Supplementary Appendix to
When Finite Learning Looks Reliable: Equilibrium Consistency and Iteration Dynamics in Learning from Prices

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Abstract

This appendix makes the article's evidentiary boundary auditable. It identifies the source-based objects under comparison, records all 20 retained claims and their limits, gives the exact symbolic certificates and finite-iteration protocol, and documents deterministic reproduction. It adds no substantive or economic claim. The all-horizon checks apply only to the prespecified starts and parameter surfaces. The deposited source was executed once for the recorded comparison and is not rerun by this reproduction package; every reproducible artifact is generated by independent exact-algebra and deterministic-analysis scripts.

1 Purpose and scope

The article distinguishes four related objects derived from or reconstructed using the replication materials for Bastianello and Fontanier [2025] and its online appendix [Bastianello and Fontanier, 2024a]: the published equilibrium equations, the fixed-point system in the deposited code, the finite path generated by the companion private-information updater, and the partially revealing updater. This appendix states precisely what evidence each object supports without extending the article's claims.

The official public replication archive is cited as Bastianello and Fontanier [2024b] and is not redistributed. The accompanying reproduction archive contains only the independent deterministic scripts, exact rational inputs, generated results, figures, tables, and verification records needed to support the article's claims. It contains no credentials, private absolute paths, account identifiers, or copies of third-party source files.

2 Recorded source execution and custody

The source comparison begins from the official article, online appendix, and Zenodo v1 archive [Bastianello and Fontanier, 2024b]. Cryptographic hashes were recorded for the archive and each source file before output inspection. The custody comparison uses one recorded, unmodified MATLAB R2023b Update 11 execution.

That execution returned normally and produced 11 figure objects and a workspace; it is not rerun by the current package.

Independent code agrees with the finite paths stored in MATLAB figure objects 10 and 11—the deposited routines corresponding to online-appendix Figures 6 and 8—with a maximum absolute plotted-value difference of 4.44×10^{-16}. It agrees with the recorded deposited-D coefficients to a maximum absolute coordinate difference of 4.94×10^{-13}. These checks do not independently reconstruct figure objects 1–9.

Seven nonlinear solves reported non-success diagnostics even though the overall source program returned normally. A wrapper also compared decimal values by exact equality; the observed representation difference was 1.11×10^{-16}, below the prespecified 2×10^{-15} interpretation tolerance. These facts limit the claim to the named finite paths and recorded coefficients rather than a full source reproduction.

The deposited driver assigns the output of `fixedpoint_ree` to `coeff_ree`. No later line reads that variable. The private-price figure instead uses the certainty-equivalent start and rows from `klevelupdating_private`, which uses the article/updater-H denominator. The denominator mismatch in the deposited fixed-point routine therefore does not feed a displayed package result.

3 Source-based objects and evidentiary boundaries

Table 1 summarizes the four objects and the boundary on what each can establish.

Table 1: Source-based objects and the claims each supports.

Object	What is established	What is not claimed
Published equilibrium equations	The private-information root implied by the published equations and companion updater exactly satisfies the independently reconstructed Gaussian-projection and market-clearing identities.	This result does not modify the authors' equilibrium theory or behavioral mechanism.
Fixed-point system in deposited code	The unique nonzero root on the source-defined domain leaves an exact mean-supply coefficient residual of 5000/116781.	This root is not interpreted as an equilibrium, and no inference is made about the authors' intent.
Companion private-information updater	The updater uses the denominator H, as do the published equations, and its recorded source-generated path agrees with the independent implementation to the stated tolerance.	The fixed-point output from the deposited private routine is assigned but not subsequently read by the plotting driver, so no displayed figure is claimed to change.
Partially revealing updater	The prespecified design yields an exact projector factorization, stability counts, finite-horizon diagnostics, and a singular-boundary limit.	No claim is made about universal orbit safety, empirical prevalence, welfare effects, or general instability of partial-equilibrium-thinking models.

4 Claim and numerical-result ledger

Table 2 records the reported claims and numerical results across the article and supplement.

Table 2: Reported claims and numerical results across the article and supplement.

ID	Claim	Exact or certified result	Boundary
C1	Unique roots of the two private fixed-point systems.	Rational substitution yields zero residuals for each implemented system.	Source-faithful domain $vDH \neq 0$.
C2	The article/updater-H root satisfies market clearing; the deposited-D root does not.	Deposited-D mean-supply residual $5000/116781$.	Benchmark calibration; no welfare interpretation.
C3	Unit-mean-supply excess demand implied by the deposited-D residual.	$15625/61104$.	Demand units; not a percentage or welfare effect.
C4	Fixed-point price gap under the original source normalization.	$125000/3544233 = 3.5268561632\%$.	$\xi = .20, .21, \ldots, 1.80$; prior mean, mean supply, and realized supply one; article/updater-H fixed-point price sup-norm denominator $2519/2075$.
C5	First-update price gap under the original source normalization.	$125000/751233 = 16.6393116383\%$.	Same grid and prescribed common start; contemporaneous article/updater-H price sup-norm denominator $3879/2575$.
C6	Ratio of normalized first-update and fixed-point price gaps.	$168773/35773 \approx 4.717888$.	Uses the horizon-specific C5 and C4 denominators; distinct from the raw coefficient-gap ratio.
C7	Ratio of the corresponding unnormalized constant-coefficient gaps.	$603/103 \approx 5.854369$.	Must not be described as the normalized price-gap ratio.
C8	Jacobian factorization of the partially revealing map.	$J = -\varepsilon\Pi$, $\Pi^2 = \Pi$, $\operatorname{rank}(\Pi) = 2$.	Local exact result at the nonzero source root; Π is generally oblique.
C9	Stability classification on the 100-cell source-package sweep.	39 unstable and 61 stable cells out of 100; no boundary cell.	Deterministic design census, not an empirical fraction.
C10	Stability classification on the prespecified 8,100-cell grid.	1,833 unstable and 6,267 stable cells out of 8,100; no boundary cell.	Prespecified variance-ratio and revelation-weight grid.
C11	Displayed $h = 0.60$ benchmark.	$\varepsilon = 10/17$; the error at iteration $K = 99$ lies below the certified bound.	Consistent with the deposited source and converged; not evidence against the published display.
C12	Independent finite paths agree with the source-generated plotted paths.	Maximum absolute difference among the plotted values for MATLAB figure objects 10 and 11: $4.440892098500626 \times 10^{-16}$.	Only the routines corresponding to Figures 6 and 8 in the deposited online appendix; MATLAB figure objects 1–9 are not independently reproduced.
C13	Deposited-system coefficients agree with the exact root.	Maximum difference from the recorded deposited-D coefficients $4.933831121434196 \times 10^{-13}$.	One recorded source execution; no rerun.

ID	Claim	Exact or certified result	Boundary
C14	Fixed-point price gap on the separate 101-point analysis grid.	$250000/6253311 = 3.997882\%$.	A separate $\xi \in [0.5, 1.5]$ normalization; does not replace C4.
C15	First-update price gap on the separate 101-point analysis grid.	$750000/3204433 = 23.405077\%$.	The same separate grid normalization; does not replace C5.
C16	Finite-horizon classification at $K = 1000$.	At $K = 1000$, 6,267 stable paths are within five percent of the nonzero root and 1,833 unstable paths are within five percent of the singular-boundary limit.	The singular boundary is not called an attractor or equilibrium.
C17	The deposited private fixed-point output is not used by later plotting code.	One assignment and zero later reads.	No published figure or conclusion is claimed to change.
C18	Novelty search of the public record.	No matching public correction or overlapping result was found in the sources searched.	No statement about undisclosed or future work.
C19	Exact no-pole result at every integer horizon.	Zero poles across 100 deposited-sweep and 8,100 grid cells.	Only the prespecified starts on the two design surfaces; not a universal claim for other starts or parameters.
C20	Exact validation of the persistent-band classifications.	244 sweep and 25,068 grid stable cell-by-band classifications; zero mismatches.	Persistence is defined through $K = 1000$ under prespecified horizons and bands.

5 Symbolic certificates

This section uses the article's coefficient notation. Let $a, b, c, A > 0$ denote the prior variance, signal-noise variance, supply-noise variance, and risk aversion, and let $h \in (0, 1]$ denote the partial-revelation weight. Define

$$E = h^2 + A^2bc, \qquad \bar{H} = h + A^2bc, \qquad P = bE + ah\bar{H}, \qquad \varepsilon = \frac{h(1-h)}{E}.$$

The partially revealing coefficient vector is (p_0, p_s, q, p_Z); the private coefficient vector is (p_0, v, q, z).

The independent symbolic verifier differentiates the deposited partially revealing map in the coefficient order (p_0, p_s, q, p_Z), substitutes the displayed nonzero root, and simplifies every entry of

$$J + \varepsilon\Pi \quad \text{and} \quad \Pi^2 - \Pi$$

to zero over positive symbolic primitives. It separately simplifies $\operatorname{tr}(\Pi) - 2$ to zero. Because an idempotent matrix has rank equal to its trace, these identities certify the stated rank-two factorization without a floating-point eigenvalue calculation. For direct verification, define

$$L = (a+b)h^2 + A^2bc(ah+b),$$
$$U = (a+b)h^2 + A^2bc(ah-b),$$
$$V = A^2bc(ah+b) - (a+b)h^2.$$

Then

$$\Pi = \begin{pmatrix} 0 & -U/L & 0 & 2Abch/L \\ 0 & U/L & 0 & -2Abch/L \\ 0 & -AbV/(hL) & 1 & -V/L \\ 0 & -AbU/(hL) & 0 & 2A^2b^2c/L \end{pmatrix}. \tag{1}$$

All entries are finite on the maintained positive domain. This explicit matrix is useful for auditing the certificate; the article keeps only the geometric interpretation needed for its main argument. The executable certificate is `replication/check_theory.py` in Online Resource 2.

For the partially revealing root, clearing the maintained positive denominators reduces the fixed-point equations to

$$Pp_0 = bE, \qquad Pp_s = ah\bar{H}, \qquad Pq = Aabh(1-h), \qquad Pp_Z = -Aab\bar{H}.$$

The coefficient matrix is PI_4, with determinant $P^4 > 0$ because $P = bE + ah\bar{H}$ and all primitives are positive. This supplies the explicit uniqueness certificate used in the article.

For the private roots, a nonzero fixed point on the source-defined domain must have $v \neq 0$; otherwise the source denominators vanish. With $t = A^2bc$ and $\kappa = a + t(a+b)$, the v and z equations imply $z = -Abv$ and reduce to $v = a(1+t)/\kappa$. The remaining equations give $p_0 = tb/\kappa$ and the distinct q_H and q_D fibers displayed in the article. The symbolic verifier substitutes each root back into its own full rational residual and requires exact zero.

6 Finite-iteration design

The horizons are fixed at

$$K \in \{1, 2, 3, 5, 10, 20, 50, 99, 100, 200, 500, 1000\},$$

with persistent relative-accuracy bands 10%, 5%, 1%, 0.1%, and 0.01%. A horizon is recorded only when the path enters a band and remains in it at every later integer iterate through $K = 1000$. Otherwise the typed result is `NOT_ATTAINED_BY_1000`. Undefined states remain typed as `EXACT_POLE` or `OUTSIDE_DEFINED_DOMAIN`; neither NaN nor infinity is serialized.

Coefficient error is normalized by the ℓ_∞ norm of the exact nonzero fixed-point coefficient vector. Price error is evaluated on the fixed 101-point grid $\xi = 0.50, 0.51, \ldots, 1.50$ and normalized by the fixed-price sup norm for the article/updater-H system. Starts specified only in the private system's scalar coordinate are used solely to test pole and domain geometry. Because the design does not prescribe a corresponding full coefficient vector, coefficient and price errors are not reported for those starts.

The full starting-value ledger is fixed in advance. It contains the deposited zero or special-branch start, the private certainty-equivalent start, the exact fixed-point control, and fixed-point scalings 0, 3/4, 1, 5/4, and 2 for every applicable map. The scalar private-core check additionally uses $w_0 \in \{-4, -2, -1/2, 0, 1/2, 2, 4\}$. The local projector experiment sets $\varepsilon = 1 + d$ for $d \in \{0, \pm 10^{-1}, \ldots, \pm 10^{-8}\}$, while the pole experiment retains the exact pole and four finite preimages at the same signed offsets. No start, horizon, band, or tolerance is selected after inspecting an outcome.

The local boundary exercise perturbs within $\mathrm{im}(\Pi)$ so that the post-projection persistence factor is exactly $|\varepsilon|^K$. It deliberately does not measure the generally oblique first projection. Pole experiments retain the true pole and four finite preimages with signed offsets; exact pole paths stop at the first undefined update.

For the deposited zero initialization, define $\delta = bh(1-h)/P$ as in the article. For every cell in the 100-point deposited sweep and the 8,100-cell grid, an exact all-horizon check evaluates the closed-orbit denominator $1 + \delta(-\varepsilon)^{K-1}$. Even exponents cannot produce a pole; odd exponents would require the exact rational equality $\delta\varepsilon^{K-1} = 1$. The check finds no finite pole at any integer update $K \geq 1$. A separate persistence certificate verifies all 244 stable sweep cell–band decisions and all 25,068 stable grid cell–band decisions. In the stable regime, positivity implies $0 \leq \delta < 1$ and $0 \leq \varepsilon < 1$. Each parity subsequence therefore decreases, so at any listed horizon the largest subsequent error occurs either at that horizon or at the next iterate. Every unstable path remains outside the 10-percent persistent-accuracy band at $K = 1000$.

7 Deterministic reproduction boundary

The deterministic build performs only the following operations:

1. check the exact symbolic identities;
2. regenerate the independent deterministic results, four figures, and four tables;
3. compile the identified and anonymous manuscripts and this appendix;
4. check that every reported claim is present and traceable, and validate typed states, citations, privacy, and anonymity; and
5. build content-addressed package and reviewer-archive manifests.

The build does not launch MATLAB, open or execute the public replication archive, or repeat the recorded source execution. The figures are supplied in vector PDF and high-resolution PNG form. Final PDF checks require embedded fonts, valid page boxes, extractable text, and a clean page-by-page visual inspection.

8 Disclosure and interpretation limits

In the log from the recorded execution, seven nonlinear solves report non-success diagnostics even though the overall program completes normally. The execution used a local environment that is not claimed to be currently supported by the software vendor. These limitations do not invalidate the reported agreement for the specified finite paths, but they rule out any broader claim of full source reproduction.

The analysis has no sampled empirical population, standard error, or welfare estimand. Grid counts therefore support implementation-specific classification only. The novelty search is limited to the public sources reviewed and cannot exclude unpublished or otherwise undisclosed work.

References

Francesca Bastianello and Paul Fontanier. Online appendix for: Expectations and learning from prices, 2024a. URL https://paulfontanier.github.io/papers/onlineappendix_PETinGE_BF_Restud.pdf. Sections D, F, and G.

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