; this behavior is consistent with PSF images that are generally smooth but have prominent central peaks.

These results show that LiRBM fitting can provide a continuous, high-precision PSF response while preserving its primary spatial characteristics. As shown in Table 2, all three performance metrics for the Linear basis function are the lowest among the six basis functions. These results are consistent with the residual distribution presented in Figure 2(b), indicating that the model achieves superior performance in terms of both overall error level and error dispersion while maintaining the primary

spatial structure of the PSF.

Figure 2(c) presents a comparison of the central profiles generated by LiRBM at $\varepsilon = 2.0$. The comparison of central profiles demonstrates that the fitting curve produced by the LiRBM model can preserve the positions of the main peaks and valleys as well as their variation trends relative to the original data. Overall, the fitting curve closely follows the variations in the original data; across multiple sidelobe fluctuation regions, the sequence of peaks and valleys in the fitting curve is essentially consistent with that of the original curve, indicating that the model successfully retains the primary structure of this cross-section. At the central peak and other extrema, although there are certain amplitude discrepancies between the fitting curve and the original data, the peak positions remain largely consistent, and the variation directions are also broadly consistent, with no significant displacement observed.

Considering the LiRBM fitting result, residual distribution, and central profile together, LiRBM not only achieves the best overall performance in the combined residual metrics but also consistently preserves the central mainlobe, sidelobe oscillations, and edge attenuation structure of the synthesized beam.

5.1.2 Thin Plate Basis Function Fitting Results

The Thin Plate model ranks as the second-best among the six basis functions considered; its mean absolute residual is 0.017637, its maximum absolute residual is 0.167062, and its residual standard deviation is 0.024888—all three metrics are higher than those of the Linear basis function. Based on the Thin Plate fitting result, the residual distribution, and the central profile shown in Supplementary Figs. S1 and S2, the Thin Plate model shows more pronounced deviations and fluctuations near local extrema, while its overall error level and spatial stability remain worse than those of the Linear basis function.

5.1.3 Fitting Results With Other Basis Functions

Results for the other basis functions show that different distance response forms match the spatial structure of the current synthesized beam to different degrees; using more complex functional forms does not improve the modeling accuracy. Under the current unified parameter settings, the Gaussian function yields significantly larger errors, and its accuracy is lower than that of LiRBM. Although the Multiquadric, Inverse, and Cubic functions can fit part of the spatial structure, their overall errors and residual dispersions are higher than those obtained with LiRBM.

5.2 Performance Advantages of the LiRBM Model

To quantitatively evaluate the performance advantages of the LiRBM model in synthesized beam modeling, this paper compares the error metrics for six RBF basis functions under various evaluation criteria with a fixed $\varepsilon = 2.0$; the results are presented in Table 2.

Table 2: Fitting Results for Different Basis Functions at $\varepsilon=2.0$

Basis function	Mean Residual	Absolute residual	Maximum absolute residual	Residual deviation	standard
Linear	0.010511		0.124657	0.015405	
Thin Plate	0.017637		0.167062	0.024888	
Multiquadric	0.020286		0.190922	0.028324	
Inverse	0.020245		0.190557	0.028268	
Cubic	0.019512		0.184992	0.027328	
Gaussian	0.020379		0.191117	0.028425	

Table 2 presents the three residual metrics for different basis functions. For the Linear basis function, the mean absolute residual is 0.010511, the maximum absolute residual is 0.124657, and the residual standard deviation is 0.015405; all three values are the smallest among the six basis functions. Compared with the second-best Thin Plate basis function, the Linear basis function reduces the mean absolute residual, maximum absolute residual, and residual standard deviation by 40.40%, 25.38%, and 38.10%, respectively; compared with the Cubic basis function, it reduces the mean absolute residual and residual standard deviation by 46.13% and 43.63%, respectively. These results show that, under the current data and parameter settings, LiRBM has a smaller overall fitting error and a more concentrated error distribution across spatial positions, enabling high-precision continuous fitting of the synthesized beam response.

The performance advantages of the LiRBM model can be explained primarily from three aspects: spatial structure matching, basis function response, and error performance.

The primary spatial characteristics of the sample in this study are a prominent central peak, gradual edge attenuation, and certain local fluctuations; however, the overall variation predominantly follows a continuous trend. The Linear basis function $\phi(r)=r$ can generate a nonlinear PSF response with peak, fluctuation, and attenuation characteristics by appropriately weighting and combining distance terms from multiple sampling centers. Analysis of the LiRBM fitting result and the central profile shows that, at the current sampling scale, LiRBM accurately tracks the positions of the major peaks and valleys as well as the overall variation trend; furthermore, no significant additional oscillations are observed near the edges or extrema. This demonstrates that the distance response form of LiRBM is well matched to the spatial structure of the current sample.

Since the Linear basis function does not explicitly include the shape parameter ϵ , its distance response scale does not require adjustment based on ϵ ; instead, the model output is determined mainly by the spatial distribution of the sampling points, the distance matrix, and the weight-calculation process, thereby reducing variation in the results caused by the choice of shape parameter. In contrast, the distance response scales of basis functions such as Gaussian, Multiquadric, and Inverse are influenced by ϵ . When ϵ is mismatched with the coordinate scale or the sample spacing, the model may exhibit insufficient local response, which can lead to an increase in errors.

The quantitative error results further demonstrate the performance advantages of the LiRBM model. The maximum absolute residual primarily identifies the local region where the error is largest; however, it is sensitive to rapid local variations and extreme deviations and therefore should not be used alone to evaluate the overall quality of continuous PSF fitting. The mean absolute residual reflects the overall deviation level of the model, while the residual standard deviation indicates the degree of dispersion of errors across different spatial locations. Given that the objective of this study is to achieve a high-precision, continuous representation of the overall response structure of the synthesized beam, model selection should consider both the overall error and spatial stability. LiRBM achieves the best results for all three overall evaluation metrics; consequently, this quantitative advantage is also reflected in its ability to fit the spatial response of the synthesized beam.

The LiRBM model not only reduces the overall modeling error but also preserves the continuity and key morphological characteristics of spatial response of the synthesized beam. Further analysis of the LiRBM fitting result, central profile, and residual distribution shows that another advantage of LiRBM is its ability to retain

critical spatial structures. This model can accurately characterize the spatial structure of EHT images without introducing significant structural deviations in regions with rapid local variations.

5.3 Final Expression of the LiRBM Fitting Function

In this experiment, the EHT synthesized beam mathematical model contains $N = 4096$ RBF centers; consequently, the function can be expressed as a weighted sum of 4096 center distance terms. This model is not a compact analytical expression composed of a small number of parameters, but rather an explicit RBF-weighted model determined by the center coordinates and their corresponding weights (Supplementary Table S1). The final LiRBM fitting function can be written as:

$$f(x,y)=\sum_{i=1}^{4096} w_i \phi(\sqrt{(x-x_i)^2+(y-y_i)^2}) \quad (24)$$

In this formula, (x_i, y_i) denotes the position of the i -th sampling point on the two-dimensional coordinate grid, w_i is the weight coefficient corresponding to this center point, and $f(x,y)$ represents the fitted response value at any spatial location (x,y) . The parameter scale of this function is determined by the number of center points after downsampling; consequently, its reproducibility depends on retaining all center coordinates and their corresponding weight coefficients. The weight coefficients w_i are obtained by solving the Linear RBF system; the complete coefficient table contains four columns: index, x_i , y_i , and w_i , and can be used to reproduce the estimated response values for experimental samples at any spatial location. The coefficients for the first 50 groups are provided in Supplementary Table S1, while the complete set of 4,096 coefficient groups is provided as Supplementary Data S1.

The functional model established in this paper is applicable only to the current experimental samples and parameter conditions. For other EHT synthesized beam images, it will be necessary to repeat the basis function selection, parameter determination, and model solution; therefore, the current set of 4,096 RBF centers and their corresponding weights are not directly generalizable.

5.4 Comparison with General Methods

Table 3 compares LiRBM, Clark CLEAN, and Asp-Clean2016 from three perspectives: PSF processing methods, evaluation targets, and their relationship to the research objectives of this paper. LiRBM proposed in this paper uses a continuous modeling approach based on the Linear basis function and the PSF response; its objective is to derive a continuous mathematical expression that describes the spatial morphology and to output the model center coordinates and corresponding weights that determine the final model. Clark CLEAN primarily reduces the computational overhead associated with component search and residual update through local beam patches within the minor cycle, while the major cycle still utilizes the full PSF for global correction. Asp-Clean2016 further adopts a single-Gaussian approximation of the PSF mainlobe to reduce the computational cost during extended source reconstruction; however, this approximation does not aim primarily at accurately describing the full sidelobe structure. The latter two methods are primarily used for deconvolution and computational acceleration in radio-interferometric imaging, whereas LiRBM focuses on providing a continuous functional representation of the full synthesized beam; consequently, the three methods differ in their intended use of the PSF and their modeling scopes.

Table 3: Comparison of PSF Processing Objectives and Results for LiRBM and the Methods in Refs. [32–33].

Method	PSF Processing Method	Evaluation Target	Relationship to This Paper
LiRBM	Linear RBF continuous function modeling of the full PSF response	The overall spatial morphology and numerical error of the full normalized PSF	Output continuous functions, center coordinates, and weights, emphasizing the continuous representation of the full PSF spatial structure and its overall preservation.
Clark CLEAN[32]	The minor cycle utilizes the truncated beam patch; the major cycle uses the full PSF to update the residual.	Computational efficiency of point-source component search and residual update	Local approximation is used for acceleration, but still relies on the full PSF for global correction.
Asp-Clean2016[33]	Approximate the PSF mainlobe as a single Gaussian, and ignore the sidelobes during component estimation.	Extended source reconstruction residual, flux recovery, and operating time	Suitable for speed enhancement under low sidelobe conditions, but does not characterize the continuous sidelobe response.

To ensure consistent evaluation across different PSF approximation methods, this study reproduces LiRBM, the Clark CLEAN local beam-patch approximation, and the Asp-Clean2016 single-Gaussian mainlobe approximation under a unified setup with the same EHT synthesized beam data, and calculates both RMS and SSIM values within the same PSF region. The results are presented in Table 4: LiRBM achieves an RMS value of 0.015405, which is lower than those of the Clark CLEAN approximation (0.138998) and the Asp-Clean2016 approximation (0.216112); its SSIM value of 0.992112 also exceeds those of the latter two methods (0.303455 and 0.229640), demonstrating that LiRBM exhibits superior fitting performance in preserving both the overall PSF numerical response and spatial structure. It should be noted that Clark CLEAN and Asp-Clean2016 were originally designed primarily for deconvolution of radio-interferometric images; the corresponding RMS and SSIM values in the table were recalculated in this study based on the underlying processing principles of these two methods, utilizing the same EHT PSF data and uniform evaluation criteria. This comparison examines how various PSF approximation methods perform in preserving the full beam structure.

Table 4: Evaluation comparison with general PSF modeling methods

method	RMS	SSIM
LiRBM	0.015405	0.992112
Clark CLEAN[32]	0.138998	0.303455
Asp-Clean2016[33]	0.216112	0.229640

5.5 Local Modeling

The local evaluation setup includes both the mainlobe region and the region of the mainlobe and first sidelobe, and examines two issues directly related to the use of the PSF: first, whether the model can maintain its peak value, width, and edge falloff within the high-gradient region of the mainlobe; second, whether the model can preserve the polarity, relative position, and oscillation sequence of its positive and negative responses when the evaluation range includes the first sidelobe.

5.5.1 LiRBM Fitting for the Mainlobe

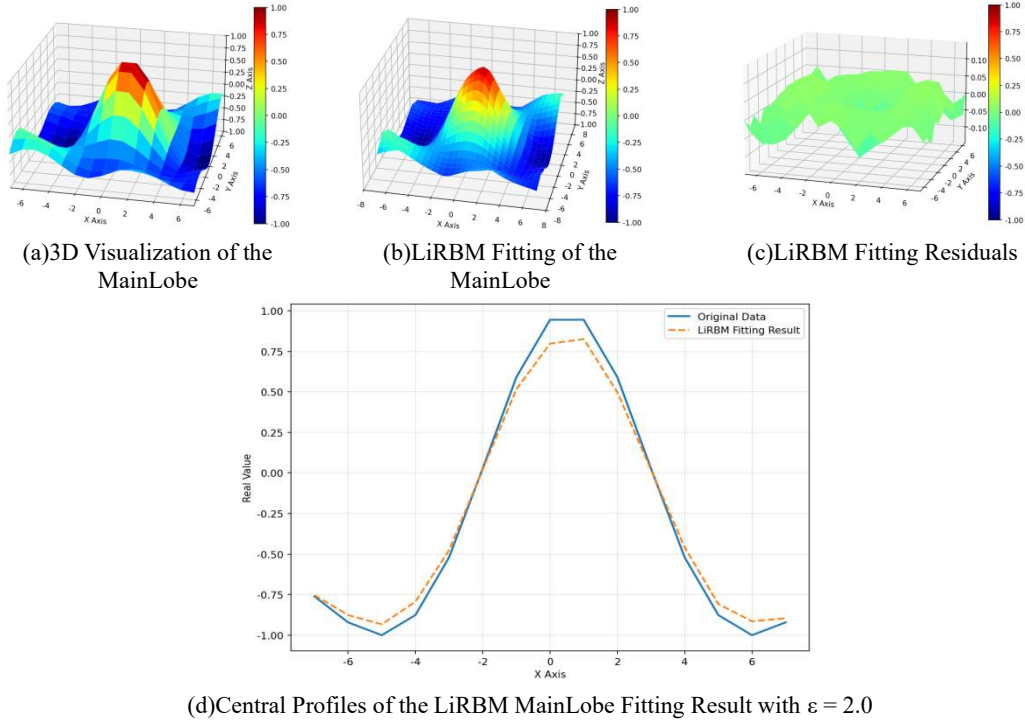


Figure 3: LiRBM-Based Modeling of the MainLobe

The mainlobe region includes the peak and the rapidly descending edges on both sides, making it the most critical region for local fitting. Figures 3(a)-(d) show the LiRBM fitting result and the central-profile comparison for the mainlobe region, respectively. In this region, the RMS value is 0.039695 and the SSIM value is 0.958808; these results indicate that the model successfully preserves the central position, overall width, and primary contour of the mainlobe, with the fitting results demonstrating high structural consistency with the original data. The local errors mainly appear as slight smoothing at the peak and reduced amplitudes at the extrema; however, the fitted width and offset of the mainlobe closely match those in the original data.

5.5.2 LiRBM Fitting for the Mainlobe and First Sidelobe

Figures 4(a)-(d) show the LiRBM fitting result and the central-profile comparison for the region of the mainlobe and the first sidelobe, respectively. When the evaluation scope is extended to include the mainlobe and the first sidelobe, the RMS value decreases to 0.030138, while the SSIM increases to 0.987305. This change does not necessarily mean that a larger fitting region yields better fitting results, because RMS and SSIM are regional statistical metrics; including more pixels with relatively smooth variations can reduce the average error. In summary, it can be concluded that within a large spatial domain, the model continues to maintain the sequential pattern of a central positive peak, adjacent negative troughs, and an outward rise, without observing any phenomena such as polarity reversal of the sidelobes, overall misalignment of peaks and troughs, or errors accumulating with distance.

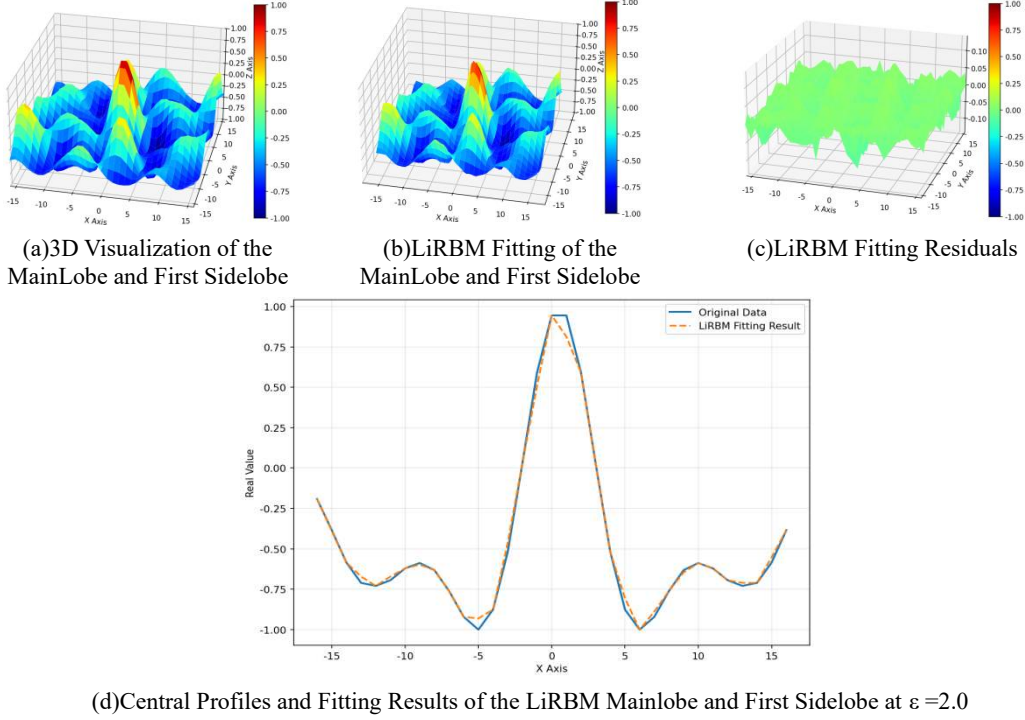


Figure 4: LiRBM-Based Modeling of the MainLobe and First Sidelobe

This demonstrates that LiRBM not only reproduces the local peak shape of the mainlobe but also preserves the relationship between the mainlobe and the first sidelobe formed by sparse (u , v) sampling. For radio-interferometric images, the sign and position of the sidelobes determine the convolution artifacts generated in adjacent regions by the point-source response; thus, preserving this structure indicates that LiRBM does not merely fit a single peak point exactly.

6 Conclusion and Future Directions

Addressing the need for a continuous functional representation of EHT synthesized beam images, this paper develops a LiRBM model based on Linear radial basis functions. The final model comprises 4096 RBF centers; under the current unified experimental conditions, it achieves a mean absolute residual of 0.010511, a maximum absolute residual of 0.124657, and a residual standard deviation of 0.015405—outperforming the other five basis function approaches across all three metrics. Compared with the second-best Thin Plate basis function, the mean absolute residual and residual standard deviation are reduced by 40.40% and 38.10%, respectively. The LiRBM fitting result, residual distribution, and central profile collectively show that the model effectively preserves the mainlobe position, sidelobe oscillations, and edge-attenuation trend.

The local evaluation further demonstrates that in the mainlobe region, the RMS is 0.039695 and the SSIM is 0.958808; in the region of the mainlobe and first sidelobe, the RMS is 0.030138 and the SSIM is 0.987305. The key feature of LiRBM is its ability to directly output a continuous PSF function, its center coordinates, and its weights, thereby providing a reproducible mathematical representation for subsequent convolution, degradation simulation, and image restoration tasks. The conclusions of this study are limited to the current samples and parameter settings; future work should use multiple synthesized beams with different structures, noise levels, and (u ,

v) coverage conditions, together with cross-validation and automatic parameter optimization, to further validate the model's stability and generalization capability.

Data availability

The EHT synthesized beam data used in this study were provided by the corresponding author and are available from the corresponding author upon reasonable request. The complete set of 4,096 RBF center coordinates and corresponding weight coefficients generated in this study is provided as Supplementary Data S1.

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Author contributions statement

Guoshengyi Wu: Software, Visualization, Formal analysis, Validation, Writing - original draft.

Li Zhang: Conceptualization, Funding acquisition, Methodology, Project administration, Resources, Supervision, Writing - original draft, Writing - review & editing.

Competing interests

The authors declare no competing interests.

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