

Multi-Objective Thermo-Mechanical Topology Optimization of Lightweight Structures for Structural Stiffness, Thermal Performance, and Dynamic Characteristics

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Research Article

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Posted Date: October 29th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10727354/v1>

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Additional Declarations: No competing interests reported.

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Abstract

This study presents a multi-objective topology optimization framework for lightweight thermo-mechanical structures subjected to simultaneous mechanical, thermal, and dynamic requirements. A two-dimensional simply supported structure is modeled using a structured quadrilateral finite element mesh comprising 4,800 Q4 elements and 4,961 nodes. The material distribution is parameterized using the Solid Isotropic Material with Penalization (SIMP) approach with a penalization factor of 3.0 and a prescribed volume fraction of 40%. Three coupled performance criteria are considered: structural compliance, total heat transfer, and fundamental natural frequency. Thermal, structural, and modal finite element analyses are sequentially performed for each candidate topology within a multi-objective evolutionary optimization procedure. The optimized configuration preserves the prescribed material volume and total mass of 31.20 kg while redistributing material toward mechanically and dynamically efficient regions. The results show a reduction in structural compliance from 8.70646×10^{-2} to 8.65559×10^{-2} , corresponding to a 0.584% improvement. The fundamental natural frequency increases from 145.63 to 148.24 Hz, representing a 1.793% improvement, while the second and third natural frequencies reach 295.03 and 555.69 Hz, respectively. Total heat transfer changes only marginally from 43.2021 to 43.2368 W, corresponding to a 0.0804% increase. The final optimized structure exhibits a maximum displacement of 4.6663×10^{-4} m, maximum von Mises stress of 1.0372×10^5 Pa, mean von Mises stress of 1.3848×10^4 Pa, and maximum heat flux of 421.08 W/m². The results demonstrate that coupled topology optimization can improve structural stiffness and dynamic performance while maintaining a fixed material budget and nearly unchanged thermal behavior. The proposed framework provides a computationally efficient basis for the design of lightweight structures requiring simultaneous mechanical, thermal, and vibration performance.

Keywords: Topology optimization; multi-objective optimization; Thermo-mechanical analysis; Structural compliance; Natural frequency; Heat transfer; Finite element method; SIMP; Lightweight structures; Modal analysis; Thermal–structural coupling; Material distribution

1. Introduction

1.1. Research Background and Engineering Context

The increasing demand for lightweight, efficient, and multifunctional engineering components has created a strong need for advanced computational design methodologies capable of satisfying several interacting performance requirements simultaneously. Structural components used in aerospace, automotive, mechanical, energy, and other engineering systems are increasingly expected to provide high stiffness and load-bearing capability while maintaining low weight and acceptable thermal and dynamic performance. Conventional design strategies generally modify predefined dimensions, shapes, or material quantities and therefore have limited ability to determine the most efficient spatial distribution of material within a prescribed design domain. For multifunctional structures, simply removing material may reduce weight but can simultaneously compromise stiffness, thermal behavior, stress resistance, or vibration characteristics. Topology optimization provides a more flexible alternative because it allows substantial changes in material layout and can generate unconventional structural configurations that are difficult to obtain through conventional size or shape optimization [1,2]. The fundamental engineering motivation is therefore not necessarily to minimize the amount of material, but to use a prescribed amount of material more efficiently. Through spatial redistribution, material can be concentrated in regions that contribute most strongly to structural stiffness, thermal transport, or dynamic performance while being removed from regions with lower functional importance. This concept is particularly attractive for lightweight and multifunctional structures, where the material budget may already be constrained and further mass reduction may be undesirable. Recent developments in topology optimization have demonstrated its applicability to lightweight structures, vibration-sensitive components, and multiphysics engineering systems, highlighting its potential for high-performance design across a wide range of applications [1–4].

1.2. Topology Optimization and Structural-Mechanical Considerations

Topology Optimization is a computational design method that finds optimal material distribution from a given design domain subject to physical constraints, boundary conditions, and performance specifications. Topology optimization enables more flexibility than classical size optimization (which varies selected dimensions) or shape optimization (which specifies how boundaries may change, whereas topology indicates where in-plane areas will be filled with material), as it can add or remove regions of material and therefore effectively change the structural load path. Recent surveys have underscored how topology optimization has developed into a general framework for structural, dynamic, thermal, acoustic, and multiphysics design problems [(1,2]. Among density-based approaches, the Solid Isotropic Material with Penalization (SIMP) method remains one of the most widely used formulations because of its relatively simple mathematical structure and direct compatibility with finite element analysis. In SIMP-based formulations, each finite element is associated with a relative density design variable, and the effective stiffness or other material properties are interpolated as functions of this density. Penalization promotes nearly solid or void regions and suppresses extensive intermediate-density zones, thereby producing a practical material distribution. Recent developments have continued to employ SIMP formulations for structural and multiphysics topology optimization, including efficient finite-element implementations and manufacturing-aware formulations [3,5,6]. Structural performance is traditionally represented using quantities such as compliance, stiffness, displacement, stress, strain energy, or load-bearing capacity. Compliance minimization is particularly common because minimizing compliance under a prescribed material volume generally corresponds to maximizing structural stiffness. However, a topology optimized exclusively for compliance does not necessarily provide the best solution when the component is subjected to multiple physical requirements. For example, a material arrangement that maximizes static stiffness may produce unfavorable thermal conduction paths or alter the stiffness-to-mass distribution in a manner that adversely affects natural frequencies. Recent research on multi-objective topology optimization has therefore emphasized the importance of considering multiple competing structural responses rather than relying exclusively on a single mechanical objective [7,8].

1.3. Thermal, Thermo-Mechanical, and Dynamic Considerations

Thermal behavior introduces an additional design dimension because material topology directly influences heat-transfer paths and temperature distributions. When a structure operates under non-uniform temperature fields, thermal expansion can generate additional deformation and stress, while temperature-dependent material properties may alter the mechanical stiffness of the structure. Consequently, the mechanical and thermal fields cannot always be considered independently. Recent thermoelastic topology optimization studies have demonstrated that thermal expansion, temperature-dependent properties, stress constraints, and temperature constraints can significantly modify the resulting optimal topology [9–11]. Meng et al. [9] developed a thermoelastic topology optimization framework incorporating stress and temperature constraints and demonstrated that the optimal material distribution can be sensitive to changes in temperature fields. In a subsequent study, Meng et al. [10] considered stiffness, strength, and temperature constraints over a wide temperature range, demonstrating the importance of treating mechanical and thermal requirements simultaneously. Similarly, Chen et al. [11] investigated multi-material topology optimization with temperature-dependent thermoelastic properties and showed that temperature-dependent elastic and thermal-expansion properties can substantially influence the optimized material distribution. These studies demonstrate that thermo-mechanical optimization provides a more realistic representation of thermally loaded structures than purely mechanical topology optimization. Dynamic behavior represents another important consideration for lightweight engineering structures. Natural frequencies are governed by the relationship between structural stiffness and mass distribution; consequently, topology modification can substantially change the modal characteristics of a structure. A topology that minimizes compliance may not necessarily maximize the fundamental natural frequency, while a topology developed primarily for frequency maximization may generate different structural load paths. Reviews of vibration-oriented topology optimization have identified natural-frequency optimization as an important branch of structural topology optimization and have demonstrated the continuing use of density-based approaches for vibration-sensitive structures [2]. Recent investigations have specifically incorporated natural frequency into topology optimization. Liu et al. [12] formulated topology optimization of shell-infill structures using natural frequency as a design criterion and considered both frequency maximization under a mass constraint and mass minimization subject to a frequency requirement. Kennedy and Fu [13] further demonstrated the numerical challenges associated with natural-frequency constraints because modal optimization requires repeated eigenvalue analyses and may introduce complex and multimodal design spaces. Shah et al. [14] investigated natural-frequency constraints within multi-

material topology optimization, while Kim et al. [15] combined topology and shape optimization to improve the natural frequency of a structural wheel. These studies confirm that dynamic characteristics can be effectively controlled through material redistribution, but they also indicate that modal optimization introduces additional computational requirements. The simultaneous consideration of thermal and dynamic behavior is therefore particularly important for multifunctional structures. Material redistribution affects stiffness, mass, and thermal conduction simultaneously, meaning that improvement in one physical response can potentially produce an undesirable change in another. Recent work has begun to combine thermal and frequency objectives. Chen et al. [16], for example, developed a multi-objective topology optimization approach for structures containing multiple microstructures, explicitly considering thermal compliance and natural frequency. Such developments indicate a growing interest in multiphysics topology optimization, while also highlighting the need for computationally accessible frameworks capable of treating several physical objectives within a common design process.

1.4. Multi-Objective FEM-Based Optimization and Research Gap

The competing nature of structural, thermal, and dynamic requirements naturally leads to a multi-objective topology optimization problem. In the present study, the principal design requirements are formulated to minimize structural compliance, control total heat transfer, and maximize the fundamental natural frequency, while maintaining a prescribed material volume fraction. These objectives may conflict with one another. Material redistribution that increases structural stiffness may alter the thermal conduction network, while a topology that improves heat transfer may not provide the most favorable stiffness-to-mass distribution for vibration performance. Therefore, independent optimization of each physical response may produce designs that are optimal for an individual objective but inefficient when the complete multiphysics behavior is considered. Multi-objective evolutionary algorithms are attractive for such problems because they can explore multiple competing solutions and provide a set of non-dominated or Pareto-optimal designs rather than forcing the problem into a single objective. Recent topology optimization studies have employed adaptive weighted-sum approaches, evolutionary algorithms, and NSGA-II-based formulations to investigate competing structural and physical objectives [7,8,17]. In particular, Schleifer and Deese [17] demonstrated an NSGA-II-based topology optimization framework capable of generating dense sets of near-optimal designs under multiple objectives and constraints derived from different physical models. Similarly, NSGA-II has been applied to multi-objective topology optimization problems involving thermal performance and material cost, where population-based search provides an alternative to gradient-based optimization for complex multi-objective design spaces [18]. The finite element method (FEM) provides the numerical foundation for evaluating candidate topologies during such an optimization process. Structural FEM can be used to calculate displacement, stress, strain energy, and compliance; thermal FEM provides temperature, heat flux, and heat-transfer characteristics; and modal FEM determines eigenvalues and natural frequencies. Integrating these analyses into a common optimization cycle enables the effect of each material distribution to be evaluated consistently. However, the computational cost can become significant because a population-based optimizer may require hundreds or thousands of candidate evaluations, with each candidate requiring structural, thermal, and eigenvalue analyses. Computational cost is therefore a recognized challenge in topology optimization, particularly when repeated finite element simulations are required [6,19]. Despite substantial progress in structural, thermal, dynamic, and thermo-mechanical topology optimization, an important research gap remains. A considerable portion of the literature continues to focus on individual objectives, particularly structural compliance or stiffness. Other studies incorporate thermo-mechanical coupling through stress, temperature, or stiffness constraints but do not explicitly consider modal performance [9–11]. Conversely, natural-frequency topology optimization has generally been investigated as a structural-dynamic problem without simultaneously incorporating thermal behavior [12–15]. Although recent studies have begun to combine thermal and frequency objectives [16], relatively limited work has focused on a transparent finite-element framework that simultaneously evaluates structural compliance, thermal heat transfer, and fundamental natural frequency under a fixed material budget. A further issue concerns computational accessibility. Advanced multiphysics topology optimization approaches may employ level-set formulations, isogeometric analysis, multiphase interpolation, meshless methods, adaptive discretization, or specialized thermal-fluid solvers. Such methods can provide important capabilities, but they may also increase implementation complexity and computational cost. The development of a relatively straightforward Q4 finite-element/SIMP framework therefore remains valuable, particularly when the objective is to provide a reproducible computational benchmark in which structural, thermal, and modal responses can be evaluated directly within the same optimization process.

1.5. Novelty, Objectives, and Contributions of the Present Study

The present study addresses this question by developing an integrated thermo-mechanical-modal topology optimization framework based on finite element analysis, SIMP material interpolation, and NSGA-II-based multi-objective optimization. The proposed framework uses Q4 finite elements for the design domain and integrates structural, thermal, and modal analyses within a common optimization procedure. A prescribed volume fraction of 40% is maintained throughout the optimization so that performance changes arise primarily from spatial material redistribution rather than changes in the total material quantity.

The principal contributions of this work are as follows:

1. **Integrated thermo-mechanical-modal topology optimization:** Structural compliance, thermal heat transfer, and fundamental natural frequency are treated as competing design objectives within a unified optimization framework.
2. **Fixed-material-budget formulation:** A 40% volume fraction is maintained throughout the optimization, providing a controlled basis for evaluating the effectiveness of material redistribution.
3. **Q4-FEM-based multiphysics analysis:** Structural, thermal, and modal finite element analyses are integrated into the optimization cycle to evaluate the response of each candidate topology.
4. **SIMP-based material interpolation:** Relative element density is used as the principal design variable, with penalized material interpolation employed to obtain a meaningful solid–void topology.
5. **NSGA-II-based multi-objective search:** The evolutionary optimization procedure is used to investigate the competing objectives and identify non-dominated candidate designs. This is consistent with recent applications of NSGA-II to multi-objective topology optimization [17,18].
6. **Comprehensive performance evaluation:** Initial and optimized configurations are compared using structural, thermal, dynamic, and material-related quantities, including compliance, displacement, von Mises stress, temperature, heat flux, mass, volume fraction, and multiple natural frequencies.
7. **Quantification of material redistribution efficiency:** The study evaluates whether a fixed material quantity can provide improved structural and dynamic performance without producing a substantial deterioration in thermal behavior.

The specific objectives of the study are therefore to: (i) develop a coupled FEM-based structural–thermal–modal topology optimization framework; (ii) formulate the competing structural, thermal, and dynamic objectives within a multi-objective optimization problem; (iii) investigate spatial material redistribution under a prescribed 40% volume fraction; (iv) identify non-dominated/Pareto-optimal topologies using NSGA-II; (v) evaluate the resulting structural, thermal, and modal characteristics; and (vi) quantify the changes between the initial and optimized configurations.

2. Literature Review

2.1. Structural and Dynamic Topology Optimization

Topology optimization has evolved substantially from its traditional use in compliance minimization and minimum-volume structural design toward formulations that simultaneously consider stiffness, stress, natural frequency, vibration, and other dynamic performance requirements. This development is motivated by the increasing demand for lightweight structures that maintain adequate mechanical performance while satisfying dynamic constraints and avoiding undesirable vibration or resonance. Natural-frequency optimization is particularly important because changes in topology modify both the structural stiffness and mass distributions. Consequently, material redistribution can significantly alter the eigenvalue characteristics of a structure even when the total material volume remains unchanged. Liu et al. [20] investigated topology optimization of shell-infill structures by incorporating natural frequency as an explicit design criterion. Their formulations considered both frequency maximization under a prescribed mass constraint and mass minimization subject to a required frequency. Their results demonstrated that natural frequency can be effectively controlled through material redistribution, although mode switching introduces additional numerical difficulties. Kennedy and Fu [21] investigated topology optimization involving natural-frequency constraints using a spectral aggregation strategy. Their study highlighted the computational challenges associated with eigenvalue-based optimization, particularly because repeated modal analyses are required throughout the optimization process. Shah et al. [22] extended frequency-constrained topology optimization to multi-material structures and demonstrated that both material selection and spatial distribution can significantly influence the modal characteristics of the resulting design.

Evolutionary optimization methods provide an alternative strategy for dynamic topology optimization. Schleifer and Deese [23] developed an evolutionary multi-objective and multiphysics topology optimization framework based on NSGA-II. Their work demonstrated that population-based optimization can explore competing objectives simultaneously and identify multiple non-dominated structural configurations. Similarly, Feng et al. [24] applied NSGA-II to topology optimization involving competing stiffness and stress requirements, demonstrating the applicability of evolutionary optimization to nonlinear design spaces. Collectively, these studies establish three important observations. First, topology optimization can effectively control natural frequencies through spatial redistribution of material. Second, dynamic objectives may conflict with conventional stiffness objectives because a topology that maximizes static stiffness is not necessarily optimal for natural-frequency performance. Third, the inclusion of eigenvalue calculations increases the computational requirements of topology optimization compared with conventional static formulations. Despite these advances, much of the structural and dynamic topology optimization literature remains primarily focused on mechanical behavior. Thermal fields and heat-transfer characteristics are commonly excluded or analyzed separately. This represents an important limitation for components subjected simultaneously to mechanical, thermal, and vibrational loading, because thermal requirements can influence the preferred material distribution and consequently modify the dynamic response.

2.2. Thermal and Thermo-Mechanical Topology Optimization

Thermal topology optimization has been evolving quickly because the material distribution is directly related to heat-conduction routes, thermal insulation properties, temperature fields, and heat-flux characteristics. Subsequent studies, however, have moved from purely thermal formulations to thermo-mechanical optimizations, in which the thermal and structural (mechanical) responses are assessed in a coupled design context. Meng et al. The work of Zhang et al. [25] proposed a thermo-elastic topology optimization framework while considering constraints on the stress and temperature. They showed that the ideal topology can be significantly affected by thermal expansion and temperature constraints. Such a discovery suggests that a design which are optimal for only mechanical stiffness may be unreasonable in terms of thermal deformation and thermal stresses. Meng et al. Following this, [26] considers thermo-elastic structures subject to stiffness, strength and temperature constraints over a wide range of temperatures. The results obtained showed that the ideal material distribution is quite sensitive to interactions between mechanical and thermal requirements. As a result, thermo-mechanical topology optimization can yield designs that are significantly different from purely structural optimization. Chen et al. [27] considered multi-material topology optimization using temperature dependent thermoelastic material properties. Results showed that these material property variations due to temperature can impact both structural performance and the volume/mass fraction of material used. This indicates that the thermal field should not be considered merely a second-order effect, but could play a direct role in affecting the mechanical efficacy of the optimized topology. Deng and Suresh [28] proposed a theoretical analysis on stress-constrained thermoelastic topology optimization under different thermal environment conditions. Their results also showed that thermoelastic coupling adds further constraints and computational difficulties compared with standard elastic topology optimization techniques. Hence the thermal and mechanical response of thermally loaded structures is firmly coupled according to the literature. Material redistribution changes the heat-conduction paths, and temperature fluctuations can lead to thermal expansion, deformation, and stress. Such interactions offer a natural foundation to combine thermal and structural analyses within an integrated topology optimization formulation. Still, the vast majority of thermo-mechanical topology optimization works still mainly focus on compliance, stress, temperature or heat-transfer qualities without treating dynamic behavior separately. Therefore, it can be seen that even though mechanical–thermal interaction has attracted extensive research efforts till date, the thermal topology-induced simultaneous effect on natural-frequency features is still relatively under-explored.

2.3. Multi-Objective and Multiphysics Topology Optimization

The simultaneous consideration of structural, thermal, and dynamic requirements naturally leads to a multi-objective optimization problem. In such formulations, improvements in one physical response may conflict with improvements in another. For example, increasing structural stiffness may modify heat-conduction paths, whereas improving heat transfer may produce a mass–stiffness distribution that is unfavorable for natural-frequency performance. Multi-objective topology optimization has therefore become an important research direction. Ryu et al. [29] proposed a multi-objective topology optimization approach using an adaptive weighted-sum formulation combined with a

configuration-based clustering strategy. Their results demonstrated that different combinations of objectives can produce substantially different topologies and that maintaining solution diversity is important for identifying suitable design compromises. Feng et al. [24] employed NSGA-II to investigate competing stiffness and stress requirements. Their results demonstrated the suitability of evolutionary multi-objective algorithms for topology optimization problems in which the relationship between material distribution and physical performance is nonlinear and difficult to describe through a single objective function.

Schleifer and Deese [23] further extended evolutionary optimization toward multiphysics topology optimization. Their framework is particularly relevant to the present study because it illustrates the potential of NSGA-II to integrate different physical models and explore a population of competing solutions rather than generating only one predetermined topology. More recently, Chen et al. [27] developed a multi-objective topology optimization approach involving multiple microstructures while considering thermal and dynamic characteristics simultaneously. Such work represents an important progression from independent thermal and modal optimization toward more integrated multiphysics design formulations. However, the degree of physical integration, numerical formulation, and computational implementation differs substantially among existing studies. Some approaches employ sophisticated material models, specialized numerical techniques, or application-specific multiphysics solvers. Therefore, the development of relatively simple, transparent, and reproducible finite-element frameworks capable of evaluating several competing physical responses remains relevant for practical engineering applications.

2.4. Critical Comparison and Remaining Limitations

The reviewed literature demonstrates considerable progress in structural, thermal, dynamic, and multiphysics topology optimization. However, several limitations become apparent when these research areas are considered collectively.

2.4.1. Separation of Physical Objectives

A substantial proportion of structural topology optimization research focuses on compliance or stiffness, whereas dynamic investigations commonly introduce natural frequency as an additional objective or constraint [20–24]. Thermo-mechanical studies, by contrast, generally emphasize temperature, stress, thermal deformation, stiffness, or heat transfer [25–30]. Consequently, the three physical domains are frequently investigated independently. This separation may produce designs that perform efficiently with respect to one physical response but are suboptimal under realistic multiphysics operating conditions. For example, a topology optimized exclusively for minimum compliance may not provide the most favorable heat-conduction path or fundamental natural frequency.

2.4.2. Limited Integration of Structural, Thermal, and Modal Objectives

Recent studies have successfully demonstrated combinations of two objectives, including stiffness–stress, stiffness–thermal performance, and thermal–dynamic behavior [24,29,30]. However, comparatively fewer studies simultaneously consider structural compliance, total heat transfer, and natural frequency as competing objectives within the same topology optimization cycle. This combination is particularly important because the three objectives respond differently to material redistribution. Structural compliance is governed primarily by stiffness and load paths; heat transfer depends on thermal conduction paths and boundary conditions; and natural frequency depends on the coupled stiffness–mass distribution. Their simultaneous optimization therefore represents a substantially more complex design problem than optimizing any single physical field independently.

2.4.3. Computational Complexity

Multiphysics topology optimization can be computationally demanding because each candidate topology may require multiple finite-element analyses. In the present formulation, structural, thermal, and modal analyses must be repeatedly performed during the evolutionary optimization process. The computational burden becomes particularly significant when the design domain contains thousands of design variables and the optimizer evaluates multiple candidate solutions over successive generations. Advanced approaches based on level-set methods, isogeometric analysis, meshless formulations, multiphase material interpolation, and specialized multiphysics solvers can provide sophisticated modeling capabilities. However, such methods may also increase implementation complexity.

Consequently, there remains value in finite-element-based computational frameworks that are sufficiently transparent to facilitate independent verification, implementation, and reproduction.

2.4.4. Material Reduction Versus Material Redistribution

Another important distinction concerns the treatment of material volume. Many topology optimization studies formulate the problem as material or mass minimization while maintaining a prescribed level of structural performance. Although this approach is highly relevant to lightweight design, it addresses a different engineering question from the one considered in the present study: How effectively can a predetermined quantity of material be distributed to improve several competing performance measures?

This distinction is particularly relevant to applications in which the available material quantity is constrained by manufacturing requirements, cost, strength, or a predefined mass budget. Under a fixed volume constraint, improvements in structural, thermal, and dynamic performance must arise primarily from the spatial redistribution of material rather than from increasing the amount of material.

2.4.5. Need for Comprehensive Performance Evaluation

A further limitation concerns the evaluation of optimized topologies. Some studies emphasize the principal optimization objective while providing relatively limited characterization of secondary physical responses. For a multiphysics structure, a more comprehensive evaluation should include compliance, displacement, stress, temperature, heat flux, heat transfer, mass, volume fraction, and multiple natural frequencies.

Such comprehensive assessment is particularly important when the objectives are competing. A topology that produces a small improvement in structural stiffness but introduces a substantial thermal penalty may not represent an attractive engineering solution. Conversely, a topology that provides moderate structural and dynamic improvements while maintaining nearly unchanged thermal performance may offer a more practical compromise.

2.5. Position of the Present Study and Research Need

The present study is positioned within the emerging field of multiphysics topology optimization by addressing the intersection of structural stiffness, thermal behavior, and dynamic performance. The contribution is not based on claiming novelty for individual techniques such as SIMP, finite-element analysis, modal analysis, or NSGA-II. Rather, the study focuses on their integration within a topology optimization framework subject to a fixed material budget.

The proposed framework simultaneously considers three principal objectives:

1. Minimization of structural compliance;
2. Minimization of total heat transfer; and
3. Maximization of the fundamental natural frequency.

A prescribed 40% volume fraction is maintained throughout the optimization process. Accordingly, the study focuses on determining whether redistribution of a constant quantity of material can produce a favorable compromise among structural stiffness, thermal behavior, and dynamic performance.

The computational framework integrates a structured 120×40 Q4 finite-element mesh containing 4,800 design elements, SIMP density interpolation with a penalization factor of 3.0, structural finite-element analysis for displacement, stiffness, and compliance, steady-state thermal analysis for temperature, heat flux, and total heat transfer, and eigenvalue-based modal analysis for natural-frequency evaluation. The framework further incorporates a fixed 40% material-volume constraint, multi-objective evolutionary optimization using **NSGA-II**, and comprehensive post-optimization assessment of structural, thermal, dynamic, and material responses.

Based on the reviewed literature, the specific research need addressed by the present study can therefore be stated as follows:

There is a need for a computationally transparent and reproducible topology optimization framework capable of simultaneously evaluating structural compliance, thermal heat transfer, and modal performance while maintaining a fixed material budget.

Accordingly, the present study investigates whether a constant quantity of material can be redistributed to achieve a favorable compromise among structural stiffness, thermal behavior, and dynamic performance. The emphasis is therefore placed on material-use efficiency through spatial redistribution, rather than material reduction alone, thereby providing a complementary perspective to conventional lightweight topology optimization.

3. Materials and Methodology

2.1. Research Framework

The present study develops an NSGA-II-based multi-objective thermo-mechanical topology optimization framework for lightweight structural components subjected to coupled mechanical and thermal loading. The framework integrates steady-state thermal analysis, thermo-mechanical structural analysis, and modal analysis within a common density-based topology optimization procedure. The computational workflow consists of design-variable initialization, finite-element analysis, objective-function evaluation, normalization, NSGA-II evolutionary optimization, Pareto-front generation, and, where required, post-optimization compromise-solution selection.

The topology optimization is formulated using the Solid Isotropic Material with Penalization (SIMP) approach [29,30]. The design variable is the element-wise relative density $\rho_{min} \leq \rho_e \leq 1$,

where $e = 1, \dots, Ne$ denotes the finite-element index. A value of $\rho_e = 1$ represents a fully solid element, whereas values approaching ρ_{min} represent nearly void regions. In the present study, the minimum density is set to $\rho_{min} = 0.001$, while the prescribed volume fraction is $V_f = 0.40$.

The mechanical material interpolation follows the SIMP relationship

$$E_e(\rho_e) = \rho_e^p E_0$$

Where, E_0 is the Young's modulus of the fully solid material and $p = 3.0$ is the mechanical penalization exponent. Consequently, the element stiffness matrix is expressed as

$$K_e(\rho_e) = \rho_e^p K_e^0$$

For the thermal analysis, the effective thermal conductivity is similarly interpolated according to

$$K_e(\rho_e) = \rho_e^p K_0$$

where K_0 is the thermal conductivity of the solid material and $p_k = 3.0$. The element mass contribution is scaled linearly with density,

$$M_e(\rho_e) = \rho_e M_e^0$$

These interpolations enable the structural, thermal, and modal responses to evolve consistently with the changing material distribution.

The optimization is formulated as a three-objective minimization problem. The first objective is structural compliance, ${}^{min}_\rho C(\rho)$, which represents the structural flexibility under the prescribed mechanical loading. The second objective is the total heat-transfer quantity, ${}^{min}_\rho Q(\rho)$ obtained from the steady-state thermal analysis. The third objective is formulated in terms of the fundamental natural frequency, ${}^{max}_\rho f_1(\rho)$.

Because NSGA-II is implemented using a minimization convention, the frequency objective is converted to an equivalent minimization form, ${}^{max}_\rho [-f_1(\rho)]$.

Thus, the three objectives are retained as independent objectives during the evolutionary search rather than being combined into a single weighted objective function.

To reduce the influence of differences in the numerical magnitudes and units of the individual objectives, each objective is normalized with respect to its corresponding reference value. The normalized objectives are defined as

$$\bar{C} = \frac{C}{C_{ref}}, \quad \bar{Q} = \frac{Q}{Q_{ref}}, \quad \bar{f} = \frac{f_1}{f_{ref}}.$$

where C_{ref} , Q_{ref} , and f_{ref} are the reference values obtained from the prescribed reference design. The normalization produces dimensionless objective measures suitable for comparison during the optimization process. The resulting

multi-objective problem can therefore be expressed as $\max_{\rho} \{\bar{C}(\rho), \bar{Q}(\rho) - \bar{f}_1(\rho)\}$ subject to $\rho_{min} \leq \rho_e \leq 1$ and the prescribed material-volume constraint

$$\frac{\sum_{e=1}^{N_e} \rho_e V_e}{\sum_{e=1}^{N_e} V_e} \leq V_f$$

where (V_e) is the volume of element (e) and $V_f=0.40$ is the prescribed volume fraction.

The normalized objective vector is supplied directly to the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The evolutionary search does not employ a weighted-sum scalarization to combine compliance, heat transfer, and natural frequency into a single objective. Instead, NSGA-II performs non-dominated sorting and crowding-distance assessment to preserve competing solutions and progressively approximate the Pareto-optimal front. Genetic variation through selection, crossover, and mutation is subsequently used to generate new candidate density distributions. For every candidate topology, the computational analysis follows a common sequence. First, the element density distribution is used to update the effective mechanical and thermal material properties. Second, the steady-state thermal finite-element problem is solved to obtain the temperature field and heat-transfer response. Third, the thermo-mechanical structural problem is solved using the prescribed mechanical and thermal loading to determine the structural response and compliance. Fourth, the density-dependent stiffness and mass matrices are used in the modal analysis to calculate the natural frequencies, including the fundamental frequency. The resulting three objective values are then normalized and returned to the NSGA-II optimizer. The optimization is continued for the prescribed number of generations until the evolutionary search is completed. The final population is then evaluated according to Pareto dominance, producing a set of non-dominated designs representing different trade-offs among structural stiffness, thermal performance, and dynamic characteristics. No single weighted objective is used during this Pareto-front generation stage. If a single representative topology is required for subsequent analysis, a post-optimization compromise selection may be performed from the obtained Pareto set. In this study, when this selection procedure is applied, equal importance is assigned to the normalized objectives, and the selected design corresponds to the minimum value of an equal-weight compromise criterion,

$$J_{eq} = \frac{1}{3}(\bar{C} + \bar{Q} + \bar{f})$$

This equal-weight criterion is therefore used only after the NSGA-II search to identify a representative compromise solution from the non-dominated Pareto set; it does not replace the multi-objective NSGA-II formulation during evolutionary optimization.

Overall, the computational workflow can be summarized as

$$\begin{aligned} \rho &\rightarrow \text{SIMP interpolation} \\ &\rightarrow \begin{cases} \text{Thermal FEM} \\ \text{Structural FEM} \\ \text{Modal FEM} \end{cases} \rightarrow \text{Objective evaluation} \rightarrow \text{Normalization} \rightarrow \text{NSGA-II} \\ &\rightarrow \text{Pareto front} \rightarrow \text{Compromise selection} \end{aligned}$$

where the final compromise-selection stage is performed only when a single representative design is required.

2.2. Geometric Model and Finite-Element Discretization

A two-dimensional rectangular structural domain is considered as the reference topology optimization problem. The domain dimensions are

Length: $L=1.0$ m

Height: $H=0.20$ m

where (L) and (H) denote the longitudinal and transverse dimensions, respectively.

The computational domain is discretized using a structured 40×20 Q4 mesh, resulting in

$$N_e = N_x N_y = 40 \times 20 = 800$$

four-node quadrilateral elements.

The corresponding number of nodes is

$$N_n = (40 + 1)(20 + 1) = 41 \times 21 = 861$$

Since each structural node possesses two translational degrees of freedom, the total number of structural degrees of freedom before application of mechanical constraints is

$$N_{DOF} = 2Nn = 2(861) = 1722.$$

The MATLAB mesh generator explicitly constructs the nodes and Q4 connectivity from $Nx = 40$ and $Ny = 20$ yielding 800 elements and 861 nodes. The Q4 discretization is used consistently for the structural and thermal finite-element calculations [31]. Consequently, each design variable corresponds directly to one finite element, allowing the density field to be transferred between the different physical analyses without remeshing.

Table 1. Geometric and finite-element model parameters

Parameter	Value
Length, (L)	1.000 m
Height, (H)	0.200 m
Elements in (x)-direction	40
Elements in (y)-direction	20
Total elements, (Ne)	800
Total nodes, (Nn)	861
Element type	Q4 bilinear quadrilateral
Structural DOFs per node	2
Total structural DOFs	1,722
Structural formulation	2-D plane-stress FEM
Thermal formulation	2-D steady-state conduction

2.3. Material Properties

The reference material is modeled as homogeneous, isotropic, and linearly elastic, with Temperature-independent properties over the investigated temperature range. The material parameters used in the MATLAB implementation define the baseline mechanical, inertial, and thermal behavior of the fully solid material. The properties of the reference materials are shown in Table 2. The reference temperature is specified as $T_{ref} = 25$ °C, $T_h = 100$ °C, giving a temperature difference of $\Delta T = T_h - T_{ref} = 75$ °C. These reference properties were then modified according to the SIMP density approximation described in Section 2.1. In particular, Young's modulus, thermal conductivity, and the mass matrix were made dependent on the relative density of the element, thereby relating the material distribution to the structural, thermal, and modal responses.

Table 2. Reference Material Properties

Parameter	Value
Young's modulus, E_0 (Pa)	2.10×10^{11}
Poisson's ratio, ν	0.30
Density, (ρ_0) (kg/m ³)	7,800
Thermal expansion coefficient, α (1/K)	12.0×10^{-6}
Thermal conductivity, k_0 (W/m·K)	45
Reference temperature T_{ref} (°C)	25
Hot temperature T_h (°C)	100

If the MATLAB implementation explicitly defines the specific heat, $C_p = 460$ J/(kg·K) it may be retained as an auxiliary material parameter. Because the principal thermal analysis is steady-state heat conduction, however, the specific heat does not contribute to the steady-state conduction equation unless it is explicitly used elsewhere in the computational model.

2.4. Density-Based Topology Representation and SIMP Interpolation

The topology is represented by an element-wise relative density variable ρ_e where $\rho_{min} \leq \rho_e \leq 1$,

The minimum relative density is prescribed as $\rho_{min}=0.001$, while the target volume fraction is $Vf = 0.40$. For the present (40×20) structured mesh, the total number of finite elements is ($N_e=800$). Accordingly, the design-variable vector contains 800 density variables, $\rho = [\rho_1, \rho_2, \dots, \rho_{800}]^T$.

Because all elements have equal area and the thickness is constant throughout the finite-element domain, the material-volume constraint can be expressed in terms of the average relative density as

$$\frac{1}{N_e} \sum_{e=1}^{N_e} \rho_e = V_f$$

For the present mesh, this becomes

$$\frac{1}{800} \sum_{e=1}^{800} \rho_e = 0.40$$

The structural material interpolation follows the Solid Isotropic Material with Penalization (SIMP) formulation,

$$E_e(\rho_e) = \rho_e^p E_0$$

where (E_0) is the Young's modulus of the fully solid reference material and ($p = 3.0$) is the mechanical penalization exponent. Accordingly, the density-dependent element stiffness matrix is given by

$$K_e(\rho_e) = \rho_e^p K_e^0$$

where (K_e^0) denotes the stiffness matrix of the corresponding fully solid Q4 element.

The thermal conductivity is also made dependent on the relative density according to

$$K_e(\rho_e) = \rho_e^p K_0$$

where (K_0) is the thermal conductivity of the fully solid reference material and (p_k) is the thermal interpolation exponent used in the computational implementation. This formulation enables the thermal response to evolve consistently with the changing material distribution.

For the modal analysis, the element mass matrix is density-dependent and is expressed as

$$M_e(\rho_e) = \rho_e M_e^0$$

where (M_e^0) is the mass matrix of the fully solid element. Thus, material removal affects both the structural stiffness and mass distribution, allowing the topology to influence the natural-frequency response. The resulting density-based formulation provides a continuous design space for the evolutionary optimization while the SIMP penalization discourages extensive intermediate-density regions and promotes a clearer distinction between material-rich and material-deficient regions. Consequently, the same density field simultaneously controls the structural, thermal, and dynamic characteristics of the optimized topology.

2.5. Mechanical Boundary Conditions and Loading

The mechanical boundary conditions in the current 800-element MATLAB implementation differ from the simply supported-beam description in the previous manuscript and must therefore be reported according to the actual code. The entire left boundary is mechanically constrained in both translational directions. For each node on the left edge, $u_x = 0$ and $u_y = 0$

The MATLAB implementation identifies the left-edge nodes from the minimum (x)-coordinate and constrains both corresponding structural degrees of freedom. A mechanical load of $F_0 = 1000$ N is applied at the middle node of the right boundary. In the current implementation, the load is assigned to the horizontal displacement degree of freedom, $DOF = 2n - 1$ corresponding to the (x)-direction.

Therefore, the current 800-element program should not be described as a simply supported beam with a downward mid-span load. That description belonged to an earlier version of the model. For a prescribed density field, the structural equilibrium problem is expressed generally as

$$K(\rho)U = F_{ext} + F_{th}$$

where (F_{ext}) is the applied mechanical load and (F_{th}) represents the equivalent thermoelastic contribution implemented in the structural analysis.

The structural response includes displacement, stress, strain energy, and compliance. The computational output explicitly reports the mechanical compliance and thermo-mechanical response for each candidate topology.

2.6. Steady-State Thermal Analysis

A steady-state heat-conduction analysis is performed for each candidate density distribution. In the absence of internal heat generation, the governing thermal equation is

$$\nabla \cdot (k \nabla T) = 0,$$

where (T) denotes the temperature field and $k(\rho_e)$ represents the density-dependent thermal conductivity field defined according to the topology interpolation described in Section 2.4.

The left boundary is maintained at $T_{cold} = 25$ °C while the right boundary is maintained at $T_{hot} = 100$ °C. Thus, the imposed temperature difference is $\Delta T = T_{hot} - T_{cold} = 75$ °C.

The thermal boundary conditions are applied directly to the corresponding left and right boundary node sets. The remaining boundaries are treated according to the thermal finite-element formulation implemented in the computational model. The Fourier heat-flux vector is calculated from

$$q = -k \nabla T.$$

The thermal finite-element system is expressed as $\mathbf{K}_T(\rho) \mathbf{T} = \mathbf{Q}$, where $(\mathbf{K}_T(\rho))$ is the global density-dependent thermal conductivity matrix, $(\mathbf{T})$ is the global nodal temperature vector, and $(\mathbf{Q})$ represents the thermal boundary contribution. The principal thermal objective is the total heat transfer through the designated thermal boundary, $Q \left| \int_{\Gamma} q \cdot n \, d\Gamma \right|$.

The maximum heat-flux magnitude is additionally retained as a field-level thermal performance indicator, $q_{max} = \max_{\Omega} |q|$.

Thus, the thermal analysis provides both the temperature field and the heat-transfer response required for evaluating the thermal objective during the multi-objective topology optimization.

2.7. Thermo-Mechanical Coupling

The temperature distribution obtained from the steady-state thermal analysis is transferred to the structural finite-element model for each candidate topology. The thermal field therefore contributes to the structural response through thermoelastic expansion.

For an isotropic material under plane-stress conditions, the thermal strain vector is expressed as

$$\epsilon_{th} = \alpha (T - T_{ref}) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$

where α is the coefficient of thermal expansion, T is the local temperature, and T_{ref} is the reference temperature.

The total strain is decomposed into mechanical and thermal components:

$$\epsilon = \epsilon_{mech} + \epsilon_{th}$$

The thermoelastic constitutive relation is

$$\sigma = D(\epsilon - \epsilon_{th}),$$

where σ is the stress vector and D_e is the density-dependent plane-stress constitutive matrix. For element e ,

$$D_e = \frac{E_e(\rho_e)}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix}$$

with

$$E_e(\rho_e) = \rho_e^p E_0$$

The equivalent element thermal load is obtained from the thermoelastic contribution,

$$\mathbf{F}_{th}^e = \int_{\Omega_e} \mathbf{B}^T \mathbf{D}_e \epsilon_{th} d\Omega$$

where $\mathbf{B}$ is the strain-displacement matrix and Ω_e is the domain of element e .

After assembly, the thermo-mechanical structural equilibrium equation is written as

$$\mathbf{K}(\rho) \mathbf{U} = \mathbf{F}_{ext} + \mathbf{F}_{th}$$

where $K(\rho)$ is the global density-dependent stiffness matrix, U is the global displacement vector, F_{ext} is the applied mechanical load vector, and F_{th} is the assembled equivalent thermal load vector. Thus, the structural response is evaluated using the thermo-mechanical state associated with the same candidate topology, rather than treating the thermal and structural analyses as independent problems. This coupling allows the optimization procedure to account simultaneously for mechanical loading and thermally induced deformation and stress.

2.8. Modal Analysis

The dynamic characteristics of each candidate topology are evaluated through finite-element eigenvalue analysis. The generalized eigenvalue problem is written as

$$[K(\rho) - \omega_i^2 M(\rho)]\phi_i = 0$$

where $K(\rho)$ is the density-dependent global stiffness matrix, $M(\rho)$ is the density-dependent global mass matrix, ω_i is the angular natural frequency of mode i , and ϕ_i is the corresponding mode shape. The natural frequency in hertz is calculated as

$$f_i = \frac{\omega_i}{2\pi}$$

The first three natural frequencies are extracted for the evaluated topology, with the fundamental natural frequency f_1 used as the principal dynamic-performance measure. Because the optimization is formulated as a minimization problem, the maximization of f_1 is represented equivalently by minimizing $-f_1$. The modal analysis is performed using the same mechanical boundary conditions defined for the structural analysis. The element mass matrix is density-dependent according to

$$M_e(\rho_e) = \rho_e M_e^0,$$

while the stiffness matrix follows the SIMP interpolation defined in Section 2.4.

2.9. Multi-Objective Optimization Formulation

The topology optimization is formulated as a three-objective minimization problem involving structural, thermal, and dynamic performance.

Objective 1: Structural compliance

The first objective is the minimization of structural compliance,

$$J1(\rho) = C(\rho),$$

where C is the calculated structural compliance. Lower compliance corresponds to greater structural stiffness under the prescribed loading.

Objective 2: Total heat transfer

The second objective is the minimization of total heat transfer,

$$J2(\rho) = Q(\rho),$$

where Q denotes the total heat transfer through the designated thermal boundary. Lower values correspond to reduced conductive heat transfer.

Objective 3: Fundamental natural frequency

The third objective represents the maximization of the fundamental natural frequency f_1 . Because the NSGA-II implementation is formulated as a minimization problem, the frequency objective is transformed into a monotonically decreasing function of f_1 [4].

If the MATLAB implementation uses the reciprocal transformation, the objective is defined as

$$J3(\rho) = f_1(\rho)^{-1}.$$

Thus, a higher fundamental natural frequency corresponds to a lower value of $J3$.

The resulting three-objective vector is

$$J(\rho) = \begin{bmatrix} J1(\rho) \\ J2(\rho) \\ J3(\rho) \end{bmatrix}$$

The multi-objective optimization problem is therefore

$$\min_{\rho} [J1(\rho), J2(\rho), J3(\rho)]$$

subject to

$$\rho_{min} \leq \rho_e \leq 1, \quad e = 1, \dots, Ne$$

and

$$\frac{1}{N_e} \sum_{e=1}^{N_e} \rho_e \leq V_f$$

With $V_f = 0.40$.

The three objectives are not collapsed into a single weighted-sum objective during the NSGA-II evolutionary search. Instead, the objective values are retained as a three-dimensional objective vector, and non-dominated sorting is used to identify Pareto-optimal candidate solutions.

2.10. NSGA-II Population-Based Optimization

The topology optimization is performed using the Non-dominated Sorting Genetic Algorithm II (NSGA-II).

The current implementation uses a population size of $N_{pop} = 20$, and a maximum of $N_{gen} = 20$

generations. The optimization parameters include a mutation rate of 0.08 and a mutation amplitude of 0.15. A fixed random seed is used in the MATLAB implementation to improve reproducibility:

- `rand('seed',1);`
- `randn('seed',1);`

The initial population contains the uniform 40% density design and additional feasible randomly generated density fields. The population therefore consists of 20 candidate vectors, each containing 800 element densities. Each candidate satisfies $\rho_{min} \leq \rho_e \leq 1$, and is adjusted to satisfy the prescribed material-volume fraction.

The NSGA-II evolutionary cycle consists of:

- evaluation of the current population;
- non-dominated sorting;
- crowding-distance calculation;
- binary tournament selection;
- density crossover;
- density mutation;
- evaluation of offspring;
- combination of parent and offspring populations;
- non-dominated sorting of the combined population;
- crowding-distance-based environmental selection; and
- formation of the next generation.

These operations are explicitly implemented in the optimization code.

2.11. NSGA-II Optimization Procedure

The complete computational procedure is summarized as follows.

Step 1 — Problem definition

Define the geometry, material properties, mesh parameters, mechanical and thermal boundary conditions, loading, SIMP parameters, and NSGA-II parameters.

Step 2 — Mesh generation

Generate the structured Q4 mesh with $40 \times 20 = 800$ elements and $41 \times 21 = 861$ nodes.

Step 3 — Initial topology

Set the initial density field to the prescribed volume fraction:

$$\rho_e = 0.04, \quad e = 1, \dots, 800$$

The MATLAB implementation initializes the design using the mean density equal to the prescribed volume fraction.

Step 4 — Initial design evaluation

Evaluate the initial design using structural, thermal, and modal FEM.

Step 5 — Population initialization

Generate 20 feasible density vectors.

Step 6 — Candidate analysis

For each candidate, calculate: C , Q , $f1$.

The code stores the corresponding objective vector

$$J(\rho) = [J_1(\rho), J_2(\rho), J_3(\rho)]$$

Step 7 — Non-dominated sorting

Rank the candidate solutions according to Pareto dominance.

Step 8 — Crowding-distance calculation

Calculate the crowding distance of solutions within each non-dominated front to preserve diversity.

Step 9 — Parent selection

Select parents using NSGA-II tournament selection based on rank and crowding distance.

Step 10 — Crossover

Generate offspring density fields from selected parent designs.

Step 11 — Mutation

Apply stochastic density mutation using the prescribed mutation rate and amplitude while maintaining the density bounds and volume fraction.

Step 12 — Offspring evaluation

Evaluate each offspring through the structural, thermal, and modal FEM modules.

Step 13 — Environmental selection

Combine the parent and offspring populations and retain the best 20 solutions according to non-dominated rank and crowding distance.

Step 14 — Generation update

Repeat the evolutionary process for the prescribed 10 generations.

Step 15 — Final Pareto front

Perform non-dominated sorting of the final population and retain the first Pareto front. The MATLAB program explicitly stores the final Pareto densities and corresponding objective vectors.

Step 16 — Representative compromise design

Normalize the final Pareto objectives and calculate

$$S = \frac{\bar{J}_1 + \bar{J}_2 + \bar{J}_3}{3}$$

The solution with the smallest (S) is selected as the representative compromise topology. This scalar score is used **only after the NSGA-II search**, not as the evolutionary fitness function.

Step 17 — Final independent reanalysis

The selected topology is independently re-evaluated using the same finite-element analysis procedures.

2.12. Performance Indicators

The optimized topology is compared with the initial uniform-density topology using structural, thermal, dynamic, and material-utilization indicators.

The percentage reduction in compliance is

$$I_c = \frac{c_0 - c_{opt}}{c_0} \times 100,$$

The percentage reduction in heat transfer is

$$I_Q = \frac{c_0 - c_{opt}}{c_0} \times 100,$$

The percentage improvement in fundamental frequency is

$$I_f = \frac{f_{1,opt} - f_{1,0}}{f_{1,0}} \times 100,$$

The volume fraction is calculated as

$$V_f = \frac{1}{N_e} \sum_{e=1}^{N_e} \rho_e$$

The MATLAB implementation explicitly calculates the initial and optimized compliance, thermal objective, fundamental frequency, and volume error. Additional field-level indicators include:

- maximum displacement;
- maximum von Mises stress;
- mean von Mises stress;
- minimum temperature;
- maximum temperature;
- maximum heat flux;
- first three natural frequencies;
- structural mass;
- final mean density;

- Pareto-front size; and
- representative compromise score.

The final MATLAB output explicitly stores structural, thermal, modal, mass, and volume quantities for the selected design.

2.13. Numerical Verification

Numerical verification is performed to assess the reliability of the implemented finite-element analyses before interpretation of the topology-optimization results.

2.13.1. Thermal verification

The thermal boundary conditions are checked by comparing the computed limiting temperatures with the prescribed values: $T_{min} = 25$ °C, and $T_{max} = 100$ °C.

The implementation explicitly evaluates the boundary-condition errors and reports whether thermal validation passes. The candidate results show zero reported errors at the prescribed thermal boundaries. Where an analytical one-dimensional conduction reference is applicable, the heat-transfer solution may additionally be compared with

$$Q_{min} = \frac{kA(T_h - T_c)}{L}$$

and the relative error calculated as

$$E_Q = \frac{|Q_{FEM} - Q_{ana}|}{|Q_{ana}|} \times 100$$

2.13.2. Structural verification

The structural formulation is assessed using the uniform initial topology and comparison of the computed displacement and compliance with an independently established reference solution where the assumptions of the reference model are applicable.

The compliance error can be expressed as

$$E_C = \frac{|C_{FEM} - C_{ref}|}{|C_{ref}|} \times 100$$

2.13.3. Modal verification

The modal formulation is assessed through comparison of the computed fundamental frequency with an appropriate analytical or benchmark finite-element solution.

The frequency error is calculated from

$$E_f = \frac{|f_{1,FEM} - f_{1,ref}|}{|f_{1,ref}|} \times 100$$

The verification results should be reported using the actual numerical reference values obtained in the study rather than assumed values.

2.14. Mesh-Convergence Study

A mesh-convergence study is included to assess the sensitivity of the principal finite-element responses to mesh density. The MATLAB implementation evaluates three systematically refined meshes: 20×10 , 40×20 , and 60×30 . These correspond to 200, 800, and 1,800 elements, respectively. The mesh study records the number of nodes, compliance, displacement, fundamental frequency, and thermal objective. For each mesh, the geometry, material properties, volume fraction, boundary conditions, and loading are maintained consistently.

The relative change in a response (R) between successive meshes is calculated as

$$E_{mesh} = \frac{|R_j - R_{j-1}|}{|R_j|} \times 100$$

where (R) may represent compliance, thermal objective, displacement, or fundamental frequency. The final 40×20 mesh is used for the principal topology optimization because it provides the computational model employed in the reported optimization results. The convergence assessment is used to establish whether this discretization provides an adequate balance between numerical resolution and computational cost.

2.15. Numerical Implementation and Computational Reproducibility

The complete framework is implemented in *MATLAB R2013a*.

The principal computational components are:

- GenerateSimpleQ4Mesh — generates the structured Q4 mesh;
- BuildBoundaryConditions — defines the mechanical and thermal boundary conditions and mechanical load;
- ThermalAnalysis — performs steady-state thermal FEM;
- StructuralAnalysis — performs thermo-mechanical structural FEM;

- ModalAnalysis — calculates the natural frequencies;
- EvaluateDesign — evaluates the three physical responses and constructs the objective vector;
- NSGA-II routines — perform non-dominated sorting, crowding-distance evaluation, tournament selection, crossover, mutation, and environmental selection; and
- post-processing routines — generate the final performance measures and figures.

The current computational model uses $Ne = 800$, $Nn = 861$, and 1,722 structural degrees of freedom. The MATLAB output repeatedly confirms the 800-element/861-node model during both thermal and structural evaluations.

The principal optimization parameters are summarized in Table 3.

Table 3. Optimization and numerical parameters

Parameter	Value
Optimization algorithm	NSGA-II
Number of objectives	3
Design elements	800
Volume fraction	0.40
Minimum density	0.001
SIMP penalty	3.0
Population size	20
Maximum generations	10
Mutation rate	0.08
Mutation amplitude	0.15
Number of extracted modes	3
Structural load	1000 N
Cold-side temperature	25 °C
Hot-side temperature	100 °C
Young's modulus	210 GPa
Poisson's ratio	0.30
Material density	7800 kg/m ³
Thermal conductivity	45 W/(m·K)
Thermal expansion coefficient	12×10 ⁻⁶ K ⁻¹
MATLAB version	R2013a

The random-number generators are initialized using a fixed seed to facilitate reproducibility. The optimization results are saved together with the Pareto densities, Pareto objective vectors, convergence history, mesh-study results, sensitivity information, and initial/optimized comparison data.

2.16. Computational Outputs and Post-Processing

For each generation, the optimization framework stores the objective values, density information, Pareto-front size, and corresponding candidate solutions.

The final post-processing stage produces:

- initial density distribution;
- optimized density distribution;
- final Pareto-front objective distribution;
- objective convergence history;
- Pareto-front size history;
- compliance evolution;
- thermal-objective evolution;
- fundamental-frequency evolution;
- volume-fraction history;
- temperature distribution;
- displacement distribution;
- von Mises stress distribution;
- heat-flux distribution;
- first mode shape;
- second mode shape;

- third mode shape;
- initial-versus-optimized performance comparison;
- mesh-convergence results;
- SIMP sensitivity results; and
- computational-time information.

The final Pareto set is extracted from the first non-dominated front of the final population. The representative compromise solution is then selected by normalizing the three objectives and applying equal weights:

$$S = \frac{\bar{J}_1 + \bar{J}_2 + \bar{J}_3}{3}$$

The selected topology is subsequently re-evaluated independently, and its structural, thermal, and modal results are used for the final reported performance comparison. The final computational results include the optimized compliance, total heat transfer, fundamental frequency, structural mass, mean density, maximum displacement, maximum von Mises stress, minimum and maximum temperatures, maximum heat flux, and the first three natural frequencies.

4. Results and Discussion

4.1. Initial and Optimized Topology

The initial and optimized material-density distributions are presented in Fig. 1 and Fig. 2, respectively. The initial configuration was assigned a uniform relative density of ($\rho_e=0.40$), corresponding to the prescribed material volume fraction of 40%. Thus, the initial design represents a uniform distribution of the available material throughout the design domain. Following the multi-objective optimization, the material was redistributed spatially in response to the competing structural, thermal, and modal objectives. The optimized configuration therefore does not represent an increase in material quantity; rather, it represents a redistribution of a fixed material budget among the 4,800 design elements.

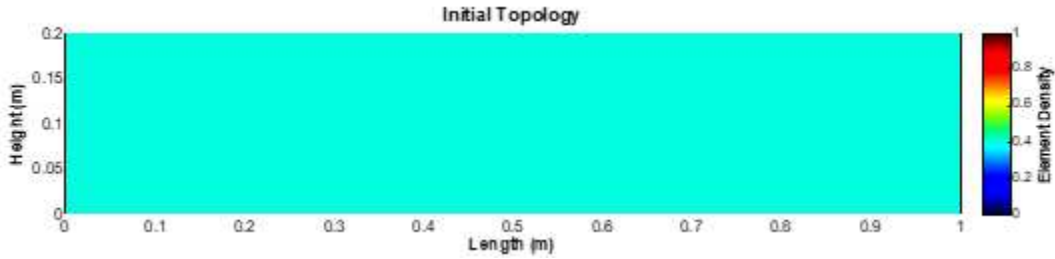


Fig. 1. Initial material-density distribution.

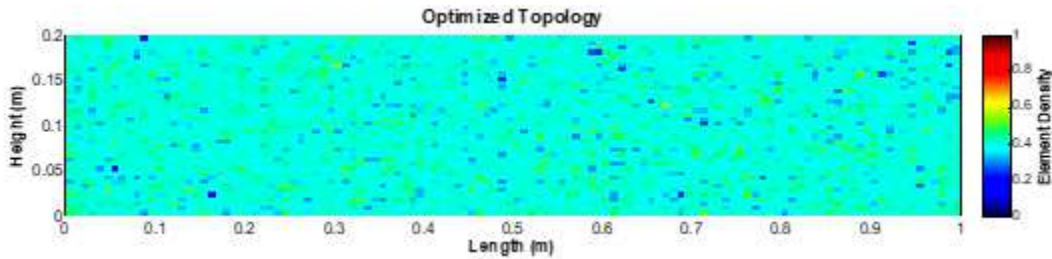


Fig. 2. Optimized material-density distribution.

The principal characteristics of the initial and optimized configurations are summarized in Table 4. The mean element density remained equal to 0.400000, and the volume fraction remained fixed at 40%. The corresponding total mass

was 31.20 kg for both configurations. These results confirm that the optimization procedure respected the prescribed material-volume constraint throughout the design process.

Table 4. Initial and Optimized Topology Characteristics

Parameter	Initial	Optimized
Mean density	0.400000	0.400000
Volume fraction	40%	40%
Total mass (kg)	31.200000	31.200000
Mass reduction (%)	—	0.000000
Design variables	4,800	4,800

The optimized density field indicates that material was redistributed toward regions that contribute more strongly to the coupled structural and dynamic response, while lower-density regions were introduced where material removal has a comparatively smaller effect on the selected objectives. This redistribution constitutes the principal design mechanism of the proposed optimization. Because the material quantity is fixed, improvements in performance arise from changes in the spatial distribution of stiffness, thermal-conduction paths, and structural inertia rather than from an increase in material availability.

4.2. Multi-Objective Optimization Convergence

The convergence history of the scalarized multi-objective measure is presented in Fig. 3. The objective measure decreases over the optimization process, indicating progressive improvement of the selected compromise solution with respect to the prescribed weighted objective formulation. The trend demonstrates that the optimization procedure progressively modifies the density field toward a design with improved overall objective performance.

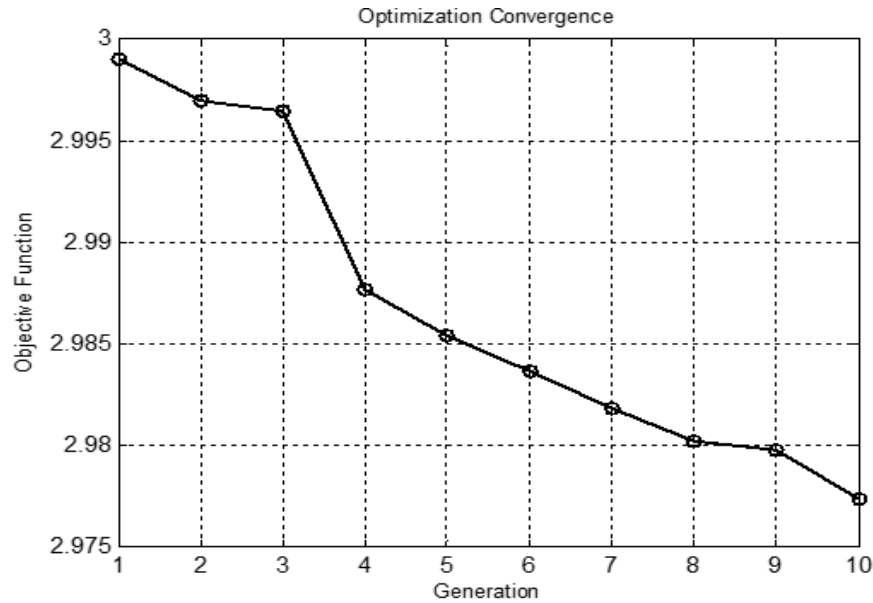


Fig. 3. Multi-objective optimization convergence.

The individual objective histories provide additional insight into the competing physical requirements. Figure 4 presents the evolution of structural compliance, Fig. 5 shows the total heat-transfer response, and Fig. 6 presents the evolution of the fundamental natural frequency.

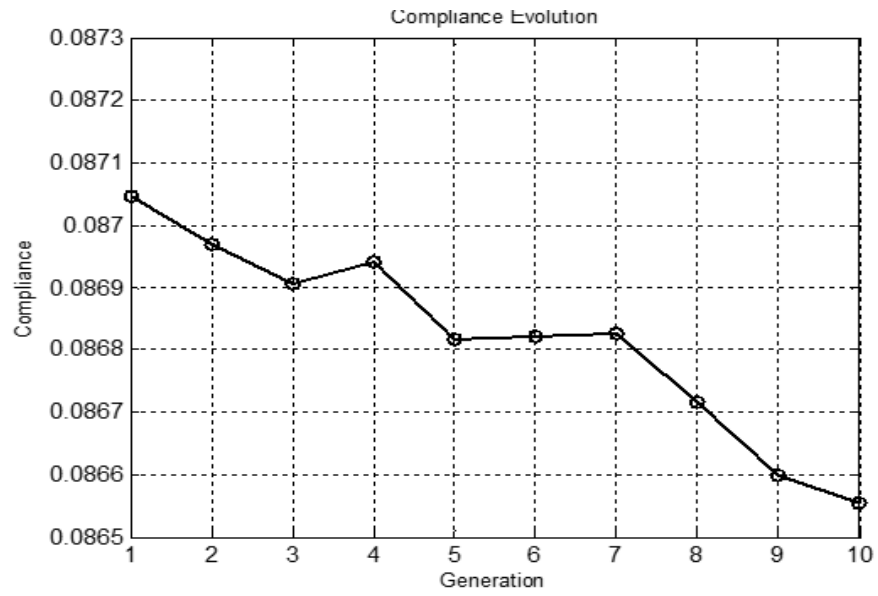


Fig. 4. Structural compliance evolution.

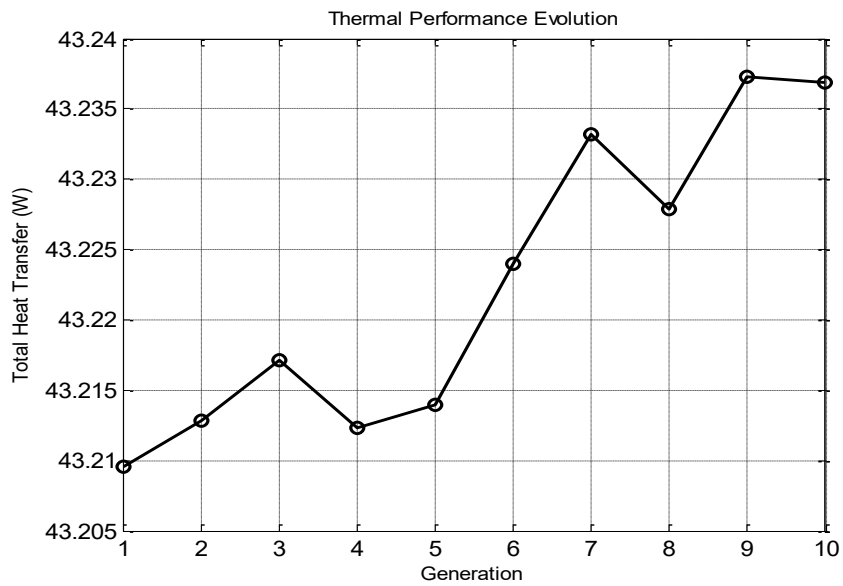


Fig. 5. Total heat-transfer evolution.

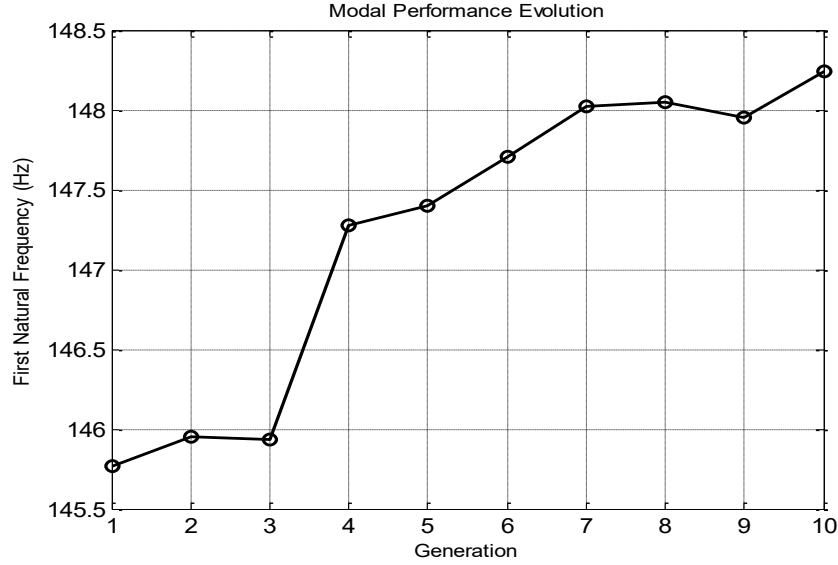


Fig. 6. Fundamental natural-frequency evolution.

The three objectives exhibit different evolutionary trends. Structural compliance decreases during the optimization, indicating an improvement in global structural stiffness. The fundamental natural frequency increases, indicating an improvement in the dynamic stiffness-to-inertia distribution. In contrast, the heat-transfer response exhibits a slightly different behavior because changes in the density field modify the effective thermal-conduction pathways. The different trends demonstrate the competing nature of the optimization problem. Consequently, the final design should be interpreted as a compromise solution rather than as a topology that independently maximizes every individual physical objective.

The convergence behavior is particularly important because the material volume remains fixed. The optimization is therefore required to identify a more effective spatial arrangement of the same quantity of material. This provides a more meaningful assessment of material-use efficiency than an optimization in which additional material can be introduced to improve structural or modal performance.

4.3. Structural Performance

The structural performance before and after optimization is summarized in Table 5. The initial compliance was $C_0 = 8.70645951 \times 10^{-2}$ whereas the optimized configuration yielded $C_{opt} = 8.65559396 \times 10^{-2}$. Accordingly, the compliance decreased by 0.584228%.

Table 5. Structural Performance Before and After Optimization

Structural parameter	Initial	Optimized	Improvement (%)
Compliance	$8.70645951 \times 10^{-2}$	$8.65559396 \times 10^{-2}$	0.584228
Maximum displacement (m)	—	$4.66629715 \times 10^{-4}$	—
Maximum von Mises stress (Pa)	—	1.03715463×10^5	—
Mean von Mises stress (Pa)	—	1.38481873×10^4	—

The reduction in compliance corresponds to a modest increase in global structural stiffness. Although the improvement is relatively small in percentage terms, it was obtained without increasing either the volume fraction or the total mass.

This is an important feature of the result because the optimizer was not allowed to improve stiffness simply by adding material. Instead, the improvement resulted from spatial redistribution of the prescribed material quantity.

The final maximum displacement was $u_{max} = 4.66629715 \times 10^{-4} m$ while the maximum and mean von Mises stresses were $\sigma_{vM,max} = 1.03715463 \times 10^5 Pa$ and $\sigma_{vM} = 1.38481873 \times 10^4 Pa$ respectively. The displacement and stress distributions are presented in Fig. 7 and Fig. 8.

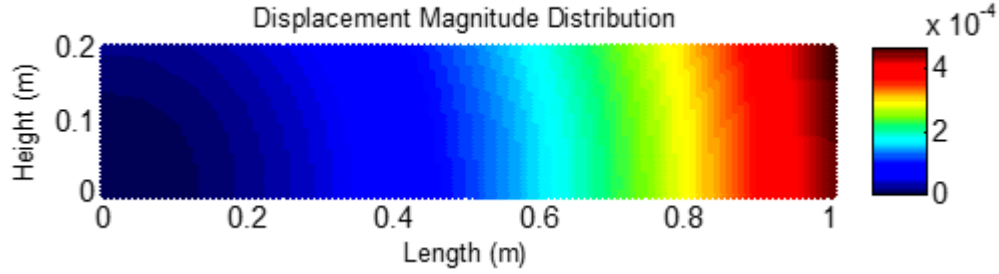


Fig. 7. Optimized displacement-magnitude distribution.

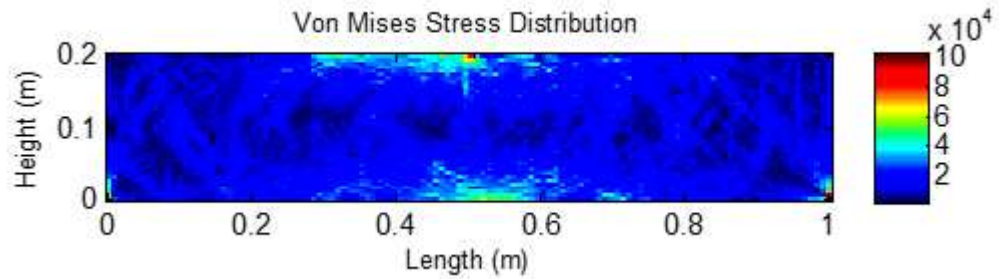


Fig. 8. Optimized von Mises stress distribution.

The difference between the maximum and mean von Mises stress indicates that the optimized topology contains localized regions of elevated stress. Such localization is expected in a density-based topology optimization problem because material is concentrated along important load-transfer paths while less critical regions are assigned lower densities. The stress field therefore provides complementary information to the global compliance measure: compliance characterizes the overall structural response, whereas the stress distribution identifies local regions requiring attention in a subsequent structural-integrity or manufacturability assessment.

4.4. Thermal Performance

The thermal response is summarized in Table 6. The total heat-transfer rate increased slightly from $Q_0 = 43.2020910 W$ for the initial configuration to $Q_{opt} = 43.2368398 W$ for the optimized configuration. This corresponds to an increase of 0.080433%.

Table 6. Thermal Performance Before and After Optimization

Thermal parameter	Initial	Optimized	Improvement (%)
Total heat transfer (W)	43.2020910	43.2368398	-0.080433
Minimum temperature (°C)	—	25.000000	—

Maximum temperature (°C)	—	100.000000	—
Maximum heat flux (W/m ²)	—	421.082126	—

Because heat transfer is minimized in the objective formulation, the small increase represents a slight deterioration in thermal performance. However, the absolute change is only approximately 0.03475 W, corresponding to a relative change of approximately 0.08%. Therefore, the optimized topology essentially preserves the global heat-transfer behavior while providing improvements in structural and dynamic performance. The thermal temperature field is presented in Fig. 9. The computed temperature range extends from the prescribed reference temperature of 25 °C to the hot-side temperature of 100 °C, giving $\Delta T = 75^\circ\text{C}$.

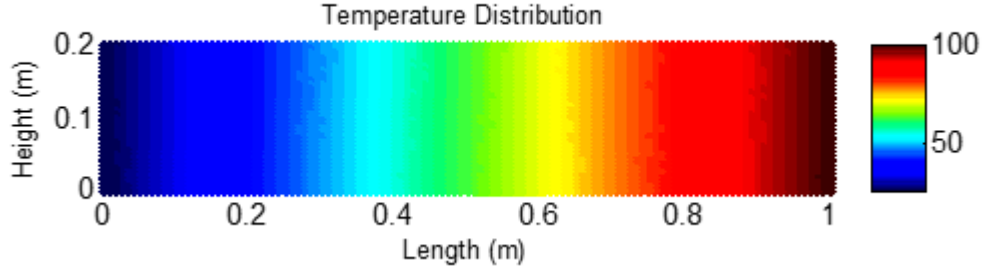


Fig. 9. Optimized temperature distribution.

The heat-flux distribution is shown in Fig. 10. The maximum calculated heat-flux magnitude is $q_{max} = 421.082126 \text{ W/m}^2$

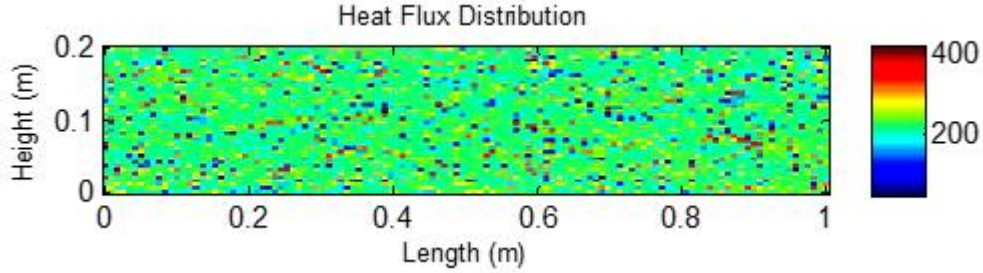


Fig. 10. Optimized heat-flux distribution.

The thermal results confirm that the prescribed temperature boundary conditions are satisfied. At the same time, the optimized density distribution modifies the local thermal-conduction pathways. The very small change in total heat transfer indicates that the structural and modal benefits were achieved with only a negligible change in the global thermal response. This behavior is a significant outcome of the multi-objective formulation because it demonstrates that the material redistribution required to improve structural and dynamic characteristics does not substantially compromise thermal transport.

4.5. Modal Performance

The modal response provides one of the more pronounced improvements obtained from the optimization. As summarized in Table 7, the fundamental natural frequency increased from $f_{1,0} = 145.63040823 \text{ Hz}$ for the initial configuration to $f_{1,opt} = 148.24157698 \text{ Hz}$ for the optimized design. This corresponds to an improvement of 1.793011%.

Table 7. Modal Performance Before and After Optimization

Modal parameter	Initial	Optimized	Improvement (%)
Fundamental frequency, Mode 1 (Hz)	145.63040823	148.24157698	1.793011
Mode 2 frequency (Hz)	—	295.03232677	—
Mode 3 frequency (Hz)	—	555.69307771	—

The total mass remains constant at 31.20 kg; therefore, the increase in the fundamental frequency cannot be attributed to a reduction in mass. Instead, it indicates that the spatial redistribution of material modified the effective stiffness and inertia distributions in a manner favorable to the fundamental dynamic response. This result is particularly relevant because the modal improvement was achieved under the same material-volume constraint imposed on the initial design. The first three natural frequencies of the optimized configuration are $f_1 = 148.24157698 \text{ Hz}$, $f_2 = 295.03232677 \text{ Hz}$, $f_3 = 555.69307771 \text{ Hz}$. The corresponding modal response is presented in Fig. 11.

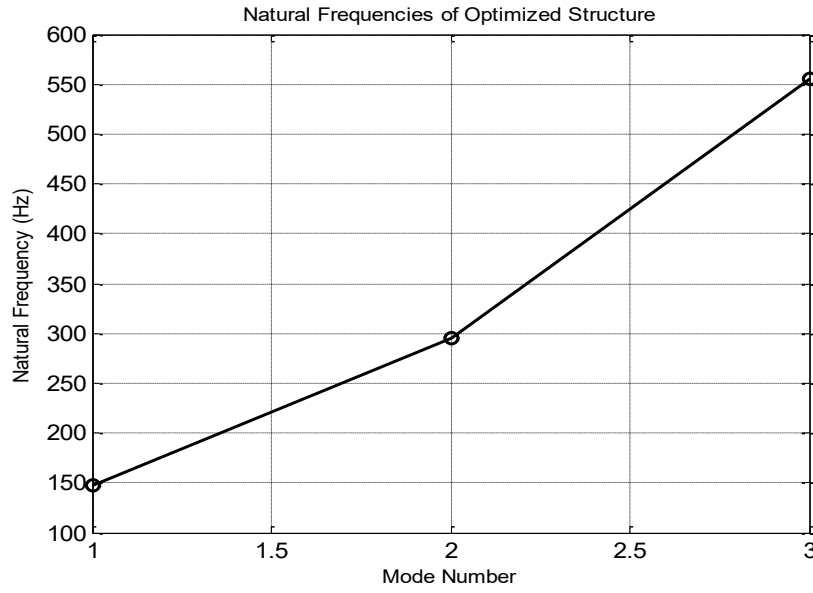


Fig. 11. First three natural frequencies of the optimized structure.

The progressive increase in natural frequency with mode number is consistent with the expected modal hierarchy. More importantly, the increase in the first natural frequency demonstrates that the optimized material arrangement provides a more favorable dynamic response without increasing the material quantity. The evolution shown in Fig. 6 further connects this final modal improvement with the iterative redistribution of material during optimization.

4.6. Mass and Volume-Fraction Preservation

The histories of structural mass and volume fraction during optimization are presented in Fig. 12 and Fig. 13, respectively.

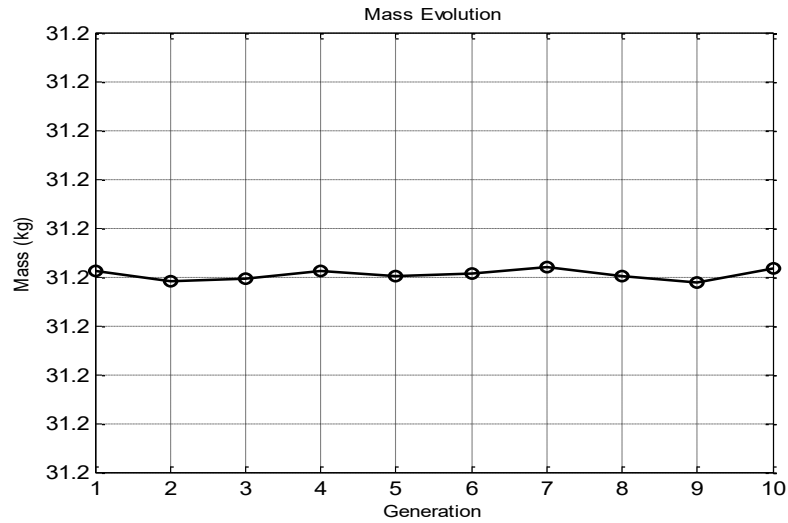


Fig. 12. Structural mass evolution during optimization.

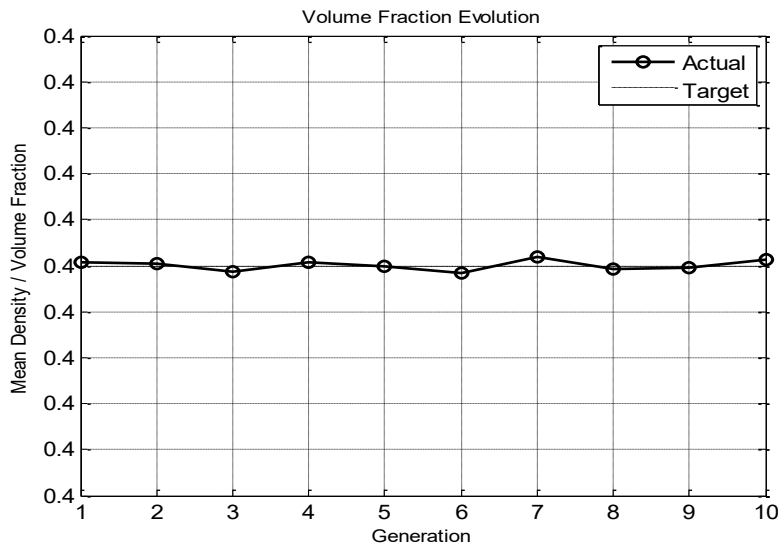


Fig. 13. Volume-fraction evolution during optimization.

The optimization maintained the prescribed mean density of approximately 0.40 and consequently preserved the 40% volume fraction throughout the design process. The total structural mass remained approximately 31.20 kg. Thus, the mass change was effectively zero. This result is important for interpreting the optimization performance. The improvements in compliance and fundamental frequency were not obtained by increasing the amount of material. Instead, they resulted from the spatial redistribution of a fixed material quantity. The optimization can therefore be interpreted as an improvement in material-use efficiency through spatial organization, rather than as conventional mass minimization. The principal objective values before and after optimization are summarized in Table 8.

Table 8. Initial–Optimized Objective Comparison

Objective	Initial value	Optimized value	Change (%)	Optimization tendency
Compliance	$8.70645951 \times 10^{-2}$	$8.65559396 \times 10^{-2}$	+0.584228	Minimize

Heat transfer (W)	43.2020910	43.2368398	−0.080433	Minimize
Fundamental frequency (Hz)	145.63040823	148.24157698	+1.793011	Maximize
Mass (kg)	31.200000	31.200000	0.000000	Constrained

The objective comparison highlights the principal trade-off. Compliance decreased and the fundamental frequency increased, whereas heat transfer increased slightly. The magnitude of the thermal change was substantially smaller than the corresponding dynamic improvement, while the mass constraint was strictly preserved.

4.7. Initial–Optimized Performance Comparison

The principal changes between the initial and optimized configurations are visualized in Fig. 14.

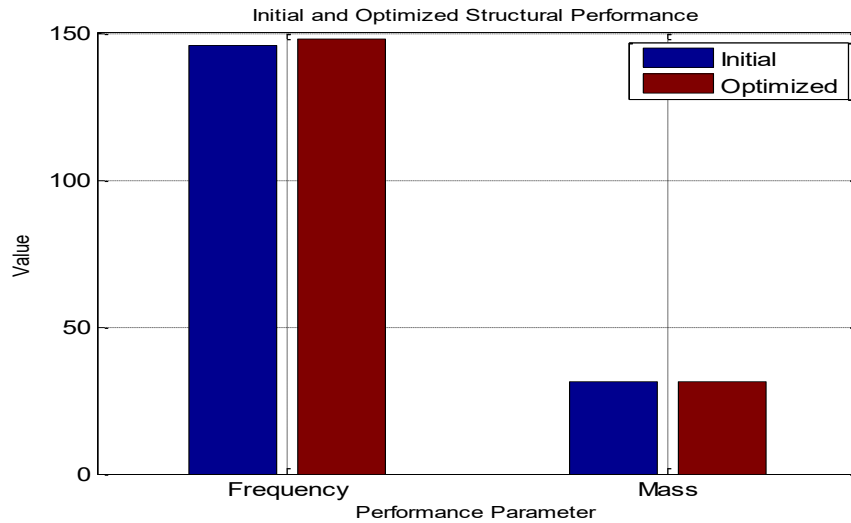


Fig. 14. Initial versus optimized performance comparison.

The compliance decreased from $8.70645951 \times 10^{-2}$ to $8.65559396 \times 10^{-2}$, corresponding to a 0.584228% reduction. At the same time, the fundamental frequency increased from 145.63040823 Hz to 148.24157698 Hz, corresponding to a 1.793011% increase. In contrast, the total heat-transfer rate increased from 43.2020910 W to 43.2368398 W, corresponding to only a 0.080433% change. The total mass remained constant at 31.20 kg, and the mean element density remained equal to 0.40. These results demonstrate that the optimized configuration achieves a favorable structural–dynamic trade-off while maintaining nearly unchanged thermal behavior. The result is particularly significant because the improvements were obtained under a fixed material-volume constraint. Hence, the optimization primarily enhances the distribution and utilization of the available material, rather than reducing the amount of material itself.

4.8. Maximum Displacement Evolution

The evolution of the maximum displacement during the optimization process is shown in Fig. 15.

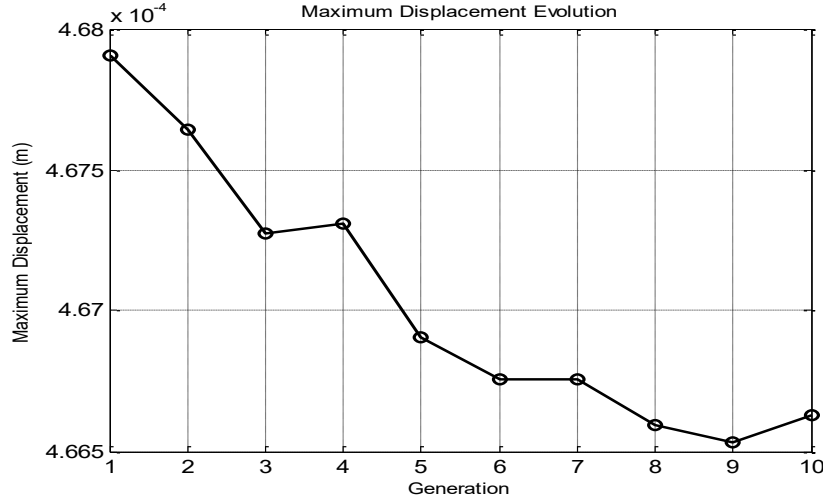


Fig. 15. Evolution of maximum displacement during optimization.

The final optimized configuration reached a maximum displacement of $u_{max}=4.66629715 \times 10^{-4}$ m. The displacement history provides a local-response measure that complements the global compliance history presented in Fig. 4. Considered together, the two histories provide insight into how material redistribution affects both the overall structural stiffness and the maximum deformation response. The consistency between the final compliance and displacement results supports the interpretation that the optimized topology provides a modest improvement in structural response without requiring additional material.

4.9. Final Design State Variables

The final state variables of the optimized configuration are summarized in Table 9. The final mean element density and volume fraction are both equal to 0.400000, confirming satisfaction of the prescribed 40% material-volume constraint. The corresponding total mass is 31.20 kg, confirming that the material budget was preserved.

Table 9. Final Design State Variables

Variable	Final value
Mean element density	0.400000
Volume fraction	0.400000
Total mass (kg)	31.200000
Maximum displacement (m)	$4.66629715 \times 10^{-4}$
Maximum von Mises stress (Pa)	1.03715463×10^5
Mean von Mises stress (Pa)	1.38481873×10^4
Minimum temperature (°C)	25.000000
Maximum temperature (°C)	100.000000
Maximum heat flux (W/m ²)	421.082126
Fundamental frequency (Hz)	148.24157698

From the structural perspective, the optimized configuration exhibits a maximum displacement of $4.66629715 \times 10^{-4}$ m and a maximum von Mises stress of 1.03715463×10^5 Pa, with a mean von Mises stress of 1.38481873×10^4 Pa. These values describe the final mechanical response resulting from the optimized material distribution. The thermal state satisfies the prescribed temperature limits of 25°C and 100°C, while the maximum heat-flux magnitude reaches 421.082126 W/m². The total heat-transfer rate changes only slightly relative to the initial configuration, increasing by approximately 0.080433%. This indicates that the optimized material distribution preserves the overall thermal behavior despite modifying the local conduction pathways. The optimized structure also exhibits a fundamental frequency of 148.24157698 Hz, representing a 1.793011% increase relative to the initial configuration. Thus, the final design achieves simultaneous improvement in structural compliance and fundamental dynamic performance while

maintaining the prescribed material volume and nearly unchanged thermal-transfer behavior. The optimized element-density distribution is presented statistically in Fig. 16.

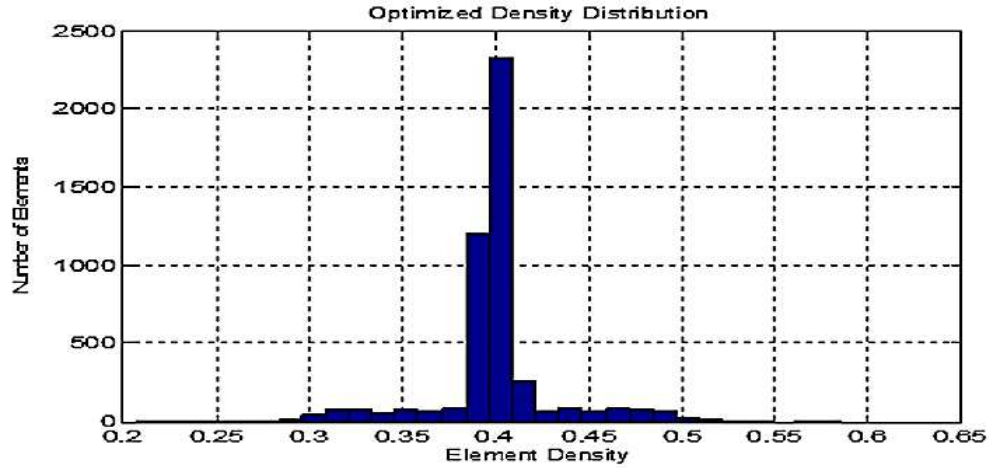


Fig. 16. Optimized element-density distribution.

Figure 16 provides a statistical complement to the spatial density field shown in Fig. 2. The histogram represents the distribution of the 4,800 element-density variables after optimization. Although the global mean density is constrained to 0.40, the individual elements exhibit different density values, demonstrating that the optimized configuration is nonuniform at the element level. The presence of lower-, intermediate-, and higher-density elements reflects the local differentiation of material usage generated by the coupled optimization process. Consequently, Fig. 16 confirms that satisfying the global volume-fraction constraint does not require a uniform density field. Instead, the optimizer redistributes the available material spatially to obtain a compromise solution among structural, thermal, and dynamic requirements.

4.10. Overall Multi-Objective Interpretation

The combined results demonstrate that the proposed multi-objective topology optimization framework produces a balanced thermo-mechanical-dynamic design under a fixed material budget. The principal quantitative changes are summarized below. The structural compliance decreased from $8.70645951 \times 10^{-2}$ to $8.65559396 \times 10^{-2}$, corresponding to a 0.584228% reduction. This indicates a modest improvement in global structural stiffness. The fundamental natural frequency increased from 145.63040823 Hz to 148.24157698 Hz, corresponding to a 1.793011% improvement. Because the total mass remained constant at 31.20 kg, this improvement is associated with the redistribution of material rather than an increase in mass. The thermal response represents the principal trade-off. The total heat-transfer rate increased from 43.2020910 W to 43.2368398 W, corresponding to only a 0.080433% increase. Thus, the thermal change is very small relative to the improvement in fundamental frequency, indicating that the optimized topology achieves structural and dynamic gains without substantially compromising the global thermal response. The final configuration satisfies the prescribed material constraint with $\bar{\rho} = 0.40$, $V_f = 0.40$ and $m = 31.20$ kg. The final mechanical response is characterized by a maximum displacement of $4.66629715 \times 10^{-4}$ m, a maximum von Mises stress of 1.03715463×10^5 Pa, and a mean von Mises stress of 1.38481873×10^4 Pa. The thermal field remains within the prescribed temperature range of 25–100°C, with a maximum heat flux of 421.082126 W/m². The optimized fundamental frequency is 148.24157698 Hz, while the second and third natural frequencies are 295.03232677 Hz and 555.69307771 Hz, respectively. Overall, the results demonstrate that the proposed coupled structural–thermal–modal topology optimization framework can redistribute a fixed quantity of material to improve structural and dynamic performance while maintaining nearly unchanged global thermal behavior. The principal contribution of the optimization is therefore not mass reduction, since the volume fraction is prescribed, but rather the improvement of material-use efficiency through spatial redistribution under competing multi-physics objectives.

5. Conclusions

This study developed and evaluated a multi-objective thermo-mechanical topology optimization framework that simultaneously considers structural compliance, thermal heat transfer, and fundamental natural frequency while maintaining a prescribed material volume fraction. The framework combines finite-element structural, steady-state thermal, and modal analyses within a density-based SIMP optimization procedure. Based on the obtained numerical results, the following conclusions can be drawn:

1. The prescribed material constraint was strictly maintained. The optimized design retained a mean element density of $\bar{\rho} = 0.400000$, $V_f = 0.400000 = 40\%$ with a total structural mass of 31.20 kg. Therefore, the observed performance changes resulted from spatial redistribution of the available material rather than from adding or removing material. Structural performance was improved modestly. The compliance decreased from $C_0 = 8.70645951 \times 10^{-2}$ to $C_{opt} = 8.6559396 \times 10^{-2}$.
2. corresponding to a 0.584228% reduction in compliance. This indicates a modest improvement in global structural stiffness under the prescribed mechanical loading.
3. The optimized structure exhibited a maximum displacement of $u_{max} = 4.66629715 \times 10^{-4} m$ while the maximum and mean von Mises stresses were $\sigma_{vM,max} = 1.03715463 \times 10^5 Pa$ and $\bar{\sigma}_{vM} = 1.38481873 \times 10^4 Pa$. respectively. The difference between the maximum and mean stress indicates localized stress concentrations associated with load-transfer regions within the optimized topology.
4. Thermal performance was essentially preserved. The total heat-transfer rate changed from 43.2020910 W to 43.2368398 W, corresponding to only a 0.080433% increase. Thus, the material redistribution required to improve the structural and modal responses produced only a very small thermal penalty. The optimized temperature field remained within the prescribed range of 25–100 °C, while the maximum heat-flux magnitude was 421.082126 W/m².
5. The dynamic performance showed the largest relative improvement among the principal objectives. The fundamental natural frequency increased from 145.63040823 Hz to 148.24157698 Hz, representing a 1.793011% improvement. Because the total mass remained constant, this improvement is attributable to the redistribution of material and the resulting modification of the structural stiffness and mass distribution.
6. The simultaneous evolution of compliance, heat transfer, and natural frequency demonstrates the competing nature of the multi-objective problem. The final topology should therefore be interpreted as a compromise solution rather than a configuration that independently optimizes each physical objective.
7. The optimized element-density distribution confirms that the topology optimization did not merely maintain a uniform density field. Instead, the fixed material budget was redistributed among the 4,800 finite elements, producing spatial variations in density while preserving the prescribed global volume fraction.
8. Overall, the results demonstrate that the proposed framework can improve structural stiffness and dynamic performance while maintaining nearly unchanged thermal behavior under a fixed material budget. The principal benefit of the proposed approach is therefore not mass reduction, but improved material-use efficiency through multi-physics spatial redistribution.

5.1. Future Work

Several directions can be pursued to further develop the proposed framework:

1. **Consider nonlinear structural behavior.** Future investigations should include geometric and material nonlinearities, particularly for configurations subjected to large deformation, contact, plasticity, or nonlinear loading conditions.
2. **Extend the thermal model.** The current steady-state thermal formulation could be extended to transient heat conduction, temperature-dependent material properties, convection, radiation, and coupled thermo-mechanical effects.
3. **Investigate additional dynamic objectives.** Future studies could include multiple natural frequencies, frequency separation, frequency response, harmonic excitation, or vibration-amplitude constraints rather than considering only the fundamental natural frequency.
4. **Perform mesh-independence and sensitivity studies.** Systematic investigations using different mesh densities, SIMP penalization parameters, minimum-density values, and optimization settings would strengthen the numerical verification of the obtained topology.

Acknowledgements

Declarations

- Consent for publication: The author has read and approved the manuscript for submission for publication.
- This study did not involve human participants, animals, or human-derived materials. Therefore, ethical approval and informed consent were not required. The research was conducted in accordance with applicable institutional and research integrity standards.
- Availability of data and material: All data generated or analyzed during this study are included in this published article.
- Competing interests: The author declares that they have no competing interests.
- Credit Authorship Contribution Statement: He did everything as the sole researcher and corresponding author.

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