

Development and Application of a Non-ergodic Ground-Motion Model for the Czech Republic

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Development and Application of a Non-ergodic Ground-Motion Model for the Czech Republic

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1 **Abstract** Peak ground accelerations (PGA) and peak ground velocities (PGV)
2 from 14 earthquakes recorded in the Czech Republic region between 2016 and
3 2019 are used to adjust the distance scaling of a reference ground-motion model
4 (GMM) to the Czech region. The residuals from the adjusted GMM are then used
5 to develop non-ergodic site and path terms for PGA and PGV for the Czech region.
6 The path terms capture strong source-site-specific path terms that can change
7 the median ground motion by up to a factor of 2 as compared to the ergodic
8 GMM. The non-ergodic GMM has a large reduction in the apparent aleatory
9 variability. The standard deviations of the within-site within-path residuals of
10 PGA and PGV are reduced to 0.33 and 0.25 natural log units as compared to the
11 standard deviations of the within-event residuals of 0.76 and 0.65 natural log units
12 for the ergodic GMM. Example hazard calculations are shown for three sites in

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the Czech Republic. The reduced apparent aleatory variability for the non-ergodic GMM leads to steeper slopes of the hazard curves for the non-ergodic GMM as compared to the slopes for the ergodic GMM. The largest effect on the seismic hazard is for long return periods. The reduced aleatory variability is accompanied by an increase in the epistemic uncertainty.

Keywords Non-ergodic, ground-motion model, seismic hazard, Czechia

1 Introduction

The Czech Republic is located in a stable continental region with low seismicity. Previous probabilistic seismic hazard analyses (PSHA) for the Czech Republic have used traditional ergodic ground-motion models (GMMs) with simple scaling based on the magnitude, distance, and site classification that is assumed to apply to all source-site pairs in the region; however, within the Czech region, there are strong systematic effects on the ground motion for specific ray paths. As recognized by Zátopek (1948) and demonstrated on instrumental data by Málek et al. (2017), the attenuation of the ground motion from Eastern Alpine earthquakes into the Bohemian Massif is weaker than for other wave paths. This is very clear in the shape of the macroseismic fields, which are pear-shaped and stretched towards the northwest (Figure 1).

Ergodic GMMs provide stable estimates of the average scaling of ground motion for a specific tectonic class; however, ergodic GMMs have limited accuracy due to unmodeled physical effects that are missing from the simple scaling in the GMM. The unmodeled effects are treated as randomness that can occur for any source-site pair. This randomness is apparent aleatory variability which is quantified by the standard deviation of the GMM. The apparent aleatory variability in a GMM reflects the limited accuracy of the GMM. To improve the accuracy of GMMs, the modeling of ground motions is currently in a transition from ergodic to non-ergodic GMMs (Abrahamson et al., 2019). This transition aims to model the systematic differences in the ground-motion scaling related to the systematic differences in the source, propagation path, and site characteristics for a given source-site pair.

In comparative analyses of ergodic and GMMs and non-ergodic GMMs in contemporary research (e.g., Lavrentiadis et al., 2023b; Sung et al., 2023a, 2024), ergodic GMMs consistently exhibits a significantly larger apparent aleatory variability than non-ergodic GMMs: non-ergodic GMMs typically have apparent aleatory standard deviations in the range of 0.35 to 0.45 ln units compared to 0.6 to 0.75 for ergodic GMMs.

In this paper, we develop non-ergodic GMMs for peak ground acceleration (PGA) and peak ground velocity (PGV) for the Czech Republic region using recorded ground motions from small-to-moderate magnitude earthquakes (M2.0 - M4.8) recorded by a temporary seismic network deployed in the Czech Republic. We first use the Czech ground-motion data to adjust the distance scaling of a reference ergodic GMM so that the scaling is consistent with the ground-motion data from the Czech region. The magnitude scaling is unchanged with preserves the magnitude scaling from the larger global data set. The adjusted GMM represents a regionalized ergodic GMM for the Czech Republic. The non-ergodic path and

site terms for specific source-site pairs are then estimated as adjustments to the Czech ergodic GMM. As examples of the effects of non-ergodic GMMs on the seismic hazard, PSHA calculations comparing the hazard for PGA and PGV using the ergodic and non-ergodic GMMs are shown for the three sites: the Dukovany (EDU) and Temelín (ETE) nuclear power plants sites in the southern part of the Czech Republic and the city of Prague.

2 Ground-Motion Data Set

Ground motions from multiple stations are needed to constrain the non-ergodic path effects. The AlpArray experiment (Hetényi et al., 2018), which operated from 2016 to 2019, deployed many temporary stations in the region. Events with magnitudes greater than 4 were selected, and in regions where no earthquake above M4 was present, smaller magnitude events were included to improve the ray coverage. A subset of 14 events was selected with magnitudes between 2.0 and 4.8.

For each event, recordings were included if the hypocentral distance was less than 400 km and if they passed the following quality control criteria:

- The power of microseisms (10 min time window; 0.1–0.3 Hz) just before the events of interest was evaluated, and the outliers (of a different order of magnitude) were removed from the analysis.
- The maximum amplitude of PKIKP / PKP / PKiKP waves for 7 teleseismic events within the magnitude range 6.3–6.9 from the Fiji region were evaluated, and the outliers (different order of magnitude) were also removed.
- Data with a signal-to-noise ratio below 5 in the vertical component were removed.

The first two criteria remove stations with obviously incorrect response functions, and the last criterion removes recordings that were too noisy to provide reliable data.

The resulting subset includes 792 recordings from 14 crustal events that occurred between 2016 and 2019. The locations of events and stations for the dataset selected for this study are shown in Figure 2. The distribution of the final dataset in terms of the magnitude, distance, and hypocentral depth is shown in Figure 3. In this study, we selected PGA and PGV for the ground-motion intensity measures.

3 Ergodic Ground-Motion Model

The statistical model for a simple ergodic GMM (GMM_{Erg}) can be expressed as

$$y_{es} = GMM_{Erg}(\mathbf{M}, \mathbf{R}, \mathbf{S}, \dots) + \delta B_e + \delta W_{es} \quad (1)$$

in which y_{es} is the ground-motion parameter (e.g., PGA), δB_e is the between-event residual for the e^{th} source, and δW_{es} is the within-event residual from the e^{th} source at the s^{th} site. The δB_e and δW_{es} are assumed to follow normal distributions with zero mean and standard deviations of τ and ϕ , respectively.

For this application, we selected the Chiou and Youngs (2014) GMM (denoted CY14) as the initial global ergodic and computed the residuals of the Czech data set using equation (4). The PGA total residuals from the CY14 GMM are shown

in Figure 4. Four events (3, 7, 10, and 11) have large negative residuals, but they show a similar trend of increasing residuals at distances greater than 50 km as seen for the other events. We do not have an explanation for the large negative residuals for these four events. To include the path effects from these four events, a separate constant term is included to account for the shift in the average residual for these four events. The large distance scaling in the CY14 GMM is adjusted to be distance scaling in the Czech region by fitting the residuals to the following form:

$$\delta_{CY14} = \begin{cases} c_0 + c_1 F_x + \delta B + \delta W & \text{for } R_{Rup} < 50 \text{ km} \\ c_0 + c_1 F_x + c_2 (R_{Rup} - 50) + \delta B + \delta W & \text{for } R_{Rup} \geq 50 \text{ km} \end{cases} \quad (2)$$

in which F_x is 1 for events 3, 7, 10, and 11, and 0 otherwise. The estimated coefficients c_0 , c_1 , and c_2 and the standard deviation terms are listed in Table 2.

The between-event residuals (δB) are shown in Figure 6a. The stars show the event terms for events 3, 7, 10, and 11 for the case in which the c_1 term is not included. The event terms for these four events are clearly outliers relative to the CY14 GMM.

The standard deviation of the PGA δB (denoted τ) is 0.66 ln units which is larger than typical τ values from GMMs. The PGA τ value from the reference CY14 GMM is 0.4 ln units for M4 events. Other GMMs for Europe, also have much smaller τ than the value computed from our subset. For example, using the Bindi et al. (2017) GMM, $\tau = 0.50$ for PGA. and the PGA τ value from the Bindi et al. (2017) GMM for Europe is 0.5 ln units. The large τ values from the Czech data set suggest that there are large uncertainties in the magnitudes in addition to the shift in the average event terms for the four outlier events. For the hazard calculations in this study, we set $\tau = 0.5$ based on the Bindi et al. (2017) GMM.

The total apparent aleatory standard deviation for the Czech ergodic GMM (σ_{Erg}) is computed from the sum of the between-event variance (τ^2) and the within-event variance (ϕ^2).

$$\sigma_{Erg} = \sqrt{\tau^2 + \phi^2} \quad (3)$$

4 Non-ergodic Ground-Motion Model

The GMM_{Erg} is used to constrain the extrapolation of the GMM to magnitudes and distances not represented in the local data set. The non-ergodic site and path terms are adjustments to the ergodic GMM. Because the non-ergodic terms are based on small-to-moderate magnitudes, only the linear effects are modeled by the non-ergodic terms.

The within-event residual from Equation (2) can be separated into a site term, $\delta S2S$, and the remaining within-site residual, δWS :

$$\delta W_{es} = \delta S2S_s + \delta WS_{es} \quad (4)$$

with this decomposition of the within-event residuals, the $\delta S2S_s$ and δWS_{es} are used to develop the models for the non-ergodic site terms and the non-ergodic path terms.

4.1 Non-ergodic Path Effects

Following the approach given in Sung et al. (2024), there are two parts for the non-ergodic path term: the effects from the 3-D Q structure denoted $\delta P2P_Q$, and the effects from the 3-D velocity structure denoted $\delta P2P_V$. The δWSP_{es} are the remaining within-site within-path residuals, which are assumed to follow normal distributions with zero mean and standard deviations of ϕ_{SP} .

The within-site residual is the sum of two path terms and the remaining within-path residual, denoted δWSP_{es} :

$$\delta WS_{es} = \delta P2P_Q(te_{cp,es}, ts_s) + \delta P2P_V(te_{cp,es}, ts_s) + \delta WSP_{es} \quad (5)$$

The ts_s is the coordinate of site s and $te_{cp,es}$ is the coordinate of the closest point on the rupture for earthquake "e" to site "s". For the small-magnitude earthquakes in the Czech data set, $te_{cp,es}$ is the hypocenter location.

4.2 Non-ergodic Site Effects

The current methodology for non-ergodic site terms incorporates two components: a site term that is spatially correlated ($\delta S2S_{reg}$) and a site-specific term that is not spatially corrected ($\delta S2S_{uncor}$) (Lavrentiadis et al., 2023a,b). The spatially correlated site term captures the similar site terms over a small region, whereas the uncorrelated site term models the systematic effect at a specific site that varies over shorter distances than the minimum separation distance between seismic stations.

$$\delta S2S_s = \delta S2S_{reg}(ts_s) + \delta S2S_{uncor}(ts_s) \quad (6)$$

The standard deviations of $\delta S2S_{reg}(ts_s)$ and $\delta S2S_{uncor}(ts_s)$ are $\phi_{S2S,reg}$ and $\phi_{S2S,uncor}$.

5 Estimation of non-ergodic terms

The non-ergodic path terms due to the 3-D velocity structure ($\delta P2P_V$) and the spatially correlated non-ergodic site are estimated using the variable coefficient model (VCM) approach. The non-ergodic path terms related to the 3-D Q structure ($\delta P2P_Q$) are estimated using the cell-specific method. These two approaches are described in the following.

5.1 VCM Approach

The varying-coefficient model (VCM) approach is used to model the non-ergodic behavior via Gaussian Process (GP) regression. A fundamental characteristic of the VCM approach is its ability to interpolate and extrapolate non-ergodic terms to source and site locations that are not represented within the dataset. Using the correlation length and variance of the non-ergodic terms, the VCM generates estimates of the median and epistemic uncertainty associated with these terms for a specific source-site pair. For ray paths or site locations that have no or sparse

nearby data, the median non-ergodic term approaches zero, and the epistemic uncertainty is large. It is essential to incorporate both the median and epistemic uncertainty of the non-ergodic terms in seismic hazard assessments when using NGMMs.

The VCM approach requires a covariance function for each non-ergodic term. We adopt the Matern covariance function ($cov(X, X')$) previously utilized by Sung et al. (2024) to estimate the non-ergodic term within the framework of R-INLA (Integrated Nested Laplace Approximation in the R programming environment) (Rue et al., 2026):

$$cov(X, X') = \Theta^2 \frac{2^{1-v}}{\Gamma(v)} (\sqrt{8v} \|X - X'\|/\rho)^v K_v(\sqrt{8v} \|X - X'\|/\rho). \quad (7)$$

Γ denotes the gamma function. The parameters ρ and Θ represent the hyperparameters associated with non-ergodic terms, specifically the correlation length and the marginal standard deviation of the spatial field, respectively. The parameter v indicates the smoothness of the function. K_v refers to the modified Bessel function of the second kind, characterized by an order v where $v > 0$.

5.2 Cell-Specific Approach

The estimation of the non-ergodic anelastic attenuation term ($\delta P2P_Q$) is performed using the cell-specific methodology introduced by Dawood and Rodriguez-Marek (2013) and refined through a Bayesian framework as described by Kuehn et al. (2019). For each record, the length of the ray path is calculated for each cell, taking into account the specific source-site pair. This method is described in detail by Sung et al. (2023a).

The functional form for the linear distance term related to the anelastic attenuation is given by:

$$\delta P2P_Q(te_{cp,es}, ts_s) = \sum_{i=1}^{N_C} c_{ca,i} \Delta R_i(te_{cp,es}, ts_s) \quad (8)$$

in which N_C is the total number of cells on the map, $c_{ca,i}$ is the coefficient for the linear distance scaling in the i^{th} cell, and ΔR_i represents the length of the ray path within the i^{th} cell. The summation computes the net attenuation along the ray path.

6 Non-ergodic GMM

The non-ergodic approach incorporates the systematic and repeatable effects related to the site and path effects into the ergodic GMM to develop the non-ergodic GMM (NGMM), resulting in the median ground motion depending on the coordinates of both the source and the site. In this study, we do not account for the non-ergodic source term due to the limited number of earthquakes considered (only 14 earthquakes) and the trends with magnitude seen in the between-event residuals.

The median non-ergodic ground-motion model is given by:

$$GMM_{NE}(M, R, t_{cp,es}, t_{ss}) = GMM_{Erg}(\mathbf{M}, \mathbf{R}) + \delta S2S_{reg}(t_{ss}) + \delta S2S_{uncor}(t_{ss}) \\ + \delta P2P_Q(t_{cp,es}, t_{ss}) + \delta P2P_V(t_{cp,es}, t_{ss}) \quad (9)$$

The non-ergodic apparent aleatory standard deviation (σ_{NonErg}) can be computed from the sum of the between-event variance (τ^2) and the within-site within-path variance (ϕ_{SP}^2):

$$\sigma_{NonErg} = \sqrt{\tau^2 + \phi_{SP}^2} \quad (10)$$

6.1 Estimated non-ergodic site terms

The between-site residuals ($\delta S2S$) of Eq. (4) are used as inputs to the VCM to estimate the correlated site term ($\delta S2S_{reg}$) and the epistemic uncertainty of the $\delta S2S_{reg}$. The estimated mean and epistemic uncertainty of ($\delta S2S_{reg}$) are shown in Figure 8. A feature of the VCM approach is that in areas with sparse or no observations, the mean value of $\delta S2S_{reg}$ approaches zero and the epistemic uncertainty approaches $\phi_{S2S,reg}$. Conversely, for sites near the observed ground motions, there can be large positive or negative values of $\delta S2S_{reg}$ with lower epistemic uncertainty.

The hyperparameters for the site term are given in Table 3. The correlation length for $\delta S2S_{cor}$ for PGA is about 50 km. This large correlation length for the site term is related to the lack of a site term in the data set. Without a site term in the GMM, the $\delta S2S$ values will reflect the surface geology. As a result, the correlation length of the $\delta S2S_{cor}$ will represent the spatial dimension of the surface geology units.

The uncorrelated site-specific term ($\delta S2S_{uncor}$) is given by the difference between the total site term ($\delta S2S$ from equation (4)) and the correlated non-ergodic site term ($\delta S2S_{reg}$)

$$\delta S2S_{uncor} = \delta S2S - \delta S2S_{reg} \quad (11)$$

The values of $\delta S2S_{uncor}$ apply only to sites with a seismic station and cannot be extrapolated to site locations that do not have recordings at the site. Therefore, the mean value of $\delta S2S_{uncor}$ is equal to zero at sites without a seismic station. Figure 9a shows the distribution of $\delta S2S_{uncor}$ terms per station. The epistemic uncertainty associated with $\delta S2S_{uncor}$ is quantified by the standard error,

$$\delta S2S_{uncor,epi,s} = \frac{\phi_{S2S}}{\sqrt{(N_s)}} \quad (12)$$

in which N_s is the number of records at station "s". The map of the epistemic uncertainty $\delta S2S_{uncor}$ is shown in Figure 9b.

6.2 Estimated Non-ergodic Path Terms for 3-D Q Structure

The cell-specific approach proposed by Kuehn et al. (2019) is used to estimate the anelastic attenuation coefficients ($c_{ca,i}$ in Equation (8)) to adjust the ergodic anelastic attenuation coefficient (c_{erg}) given by the Czech region ergodic GMM. To ensure the net anelastic attenuation coefficient remains physically realizable, the sum of the ergodic component (c_{erg}) and the cell-specific component ($c_{ca,i}$) is constrained to be negative for each cell. Therefore, the maximum value of $c_{ca,i}$ is constrained to be less than $-c_{erg}$.

The source-site pairs from the dataset are shown in Figure 2. The region is divided into 20 km x 20 km cells. The number of rays that pass through each cell is shown in Figure 10. The higher number of paths that pass through a cell indicates better constraints on the $\delta P2P_Q$. Figure 11 shows the mean $c_{ca,i}$ and epistemic uncertainty of the posterior distribution of the $c_{ca,i}$ per cell at PGA and PGV. Strong negative anelastic attenuation coefficients (lower Q) occur in the Carpathians mountains on the border of Czechia and Slovakia, whereas smaller values (higher Q) occur in a significant part of the southern Czech Republic, excluding its southwest and easternmost regions. These results are similar to the Czech Republic Qs from Málek et al. (2017). If the cell does not include any rays, the non-ergodic cell-specific anelastic attenuation coefficient (adjustment) goes to zero. That is, for ray paths without observations, the non-ergodic adjustment to the cell-specific anelastic attenuation goes to zero, and the mean of the net linear R coefficient is equal to the linear R term in the Czech region ergodic GMM but with large epistemic uncertainty.

6.3 Estimated Non-ergodic Path Terms for 3-D Velocity Structure

The non-ergodic cell-specific anelastic attenuation terms can capture significant path effects on the short-period median ground motion at large distances. For long periods ($T > 1$ s or PGV), the anelastic attenuation coefficient is small, and there is little or no effect on the ground motion (Kuehn et al., 2019; Sung et al., 2023a). The limitation of the cell approach is that it does not capture 3D path effects at short distances or for long spectral periods that can occur due to wave propagation in a complex 3-D velocity structure. Therefore, a second term, $\delta P2P_V$, is needed to capture the additional path effects due to the 3D velocity structure.

The path terms related to the 3-D velocity structure are more difficult to estimate than the path terms related to the 3-D Q structure because the $\delta P2P_V$ is not cumulative along the ray path. The ray path can be represented by a vector from the source to the site. The "separation distance" between two ray paths is the difference between vectors which complicates the VCM methodology.

We use a simplified method for assessing the spatial variability of the $\delta P2P_V$ term. Rather than using path pairs from all sites collectively to execute the VCM simultaneously, the VCM is computed in two steps. In the first step, we evaluate the within-site residuals (δWS) for each site individually using the VCM. Considering one site at a time allows the "separation distance" between two ray paths to become simply the distance between the earthquakes. The results of the VCM from this first step are used in forward modeling to estimate the path terms for a given source for a grid of site locations (not just the locations of the seismic

stations). The VCM approach is then applied to one earthquake at a time, and the "separation distance" is given by the distances between sites. Combining these two steps yields a model for the $\delta P2P_V$ for any source-site pair for future earthquakes. The method is explained in more detail by Sung et al. (2023b).

Figures 12 and 13 show the $\delta P2P_V$ for PGA and PGV for the three example sites. The correlation length of the path effect is about 40 km. The epistemic uncertainty of the $\delta P2P_V$ is also shown in Figures 12 and 13. Similar to the non-ergodic site term, for paths with limited or no data, the mean $\delta P2P_V$ tends towards zero, and the epistemic uncertainty of the $\delta P2P_V$ becomes equal to ϕ_{P2P_V} .

7 Non-ergodic residuals and standard deviations

The within-path residuals of the non-ergodic GMMs for PGA and PGV are shown as a function of distance in Figure 14. The residuals have a small range and do not have a trend with distance.

The various components of the standard deviation are listed in the lower section of Table 2. The ϕ_{S2S} values are 0.50 and 0.41 for PGA and PGV, respectively. These values are somewhat inflated compared to previous studies because site parameters, such as the time-averaged shear-wave velocity in the top 30 m (V_{S30}), were not available for this data set. Without a site parameter for computing the residuals, the non-ergodic site terms include the effect of the different site classes on the ground motion in addition to differences in the site response for sites with the same simple site parameters.

The standard deviations of the combined path terms for PGA and PGV are 0.40 and 0.37, respectively. These values are comparable to the standard deviations of the $\delta P2P$ from other regions. For example, the standard deviation of the $\delta P2P$ for Japan ranges from 0.25 to 0.40 natural-log units over the period range of 0–10 sec. (Sung et al., 2024).

By modeling the site and path terms in the non-ergodic GMM, the remaining apparent aleatory standard deviations of the within-path residuals are reduced to 0.33 and 0.25 for PGA and PGV. These small standard deviations reflect the potential increase in accuracy of non-ergodic GMMs as compared to ergodic GMMs.

8 Median PGA and PGV Models

Because the adjustment terms of non-ergodic GMMs differ by geographical location, the median predictions at a specific site also vary depending on the locations of the events. The final non-ergodic term (Δ_{NE}) in this study is the sum of $\delta S2S_{reg}$, $\delta S2S_{uncor}$, $\delta P2P_Q$, and $\delta P2P_V$:

$$\Delta_{NE} = \delta S2S_{reg} + \delta S2S_{uncor} + \delta P2P_Q + \delta P2P_V \quad (13)$$

The spatial variation of Δ_{NE} across three different sites is shown in Figures 15 and 16. Sites 1 and 2 are located in the southern region of the Czech Republic, near the Dukovany and Temelín Nuclear Power Plants, respectively. Site 3 is located in the Prague city center. The details of the three sites and the non-ergodic site terms are listed in Table 4.

The site term applies to the ground motion from all source locations. Due to the strong positive non-ergodic site terms, the Δ_{NE} are positive across most of the grids for Site 1. In contrast, the other two sites have negative Δ_{NE} values. For Site 2, there is a significant positive (Δ_{NE}) observed from the southwest of the station, which is attributed to the $\delta P2P_V$ path term. We selected four directions (A, B1, B2, C) to show the systematic differences in PGA and PGV scaling with distance. The distance scaling for the ergodic and non-ergodic models for the three sites is compared in Figure 17. By definition, the ergodic GMM has the same distance scaling for all sites and for all directions, whereas the distance scaling for the non-ergodic models can differ by location and by direction. The non-ergodic path terms, particularly the $\delta P2P_V$ associated with the velocity structure, significantly influence the results at shorter distances. The $\delta P2P_Q$ term can be significant at large distances. Due to the negative non-ergodic site terms, Site 3 shows a reduction in the non-ergodic GMM relative to the ergodic model. For Site 2, we chose two directions (B1 and B2), which show increased and decreased scaling compared with the ergodic model, respectively, from a distance range of 80-150 km. Site 1 shows an increase in the non-ergodic GMM, attributable to the positive non-ergodic path terms at longer distances, which relate to the Q structure ($\delta P2P_Q$).

To capture the spatial correlation of the epistemic uncertainty in the non-ergodic terms for a given site location, we generate 100 realizations for each non-ergodic term, utilizing its posterior distribution for source location. The 100 non-ergodic ground motion attenuation curves are shown by thin lines in Figure 17. The variability among the 100 curves reflects the epistemic uncertainty associated with Δ_{NE} . A narrower range of the 100 curves indicates that the non-ergodic terms are better constrained from the available local data.

9 Seismic Hazard Example

For traditional GMMs, the seismic hazard for the i^{th} source is given by:

$$Haz_i(Y > z) = N_i(M_{min}) \int_M \int_R f_{m_i}(M) f_{R_i}(R, M) P(Y > z | M, R, S) dR dM \quad (14)$$

in which $N_i(M_{min})$ is the rate of earthquake with $M > M_{min}$, $f_{m_i}(M)$ is the magnitude probability density function, and $f_{R_i}(R, M)$ is the distribution of distances for a given magnitude for the i^{th} source.

The aleatory variability of ground motion values for a specific scenario and site location is typically represented by a lognormal distribution. For an ergodic GMM, the conditional probability of exceedance is expressed as follows:

$$P(PSA_{erg,es} > z | M_e, R_{es}, S_s) = 1 - \Phi \left(\frac{\ln(z) - \mu_{erg}(M, R, S)}{\sigma_{erg}} \right) \quad (15)$$

in which μ_{erg} and σ_{erg} are the median and aleatory standard deviation of the ergodic GMM, respectively. The function $\Phi()$ denotes the cumulative standard normal distribution.

For a non-ergodic GMM, the $P(Y > z | M, R, S)$ term is modified as follows:

$$P(Y > z | M, R, S) = 1 - \Phi \left(\frac{\ln(z) - [\mu_{erg}(M, R, S) + \Delta\mu_{NE}(te_{cp,es}, ts_s)]}{\sigma_{NE}} \right) \quad (16)$$

in which σ_{NE} is the apparent aleatory variability for the non-ergodic GMM.

9.1 Seismic Source Models

The seismicity in the region is shown in Figure 18. To model the seismicity, we adopted the A3_LASZ_v25 source model developed by Málek et al. (2022). The model covers seismic sources within 100 km of the border of the Czech Republic. The A3_LASZ_v25 model is based on large areal source zones that respect the division of the area of interest into seismotectonic domains and the geological structure within the Bohemian Massif, the Eastern Alps, and the Western Carpathians. The source model contains 26 areal zones shown in Figure 19.

The capable faults in the region are shown in Figure 19. A source model that explicitly models the faults, called F3, is also available. The M_{max} values from the F3 model are given in Table 6 with the names of fault systems and the number of areal seismic zones. For this example, the F3 source model is not implemented because of the difficulties in characterizing the fault parameters and because the example sites are not near the faults. Instead, the areal source model was used to model the seismicity from both background earthquakes on unknown faults and from the identified capable fault.

This source model primarily represents an expression of historical seismicity, and it assumes spatial stationarity for the occurrence of future earthquakes. The maximum magnitude (M_{MAX}) was derived from the maximum observed magnitude (M_{MAX}^{OBS}) plus an increment:

$$M_{MAX} = M_{MAX}^{OBS} + \sigma_M + 0.5 \quad (17)$$

in which the σ_M values range from 0.25 to 0.34, depending on the source region. If the maximum observed magnitude was less than 4.15, the minimum value of $M_{MAX} = 4.9$ was used.

Models A3 and F3 were combined into one seismic source model. If M_{MAX} values were available for both the areal source and the capable faults, the higher one was conservatively chosen. Table 5 lists the names of the individual source zones as well as the maximum observed magnitude (M_{MAX}^{OBS}), the safety margin ($\Delta = \sigma_M + 0.5$) used, and the expert-estimated M_{MAX} . The M_{MAX} F3 values are also listed, and in the last column, the resulting estimated M_{MAX} A3.

9.2 Hazard Curves

The hazard was computed for the three sites listed in Table 4. The mean and epistemic fractiles (16th and 84th) hazard curves for the ergodic and non-ergodic GMMs for PGA and PGV are compared in Figures 20, 21, and 22 for the three sites. The hazard was computed for a single source model. Therefore, the epistemic fractiles shown in these figures only reflect the epistemic uncertainty in the GMM.

The non-ergodic site term applies to the ground motion at the site from all sources. By itself, the non-ergodic site term leads to a constant shift of the hazard curve along the $\log(GM)$ axis. The non-ergodic path terms vary by source location. The effect of the non-ergodic path term on the hazard depends on the relative contribution to the hazard from each source location.

For site 1 (Temelin), the mean site term is a moderate positive value of 0.103 ln units (Table 4). At the 1E-3 annual hazard level, the increase in the median ground motion due to the non-ergodic site term is offset by the reduction in the apparent aleatory standard deviation such that the mean hazard is similar for the ergodic and non-ergodic cases, but the epistemic fractiles are wider for the non-ergodic case. Particularly for the 16th fractile. At the 1E-5 hazard level, the reduction in the apparent aleatory variability has a larger effect on the hazard than the increase in the median ground motion from the site term, resulting in a 15-20 percent reduction in the PGA and PGV.

For site 2 (Dukovany), the site term is a small negative value of -0.041 ln units. With a small negative site term, the non-ergodic hazard shows a larger reduction compared to the Temelin site.

For site 3 (Prague), the site term is a large negative value of -0.32 ln units. With this large negative site term combined with the reduced apparent aleatory variability, the hazard curves for the non-ergodic case are much lower than the ergodic hazard curves for the ergodic case. For the 1E-3 and 1E-5 annual hazard levels, the PGA for the mean hazard is reduced by factors of 1.5 and 2.0, respectively.

The reduction in the apparent aleatory variability for the non-ergodic GMM will lead to steeper hazard curves for individual branches of the logic tree for all cases, but the net effect on the hazard depends on the non-ergodic site and path terms in addition to the size of the apparent aleatory standard deviation. The mean hazard will also depend on the size of the epistemic uncertainty in the non-ergodic site and path terms.

10 Conclusions

In low activity regions, such as the Czech Republic, there are few ground-motion recordings from large-magnitude earthquakes, but recordings from small- and moderate-magnitude earthquakes can be used to constrain the linear non-ergodic effects. The successful application of the non-ergodic GMM approach to the Czech Republic region using four years of data from a temporary deployment shows that ground-motion data from temporary deployments can be valuable for estimating the non-ergodic terms.

By including additional physical effects, non-ergodic GMMs have improved accuracy compared to ergodic GMMs as evidenced by the reduction from the apparent aleatory standard deviation of the ergodic within-event PGA residuals of 0.76 ln units to a standard deviation of the non-ergodic within-path residuals of 0.33 ln units. There is a similar reduction from 0.65 to 0.25 for PGV.

The reduced apparent aleatory variability for the non-ergodic GMM leads to steeper slopes of the hazard curves for the non-ergodic GMM as compared to the slopes for the ergodic GMM. The largest effect on the seismic hazard is for long return periods. Typically, the reduction in the apparent aleatory variability of the GMM will be accompanied by an increase in the epistemic uncertainty because more parameters are estimated from the same data. Over time, as more local data are collected, the epistemic uncertainty for the non-ergodic GMM will be reduced, and the apparent aleatory variability will be refined; however, if additional model

complexity is incorporated, then there will be a reduction in the aleatory variability and an increase in the epistemic uncertainty.

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11 Statements and Declarations

11.1 Funding

This study was partially supported by Strategy AV21 program Sustainable Energy.

11.2 Competing Interests

The authors have no relevant financial or non-financial interests to disclose

11.3 Author Contributions

All authors contributed to the study conception and design. The compilation of the ground-motion data and seismic source characterization were performed by Jiří Vackář, Jiří Málek, and Ivan Prachař. The development of the Czech ergodic GMM and the non-ergodic hazard calculations were performed by Norman Abrahamson. The development of the non-ergodic GMM was performed by Chih-Hsuan Sung. The first draft of the manuscript was written by Chih-Hsuan Sung and Norman Abrahamson. All authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

12 Data Availability

The data set of PGA and PGV values used in this study is available in the electronic supplement file "Czech_PGA_PGV_data.csv".

Table 1 Earthquakes in the Selected Data Set.

Epk Num	Date	Time	Mag	Long	Lat	Dep (km)	Num of Rec	Min,Max Distance (km)
1	2016-04-25	10:28	4.2	16.16	48.10	5	83	42,385
2	2017-02-17	15:17	2.3	13.74	50.59	3	11	22,150
3	2017-04-08	22:23	4.7	16.07	51.57	10	95	82,396
4	2017-04-29	00:56	3.1	12.18	51.30	10	49	28,272
5	2017-11-10	16:40	3.2	16.16	47.73	5	28	60,274
6	2017-12-10	02:52	3.6	18.17	49.95	10	42	13,292
7	2018-05-05	08:58	4.3	18.63	50.02	10	43	41,388
8	2018-05-14	01:54	3.7	12.44	50.28	5	98	5,292
9	2018-05-21	21:04	4.2	12.42	50.26	4	102	4,371
10	2018-07-20	03:31	4.8	16.05	51.55	5	102	81,394
11	2018-08-11	11:53	4.1	18.79	50.39	10	39	77,401
12	2018-08-17	01:25	3.1	18.18	48.91	5	31	24,262
13	2018-12-03	13:32	2.0	14.12	48.80	2	8	14,106
14	2019-10-22	23:35	4.3	12.27	47.62	3	61	55,378

Table 2 Coefficients and standard deviation terms for PGA and PGV for the Czech ergodic GMM and the standard deviation terms for two global ergodic GMMs. The standard deviation terms for the Czech non-ergodic GMM are also listed.

GMM	Coefficient	PGA	Std Err	PGV	Std Err
Czech Ergodic	c_0	-0.11	0.22	-0.13	0.20
	c_1	-2.90	0.40	-2.24	0.37
	c_2	0.00809	0.00034	0.00527	0.00029
	τ	0.66		0.62	
	ϕ	0.77		0.65	
CY14	τ (M=4)	0.40		0.39	
	ϕ (M=4)	0.66		0.63	
Bindi et al. (2017)	τ	0.50			
	ϕ	0.64			
Czech Partial Non-ergodic	ϕ_{S2S}	0.50		0.41	
	ϕ_{SS}	0.52		0.50	
Czech Non-ergodic	ϕ_{P2P}	0.40		0.37	
	$\phi_{S2S,reg}$	0.273		0.228	
	$\phi_{S2s,uncor}$	0.269		0.233	
	ϕ_{SP}	0.33		0.25	

Table 3 Hyperparameters for PGA and PGV.

Parameter	PGA		PGV	
	Variance	Correlation Length (km)	Variance	Correlation Length (km)
$\delta S2S_{Reg}$	0.052	50	0.036	48
$\delta P2P_V$	0.125	35	0.332	35

Table 4 Latitude, longitude, $\delta S2S_{cor}$, and $\delta S2S_{uncor}$ for three sites.

	Temelin	Dukovany	Prague
Longitude	14.37622	16.14888	14.42138
Latitude	49.18013	49.085	50.0875
$\delta S2S_{cor}$	0.103	-0.041	-0.320
$\delta S2S_{uncor}$	0	0	0

Table 5 Maximum magnitudes for the areal source zones.

Id	M_{MAX}^{OBS}	Δ ($\sigma_M \pm 0.5$)	M_{MAX} (Eq (17))	M_{MAX} (Faults)	M_{MAX} (Model A3)
300	6.26	0.84	7.1	6.95 6.13 6.5	7.1
301	5.73	0.84	6.57	6.53	6.6
302	6.3	0.84	7.14	7.12 7.15	7.2
303	5.7	0.84	6.54	6.6 6.22 6.13	6.6
304	5.6	0.75	6.35	6.68	6.7
305	5.85	0.75	6.6	6.42	6.6
306	5.14	0.81	5.95		6.0
307	4.49	0.75	5.24		5.2
308	4.5	0.75	5.25		5.3
309	5.75	0.75	6.5	6.74	6.7
310D	3.57	0.75	4.32		4.9*
311D	2.89	0.75	3.64		4.9*
312	4.59	0.75	5.34		5.3
313	4.93	0.75	5.68		5.7
314	5.65	0.81	6.46	6.64	6.6
315	4.89	0.75	5.64		5.6
316D	3.95	0.75	4.7		4.9*
317	4.45	0.75	5.2		5.2
318D	3.23	0.75	3.98		4.9*
319D	3.42	0.75	4.17		4.9*
320	4.87	0.75	5.62	5.73	5.7
321	3.78	0.75	4.53		4.9*
322D	3.19	0.75	3.94		4.9*
323	5.28	0.75	6.03		6.0
324	4.87	0.75	5.62	6.5	6.5
324	4.87	0.75	5.62	6.5	6.5
325	4.25	0.75	5		5.0

Table 6 Maximum magnitudes for identified capable faults.

F_Id	A_Id	Fault Zone Name	M_{MAX} (F3)	Procedure
[1]	324	Marianske Lazne Fault Zone	6.50	Geol. Record
[2]	320	Hronov-Porici Fault Zone	5.73	LEO.SCR
[3]	309	Tullnerbach Fault	6.74	LEO.SCR
[4]	314	Inntal Fault System	6.64	LEO.SS
[5]	300	Hochstuhl Fault Zone	6.95	LEO.SS
[6]	300	Trogscharten - Gurk Fault Zone	6.13	LEO.SS
[7]	300	Pöls-Lavanttal Fault System	6.50	LEO.SS
[8]	301	Mur-Murzf Fault System	6.53	LEO.SS
[9a]	302	Markgrafneusiedl Fault	7.12	Geol. Record
[9b]	302	Wiener-Becken Fault System		
		Lasse Fault	7.15	LEO.SS
[10]	303	Plavec-Dobra Voda Fault System	6.60	LEO.DS
[11]	303	Klippen Belt Fault System	6.22	LEO.DS
[12]	303	Podcerwone-Mszana Dolna Fault System	6.13	LEO.SS
[13]	303	Stara Ves Fault	6.04	LEO.DS
[14]	303	Poprad Fault	5.91	LEO.SCR
[15]	305	Kremnica Fault Zone	6.42	LEO.SCR
[16]	304	Komarno Zone	6.68	LEO.SS

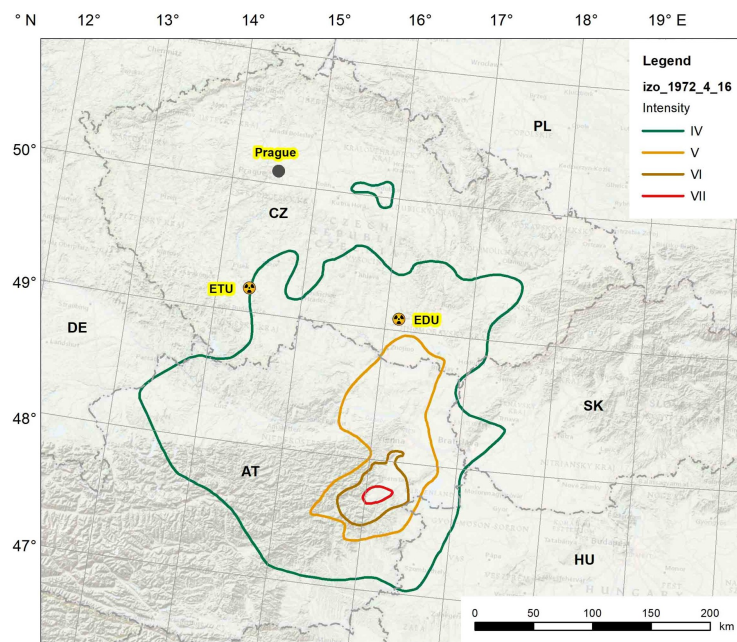


Fig. 1 Isoseismal map of the earthquake of April 16, 1972 with an epicenter near Seebenstein/Pitten. The map shows the extension of the macroseismic field from the Alpine source area deep into the Bohemian Massif. The elongation of the isoseismals to the north indicates strong path effects. (from Málek et al., 2022)

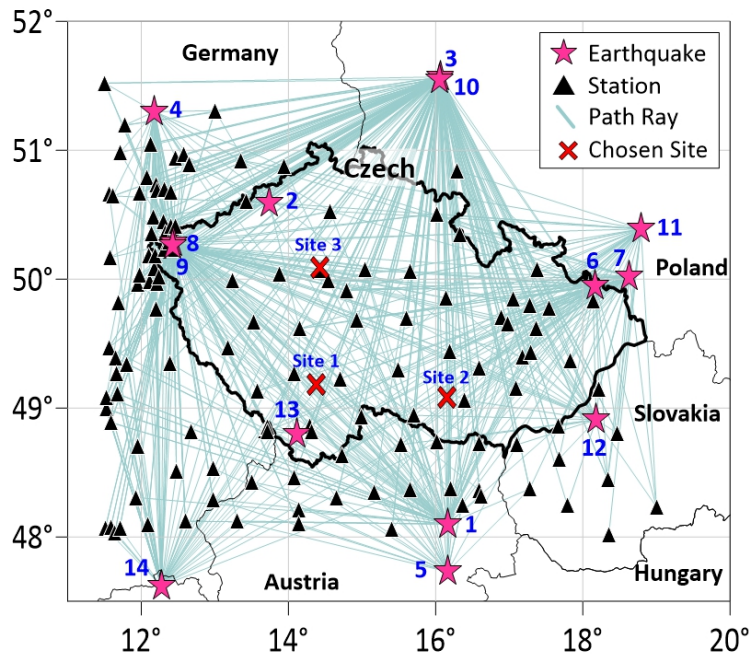


Fig. 2 Map showing the locations of strong-motion stations and crustal earthquakes in the Czech Republic that were utilized for this study. The stars denote the epicenters of the earthquakes, whereas the triangles indicate the positions of the strong-motion stations. Numbers plotted next to the earthquake symbols represent their event IDs. The lines illustrate the path ray included in the data set. The cross symbols show the locations of target sites used for the PSHA example.

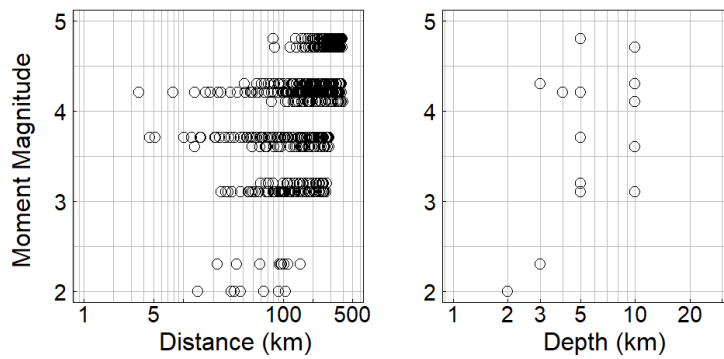


Fig. 3 Distribution of earthquake magnitudes versus rupture distance and earthquake magnitude versus hypocentral depth for the selected subset ground motions for the Czech Republic.

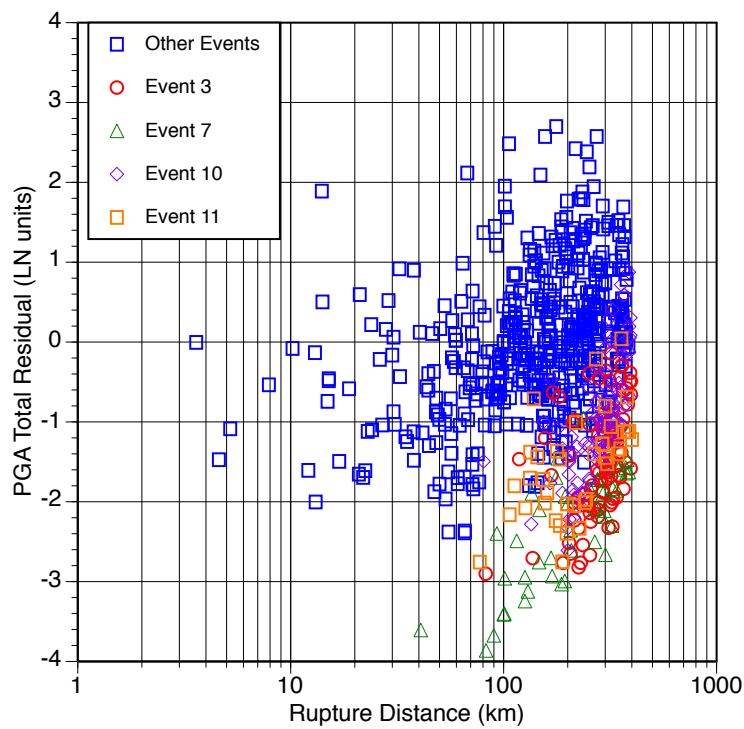


Fig. 4 Total Residuals for PGA relative to the CY14 GMM. Four events (3, 7, 10, and 11) have large negative residuals compared to the other events at the same distance. For distances greater than 50 km, the residuals increase with distance.

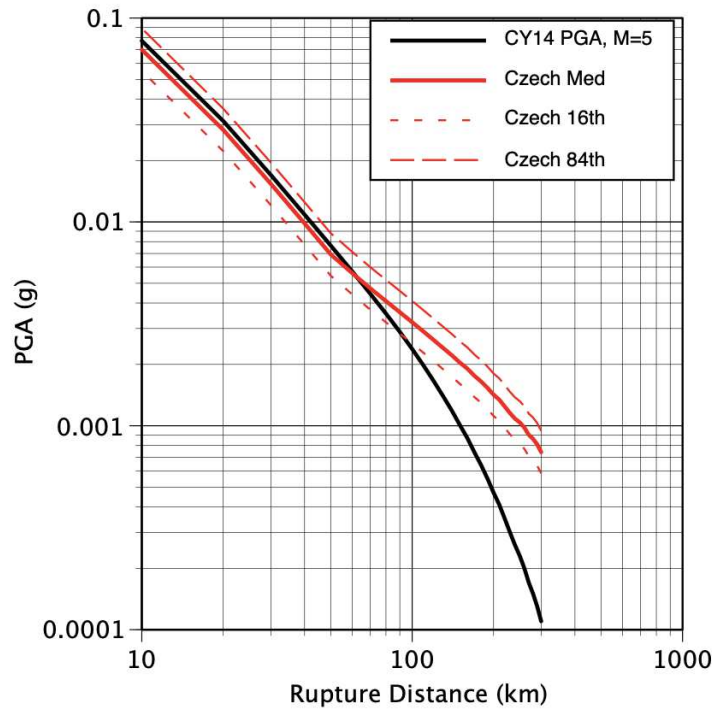


Fig. 5 Comparison of the distance scaling of PGA for $M=5$ events for the CY14 GMM and the GMM adjusted to the distance scaling for the Czech Republic region which has weaker attenuation with distance.

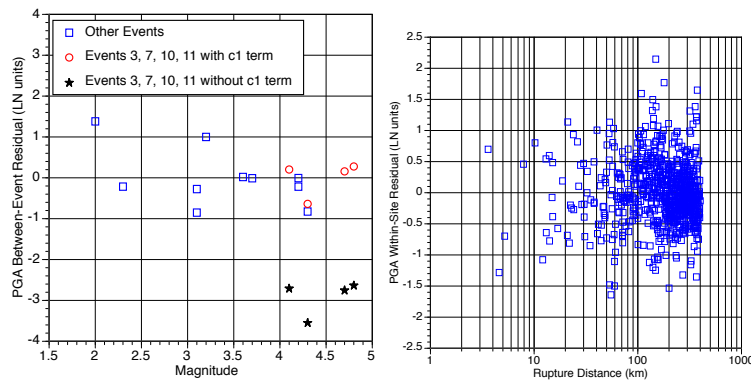


Fig. 6 The left frame shows the between-event residuals for the PGA after the modification to the CY14 GMM. Events 3, 7, 10, and 11 (red circles) have large negative event terms of about a factor of 10. The stars show the event terms for these four events with a factor of 10 increase applied. The right frame shows the PGA within-event residuals for the adjusted CY14 GMM. The inclusion of the Czech region distance term removed the distance trend seen in the total residuals in Figure 4.

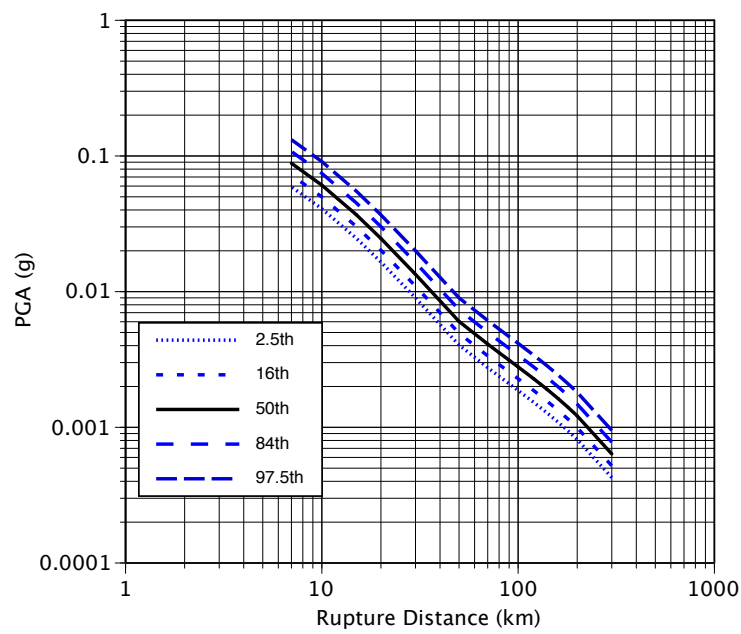


Fig. 7 Scaled backbone GMM with five alternative branches to model the epistemic uncertainty of the ergodic GMM for PGA for the Czech Region.

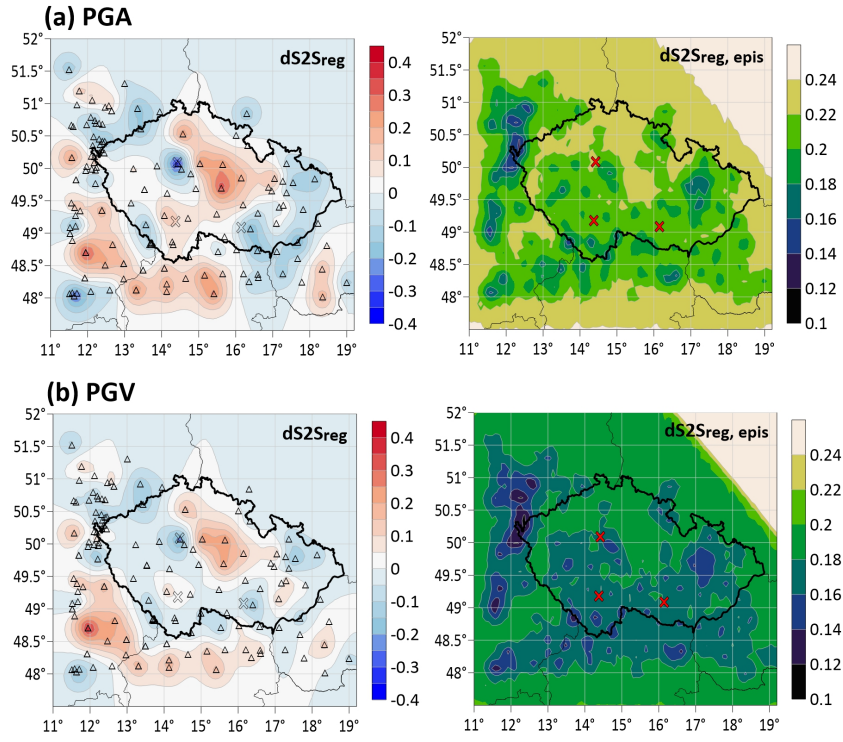


Fig. 8 Maps of the spatially correlated site term ($\delta S2S_{reg}$) and the epistemic uncertainty for (a) PGA and (b) PGV. The left frames show the median values of the $\delta S2S_{reg}$ and the right frames show the epistemic uncertainty of the $\delta S2S_{reg}$.

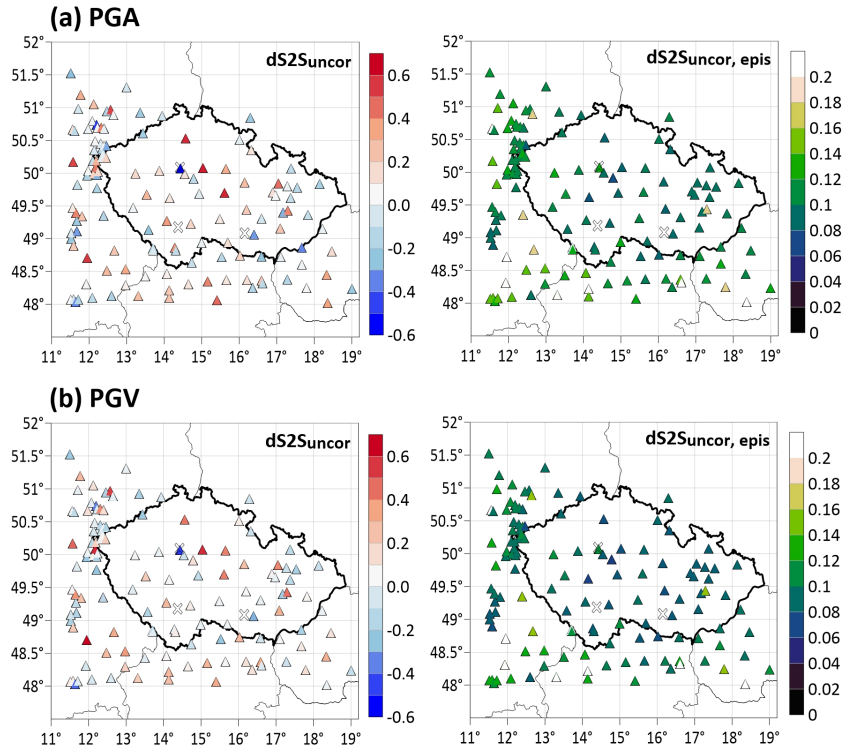


Fig. 9 Maps of the site-specific site term ($\delta S2S_{uncor}$) and the epistemic uncertainty for (a) PGA and (b) PGV. The left frames show the median values, and the right frames show the epistemic uncertainty.

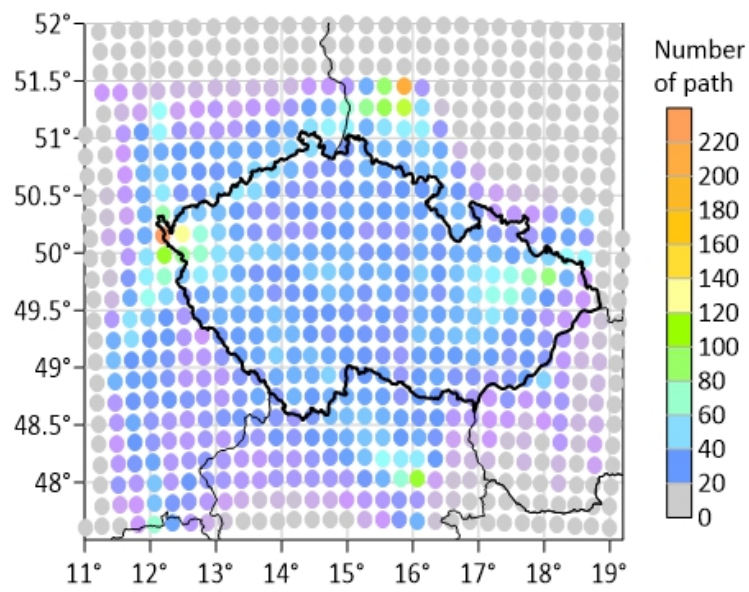


Fig. 10 Map of the number of rays per cell based on a cell size of 20 km x 20 km.

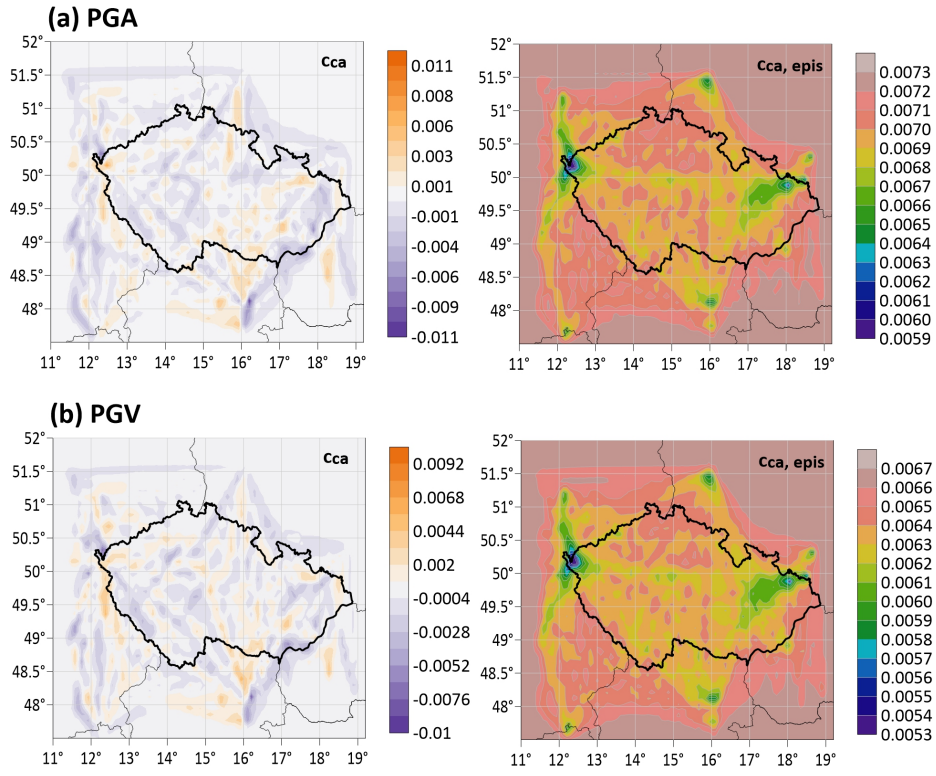


Fig. 11 Mean values of the posterior distribution of the non-ergodic anelastic attenuation (c_{ca}) and the epistemic uncertainty of c_{ca} per cell for (a) PGA and (b) PGV.

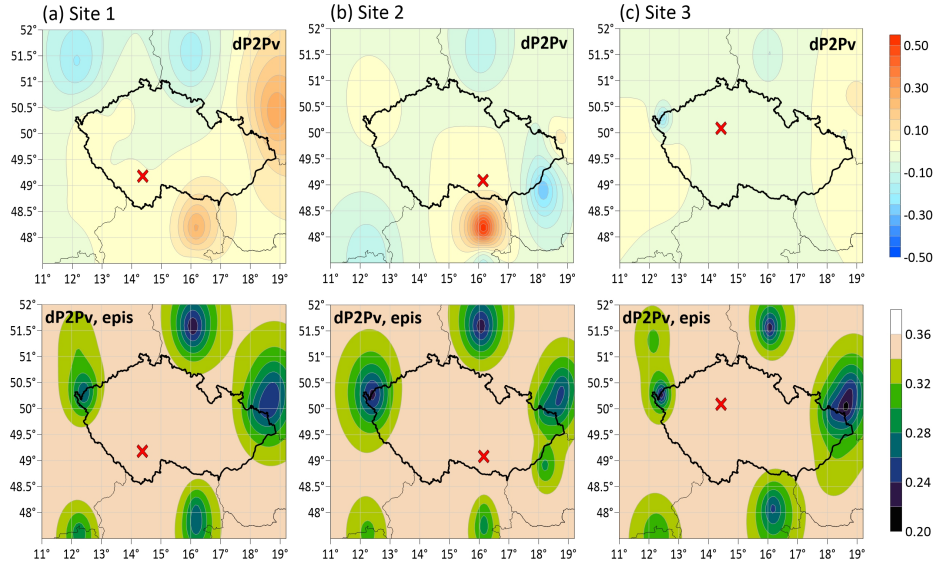


Fig. 12 Non-ergodic path term ($\delta P2P_V$) and epistemic uncertainty at PGA for (a) site 1, (b) site 2, and (c) site 3.

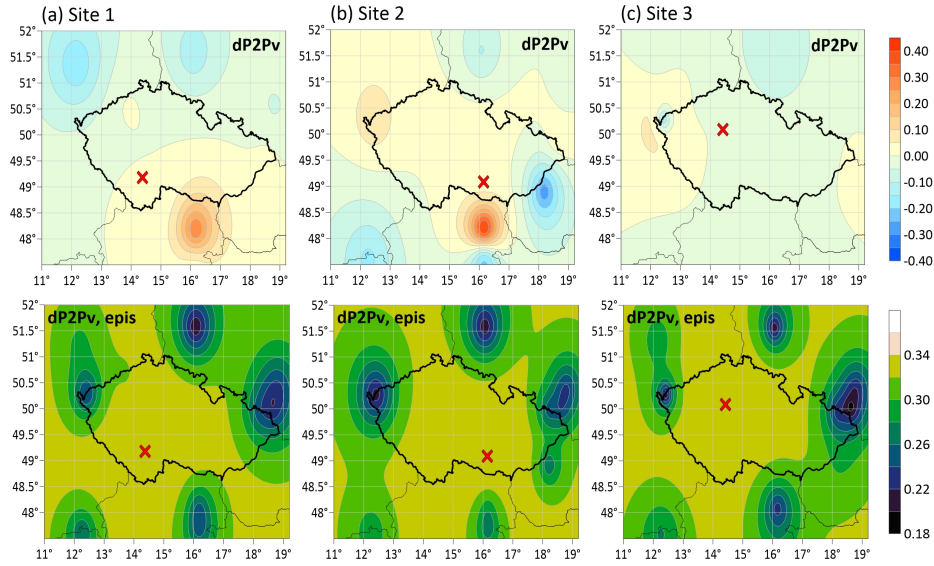


Fig. 13 Non-ergodic path term ($\delta P2P_V$) and epistemic uncertainty at PGV for (a) site 1, (b) site 2, and (c) site 3.

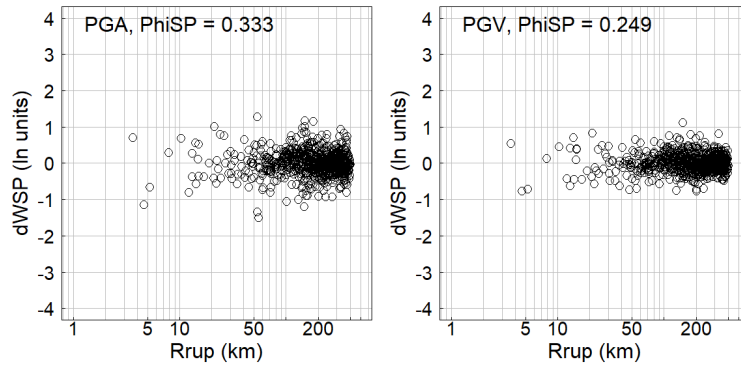


Fig. 14 Within-path residuals of the non-ergodic GMM for PGA and PGV.

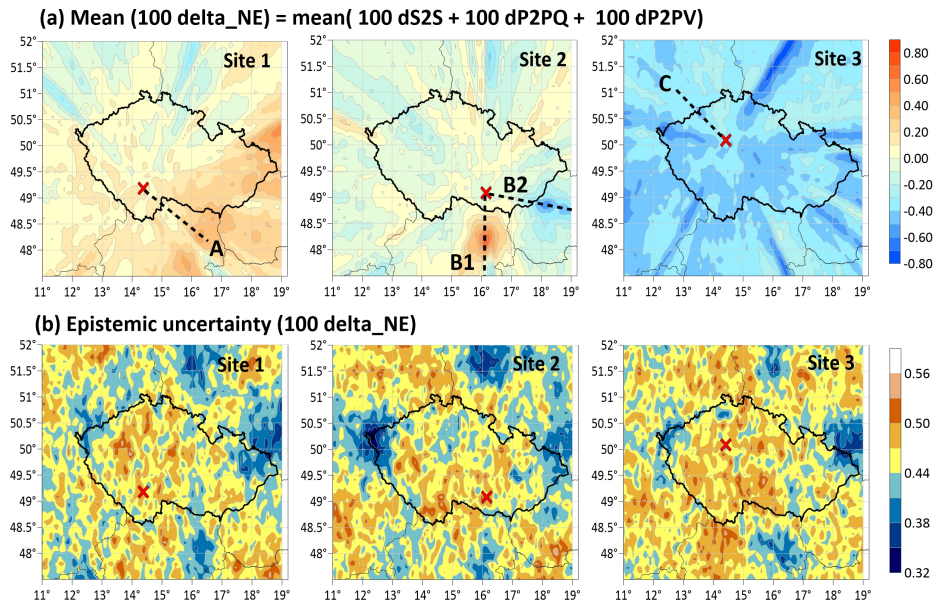


Fig. 15 Mean and epistemic uncertainty of PGA for 100 Δ_{NE} per grid for (a) Site 1, (b) Site 2, and (c) Site 3.

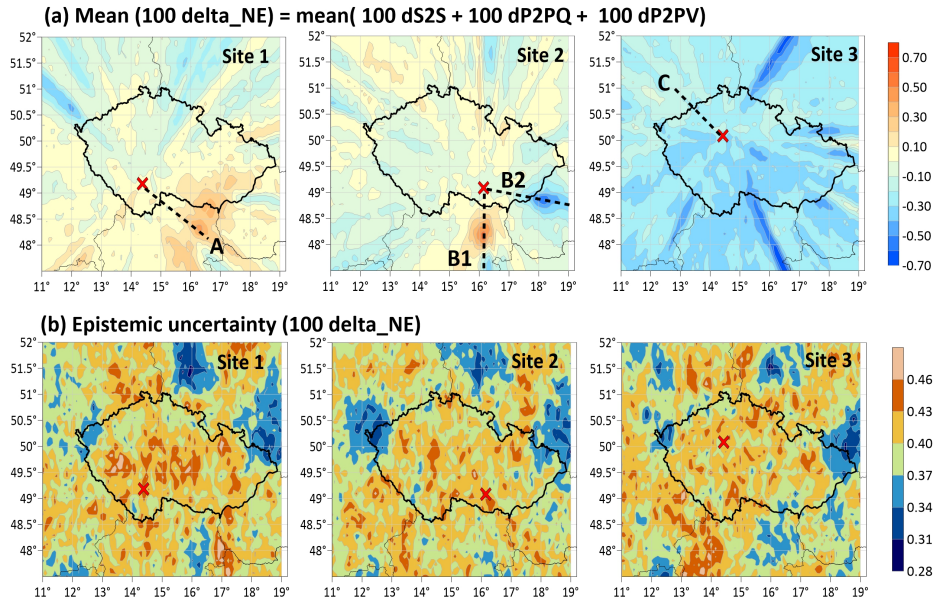


Fig. 16 Mean and epistemic uncertainty of PGV for $100 \Delta_{NE}$ per grid for (a) Site 1, (b) Site 2, and (c) Site 3.

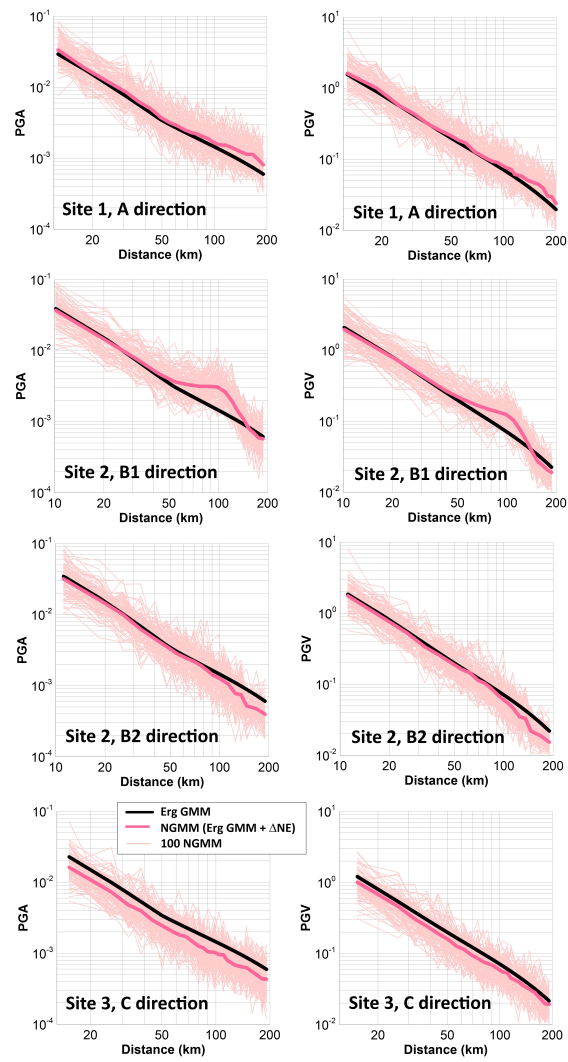


Fig. 17 PGA and PGV ergodic and non-ergodic GMM for four directions at three target sites.

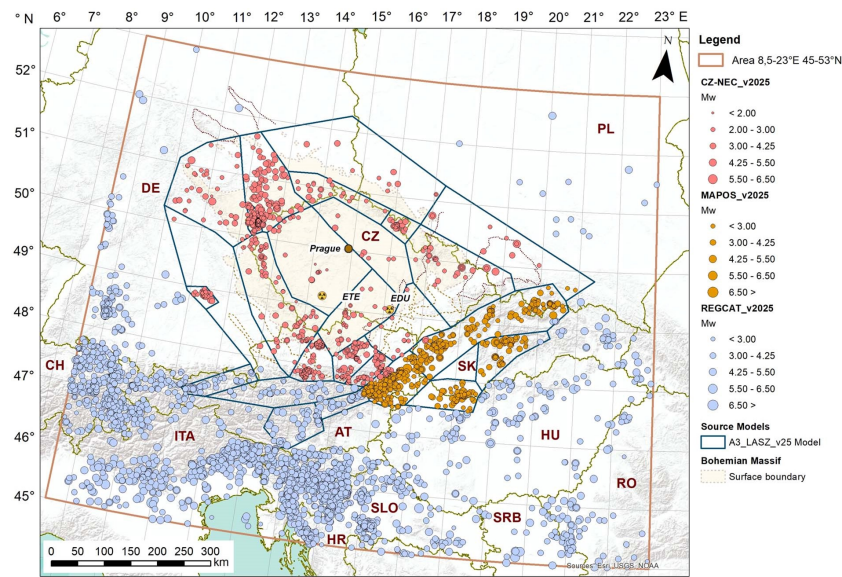


Fig. 18 Seismicity based on the earthquake catalogs CZ-NEC.v25, MAPOS.v25 and REG-CAT.v25 in the mapped area 8.5-23°E and 45-53°N. The areal source zones from A3_LASZ.v25 used in the PSHA are shown by the polygons.

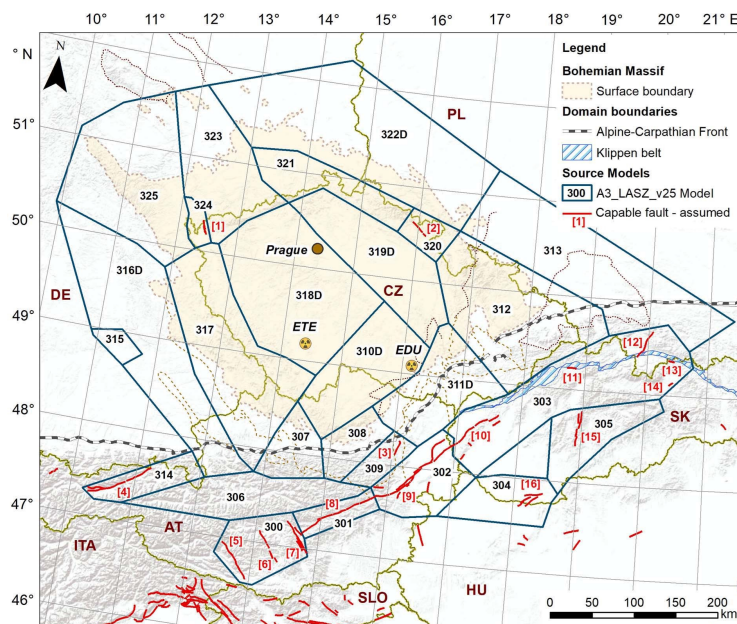


Fig. 19 Map of areal seismic sources (A3_LASZ.v25 model) with the known capable faults (red lines). The locations of the Temelín and Dukovany nuclear power plants and the capital of the Czech Republic, Prague. The numbers indicate the areal source zones listed in Table 5. The red numbers in square brackets indicate the assumed, or verified, capable faults listed in Table 6.

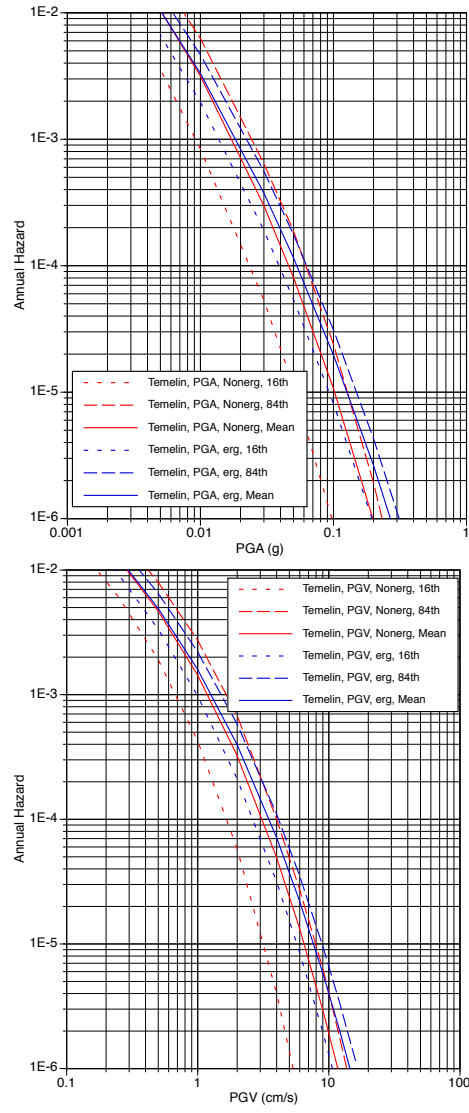


Fig. 20 Comparison of the mean hazard and 16th and 84th fractiles of the hazard for the ergodic GMM and the non-ergodic GMM for the Temelin site. (a) PGA (b) PGV. The epistemic uncertainty only includes the uncertainty in the GMMs.

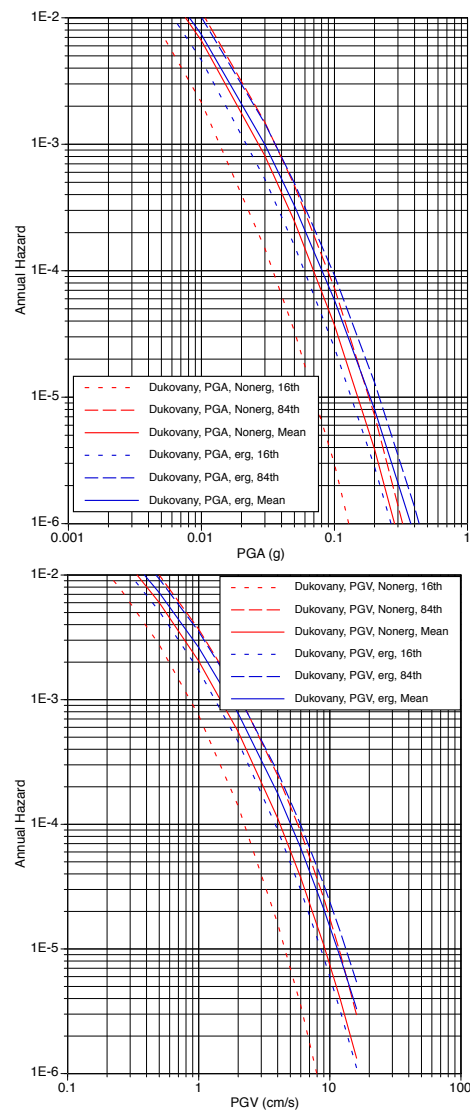


Fig. 21 Comparison of the mean hazard and 16th and 84th fractiles of the hazard for the ergodic GMM and the non-ergodic GMM for the Dukovany site. (a) PGA (b) PGV. The epistemic uncertainty only includes the uncertainty in the GMMs.

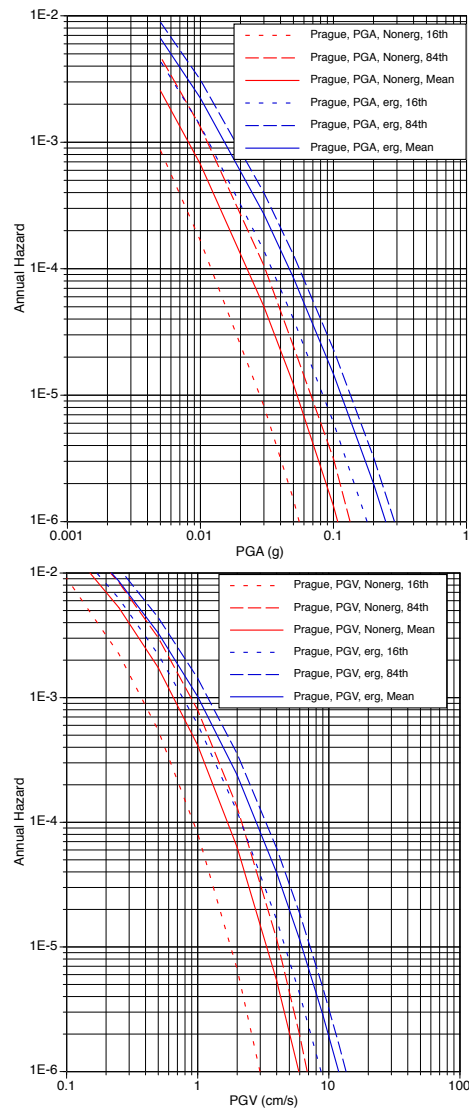


Fig. 22 Comparison of the mean hazard and 16th and 84th fractiles of the hazard for the ergodic GMM and the non-ergodic GMM for the Prague site. (a) PGA (b) PGV. The epistemic uncertainty only includes the uncertainty in the GMMs.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [CzechPGAPGVdata.csv](#)