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Abstract: In many theoretical studies, the assumption that only the output contains errors is a common idealization that simplifies the design and analysis of input signals. However, systems in engineering practice often encounter errors in both inputs and outputs, significantly diminishing the effectiveness of these theoretical approaches in real-world applications. This paper employs two approaches nonparametric design and harmonic design to design the input signals for closed loop errors-in-variables system. By analyzing the relationships among the system's parameters and applying power spectrum correlation knowledge, a nonparametric design expression for the input signal was derived. Following this, a method for designing the input signal's harmonics was proposed. Through designing the input waveform, the system's input signal achieves optimal performance. Finally, the validity of the designed input signal was verified through simulation. Observation of the experimental data revealed that the design achieved satisfactory results.

Key words: Input signal; closed loop errors-in-variables system; Nonparametric design; Harmonic design

Paper type Technical paper

1. Introduction

When both input and output contain errors or measurement noise, the system is referred to as an error-in-variables system, commonly abbreviated as EIV[1]. In the real world, sensor noise, actuator drift, and environmental interference are ubiquitous; a perfectly “clean” system does not exist. Error-in-variables system provide a more realistic and practical description of real-world systems, enabling more effective analysis and control of those systems where inherent errors are unavoidable. They acknowledge that in any practical system, the measurable output signal is not perfectly determined by the input signal we apply, but is inevitably influenced by the combined effects of various internal and external uncertainties. Research on error-in-variables system serves as the theoretical foundation for transitioning engineering systems from idealized models to practical, reliable applications. Its ultimate goal is to achieve precise and robust control and estimation in uncertain environments.

In reality, experiments always have constraints, such as: input amplitude cannot be infinitely large, energy cannot be infinitely abundant, and time cannot be infinitely long. Optimal signal design[2-7] enables the achievement of desired experimental outcomes with minimal experimental

costs under these constraints, thereby avoiding failures and resource wastage caused by blind experimentation. By imposing physical constraints on input amplitude, power, and rate of change, optimal signal design maximizes information acquisition without exceeding device safety limits. This reduces experimental iterations, energy consumption, and excessive excitation of the test object. Optimal input signal design[8-10] maximizes the information matrix of the parameters to be estimated in the system model by optimizing its energy distribution, spectral characteristics, and timing structure, thereby enhancing parameter identification accuracy and efficiency. Particularly in complex, nonlinear, and high-noise systems, applying random or step inputs may fail to adequately excite all modes. Optimal design avoids under-excitation or overload, significantly reducing the time required for system identification. Shaping the spectrum for specific frequency bands or dynamic characteristics concentrates signal energy within the system's sensitive frequency range while avoiding areas dominated by measurement noise or strong external interference. This approach effectively enhances the signal-to-noise ratio, reduces the need for filtering in subsequent data processing, and improves data quality. In short, the design of optimal input signals can stimulate the system's key characteristics in the most efficient and reliable manner, thereby achieving performance maximization and resource optimization in both engineering practice and scientific research.

Piotr Bania et al. proposed an algorithm to reduce the dimension of the matrix to be inverted to a factor of N . They compared the proposed Bayesian method with the mean D-optimal design method and demonstrated its superiority[11]. Han Xiumei et al. proposed an optimization problem incorporating transmission costs into the cost function for completely unknown discrete systems[12]. Liu Yuqing et al. designed a signal vector with a pre-specified desired covariance matrix for the detection signal of a MIMO radar, while ensuring a small sidelobe level of the fuzzy function[13]. Deshpande Sunil et al. proposed a data-centric estimation method and investigated the design of linear time-invariant systems under input amplitude constraints[14]. A KRÓLIKOWSKI considered an alternative approach to indirect adaptive control, utilizing input signal design in parameter estimation to control linear systems with unknown parameters[15]. Jakowluk W. proposed a novel optimal input design method for fractional-order system identification, thereby maximizing the information obtained regarding the estimated fractional-order model parameters[16]. Wang Qing et al. proposed an optimization design method for controlling inputs, with the objective function defined as the volume of the uncertainty ellipsoid for parameter estimation. The optimal solution search employs a particle swarm optimization algorithm[17]. Prakash Kotecha et al. effectively modeled and solved the original nonlinear problem using a constrained programming approach, guaranteeing global optimality[18]. Optimal input design can flexibly adapt to different objectives by adjusting optimization criteria. For example: designing signals to maximize sensitivity to defect features in fault diagnosis, generating signals to balance exploration and exploitation in adaptive control, and optimizing signals to reduce channel interference in communication systems.

This paper employs two methods nonparametric design and harmonic design to achieve optimal input signal design for closed loop errors-in-variables system. The nonparametric design method first derives the prediction error under noisy output signals, then analyzes the case where the input signals are also noisy. It combines the prediction errors of both into a composite cost function, jointly deriving the nonparametric expression for the optimal input signal. Harmonic design constructs time-domain waveforms by manipulating four key parameters: frequency, harmonic

order, amplitude, and phase. The optimal input signals obtained from both design methods perform well for this system.

2. Closed loop errors-in-variables system Description

In closed loop errors-in-variables system, both input and output signals are subject to noise interference. This complicates input signal design and may render certain parameters unidentifiable with limited data due to input errors. Therefore, input signal design must ensure model identifiability. The presence of input errors and output errors means that simply increasing the input amplitude does not necessarily improve accuracy, as input errors may increase accordingly. For dynamic systems, the input signal spectrum must cover the system's critical frequency bands. However, due to input errors, high-frequency measurement noise tends to be significant. Over-exciting these high-frequency regions can substantially increase noise interference. Therefore, designing the input signal for a closed loop errors-in-variables system can effectively reduce the impact of input and output noise on the system, thereby improving system performance.

During the design phase of input signals, uncertainties in models and noise interference must be explicitly considered to ensure that the designed input signals can effectively suppress the impact of external noise on system performance. This paper employs two methods nonparametric design and harmonic design to design input signals for closed loop errors-in-variables system.

Analysis is conducted under a closed-loop system with noise present at both the input and output. The model of this system is shown in Figure 1.

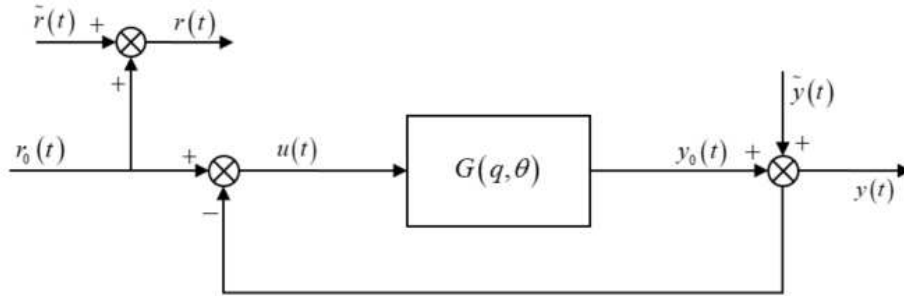


Figure.1: Errors-in-variables Closed-loop System Model

In this model, $r_0(t)$ and $y_0(t)$ represent noise-free input and output signals, respectively. $\tilde{r}(t)$ and $\tilde{y}(t)$ denote noise signals at the input and output terminals, respectively. $r(t)$ and $y(t)$ denote the input and output signals affected by noise, respectively. $G(q, \theta)$ represents the system's transfer model.

Transmission model $G(q, \theta) = \frac{A(q)}{B(q)}$, where θ represents parameters in $A(q)$ and $B(q)$. $A(q)$,

$B(q)$ and θ are respectively defined as:

$$\begin{aligned} A(q) &= 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a}; B(q) = b_0 + b_1 q^{-1} + \dots + b_{n_b} q^{-n_b} \\ \theta &= [a_1, a_2, \dots, a_{n_a}, b_0, b_1, \dots, b_{n_b}]^T \end{aligned} \quad (1)$$

n_a and n_b denote the degrees of the polynomials, and q represents the time-shift operator.

By applying both nonparametric design and harmonic design methods to the input signal of this closed loop errors-in-variables system, the optimal input signal is obtained.

3 Optimal Input Signal Design

3.1 Nonparametric Design

By observing Figure 1, we can derive some input-output relationships for this errors-in-variables closed-loop system with error:

$$\begin{aligned} r(t) &= r_0(t) + \tilde{r}(t); y(t) = y_0(t) + \tilde{y}(t) \\ y_0(t) &= G(q, \theta) [r_0(t) - y(t)] \end{aligned} \quad (2)$$

By combining the formulas in equation (2), the transfer function expression for this system can be obtained as:

$$y(t) = \frac{G(q, \theta)}{1 + G(q, \theta)} r(t) + \frac{\tilde{y} - G(q, \theta) \tilde{r}(t)}{1 + G(q, \theta)} \quad (3)$$

Its one-step-ahead predictive expression can be obtained as:

$$\begin{aligned} \hat{y}_1(t) &= (1 + G(q, \theta)) \frac{G(q, \theta)}{1 + G(q, \theta)} r(t) + (1 - 1 - G(q, \theta)) y(t) \\ &= G(q, \theta) r(t) - G(q, \theta) y(t) \end{aligned} \quad (4)$$

Prediction error is:

$$\begin{aligned} \varepsilon_1(t) &= y(t) - \hat{y}_1(t) = y(t) - G(q, \theta) r(t) + G(q, \theta) y(t) \\ &= (1 + G(q, \theta)) y(t) - G(q, \theta) r(t) \end{aligned} \quad (5)$$

After obtaining the prediction error, define an objective function $\mathbf{J}_1(z)$:

$$\mathbf{J}_1(z) = \arg \min \frac{1}{N} \sum_{t=1}^N \varepsilon_1^2(t) \quad (6)$$

By minimizing this objective function, we obtain an estimator for the prediction error. Taking the derivative with respect to $G(q, \theta)$ and setting it to zero yields:

$$\frac{2}{N} \sum_{t=1}^N [(1 + G(q, \theta)) y(t) - G(q, \theta) r(t)] [y(t) - r(t)] = 0 \quad (7)$$

Simplified to:

$$(1 + G(q, \theta)) E y^2(t) - (1 + 2G(q, \theta)) E y(t) r(t) + G(q, \theta) E r^2(t) = 0 \quad (8)$$

Applying the knowledge of power spectrum correlation to equation (8) yields:

$$(1 + G(q, \theta)) \phi_y(\omega) - (1 + 2G(q, \theta)) \phi_{ry}(\omega) + G(q, \theta) \phi_r(\omega) = 0 \quad (9)$$

Among them:

$$\phi_y(\omega) = E y^2(t); \phi_r(\omega) = E r^2(t); \phi_{ry}(\omega) = E y(t) r(t) \quad (10)$$

Equation (9) yields:

$$\mathbf{J}_1(z) = \frac{\phi_{ry}(\omega) - \phi_y(\omega)}{\phi_y(\omega) - 2\phi_{ry}(\omega) + \phi_r(\omega)} \quad (11)$$

Observing equation (4), we find that its one-step-ahead prediction $\hat{y}_1(t)$ considers only external noise $\tilde{y}(t)$ while neglecting input noise $\tilde{r}(t)$. Here, we incorporate both together to derive the

one-step-ahead prediction expression $\hat{y}_2(t)$:

$$\begin{aligned}\hat{y}_2(t) &= -\frac{1+G(q,\theta)}{G(q,\theta)} \frac{G(q,\theta)}{1+G(q,\theta)} r(t) + \left(1 + \frac{1+G(q,\theta)}{G(q,\theta)}\right) y(t) \\ &= -r(t) + \frac{1+2G(q,\theta)}{G(q,\theta)} y(t)\end{aligned}\quad (12)$$

Prediction error is:

$$\varepsilon_2(t) = y(t) - \hat{y}_2(t) = r(t) - \frac{1+G(q,\theta)}{G(q,\theta)} y(t) \quad (13)$$

Performing the expectation operation on $\varepsilon_1(t)$ and $\varepsilon_2(t)$ yields:

$$\begin{aligned}E\varepsilon_1(t) &= E\left[(1+G(q,\theta))y(t) - EG(q,\theta)r(t)\right] = E\left[\hat{y}(t) - G(q,\theta)\hat{r}(t)\right] \\ E\varepsilon_2(t) &= E\left[r(t) - \frac{1+G(q,\theta)}{G(q,\theta)} y(t)\right] = E\left[\hat{r}(t) - \frac{1}{G(q,\theta)} \hat{y}(t)\right]\end{aligned}\quad (14)$$

To simplify the calculations, assume that both the input and output noise are zero-mean white noise, i.e.:

$$E[\hat{r}(t)] = E[\hat{y}(t)] = 0 \quad (15)$$

After obtaining the prediction errors, construct a composite cost function $\mathbf{P}(z)$ using these two prediction errors, expressed as:

$$\begin{aligned}\mathbf{P}(z) &= \arg \min \frac{1}{N} \sum_{t=1}^N \varepsilon_1^2(t) + \varepsilon_2^2(t) \\ &= \arg \min \frac{1}{N} \sum_{t=1}^N \left[(1+G(q,\theta))y(t) - G(q,\theta)r(t) \right]^2 + \left[r(t) - \frac{1+G(q,\theta)}{G(q,\theta)} y(t) \right]^2\end{aligned}\quad (16)$$

To simplify calculations, the composite cost function $\mathbf{P}(z)$ is streamlined. By examining the relationship between $\varepsilon_1(t)$ and $\varepsilon_2(t)$, we can derive that:

$$\begin{aligned}\varepsilon_1(t) &= (1+G(q,\theta))y(t) - G(q,\theta)r(t) \\ &= -G(q,\theta) \left(r(t) - \frac{1+G(q,\theta)}{G(q,\theta)} y(t) \right) = -G(q,\theta) \varepsilon_2(t)\end{aligned}\quad (17)$$

$$\begin{aligned}\varepsilon_2(t) &= r(t) - \frac{1+G(q,\theta)}{G(q,\theta)} y(t) = -\frac{1}{G(q,\theta)} \left[(1+G(q,\theta))y(t) - G(q,\theta)r(t) \right] \\ &= -\frac{1}{G(q,\theta)} \varepsilon_1(t)\end{aligned}\quad (18)$$

After obtaining the functional relationship between the two, simplify the composite cost function:

$$\begin{aligned}\mathbf{P}(z) &= \arg \min \frac{1}{N} \sum_{t=1}^N \varepsilon_1^2(t) + \varepsilon_2^2(t) = \arg \min \frac{1}{N} \sum_{t=1}^N \left(1 + \frac{1}{G(q,\theta)^2} \right) \varepsilon_1^2(t) \\ &= \arg \min \frac{1}{N} \sum_{t=1}^N \left(1 + G(q,\theta)^2 \right) \varepsilon_2^2(t)\end{aligned}\quad (19)$$

Here we select one of them for analysis, i.e.:

$$\mathbf{P}(z) = \arg \min \frac{1}{N} \sum_{t=1}^N \left(1 + G(q, \theta)^2\right) \varepsilon_2^2(t) \quad (20)$$

Differentiate equation (20) with respect to $G(q, \theta)$ and set the derivative equal to zero to obtain:

$$\begin{aligned} & 2G(q, \theta) \sum_{t=1}^N \left[y(t) - G(q, \theta)(r(t) - y(t)) \right]^2 + \\ & 2 \left(1 + G(q, \theta)^2\right) \sum_{t=1}^N \left[y(t) - G(q, \theta)(r(t) - y(t)) \right] [r(t) - y(t)] = 0 \end{aligned} \quad (21)$$

Simplify it to obtain:

$$G(q, \theta) \left[\sum_{t=1}^N y^2(t) - \sum_{t=1}^N (r(t) - y(t))^2 \right] + \sum_{t=1}^N [y(t)(r(t) - y(t))] = 0 \quad (22)$$

Applying power spectrum theory to equation (22) yields:

$$G(q, \theta) (\phi_y(\omega) - \phi_r(\omega) + 2\phi_{ry}(\omega) - \phi_y(\omega)) = \phi_y(\omega) - \phi_{ry}(\omega) \quad (23)$$

Through equation (23), we obtain:

$$\mathbf{P}(z) = \frac{\phi_y(\omega) - \phi_{ry}(\omega)}{2\phi_{ry}(\omega) - \phi_r(\omega)} \quad (24)$$

Assuming the variances of input noise $\varepsilon_1(t)$ and output noise $\varepsilon_2(t)$ are σ_r^2 and σ_y^2 respectively, through simple yet complex calculations, we obtain:

$$\begin{aligned} \phi_r(\omega) &= \phi_{r_0}(\omega) + \sigma_r^2; \phi_y(\omega) = \frac{|G(q, \theta)|^2 \phi_{r_0}(\omega) + \sigma_y^2}{|1 + G(q, \theta)|^2} \\ \phi_{ry}(\omega) &= \frac{G(q, \theta)}{1 + G(q, \theta)} \phi_{r_0}(\omega) \end{aligned} \quad (25)$$

Substituting equation (25) into both the numerator and denominator of equation (24) yields:

$$\begin{aligned} \phi_y(\omega) - \phi_{ry}(\omega) &= \frac{|G(q, \theta)|^2 \phi_{r_0}(\omega) + \sigma_y^2}{|1 + G(q, \theta)|^2} - \frac{G(q, \theta)}{1 + G(q, \theta)} \phi_{r_0}(\omega) \\ &= \frac{\sigma_y^2 - G(q, \theta) \phi_{r_0}(\omega)}{|1 + G(q, \theta)|^2} \end{aligned} \quad (26)$$

$$\begin{aligned} 2\phi_{ry} - \phi_r &= 2 \frac{G(q, \theta)}{1 + G(q, \theta)} \phi_{r_0}(\omega) - \phi_{r_0}(\omega) - \sigma_r^2 \\ &= \frac{(G(q, \theta) - 1) \phi_{r_0} - (1 + G(q, \theta)) \sigma_r^2}{1 + G(q, \theta)} \end{aligned} \quad (27)$$

Substituting equations (26) and (27) back into equation (24) yields:

$$\mathbf{P}(z) = \frac{\sigma_y^2 - G(q, \theta) \phi_{r_0}(\omega)}{(G(q, \theta)^2 - 1) \phi_{r_0} - (1 + G(q, \theta)) \sigma_r^2} \quad (28)$$

To ensure that $\mathbf{P}(z)$ is an unbiased estimator, i.e., $E[\mathbf{P}(z)] = G(q, \theta)$, we can obtain:

$$E[\mathbf{P}(z)] = \frac{\sigma_y^2 - G(q, \theta)\phi_{r_0}(\omega)}{(G(q, \theta)^2 - 1)\phi_{r_0} - (1 + G(q, \theta))^2 \sigma_r^2} = G(q, \theta) \quad (29)$$

By combining like terms in equation (29), we obtain an expression for the nonparametric design of the optimal input signal:

$$\phi_{r_0} = \frac{\sigma_y^2 - G(q, \theta)(1 + G(q, \theta))^2 \sigma_r^2}{G(q, \theta)^3} \quad (30)$$

3.2 Harmonic Design

Previously, nonparametric design was applied to the input signal of a closed loop errors-in-variables system. Now, adopting a different approach, we focus on designing the harmonic signals of its input. By selecting four parameters frequency, harmonic order, amplitude, and phase the time-domain waveform is constructed. During harmonic design, waveform design must be reasonable to prevent signal distortion during operation.

Optimal input signal design by analyzing the waveform of the input signal enables the system to be fully excited within a specific frequency range, thereby allowing more accurate estimation of model parameters while reducing experimental time and costs. Common types of optimal input signals include pseudorandom binary sequences, multisine signals, swept-frequency signals, and optimal continuous excitation signals. Here, the Schroeder scan method is employed to design the optimal input signal for this system. Schroeder scanning is a deterministic, periodic multi-harmonic signal that optimizes the initial phase of each harmonic component. This results in a composite signal with a low peak-to-average power ratio in the time domain, enabling the injection of maximum average power within the device's permissible peak power limits. Consequently, it significantly enhances the signal-to-noise ratio of system excitation.

The input signal formula designed using the Schroeder scan method is:

$$r(i) = \sum_{k=1}^M A_k \cos\left(\frac{2\pi k t(i)}{T} + \phi_k\right), i = 0, 1, 2, \dots, N-1 \quad (31)$$

The amplitude A_k of the harmonic component is:

$$A_k = \sqrt{P/M} \quad (32)$$

P represents total power, M represents the number of harmonics. The amplitude of the input signal can be controlled by adjusting the total power P .

The phase recurrence relation is:

$$\phi_k = \phi_{k-1} - \frac{\pi k^2}{M}, k = 2, 3, \dots, M; \phi_1 = 0 \quad (33)$$

where $r(i)$ is the Schroeder scan signal value generated at the i th time point. Setting the amplitude A_k ensures that all M harmonic components have equal amplitudes, guaranteeing uniform distribution of signal energy across the frequency domain. The phase ϕ_k setting makes the signal's autocorrelation approximate a pulse, with a small cross-correlation, enabling effective separation of each frequency response. The Schroeder phase is a deterministic quadratic recursive sequence that shifts the peaks of each harmonic in the time domain. This results in a more uniform distribution of the composite signal's amplitude throughout the entire cycle, with a peak-to-average power ratio approaching the theoretical minimum. Consequently, under the same equipment peak power limitations, higher-power signals can be safely injected. Input signals designed using this method maximize system excitation while suppressing disturbance effects,

enabling efficient and high-precision system identification.

After completing the design for input signal $r(t)$, to identify signals that yield greater benefits for the system, parameter identification is performed on this errors-in-variables closed-loop system using system identification methods. The quality of this input signal is assessed by observing the convergence behavior of parameter θ .

Having previously calculated the prediction error for the closed loop errors-in-variables system, we now derive the power spectrum of equation (13) to obtain:

$$\phi_\varepsilon(\omega, \theta) = \phi_r(\omega) - \frac{|1 + G(q, \theta)|^2}{G(q, \theta)^2} \phi_y(\omega) \quad (34)$$

Then through the expression:

$$E[S^2(t)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_s(\omega) d\omega \quad (35)$$

You can obtain function $V(\theta)$ regarding θ :

$$V(\theta) = E\left[\frac{1}{2} \varepsilon^2(t, \theta)\right] = \frac{1}{4\pi} \int_{-\pi}^{\pi} \phi_\varepsilon(\omega, \theta) d\omega \quad (36)$$

Substitute equation (34) into equation (36), then solve for θ using the conjugate gradient method. The conjugate vector P_0 is taken as the negative gradient $-g^0 = -\nabla V(\theta^0)$ at point θ^0 , setting $Q = \nabla^2 V(\theta^0)$. The expression is:

$$\begin{cases} \theta^{k+1} = \theta^k + \alpha_k P_k \\ \alpha_k = -\frac{\nabla^T V(\theta^k) P_k}{P_k^T Q P_k} \\ P_{k+1} = -\nabla V(\theta^{k+1}) + \beta_k P_k \\ \beta_k = \frac{\nabla^T V(\theta^{k+1}) Q P_k}{P_k^T Q P_k} \end{cases} \quad (37)$$

The main steps for designing the optimal input signal using the harmonic method are as follows:

- (1) Select the appropriate total power P and number of harmonics M , then calculate the input signal $r(t)$ using the formula.
- (2) Measure and collect input and output signals using physical devices.
- (3) After obtaining the relevant data, perform parameter identification on the system.
- (4) Observe the identification results to determine whether the input signal is the optimal input signal. If it is, stop; otherwise, continue repeating the above steps until the optimal input signal is obtained.

4 Simulation Example

The purpose of this paper is to design an optimal input signal for a closed loop errors-in-variables system. To verify whether the designed optimal input signal has a positive impact on the system, its performance is evaluated through simulation^[19].

Assume that transmission model $G(q, \theta)$ is of the following form:

$$G(q, \theta) = \frac{B(q)}{A(q)} = \frac{0.8z^{-1} + 0.35z^{-2} + 0.4z^{-3}}{1 - 0.55z^{-1} - 0.3z^{-2}} \quad (38)$$

In the simulation experiment, the sampling time was set to 50 seconds. Both the input interference noise and output interference noise employed white noise with a mean of 0 and variance of 0.5. By observing Figures 2 and 3, it can be seen that the parameters in θ gradually converge to the target values as the simulation time increases and remain stable. Observing Figure 4 reveals that the similarity between the measured output and the identified output is relatively high, indicating that the designed input signal used for identification effectively reflects the system characteristics. Based on the above data, it can be concluded that the optimal input signal designed for this closed loop errors-in-variables system is effective.

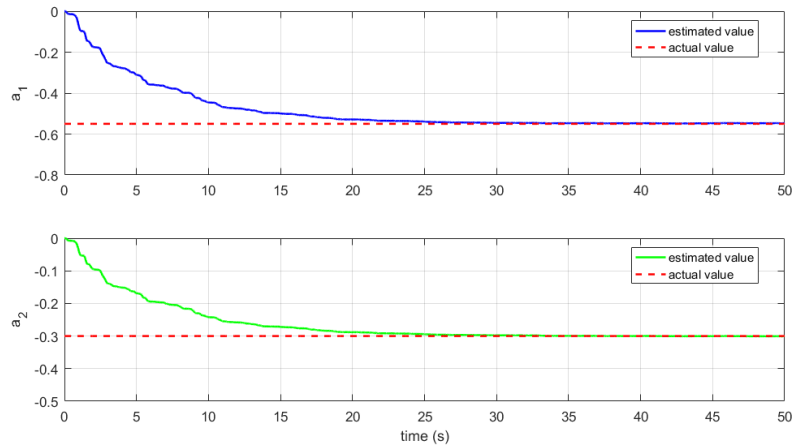


Figure.2: Parameter estimate for A(q)

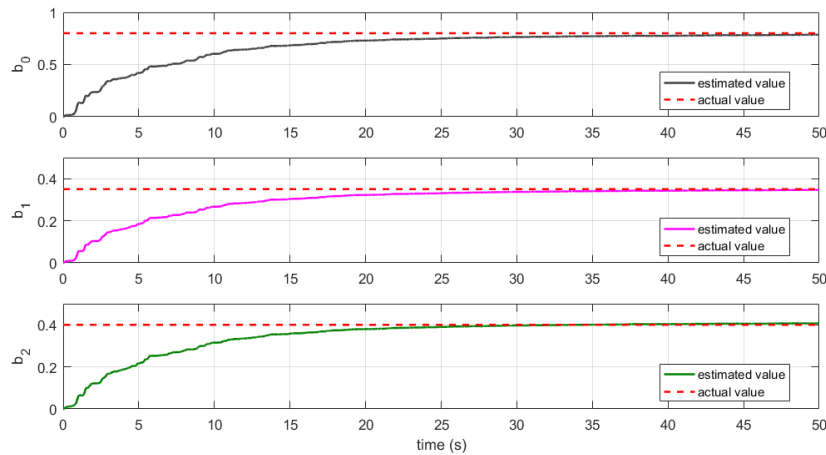


Figure.3: Parameter estimate for B(q)

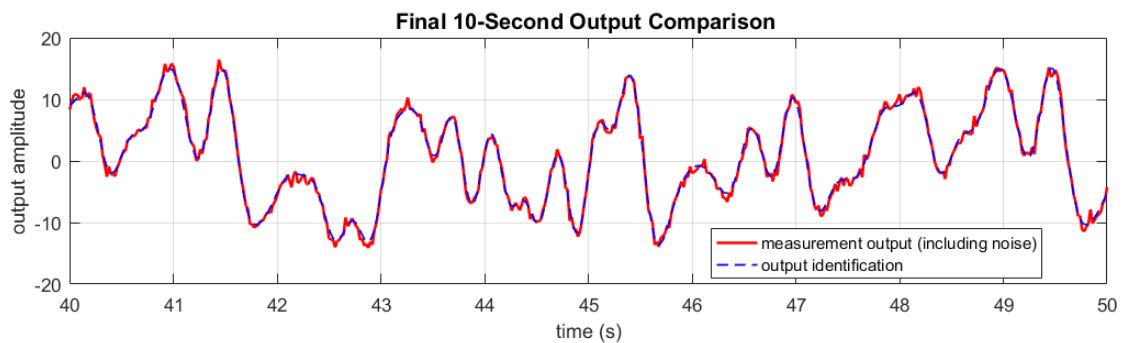


Figure.4: Measurement Output and Identification Output

5 Conclusion

This paper employs two methods nonparametric design and harmonic design to design input signals for closed loop errors-in-variables system. Through simulation observations of parameter convergence, it demonstrates the validity of the designed optimal input signals. Even in the presence of external noise interference, parameters can still be identified effectively and accurately. The current research is based on the assumption of linear time-invariant parameters. However, the nonlinear and time-varying characteristics present in actual engineering systems can degrade the performance of existing methods. Therefore, future work will focus on investigating nonlinear systems.

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Author contributions

Methodology and Conceptualization, Wen Xiangyun and Guo Xiaoyong; writing-original draft preparation, Wang Xiangjun and Guo Xiaoyong; writing-review and editing, Wen Xiangyun and Guo Xiaoyong; software and validation, Wang Xiangjun and Guo Xiaoyong. All authors have read and agreed to the published version of the manuscript.

Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Competing interests

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