

Multiphysics-Informed Numerical Characterization of Bearing Behaviour in Sand–Clay Mixtures: Linking Clay Content, Strength–Stiffness Evolution, and Foundation Response

Muhammed Yeasin Arafat

muhammed1804177@gmail.com

Bangladesh University of Engineering and Technology

Research Article

Keywords: sand–clay mixture, bearing capacity, finite-element method, limit analysis, strength–stiffness transition, shallow foundation

Posted Date: August 27th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10756415/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Multiphysics-Informed Numerical Characterization of Bearing Behaviour in Sand–Clay Mixtures: Linking Clay Content, Strength–Stiffness Evolution, and Foundation Response

Muhammed Yeasin Arafat¹

Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh

E-mail: muhammed1804177@gmail.com

Abstract

Mixed sand–clay foundations are governed by a moving balance between two load-bearing systems: a sand skeleton that mobilizes friction and dilation, and a fine-grained matrix that mobilizes cohesion while changing stiffness. This transfer of dominance was traced across 11 mixtures containing 0–100% clay at 10% intervals. The complete property matrix was carried into a common 2 m surface strip-footing problem evaluated by Vesic, Hansen, Meyerhof, and Terzaghi-type solutions, numerical-limit-analysis-informed bounds, an upper-bound-limit-analysis benchmark, and an independently executed nonlinear plane-strain finite-element model. Effective friction angle declined from 41.22° to 14.28° , whereas effective cohesion increased from 0 to 34.03 kPa. Vesic capacity fell from 2165.8 kPa to a minimum of 258.3 kPa at 80% clay and then recovered to 401.4 kPa at 100% clay; all collapse-oriented methods identified the same minimum. The FEM pressure mobilized at $s/B = 3\%$ followed the same transition, decreasing from 545.0 to 288.6 kPa and recovering to 352.2 kPa, with representative mesh differences of 5.0–6.9%. Resistance partitioning reveals the deeper mechanism: cohesive and frictional contributions become comparable near 50% clay, yet total resistance continues to fall until about 80% clay. This delayed minimum defines a resistance-deficit window in which granular resistance has largely decayed before cohesive resistance fully compensates. The resulting composition–property–response framework provides a mechanics-based interpretation of foundation behaviour in mixed-grained soils rather than a clay-content-only correlation.

Keywords: sand–clay mixture; bearing capacity; finite-element method; limit analysis; strength–stiffness transition; shallow foundation

1 Introduction

Natural foundation soils seldom respect the tidy boundary between sand and clay. Alluvial deposits, reworked fills and young sedimentary environments commonly contain both coarse and fine fractions, and their engineering behaviour is controlled less by the nominal percentage of each fraction than by which fraction forms the load-bearing skeleton. In a sand-dominated state, stress is transmitted through grain contacts and resistance is governed chiefly by friction, interlocking and dilation. As fines accumulate, they first occupy voids, then interrupt coarse-grain contacts and finally form a continuous matrix in which sand grains become inclusions. The mechanical consequences are therefore transitional rather than linear (Thevanayagam 1998).

The transition becomes especially important when strength and stiffness are examined together. Granular soils can mobilize high friction and density-dependent dilatancy, whereas clay-rich mixtures progressively suppress dilation and develop a larger cohesive component (Bouchemella and Taibi 2022). (Bolton 1986) demonstrated the central role of friction–dilatancy coupling in sand, while (Salgado et al. 2000) showed that fines can alter both stiffness and shear resistance before the material becomes fully fines-controlled. Plastic fines add a further layer of complexity because mineralogy, water affinity, compaction state and stress history influence the extent to which the fine matrix participates in load transfer. More recent experimental and numerical work continues to show that fines content alone cannot capture this evolution (Goudarzy et al. 2021).

The foundation response inherits this internal competition. The friction angle influences the bearing-capacity factors N_q and N_γ exponentially, while cohesion contributes through c' N_c (Gendy 2025). A comparatively small loss in ϕ' can therefore remove a large part of the granular bearing contribution, particularly in sand-rich states, whereas the benefit of increasing c becomes dominant only after a sufficiently strong clay matrix develops. Stiffness governs a different part of the problem: it controls how rapidly resistance is mobilized with settlement and how yielding spreads beneath the footing (Osman and Bolton 2005). A useful analysis must consequently distinguish collapse resistance from serviceability mobilization rather than forcing both into a single bearing-capacity number (Vardanega and Bolton 2015).

Classical bearing-capacity equations, limit analysis and nonlinear continuum modelling provide complementary views of the same mechanics. (Terzaghi 1943; Meyerhof 1963; Hansen 1970; Vesić 1973) express collapse resistance through alternative bearing-capacity factors and correction schemes. Numerical limit analysis approaches collapse using the plasticity theorems and can bracket or approximate the limiting load (Hjiaj et al. 2004; Khatri and Kumar 2019). Finite element analysis, in contrast, resolves the progressive relationship between footing pressure, settlement and distributed plasticity (Conte et al. 2013). Agreement in trend across these different formulations is more informative than numerical coincidence, because each formulation weighs strength, stiffness and failure mechanism differently.

The present work addresses a gap between material-scale mixture characterization and foundation-scale response. The source material was collected from the Purbachal New Town Project area of Bangladesh and represented by 11 sand–clay states spanning the full 0–100% composition range. Rather than fitting bearing capacity directly to clay content, the analysis follows the chain from composition to index properties, effective strength, stiffness, and constitutive descriptors, and then to footing response. The term multiphysics-informed is used in this material-state sense: physical and mechanical descriptors are interpreted jointly, while the numerical solver itself remains a mechanical continuum model rather than a fully coupled transient hydro-mechanical formulation.

Three questions guide the paper. First, how does the sand-to-clay transition redistribute the frictional and cohesive components of bearing resistance? Second, does the composition associated with minimum collapse capacity also appear in settlement-based FEM response? Third, can the separation between the friction–cohesion crossover and the minimum total capacity be interpreted as a mechanically meaningful transition interval? The resulting framework is intended to expose the mechanism behind the non-monotonic response, not to propose a universal empirical threshold.

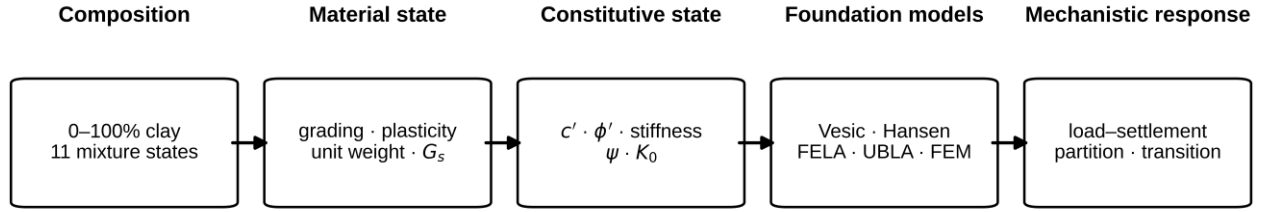


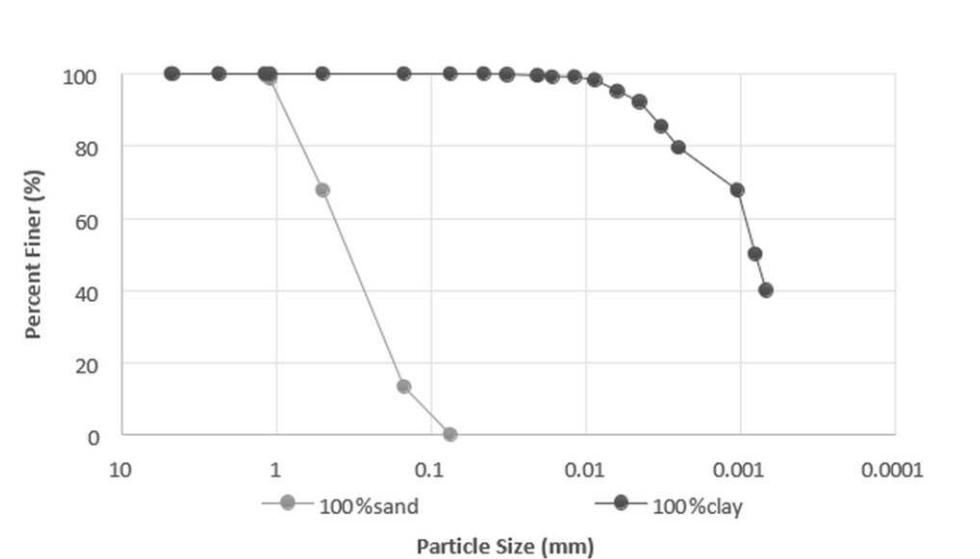
Fig. 1 Composition–property–response framework adopted for the sand–clay mixture and strip-footing analyses

2 Materials and experimental characterization

2.1 Source soil and mixture framework

The source soil was collected from the Purbachal New Town Project area, extending through Rupganj in Narayanganj District and Kaliganj in Gazipur District, Bangladesh. The development lies between the Balu and Shitalakhya river systems, approximately 16 km from central Dhaka. Three source samples were available for each broad soil fraction identified as sand, silt, and clay. Because direct physical separation can alter the original fabric, the collected material was prepared through particle-size separation and dispersion procedures before the sand- and clay-rich fractions were used to establish the controlled mixture matrix.

Eleven nominal clay contents were considered: 0, 10, 20, ..., 100%. The end-member states represent the sand-rich and clay-rich materials, while the intermediate states capture the progressive replacement of the granular skeleton by fines. All reported numerical analyses use the same composition sequence, allowing property evolution and foundation response to be compared without interpolation between nominal mixture states



appear from 10% clay onward. The liquid limit increases from 14% at 10% clay to 88% at 100% clay, while the plasticity index rises from 5% to 57%. The plasticity chart therefore records a genuine change in material state rather than a simple shift in particle density.

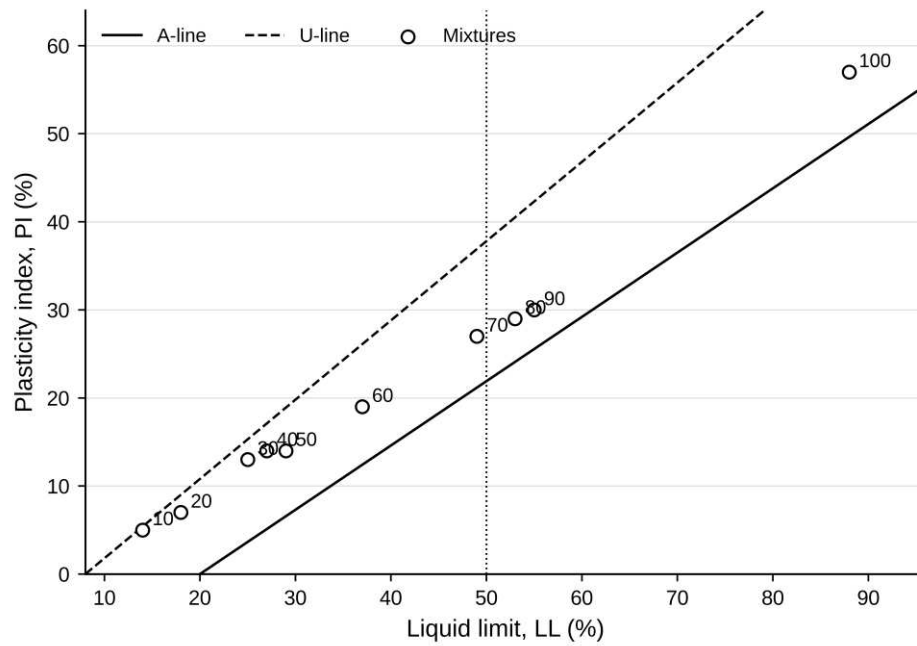


Fig. 3 Plasticity evolution of the sand–clay mixtures relative to the Casagrande A-line and U-line



Fig. 4 Representative volumetric-flask testing during index-property characterization of the mixture states

Table 1 Composition-dependent index, strength and unit-weight properties

Clay (%)	G _s	LL (%)	PL (%)	PI (%)	c' (kPa)	φ' (°)	γ _{unsat} (kN/m ³)	γ _{sat} (kN/m ³)
0	2.65	NP	NP	NP	0.000	41.22	16.0	18.0

Clay (%)	G _s	LL (%)	PL (%)	PI (%)	c' (kPa)	φ' (°)	γ _{unsat} (kN/m ³)	γ _{sat} (kN/m ³)
10	2.68	14	9	5	0.932	35.67	16.2	18.2
20	2.69	18	11	7	2.936	32.30	16.4	18.4
30	2.69	25	12	13	4.357	29.99	16.6	18.6
40	2.69	27	13	14	6.261	26.31	16.8	18.8
50	2.70	29	15	14	8.995	24.23	17.0	19.0
60	2.70	37	18	19	12.321	22.76	17.2	19.2
70	2.71	49	22	27	14.842	19.32	17.4	19.4
80	2.71	53	24	29	14.842	17.64	17.6	19.6
90	2.71	55	25	30	22.416	15.44	17.8	19.8
100	2.72	88	31	57	34.033	14.28	18.0	20.0

2.3 Strength, stiffness and constitutive descriptors

The effective strength parameters move in opposite directions with clay content (Fig. 5). The effective friction angle decreases almost continuously from 41.22° to 14.28°, whereas the effective cohesion increases from 0 to 34.03 kPa. The decrease in ϕ' is especially consequential because N_q and N_γ are highly nonlinear functions of friction angle (Meyerhof 1951). Cohesion develops more gradually through the mid-composition range, remains unchanged between 70 and 80% clay in the working dataset, and then rises sharply at 90–100% clay. That phase lag between friction loss and cohesion gain later proves central to the bearing-capacity minimum.

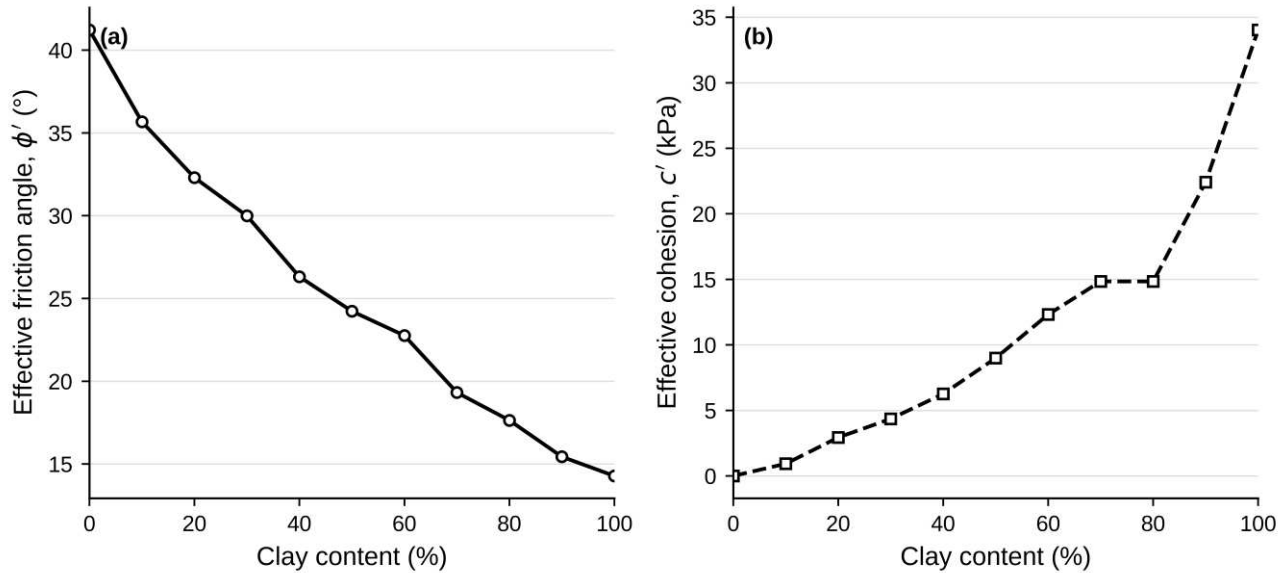


Fig. 5 (a) Evolution of effective friction angle and (b) effective cohesion with clay content

The stiffness descriptors do not exhibit a uniform trend. E_{50ref} decreases from 43 to 35 MPa and E_{ur} from 129 to 105 MPa, whereas E_{oedref} increases from 22 to 33 MPa. Thus, increasing clay content reduces the adopted loading/unloading stiffness descriptors while increasing the oedometer-reference stiffness in the prepared property matrix. The dilation angle falls from 11.22° to 0° by 30% clay; K_0 increases from 0.341 to 0.753; and the stress-dependency exponent m rises from 0.50 to 1.00. These simultaneous shifts confirm that the transition cannot be represented by ϕ' or c' alone.

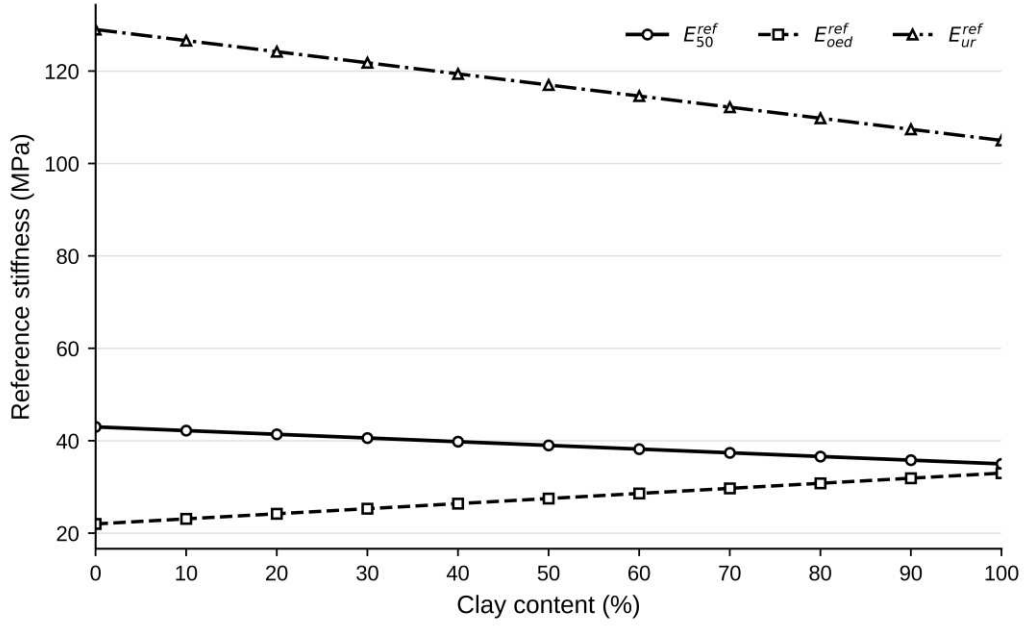


Fig. 6 Evolution of the reference stiffness parameters across the mixture spectrum

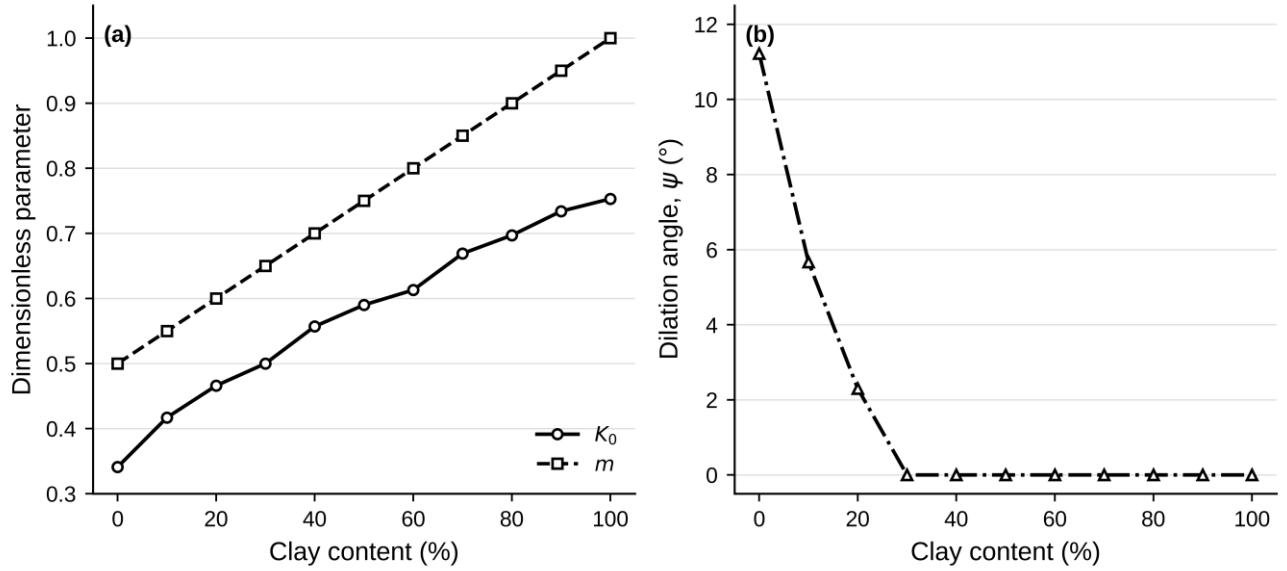


Fig. 7 Composition-dependent evolution of (a) K_0 and stress-dependency exponent m and (b) dilation angle

Table 2 Stiffness and constitutive property matrix

Clay (%)	E_{50ref} (MPa)	E_{oedref} (MPa)	E_{urref} (MPa)	ν	ψ (°)	m	K_0	Approx. peak stress
0	43.0	22.0	129.0	0.30	11.22	0.50	0.341	~0 psi
10	42.2	23.1	126.6	0.30	5.67	0.55	0.417	~2.3 psi
20	41.4	24.2	124.2	0.30	2.30	0.60	0.466	~14.3 psi
30	40.6	25.3	121.8	0.30	0.00	0.65	0.500	~20.7 psi
40	39.8	26.4	119.4	0.30	0.00	0.70	0.557	~22.8 psi
50	39.0	27.5	117.0	0.30	0.00	0.75	0.590	~22.6 psi

Clay (%)	E50ref (MPa)	Eoedref (MPa)	Eurref (MPa)	ν	ψ (°)	m	K_0	Approx. peak stress
60	38.2	28.6	114.6	0.30	0.00	0.80	0.613	~22.5psi
70	37.4	29.7	112.2	0.30	0.00	0.85	0.669	~21.6 psi
80	36.6	30.8	109.8	0.30	0.00	0.90	0.697	~20.5 psi
90	35.8	31.9	107.4	0.30	0.00	0.95	0.734	~18.8 psi
100	35.0	33.0	105.0	0.30	0.00	1.00	0.753	~16.5 psi

3 Analytical and numerical methodology

3.1 Common footing idealization

Every formulation was evaluated for the same reference geometry: a rigid, smooth strip footing of width $B = 2.0$ m placed at the ground surface ($= 0$) and subjected to vertical centric loading. The numerical problem was treated in plane strain. Holding geometry and loading fixed makes the clay-content-dependent property matrix the only changing physical input and prevents method comparisons from being contaminated by footing-shape or embedment effects.

3.2 Classical bearing-capacity formulations

Classical strip-footing theory represents general shear failure as a coupled mechanism in which a highly stressed zone forms beneath the footing and transfers load into adjacent radial and passive zones (Pantelidis 2024). The resulting ultimate pressure is resolved into three physically distinct contributions: cohesion, surcharge at foundation level, and the self-weight of the soil participating in the failure mechanism (Pantelidis 2024). This decomposition is useful for mixed soils because clay content changes the relative importance of the cohesive and frictional components rather than scaling all three terms uniformly.

For the vertically and centrally loaded strip footing considered here, the classical ultimate bearing-capacity expression is (Pantelidis 2024):

$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma \quad (1)$$

where the surcharge at foundation level is

$$q = \gamma D_f \quad (2)$$

For the present surface footing, $D_f = 0$ and hence $q = 0$, giving

$$q_u = c'N_c + \frac{1}{2}\gamma BN_\gamma \quad (3)$$

The common friction-dependent factors were evaluated from

$$N_q = e^{\pi \tan \phi'} \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) \quad (4)$$

$$N_c = (N_q - 1) \cot \phi' \quad (5)$$

The alternative unit-weight factors used in the analytical comparison were

$$N_{\gamma,V} = 2(N_q + 1) \tan \phi' \quad (6)$$

$$N_{\gamma,H} = 1.5(N_q - 1) \tan \phi' \quad (7)$$

$$N_{\gamma,M} = (N_q - 1) \tan(1.4\phi') \quad (8)$$

The factors N_c , N_q , and N_γ are dimensionless amplifiers of the corresponding strength and overburden terms. Their dependence on ϕ' is strongly nonlinear, especially for N_γ (Pantelidis 2024). Consequently, a moderate decrease in ϕ' can result in a disproportionate reduction of the bearing resistance in sand-rich mixtures, whereas the same change has much

less influence once the response becomes cohesion-dominated. This theoretical sensitivity provides the basis for interpreting why the analytical methods diverge most strongly at low clay content and progressively converge toward the clay-rich end. The subscripts V, H, and M denote Vesic, Hansen, and Meyerhof, respectively. The Terzaghi-type sensitivity series retained the same strip-footing form but used the conventional Terzaghi N_γ factor values represented in the computational table rather than imposing an additional closed-form approximation. An allowable pressure based on a global factor of safety was calculated as

$$q_{allow} = \frac{q_u}{FS} \quad (9)$$

3.3 Nonlinear plane-strain finite-element formulation

A custom two-dimensional finite-element solver was implemented independently of the closed-form bearing-capacity equations. The model used a $10 \text{ m} \times 10 \text{ m}$ symmetry half-domain. The 2 m full footing therefore occupied $x = 0$ –1 m in the half-model. The production mesh contained 361 four-node quadrilateral plane-strain elements and 400 nodes, with local refinement beneath the footing; a 1089-element mesh was used for sensitivity checks. Symmetry enforced $u_x = 0$ at $x = 0$; the far vertical boundary was horizontally restrained, and the base was fixed in both directions. The ground surface outside the footing remained traction-free. Plane strain is consistent with an idealized strip footing whose longitudinal dimension is sufficiently large for out-of-plane strain to be negligible.

In the finite-element formulation, the soil continuum is replaced by an assemblage of elements connected at nodes (Keller et al. 2012). The nodal displacements are the primary unknowns; the displacement field within each element is interpolated through its shape functions, after which strains, stresses, and internal nodal forces are recovered (Franssen 2005). Global equilibrium is then satisfied incrementally. This displacement-based formulation is especially useful for foundation problems because the same calculation resolves both the mobilized footing reaction and the deformation required to develop it.

The small-strain equilibrium equation is (Chen et al. 2024)

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad (10)$$

and its finite-element weak form is written as

$$\int_{\Omega} \mathbf{B}^T \boldsymbol{\sigma} d\Omega - \mathbf{f}_{ext} = \mathbf{0} \quad (11)$$

The element displacement interpolation and associated strain–displacement relation are

$$\mathbf{u}(\mathbf{x}) = \mathbf{N}(\mathbf{x})\mathbf{d}_e \quad (12)$$

$$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{d}_e \quad (13)$$

The incremental elastic predictor uses

$$\Delta \boldsymbol{\sigma} = \mathbf{D}^e \Delta \boldsymbol{\varepsilon} \quad (14)$$

where the shear and bulk moduli are

$$G = \frac{E}{2(1 + \nu)}, \quad K = \frac{E}{3(1 - 2\nu)} \quad (15)$$

The model uses $E = E_{50ref}$ from the property matrix and $\nu = 0.30$. This is an explicit modelling approximation: E_{50ref} is treated as the operative pre-yield modulus rather than invoking a full hardening-soil stiffness law. Initial stresses were generated from γ_{unsat} and K_0 . For the cohesionless 0% clay case only, K_0 was increased from 0.341 to 0.3652 to place the initial stress state inside the smooth Drucker–Prager yield surface; all other states used the supplied K_0 directly.

Stress invariants are defined as

$$p = -\frac{1}{3}\text{tr}(\boldsymbol{\sigma}), \quad \mathbf{s} = \boldsymbol{\sigma} + p\mathbf{I} \quad (16)$$

$$J_2 = \frac{1}{2}\mathbf{s}:\mathbf{s}, \quad q = \sqrt{3J_2} \quad (17)$$

The elastic-perfectly plastic Drucker–Prager yield condition was

$$f = q - Mp - d \leq 0 \quad (18)$$

The Drucker–Prager surface provides a smooth pressure-dependent approximation to Mohr–Coulomb strength (Sanei et al. 2020). In this framework, confinement enters through the mean effective stress p and distortional loading through the deviatoric invariant q . The strength parameters were mapped from the measured c' , $-\phi'$ pairs under plane-strain conditions, while ψ controlled the direction of plastic flow. A non-associated potential was retained because associated flow would impose unrealistically large dilation on frictional soils, particularly at the sand-rich end (Lu et al. 2019).

The plane-strain matching parameters were

$$M = \frac{9\sin\phi'}{\sqrt{3(9 - \tan^2\psi)} + \sin\phi'\tan\psi} \quad (19)$$

$$d = \frac{c'\cos\phi'(9 - M\tan\psi)}{\sqrt{3(9 - \tan^2\psi)}} \quad (20)$$

Non-associated flow was introduced through

$$g = q - M_\psi p, \quad M_\psi = \tan\psi \quad (21)$$

$$d\boldsymbol{\varepsilon}^p = d\lambda \frac{\partial g}{\partial \boldsymbol{\sigma}} \quad (22)$$

For a plastic correction, the return-mapping increment used in the implemented algorithm was

$$\Delta\lambda = \frac{f^{tr}}{3G + MKM_\psi} \quad (23)$$

Element and global Newton equations were assembled from

$$\mathbf{K}_e = \int_{\Omega_e} \mathbf{B}^T \mathbf{D}_t \mathbf{B} d\Omega \quad (24)$$

$$\mathbf{K}_T \Delta \mathbf{u} = -\mathbf{R}, \quad \mathbf{R} = \mathbf{f}_{int} - \mathbf{f}_{ext} \quad (25)$$

The footing was driven in displacement control to $s/B = 10\%$ in 20 increments. Average pressure was obtained from the total vertical reaction on the half-footing,

$$q_{FEM} = \frac{\sum R_z}{B/2} \quad (26)$$

and normalized settlement together with the primary comparison criterion were defined by

$$\frac{s}{B} = \frac{\delta}{B}, \quad q_{3\%} = q\left(\frac{s}{B} = 0.03\right) \quad (27)$$

The 3% criterion is intentionally described as mobilized bearing pressure rather than ultimate capacity. At larger settlements the perfect-plastic model develops extensive localization and becomes more sensitive to mesh and regularization.

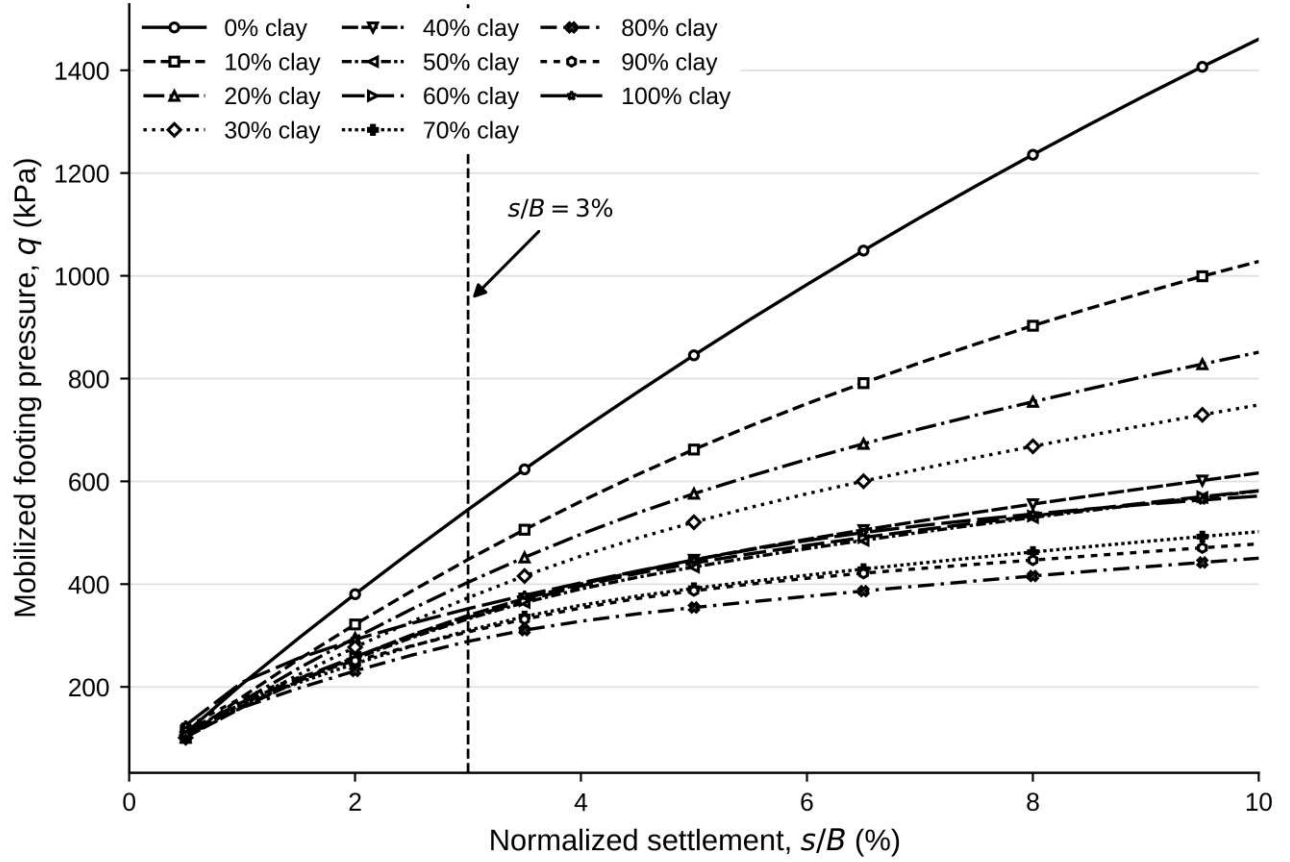


Fig. 8 FEM load–settlement response for all eleven clay contents; the dashed line and arrow identify the $s/B = 3\%$ comparison criterion

3.4 Limit-analysis-informed collapse benchmarks

Limit analysis evaluates collapse without tracing the complete stress–strain path. The lower-bound theorem searches for a statically admissible stress field that satisfies equilibrium and remains everywhere within the yield surface, so the resulting load cannot exceed the true collapse load (Füssl et al. 2017). The upper-bound theorem instead uses a kinematically admissible velocity field and equates external work to internal plastic dissipation; the corresponding load cannot fall below the exact collapse solution. When the two bounds are close, they provide a mechanically rigorous bracket on foundation capacity (Merifield et al. 1999).

The limit-analysis comparison was constructed separately from the displacement-based FEM. In a rigorous plastic limit analysis, lower and upper solutions bracket the collapse load (Merifield et al. 1999),

$$q_{LB} \leq q_u \leq q_{UB} \quad (28)$$

For the FELA-informed series, published smooth-footing lower- and upper-bound N_γ values were interpolated at each ϕ' and summarized by

$$N_{\gamma,FELA} = \frac{N_{\gamma,LB} + N_{\gamma,UB}}{2} \quad (29)$$

so that the representative FELA-informed capacity becomes

$$q_{FELA} = c'N_c + \frac{1}{2}\gamma BN_{\gamma,FELA} \quad (30)$$

The relative width of a lower–upper bracket can be expressed as

$$\delta_{LA} = 100 \frac{q_{UB} - q_{LB}}{q_{UB} + q_{LB}} \quad (31)$$

The upper-bound theorem is based on equality of external work rate and internal plastic dissipation,

$$W_{ext} = W_{int} \quad (32)$$

$$\int_{\Omega} \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^p d\Omega = \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{v} d\Gamma + \int_{\Omega} \mathbf{b} \cdot \mathbf{v} d\Omega \quad (33)$$

The Michalowski-based kinematic series was therefore expressed, under the same superposition used in the working comparison, as

$$q_{UBLA} = c' N_c + \frac{1}{2} \gamma B N_{\gamma, UB}^{(M)} \quad (34)$$

The FELA and UBLA values are literature-informed limit-analysis benchmarks rather than solver outputs from the custom FEM. Keeping this distinction explicit prevents a settlement-based continuum response from being mistaken for a plastic collapse bound.

3.5 Resistance partition and cross-method comparison

A direct comparison of total bearing capacity can conceal the internal exchange of resistance between the granular skeleton and the clay matrix. The analytical solution was therefore partitioned into a cohesion-controlled term and a self-weight/friction-controlled term. The partition does not imply that the mechanisms act independently in the soil; rather, it is a diagnostic decomposition of the closed-form equation that reveals which strength contribution dominates as composition changes.

To expose the mechanism rather than only the total capacity, the Vesic solution was decomposed into cohesive and self-weight/friction components (Santos and Freitas 2020):

$$q_c = c' N_c, \quad q_\gamma = \frac{1}{2} \gamma B N_\gamma \quad (35)$$

so that the total resistance is

$$q_u = q_c + q_\gamma \quad (36)$$

A dimensionless cohesive resistance share was then defined as

$$I_c = \frac{q_c}{q_c + q_\gamma} \quad (37)$$

and the companion contribution ratio as

$$\chi = \frac{q_c}{q_\gamma} \quad (38)$$

$I_c = 0.5$ (or $\chi = 1$) marks equal contributions from $c' N_c$ and $0.5 \gamma B N_\gamma$. These diagnostics are not proposed as universal soil indices; they are used only to locate the internal transfer of bearing resistance in the present mixture series.

Because the retained models do not all represent the same physical quantity, the comparison was based on both magnitude and trend. Normalization removes the different absolute scales of collapse-based and settlement-based quantities while preserving their composition-dependent shape; relative divergence then quantifies how far each collapse-oriented formulation departs from the Vesic reference (Al-Mosawe et al. 2011). Correlation metrics are interpreted as measures of trend consistency rather than proof of numerical equivalence.

First, each response was normalized by its own sand end-member value,

$$R_m^*(CC) = \frac{R_m(CC)}{R_m(0)} \quad (39)$$

Second, the collapse-oriented methods were compared with the Vesic series through the signed relative difference

$$\Delta_i^{(m)} = 100 \frac{R_i^{(m)} - q_{u,i}^{(V)}}{q_{u,i}^{(V)}} \quad (40)$$

and mean absolute divergence

$$MAD^{(m)} = \frac{1}{n} \sum_{i=1}^n |\Delta_i^{(m)}| \quad (41)$$

Trend consistency was quantified by Pearson correlation,

$$r_{xy} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}} \quad (42)$$

while Spearman rank correlation was used as an additional monotonic-order check. Mesh sensitivity of $q_{3\%}$ was summarized as

$$\eta_{mesh} = 100 \frac{|q_{3\%,fine} - q_{3\%,prod}|}{q_{3\%,fine}} \quad (43)$$

4 Results

4.1 Composition-driven change in material state

The end-member grading curves and consistency data show that the mixture series traverses a genuine granular-to-plastic transition. Specific gravity changes by only about 2.6% across the full composition range, so particle density cannot explain the much larger changes in mechanical response. By contrast, LL, PL, and PI increase systematically once clay is introduced (Alnmr and Ray 2023). At 100% clay, LL = 88% and PI = 57%, compared with a non-plastic sand end member. The mixture therefore evolves not only in grain-size composition but also in water-sensitive consistency and constitutive character.

The approximate peak-stress indicator in the supplied matrix is non-monotonic, rising from nearly zero in sand to approximately 22–23 psi through the 40–60% range before decreasing toward 16.5 psi at 100% clay (Nagaraj 2016). This behaviour is consistent with a mixed state in which frictional contacts and developing cohesion can coexist, even though a different combination of strength factors and deformation response governs footing bearing resistance.

4.2 Strength–stiffness evolution

Across the eleven states, ϕ' decreases from 41.22° to 14.28° , a reduction of 65.4%. Over the same interval, c' increases from 0.000 to 34.033 kPa. The strongest mechanistic feature is not simply that the two variables move in opposite directions, but that they do so at different rates. Most of the frictional loss occurs before the large cohesion increase observed at 90–100% clay. The 70–80% interval is especially important because c' remains at 14.842 kPa while ϕ' falls from 19.32° to 17.64° (Kim et al. 2018).

Stiffness evolution is similarly non-uniform. E_{50ref} and E_{ur} decrease by approximately 18.6% from sand to clay, whereas E_{oed} increases by 50%. The disappearance of dilation by 30% clay and the rise of K_0 toward 0.75 further shift the mechanical response away from a dilative granular state. These trends help explain why collapse-oriented and settlement-oriented predictions remain correlated but not numerically identical (Goudarzy et al. 2021).

4.3 Internal transfer of bearing resistance

The resistance partition provides the clearest explanation of the non-monotonic capacity curve (Fig. 9). At 0% clay, $c' = 0$ and the Vesic resistance is entirely carried by the self-weight/friction component. At 10% clay, the cohesive share I_c is only 0.050. It rises to 0.393 at 40% and reaches 0.516 at 50%, where the two components are effectively balanced. The total capacity, however, does not reach its minimum there.

Beyond the crossover, the granular contribution continues to collapse. At 60% clay, the cohesive share has already reached 61.6%, but the total Vesic capacity is only 355.2 kPa. At 80% clay, I_c reaches 73.7%; nevertheless, total capacity falls further to 258.3 kPa because q_{γ} has dropped to 68.0 kPa and the cohesive contribution also decreases to 190.3 kPa as N_c falls while c' remains unchanged from 70 to 80%. The minimum therefore occurs after cohesion has become dominant. Only the sharp increase in c' at 90–100% reverses the trend.

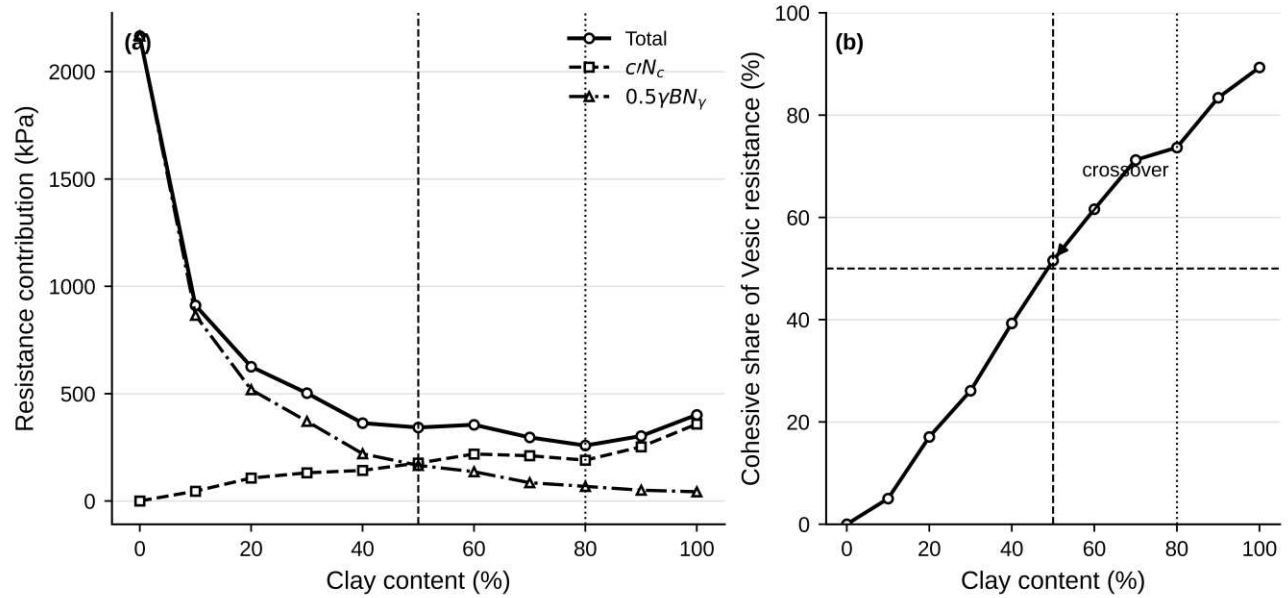


Fig. 9 (a) Partition of Vesic bearing resistance and (b) evolution of the cohesive resistance share; arrows and guide lines identify the component crossover and 80% clay transition

This separation between the 50% contribution crossover and the 80% capacity minimum defines a resistance-deficit window. Within that interval, the sand skeleton has lost most of its high- ϕ' advantage, but the clay matrix has not yet accumulated enough cohesive resistance to compensate. The concept is specific to the present material and loading idealization, yet it gives a physically interpretable marker for the transition rather than assigning the minimum to clay percentage alone.

4.4 Analytical bearing-capacity response

All four analytical series reproduce the same broad shape: a steep initial loss, a low-capacity transition zone, and a recovery at the clay-rich end. Vesic is most sensitive to the sand-rich friction angle because its N_γ term grows rapidly with ϕ' . Capacity decreases from 2165.8 kPa at 0% clay to 911.1 kPa at 10%, 625.5 kPa at 20%, and 258.3 kPa at 80%, equivalent to an 88.1% reduction from the sand end member. It then rises to 302.6 kPa at 90% and 401.4 kPa at 100%.

Hansen is consistently lower than Vesic but converges toward it as the material becomes clay rich: the difference narrows from 26.9% at 0% clay to 6.1% at 100%. This is mechanically expected. The methods share the same N_c and N_q expressions in the present comparison, whereas their N_γ treatment differs (Pantelidis 2023); as ϕ' decreases and cohesion dominates, the influence of N_γ diminishes. Meyerhof- and Terzaghi-type sensitivity calculations preserve the same minimum near 80%, showing that the location of the transition is not an artifact of one analytical N_γ expression.

Table 3 Analytical bearing-capacity calculations for the 2 m surface strip footing

Clay (%)	N_q	N_c	Vesic (kPa)	Hansen (kPa)	Meyerhof (kPa)	Terzaghi-type (kPa)	Vesic/FS=3 (kPa)
0	76.26	85.91	2165.8	1582.3	1905.3	1826.9	721.9
10	36.21	49.05	911.1	659.8	724.0	746.2	303.7
20	24.01	36.40	625.5	464.7	487.1	524.0	208.5
30	18.38	30.12	502.5	381.0	390.9	431.0	167.5
40	12.25	22.76	362.7	282.7	284.1	323.6	120.9
50	9.84	19.63	342.4	278.0	277.6	315.4	114.1
60	8.45	17.76	355.2	299.5	298.5	334.8	118.4
70	5.98	14.21	296.2	256.6	255.2	286.8	98.7

Clay (%)	Nq	Nc	Vesic (kPa)	Hansen (kPa)	Meyerhof (kPa)	Terzaghi-type (kPa)	Vesic/FS=3 (kPa)
80	5.08	12.82	258.3	224.5	223.3	252.5	86.1
90	4.11	11.26	302.6	275.3	274.3	300.2	100.9
100	3.68	10.54	401.4	377.0	376.1	400.3	133.8

4.5 FEM mobilization, yielding and mesh sensitivity

The FEM results provide a settlement-dependent view of the same transition. At $s/B = 1\%$, the mobilized pressure ranges from 205.6 kPa for sand to 159.9 kPa at 80% clay and 209.2 kPa at 100% clay. At the primary $s/B = 3\%$ criterion, the response decreases from 545.0 kPa at 0% clay to 288.6 kPa at 80% and recovers to 352.2 kPa at 100%. The minimum composition therefore agrees with the collapse-oriented methods even though the FEM quantity is not an ultimate capacity.

Figure 8 presents the complete load–settlement response for all 11 mixture states from 0 to 100% clay. The family of curves shows that the influence of composition is not confined to a single settlement criterion: sand-rich mixtures mobilize resistance rapidly and continue to harden over the simulated settlement range, whereas intermediate-to-high clay mixtures form a lower and more closely grouped response band. The 80% clay curve remains the weakest over most of the loading path, while the 90% and 100% states progressively recover as the cohesive contribution strengthens.

At $s/B = 10\%$, the model carries larger pressures, but 62–85% of the Gauss points are plastic depending on composition. This post-yield region is more sensitive to perfect-plastic localization and numerical regularization (Sánchez et al. 2007); it is retained to show the shape of the load–settlement curves, not as the principal capacity metric. The 3% response is more suitable for cross-composition comparison because it lies earlier in the mobilization process and was directly checked on a finer mesh.

Table 4 FEM mobilized pressure at selected normalized settlements

Clay (%)	q1% (kPa)	q2% (kPa)	q3% (kPa)	q5% (kPa)	q10% (kPa)	Plastic GP at 10% (%)
0	205.6	380.7	545.0	845.5	1460.5	84.2
10	180.3	321.7	447.9	662.1	1027.9	84.6
20	172.3	295.5	403.6	576.0	851.2	82.5
30	165.9	277.3	373.5	520.6	748.8	79.6
40	159.9	258.2	339.1	447.5	616.4	77.6
50	162.8	255.3	332.5	433.4	581.3	74.9
60	170.2	258.9	334.7	440.9	581.8	72.3
70	167.3	245.2	309.9	391.9	502.1	70.3
80	159.9	231.2	288.6	354.5	450.5	68.7
90	179.2	251.0	307.0	387.1	478.4	65.1
100	209.2	292.2	352.2	447.2	571.4	62.2

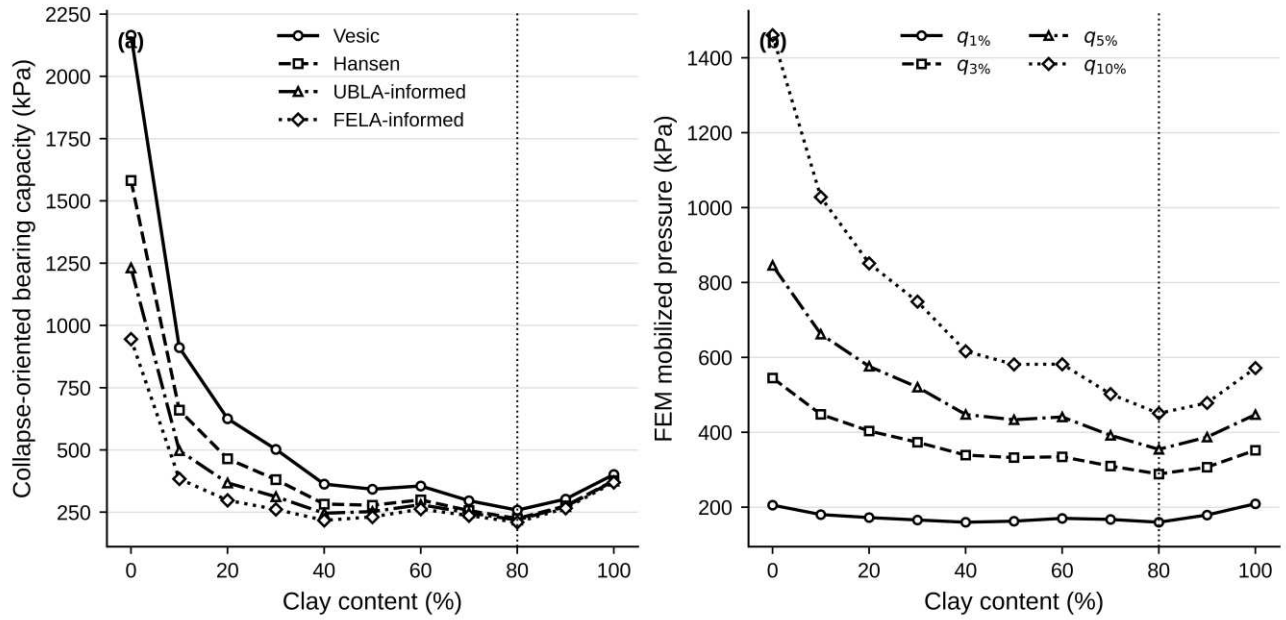
The finer mesh produced $q3\% = 311.1$ kPa at 50% clay, 270.7 kPa at 80% and 335.4 kPa at 100%, compared with 332.5, 288.6 and 352.2 kPa on the production mesh. The corresponding differences are approximately 6.85, 6.60 and 5.02%. The similarity of the percentage change across the crossover, minimum and clay-rich recovery states supports the use of $q3\%$ as a comparative response variable while still cautioning against interpreting the large-settlement branch as a mesh-converged collapse solution.

Table 5 Mesh sensitivity of the FEM q3% response

Clay (%)	Production mesh (kPa)	Fine mesh (kPa)	Difference (%)
50	332.45	311.15	6.85
80	288.58	270.72	6.60
100	352.23	335.40	5.02

4.6 Limit-analysis and cross-method comparison

The FELA-informed and UBLA-informed collapse series sit below the classical analytical predictions (Ukritchon et al. 2003), particularly in sand-rich states where N_γ dominates. The FELA-informed best estimate decreases from 945.0 kPa at 0% clay to 209.7 kPa at 80% and then recovers to 369.8 kPa at 100%. The UBLA-informed series changes from 1230.8 to 218.5 and then to 375.1 kPa over the same compositions. Both therefore identify the same minimum despite using different limit analysis information.

**Fig. 10** Comparison of (a) collapse-oriented bearing-capacity methods and (b) settlement-based FEM mobilized pressures

Normalizing each method by its own 0% value removes the mismatch in physical scale and exposes the common composition signal (Fig. 11). All retained series descend toward the high-clay transition and recover after 80% clay. The collapse methods lose a larger fraction of their sand-end-member capacity because their N_γ contribution is extremely sensitive to ϕ' (Díaz-Segura 2013). The FEM q3% curve is flatter because stiffness and a fixed settlement criterion limit the mobilized strength (AL-Shyokhi et al. 2023).

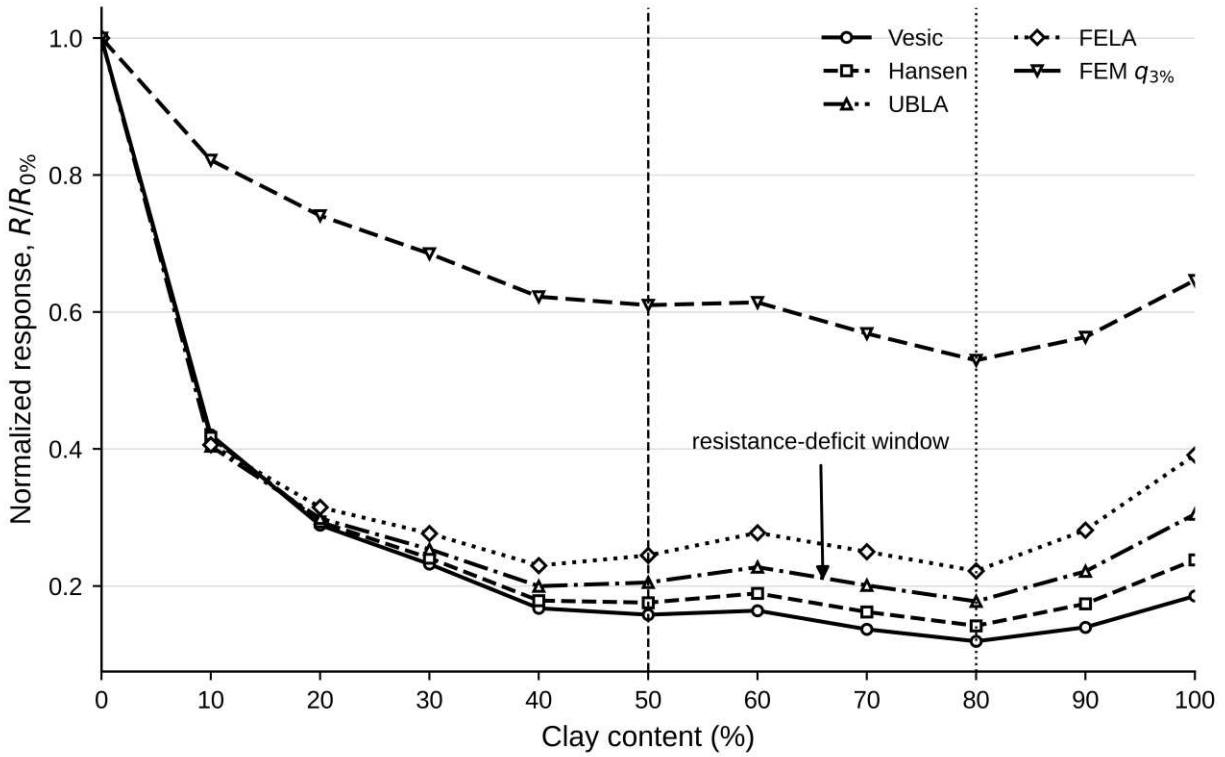


Fig. 11 Normalized response of the analytical, limit-analysis-informed and FEM series; the arrow identifies the resistance-deficit window

Hansen has the smallest mean absolute divergence from Vesic among the collapse-oriented alternatives (18.4%), followed by UBLA-informed (26.9%) and FELA-informed (33.8%). These values measure method divergence, not prediction error, because no experimental footing capacity is available as ground truth. Pearson correlations with Vesic remain high: 0.998 for Hansen, 0.991 for UBLA-informed, 0.973 for FELA-informed, and 0.947 for FEM $q_{3\%}$. The FEM rank correlation with Vesic is 0.991, confirming that the independent continuum solver reproduces the ordering of the composition response even though it represents a different mechanical quantity.

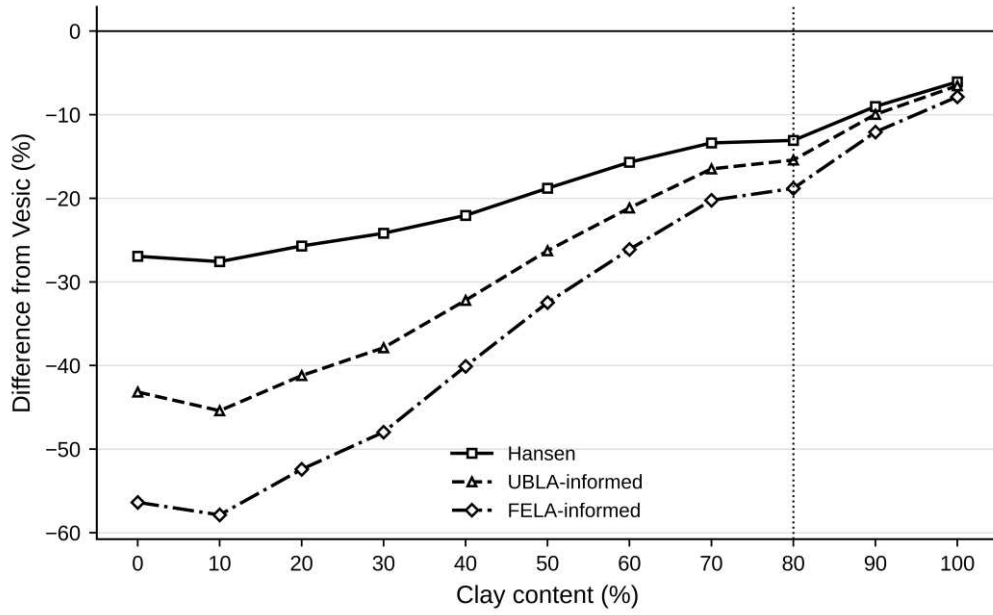


Fig. 12 Relative divergence of collapse-oriented alternatives from the Vesic series

Table 6 Limit-analysis-informed bearing-capacity benchmarks

Clay (%)	FELA LB	FELA best	FELA UB	UBLA	FELA-Vesic (%)	UBLA-Vesic (%)
0	765.9	945.0	1124.1	1230.8	-56.4	-43.2
10	332.5	383.9	435.2	497.4	-57.9	-45.4
20	275.6	297.6	319.7	367.8	-52.4	-41.2
30	250.2	261.5	272.8	312.2	-48.0	-37.9
40	210.9	217.2	223.6	245.9	-40.1	-32.2
50	226.8	231.2	235.6	252.5	-32.5	-26.3
60	259.1	262.4	265.7	280.1	-26.1	-21.2
70	234.7	236.2	237.8	247.4	-20.2	-16.5
80	208.6	209.7	210.8	218.5	-18.8	-15.4
90	265.4	266.0	266.7	272.4	-12.1	-10.0
100	369.3	369.8	370.4	375.1	-7.9	-6.5

Table 7 Cross-method trend diagnostics relative to the Vesic series

Method	Response type	Mean difference (%)	Pearson r	Spearman ρ	Minimum clay (%)	Minimum response (kPa)
Hansen	collapse	18.4	0.998	0.991	80	224.5
UBLA-informed	collapse	26.9	0.991	0.873	80	218.5
FELA-informed	collapse	33.8	0.973	0.764	80	209.7
FEM q3%	settlement	21.1	0.947	0.991	80	288.6

5 Discussion

5.1 A load-bearing skeleton that changes identity

The results are best understood as a change in the identity of the load-bearing skeleton. At the sand-rich end, ϕ' is high, dilation is present, and nearly all analytical bearing resistance comes from the self-weight/friction term. Clay addition progressively interrupts that network. The immediate consequence is not a proportional increase in cohesion, but a rapid loss of the granular advantage embedded in $N\gamma$. This explains why the first 10–20% clay produces a far larger drop in Vesic capacity than the same absolute change in clay content near the clay-rich end.

Once the fine phase becomes mechanically continuous, the logic reverses. Friction angle is already low and the $N\gamma$ term has little room left to decline, whereas c' can still increase substantially. The sharp rise of c' at 90–100% therefore produces a recovery even though ϕ' continues to fall. The non-monotonic capacity curve is consequently not contradictory; it is the expected outcome when two different strength mechanisms exchange dominance at different rates.

5.2 Why the minimum is delayed beyond the friction–cohesion crossover

The most distinctive finding is the separation between equal contribution and minimum total resistance. Equal partition occurs at about 50% clay, where $I_c \approx 0.516$. Yet the minimum capacity is delayed to 80%. The intervening 50–80% region is mechanically vulnerable because the granular contribution continues to collapse after it has ceased to dominate, while the cohesive contribution is constrained by two effects: c' grows slowly and N_c decreases with ϕ' . The 70–80% plateau in c' makes that delay especially visible in the present dataset.

This resistance-deficit window should not be interpreted as a universal critical fines content. Packing density, particle-size ratio, clay mineralogy, water state, and preparation method can all shift the transition (Wu et al. 2025),(Goudarzy et al. 2021). Its value lies in the diagnostic logic: separating component crossover from total-capacity minimum prevents the common assumption that the point at which cohesion becomes dominant must also be the point at which capacity begins to recover.

5.3 Serviceability mobilization and collapse are related, not interchangeable

The FEM and collapse-oriented models converge on the same minimum composition but not the same pressure magnitude. That difference is physically appropriate. Analytical and limit-analysis methods estimate a limiting state, while $q_3\%$ records how much resistance has been mobilized at a finite settlement under a stiffness-dependent continuum response (Merifield et al. 1999). In sand, where analytical $N\gamma$ becomes very large, the gap is especially pronounced because a 3% settlement has not mobilized the full collapse mechanism assumed by the bearing-capacity equations. In the clay-rich states, the gap narrows because cohesion dominates and the contrast between alternative $N\gamma$ formulations becomes less important.

The stiffness data reinforces this point. E_{50ref} and E_{ur} decrease with clay content while E_{oedref} rises, so the material cannot be represented by a single monotonic 'stiffness versus clay' rule. The present FEM deliberately uses E_{50ref} as the pre-yield elastic modulus; a hardening-soil or small-strain formulation would permit stress-dependent stiffness and unloading-reloading behaviour to evolve more realistically. The current model should therefore be read as a transparent comparative continuum model, not as a substitute for calibrated constitutive testing.

5.4 Engineering significance

For the assumed 2 m surface strip footing, the transition zone is the least forgiving region for design. Using the clean-sand end member as a proxy would grossly overstate collapse resistance for mixtures containing moderate-to-high clay. Conversely, treating the clay-rich end member as representative would miss the lower-capacity states around 70–80% clay. The practical lesson is that mixed-grained foundation soils require direct characterization of both ϕ' and c' ; classification by fines percentage alone can conceal the composition at which the foundation is most vulnerable.

The multi-method agreement strengthens that interpretation. Vesic, Hansen, Meyerhof-type, Terzaghi-type, FELA- and UBLA-informed calculations all place the capacity minimum at 80% clay, and the independently executed FEM $q_3\%$ response does the same. The absolute differences between methods are therefore less important than their common indication that the governing mechanism changes across the composition range.

5.5 Scope and limitations

Several limitations define the boundary of the conclusions. The working dataset contains one property state for each nominal clay content and does not provide replicate uncertainty for every parameter. The footing is idealized as rigid, smooth, surface-founded and vertically loaded. The custom FEM uses a smooth elastic–perfectly plastic Drucker–Prager approximation, a single operative elastic modulus and no constitutive hardening, anisotropy or coupled pore-pressure calculation. The FELA- and UBLA-informed series are assembled from published limit-analysis N_γ information (Ukritchon et al. 2003) and are not independent runs of the custom solver. Finally, the available hydraulic observations were not sufficiently complete for quantitative hydro-mechanical coupling; consequently, the present numerical interpretation is restricted to mechanical response.

These restrictions do not alter the central mechanistic result, but they identify the most useful next experiments: replicate strength testing, model-footing or plate-load tests, strain-dependent stiffness measurements and, if the multiphysics scope is expanded, complete hydraulic-conductivity and water-retention data for coupled hydro-mechanical modeling.

6 Conclusions

The complete 0–100% composition series reveals a continuous but strongly nonlinear transition from a friction- and dilation-controlled sand skeleton to a cohesion-controlled clay matrix. Effective friction angle decreases from 41.22° to 14.28° , whereas effective cohesion rises from 0 kPa to 34.033 kPa. The stiffness and constitutive descriptors evolve at different rates as well: E50ref and Eurref decrease, Eoedref increases, dilation disappears, and K0 rises. The foundation response is therefore governed by an evolving material state rather than clay content as an isolated scalar variable.

The analytical resistance partition identifies a mechanistic lag between the loss of granular resistance and the development of cohesive resistance. The cohesive and self-weight/friction components of the Vesic expression become approximately equal near 50% clay, yet the minimum total capacity is delayed until 80% clay. Vesic capacity falls from 2165.8 kPa for the sand end member to 258.3 kPa at 80% clay before recovering to 401.4 kPa at 100%. Hansen, Meyerhof-type, Terzaghi-type, FELA-informed, and UBLA-informed collapse estimates reproduce the same minimum composition. For the present material, the 50–80% interval can therefore be interpreted as a resistance-deficit window in which the high-friction sand skeleton has largely lost its advantage before the clay matrix has developed sufficient cohesion to compensate.

The independently executed nonlinear FEM reaches the same composition-level conclusion through a different physical route. At $s/B = 3\%$, mobilized pressure decreases from 545.0 kPa at 0% clay to 288.6 kPa at 80% and then increases to 352.2 kPa at 100%. Representative mesh differences of approximately 5.0–6.9% support the use of $q_{3\%}$ as a comparative settlement-based response measure, while the large-settlement branch is retained only as a description of post-yield mobilization rather than a mesh-converged ultimate capacity. The agreement among methods is consequently strongest in the location and shape of the transition, not in absolute pressure, because collapse solutions and displacement-controlled FEM represent different stages of soil resistance mobilization.

For foundation assessment, the results demonstrate that mixed-grained soils should not be represented by either end member or by fines content alone. Direct characterization of ϕ' , c' and stiffness within the transition range is essential, particularly where the granular skeleton is being lost but cohesive resistance has not yet fully developed. The proposed composition–property–response framework provides a reproducible way to identify that vulnerable state, while future model-footing tests, replicated strength measurements and more advanced stress-dependent constitutive calibration would provide the experimental validation needed to generalize the transition beyond the present soil system.

References

- Al-Mosawe MJ, Albusoda BS, Yaseen AS (2011) BEARING CAPACITY OF SHALLOW FOOTING ON SOFT CLAY IMPROVED BY COMPACTED FLY ASH. *Journal of Engineering* 17:1473–1482. <https://doi.org/10.31026/j.eng.2011.06.13>
- Alnmr A, Ray R (2023) Investigating the Impact of Varying Sand Content on the Physical Characteristics of Expansive Clay Soils from Syria. *Geotechnical and Geological Engineering* 42:2675–2691. <https://doi.org/10.1007/s10706-023-02698-w>
- AL-Shyoukhi T, Elmeligy M, Altahrany A (2023) Experimental and Numerical Parametric Studies on Inclined Skirted Foundation Resting on Sand. *Civil Engineering Journal* 9:1795–1807. <https://doi.org/10.28991/cej-2023-09-07-017>
- Bolton MD (1986) The strength and dilatancy of sands. *Géotechnique* 36:65–78. <https://doi.org/10.1680/geot.1986.36.1.65>

- Bouchemella S, Taibi S (2022) Effect of suction on the mechanical behaviour of unsaturated compacted clay–sand mixtures. *Studia Geotechnica et Mechanica* 44:175–189. <https://doi.org/10.2478/sgem-2022-0016>
- Chen YF, Monteiro É, Koutiri I, Favier V (2024) Stress-constrained topology optimization using the constrained natural element method. *SPIRE - Sciences Po Institutional REpository* 67:. <https://doi.org/10.1007/s00158-024-03786-y>
- Conte E, Donato A, Troncone A (2013) Progressive failure analysis of shallow foundations on soils with strain-softening behaviour. *Computers and Geotechnics* 54:117–124. <https://doi.org/10.1016/j.compgeo.2013.07.002>
- Díaz-Segura EG (2013) Assessment of the range of variation of N_f from 60 estimation methods for footings on sand. *Canadian Geotechnical Journal* 50:793–800. <https://doi.org/10.1139/cgj-2012-0426>
- Franssen J (2005) SAFIR: A Thermal and Structural Program for Modeling Structures Under Fire. *Engineering Journal* 42:143–158. <https://doi.org/10.62913/engj.v42i3.856>
- Füssl J, Li M, Lukacevic M, et al (2017) Comparison of unit cell-based computational methods for predicting the strength of wood. *Engineering Structures* 141:427–443. <https://doi.org/10.1016/j.engstruct.2017.03.005>
- Gendy ME (2025) A comparative machine and deep learning approach for predicting ultimate bearing capacity of shallow foundations in cohesionless soil. *Scientific Reports* 15:40402–40402. <https://doi.org/10.1038/s41598-025-22812-7>
- Goudarzy M, Sarkar D, Lieske W, Wichtmann T (2021) Influence of plastic fines content on the liquefaction susceptibility of sands: monotonic loading. *Acta Geotechnica* 17:1719–1737. <https://doi.org/10.1007/s11440-021-01283-w>
- Hansen JB (1970) A revised and extended formula for bearing capacity. *Medical Entomology and Zoology* 28:5–11
- Hijaj M, Lyamin AV, Sloan SW (2004) Numerical limit analysis solutions for the bearing capacity factor N_γ . *International Journal of Solids and Structures* 42:1681–1704. <https://doi.org/10.1016/j.ijsolstr.2004.08.002>
- Keller T, Lamand M, Peth S, et al (2012) An interdisciplinary approach towards improved understanding of soil deformation during compaction. *HAL (Le Centre pour la Communication Scientifique Directe)* 128:61–80. <https://doi.org/10.1016/j.still.2012.10.004>
- Khatri VN, Kumar J (2019) Finite-Element Limit Analysis of Strip and Circular Skirted Footings on Sand. *International Journal of Geomechanics* 19:. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0001370](https://doi.org/10.1061/(asce)gm.1943-5622.0001370)
- Kim D, Nam BH, Youn H (2018) Effect of clay content on the shear strength of clay–sand mixture. *International Journal of Geo-Engineering* 9:. <https://doi.org/10.1186/s40703-018-0087-x>
- Li M, Füssl J, Lukacevic M, et al (2018) An algorithm for adaptive introduction and arrangement of velocity discontinuities within 3D finite-element-based upper bound limit analysis approaches. *Computer Methods in Applied Mechanics and Engineering* 348:875–911. <https://doi.org/10.1016/j.cma.2018.12.017>
- Lu K, Coombs WM, Augarde CE, Hu Z (2019) An implicit boundary finite element method with extension to frictional sliding boundary conditions and elasto-plastic analyses. *Computer Methods in Applied Mechanics and Engineering* 358:112620–112620. <https://doi.org/10.1016/j.cma.2019.112620>
- Merifield RS, Sloan SW, Yu H (1999) Rigorous plasticity solutions for the bearing capacity of two-layered clays. *Géotechnique* 49:471–490. <https://doi.org/10.1680/geot.1999.49.4.471>
- Meyerhof GG (1963) Some Recent Research on the Bearing Capacity of Foundations. *Canadian Geotechnical Journal* 1:16–26. <https://doi.org/10.1139/t63-003>
- Meyerhof GG (1951) The Ultimate Bearing Capacity of Foundations. *Géotechnique* 2:301–332. <https://doi.org/10.1680/geot.1951.2.4.301>
- Nagaraj HB (2016) Influence of gradation and proportion of sand on stress–strain behavior of clay–sand mixtures. *International Journal of Geo-Engineering* 7:. <https://doi.org/10.1186/s40703-016-0033-8>
- Osman AS, Bolton MD (2005) Simple plasticity-based prediction of the undrained settlement of shallow circular foundations on clay. *Géotechnique* 55:435–447. <https://doi.org/10.1680/geot.2005.55.6.435>
- Pantelidis L (2024) Bearing capacity of shallow foundations: a focus on the depth factors in combination with the respective N-factors. *Arabian Journal of Geosciences* 17:. <https://doi.org/10.1007/s12517-024-11976-7>
- Pantelidis L (2023) Finite element and regression analysis for improving Terzaghi's, Meyerhof's, Hansen's and Vesic's bearing capacity methods for strip foundations. *Research Square*
- Regueiro RA, Borja RI (1999) A finite element model of localized deformation in frictional materials taking a strong discontinuity approach. *Finite Elements in Analysis and Design* 33:283–315. [https://doi.org/10.1016/s0168-874x\(99\)00050-5](https://doi.org/10.1016/s0168-874x(99)00050-5)
- Salgado R, Bandini P, Karim AM (2000) Shear Strength and Stiffness of Silty Sand. *Journal of Geotechnical and Geoenvironmental Engineering* 126:451–462. [https://doi.org/10.1061/\(asce\)1090-0241\(2000\)126:5\(451\)](https://doi.org/10.1061/(asce)1090-0241(2000)126:5(451))
- Sánchez PJ, Huespe AE, Oliver J (2007) On some topics for the numerical simulation of ductile fracture. *International Journal of Plasticity* 24:1008–1038. <https://doi.org/10.1016/j.ijplas.2007.08.004>
- Sanei M, Forti TLD, Santos E (2020) Étalonnage automatique de modèles élastoplastiques basés sur la physique à l'aide d'algorithmes d'optimisation pour la roche poreuse, Partie II. *HAL (Le Centre pour la Communication Scientifique Directe)*
- Santos LF dos, Freitas AC de (2020) Orientational Analysis of the Vesic's Bearing Capacity of Shallow Foundations. *Soils and Rocks* 43:3–9. <https://doi.org/10.28927/sr.431003>
- Terzaghi K (1943) *Theoretical Soil Mechanics*. Andalus University

- Thevanayagam S (1998) Effect of Fines and Confining Stress on Undrained Shear Strength of Silty Sands. *Journal of Geotechnical and Geoenvironmental Engineering* 124:479–491. [https://doi.org/10.1061/\(asce\)1090-0241\(1998\)124:6\(479\)](https://doi.org/10.1061/(asce)1090-0241(1998)124:6(479))
- Ukritchon B, Whittle AJ, Klangvijit C (2003) Calculations of Bearing Capacity Factor N_γ Using Numerical Limit Analyses. *Journal of Geotechnical and Geoenvironmental Engineering* 129:468–474. [https://doi.org/10.1061/\(asce\)1090-0241\(2003\)129:6\(468\)](https://doi.org/10.1061/(asce)1090-0241(2003)129:6(468))
- Vardanega PJ, Bolton MD (2015) Design of Geostuctural Systems. Bristol Research (University of Bristol) 2:. <https://doi.org/10.1061/ajrua6.0000849>
- Vesić AS (1973) Analysis of Ultimate Loads of Shallow Foundations. *Journal of the Soil Mechanics and Foundations Division* 99:45–73. <https://doi.org/10.1061/jsfeaq.0001846>
- Wu K, Lv B, Baoyin H, et al (2025) Experimental and DEM Study on the Mechanical Behaviors of Sand–Fines Mixtures with Different Fines Contents and Particle Size Ratios. *Materials* 18:4929–4929. <https://doi.org/10.3390/ma18214929>
- Wu W, Li J, Zhao J (2014) Role of filling materials in a P-wave interaction with a rock fracture. *Engineering Geology* 172:77–84. <https://doi.org/10.1016/j.enggeo.2014.01.007>

Statements and Declarations

Funding: No funding received.

Competing interests: N/A

Data availability: The datasets generated and/or analysed during the current study are available from the corresponding author on reasonable request.