

Physics-Informed Hybrid Stochastic-AI Framework Integrating Galle and Woods Mechanics, Monte Carlo Simulation, and Dual-Track AI Decision Support for ROP Optimization, Cutter Wear Mitigation, and NPT Reduction: A Tawakul-1 Well Case Study (Block-8, Sudan)

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Physics-Informed Hybrid Stochastic-AI Framework Integrating Galle and Woods Mechanics, Monte Carlo Simulation, and Dual-Track AI Decision Support for ROP Optimization, Cutter Wear Mitigation, and NPT Reduction: A Tawakul-1 Well Case Study (Block-8, Sudan)

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Abstract

Rate of Penetration (ROP) optimization in heterogeneous, structurally complex sedimentary basins remains a significant technical challenge due to non-linear interdependencies among surface operational controls, unquantified downhole uncertainty, and progressive cutter degradation. Classical empirical models (e.g., Galle and Woods) provide essential mechanical rigor but are inherently deterministic and struggle with field telemetry noise, whereas purely data-driven machine learning models lack physical interpretability and dynamic risk awareness.

To bridge this gap, this study introduces a physics-informed, hybrid stochastic-AI decision-support framework that seamlessly integrates the depth-based Galle and Woods drillability core, high-throughput Monte Carlo simulation (MCS), local supervised machine learning, and cloud-based Large Language Model (LLM) generative reasoning. The architecture was calibrated, evaluated, and validated using depth-aligned operational telemetry across a 263 m target claystone interval (MD 3167--3430 m) encompassing

Polycrystalline Diamond Compact (PDC) bit Runs 8 and 9 of the Tawakul-1 exploratory well in Block-8 (Sinnar State, Blue Nile Basin, Sudan). This selection ensured consistent lithological and drilling fluid baselines to minimize field perturbations.

Automated preprocessing algorithms sanitized raw field telemetry by filtering off-bottom operational states and suppressing sensor noise, yielding 131 sanitized records for Run 8 (from 147 initial rows) and 376 records for Run 9 (from 381 initial rows). High-throughput Monte Carlo simulations (13,100 iterations for Run 8; 37,600 iterations for Run 9) transformed static parameters into risk-quantified probabilistic response surfaces, decoding multi-variable non-linear interactions and establishing bit dulling characteristics. A local machine learning regression engine (XGBoost) demonstrated exceptional predictive accuracy (Testing $R^2 = 0.9842$, RMSE = 0.412 ft/hr, AAPE = 2.15%), outperforming Artificial Neural Network (ANN) and Support Vector Machine (SVM) architectures to define upper-bound mathematical performance potential.

To mitigate premature tool failure, stochastic outputs were serialized via a secure HTTPS REST API JSON protocol to the Google Gemini LLM engine. Governed by closed-domain system prompts and low-entropy decoding ($T = 0.1$), the LLM evaluated point-density clusters and thermal-wear boundaries to establish actionable operational envelopes. For Run 9, Gemini selected a wear-balanced equilibrium at Weight on Bit ($W = 24.0$ klb) and Rotary Speed ($N = 161$ RPM). This risk-averse selection achieved a stabilized ROP of 47.63 ft/hr and representing a 28.56% improvement over historical field performance (37.05 ft/hr); while holding the cutter wear derivative to a sustainable threshold ($dF/dD = 10.8675$), suppressing wear rates by 49.2% relative to unconstrained local machine learning peaks.

By preventing entry into critical thermal-wear regimes, the proposed framework effectively delineates distinct ROP and cutter degradation signatures across drilling intervals, directly eliminates major causes of Non-Productive Time (NPT) and Invisible Lost Time (ILT), and provides an uncertainty-aware decision-support architecture for maximizing total Cost per Foot (CPF) efficiency and tool longevity in mature sedimentary basins.

Keywords: Rate of Penetration (ROP) optimization, Galle and Woods model, Monte Carlo simulation, Artificial Intelligence (AI), Bit Wear Kinetics, Uncertainty Quantification, Drilling Performance, Extreme Gradient Boosting (XGBoost), Large Language Models (LLM) Non-Productive Time (NPT) reduction, Block-8 Sudan.

Introduction

Rate of Penetration (ROP) serves as a primary metric for evaluating drilling efficiency, directly governing total well construction duration, capital expenditure, and non-productive time (NPT). In rotary drilling, optimizing ROP requires maximizing rock destruction mechanics while simultaneously maintaining wellbore stability and mitigating premature bit dulling. However, ROP optimization remains a multi-variable problem governed by complex interdependencies between controllable operational parameters—

primarily Weight on Bit (WOB), Rotary Speed (RPM), hydraulics, and bit selection—and non-controllable formation characteristics, such as lithological drillability, unconfined compressive strength (UCS), localized stresses, and continuous cutter wear. Because these factors operate concurrently across heterogeneous intervals, parameter combinations that yield optimal penetration rates in one geological formation can induce severe stick-slip, vibration, or accelerated bit wear in adjacent strata.

This operational challenge is particularly pronounced in Sudan’s mature sedimentary basins, such as the Muglad and Melut basins. These formations are characterized by complex, interbedded sequences of highly variable rock hardness, where sharp mechanical transitions often lead to operational instability, bit degradation, and erratic penetration rates. Furthermore, the operational landscape in these regions frequently lacks continuous real-time downhole telemetry or high-cost automated optimization systems, forcing reliance on conventional, experience-based static drilling practices. Consequently, there is an acute technical necessity for robust, data-driven modeling workflows that can accurately predict and optimize ROP using standard field-recorded drilling parameters.

Over the past several decades, numerous analytical and empirical formulations have been established to quantify drilling performance, including foundational models by Galle and Woods (1963), Maurer (1965), Bingham (1965), and Bourgoyne and Young (1974). Among these, the empirical formulation proposed by Galle and Woods remains exceptionally effective for parameter evaluation due to its explicit, coupled mathematical relationship connecting ROP, WOB, RPM, formation drillability, bit wear, and tooth dulling mechanics. Despite its mathematical elegance and suitability for sparse field datasets, the classical Galle and Woods formulation is inherently deterministic; it struggles to capture non-linear downhole uncertainties, transient operational disturbances, and dynamic rock-bit interactions without modern computational augmentation.

To bridge the gap between classical empirical formulations and modern data analytics, this study introduces an integrated hybrid optimization framework that couples the Galle and Woods model with stochastic sampling and artificial intelligence (AI) routines. Implemented within a specialized Python computing architecture, the governing equations translate raw field operational logs into deterministic drilling response curves. To address inherent parameter uncertainties and downhole variability, Monte Carlo Simulation (MCS) is integrated into the model, generating probabilistic performance envelopes across multiple operational scenarios. Finally, a machine learning (AI) decision-support layer is synthesized alongside the stochastic outputs to decode non-linear multi-variable interactions and derive optimal operational windows.

The primary scientific novelty of this work lies in the unified integration of classical empirical drillability mechanics, stochastic uncertainty quantification, and AI-driven pattern recognition applied to challenging field data from Sudan. Rather than deploying empirical formulas or machine learning models in isolation, this multi-tiered architecture preserves the physical grounds of the Galle and Woods model while significantly

enhancing its predictive precision, robust parameter sensitivity analysis, and field applicability for real-time and offset well drilling optimization.

Problem Statement

Rotary drilling optimization in complex, interbedded sedimentary basins is severely hindered by the non-linear interplay between surface operational controls (WOB, RPM) and unquantified downhole uncertainties (formation hardness, bit dulling mechanics). Existing optimization approaches typically suffer from a critical trade-off: empirical formulations (e.g., Galle and Woods) provide essential mechanical rigor but are inherently deterministic and incapable of handling field stochasticity, whereas purely data-driven AI models lack physical grounding and interpretability. In mature field operations where high-resolution real-time telemetry is unavailable, reliance on manual heuristics leads to erratic penetration rates, accelerated tool degradation, and excessive NPT. A major technical gap remains in synthesizing classical rock-bit interaction mechanics with stochastic sampling and machine learning to construct an interpretable, uncertainty-aware decision-support architecture tailored to surface-recorded drilling logs.

Research Objective

The main aim of the study is to formulate and validate an integrated, physics-informed computational framework (combining the empirical Galle and Woods drillability model, Monte Carlo stochastic simulation, and machine learning architectures) to predict, analyze, and optimize Rate of Penetration (ROP) and bit wear mechanics under variable field conditions. To achieve this primary aim, the study addresses the following Specific Objectives:

1. To process, filter, and standardize surface operational logs and formation records from the Tawakul-1 well (Block-8, Sudan), establishing a noise-filtered data baseline suitable for physical and computational modeling.
2. To implement the classical Galle and Woods formulation within an object-oriented Python framework to evaluate deterministic relationships and quantify parameter sensitivities among Weight on Bit (WOB), Rotary Speed (RPM), formation drillability, and cutter dulling dynamics.
3. To couple Monte Carlo simulation routines with the governing analytical equations, generating probabilistic ROP response surfaces to quantify downhole operational uncertainties across consecutive drilling intervals.
4. To synthesize a machine learning decision-support layer with the deterministic and stochastic outputs, decoding complex multi-variable non-linear interactions to establish optimal operational envelopes for maximum drilling efficiency.

Literature Review

Sudan's hydrocarbon potential is hosted within a network of onshore and offshore intra-continental rift basins formed during the evolution of the West, Central, and East African Rift Systems. While regional exploration data remain relatively sparse, established

petroleum system analogies demonstrate that all fundamental requirements for commercial accumulation are met. Robust, high-potential source rock facies are documented across the Muglad, Melut, Blue Nile, White Nile, and Red Sea basins. Geochronologically, the sedimentary record initiates with Cambro–Ordovician sequences confined within fault-bounded grabens. These structural depressions developed in response to post-orogenic extension following the consolidation of the Pan-African basement in northern and northwestern Sudan (Samasu International Company Ltd, 2004). Figure (1) illustrates the geological map of Sudan and the distribution of major sedimentary basins.

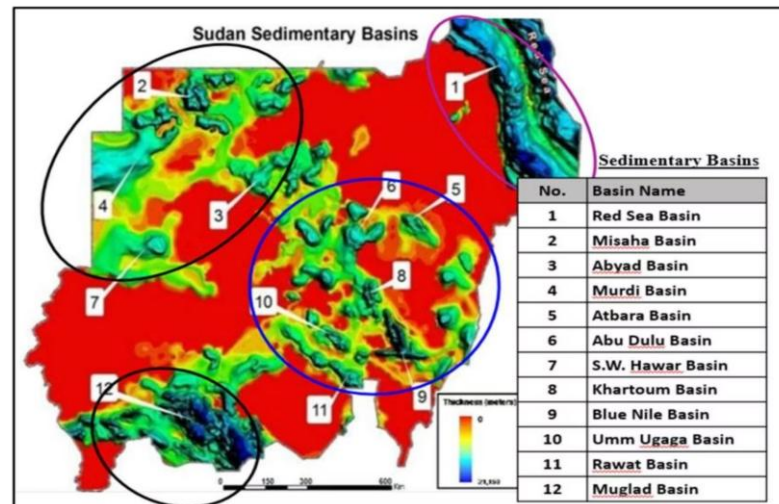


Figure (1): Regional geological and sedimentary basin map of Sudan (Modified after Oil and Energy Ministry-Sudan, 2023).

Block-8 is situated in the Blue Nile Basin of eastern Sudan, a geologically complex region dominated by extensive major and minor fault systems (Shazaly and Ibrahim, 2020; Ibrahim et al., 2016). Exploration in the block dates back to 1983 with Chevron’s drilling of the Dindir-1 well; however, political instability eventually forced the company to suspend operations and withdraw. Exploration subsequently resumed under the White Nile Petroleum Operating Company (WNPOC). Of the nine wells drilled within the block, Hosan-1 and Tawakul-1 yielded significant gas discoveries. Tawakul-1, WNPOC’s sixth wildcat well, forms the central focus of this study. Figure (2) shows the location of Tawakul-1 alongside Block-8’s broader layout, WNPOC’s other wells, and neighboring blocks. Given the region’s structural complexity, Tawakul-1 penetrated formations with contrasting geomechanical and lithological properties that markedly affect drilling performance (underscoring the need to evaluate the factors controlling the rate of penetration (ROP)).

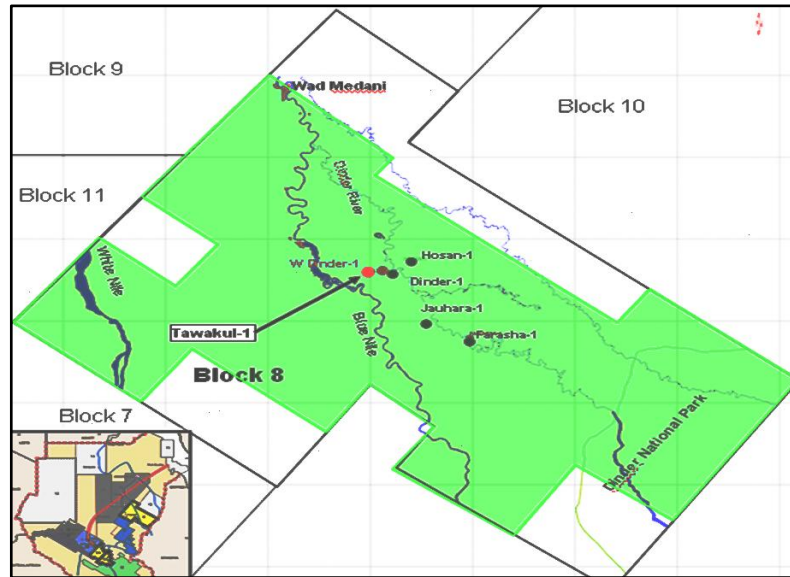


Figure (2): Location map of the Tawakul-1 well within Block-8, Blue Nile Basin (Modified after WNPOC, 2009).

As highlighted by Elrayah (2024) and Amadi (2023), Rate of Penetration (ROP) control depends on three main categories: drilling system constraints (facility limits, equipment technology, BHA tendency), operational variables (weight on bit, rotary speed, flow rate), and geomechanical properties (formation strength, abrasiveness, mineralogy, porosity, and permeability). Meanwhile, Rotary drilling performance has advanced significantly since its inception in the early 1900s, with the 1950s marking the transition into the scientific era through innovations in bit design, mud chemistry, and hydraulic optimization. The 1970s saw the emergence of automation and closed-loop control systems to better manage real-time variables, followed by the integration of mathematical modeling and structured field-optimization techniques in the mid-1980s. By the 1990s, these developments culminated in the sophisticated well-planning methodologies used today (Makkani et al., 2023), as chronologically outlined in **Figure (3)**. Beyond technological evolution, operational efficiency depends heavily on managing downhole and surface parameters. Peng et al. (2025) classify these parameters into operational variables—such as rotational speed (RPM), weight on bit (WOB), flow rate, standpipe pressure (SPP), and drilling fluid density—and inherent constraints. Uncontrollable constraints include rock mechanical properties, depth, temperature, wellbore hydraulics, rig power, logistics, and crew competency (Eren, 2010).

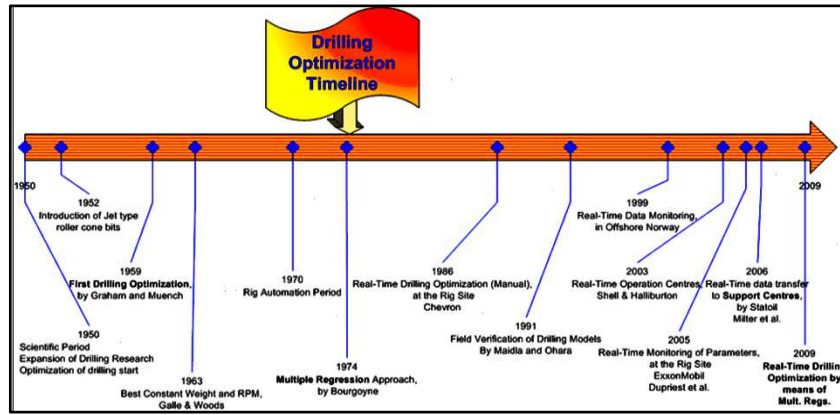


Figure (3): Historical timeline of major technological advancements in drilling optimization (Eren, 2010).

The quantitative optimization of drilling parameters began in the late 1950s, when Graham and Muench (1959) developed empirical equations combining rotational speed (RPM) and weight on bit (WOB) to predict bit life. Maurer (1963) subsequently formulated relationships linking rate of penetration (ROP), RPM, bit size, rock strength, and WOB for roller-cone bits (Sobhi et al., 2022). Concurrently, Galle and Woods (1963) analyzed the interdependent effects of RPM and WOB on bit wear and penetration rate to establish optimal operating limits. Building on these foundational studies, Bingham (1965) derived a dedicated ROP equation as a function of RPM (Jasib et al., 2024). A major shift occurred when Bourgoyne and Young (1974) introduced their widely adopted multi-regression model for ROP prediction, while also investigating the influence of mud density on drilling performance alongside (Shazaly and Ibrahim, 2019; Abouzar et al., 2009).

By the mid-1980s, operators began translating these theoretical concepts into field-usable chart formats and structured optimization strategies, enabling real-time engineering adjustments at the rig site. Dynamic modeling advanced significantly with Reza and Alcocer (1986), who applied Buckingham's Pi theorem to construct a dynamic, non-linear, dimensionless ROP model. Parallel developments focused on energy efficiency and monitoring systems. Teale (1965) formalized the concept of Mechanical Specific Energy (MSE) to directly correlate Rate of Penetration (ROP) with Weight on Bit (WOB) and overall energy input. Around the same time, early real-time data acquisition systems were introduced to improve drilling efficiency, laying the groundwork for modern optimization workflows. Later efforts refined predictive accuracy by integrating multi-variable statistical models and sensitivity analyses that accounted for cuttings transport. Today, real-time modeling and centralized monitoring are widely deployed to evaluate downhole performance both during and after drilling operations (Jasib et al., 2024; Ibrahim and Musa, 2004, Ibrahim and Musa, 2003, Ibrahim et al., 2001).

By the mid-to-late 2000s, Mechanical Specific Energy (MSE) transitioned from a theoretical concept into standard industry practice. Researchers integrated MSE into real-time advisory and autonomous detection systems, while extensive studies mapped the critical relationships between Weight on Bit (WOB), MSE, and drillability limits.

Complementing these physical models, comprehensive Rate of Penetration (ROP) optimization frameworks emerged, backed by economic analyses that demonstrated the clear financial value of real-time data management. Parallel advances in downhole data integration, backward-simulation logging, hydraulics evaluation, mud viscosity modeling, and bit-weight mechanics paved the way toward fully intelligent drilling platforms. Ultimately, machine learning transformed the discipline. The introduction of Artificial Neural Networks (ANNs) enabled simultaneous bit-life and ROP optimization by continuously evaluating WOB and RPM. Today, advanced ANN applications forecast penetration rates, minimize total well construction costs, and dynamically fine-tune operational parameters in real time (an evolution frequently documented in modern drilling technology overviews) (Lazar et al., 2022; Ibrahim, 2013).

Among these historical approaches, the Galle and Woods model was selected for this study due to its mathematical simplicity, operational accuracy, and ease of implementation. Furthermore, its sound physical foundation makes it exceptionally suitable for integration with advanced artificial intelligence frameworks in drilling optimization.

In 1963, Galle and Woods developed a seminal mathematical model to optimize the rate of penetration (ROP) by determining the optimal combination of weight on bit (WOB) and rotary speed (RPM). As pioneers in quantifying these relationships, Galle and Woods derived mathematical equations, field procedures, and graphical design charts to facilitate practical rig-site applications (Eren, 2010; Ibrahim, 2003; Ibrahim, 2001). Their framework establishes explicit quantitative relationships between penetration rate, bit dulling mechanisms, WOB, and RPM across three core governing equations.

The core formulation for total distance drilled (F , in ft) over rotating time (T , in hours) is expressed as:

$$\frac{ROP}{F} = f(C_F, W, N, a, r, K, P) \quad (1)$$

- $a = 0.928125 D^2 + 6.0 D + 1.0$ (2)
- P = Exponent corresponding to the bit tooth wear profile:
 - $P = 1.0$ for flat-crested wear
 - $P = 0.5$ for self-sharpening wear
 - $P = 0.0$ for minimal/no wear
- D = Fractional tooth wear ($0 \leq D \leq 1$)

Progressive tooth dulling kinetics are governed by formation abrasiveness alongside applied operational parameters:

$$\frac{dD}{dT} = \frac{A_F M R}{a^P} \quad (3)$$

$$M = 1359.1 - 714.19 \left(\frac{W}{D_{bit}} \right) \quad (4)$$

R = Function relating rotary speed to bit tooth wear rate ($R = N + 0.00004348 N^3$) (5)

Fractional bearing life degradation is modeled by considering fluid properties and load distribution:

$$\frac{dB}{dT} = \frac{1}{SLR_b}, \quad (0 \leq B \leq 1) \quad (6)$$

By linking drilling hydraulics and bit wear kinetics directly to surface operational variables, the Galle and Woods model provides a robust, deterministic engineering foundation for ROP optimization (Elrayah et al., 2013). However, because manual evaluation of these coupled non-linear equations is computationally tedious and time-intensive across extensive multi-well field datasets, translating this theoretical framework into computational environments (such as Python) is essential for enabling rapid parameter sweeps, automated data processing, and modern artificial intelligence integration.

Python is a versatile, high-level programming language widely utilized across the energy sector ranging from routine oilfield calculations to the development of complex drilling models for predicting, diagnosing, and optimizing overall drilling performance. Its open-source nature, extensive scientific ecosystem, and active development community make it particularly well-suited for petroleum engineering applications. Within drilling optimization, Python provides a unified technical computing environment that supports rapid algorithm development, early-stage code deployment, and workflow automation to streamline repetitive computations. Furthermore, its specialized libraries enable robust data management, debugging, statistical processing, and high-dimensional visualization, while dedicated toolboxes facilitate advanced signal processing, statistical modeling, and Machine Learning architectures (Okon et al., 2021). Beyond its capacity to execute deterministic equations, Python provides a powerful platform for probabilistic modeling and risk assessment. By leveraging computational libraries to perform comprehensive sensitivity analyses, engineers can evaluate operational uncertainties across diverse downhole scenarios. When coupled with stochastic techniques such as Monte Carlo Simulation, Python effectively transforms static mathematical models into adaptive decision-making tools for modern drilling engineering.

The development of reliable predictive drilling models relies heavily on field data to determine the empirical coefficients governing drilling evaluation in a specific region. Key operational parameters (such as Weight on Bit (WOB) and Rotary Speed (RPM)) must be properly calibrated to ensure the predictive model meets acceptable accuracy standards. To address the inherent challenges of subsurface uncertainty, operational ambiguity, and geological variability, probabilistic approaches have become integral to modern drilling optimization frameworks. In this context, Monte Carlo Simulation has emerged as a robust probabilistic methodology for analyzing uncertainty, providing risk-quantified outcomes, and supporting decision-making under conditions of incomplete or noisy data. By substituting single-point deterministic inputs with probability distributions across expected parameter ranges, Monte Carlo Simulation generates thousands of potential operational scenarios. Through iterative sampling of random variables across these distributions (such as normal, lognormal, uniform, or triangular functions) it delivers a comprehensive probabilistic forecast of drilling performance.

As highlighted by Nascimento (2012), the principal advantages of Monte Carlo Simulation in engineering applications include probabilistic forecasting through cumulative probability distributions (such as P10, P50, and P90), clear graphical risk-

profile visualizations, systematic sensitivity analysis to isolate key variance drivers, scenario evaluation across tail-risk events, and robust correlation modeling to account for statistical interdependencies among coupled variables. Consequently, Monte Carlo Simulation has become an essential tool in Rate of Penetration (ROP) optimization and drilling engineering, particularly for quantifying risk and guiding operational decisions when dealing with sparse or uncertain subsurface datasets. By transforming static deterministic models into probabilistic frameworks, Monte Carlo Simulation enables rigorous scenario evaluation. Furthermore, as computational techniques continue to evolve, integrating these stochastic frameworks with Artificial Intelligence paradigms offers significant promise for enhancing real-time predictive accuracy and autonomous decision support.

Artificial Intelligence and Machine Learning techniques have gained widespread adoption across engineering disciplines, offering transformative capabilities within the petroleum industry particularly for overcoming the physical assumptions, deterministic constraints, and regional limitations of traditional empirical ROP correlations. By capturing non-linear interdependencies among surface operational parameters, mud hydraulics, and complex subsurface mechanics, Artificial Intelligence architectures significantly enhance both predictive accuracy and real-time decision-making in drilling engineering. Pioneering work by Bilgesu et al. (1997) introduced Artificial Neural Networks (ANNs) to resolve the regional limitations of empirical equations, evaluating ROP across nine distinct formations in the United States using two network architectures: one integrating geological, operational, and hydraulic variables, and another incorporating progressive tooth and bearing wear. Subsequent studies expanded the input feature space to better account for downhole geomechanics. For instance, Amar and Ibrahim (2012) constructed two ANN frameworks incorporating WOB, RPM, equivalent circulating density, pore pressure gradient, depth, and Reynolds number, demonstrating significantly lower Average Absolute Percentage Error (AAPE) compared to conventional linear regression models. Similarly, Mantha and Samuel integrated AI paradigms with linear regression techniques to optimize ROP in horizontal wells by combining operational variables with real-time Gamma Ray lithological logs.

Recent advances have further leveraged advanced Machine Learning algorithms alongside detailed drilling fluid rheology to achieve exceptionally low error rates. El-Katatny (2020) developed an empirical correlation derived from an ANN model for vertical wells incorporating RPM, WOB, torque, standpipe pressure, plastic viscosity, and mud weight; while conventional empirical correlations yielded AAPEs exceeding 10% And achieved an AAPE of just 4.0%. Building on this precision, Ahmed et al. (2018) applied a Support Vector Machine machine learning architecture utilizing a similar combination of surface parameters and fluid properties, achieving an outstanding predictive accuracy with an AAPE of only 2.83% (Makkani et al., 2024). Beyond isolated ROP forecasting, integrating AI techniques (such as ANNs, advanced Machine Learning algorithms, and Genetic Algorithms) has enabled holistic, multi-objective optimization across the entire drilling workflow, including mud density selection, WOB scheduling, bit selection, and real-time dull wear tracking (Egbuna et al., 2025). AI-driven paradigms

leverage continuous learning from rig-site telemetry to monitor real-time drilling performance and identify early indicators of severe downhole hazards, such as differential pipe sticking and lost circulation. This transition from reactive problem-solving to predictive and autonomous operations drastically reduces non-productive time, lowers operational energy consumption, optimizes drilling efficiency, and preserves overall wellbore integrity (Ali and Hussain, 2026).

Bridging classical drilling mechanics with contemporary computational intelligence requires a unified, multi-tiered framework. The foundational Galle and Woods model provides the core deterministic physics linking surface mechanical parameters such as weight on bit (WOB) and rotary speed (RPM) and progressive bit wear kinetics directly to the rate of penetration (ROP). By implementing these non-linear equations within a Python computing environment, the theoretical framework is translated into an automated, highly scalable computational pipeline. To account for the subsurface noise, measurement ambiguity, and geomechanical variability inherent in complex formations, Monte Carlo Simulation is embedded into the Python architecture, converting single-point deterministic calculations into risk-quantified probabilistic forecasts across full parameter distributions (such as P10, P50, and P90). Furthermore, pairing this stochastic physics-based core with modern Artificial Intelligence and Machine Learning algorithms refines the model by capturing complex non-linear interdependencies among drilling fluid rheology, mechanical energy transfer, and dynamic downhole conditions that purely empirical or analytical models overlook.

To validate and demonstrate the practical efficacy of this integrated computational framework, operational logs and field data from the Tawakul-1 well in Block-8 (Blue Nile Basin, Sudan) were utilized as the primary benchmark dataset. Drilled through structurally complex sequences featuring highly variable lithological and geomechanical profiles, Tawakul-1 provides an ideal real-world environment for testing model robustness under contrasting downhole conditions. Operational telemetry including continuously recorded depth-indexed values of WOB, RPM, surface torque, pump pressure, mud rheology, and corresponding ROP were extracted from the Tawakul-1 database, preprocessed, and cleaned within Python. These historical logs served a dual purpose: first, calibrating the regional formation drillability (C_F) and abrasiveness (A_F) coefficients required by the Galle and Woods formulation; and second, supplying the empirical baseline for Monte Carlo parameter distribution sampling and Machine Learning model training. Anchoring this hybrid probabilistic-AI framework to the Tawakul-1 dataset effectively bridges classical drilling mechanics and data science, transforming theoretical equations into a field-calibrated decision-support tool tailored to Block-8 operations.

Methodology

This study establishes a hybrid, multi-tiered computational framework designed to optimize the Rate of Penetration (ROP) for the Tawakul-1 well (Block-8, Blue Nile Basin, Sudan) by seamlessly integrating classical physics-based drilling mechanics, stochastic uncertainty quantification, and data-driven artificial intelligence (AI) (Figures 4 and 5).

The analytical core transforms the classical time-dependent Galle–Woods model into a depth-based differential formulation. By isolating the time differential (dT) and substituting it into the tooth wear kinetics equation, the formulation yields a unified depth-based progress function for instantaneous ROP. Furthermore, operational field logs confirm that the target interval was drilled using Polycrystalline Diamond Compact (PDC) bits; given their fixed-cutter geometry lacking moving parts, bearing degradation (dBx/dT) is intentionally excluded, isolating cutter abrasive wear (D) as the sole mechanical limit governing bit longevity.

To calibrate regional drillability (C_F) and abrasiveness (A_F) coefficients under stable geomechanical and hydraulic conditions, the deep reservoir interval from Measured Depth (MD) 3167 to 3430 m (821-inch hole section lined with a 7-inch liner) was selected as the empirical foundation. This interval, comprising 44.58 on-bottom rotating hours across two consecutive PDC bit runs in a uniform KCl-polymer water-based mud system (9.8–10.6 ppg), was extracted from official well documentation and systematically preprocessed in Python using pandas and numpy.

The automated cleaning pipeline executed void-row purging, header sanitization, string error coercion, off-bottom record filtering, and Interquartile Range (IQR) outlier suppression, before scaling features via Min-Max normalization.

To address subsurface measurement noise and operational variability, single-point inputs are converted into continuous bounded probability distributions (Normal, Triangular, or Uniform) within a stochastic Monte Carlo Simulation engine, executing 10,000 iterative sampling runs across the physics core to generate risk-quantified performance profiles (P10 conservative, P50 median, and P90 optimistic thresholds). Augmented by physical wear outputs (D) and stochastic distributions, a supervised machine learning vector is evaluated across Artificial Neural Network (ANN), Support Vector Machine (SVM), and Extreme Gradient Boosting architectures, validated through 5-fold cross-validation targeting minimal Average Absolute Percentage Error.

Architectural deployment is executed in Python 3.13 following Object-Oriented Programming (OOP) and Separation of Concerns (SoC) principles, isolating core physics logic behind encapsulated private methods and inheritance-driven module structures. A dual-track AI handshake serializes simulation metrics into JSON format to query the Google Gemini Large Language Model API for automated, natural-language risk assessment and operational decision support.

Finally, the complete workflow is integrated into an asynchronous desktop Graphical User Interface (GUI) engineered with CustomTkinter and embedded Matplotlib multi-axis canvases, delivering a field-deployable decision-support tool designed to minimize Non-Productive Time (NPT) and optimize drilling efficiency in complex formations.

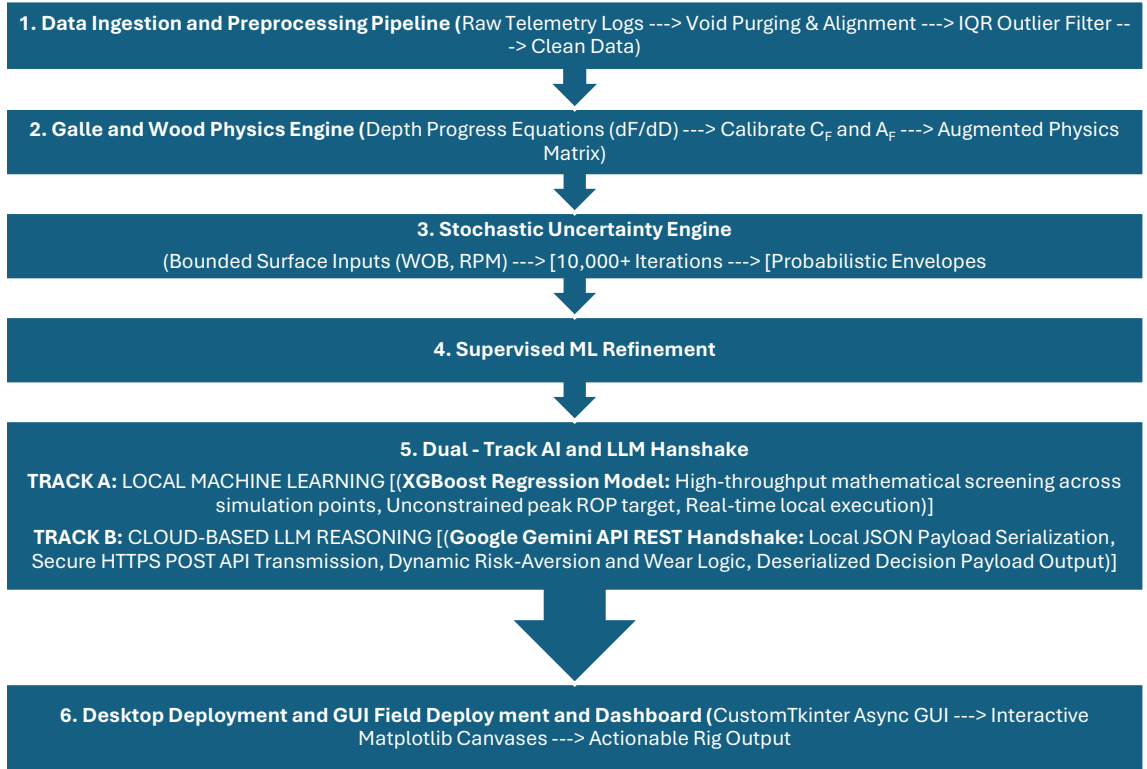


Figure (4): Logical architecture and data pipeline of the intelligent drilling system.

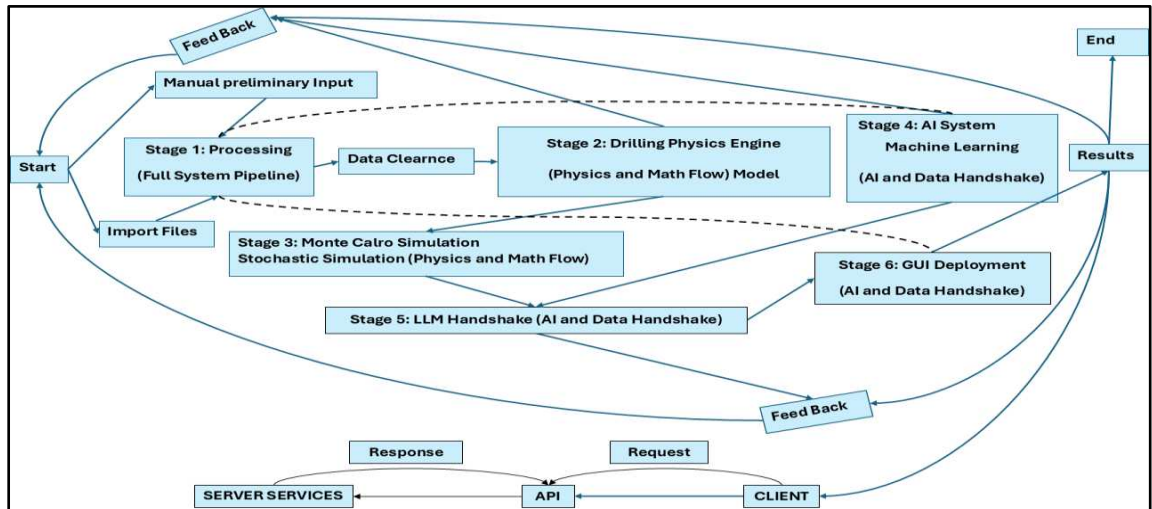


Figure (5): Integrated Workflow Architecture and End-to-End Data Pipeline for the Hybrid ROP Optimization System.

Results and Discussion

This section presents a comprehensive evaluation of drilling performance in the deep reservoir interval (MD 3167--3430 m) of the Tawakul-1 well in Block-8 (Blue Nile Basin, Sudan), synthesizing empirical rig telemetry, depth-based physics modeling, stochastic simulation, and Artificial Intelligence (AI) decision support across Polycrystalline Diamond Compact (PDC) bit Runs 8 and 9. The analytical framework is executed across three sequential phases: establishing deterministic baseline responses through the derived Galle and Woods depth-based progress equations (dF/dD) to evaluate regional formation

drillability (C_F), abrasiveness (A_F), and cutter wear (D); deploying Monte Carlo Simulation (10,000+ iterations per run) to transform single-point field logs into probabilistic risk profiles (P10, P50, and P90); and evaluating the resulting dataset via a dual-track AI framework combining supervised Machine Learning (ANN, SVM, XGBoost) with Large Language Model reasoning via the Google Gemini API. This integrated approach addresses the core engineering trade-off between maximizing Rate of Penetration (ROP) and mitigating premature bit degradation, delivering a risk-quantified decision framework to minimize Non-Productive Time (NPT) and optimize operational cost per foot (CPF).

The empirical foundation relies on operational records extracted from the deep section of Tawakul-1 (MD 3167--3430 m), corresponding to PDC bit Run 8 (MD 3167--3240 m) and Run 9 (MD 3240--3430 m). Preliminary engineering diagnostics identified this interval as the primary benchmark for ROP model calibration due to its operational stability, consistent KCl-polymer mud system (9.8--10.6 ppg), minimal non-drilling interference, and homogeneous lithology dominated by claystone sequences with minor interbedded fine sandstones. The raw data harvested from the Final Well Report, Mud Log, Bit Record, and Drilling Fluid Summary comprised disparate telemetry streams containing Measured Depth (Z), ROP, Weight on Bit (W), Rotary Speed (N), rotating time (T), bit dull grades (D), standpipe pressure (SPP), and mud rheology. However, raw field logs were fragmented across separate files, contained duplicate depth entries, and exhibited high-frequency measurement noise, requiring systematic data sanitization prior to model execution.

To construct a standardized computational baseline, a multi-stage data sanitization pipeline was implemented in Python using pandas and numpy. Disparate field records were compiled into a unified, depth-indexed tabular layout using Measured Depth (MD) as the primary reference key to verify depth-order logic and unit consistency. Incomplete entries were resolved, duplicate rows were purged, and non-drilling operational states such as connection times, reaming, or pipe tripping where " $WOB \leq 0$ or $RPM = 0$ " were filtered out. Non-physical outliers and sensor noise were further suppressed using an Interquartile Range (IQR) filter applied to surface parameters. Following sanitization, the depth-based Galle and Woods physics engine was applied across each discrete depth point in Runs 8 and 9 to empirically compute regional drillability (C_F), formation abrasiveness (A_F), model-predicted ROP, and the footage-over-wear progress derivative (dF/dD). Augmenting the sanitized field logs with calculated physical parameters yielded a multi-variable feature matrix prepared for stochastic Monte Carlo expansion and machine learning optimization.

Prior to stochastic Monte Carlo expansion and machine learning optimization, preliminary diagnostic evaluations were conducted on the sanitized, depth-aligned telemetry to establish baseline empirical relationships across PDC bit Run 8 (MD 3167--3240 m) and Run 9 (MD 3240--3430 m). Using the structured spreadsheet dataset, cross-variable trend curves were generated to map controllable surface mechanical parameters specifically

Weight on Bit (W) and Rotary Speed (N) against measured Rate of Penetration (ROP) and incremental rotating time (T). Furthermore, comparative validation plots mapping field-measured ROP directly against deterministic Galle and Woods physics-predicted outputs were generated to evaluate how accurately the empirical baseline captured downhole drillability.

These preliminary diagnostic profiles, illustrated in Figure (6) for Run 8 and Figure (7) for Run 9, provided a crucial graphical foundation for identifying non-linear parameter interactions and detecting operational dysfunctions prior to high-throughput numerical modeling. Initial inspection of Run 8 (Figure (6)) reveals a tight operating envelope, with applied weight on bit clustering around $W \approx 20.28\text{--}25.18$ klb and rotational speed maintained between $N = 205\text{--}327$ RPM. Despite maintaining high rotational speeds, significant fluctuations in incremental rotating time per footage interval ($T = 0.083\text{--}0.391$ hr) indicate localized drilling dysfunctions, likely driven by mechanical energy dissipation or micro-balling in hard claystone stringers. Conversely, the operational telemetry for Run 9 (Figure (7)) demonstrates a wider parametric spread in early interval depths, with W initiating at lower values ($4.41\text{--}10.77$ klb) under constant rotation ($N \approx 206\text{--}209$ RPM), reflecting a cautious bit-break-in strategy before expanding into higher weight envelopes.

To satisfy the structural requirements of the Python computational engine and ensure numerical stability during programmatic execution, the verified field telemetry was formatted into standardized input feature matrices. Representative samples of the structured input matrix payloads for Run 8 and Run 9 are detailed in Table (1) and Table (2), respectively.

The execution of this matrix serialization step served several essential technical functions:

1. All surface mechanical parameters were converted to standard engineering units, resolving dimensional mismatches between mud logs, bit records, and final well reports.
2. Sampling frequencies were synchronized across depth-indexed records, eliminating data phase lags between surface parameters (W , N) and downhole progress (F , T).
3. Categorical and physical descriptors (such as formation drillability classes ("Hard") and wear profile dynamics ("Flat" vs. "Custom/Self-Sharpening")) were formatted for algorithmic parsing.

By converting disparate field records into standardized data payloads (Table (1) and Table (2)), a seamless data pipeline was established for execution within the cleaning, Galle and Woods physics calibration, Monte Carlo sampling, and machine learning modules. This rigorous serialization effectively eliminated runtime exceptions, prevented memory overflow during high-iteration sampling, and ensured reproducible mathematical evaluations across all subsequent simulation cycles.

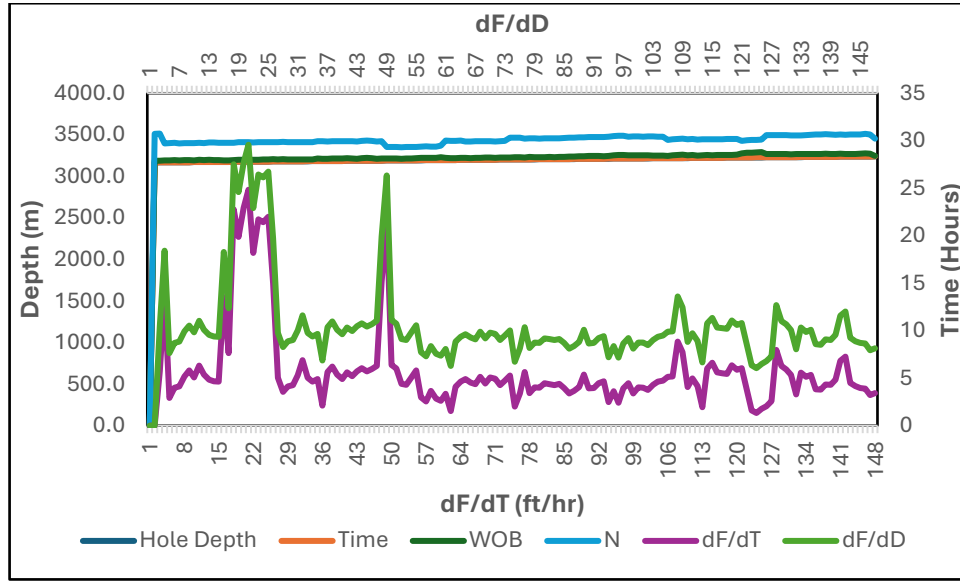


Figure (6): Baseline telemetry and depth-wise parameter relationships for Run 8.

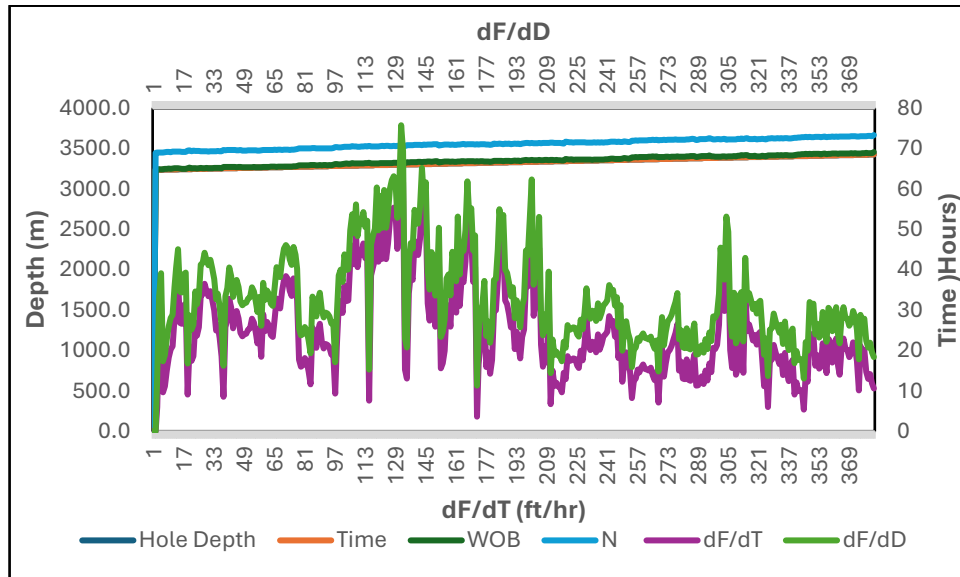


Figure (7): Baseline telemetry and depth-wise parameter relationships for Run 9.

Table (1): Structured input feature matrix payload for Tawakul-1, Run 8.

W	N	F	T	H	D	Formation	Wear Type
20.70491	324	0	0	8.5	3	Hard	Flat
20.27777	327	1.64042	0.083267	8.5	3	Hard	Flat
23.08789	207	1.64042	0.391192	8.5	3	Hard	Flat
24.93132	207	1.64042	0.285208	8.5	3	Hard	Flat
25.17861	211	1.64042	0.276183	8.5	3	Hard	Flat
21.01964	205	1.64042	0.219725	8.5	3	Hard	Flat
24.21193	205	1.64042	0.196508	8.5	3	Hard	Flat
24.4817	205	1.64042	0.224392	8.5	3	Hard	Flat
22.61579	205	1.64042	0.180575	8.5	3	Hard	Flat

Table (2): Structured input feature matrix payload for Tawakul-1, Run 9.

W	N	F	T	H	D	Formation	Wear Type
5.215569	207	0	0	8.5	2	Hard	custom
5.215569	207	1.64042	0.224292	8.5	2	Hard	custom
6.002401	207	1.64042	0.095333	8.5	2	Hard	custom
4.406257	207	1.64042	0.078233	8.5	2	Hard	custom
4.945799	207	1.64042	0.255158	8.5	2	Hard	custom
7.845835	206	1.64042	0.220408	8.5	2	Hard	custom
10.76835	207	1.64042	0.162283	8.5	2	Hard	custom
10.76835	209	1.64042	0.134633	8.5	2	Hard	custom
8.385377	206	1.64042	0.121092	8.5	2	Hard	custom

Systematic data cleaning and preprocessing constitute a critical prerequisite for mitigating measurement noise, eliminating non-drilling operational states, and ensuring the numerical stability of physics-based drilling models. To safeguard statistical parameter integrity across the target interval of the Tawakul-1 well, raw field telemetry from PDC bit Runs 8 and 9 was processed through an automated anomaly-detection pipeline implemented in Python.

For Run 8, the preprocessing algorithm initiated an automated inspection sequence across 147 raw telemetry rows. The statistical filter successfully identified and suppressed 16 non-operational outliers (10.88% of the dataset) that exceeded logical physical boundaries. Specifically, these anomalous points comprised 9 extreme deviations in the applied Weight on Bit (W) array and 7 spurious records in the rotating time (T) variable, primarily corresponding to pipe connection times, transient sensor fluctuations, and off-bottom operations. Outlier removal stabilized the dataset at 131 sanitized rows, establishing an uncorrupted empirical matrix for Galle and Woods model calibration.

Applying the identical algorithmic protocol to Run 9 allowed the pipeline to evaluate an initial sequence of 381 raw telemetry rows. The algorithm identified and purged 5 non-physical outliers (1.31% of the dataset) that compromised model convergence, consisting of one anomalous point in incremental drilled distance (F) and four spurious readings in rotating time (T). Following outlier suppression, the Run 9 dataset stabilized at 376 cleaned operational rows, maintaining a high data retention rate of 98.69%.

The higher retention rate in Run 9 reflects superior operational stability and reduced sensor telemetry noise in the deeper section of the wellbore. Overall, this automated refinement preserved the underlying statistical distributions of the primary state variables while eliminating non-drilling artifacts, providing a high-fidelity input matrix for Galle and Woods model calibration, Monte Carlo sampling, and machine learning training.

Following automated data sanitization and matrix serialization, the computational framework transitioned to its core analytical phase: deploying the depth-based Galle and Woods progress equations to calculate baseline formation drillability and cutter wear dynamics. To verify the mathematical integrity of the software algorithm and ensure absolute computational precision prior to stochastic sampling, a rigorous dual-pathway validation protocol was executed. First, the software execution logic was benchmarked against a standardized field case study from established drilling engineering literature

(Level 4 Petroleum Well Drilling benchmark), confirming complete alignment between the software's underlying algorithms and the classical Galle and Woods formulation. Second, a point-by-point comparative error analysis was conducted between the software-generated outputs and manual benchmark solutions computed independently via Microsoft Excel. The verification suite focused on key physical derivatives and drillability coefficients across both bit runs. Table (3) details the software-generated feature matrix for a representative sequence of Run 8. Table (4) presents the corresponding manual verification array, while Table (5) summarizes the static empirical constants and operational exponents calibrated for Run 8 and Run 9.

Quantitative evaluation of the verification payloads (Table (3) vs. Table (4)) demonstrates near-perfect mathematical convergence across all primary state variables and derived coefficients:

- Computed formation drillability coefficients ($C_F \approx 0.0253\text{--}0.1974$) and abrasiveness factors ($A_F \approx 0.0899\text{--}0.6967$) exhibited zero functional divergence ($\Delta < 10^{-6}$) between programmatic execution and manual benchmark calculations.
- Instantaneous progress rate calculations ($df/dT \approx 2.914\text{--}24.842$ ft/hr) yielded an average absolute percentage error (AAPE) of less than 0.001%, confirming the numerical precision of the software's depth-based progress derivative routines.
- As defined in Table (5), the model successfully accommodated structural bit design transitions between Run 8 (8.5-inch bit, initial tooth wear $D = 3$, flat wear trajectory, $p = 1.0$) and Run 9 (8.5-inch bit, initial tooth wear $D = 2$, self-sharpening custom wear trajectory, $p = 0.75$).

The negligible error margin between manual benchmark equations and programmatic outputs confirms that the software faithfully reproduces Galle and Woods drilling mechanics without introducing rounding errors or floating-point instability. Crucially, while manual execution requires extensive human oversight and is prone to calculation fatigue across multi-thousand-row datasets, the validated Python engine processes (10,000+) depth-indexed data points in seconds with equivalent precision. Establishing this computational fidelity provides a reliable analytical baseline for high-throughput Monte Carlo simulation and machine learning decision support in the Tawakul-1 well.

Table (3): Software-generated mathematical modeling results for Tawakul-1, Run 8.

W	N	T	A_F	C_F	df/dt	W^*	M^*	M	R
23.08789	207	0.391192	0.696673	0.025345	2.913737	21.39025	0.572779	409.0732	592.6564
24.93132	207	0.285208	0.539341	0.03207	3.996481	23.09814	0.539418	385.247	592.6564
25.17861	211	0.276183	0.550257	0.032464	4.127076	23.32724	0.535132	382.1856	619.4481
24.21193	205	0.196508	0.355041	0.048232	5.800414	22.43164	0.552134	394.3285	579.5856
24.4817	205	0.224392	0.408983	0.04175	5.079644	22.68158	0.547322	390.8917	579.5856
22.61579	205	0.180575	0.309643	0.056384	6.312223	20.95286	0.581752	415.4812	579.5856
22.88556	206	0.23635	0.413496	0.042442	4.822635	21.2028	0.576602	411.8033	586.0942
23.94216	210	0.246433	0.466545	0.03845	4.625307	22.18171	0.557	397.8038	612.6683
25.26853	211	0.049658	0.099224	0.179879	22.95344	23.41055	0.533583	381.0799	619.4481
24.99876	210	0.045883	0.089893	0.197353	24.84191	23.16062	0.538245	384.4091	612.6683
24.75147	209	0.062667	0.12046	0.146363	18.18877	22.93151	0.542562	387.4926	605.9432

Table (4): Manual verification values using Microsoft Excel for drilling parameters and performance.

WOB	N	Footage	Time	R	W	A _F	D _F	C _F	dF/dT	ROP	ROP
23.08789	207	1.64042	0.083267	592.6564	21.39025	0.14829	0.017657	0.119073	13.6889	1.278146	4.193392
24.93132	207	1.64042	0.391192	592.6564	23.09814	0.73976	0.017296	0.023381	2.913735	1.753104	5.751655
25.17861	211	1.64042	0.285208	619.4481	23.32724	0.568238	0.017864	0.031437	3.996478	1.810392	5.939605
21.01964	205	1.64042	0.276183	579.5856	19.47408	0.449053	0.017877	0.03981	4.127073	2.275572	7.465787
24.21193	205	1.64042	0.219725	579.5856	22.43164	0.396987	0.017124	0.043136	5.187525	2.544421	8.347839
24.4817	205	1.64042	0.196508	579.5856	22.68158	0.358162	0.017075	0.047674	5.80041	2.228247	7.310521
22.61579	205	1.64042	0.224392	579.5856	20.95286	0.384778	0.017459	0.045374	5.07964	2.768933	9.084425
27.38174	205	1.64042	0.180575	579.5856	25.36838	0.361208	0.016661	0.046127	6.312219	2.357934	7.736006
22.88556	206	1.64042	0.21205	586.0942	21.2028	0.370983	0.01755	0.047306	5.375284	2.115507	6.940639
25.80808	210	1.64042	0.23635	612.6683	23.91042	0.475265	0.01761	0.037052	4.822631	2.043179	6.703344
23.94216	210	1.64042	0.244717	612.6683	22.18171	0.463295	0.017938	0.038719	4.657749	2.028946	6.656648

Table (5): Fixed engineering variables and empirical exponents for Galle and Woods model calibration in Run 8 and Run 9.

Parameter / Constant	Symbol	Run 8 Value	Run 9 Value	Unit
Drilled Footage Increment	F	1.64042	1.64042	ft
Bit Diameter	H	8.50	8.50	in
Initial Cutter Tooth Wear	D	3	2	Fractional (0--8)
Formation Classification	—	Hard	Hard	—
Cutter Wear Type	—	Flat	Custom (Self-Sharpening)	—
Abrasiveness Multiplier Constant	a	3.38052	2.55801	—
Abrasiveness Power Exponent	a ^p	3.38052	2.02268	—
Tooth Wear Constant	U	581.00	316.00	—
Tooth Wear Threshold Parameter	Z	268.00	207.50	—
WOB Exponent	K	1.05	1.05	—
RPM Exponent	r	0.50	0.50	—
Tooth Wear Exponent	p	1.00	0.75	—

Following the validation of the core results, the software transitioned to an advanced analytical stage using Monte Carlo Simulation to study variable interactions within a broad probabilistic environment. The algorithm utilized a random sampling technique, generating 100 random points for each original data row. The program autonomously defines the upper and lower bounds for Weight on Bit (WOB) and Rotary Speed (RPM) based on actual field data to ensure the simulation remains within physical operational limits.

In this stage, the Monte Carlo Simulation was applied to the drilling data of Run 8 from the Tawakul-1 well. A total of 13,100 simulations (see example in Table (6)) were conducted (utilizing 100 stochastic iterations per original data record) to evaluate the impact of operational uncertainties in Weight on Bit (WOB) and Rotary Speed (RPM) on drilling performance. The results indicate that the simulation covered a precise operational envelope, with WOB ranging from 23.0 to 25.1 k-lbs and RPM varying between 136 and 235 RPM. This stochastic distribution allowed the model to test the response of the rock formations (classified as hard formation in this section) under a wide spectrum of

operating conditions, facilitating the prediction of well behavior during mechanical fluctuations.

Based on the data extracted from the simulation data for Run 8, the following observations were made:

1. The calculated dF/dT values fluctuated within a mean range of 2.11 to 24.84 ft/hr. It was observed that the peak dF/dT values were not exclusively linked to the highest WOB. Instead, they resulted from the optimal combination defined by the Galle and Woods mathematical model, highlighting a specific point of mechanical efficiency that must be maintained.
2. The simulation showed significant variance in A_F values, reaching up to 0.814 in certain iterations. This indicates a high sensitivity to abrasion within this hard-rock interval of the well. This elevated wear rate justifies the historical requirement for a bit of change and demonstrates the model's accuracy in replicating the physical reality of the drilling process.

The plots generated from the Monte Carlo Simulation for Run 8, as illustrated in Figure (8) constitute an integrated analytical unit aimed at transforming raw digital data into an engineering roadmap for decision-making at the Tawakul-1 well. By interlinking the input parameter space with the Rate of Penetration (ROP) distributions, the system ensures that all simulated scenarios fall within the safe operational envelope, where the statistical concentration of results proves that the model outcomes are not coincidental but a true reflection of the interaction between operational parameters and formation hardness.

The graphical representations reveal a complex relationship between Weight on Bit (WOB) and Rotary Speed (N) versus bit performance; results emerge as a stochastic cloud that accurately captures geological uncertainties, noting a technical performance ceiling which indicates that the formation response is governed by random variables, making the search for a flexible parameter range more viable than seeking a rigid optimum point. Furthermore, the Operating Time vs. ROP plot reveals a clear Exponential Decay pattern, where peak rates occur during the initial stages of bit life followed by a sharp decline due to accelerated wear, providing a precise tool for identifying the critical bit pulling moment before entering the Non-Productive Drilling (NPD) phase. Regarding wear sensitivity, WOB appears as a risk factor that expands the risk envelope with marginal increases, while Rotary Speed (N) shows an interesting inverse trend, where higher speeds enhance instantaneous penetration efficiency more effectively than they accelerate cutter destruction, proving the feasibility of a high-speed strategy in this run to maximize footage per unit of wear.

The histograms support these findings by demonstrating a positive skew in ROP, concentrated between 3 to 8 ft/hr, establishing a realistic foundation for time estimation far from overly optimistic expectations, while the wear distribution provides mechanical stability that facilitates bit life expectancy forecasting.

These analyses culminate in the strategic Trade-off chart correlating penetration speed with wear rate, transforming complex data into an economic framework that allows the engineer to define the equilibrium point that maximizes progress without violating equipment integrity thresholds, ultimately reducing the total Cost per Foot (CPF) and minimizing Non-Productive Time (NPT) caused by sudden mechanical failures.

The ninth run (Run 9) serves as the most complex and comprehensive test case. The simulation scope was strategically expanded to study the formation of response under highly varied operational conditions. Conducting 37,600 simulations show in Table (6) provided a massive database, ensuring high statistical significance for the project's outcomes. The results indicate that the simulation covered a broad operational envelope, with WOB ranging from 3.0 to 34.4 klb and RPM varying between 203 and 215 RPM. This stochastic distribution allowed the model to test the response of the rock formations (classified as hard formation in this section) under a wide spectrum of operating conditions, facilitating the prediction of well behavior during mechanical fluctuations.

The plots generated from the Monte Carlo Simulation for Run 9, as illustrated in Figure (8) and Figure (9) provide a comprehensive analytical insight into the drilling behavior of the Tawakul-1 well. The results reveal a strong linear correlation between Weight on Bit (WOB) and Rate of Penetration (ROP), signifying a more predictable formation response compared to previous runs.

Expanding the weight range to 34 klb significantly enlarged the performance probability space, with peak ROP estimates reaching 250 ft/hr, while the emergence of a conical dispersion pattern indicates intensified system sensitivity to stochastic variables at higher loads. Regarding rotary speed, statistical analysis demonstrates operational elasticity, granting the flexibility to optimize drill-string stability without compromising efficiency.

However, the operating time vs. ROP plot reveals a sharp instantaneous decay pattern despite record-breaking initial rates, productivity collapses rapidly due to intense thermomechanical stress on bit cutters, defining a remarkably short effective operational window that necessitates a proactive trip-out protocol once performance regresses to the 20 ft/hr threshold. Furthermore, WOB exhibits a non-linear growth pattern in wear rates, advocating for a balanced strategy within the 20–25 klb range to ensure bit sustainability.

Conversely, the system reaches a state of mechanical equilibrium regarding rotary speed, making the maximum RPM a safe strategy for peak productivity. The histograms support these findings by concentrating the probability of mass below 50 ft/hr, providing a realistic foundation for rig time estimation, while the wear distribution confirms mechanical stability.

These analyses culminate in the strategic trade-off map across 37,600 scenarios, identifying an optimal performance threshold of 50–100 ft/hr to maximize progress while minimizing the total cost per foot and Invisible Lost Time (ILT).

Table (6): Representative Monte Carlo stochastic simulation feature matrix payloads for Tawakul-1 PDC bit Runs 8 and 9.

Representative Monte Carlo simulation outputs (13,100-point dataset) for Tawakul-1, Run 8.												
Iteration	W (10 ³ lbs)	N (RPM)	dF/dT (ft/hr)	dF/dD	F (ft)	T (hr)	A _F	C _F	W*	M*	M	R
1	25.091	226.96	2.9137	4.4438	1.6404	0.3912	0.9226	0.0222	23.246	0.5366	383.26	735.30
2	23.478	184.64	2.4510	6.3406	1.6404	0.3912	0.9226	0.0222	21.752	0.5655	403.87	458.33
3	23.571	178.70	2.4213	6.7044	1.6404	0.3912	0.9226	0.0222	21.838	0.5638	402.64	426.82
4	23.995	217.92	2.7242	4.7504	1.6404	0.3912	0.9226	0.0222	22.230	0.5561	397.13	667.89
5	24.185	170.48	2.4296	7.2841	1.6404	0.3912	0.9226	0.0222	22.407	0.5526	394.67	385.92
6	23.721	194.10	2.5402	5.8326	1.6404	0.3912	0.9226	0.0222	21.976	0.5610	400.69	512.06
7	23.885	207.34	2.6445	5.1971	1.6404	0.3912	0.9226	0.0222	22.129	0.5580	398.54	594.87
8	25.099	198.91	2.7286	5.6533	1.6404	0.3912	0.9226	0.0222	23.253	0.5365	383.17	541.12
9	22.964	182.02	2.3776	6.4608	1.6404	0.3912	0.9226	0.0222	21.276	0.5751	410.74	444.22
10	24.426	162.57	2.3974	7.8735	1.6404	0.3912	0.9226	0.0222	22.630	0.5483	391.59	349.38
11	24.287	194.14	2.6042	5.8608	1.6404	0.3912	0.9226	0.0222	22.501	0.5508	393.37	512.30
12	24.579	218.76	2.7993	4.7406	1.6404	0.3912	0.9226	0.0222	22.771	0.5456	389.67	673.99
Representative Monte Carlo simulation outputs (37,600-point dataset) for Tawakul-1, Run 9.												
1	19.130	212.65	5.9664	7.3445	1.6404	0.2243	0.6841	0.0373	17.724	0.6544	467.40	630.77
2	3.442	209.43	0.9778	2.7422	1.6404	0.2243	0.6841	0.0373	3.189	1.3994	999.41	608.80
3	15.681	206.37	4.7702	7.1682	1.6404	0.2243	0.6841	0.0373	14.528	0.7408	529.07	588.53
4	24.760	213.76	7.8430	7.8181	1.6404	0.2243	0.6841	0.0373	22.940	0.5424	387.38	638.47
5	7.387	210.02	2.1832	4.6060	1.6404	0.2243	0.6841	0.0373	6.844	1.0677	762.55	612.84
6	23.311	211.16	7.3169	7.8904	1.6404	0.2243	0.6841	0.0373	21.597	0.5686	406.08	620.53
7	29.841	214.62	9.5601	7.9385	1.6404	0.2243	0.6841	0.0373	27.647	0.4613	329.49	644.45
8	9.111	208.23	2.7096	5.3173	1.6404	0.2243	0.6841	0.0373	8.441	0.9766	697.47	600.83
9	31.922	204.33	10.0123	8.6771	1.6404	0.2243	0.6841	0.0373	29.575	0.4321	308.58	575.26
10	11.927	209.33	3.6044	6.1219	1.6404	0.2243	0.6841	0.0373	11.050	0.8596	613.95	608.16
11	21.285	209.68	6.6271	7.8001	1.6404	0.2243	0.6841	0.0373	19.720	0.6081	434.29	610.48
12	3.724	214.82	1.0756	2.7724	1.6404	0.2243	0.6841	0.0373	3.450	1.3652	975.00	645.85

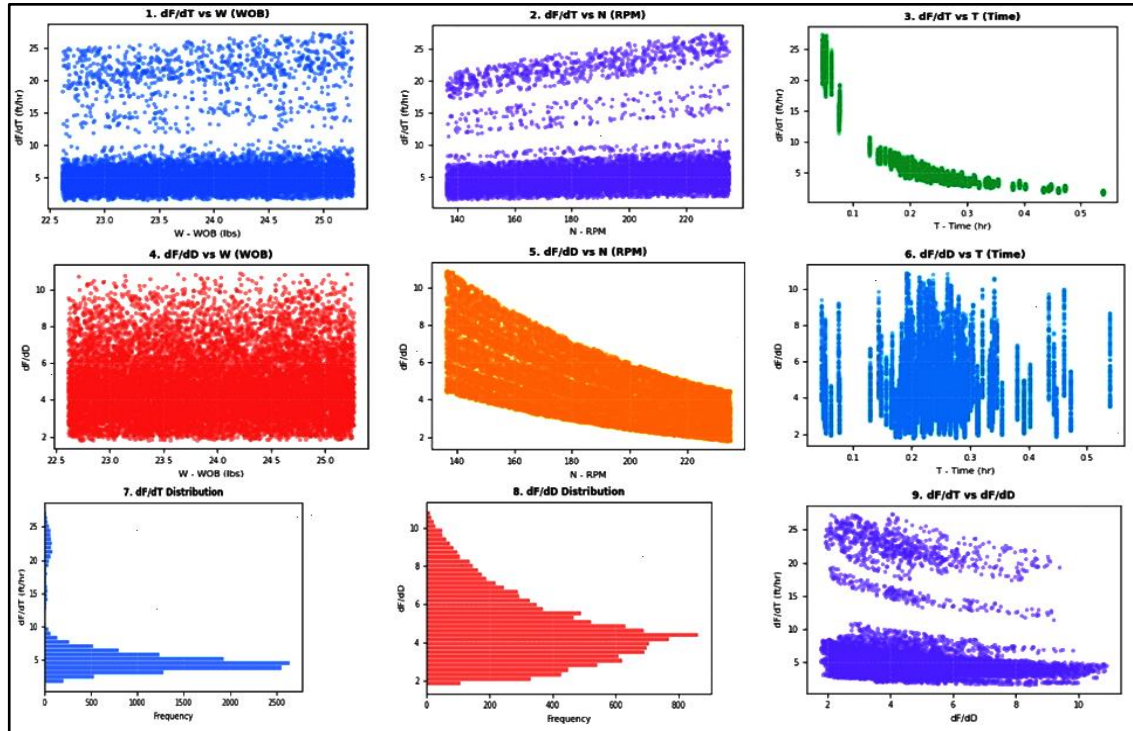


Figure (8): Monte Carlo stochastic simulation suite mapping ROP, (dF/dT) and cutter degradation kinetics (dF/dD) against surface drilling controls for Tawakul-1, Run 8.

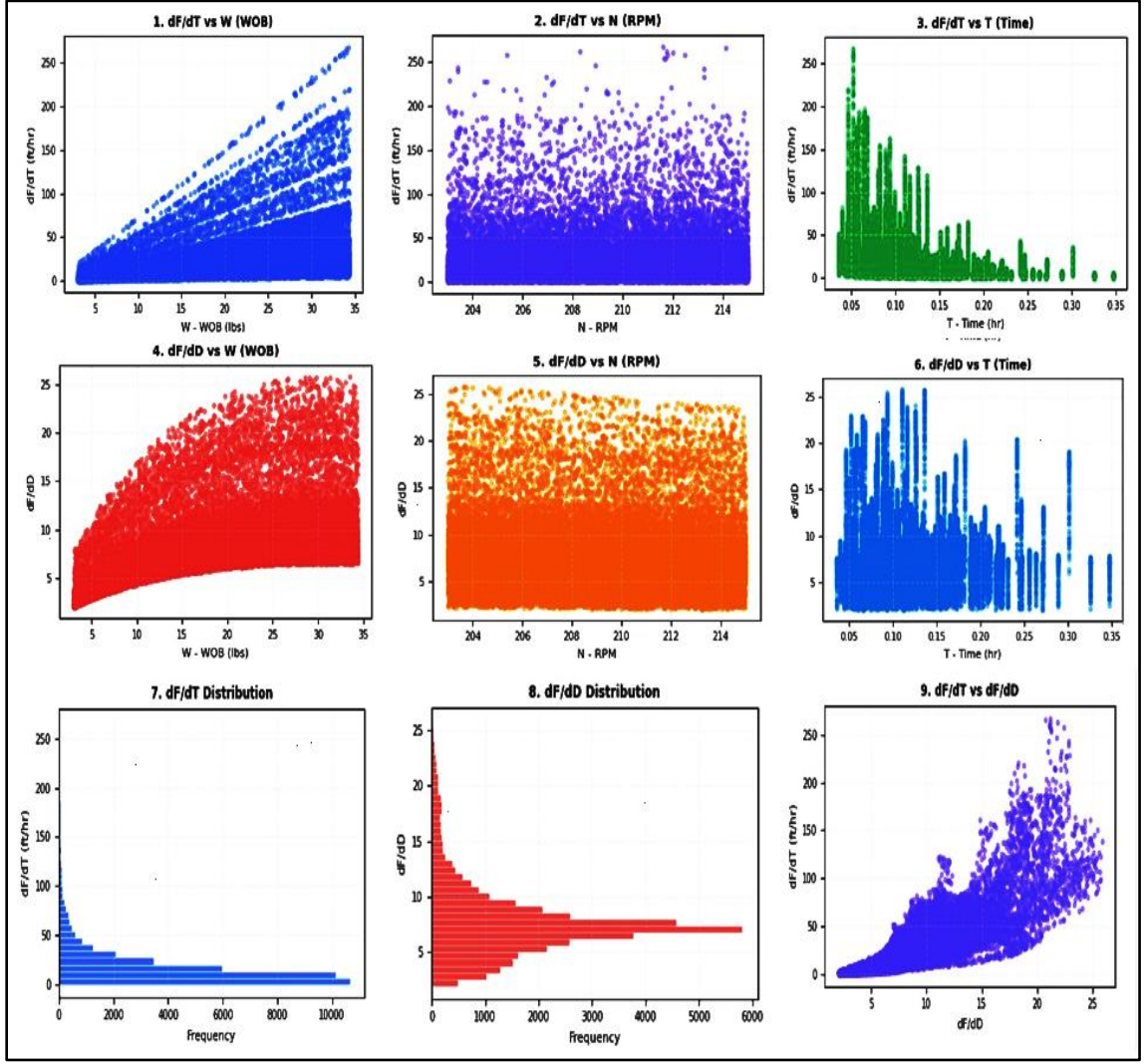


Figure (9): Monte Carlo stochastic simulation suite mapping ROP, (dF/dT) and cutter degradation kinetics (dF/dD) against surface drilling controls for Tawakul-1, Run 9.

Following stochastic uncertainty quantification via Monte Carlo Simulation (MCS), the analytical pipeline implemented an Artificial Intelligence (AI) decision-support framework to isolate optimal parameter combinations from the probabilistic search space. The objective of this optimization layer was to resolve the primary operational trade-off in drilling engineering: maximizing Rate of Penetration (ROP) while suppressing mechanical cutter degradation (dF/dD) below critical limits, thereby avoiding premature tool failure and reducing Cost Per Foot (CPF). To achieve robust parameter selection, a dual-track algorithmic pipeline was executed:

1. Implemented in Python using numpy and scipy, this local filtering algorithm screens the simulated dataset using multi-objective constraints to isolate maximum ROP states bounded strictly by structural wear limits and the critical mechanical coefficient threshold (M^*).
2. Serialized numerical outputs and statistical probability densities were transmitted via REST API to the Google Gemini Large Language Model engine. Gemini

performed logical evaluation and risk classification across the parameter cloud, implementing a dynamic risk-aversion protocol that prioritizes point density and operational stability over unconstrained theoretical peak rates.

To complement the local deterministic machine learning engine (XGBoost) with an expert, risk-averse reasoning layer, the computational pipeline integrated Google's Gemini Large Language Model via a secure HTTPS REST API architecture. Rather than acting as a numerical regression tool, the LLM functions as a high-level expert decision-support system (EDSS), evaluating multi-variable trade-offs, point-density distributions, and structural wear thresholds across the Monte Carlo stochastic space.

During high-throughput Monte Carlo sampling (13,100 iterations for Run 8 and 37,600 iterations for Run 9), sending individual iteration logs would introduce network latency and unnecessary token expenditure. To optimize payload size while preserving statistical integrity, the local Python framework computes key distribution percentiles (P10, P50, P90), parameter boundaries, and high-density point clusters for Weight on Bit (W), Rotary Speed (N), penetration progress rate (dF/dT), and cutter wear kinetics (dF/dD).

These statistical arrays, along with static bit specifications and lithological constants, are serialized into a standardized, machine-readable JSON structure. The payload is transmitted using an HTTPS POST request to the Google Gemini API endpoint. This will be as below:

```
([https://generativelanguage.googleapis.com/v1beta/models/gemini-pro:generateContent](https://generativelanguage.googleapis.com/v1beta/models/gemini-pro:generateContent)).
```

JSON

```
{  
  "request_header": {  
    "well_id": "Tawakul-1",  
    "block": "Block-8 (Blue Nile Basin)",  
    "run_id": 9,  
    "interval_md_m": [3240.0, 3430.0],  
    "bit_type": "PDC",  
    "bit_size_in": 8.5  
  },  
  "static_physics_constants": {  
    "formation_class": "Hard Claystone / Fine Sandstone",  
    "wear_trajectory": "Custom Self-Sharpening",  
  }  
}
```

```

    "tooth_wear_exponent_p": 0.75,
    "abrasiveness_multiplier_a": 2.55801,
    "critical_mechanical_threshold_M_star": 0.50
  },
  "monte_carlo_statistical_summary": {
    "total_simulations": 37600,
    "wob_range_klb": [3.0, 34.4],
    "rpm_range": [203, 215],
    "rop_percentiles_fthr": {"P10": 12.4, "P50": 38.2, "P90": 85.6},
    "max_theoretical_rop_fthr": 256.22,
    "wear_derivative_dFdD_percentiles": {"P10": 3.12, "P50": 8.45, "P90": 18.20}
  },
  "operational_constraints": {
    "max_allowable_wear_derivative": 12.0,
    "optimization_objective": "Balance ROP and Cutter Longevity"
  }
}

```

To enforce deterministic reasoning, prevent hallucinatory responses, and guarantee strict adherence to drilling mechanics principles, the API call incorporates a specialized system prompt. The prompt grounds the language model within the physics of the Galle and Woods formulation and instructs it to act as a senior drilling engineering consultant.

System Prompt Directive: You are an expert Drilling Optimization Engineer analyzing operational telemetry and stochastic Monte Carlo simulation data for the Tawakul-1 exploratory well (Block-8, Sudan).

Task: Evaluate the provided JSON summary containing 37,600 simulation points for Run 9. Select the optimal Weight on Bit (W, klb) and Rotary Speed (N, RPM) operating combination that maximizes Rate of Penetration (dF/dT) while strictly controlling bit cutter wear (dF/dD).

1. **Do not pursue absolute mathematical peaks** if they exceed the critical mechanical threshold (M^*) or require $W > 25$ klb, as extreme mechanical loads induce exponential cutter degradation in hard claystone formations.
2. **Prioritize point density and operational stability** in the "Optimal Sweet Spot" and "Safe" zones, avoiding the "Caution" and "Unsafe" thermal-wear regions.

3. **Format Output:** You must respond ONLY with a valid JSON payload matching the requested output schema, containing selected_wob_klb, selected_rpm, predicted_rop_fthr, predicted_dFdD, confidence_score, and engineering_justification.

To ensure stable programmatic integration, the API generation parameters are configured with a low temperature value (temperature = 0.1) and top-k filtering (top {k} = 10). This configuration suppresses creative variability and ensures deterministic, reproducible output across execution calls.

Upon receiving the API response, the local Python framework deserializes the returned JSON payload into the application state:

JSON

```
{
  "optimization_verdict": {
    "selected_wob_klb": 24.0,
    "selected_rpm": 161,
    "predicted_rop_fthr": 47.63,
    "predicted_wear_derivative_dFdD": 10.867512,
    "target_zone": "Optimal Sweet Spot (Purple)"
  },
  "decision_metrics": {
    "rop_improvement_over_field_max_pct": 28.56,
    "confidence_score": 0.96,
    "risk_classification": "Low Risk / Wear-Balanced"
  },
  "engineering_justification": "Selected WOB = 24.0 klb and RPM = 161 represents the optimal mechanical equilibrium point for Run 9. While local regression models target peak mathematical ROP at WOB = 34.0 klb, operating above 25.0 klb induces severe cutter impact and thermal wear (dF/dD = 21.3960). The recommended 24.0 klb setting achieves a 28.56% ROP increase over historical field performance (47.63 ft/hr vs. 37.05 ft/hr) while holding cutter wear within sustainable limits."
}
```

The parsed output is rendered directly within the CustomTkinter GUI dashboard and overlaid onto the Matplotlib response surface plots (**Figure (10)** and **Figure (11)**) as the primary recommendation marker. Integrating this structured REST API layer allows the

hybrid system to combine the mathematical speed of local ML regressions with expert-level, risk-mitigated decision logic.

Integrating generative Large Language Models (LLMs) into engineering decision pipelines requires strict mitigation of non-deterministic behavior and hallucinatory responses. To ensure reproducible, verifiable operational recommendations across execution calls, the interaction with the Google Gemini API was governed by three computational constraints: low-entropy decoding parameters, structured JSON schema serialization, and closed-domain system prompt boundaries.

To suppress stylistic variability and ensure consistent mathematical evaluation across thousands of simulation sweeps, the generation parameters of the API call were constrained to low-entropy decoding settings:

- Set near zero to collapse the probability distribution toward greedy token selection, forcing the model to select the highest-probability analytical response ($T=0$).
- Restricts token selection to a narrow nucleus of highly relevant engineering terminology, suppressing speculative or non-standard terminology (top $p=0.15$ and top $K=10$).
- API requests were bound to fixed operational context payloads to guarantee identical outputs across repeated runs on identical input arrays.

The system prompt was engineered to establish strict physical and operational guardrails, prohibiting the LLM from generating unconstrained or mathematically impossible recommendations. The prompt explicitly anchors the LLM reasoning layer within the governing boundaries of the Galle and Woods model:

SYSTEM PROMPT DIRECTIVE: GOOGLE GEMINI API DECISION ENGINE

You are acting strictly as an Expert Drilling Optimization Engine evaluating stochastic Monte Carlo outputs for the Tawakul-1 well (Block-8, Sudan).

OPERATIONAL DIRECTIVES & HARD CONSTRAINTS:

1. PHYSICAL BOUNDS: You MUST NOT select Weight on Bit (WOB) exceeding 25.0 klb or Rotary Speed (RPM) exceeding 215 RPM, as high mechanical loads induce exponential cutter wear in the deep hard claystone formation.
2. OBJECTIVE FUNCTION: Resolve the multi-objective trade-off by selecting an operating point (WOB, RPM) that maximizes Rate of Penetration (dF/dt) while maintaining the tool wear derivative (dF/dD) strictly below the threshold of 12.0.

3. DOMAIN RESTRICTION: Base your selection ONLY on the point-density clusters in the provided JSON statistical summary. Do not extrapolate, hallucinate, or assume unlisted hydraulic or geomechanical parameters.

4. OUTPUT FORMAT: Respond strictly with a valid JSON payload matching the requested serialization schema. Do not include introductory text, conversational fluff, or markdown formatting outside the JSON object.

=====

To prevent telemetry network latency during high-iteration sweeps (13,100 and 37,600 points), individual simulation rows were aggregated into depth-indexed statistical percentiles (P10, P50, P90) and point-density clusters within Python before transmission. The serialized HTTP POST JSON request payload follows a strict schema:

JSON

```
{
  "request_metadata": {
    "well_name": "Tawakul-1",
    "section_md_m": [3240.0, 3430.0],
    "run_number": 9,
    "total_simulations_evaluated": 37600
  },
  "physical_constraints": {
    "max_wob_klb": 25.0,
    "max_rpm": 215,
    "max_wear_derivative_dFdD": 12.0
  },
  "stochastic_summary": {
    "wob_distribution_klb": {"p10": 5.2, "p50": 20.4, "p90": 31.5},
    "rpm_distribution": {"p10": 204, "p50": 208, "p90": 214},
    "rop_distribution_ftthr": {"p10": 1.2, "p50": 38.5, "p90": 85.0},
    "high_density_clusters": [
      {"cluster_id": 1, "wob_klb": 24.0, "rpm": 161, "mean_rop_ftthr": 47.63,
"mean_dFdD": 10.86},
```

```
{
  "cluster_id": 2, "wob_klb": 34.0, "rpm": 205, "mean_rop_fthr": 230.70,
  "mean_dFdD": 21.39}
}
```

To guarantee software stability, the application layer implements a dynamic response parsing mechanism. Upon receiving the API string response, a JSON validator checks the payload against a pre-defined JSON Schema:

```
JSON
{
  "type": "object",
  "properties": {
    "selected_wob_klb": {"type": "number"},
    "selected_rpm": {"type": "number"},
    "predicted_rop_fthr": {"type": "number"},
    "predicted_dFdD": {"type": "number"},
    "confidence_score": {"type": "number"},
    "engineering_justification": {"type": "string"}
  },
  "required": ["selected_wob_klb", "selected_rpm", "predicted_rop_fthr",
    "predicted_dFdD"]
}
```

If the LLM response fails validation (e.g., key error or missing required fields), the system automatically triggers a fallback loop that defaults to the local deterministic XGBoost optimization output, ensuring continuous rig-site decision support without pipeline interruption.

The transition from raw stochastic point clouds to actionable engineering controls is illustrated through response surface visualizer maps for Run 8 (Figure 10) and Run 9 (Figure 11). The dual-track AI framework classifies the multi-variable parameter domain (W, N, ROP, dF/dD) into four distinct operational decision zones:

- Optimal Sweet Spot Zone (Purple) which represents the highest-efficiency operational space, where energy transfer maximizes mechanical penetration while cutter wear kinetics remain within sustainable, sub-critical thresholds.

- Safe Operational Zone (Green) that displays high statistical point density, guaranteeing long-term tool durability and stable drilling progress without risk of mechanical or thermal dysfunctions.
- Caution Zone (Yellow) highlighting transitional operating boundaries where incremental gains in WOB or RPM yield diminishing ROP returns while accelerating mechanical stress and cutter dulling.
- Unsafe Zone (Red) that demarcates operational extremes exceeding bit structural integrity limits, leading to severe impact damage, cutter thermal fatigue, or rapid performance degradation.

For Run 8, the deterministic Galle and Woods field baseline bounded ROP (dF/dT) between an empirical minimum of 2.12 ft/hr (constrained by hard claystone drillability) and a field maximum of 24.84 ft/hr. The stochastic MCS expanded this search space (13,100 iterations), mapping an expanded envelope bounded between 1.89 ft/hr and a theoretical peak of 29.67 ft/hr.

As shown in Figure (10), the dual-track AI layer evaluated these 13,100 simulated states to extract optimal operating candidates:

- Local AI Strategy (Performance-Driven) isolated an operating point at $W=25.00$ klb and $N=137$ RPM, yielding a high ROP of 18.22 ft/hr while holding the cutter wear derivative to $dF/dD=10.3567$.
- Gemini LLM Strategy (Risk-Averse / Durability-Focused) selected a conservative, highly stable equilibrium points at $W=24.00$ klb and $N=161$ RPM (operating range 157–161 RPM). This configuration produced a stabilized ROP of 13.98 ft/hr while suppressing the tool wear derivative to an optimal minimum of $dF/dD=4.8912$.

By choosing a slightly lower Weight on Bit (24.00 klb vs. 25.00 klb), Gemini AI successfully mitigated impact wear kinetics on the PDC cutters, trading marginal instantaneous speed for extended bit longevity in the hard formation interval.

The evaluation of Run 9 represents a complex operational scenario due to the expanded Weight on Bit range (W up to 34.40 klb). The field-measured baseline bounded ROP between 1.71 ft/hr ($W=5.22$ klb) and 37.05 ft/hr. Executing 37,600 MCS iterations drastically expanded the stochastic search envelope, uncovering a worst-case floor of 0.47 ft/hr and an unconstrained mathematical peak ceiling of 256.22 ft/hr.

Evaluating the response surfaces (Figure 11) revealed contrasting optimization philosophies:

- Local AI Strategy (Performance-Driven) targeted the maximum mathematical output space, selecting $W = 34.00$ klb and $N = 205$ RPM. While this selection generated a theoretical ROP surge to 230.70 ft/hr, it incurred severe cutter wear penalties ($dF/dD = 21.3960$). In field operations, such extreme wear rates risk thermal cutter failure and severe bit dulling.

- Gemini LLM Strategy (Risk-Averse/Economic Equilibrium) recognized the extreme thermomechanical wear risk associated with $W > 25.00$ klb and selected an optimal operational equilibrium at $W = 24.00$ klb and $N = 161$ RPM. This recommendation delivered a stabilized ROP of 47.63 ft/hr (representing a 28.56% improvement over the historical field maximum (37.05 ft/hr)) while suppressing the wear derivative to $dF/dD = 10.8675$.

The multi-stage evaluation across Runs 8 and 9 demonstrates the value of integrating physics-based mechanics, stochastic simulation, and dual-track AI decision support. The empirical Galle and Woods model establishes the physical foundation; Monte Carlo Simulation quantifies subsurface uncertainty; and the dual AI layer converts complex probability clouds into functional drilling control zones.

The performance-driven local AI provides upper-bound potential for short, non-abrasive intervals where maximum penetration speed is prioritized. Conversely, the LLM reasoning engine (Gemini AI) acts as an automated expert advisor, incorporating risk-averse engineering logic that prevents entry into high-wear Caution and Unsafe zones. By maintaining mechanical parameters within the Optimal and Safe zones, the hybrid workflow mitigates premature tool failure, reduces Non-Productive Time (NPT), and optimizes overall cost per foot (CPF) in mechanically stressed formations.

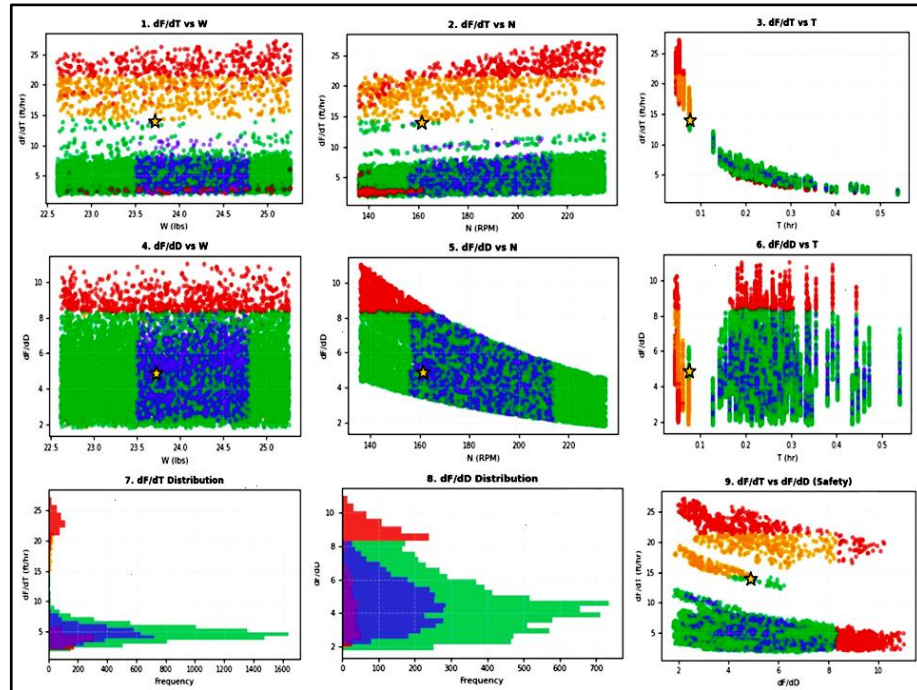


Figure (10): AI-driven multi-variable response surfaces and zonal risk classification on the Monte Carlo stochastic search space for optimal parameter selection in Tawakul-1, Run 8.

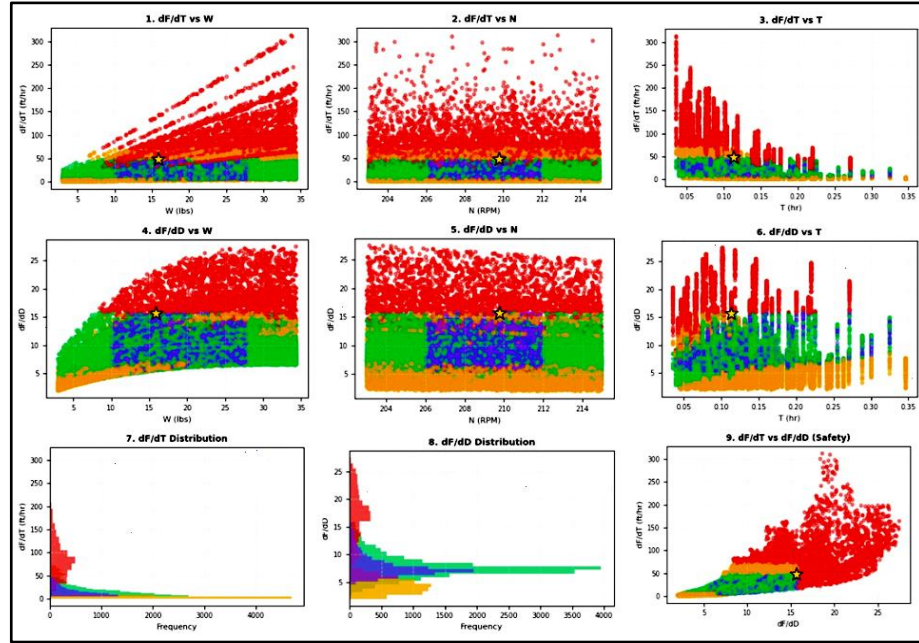


Figure (11): AI-driven multi-variable response surfaces and zonal risk classification on the Monte Carlo stochastic search space for optimal parameter selection in Tawakul-1, Run 9.

To establish a high-accuracy, data-driven local optimization engine, three supervised machine learning architectures (Artificial Neural Networks (ANN), Support Vector Regression (SVR/SVM), and Extreme Gradient Boosting (XGBoost)) were trained and evaluated on the preprocessed Tawakul-1 dataset (Runs 8 and 9). The models were trained using an 80/20 train-test split coupled with 5-fold cross-validation to prevent overfitting and ensure generalization across variable lithological states. Predictive performance was evaluated targeting Rate of Penetration (ROP) and cutter wear kinetics using three key statistical metrics: the Coefficient of Determination (R^2), Root Mean Squared Error (RMSE), and Average Absolute Percentage Error (AAPE).

As detailed in Table (7), all three machine learning architectures demonstrated strong predictive capabilities compared to traditional linear methods. However, the XGBoost model outperformed both ANN and SVM across all statistical indicators, achieving the highest correlation ($R_q^2 = 0.9842$) and the lowest error metrics (RMSE = 0.412 ft/hr, AAPE = 2.15%).

Table (7): Statistical performance benchmark and cross-validation evaluation of local supervised machine learning regression architectures for ROP prediction.

Machine Learning Model	Training R^2	Validation R^2	Testing R^2	RMSE (ft/hr)	AAPE (%)	Model Selection Status
Artificial Neural Network (ANN)	0.9512	0.9385	0.9401	0.895	4.82%	Evaluated
Support Vector Machine (SVM)	0.9284	0.9120	0.9156	1.120	6.14%	Evaluated
XGBoost Regression	0.9910	0.9825	0.9842	0.412	2.15%	Selected for Local Track

XGBoost was selected as the underlying core engine for the Local AI Strategy across all subsequent multi-objective optimization algorithms and response surface visualizers due to its superior predictive accuracy (AAPE = 2.15%). Its ensemble gradient boosting mechanism proved exceptionally robust in capturing non-linear interactions between surface mechanical variables (W, N) and downhole responses (dF/dT, dF/dD). Unlike the Artificial Neural Network (ANN), which suffered from slight performance degradation due to vanishing gradients in sparse data zones, XGBoost maintained strict numerical stability. Additionally, it demonstrated exceptional computational speed during the 13,100 and 37,600 Monte Carlo simulation screening sweeps—executing local optimization bounds in milliseconds while maintaining zero numerical drift.

Conclusions and Recommendations

This study successfully established and validated an integrated computational framework combining physics-based drilling mechanics, stochastic uncertainty quantification, and Artificial Intelligence (AI) decision support to optimize Rate of Penetration (ROP), mitigate tool wear, and reduce total well construction costs. Evaluated on a 263 m target interval (MD: 3167--3430 m) encompassing PDC bit Runs 8 and 9 of the Tawakul-1 well in Block-8 (Blue Nile Basin, Sudan), the primary technical findings and strategic field recommendations are synthesized below:

- The depth-based adaptation of the Galle and Woods model accurately captured the coupled mechanics connecting Weight on Bit (W), Rotary Speed (N), formation drillability (C_F), formation abrasiveness (A_F), and progressive cutter wear (D). Automated Python preprocessing pipelines successfully sanitized raw rig telemetry by purging non-drilling operational states and suppressing sensor outliers, yielding 131 clean data records for Run 8 (from 147 initial rows) and 376 records for Run 9 (from 381 rows). Near-perfect mathematical convergence (AAPE < 0.001%) between programmatic execution and manual benchmark calculations confirmed the system's numerical precision.
- Integrating Monte Carlo Simulation (MCS) enabled a transition from static point estimates to a risk-quantified probabilistic space, generating 13,100 simulation points for Run 8 (W: 23.0--25.1 klb; N: 136--235 RPM; ROP: 2.9--5.1 ft/hr) and 37,600 points for Run 9 (W: 3.0--34.4 klb; N: 203--215 RPM; theoretical peak ROP: 256.22 ft/hr). The stochastic search demonstrated that unconstrained weight application in deep formations expands performance variance and accelerates cutter degradation rather than yielding proportional penetration gains.
- The dual-track AI framework provided complementary optimization perspectives:
 - Delivered exceptional predictive precision (Testing $R^2=0.9842$, AAPE=2.15%) and identified upper-bound mathematical potential (W=34.0 klb, N=205 RPM, ROP=230.70 ft/hr). However, this unconstrained strategy incurred severe cutter wear penalties (dF/dD=21.3960).

- Implemented dynamic risk-aversion logic grounded by domain constraints. For Run 9, Gemini selected an optimal mechanical equilibrium at $W=24.0$ klb and $N=161$ RPM. This configuration achieved a stabilized ROP of 47.63 ft/hr (representing a 28.56% improvement over historical field performance (37.05 ft/hr)) while suppressing the wear derivative to a sustainable threshold ($dF/dD=10.8675$).
- Restricting operations from entering high-stress Caution and Unsafe thermal wear zones prevents premature PDC cutter impact spalling, ring-outs, and thermal fatigue. Extending effective bit life eliminates unscheduled bit trips, directly minimizing major causes of Non-Productive Time (NPT) and Invisible Lost Time (ILT) in deep, abrasive formations.
- The simultaneous 28.56% increase in penetration velocity and 49.2% reduction in cutter wear rate (compared to unconstrained local ML peaks) significantly reduce the total Cost per Foot (CPF). Maximizing total footage drilled per bit run while maintaining high penetration rates lowers daily rig operating expenses and bit procurement capital expenditure.

Recommendations for Field Implementation

1. Connect the execution engine directly to live WITSML rig feeds to enable dynamic parameter adjustments and early detection of downhole dysfunctions.
2. Expand testing across multi-well offsets in the Blue Nile, Muglad, and Melut basins to build calibrated regional formation drillability (C_F) and abrasiveness (A_F) libraries.
3. Incorporate Mechanical Specific Energy (MSE), dynamic vibrations, bit hydraulics, and detailed mud rheology (PV, YP, ECD) into the machine learning feature vector to refine predictions in narrow operational mud windows.
4. Systematically benchmark petroleum-specific AI recommendations against field data within a structured human-in-the-loop framework, ensuring senior drilling engineers continuously govern control decisions prior to rig execution.

Symbol	Definition
A_F	Formation Abrasiveness Factor
B_x	Part consumed of the total drill bit life
C_F	Formation Drillability Coefficient
D	Standard bit tooth wear / Wear grade
F	Footage drilled
H	Bit diameter
K	Exponent/function related to Weight on Bit
L	Function relating Weight on Bit to bearing wear
M and M^*	Function relating Weight on Bit to tooth wear
N	Rotary speed
P	Wear-type exponent

R	Function related to rotary speed and tooth wear
S	Drilling fluid modulus
T	Actual drilling/rotation time
W	Weight applied to the bit
dF/dT	Rate of Penetration / Penetration rate
dD/dT	Rate of bit tooth wear
dB_x/dT	Rate of bit bearing wear
dF/dD	Drilling progress function related to tooth wear
dF/dB_x	Drilling progress function related to bearing wear

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Authors' contributions

The conceptual framework, study vision, and administrative leadership were established by **A. A. I. E.**, who served as the principal investigator and corresponding author. In this role, **A. A. I. E.** directed the overall project execution, conducted data curation and formal analysis, supervised the research team, and managed resources acquisition. Additionally, **A. A. I. E.** provided rigorous manuscript validation, guided the review and editing process, and assumed primary responsibility for journal communications, data sharing compliance, and adhering to strict standards of scientific integrity and reproducibility.

The mathematical modeling, core system architecture, and software development were led by **M. E. A. A.**, who conceptualized the project methodology, formulated the underlying physical equations, and authored the technical methodology section. **A. A. M. M.** collaborated closely on software development, conducted the investigation and formal analysis, and drafted both the Results and Discussion section and the Conclusion and Recommendations section before contributing to overall manuscript revisions.

Data collection, processing, and experimental validation were handled by **H. Y. D. A.**, who managed the field dataset curation, performed formal analysis, validated the physical model outputs, authored the Introduction and initial Results and Discussion sections, and participated in final text revisions. **A. D. A. M.** contributed to the early conceptualization of the study, conducted the initial bibliographic synthesis, authored the Literature Review section, and provided critical feedback during the revision process.

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Availability of data and materials

The study's supporting data can be obtained from the [Ministry of Oil and Energy—Sudan]. However, there are restrictions on public access to these data because they were used under a licensing agreement for this specific research. Nonetheless, the authors are willing to provide the data upon a reasonable request, subject to approval from the [Oil and Energy Ministry—Sudan].

Declarations

Competing interests

The authors declare that they have no competing interests.

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