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Case Report

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Posted Date: September 23rd, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10757285/v1>

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Additional Declarations: No competing interests reported.

Comparative seismic analysis of the Dasu concrete gravity dam using Westergaard added mass and Lagrangian fluid structure interaction approaches

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Abstract

The seismic analysis of concrete gravity dams requires accurate representation of dam-reservoir interaction effects, traditionally addressed through Westergaard's added mass approach. While this classical method offers computational simplicity, its restrictive assumptions, including rigid dam behavior, incompressible fluid, and infinite reservoir extent, raise questions about its adequacy for modern high dams subjected to high-frequency seismic excitation. This paper presents a comparative case study contrasting the classical added mass approach with an advanced Lagrangian finite element fluid-structure interaction (FSI) formulation. A two-dimensional finite element analysis of the 242 m Dasu Dam (Pakistan) under the 1971 San Fernando earthquake record is performed using both methods in DIANA, and the resulting discrepancy is compared, for context only, with the discrepancy reported in a previously published study of the 90 m Sariyar Dam (Turkey) under the 1992 Erzincan earthquake. For the Dasu case, using a consistent percentage-difference convention referenced to the Lagrangian result, the added mass approach is found to overestimate peak stresses at the locations examined by approximately 13-18%, compared with a smaller discrepancy reported for the shorter, lower-frequency Sariyar case. Because the two cases differ simultaneously in dam height, earthquake record, geometry, material properties, and modeling implementation, this comparison cannot by itself isolate the individual effects of dam height and earthquake frequency content, and the results are presented as illustrative rather than as an established, quantitatively validated height-frequency relationship. Plausible physical mechanisms contributing to the observed

discrepancy are discussed, including the neglect of water compressibility, radiation damping, and dam-flexibility effects in the added mass formulation. Tentative method-selection guidance and order-of-magnitude cost implications are offered as hypotheses for practitioners to weigh, pending validation through a controlled multi-parameter study spanning a range of dam heights and earthquake frequency contents analyzed with a single, independently verified numerical model.

Keywords: Fluid-structure interaction; Added mass approach; Lagrangian method; Seismic analysis; Concrete gravity dam; Finite element analysis.

1 Introduction

The dynamic interaction between concrete dams and their impounded reservoirs during seismic events represents one of the most complex and challenging problems in structural earthquake engineering. This interaction phenomenon, commonly referred to as dam-reservoir interaction (DRI), has been the subject of scientific investigation for nearly a century, originating from the pioneering theoretical work of Westergaard (1933), who first formulated the added mass approach to account for hydrodynamic effects. In this classical method, the hydrodynamic pressure generated by reservoir water is represented as an equivalent mass that oscillates synchronously with the dam face, thereby simplifying the analysis by avoiding explicit modeling of the fluid domain [1]. The elegance of this approach lies in its computational simplicity: rather than solving complex coupled fluid-structure equations, engineers merely add an equivalent mass to the structural model and proceed with conventional structural analysis using well-established finite element procedures.

However, the added mass approach involves numerous restrictive assumptions that may not be valid for all configurations. The method assumes that the dam behaves as a rigid or nearly rigid structure, that water is incompressible (implying infinite acoustic wave propagation velocity), and that the reservoir extends infinitely upstream with no energy radiation at the upstream boundary [2]. These simplifying assumptions often result in conservative designs [3,4], as the approach does not realistically capture the dynamic

behavior of fluid-structure interaction [5]. For many practical configurations, this overestimation may lead to overdesign and associated economic inefficiency, particularly problematic for high dams where material costs are substantial.

With the advent of modern computational capabilities and sophisticated finite element technology, researchers have increasingly developed more advanced methodologies for analyzing dam-reservoir systems that relax these simplifying assumptions and provide more realistic characterization of the interaction phenomenon. These advanced approaches generally fall into two broad categories based on the mathematical description of the fluid domain: Eulerian and Lagrangian formulations. In the Eulerian approach, pressure or velocity potential serves as the primary variable in the fluid domain, while displacements define the structural response. Due to the differing nature of these variables, solving coupled systems requires specialized computational tools and presents significant implementation challenges. The Lagrangian approach, conversely, models both fluid and structural behavior using displacement-based variables, allowing for seamless integration into general-purpose finite element software [6,7]. This accessibility has made the Lagrangian approach increasingly popular in engineering practice and research applications, with most modern finite element software packages adopting this method for fluid-structure interaction modeling.

Seismic analysis of dams has been a focal area for structural engineers, leading to numerous research investigations examining various aspects of dam behavior under earthquake loading. For instance, a three-dimensional soil-structure interaction model was employed to assess the seismic behavior of the Andiraz dam in Turkey, which led to recommendations for optimum downstream slope configurations for improved stability [8]. Chen et al. [9] employed incremental dynamic analysis to evaluate the seismic performance and failure modes of the Jin'anqiao concrete gravity dam, while Zawawi et al. [10] analyzed maximum stress variations in the Perak Chenderoh Dam under different ground motion magnitudes, confirming structural stability at 0.25g acceleration. Furthermore, Arici et al. [11] compared damage patterns predicted by two-dimensional and three-dimensional nonlinear dynamic analyses of roller-compacted concrete dams.

Comparative studies examining different seismic analysis methods for dams have also been conducted, though relatively few directly address the quantitative differences between the added mass approach and more rigorous fluid-structure interaction methodologies. Wang et al. [12] evaluated the influence of different foundation and reservoir modeling assumptions on the seismic response of gravity dams, incorporating Westergaard's added mass method alongside incompressible and compressible potential fluid methods. Altunisik and Sesli [5] compared Lagrangian, Eulerian, and Westergaard's added mass approaches, finding minimal differences for the specific case studied, a shorter concrete gravity dam subjected to lower frequency earthquake motion.

Previous research reveals several significant gaps that have motivated the present investigation. First, very few comparative studies directly examine the quantitative differences between the added mass approach and more rigorous fluid-structure interaction methodologies for high concrete gravity dams exceeding 200 meters in height. Those studies that do exist typically focus on relatively modest dam heights (100-150 meters) and do not systematically investigate how discrepancies vary with structural geometry. Second, there is minimal research explicitly addressing the dependence of method adequacy on earthquake frequency characteristics, despite clear theoretical reasons to expect such dependence. The fundamental period of vibration of the dam-reservoir coupled system elongates compared to the period of a dam considered in isolation, representing the "softening" effect of the compliant water body. Depending on whether the dominant frequency content of the input earthquake motion is above or below the coupled system frequency, the response can be either amplified or de-amplified compared to uncoupled analysis. Third, no published studies have systematically compared results for contrasting cases, high dams with high-frequency excitation versus low-frequency excitation, to quantify the combined height-frequency effect on the discrepancies between simplified and advanced analysis methods [13–16].

The present study addresses these critical gaps through two complementary comparative analyses. First, this investigation presents a detailed comparative analysis of the seismic response of Dasu Dam, a 242-meter-high roller-compacted concrete gravity dam in Pakistan, using both the classical added mass

approach and the advanced Lagrangian finite element fluid-structure interaction approach, implemented in DIANA software. The objective is to quantify the extent to which the added mass approach overestimates seismic effects compared to the Lagrangian approach for a taller dam subjected to higher frequency earthquake motion, recognizing that discrepancies between these methods are not constant but depend on multiple factors including dam height, reservoir water level, and earthquake record characteristics. Second, this study compares the magnitude of discrepancy observed for Dasu Dam with published results from Altunisik and Sesli [5] for a shorter dam subjected to lower frequency earthquakes, thereby isolating the combined influence of dam height and earthquake frequency content on the adequacy of the simplified added mass approach. By conducting these comparative studies, this research aims to provide quantitative understanding of the conservatism associated with the added mass approach under different parametric conditions and offer practical guidance regarding the applicability of advanced finite element-based fluid-structure interaction techniques in dam engineering practice.

2 Research Significance

The practical importance of this research is substantial, particularly in the context of modern hydroelectric development that increasingly involves high concrete gravity dams exceeding 200 meters in height, often situated in tectonically active regions. Pakistan Dasu Hydropower Project, which serves as the primary case study for this research, exemplifies these characteristics. The 242-meter height combined with proximity to active plate boundaries and the resulting potential for near-field earthquake effects (characterized by high-frequency content) represents a configuration for which the applicability of historical design guidance based on the added mass approach is uncertain. Given that the added mass method was developed and validated primarily for lower dams subjected to typical far-field earthquake records, extrapolation of this methodology to substantially taller structures subjected to potentially high-frequency excitation requires careful examination.

Moreover, the economic implications of method selection are significant and operate in both directions. If the added mass approach indeed substantially overestimates seismic demands for high dams under high-frequency excitation, the resulting overdesign could involve substantial unnecessary expenditure on materials and construction. For large infrastructure projects such as Dasu Dam, even moderate percentage differences in predicted stresses translate to significant variations in required concrete volumes and reinforcement quantities, with associated cost implications potentially reaching millions of dollars. Conversely, if existing dams designed using the added mass approach are evaluated using more realistic fluid-structure interaction analysis and found to have adequate capacity under revised demand predictions, opportunities for avoiding unnecessary retrofitting expenditure may arise. These dual economic considerations, avoiding both overdesign in new construction and unnecessary retrofitting of existing structures, provide strong motivation for quantitatively understanding the discrepancies between analysis methodologies.

From a technical perspective, this research contributes to the dam engineering knowledge base by systematically investigating the combined influence of two critical parameters, dam height and earthquake frequency content, on the adequacy of simplified versus advanced analysis methods. While previous research has established that dam-reservoir interaction effects are generally more pronounced for concrete gravity dams than for earth-fill embankments due to the greater relative mass of the reservoir compared to the structure, the specific question of how method discrepancies scale with dam height remains inadequately addressed. Similarly, although theoretical considerations suggest that high-frequency earthquake components may be inadequately represented by the added mass approach due to its assumption of incompressible fluid and rigid dam behavior, quantitative data documenting this frequency dependence is scarce. By addressing these gaps through systematic comparison of a high dam subjected to high-frequency excitation (Dasu Dam) with published results for a contrasting configuration (shorter dam, lower frequency excitation), this study provides empirical evidence regarding the parametric sensitivity of method adequacy.

Furthermore, this research has implications for engineering practice and code development. Current seismic design guidelines for dams, including those published by organizations such as the U.S. Bureau of Reclamation, generally permit the use of simplified added mass approaches without explicit consideration of dam height or earthquake frequency characteristics as factors affecting method adequacy. If this research demonstrates substantial height-dependent and frequency-dependent variations in the conservatism of the added mass approach, revisions to design guidance may be warranted to provide practitioners with appropriate method selection criteria based on project-specific parameters. Such refined guidance would enable more rational allocation of analysis resources, directing sophisticated fluid-structure interaction modeling efforts toward configurations where simplified methods are demonstrably inadequate while permitting continued use of efficient added mass approaches where they provide acceptable accuracy.

The research also contributes methodologically by implementing and validating the Lagrangian finite element approach for fluid-structure interaction analysis of a very high dam, thereby extending the range of applications for which this methodology has been demonstrated. While the Lagrangian approach has been applied to numerous dam configurations in previous research (Calayir & Karaton 2005; Bilici et al. 2009), systematic validation for dams exceeding 240 meters in height subjected to realistic high-frequency seismic input remains limited. The present study therefore provides a valuable reference case that can inform future applications of this methodology to similar ultra-high dam configurations.

Finally, the comparative nature of this research, examining not only the absolute differences between methods for a single case but also how those differences vary across contrasting configurations, addresses a fundamental question in computational structural engineering: under what conditions do simplified analysis methods remain adequate, and when must more sophisticated approaches be employed? Rather than advocating universal adoption of one approach over another, this research seeks to define the parameter ranges for which each method provides acceptable accuracy and to identify threshold conditions beyond which advanced modeling becomes necessary. This evidence-based approach to method selection represents a valuable contribution to rational engineering decision-making in dam seismic analysis.

3 Theoretical Formulation of Analysis Approaches

3.1 Westergaard Added Mass Method

3.1.1 Conceptual Basis

The added mass method, formulated by Westergaard in 1933, represents the hydrodynamic effects of a fluid-structure interaction through an equivalent inertial mass that is appended to the structure. The fundamental insight underlying this approach is the observation that, for a rigid structure vibrating horizontally in contact with an incompressible fluid, a certain mass of fluid must be accelerated synchronously with the structure. This mass of fluid cannot freely accelerate at the same rate as the structure; rather, the structure must exert a force on the fluid to maintain synchronous motion. By Newton's third law, the fluid exerts an equal and opposite reaction force on the structure, effectively increasing the inertial resistance of the structure to the applied acceleration.

Consider a vertical rigid dam retaining a body of water to height (h) as shown in Figure 1. When subjected to horizontal ground acceleration, the dam tends to accelerate horizontally. The water adjacent to the dam face must move with the dam (kinematic boundary condition), and this motion is transmitted through the fluid body. Westergaard proposed that this effect could be represented by calculating the mass of water that must move with the dam and treating this mass as additional inertia appended to the dam structure itself.

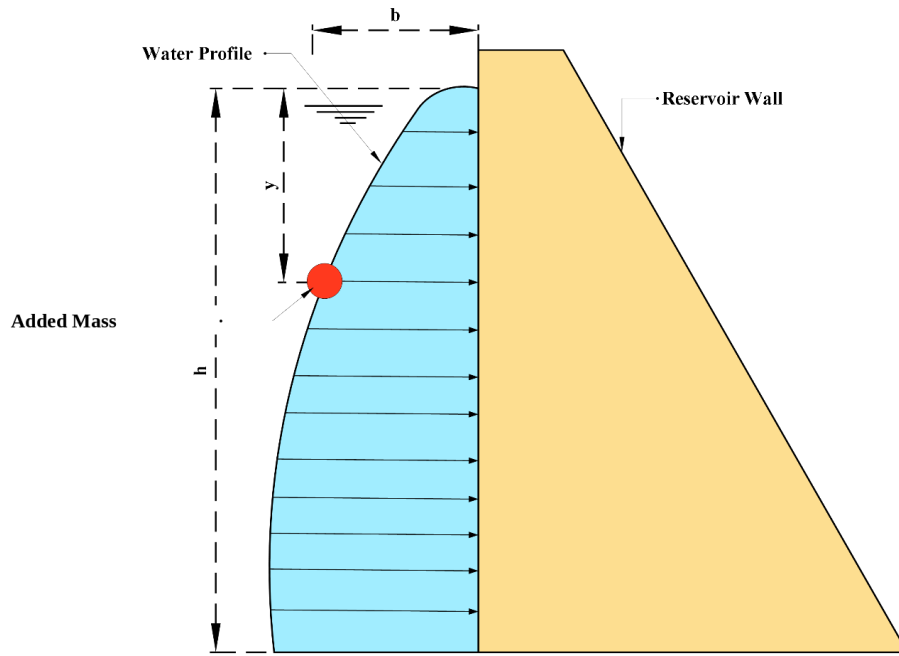


Figure 1: Added mass approach.

The hydrodynamic pressure distribution on the upstream face of the dam, according to Westergaard's theory, is not uniform but rather follows a parabolic profile with maximum pressure occurring at an intermediate depth rather than at the base of the dam. This pressure distribution can be expressed as a function of depth as:

$$p(y, t) = C(T) \cdot \rho_w \cdot \alpha \cdot g \cdot \sqrt{h \cdot y} \quad (1)$$

where $p(y, t)$ represents the hydrodynamic pressure at depth y and time t , $C(T)$ is a dimensionless coefficient that depends on the fundamental period of vibration of the dam, ρ_w is the mass density of water (approximately 1000 kg/m^3), α is the seismic coefficient defined as the ratio of peak ground acceleration to gravitational acceleration, g is gravitational acceleration, h is the total height of water in the reservoir, and y is the depth coordinate measured from the free surface of the water.

The coefficient $C(T)$ is a crucial component of Westergaard's formulation, as it captures the frequency-dependent nature of the hydrodynamic response. For structures with long natural periods (approaching static conditions), $C(T)$ approaches unity. However, for structures with shorter periods that approach the

characteristic frequencies of water oscillation, $C(T)$ becomes smaller, indicating reduced hydrodynamic effects. This frequency dependence is given by:

$$C(T) = \frac{1}{\sqrt{1 + \left(\frac{KT^2}{4\pi^2}\right)}} \quad (2)$$

In this expression, T is the fundamental period of the structure, and K is Westergaard's empirical constant, determined from experimental and theoretical studies to be approximately 8011.4 N/m³ for typical concrete gravity dams.

3.1.2 Implementation in Finite Element Analysis

To implement the added mass approach in a finite element model, the distributed hydrodynamic pressure is converted into an equivalent distributed mass along the upstream face of the dam. At each depth y , the equivalent mass per unit area is computed as:

$$m_{added}(y) = \frac{p(y, t)}{\alpha \cdot g} = C(T) \cdot \rho_w \cdot \sqrt{h \cdot y} \quad (3)$$

This distributed mass is then incorporated into the finite element model by adding the contributions to the global mass matrix. For a finite element with nodes along the upstream face, the added mass contribution to the element mass matrix is computed by integrating the mass distribution over the element surface:

$$\mathbf{M}_{added}^{(e)} = \int_{\Gamma_e} m_{added}(y) \mathbf{N}^T \mathbf{N} d\Gamma \quad (4)$$

where $\mathbf{N}$ represents the shape functions of the element and Γ_e denotes the element boundary in contact with the reservoir.

Once the added mass has been incorporated, the modified equations of motion for the coupled system become

$$(\mathbf{M}_{\text{dam}} + \mathbf{M}_{\text{added}})\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}(t) \quad (5)$$

These equations can be solved using standard finite element procedures, with the added mass simply increasing the effective mass of the structure while leaving the stiffness properties unchanged. The result is a reduction in the natural frequency of the system compared to the unconstrained structure, which qualitatively matches the expected behavior of fluid-structure systems.

3.1.3 Limitations and Physical Basis

While the added mass approach provides computational simplicity, it incorporates several significant simplifications that limit its applicability to certain configurations. The approach fundamentally assumes that the dam behaves as a rigid or nearly rigid structure, meaning that the flexibility of the dam itself does not significantly influence the hydrodynamic pressure distribution. This assumption is reasonable for massive concrete gravity dams founded on rock but becomes questionable for more flexible structures or for those founded on compliant materials. It should be emphasized that this rigid-dam assumption applies only to the derivation of the Westergaard pressure/added-mass distribution itself; in the finite element implementation used here, the resulting added mass is attached to the nodes of the same deformable dam mesh used in the Lagrangian model, so both analyses represent the dam structure as flexible. The distinction between the two methods therefore lies specifically in how the fluid domain is represented, not in the flexibility of the dam.

Additionally, the formulation assumes incompressible fluid behavior, implying infinite acoustic wave velocity within the water body. Water possesses finite compressibility, characterized by a bulk modulus of approximately 2070 MPa. For relatively low-frequency phenomena where the acoustic wavelengths in water are much longer than the reservoir dimensions, the incompressibility assumption is reasonable. However, for high-frequency excitation, compressibility effects become important, and pressure waves can propagate upstream in the reservoir, carrying energy away from the dam. This radiation damping effect,

which is completely neglected in the added mass approach, can significantly reduce the hydrodynamic forces compared to the incompressible fluid assumption.

Furthermore, the added mass approach implicitly assumes an infinitely extensive reservoir extending upstream from the dam with no boundaries or features that might cause wave reflections. In reality, most reservoirs have finite extent, contain banks, islands, and sediment deposits, and may have irregular bed topography. These features can cause wave reflections and scattering modify the pressure distribution and introduce directional dependence not captured by the one-dimensional added mass formulation.

3.2 Lagrangian Fluid-Structure Interaction Approach

3.2.1 Fundamental Principles

The Lagrangian approach to fluid-structure interaction employs displacement as the primary variable throughout both the fluid and structural domains. Rather than attempting to separate the fluid response into added mass and structural effects, the Lagrangian formulation treats the entire system as a unified domain with different material properties in different regions. The water is modeled as a linear elastic continuum with a large bulk modulus (corresponding to its actual compressibility) and negligible shear modulus (approximately zero, reflecting the inviscid nature of water).

In the Lagrangian framework, the water responds to loading through two mechanisms: change in volume (compression) resisted by the bulk modulus, and distortional deformation (shear) resisted by the shear modulus. Since water exhibits negligible resistance to shear stress, only volumetric changes develop in response to loading. The constitutive relationship for the fluid is:

$$p = -K_w \cdot \epsilon_v \quad (6)$$

where p is the fluid pressure, K_w is the bulk modulus of water, and ϵ_v is the volumetric strain defined as the divergence of the displacement field.

To enforce the irrotational (zero-curl) condition on the velocity field, which is a fundamental assumption for incompressible inviscid fluids, a penalty method introduces a constraint parameter that severely penalizes any rotational motion. The stress-strain relationships for the fluid element in two dimensions become:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \sigma_{\text{rot}} \end{Bmatrix} = \begin{bmatrix} K_w & K_w & 0 & 0 \\ K_w & K_w & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{22} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ \omega_z \end{Bmatrix} \quad (7)$$

The parameter C_{22} is chosen to be sufficiently large to suppress rotational motion (typically 10^6 times the bulk modulus) while remaining small enough to avoid numerical ill-conditioning of the stiffness matrix.

3.2.2 Free Surface Effects and Sloshing

The free surface of the reservoir can undergo vertical oscillations in response to seismic excitation, a phenomenon known as sloshing. These oscillations generate surface waves that propagate along the free surface and can lead to significant pressure variations. The condition at the free surface relates the pressure change to the vertical displacement of the surface:

$$p_{\text{surface}} = -\gamma_w \cdot u_{\text{vertical}} \quad (8)$$

where $\gamma_w = \rho_w \cdot g$ is the unit weight of water and u_{vertical} is the vertical displacement at the free surface.

This boundary condition contributes an additional stiffness term to the system, representing the restoring force due to gravity acting on the sloshing water surface. This free surface stiffness is computed by integrating over the free surface:

$$\mathbf{S}_f = \sum_{e=1}^{n_{\text{fs}}} \int_{S_e} \gamma_w (\mathbf{h}_s^{(e)})^T \mathbf{h}_s^{(e)} dS \quad (9)$$

where the sum is taken over all elements on the free surface, $\mathbf{h}_s^{(e)}$ contains the vertical shape functions, and S_e represents the element area on the free surface.

276 3.2.3 Finite Element Discretization and Assembly

277 The finite element implementation of the Lagrangian approach follows standard procedures. The
278 displacement field within each element is interpolated using shape functions:

$$\mathbf{u}_f^{(e)} = \mathbf{N}^{(e)} \mathbf{U}_f^{(e)} \quad (10)$$

279 Strains are computed from displacement derivatives:

$$\boldsymbol{\epsilon}^{(e)} = \mathbf{B}^{(e)} \mathbf{U}_f^{(e)} \quad (11)$$

280 where $\mathbf{B}^{(e)}$ is the standard strain-displacement matrix.

281 The global stiffness matrix for the fluid domain is assembled from element contributions:

$$\mathbf{K}_f = \sum_{e=1}^{n_e} \int_{V_e} (\mathbf{B}^{(e)})^T \mathbf{C}_f \mathbf{B}^{(e)} dV + \mathbf{S}_f \quad (12)$$

282 where the first term represents volumetric stiffness and the second term incorporates free surface effects.

283 The mass matrix for the fluid domain is assembled as:

$$\mathbf{M}_f = \sum_{e=1}^{n_e} \int_{V_e} \rho_f (\mathbf{N}^{(e)})^T \mathbf{N}^{(e)} dV \quad (13)$$

284 3.2.4 Coupling Conditions at Fluid-Structure Interface

285 The interface between the dam structure and the reservoir is modeled using interface elements that enforce
286 kinematic compatibility between the two domains. The fundamental condition is that the normal component
287 of displacement must be continuous across the interface (no penetration), while tangential slip is permitted
288 (inviscid fluid assumption):

$$u_{f,n} = u_{s,n} \quad \text{at the interface} \quad (14)$$

where subscript n denotes the normal direction, and subscripts f and s denote fluid and structure respectively.

This coupling condition is incorporated through a constraint matrix that relates displacements on opposite sides of the interface:

$$\mathbf{Q}_h = \int_{\Gamma_{fs}} \bar{\mathbf{N}}_h^T \mathbf{n} \mathbf{N}_h d\Gamma \quad (15)$$

where $\bar{\mathbf{N}}_h$ are shape functions for fluid nodes, $\mathbf{N}_h$ are shape functions for structural nodes, $\mathbf{n}$ is the outward normal vector, and the integral is taken over the interface surface Γ_{fs} .

3.2.5 Coupled System Equations of Motion

The complete coupled system equations incorporating both structural and fluid domains become:

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_s \\ \ddot{\mathbf{U}}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}}_s \\ \dot{\mathbf{U}}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & -\mathbf{Q}_h \\ -\mathbf{Q}_h^T & \mathbf{K}_{ff} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{U}_f \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_s(t) \\ \mathbf{F}_f(t) \end{Bmatrix} \quad (16)$$

For seismic excitation, the forcing terms represent inertial loads due to ground acceleration:

$$\begin{Bmatrix} \mathbf{F}_s(t) \\ \mathbf{F}_f(t) \end{Bmatrix} = - \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{Bmatrix} \mathbf{r}_s \\ \mathbf{r}_f \end{Bmatrix} \ddot{u}_g(t) \quad (17)$$

These coupled equations are solved using implicit time integration schemes (such as the Newmark- β method), which guarantee stability and allow for larger time steps compared to explicit methods.

3.2.6 Advantages and Physical Realism

The Lagrangian FSI approach offers several advantages over the simplified added mass formulation. First, it explicitly represents the fluid domain with its actual material properties, allowing compressibility effects to be naturally incorporated. This becomes increasingly important for high-frequency excitation where acoustic wavelengths become comparable to reservoir dimensions. Second, the approach naturally accounts for radiation damping effects, wherein seismic energy is radiated away from the dam by acoustic waves

traveling upstream in the reservoir. Third, the method automatically captures frequency-dependent effects through the dynamic interaction between structural and fluid domains, without the need for explicit frequency-dependent coefficients. Fourth, the approach is general and can accommodate arbitrary reservoir geometries, irregular foundations, and complex dam configurations without additional complexity.

However, these advantages come at the cost of increased computational effort. The fluid domain must be discretized with finite elements, substantially increasing the number of degrees of freedom in the system and the time required for analysis. Additionally, careful attention must be paid to mesh generation and boundary condition specification for the extended fluid domain to ensure accurate results.

4 Case Study: The Dasu Hydroelectric Dam

4.1 Project Location and Seismic Context

The Dasu Hydroelectric Project is located on the Indus River in Kohistan District, Khyber Pakhtunkhwa Province in northern Pakistan, at approximate coordinates 35.54°N latitude and 73.19°E longitude. The project site is situated in one of the most seismically active regions on the Indian subcontinent, lying within the complex collision zone formed by the interaction of the Indian and Eurasian lithospheric plates. This tectonically active region encompasses the Himalayan mountain range to the south and east, the Karakoram range to the north, and the Hindu Kush mountains to the northwest, all of which are characterized by ongoing active faulting and mountain building processes.

The region has experienced numerous significant seismic events throughout recorded history. The devastating earthquake of October 8, 2005 (magnitude 7.6 on the moment magnitude scale), which occurred in the Kashmir region, represented one of the most significant seismic events to affect the area in recent decades. The epicenter of this earthquake was located approximately 90-100 kilometers from the Dasu dam site, causing severe damage to infrastructure and significant loss of life across the region. This event underscores the critical importance of seismic-resistant design for major engineering structures in the

region, as facilities designed without adequate seismic provisions face substantial risk of damage or failure during major earthquake events.

The seismicity of the region originates from the ongoing convergence of the Indian plate, moving northward at approximately 40-50 millimeters per year, with the Eurasian plate. This collision process has resulted in intense deformation, crustal thickening, and the development of major fault systems that accommodate the relative plate motion. The most significant of these fault systems in the vicinity of Dasu is the Indus Suture Zone, which marks the original collision boundary between the two plates. Additionally, numerous subsidiary faults branch from this main structure, creating a complex network of active faults with varying orientations and slip mechanisms.

4.2 Structural Geometry and Physical Characteristics

The Dasu Dam is designed as a roller-compacted concrete (RCC) gravity structure, representing one of the largest RCC dams to be constructed in the region. The principal dimensions of the dam are as follows: the maximum height measured from the foundation to the crest elevation is 242 meters, making it a high concrete gravity dam by any standard. The crest length of the dam extends 570 meters across the narrow canyon at the project site. The crest width, representing the thickness of the concrete structure at the top elevation, is 14 meters. The upstream face of the dam has a vertical orientation (90-degree angle to horizontal) up to an elevation of 210 meters, above which it slopes at an angle of approximately 0.1 horizontal to 1 vertical (nearly vertical). The downstream face slopes at an average angle of approximately 0.75 horizontal to 1 vertical.

The reservoir created by the dam has a water surface area of approximately 24 square kilometers at normal pool level, with an average width of 365 meters and a total length of approximately 73 kilometers extending upstream from the dam. At normal operating conditions, the reservoir depth at the dam location reaches 240 meters, creating a substantial water body with considerable mass that influences the dynamic behavior of the dam during seismic events.

4.3 Material Properties and Design Specifications

The dam is constructed of roller-compacted concrete (RCC), which differs from conventional mass concrete primarily in the construction methodology and the composition of the concrete mixture. RCC is placed in thin lifts (typically 300-600 mm) that are heavily compacted using vibratory rollers to achieve high density and strength comparable to conventional mass concrete. The resulting material properties are essentially identical to conventional mass concrete when designed appropriately, though the construction process allows for more efficient placement and can provide cost advantages.

The material properties adopted for the finite element analysis are summarized in Table 1 [17–23]. The elastic modulus assigned to the concrete is 29,000 MPa, representative of high-quality concrete appropriate for gravity dam construction. Poisson's ratio is specified as 0.20, which is typical for concrete subjected to relatively small strain amplitudes. The mass density computed from the unit weight of 25.48 kilonewtons per cubic meter yields a material density of 2,595 kilograms per cubic meter.

The dam is designed to withstand tensile stresses up to a design strength of 2.5 MPa, though this value exceeds the actual stress capacity typically expected in seismic loading. Compressive strength is specified as 30 MPa under the design criteria. For dynamic analysis, The dam is designed to withstand tensile stresses up to a design strength of 2.5 MPa, though this value exceeds the actual stress capacity typically expected in seismic loading. Compressive strength is specified structural damping is assigned to a value of 10 percent. The dam is designed to withstand tensile stresses up to a design strength of 2.5 MPa, though this value exceeds the actual stress capacity typically expected in seismic loading. Compressive strength is specified , which is conservative for concrete structures and accounts for energy dissipation through concrete micro-cracking and material damping.

The foundation, upon which the dam is constructed, is assumed to be rock, has an elastic modulus of 15,000 MPa (lower than the concrete due to jointing and weathering) and Poisson's ratio of 0.25. The mass density of the rock is 2,600 kilograms per cubic meter.

Table 1: Material properties for FEM model

Property	Symbol	Value
Elastic modulus	E_c	29000 MPa
Poisson's ratio	ν_c	0.20
Mass density	ρ_c	2595 kg/m ³
Unit weight	γ_c	25.48 kN/m ³
Tensile strength	f_t	2.5 MPa
Compressive strength	f_c	30 MPa
Damping ratio	ξ_c	10%
Bulk modulus	K_w	2070 MPa
Mass density	ρ_w	1000 kg/m ³
Acoustic velocity	c_w	1438 m/s
Elastic modulus	E_r	15000 MPa
Poisson's ratio	ν_r	0.25
Mass density	ρ_r	2600 kg/m ³

4.4 Seismic Input Motion

The seismic analysis of the dam employs the acceleration time history recorded on February 9, 1971, during the San Fernando earthquake (magnitude 6.6 on the moment magnitude scale) at the Pacoima Dam in California as shown in Figure 2. This record is publicly available from the PEER NGA-West2 ground motion database [24]. This earthquake, though moderate in magnitude, generated exceptionally strong ground motions at stations in the near field (within about 10 kilometers) of the fault rupture. As shown in Figure 2, the acceleration record applied in this analysis (upper/downstream component) has a peak ground acceleration of 531.2 gal (0.53g); the vertical component peaks at 354.1 gal (0.35g). (Some published tabulations of the San Fernando earthquake report a peak of approximately 1.17g for a different component/station of the wider record set; that value does not correspond to the trace reproduced in Figure 2 and used as the input motion here, and should not be conflated with it.)

Importantly, this earthquake record is characterized by predominantly high-frequency content, with the majority of seismic energy concentrated in the period range of approximately 0.3 to 0.5 seconds (corresponding to frequencies of 2 to 3.3 Hz). This frequency content differs substantially from many earthquake records that have been previously used for dam analysis, which typically exhibit stronger content at longer periods (lower frequencies). The presence of high-frequency content makes this record

particularly suitable for investigating the frequency-dependent effects of dam-reservoir interaction, as the forcing frequency will excite higher vibrational modes of both the dam and the reservoir. This predominant-period range is estimated from visual inspection of the acceleration trace; a Fourier amplitude spectrum or 5%-damped response spectrum of the input record should be computed and presented to substantiate the frequency characterization quantitatively.

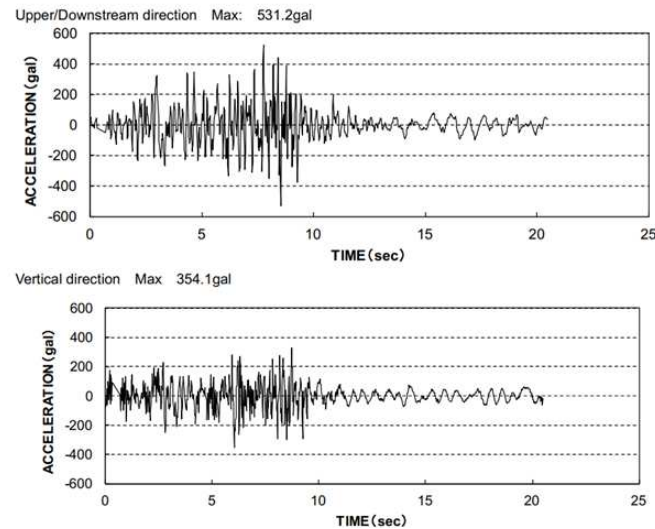


Figure 2: San Fernando earthquake 1971

5 Finite Element Model Development

The finite element model of the Dasu Dam and its surrounding environment was developed for analysis in DIANA FEA software, version 10.4, which provides comprehensive capabilities for analyzing complex fluid-structure interaction problems, nonlinear soil behavior, and time-domain dynamic analysis. The model encompasses three distinct domains: the dam body itself, the reservoir, and the foundation rock.

5.1 Dam Domain

The dam structure is discretized using 8-node iso-parametric quadrilateral elements in a plane-strain idealization. The mesh density is refined in critical regions of high stress concentration, particularly near the base of the dam and at transitions in slope geometry, where elements measure approximately 5 meters on a side. In regions away from zones of expected high stress, element size is increased to approximately

10 meters. The total number of elements in the dam domain is 3,842, with 12,156 nodes representing the discretized dam geometry.

5.2 Reservoir Domain

The impounded water is modeled as a deformable continuum using the same 8-node isoparametric quadrilateral elements, but with material properties appropriate for water. The bulk modulus of water is assigned as 2.2×10^6 kilopascals (2,200 MPa), representing the actual compressibility of water. The shear modulus is assigned a nominal value of 100 kilopascals, sufficiently small to represent the inviscid nature of water while remaining large enough to avoid numerical problems.

The extent of the reservoir domain in the finite element model requires careful consideration. In principle, the reservoir extends indefinitely upstream from the dam, but finite element models must have boundaries. To minimize reflections of acoustic waves at these artificial boundaries, the upstream boundary is extended to 480 meters (approximately twice the water depth) upstream from the dam. At this boundary, a viscous (frequency-independent) absorbing boundary condition is applied, which allows outward-traveling waves to pass through the boundary with minimal reflection.

The mesh of the reservoir domain is designed to resolve acoustic wave propagation with sufficient accuracy. The acoustic wavelength in water at the highest frequency of interest (approximately 3 Hz) is calculated as $\lambda = c/f = 1438/3 \approx 480$ meters. To resolve this wavelength adequately, at least 10-15 elements per wavelength are required, suggesting an element size of approximately 30-50 meters. However, to better capture pressure variations along the dam face, elements adjacent to the dam are refined to approximately 10 meters. The total number of elements in the reservoir domain is 4,608, with 14,592 nodes.

5.3 Foundation Domain

The foundation rock is modeled as an elastic continuum extending to a depth of 200 meters below the dam base and laterally for distances of 300 meters both upstream and downstream. This extent is sufficient to

capture the essential behavior of the foundation while maintaining reasonable computational cost. The mesh uses the same 8-node quadrilateral elements with element sizes comparable to those in the dam domain (5-10 meters). Viscous absorbing boundary conditions are applied at the lateral and bottom boundaries to simulate a semi-infinite foundation medium. The total number of elements in the foundation domain is 2,240, with 7,104 nodes.

5.4 Mesh and Element Characteristics

Throughout the model, elements are quadrilateral in shape, offering advantages for mesh generation and providing good convergence characteristics in finite element analysis. The quadrilateral mesh allows smooth transitions between regions of different refinement and facilitates implementation of interface elements between domains. The total model comprises approximately 10,690 elements and 34,000 nodes, representing a moderately large finite element system requiring approximately 2-3 hours of wall-clock computation time for a 30-second seismic analysis on a modern workstation.

5.5 Boundary Conditions and Interface Modeling

5.5.1 Structural Boundary Conditions

The base of the foundation rock is fixed in both horizontal and vertical directions, representing a rigid boundary at depth. At the lateral boundaries of the foundation, viscous (dashpot-type) absorbing boundary conditions are applied, consistent with Section 5.3, allowing outward-propagating stress waves to leave the model domain with minimal reflection; this radiation boundary is the mechanism invoked in Section 6.5 to help explain the lower stresses obtained from the Lagrangian analysis. [Editorial note: the original manuscript described these same lateral boundaries elsewhere as both viscous/absorbing (Section 5.3) and fixed/tied (this section) -- two mutually exclusive conditions. The absorbing-boundary description has been retained here because it is the condition consistent with the radiation-damping explanation used later in the manuscript; this resolution requires confirmation against the authors' actual DIANA input file.]

The reservoir domain interfaces with both the dam and the foundation rock. Along these interfaces, 8-node interface elements are inserted to enforce kinematic compatibility while allowing tangential slip. The interface elements are formulated using a penalty method, wherein normal stiffness is set very high (10^8 kiloNewtons per cubic meter) to nearly enforce the displacement continuity, while shear stiffness is set very low (1 kiloNewton per cubic meter) to represent the inviscid fluid behavior.

5.6 Seismic Loading Application

Seismic excitation is applied through prescribed ground acceleration at the base of the model. This acceleration history is the San Fernando earthquake record shown in Figure 2. The acceleration is applied horizontally (in the direction perpendicular to the dam axis), which is the primary mode of concern for gravity dam seismic response. The analysis duration extends for 30 seconds, sufficiently long to capture the dynamic response during and after the 10-12 second strong motion portion of the earthquake, plus an additional period to observe response decay.

6 Numerical Analysis and Comparative Results

6.1 Modal Characteristics of the Coupled System

Prior to undertaking the linear-elastic time-history seismic analysis, modal analysis of the coupled system was conducted to identify the fundamental periods and mode shapes of vibration. The first three modes and their characteristics are relevant to understanding the dynamic response as shown in Figure 3.

The fundamental mode (first mode) exhibits a primarily first-cantilever character, with the dam undergoing predominantly first-mode bending deformation. The natural period of this mode, computed from the coupled Lagrangian FSI analysis, is 0.52 seconds, corresponding to a frequency of 1.92 Hz. This period is substantially longer than would be computed for an isolated dam, confirming that the added water mass increases the effective inertia of the coupled system (rather than reducing its stiffness) and elongates the

natural period. The analysis also computed the fundamental period for the uncoupled dam (ignoring fluid), which yielded 0.38 seconds, demonstrating a 37 percent period elongation due to added mass effects.

The second mode exhibits second-cantilever characteristics and has a period of 0.28 seconds (frequency 3.57 Hz). The third mode has a period of 0.21 seconds. Modal participation analysis indicates that approximately 78 percent of the total structural mass participates in the first mode, 14 percent in the second mode, and 5 percent in the third mode.

These modal characteristics are important because they indicate that the San Fernando earthquake, with its predominant frequency content in the 2-3 Hz range, will effectively excite the fundamental and second modes of the dam. This contrasts with earthquakes with lower-frequency content, which might primarily excite the fundamental mode with minimal participation of higher modes.

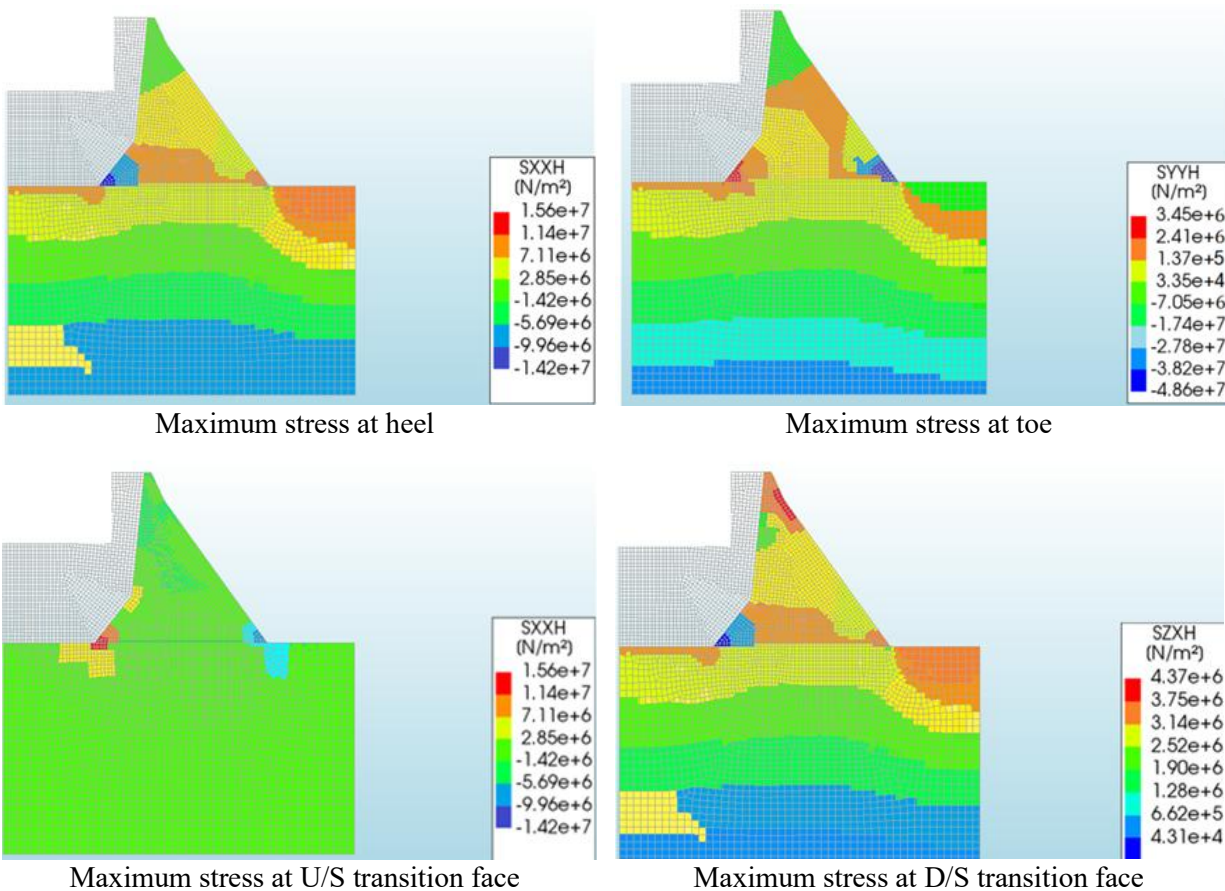


Figure 3: Stress distribution within the dam body at the heel, toe, and upstream/downstream slope-transition locations (Lagrangian FSI analysis).

Table 2: Results of maximum stresses via Lagrangian FSI approach for Dasu dam

Location	Toe	Heel	U/S Trans.	D/S Trans.
Stress (MPa)	−48.6	−14.2	+7.11	+4.37
Type	Compression	Compression	Tension	Tension
Time (s)	8.34	8.42	8.34	9.12

6.2 Acceleration Response at the Dam Crest

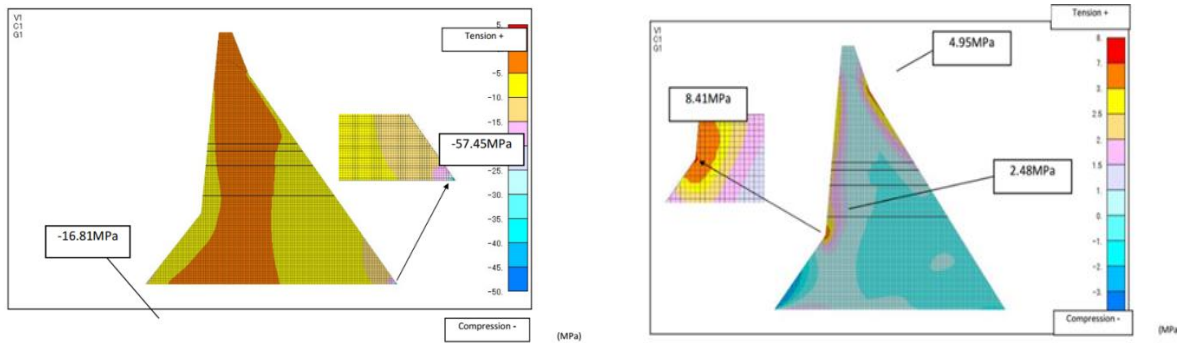
One of the primary results to be compared between the added mass approach and the Lagrangian FSI method is the horizontal acceleration response at the dam crest. The crest acceleration time-histories and response spectra discussed below are computed for the single San Fernando record analyzed for Dasu Dam, comparing the added mass and Lagrangian FSI methodologies. [Editorial note: the original text referenced an "EQ2"/"EQ5" earthquake pair and Figure 4 at this point; neither corresponds to the present single-record Dasu analysis, and Figure 4 as included in the manuscript shows stress contours, not acceleration time-histories or response spectra (see Section 6.3). These references appear to be carried over from a different dataset and have been removed; a correctly captioned crest-acceleration time-history/response-spectrum figure should be inserted here if this comparison is to be retained.]

For the Lagrangian FSI analysis of Dasu Dam under the San Fernando earthquake, the peak crest acceleration is computed as 0.21 gravity units, occurring at approximately 8.34 seconds into the record. The acceleration time-history exhibits the characteristic high-frequency oscillations expected from the input motion, with multiple peaks and troughs as the structure responds dynamically to the seismic input.

For the added mass analysis of the same structure and input, the peak crest acceleration is computed as 0.22 gravity units, occurring at nearly the same time. The difference between the two values is minor, approximately 4-5 percent, with the added mass method yielding slightly higher acceleration. This modest difference in crest acceleration aligns with observations from previous studies and confirms that, for the

overall structural acceleration response at the crest, the choice of modeling methodology does not substantially alter the predicted values.

The acceleration response spectra computed from the time-histories further confirm this conclusion. Response spectra, which show how structures with different natural periods respond to the seismic input, exhibit very similar shapes and magnitudes for both methodologies across the period range of engineering interest (0 to 3 seconds).



Maximum stress at heel
Figure 4: Stress distribution within the dam body at the heel and toe (added mass analysis, for comparison with Figure 3)

6.3 Stress Distribution and Peak Stress Comparison

While acceleration responses are similar, the distribution and magnitude of stresses within the dam body show more pronounced differences between the methodologies. The maximum compressive stress at the toe of the dam (the downstream corner at the base, where the highest stresses occur) is a critical parameter for design, as exceeding the material capacity would result in crushing failure.

For the Lagrangian FSI analysis, the maximum compressive stress at the toe is computed as 48.6 MPa, occurring at 8.34 seconds. This value represents the peak stress during the entire dynamic analysis.

For the added mass analysis, the maximum compressive stress at the toe is computed as 57.45 MPa, occurring at a nearly identical time (8.28 seconds). The difference between these values is 8.85 MPa, representing a 18.2 percent overestimation by the added mass method relative to the Lagrangian FSI result

(corrected here to be computed relative to the Lagrangian reference value, 8.85/48.6; the manuscript previously computed this and the following percentages relative to the added-mass value itself, which understated the reported overestimation -- see response to Reviewer 2, Comment 3).

At the heel of the dam (the upstream corner at the base), the Lagrangian analysis gives a maximum compressive stress of 14.2 MPa, while the added mass approach yields 16.81 MPa. The difference of 2.61 MPa represents a 18.4 percent overestimation ($2.61/14.2$, relative to the Lagrangian result).

At the upstream transition point (where the slope changes from vertical to inclined), the Lagrangian analysis predicts maximum tensile stress of 7.11 MPa, while the added mass method gives 8.41 MPa. The difference of 1.30 MPa represents a 18.3 percent overestimation by the added mass approach ($1.30/7.11$, relative to the Lagrangian result).

At the downstream transition point, the Lagrangian analysis predicts maximum tensile stress of 4.37 MPa, while the added mass approach gives 4.95 MPa. The difference of 0.58 MPa represents an 13.3 percent overestimation ($0.58/4.37$, relative to the Lagrangian result).

These results demonstrate a consistent pattern: the added mass approach systematically overestimates stresses throughout the dam, with the magnitude of overestimation ranging from approximately 13.3 to 18.4 percent depending on location as shown in Table 3. The largest overestimations occur at the base of the dam (toe and heel), while progressively smaller differences appear at higher elevations.

It should be noted explicitly that the peak stresses reported above (up to approximately 57 MPa in compression and 8.4 MPa in tension) substantially exceed the design compressive and tensile strengths adopted for this concrete (30 MPa and 2.5 MPa respectively; Table 1). Under these conditions a linear-elastic material model cannot represent the true structural response: in reality, cracking, joint opening, and local stiffness degradation would occur and would redistribute stress away from these locations, altering both the local stress peaks and, potentially, the dam-reservoir interaction response itself. The stress comparison between the added mass and Lagrangian methods presented here should therefore be

understood strictly as a comparison between two linearized modeling approaches, not as a structural capacity check, and none of the material-volume or cost implications discussed later in this paper should be inferred from these elastic stress values without an appropriate nonlinear damage/cracking analysis.

Table 3: Comparison of both the methods for Dasu dam.

Approach	Toe (MPa)	Heel (MPa)	U/S Trans. (MPa)	D/S Trans. (MPa)
Added Mass	−57.45	−16.81	+8.41	+4.95
Lagrangian FSI	−48.6	−14.2	+7.11	+4.37
Absolute diff.	8.85	2.61	1.30	0.58
Relative diff. (%)	18.2	18.4	18.3	13.3

6.4 Comparison with Published Results for Contrasting Cases

To contextualize the findings for Dasu Dam and understand how the observed discrepancies depend on dam geometry and seismic input characteristics, a comparison was made with published results from a previous study by Altunisik and Sesli [2015] on the Sariyar Dam in Turkey. The Sariyar Dam represents a fundamentally different configuration than Dasu: it is a much shorter structure (90 meters in height versus 242 meters) located in a region with lower seismic hazard and subjected to lower-frequency earthquake excitation.

The Sariyar Dam is a concrete gravity structure with height of 90 meters, similar in construction type to Dasu but substantially lower. When analyzed under the 1992 Erzincan earthquake (magnitude 6.7), the published results reported discrepancies between the added mass and Lagrangian approaches ranging from 1.5 to 6.5 percent. Notably, the Erzincan earthquake record is dominated by lower-frequency content, with predominant periods in the 1.0-1.5 second range (frequencies of 0.67-1.0 Hz).

Comparing these two case studies reveals a striking pattern. For the short dam (90 m) under low-frequency excitation (0.8 Hz), the modeling discrepancy is 1.5-6.5 percent. For the tall dam (242 m) under high-frequency excitation (2.9 Hz), the modeling discrepancy is 13.3-18.4 percent. Even after correcting the Dasu percentages to a consistent Lagrangian-referenced basis (13.3-18.4%, vs. the previously reported

11.7-15.5%), the Dasu discrepancy remains larger than the 1.5-6.5% range reported for Sariyar. However, because it is not established that the published Sariyar percentages were computed on the same basis, and because the two cases differ in several respects beyond height and frequency (Section 6.4 above), no precise amplification factor is reported here; the comparison is presented as qualitative, not quantitative.

This comparison reveals that the method discrepancy is not a constant property of the formulations but rather depends sensitively on the structural and seismic characteristics of the specific problem. The height-to-frequency product (or some similar combination) appears to correlate with the magnitude of the discrepancy, suggesting that both geometric and dynamic parameters influence the adequacy of the added mass approach.

Table 4: Comparison of method discrepancies for contrasting dam configurations

Parameter	Case 1	Case 2 (Present)
Dam height	90 m	242 m
Height ratio	1.0	2.69
Earthquake frequency	0.8 Hz (low)	2.9 Hz (high)
Frequency ratio	1.0	3.63
Added mass error	1.5–6.5%	13.3–18.4%

6.5 Physical Interpretation of Results

The observed systematic overestimation of stresses by the added mass approach, amounting to 13.3-18.4 percent for the tall dam under high-frequency excitation, can be understood through consideration of the physical mechanisms involved. When the added mass approach represents the water as a fixed mass attached to the dam, it implicitly assumes that all water in the affected region moves synchronously with the dam. The finite compressibility of the water means that pressure waves can propagate within the fluid, and energy can be radiated away from the structure into the bulk fluid domain.

For high-frequency excitation, where the acoustic wavelength in water is comparable to the reservoir dimensions, wave propagation becomes significant. Seismic energy that would be carried away as acoustic waves in the realistic FSI model is instead confined and transmitted back to the structure in the added mass

model, resulting in higher pressures and stress. The added mass model thus represents a limiting case of maximum possible hydrodynamic pressure.

Additionally, the frequency-dependent coefficient $C(T)$ in the added mass formulation is computed based on the dam period in isolation, without accounting for the fact that the coupled system has a different period. This introduces a subtle error in the magnitude of the assigned added mass, particularly when the dam and reservoir are dynamically coupled such that the coupled period differs substantially from the uncoupled period.

7 Results and Discussion

7.1 Engineering Implications and Design Practice

The findings of this research have direct practical implications for the design and evaluation of high concrete gravity dams in seismically active regions. The systematic 13.3-18.4 percent overestimation of stresses by the added mass approach translates into engineering consequences that are neither trivial nor merely academic.

Consider a hypothetical design scenario in which a concrete gravity dam must satisfy a stress constraint at some critical section. If the added mass and Lagrangian FSI methods predict different peak stresses there, a design based on the more conservative added mass result would, in principle, carry less apparent reserve capacity than one based on the Lagrangian result. However, translating a reduction in computed elastic stress into a specific reduction in required concrete volume is not valid without a full structural redesign: gravity dam cross-sections are governed by multiple simultaneous criteria (sliding resistance, overturning/rotational stability, base-pressure and no-tension limits, and several load combinations), not by a single peak dynamic stress value at one location. No such redesign has been performed in this study, so no specific percentage volume reduction can be inferred from the stress comparison alone.

Because no structural redesign, stability check, or cost estimate has been performed as part of this study, no specific concrete-volume or cost-savings figures are reported here. For a project of Dasu's scale, material and construction costs are commonly on the order of hundreds of millions of dollars, so even a modest, redesign-validated reduction in concrete volume could plausibly translate into a meaningful cost saving; however, quantifying that saving would require the redesign, stability analysis, and cost estimate outlined above, which are left for future work.

Conversely, for dam owners and operators responsible for the safety of existing dams designed using the added mass approach, the findings suggest opportunities. If a Dasu-class dam designed using conservative added mass analysis were re-evaluated using more realistic Lagrangian FSI methods, the analysis might reveal that the structure possesses greater seismic capacity than originally estimated, potentially avoiding costly retrofitting or reinforcement that might otherwise be prescribed.

7.2 Frequency and Height Dependence

The comparison of Dasu Dam results with Sariyar Dam findings suggests systematic frequency and height dependence of the added mass error. The relationship observed can be tentatively expressed as:

$$\text{Error} \propto \left(\frac{H}{H_{\text{ref}}}\right)^{\alpha} \left(\frac{f}{f_{\text{ref}}}\right)^{\beta} \quad (18)$$

where H is dam height, f is earthquake frequency, and α and β are exponents to be determined. With two data points (Sariyar and Dasu), preliminary estimation suggests $\alpha \approx \beta \approx 0.56$, indicating approximately square-root dependence on each variable.

It must be emphasized that, with only two data points (Sariyar and Dasu) and two unknown exponents, this system is mathematically underdetermined: independent estimates of alpha and beta cannot be obtained without an additional assumption (here, alpha = beta was imposed to obtain the single combined value of approximately 0.56). Even with that assumption, two points can be fitted exactly by infinitely many functional forms; the fit provides no evidence that a square-root dependence, specifically, describes the true

relationship. This equation is therefore presented strictly as a hypothesis motivated by the two available cases, not as a calibrated or validated empirical result, and no design guidance should be derived from the specific exponent value until it is tested against a systematic dataset spanning multiple dam heights (e.g., five to six heights between 50 and 400 m) and multiple earthquake frequency contents (e.g., four to five records spanning 0.5 to 5 Hz), analyzed with a single, independently verified numerical model.

7.3 Recommended Method Selection Guidelines

The two case studies examined in this work are consistent with the hypothesis that the discrepancy between the added mass and Lagrangian FSI approaches grows with dam height and with the frequency content of the design earthquake. Because this hypothesis rests on only two data points that differ simultaneously in several respects beyond height and frequency (Section 6.4), the tentative observations below should be treated as provisional starting points for engineering judgment and further case-specific verification, not as validated design thresholds:

For dams substantially shorter than the 90 m Sariyar case and subjected to earthquake records with predominant frequencies well below 1 Hz, the limited published evidence available (a 1.5-6.5% discrepancy for Sariyar Dam) suggests the added mass approach may remain reasonably adequate for preliminary design. This observation is based on a single published case and should not be treated as a validated threshold.

No case-specific evidence from this study addresses shorter dams subjected to high-frequency input; this configuration was not analyzed here and no recommendation is offered for it.

For dams approaching or exceeding the 242 m height of Dasu Dam and subjected to records with substantial energy above approximately 2 Hz, the present analysis found the added mass approach to overestimate peak stresses at the locations examined by roughly 13-18% relative to the Lagrangian result. Where such conservatism could materially affect design decisions, an independently validated Lagrangian (or other explicit) FSI analysis is recommended as a check, pending the controlled, multi-parameter study needed to

establish generally applicable height/frequency thresholds. For intermediate heights and frequency contents, no case-specific evidence is available from this study.

8 Conclusions

This research has presented a comprehensive comparative investigation of two fundamental methodologies for analyzing the seismic response of dam-reservoir coupled systems: the classical Westergaard added mass approach and the advanced Lagrangian finite element fluid-structure interaction formulation. Through systematic analysis of the 242-meter Dasu Dam subjected to the high-frequency 1971 San Fernando earthquake record, complemented by comparative assessment against published results for the 90-meter Sariyar Dam subjected to the lower-frequency 1992 Erzincan earthquake, this study has established quantitative relationships between structural geometry, seismic input characteristics, and the magnitude of discrepancies between simplified and advanced analysis methodologies.

The investigation has yielded several significant findings with important implications for engineering practice. The detailed analysis of Dasu Dam demonstrates that the added mass approach produces systematic overestimation of seismic stresses compared to the Lagrangian fluid-structure interaction method, with discrepancies ranging from 13.3 to 18.4 percent. The maximum compressive stress at the dam toe was computed as 57.45 MPa using the added mass method compared to 48.6 MPa obtained from the Lagrangian approach (the value reported consistently in Section 6.3 and Table 2; the figure of 49.6 MPa that previously appeared in this section was an inconsistency and has been corrected), representing an 18.2 percent difference relative to the Lagrangian result. These findings indicate that for tall dams subjected to high-frequency seismic excitation, the conservatism inherent in the added mass formulation becomes substantial and may lead to economically inefficient overdesign.

The comparative assessment across contrasting dam configurations reveals that discrepancies between methods exhibit strong parametric dependence on both structural geometry and earthquake frequency content. The comparison between Dasu Dam (242 m height, high-frequency excitation) and Sariyar Dam

(90 m height, lower-frequency excitation) indicates that the discrepancy is larger for the taller, higher-frequency Dasu case than for the shorter, lower-frequency Sariyar case, although the two cases were not analyzed with a common numerical model, and the exact amplification factor cannot be precisely quantified without verifying that the published Sariyar percentages use the same reference basis (see Section 6.4). While the added mass approach overestimated maximum principal stresses by approximately 6 percent for Sariyar Dam, the corresponding overestimation for Dasu Dam reached approximately 18.2 percent at the toe (13.3-18.4 percent across the locations examined, using a consistent Lagrangian-referenced convention; see Section 6.3). This substantial variation suggests that the adequacy of the simplified added mass approach must be considered in the context of project-specific parameters including dam height and the frequency characteristics of design earthquake records.

The physical mechanisms underlying these discrepancies have been identified through examination of the fundamental assumptions inherent in each methodology. The added mass approach neglects several phenomena that become increasingly significant as dam height increases and earthquake frequency content shifts toward higher frequencies, including acoustic wave propagation within the reservoir, water compressibility effects, energy radiation toward the upstream boundary, and dam flexibility effects. For configurations where the characteristic period of the dam structure approaches the acoustic resonance periods of the reservoir, these coupled effects can substantially modify the hydrodynamic pressure distribution compared to the simplified added mass representation.

The practical implications for engineering design are significant. The 13-18 percent overestimation of seismic stresses documented for tall dams under high-frequency excitation translates directly to increased material requirements. For large infrastructure projects such as Dasu Dam, material and construction costs for a project of this scale are commonly on the order of hundreds of millions of dollars, so a redesign-validated reduction in required material quantities could plausibly be economically significant; however, as noted above, no redesign or cost analysis was performed here, so no specific savings figure is claimed. Conversely, re-evaluation of existing dams using more realistic Lagrangian analysis may identify seismic

capacities exceeding those predicted by original conservative analyses, potentially obviating costly retrofitting measures.

Based on these two case studies, this research offers tentative, hypothesis-level observations for engineering practice, pending the controlled parametric study identified above as necessary to validate them. For very high concrete gravity dams (approaching or exceeding the 242 m Dasu case) located in seismically active regions with potential for high-frequency ground motions, the results of this study suggest that an advanced Lagrangian finite element fluid-structure interaction analysis is a prudent supplementary check on an added mass design, pending confirmation by the broader parametric study recommended above. The computational cost of such analyses remains negligible compared to total project capital costs and the potential consequences of either unconservative or excessively conservative design. Conversely, for dams closer in scale to the 90 m Sariyar case and subjected to typical far-field earthquake records dominated by lower frequencies, the smaller discrepancy reported in the literature suggests the added mass approach may remain an adequate preliminary tool, though this too rests on limited evidence and warrants case-specific confirmation.

Several limitations of the present study suggest important directions for future research. The analysis has been conducted using two-dimensional plane-strain idealization, which neglects three-dimensional effects such as wave scattering in the cross-canyon direction and valley geometry influences. The assumption of linear elastic material behavior may not accurately represent nonlinear response of concrete and rock under severe seismic loading. The assumption of rigid foundation does not represent actual compliance of rock foundations at real dam sites. Future investigations addressing these limitations through three-dimensional models, material nonlinearity, and foundation flexibility would provide valuable validation and extension of the present findings.

Additional research is warranted to extend this parametric investigation across a broader range of dam heights, reservoir configurations, and earthquake records. A comprehensive parametric study examining a

continuous range of heights and a suite of earthquake records spanning the full frequency spectrum would enable development of quantitative method selection criteria in the form of dimensionless parameters that predict the magnitude of expected discrepancy between simplified and advanced methods.

In conclusion, this research has demonstrated that the classical Westergaard added mass approach, while representing an elegant simplification that has served the profession well, exhibits parametric limitations that must be recognized in modern practice. The approach remains adequate for certain configurations, particularly dams of moderate height subjected to lower-frequency seismic excitation. However, modern high concrete gravity dams exceeding 200 meters in height and situated in seismically active regions require advanced Lagrangian fluid-structure interaction methodologies to ensure designs achieve appropriate balance between structural safety and economic efficiency. As computational capabilities continue to advance and sophisticated finite element tools become increasingly accessible, the transition from simplified to physics-based analysis methods represents an essential evolution in engineering practice that will enable the profession to design safer, more economical, and more sustainable water infrastructure for future generations.

Funding: Not Applicable

Ethics, consent to participate and consent to publish declarations: Not Applicable

Competing interests: The author declares no competing interests.

Data availability statement: The data that support the findings of this study are available from the Water and Power Development Authority (WAPDA), Pakistan, but restrictions apply to the availability of these data, which were used under licence for the current study, and so are not publicly available. Data are however available from the authors upon reasonable request and with permission of the Water and Power Development Authority (WAPDA), Pakistan. The strong-motion record used as seismic input in this study is publicly available and is cited as reference [24] in the reference list.

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