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Rotation-Induced Electron Flow as a Driver of Localized Magnetic Activity in Neutron Star Surface Layers

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Abstract

Neutron stars are highly magnetized, rapidly rotating compact remnants produced primarily through the core collapse of massive stars. In this study, the impact of neutron star rotation on the surface magnetic structure through rotation-driven electron flow and its resulting local magnetic activity is investigated using magnetohydrodynamic equations. Specifically, an analytical formulation, complemented by numerical methods, is developed to derive the rotation-induced toroidal component and resulting effective surface magnetic fields, accounting for electron degeneracy pressure, gravitational, centrifugal, Lorentz and Coriolis forces. The results show that the local radial electron current density decreases radially. Furthermore, rotation-driven electron current generates additional local magnetic components that modify the surface field distribution. Consequently, the effective surface magnetic field exhibits distinct latitudinal variations relative to the unperturbed field. This redistribution may provide a physical pathway to create magnetic interaction zones and emphasizes the crucial role of rotation in producing surface magnetic anisotropy and localized magnetic activities, such as micro-flares, bursts, and hotspots. Ultimately, our model provides an analytical framework for investigating the role of rotation-driven electron transport in neutron star surface magnetic evolution.

Keywords: Rotating neutron stars, Surface magnetic fields, Magnetic activity, Magnetohydrodynamics, Magnetic anisotropy

1 Introduction

Neutron stars are compact stellar remnants formed during the core-collapse supernovae of massive stars, containing masses approximately $1.1\text{--}2.3 M_{\odot}$ within radii of about $10\text{--}13\text{ km}$ [1–3]. These extreme objects typically rotate with spin periods ranging from milliseconds to seconds [4, 5]. Their surface magnetic fields span an extraordinary range, from approximately $10^8\text{--}10^9\text{ G}$ in recycled millisecond pulsars to $10^{11}\text{--}10^{13}\text{ G}$ in ordinary radio pulsars, reaching $10^{14}\text{--}10^{15}\text{ G}$ or even higher in magnetars [6, 7]. The coexistence of rapid rotation and strong magnetization governs many observable properties of neutron stars, including their timing behavior, pulse morphology, thermal emission, particle acceleration, and long-term magnetic evolution [8, 9].

The rapid rotation of a neutron star profoundly influences its global structure and surface properties. Rotational forces induce an oblate deformation, increasing the equatorial radius relative to the polar radius [10, 11]. This asymmetry induces latitude-dependent variations in local surface gravity, which consequently alter the local effective temperature and radiative flux distributions across the crust and the surrounding magnetosphere [12, 13]. Beyond these structural influences, rotation mediates the complex coupling between the stellar surface and its magnetic field. In rapidly rotating, highly magnetized neutron stars, the interplay between rotation and magnetic stress produces significant deformations that is expected to influence the electrosphere and surface charge distribution. Force-free magnetospheric simulations of oblate, rapidly rotating neutron stars demonstrate that increased oblateness modifies polar-cap geometry and current-density profiles. Specifically, it yields larger polar cap areas and altered spin-down luminosities compared to spherical, non-rotating counterparts, directly influencing the configurations of surface hotspots and subsequent X-ray emission profiles [14, 15].

Accurately modeling rotation-driven surface magnetic activity is essential for interpreting modern, high-precision astrophysical observations. X-ray timing missions, such as the Neutron Star Interior Composition Explorer (*NICER*), constrain neutron star masses and radii using pulse-profile modeling that accounts for rotational deformation and localized magnetic field contributions [10, 16]. Observations of millisecond pulsars (MSPs) like PSR J0030+0451 and PSR J0740+6620 have successfully delivered simultaneous mass-radius measurements through such modeling [17, 18]. Detailed analysis of the *NICER* thermal X-ray light curves for PSR J0030+0451 has provided strong evidence for the existence of non-dipolar, multipolar magnetic fields [19]. The surface hotspots on PSR J0030+0451 are asymmetric and localized within a single hemisphere, deviating significantly from standard antipodal dipolar geometries [20]. Concurrently, gravitational-wave detectors probe rotational instabilities and field-induced asymmetries, offering complementary constraints on surface dynamics. Theoretical frameworks indicate that rotation can significantly modify the global magnetic topology, yielding non-dipolar components, surface anisotropies, and eruptive activity such as flares and bursts. On larger scales, magnetic stress releases in magnetars trigger catastrophic crustal ruptures and local field realignments, powering giant flares. For instance, the magnetar SGR 1806-20 released approximately $10^{39} - 10^{40}\text{ J}$, as shown in Figure 1.



Fig. 1 Magnetar burst showing the release of a vast amount of energy. *Image credit: NASA/Goddard/CI Lab*

While standard Hall magnetohydrodynamic (MHD) models traditionally focus on magnetic-driven electron currents in neutron star crusts [21–23], our model incorporates localized, rotation-driven electron dynamics influenced by the Coriolis force and small-scale surface perturbations. We develop a self-consistent analytical framework using steady-state continuity and momentum equations to investigate how rotation effects alter surface magnetic activity. This approach simultaneously captures the redistribution of pre-existing large-scale fields and the generation of rotation-induced local micro-magnetic structures. By analysing the interplay between rotational and magnetic forces, this work establishes a physical bridge between small-scale rotation-driven electron currents and macroscopic surface field evolution, offering deeper insight into how neutron star rotation shapes observable local surface magnetic activity.

2 Method

2.1 Governing MHD Equations

The neutron star’s surface behaves like an electrically conducting electron fluid governed by MHD [24]. The complete system of MHD equations unifies the Navier-Stokes equations of fluid dynamics with Maxwell’s equations of electromagnetism, requiring simultaneous solution either analytically or numerically [25]. In this work, we focus specifically on the MHD equations governing the flow of electrons at the surface shell of a rotating neutron star.

The conservation of mass is governed by the standard continuity equation for the mass density ρ and the fluid velocity $\mathbf{v}$ [26]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (1)$$

For an ideal plasma fluid subject to electromagnetic, gravitational, and inertial forces in a rotating frame, the steady-state momentum equation is expressed as [27, 28]

$$(\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla P - \nabla \Phi + \nabla \Phi_c + \frac{1}{\mu_0 \rho} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (2)$$

where: the centrifugal potential Φ_c for a uniform angular velocity (Ω) is

$$\Phi_c = -\frac{1}{2} \Omega^2 r^2 \sin^2 \theta, \quad (3)$$

and the gravitational potential Φ for a star of mass M is

$$\Phi = -\frac{GM}{r}, \quad (4)$$

Using standard vector identities, the Lorentz force density term $\mathbf{f}_{\text{Lorentz}}$ can be decomposed into magnetic pressure and magnetic tension components [25, 28]

$$\mathbf{f}_{\text{Lorentz}} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = (\mathbf{B} \cdot \nabla) \frac{\mathbf{B}}{\mu_0} - \nabla \left(\frac{B^2}{2\mu_0} \right). \quad (5)$$

Under the uniform rotation approximation of our model, differential rotation within the thin shell is neglected, as a result magnetic field lines experience minimal localized shearing or stretching across the layer thickness. Consequently, the magnetic tension force is small compared to the magnetic pressure gradient and can be approximated as [26]

$$\frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} \approx 0. \quad (6)$$

2.2 Formulation of Theoretical Model

The neutron star is modeled as a self-gravitating, non-accreting compact sphere surrounded by a vacuum. Dissipative processes—such as viscosity, turbulence, and resistivity—are assumed to be negligible compared to the dominant surface forces, which include electron degeneracy pressure, gravity, centrifugal and Coriolis inertial forces, and the Lorentz force.

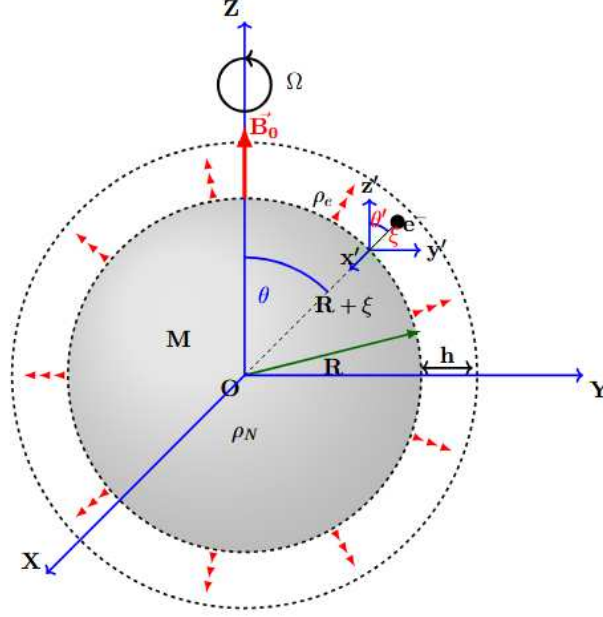


Fig. 2 Schematic representation of the theoretical model illustrating the thin electron fluid shell at the surface of a rotating neutron star, alongside both the local and global spherical coordinate systems.

To analyze the thin outermost shell, we establish a local spherical coordinate system (ξ, θ', ϕ') , where $\xi = r - R \in [0, h]$ with layer thickness $h \ll R$. Here, R is the neutron star's radius and h is the thickness of the electron fluid layer. The system is assumed to be axisymmetric ($\partial/\partial\phi' = 0$), thereby omitting explicit azimuthal variations in the electron flow.

Our approach centres on a dominant radial electron motion. Electrons possess much lower inertia and substantially higher mobility than the ions forming the crustal lattice in the thin layer. Consequently, under the combined influence of electron degeneracy pressure, electromagnetic forces, gravity, and rotational inertial effects, electrons can undergo a net outward drift relative to the nearly stationary ionic background. As this radial electron flow develops, the Coriolis force deflects the moving electrons in the azimuthal direction, producing electric currents that generate localized magnetic fields.

The global unperturbed magnetic field is assumed to be an axis-aligned dipolar field. To ensure regularity at the poles, we impose the boundary condition that the induced azimuthal magnetic field vanishes along the polar axis, $B'_{\phi'}(\xi, 0) = 0$.

2.3 Derivation of the Electron Current Density

For a predominantly radially dominated outflow velocity profile $\vec{v}_e(\xi, \theta') \approx v_\xi(\xi, \theta') \hat{\xi}$, the radial mass flux magnitude profile is given by

$$\rho_e(\xi, \theta') \vec{v}_e(\xi, \theta') = f(\xi, \theta'). \quad (7)$$

Expanding the divergence of mass flux in spherical coordinates under axisymmetry from Equation (1), yields

$$\frac{1}{(R + \xi)^2} \frac{\partial}{\partial \xi} ((R + \xi)^2 f(\xi, \theta')) + \frac{1}{(R + \xi) \sin \theta'} \frac{\partial}{\partial \theta'} (\sin \theta' f(\xi, \theta')) = 0, \quad (8)$$

For a small parameter in the thin shell away from the immediate polar axis, $h \ll R$, the angular term scales like $f/(R) \times (\cot \theta')$, while the radial derivative term scales like f/ξ where $\xi \sim h \ll R$; hence the ratio $(h/R) \cot \theta' \ll 1$. Applying the product rule and neglecting $\frac{\partial}{\partial \theta'} f(\xi, \theta')$ compared with the geometric $\sin \theta'$ term, will be simplified into a first-order ordinary differential equation with respect to ξ

$$\frac{df(\xi, \theta')}{d\xi} + \frac{2 + \cot \theta'}{(R + \xi)} f(\xi, \theta') = 0, \quad (9)$$

Separating variables and integrating from the inner boundary ξ_0 yields the electron mass density profiles

$$\rho_e(\xi, \theta') = \rho_{e0} \left(\frac{R + \xi}{R + \xi_0} \right)^{-(2 + \cot \theta')}. \quad (10)$$

where: ρ_{e0} and v_{e0} are the out-streaming electron mass density and its velocity at the inner boundary, respectively. The corresponding radial electron current density becomes

$$\vec{J}_e(\xi, \theta') = J_{e0} \left(\frac{R + \xi}{R + \xi_0} \right)^{-(2 + \cot \theta')} \hat{\xi}, \quad (11)$$

where: $J_{e0} = -e\rho_{e0}v_{e0}/m_e$ represents the electron current density at the base of the layer.

2.4 Derivation of Local Induced Magnetic Field

The radial current density from Equation (11) generates an induced azimuthal (toroidal) magnetic field component. Applying the radial component of Ampere's Law under axisymmetric condition gives

$$\frac{1}{(R + \xi) \sin \theta'} \frac{d}{d\theta'} (\sin \theta' B'_{\phi'}(\xi, \theta')) = \mu_0 J_\xi(\xi, \theta'), \quad (12)$$

Integrating with respect to the local polar angle θ' and applying the boundary condition ($B'_{\phi'}(\xi, 0) = 0$), the locally induced toroidal magnetic field becomes

$$B'_{\phi'}(\xi, \theta') = \frac{(R + \xi)\mu_0 J_{e0}}{\sin \theta'} \int_0^{\theta'} \sin \theta' \left(\frac{R + \xi_0}{R + \xi} \right)^{2+\cot \theta'} d\theta'. \quad (13)$$

This Equation is non-elementary due to the non-linear angular function in the exponent and will be evaluated numerically.

2.5 Total Effective Surface Magnetic Field Formulation

To evaluate the total effective surface magnetic field magnitude, B_s , we apply the steady-state radial force balance equation (2) together with the relativistic degenerate electron equation of state ($P_e = K\rho_e^{4/3}$)

$$\frac{d}{d\xi} \left[P_e(\xi, \theta') + \frac{|B_s|^2 + |B'_{\phi'}|^2}{2\mu_0} \right] = -\rho_e(\xi, \theta') \left[\frac{GM}{(R + \xi)^2} - \Omega^2(R + \xi) \sin^2 \theta \right], \quad (14)$$

Where: $P_e(\xi, \theta')$ is the electron degeneracy pressure in the surface layer.

Integrating from the inner boundary ξ_0 to ξ yields the effective surface magnetic field magnitude as

$$B_s = [B_0^2(\theta) - B_{\text{ind}}^2(\theta) + B_{\text{deg}}^2(\theta) + B_{\text{grav}}^2(\theta) + B_{\text{cent}}^2(\theta)]^{1/2}. \quad (15)$$

Where: The first term

$$B_0^2(\theta) = \frac{\mu_0^2 Q^2 \Omega^2}{16\pi^2 R^2} \left[\left(\frac{2}{3} \cos \theta + \frac{3}{4} (\cos^2 \theta - 1) \right)^2 + \sin^2 \theta \left(\frac{1}{3} + \frac{3}{2} \cos \theta \right)^2 \right], \quad (16)$$

is the unperturbed magnetic field emanating from the inner boundary of the rotating neutron star, which we derived by taking the sum of the low-order multipole fields (dipole + quadrupole) produced by a thin, uniformly charged shell of total charge Q [29].

The second term

$$B_{\text{ind}}^2(\theta) = \left[\frac{(R + \xi)\mu_0 J_{e0}}{\sin \theta'} \int_0^{\theta'} \sin \theta' \left(\frac{R + \xi_0}{R + \xi} \right)^{2+\cot \theta'} d\theta' \right]^2, \quad (17)$$

represents the local induced magnetic field due to the radial electron current density from Equation 13 in the thin shell.

The third term

$$B_{\text{deg}}^2(\theta) = 2\mu_0 K \rho_{e0}^{4/3} \left[1 - \left(\frac{R + \xi}{R + \xi_0} \right)^{-\frac{4}{3}(2+\cot \theta')} \right], \quad (18)$$

represents the surface magnetic field contribution from the electron degeneracy pressure difference between the inner boundary (R) and the outer boundary ($R + \xi$) of the surface layer.

The fourth term

$$B_{\text{grav}}^2(\theta) = -\frac{2\mu_0\rho_{e0}GM}{(R + \xi_0)(3 + \cot\theta')} \left[1 - \left(\frac{R + \xi}{R + \xi_0} \right)^{-(3+\cot\theta')} \right], \quad (19)$$

is the gravitational correction, which shows how the magnetic field is modified by the immense gravitational pull of the neutron star.

The fifth term

$$B_{\text{cent}}^2(\theta) = \frac{2\mu_0\rho_{e0}\Omega^2(R + \xi_0)^2 \sin^2\theta}{\cot\theta'} \left[1 - \left(\frac{R + \xi}{R + \xi_0} \right)^{-\cot\theta'} \right], \quad (20)$$

is the centrifugal correction, showing the effect of the neutron star's rotation on the surface magnetic field.

2.6 The Coriolis Force Profile

The radial electron fluid moving through the surface of rotating neutron star experiences Coriolis force density ($f_{\text{Coriolis}} = -2\rho_e \vec{\Omega} \times \vec{v}_e$). Substituting uniform angular velocity $\Omega \hat{z}$ and the derived mass density profile from Equation 10 results in a strictly azimuthal Coriolis force vector

$$\vec{f}_{\text{Coriolis}}(R + \xi, \theta') = -2\rho_{e0}\Omega v_{e0} \sin\theta' \left(\frac{R + \xi}{R + \xi_0} \right)^{-(2+\cot\theta')} \hat{\phi}'. \quad (21)$$

This force exhibits a strong angular dependence, vanishing at the poles ($\theta' = 0^\circ, 180^\circ$) and reaching its maximum at the equator ($\theta' = 90^\circ$) due to its proportionality to $\sin\theta'$. Consequently, electron motion near the poles remains predominantly radial, whereas equatorial flows experience significant azimuthal deflection, generating toroidal currents capable of modifying the local magnetic-field structure. The force also decreases with radial distance according to the power-law factor $\left(\frac{R + \xi}{R + \xi_0} \right)^{-(2+\cot\theta')}$, implying that Coriolis effects are strongest near the stellar surface and weaken outward. Because the radial decay is less severe near the equator than near the poles, equatorial regions maintain a stronger rotational influence over larger distances. Furthermore, the Coriolis force scales linearly with the stellar angular velocity Ω , indicating that rapidly rotating neutron stars, particularly millisecond pulsars, can develop enhanced toroidal currents and more pronounced magnetic-field restructuring through rotationally induced electron deflection.

2.7 Computational Implementation

Numerical evaluation of the non-elementary integrals in Equation 13 and spatial field configurations was conducted via Python utilizing the trapezoidal rule ($N = 1000$

points). Physical parameter choices follow standard typical values for neutron stars, as listed in Table 1. Standard quantities, including mass, radius, surface magnetic field strength, and rotation rate, are taken from widely accepted values reported in the neutron-star literature [1, 3]. In contrast, the shell thickness, effective surface charge, electron-flow velocity, and related quantities are model-specific parameters introduced to satisfy the assumptions of the present thin-shell analytical framework. These values are chosen to investigate the influence of rotation-driven electron flows on localized surface magnetic-field evolution rather than to represent unique observational measurements. Consequently, the numerical results should be interpreted as illustrating the qualitative behavior and underlying physical trends predicted by the model.

Table 1 Typical surface parameter values for the rotating neutron star and its thin surface layer adopted in our computational analysis.

Parameter	Symbol	Adopted value
Neutron star mass	M	$2.78 \times 10^{30} \text{ kg}$
Neutron star radius	R	$1.0 \times 10^4 \text{ m}$
Angular velocity	Ω	100 rad s^{-1}
Surface magnetic field at pole	B_{pole}	$1.0 \times 10^8 \text{ T}$
Total effective surface charge	Q	$1.5 \times 10^{17} \text{ C}$
Shell thickness	h	1.0 m
Base electron number density	n_{e0}	$1.0 \times 10^{20} \text{ m}^{-3}$
Base electron mass density	$\rho_{e0} = m_e n_{e0}$	$9.11 \times 10^{-11} \text{ kg m}^{-3}$
Electron drift velocity	v_{e0}	$1.0 \times 10^6 \text{ m s}^{-1}$
Electron current density	$J_{e0} = n_{e0} e v_{e0}$	$1.60 \times 10^7 \text{ A m}^{-2}$
Polytropic constant	K	$4.93 \times 10^9 \text{ m}^3 \text{ kg}^{-1/3} \text{ s}^{-2}$
Magnetic permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Gravitational constant	G	$6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

3 Result and Discussions

3.1 Current Density Anisotropy

The electron current density given by Equation 11 describes the radial flow of electrons within the thin surface shell of a rotating neutron star. The current density undergoes a systematic power-law dilution due to the increasing shell volume available for the flow. This geometric attenuation is intrinsically coupled to the local polar angle through the $\cot \theta'$ term. As shown in Figure 3, the radial current density decreases with increasing radial distance from the surface. Near the magnetic poles ($\theta' \rightarrow 0^\circ$), the large magnitude of $\cot \theta'$ strongly suppresses the radial current density, producing a rapid radial decay. In contrast, toward the equatorial region ($\theta' \rightarrow 90^\circ$), where $\cot \theta' \rightarrow 0$, the radial electron flow experiences minimal geometric suppression and therefore remains significant over radial distances. Consequently, the radial current distribution becomes highly anisotropic, with a pronounced concentration toward the equatorial plane.

The angular dependence of the current density is illustrated in Figure 4, which shows that the strongest local electron outflows are concentrated around the equatorial

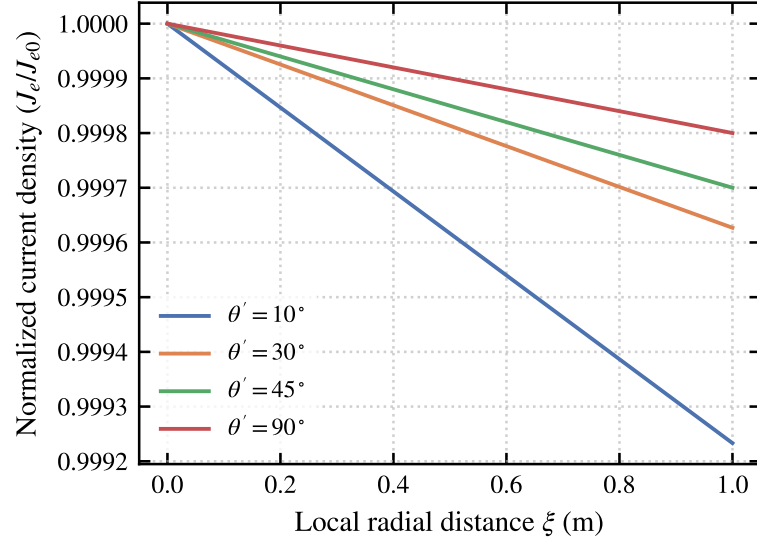


Fig. 3 Radial variation of the electron current density within the thin surface layer of a rotating neutron star at different fixed local polar angles.

region, whereas the polar regions contribute comparatively little to the total current transport.

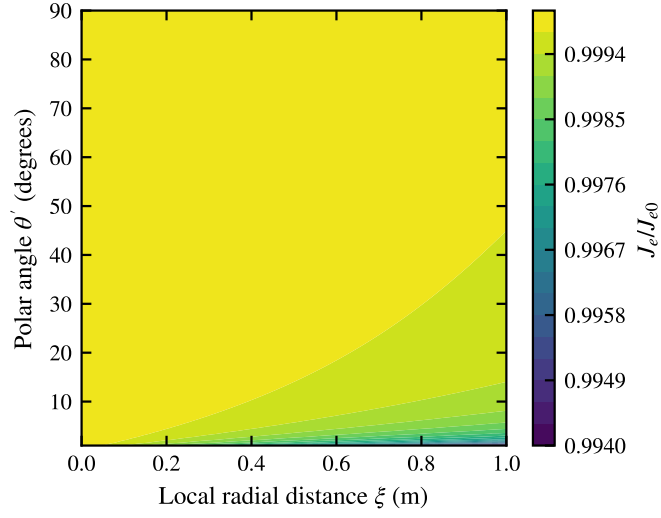


Fig. 4 Contour map showing the normalized radial electron current density within the thin surface layer of a rotating neutron star.

This equatorially enhanced outflow naturally promotes the formation of non-dipolar surface magnetic multipoles through the uneven redistribution of surface currents. These results are broadly consistent with previous studies of neutron-star magnetospheres, pulsar winds, and magnetar magnetic-field evolution [30, 31].

Our analysis indicates that neutron star rotation strongly influences the behavior of local electrons flowing within the thin surface layer. The radial current density decreases with radial distance and exhibits a pronounced angular dependence, becoming concentrated near the equator while remaining substantially weaker near the poles. This anisotropy arises from rotationally induced forces and the combined effects of geometry, which preferentially channel local electron outflows toward lower latitudes.

The resulting non-uniform current distribution modifies the surface magnetic-field topology, facilitating the generation of toroidal and higher-order multipolar magnetic components. Consequently, rotation not only drives anisotropic surface currents but also plays a significant role in the long-term evolution of neutron-star magnetic fields. This mechanism provides a plausible explanation for magnetic-field distortions, localized surface magnetic structures, and the formation of X-ray hot spots observed in highly magnetized neutron stars.

3.2 Induced Toroidal Field Evolution

The numerically integrated induced magnetic-field profile, $B'_{\phi'}$, in Equation (13) demonstrates a clear monotonic increase from zero at the poles to a pronounced maximum near the equatorial plane as shown in Figure 6. This accumulation corresponds to a localized toroidal magnetic structure generated by the Coriolis deflection of the radial electron current. Figure 5 illustrates the radial dependence of $B'_{\phi'}$. For a fixed polar angle θ' , there is a decrease in field strength with increasing ξ . This variation is a small range and almost negligible within the thin shell approximation of our system; it strongly depends on the polar angle. The radial decrease becomes appreciable only when ξ approaches a significant fraction of R , in thicker shells. Although the magnitude of this induced magnetic component remains a relatively small perturbation compared to the underlying stellar magnetic field, its associated magnetic pressure becomes strongly concentrated near the base of the crust ($\xi \rightarrow 0$).

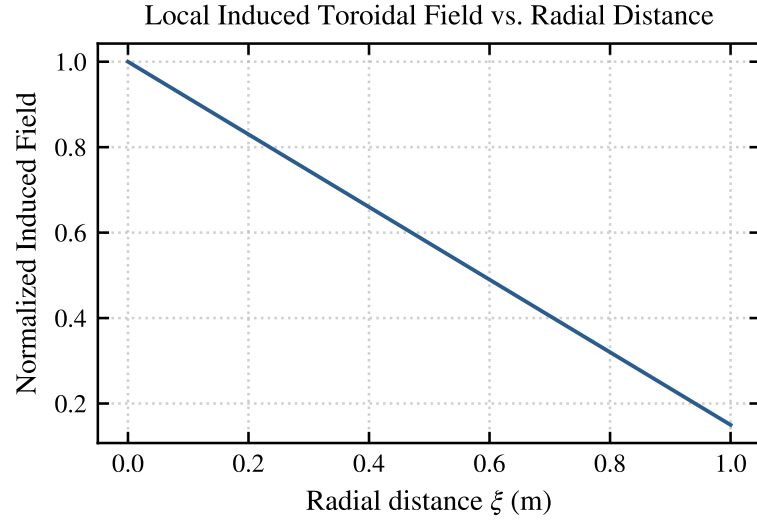


Fig. 5 Radial variation of the induced magnetic field.

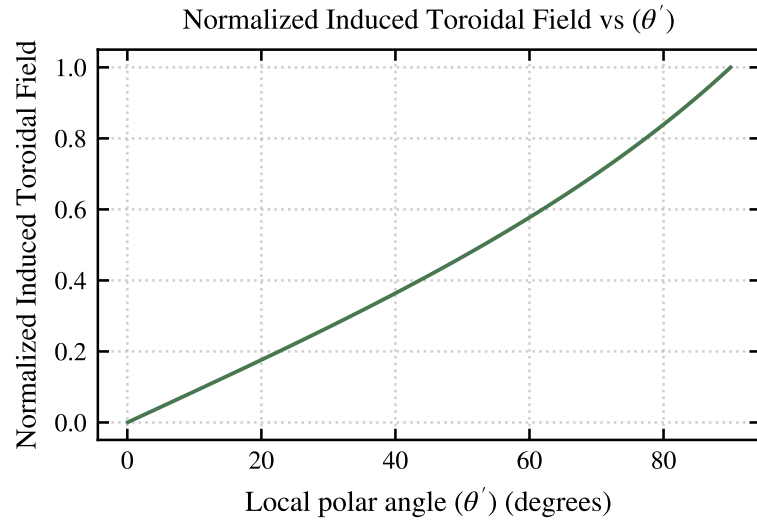


Fig. 6 Radial variation of the induced magnetic field.

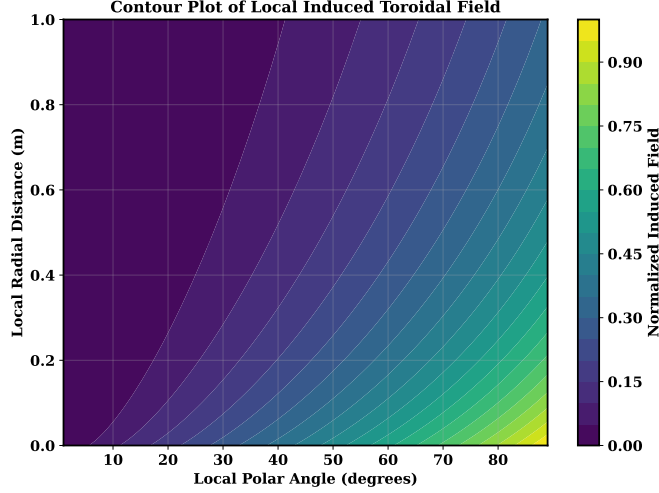


Fig. 7 Two-dimensional contour map showing the spatial distribution of the locally induced surface magnetic field within the neutron star's surface layer.

The contour map in Figure 7 provides a clear coloured visualization of how the induced magnetic field $B'_{\phi'}$ varies across both the radial (ξ) and angular (θ') directions. The field strength is most pronounced in the area closest to the surface ($\xi \rightarrow 0$) and near the equatorial plane ($\theta' \rightarrow 90^\circ$).

Such localized magnetic-field enhancement is sufficient to break the original magnetic symmetry, distort the pre-existing field-line topology, and establish localized magnetically active regions. Consequently, rotation serves as a key physical mechanism that sustains and modulates localized magnetic activity on neutron star surfaces, consistent with previous studies [32–35].

3.3 Total Effective Surface Magnetic Field Redistribution

The calculated total surface magnetic profile B_s in Equation (15) highlights that the surface magnetic field is modified by rotational configurations rather than remaining a static fossil field. The key finding is that neutron star rotation introduces measurable local perturbations that modulate the surface magnetic strength and structure, thereby linking rotational dynamics to observable surface magnetic activity in neutron stars.

The first term from Equation (16) is the global, large-scale magnetic field anchored in the neutron star's crust, modeled as a combination of dipole and quadrupole multipoles, consistent with observed multipolar surface fields of neutron stars. This term represents the reference field before rotationrotationally induced are considered, and it is a peak around mid-latitudes and weakens toward poles/equator as shown in Figure 8(a). The prefactor $\frac{\mu_0}{4\pi} \frac{Q\Omega}{R}$ directly links the magnetic field strength to the star's rotation, which demonstrates that faster-rotating stars naturally generate a stronger surface magnetic field. Taking both dipole and quadrupole contributions provides a

more realistic and detailed description of the local surface magnetic field of neutron stars compared to the global simple dipole approximation [29, 34, 36].

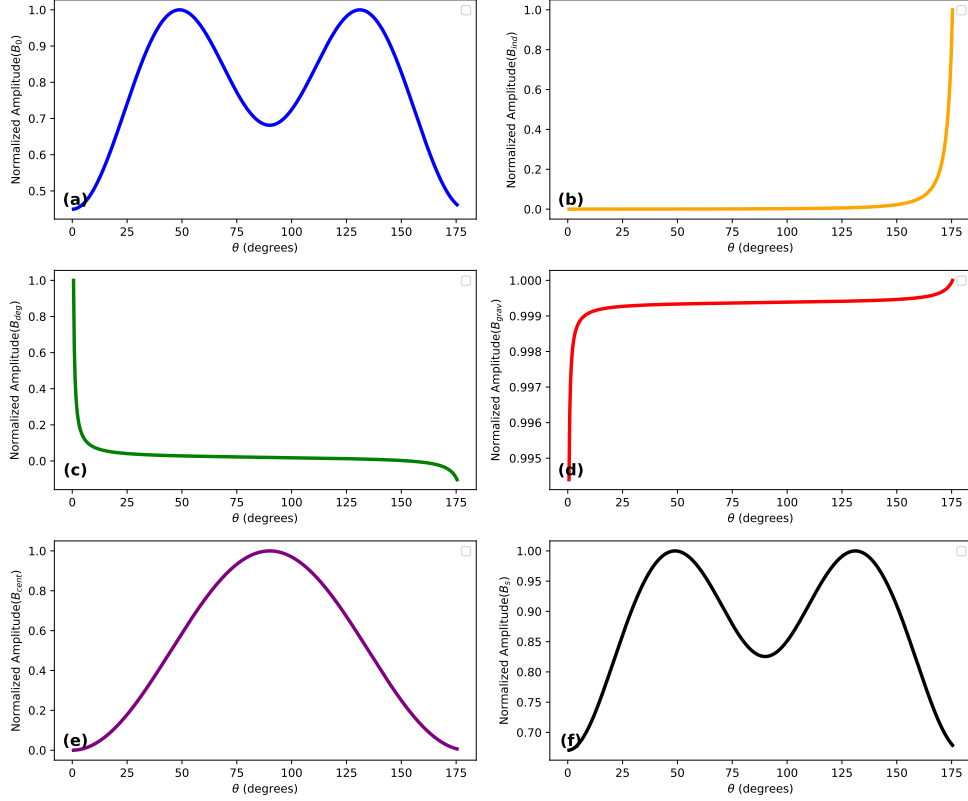


Fig. 8 Individual surface force contributions to the effective surface magnetic field (B_s), all plotted as a function of polar angle at fixed radial distance.

The second contributing term to the effective field from Equation (17) represents micro-scale toroidal field loops that arise from the local radial currents flowing within the neutron star's surface layer. The magnitude increases sharply near the equator, as shown in Figure 8 (b), where $\sin \theta'$ is largest, explaining why field suppression or inversion from rotationally driven local flow can occur at mid-latitudes and equator, a feature consistent with equatorial magnetic spot formation [37, 38].

The third contribution term from Equation (18) originates from the pressure gradient of the degenerate flowing electrons. It scales as $\rho_{e0}^{4/3}$, highlighting the importance of the dense surface plasma in maintaining structural support. Rotation indirectly affects this behavior through the density distribution, which contains the geometric dependence $\cot \theta'$ arising from local flow conditions. In regions with steep density gradients, particularly near the inner boundary, this pressure term enhances the effective

surface magnetic field B_s . The explicit angular dependence reflects its latitudinal variation as shown in Figure 8 (c). Consequently, rotational effects can strengthen the effective magnetic support provided by electron degeneracy pressure, contributing to the stabilization of surface magnetic structures.

The fourth term in Equation (19) introduces a correction term to the effective surface magnetic field due to the gravitational confinement of the surface electrons. Stronger gravity reduces magnetic expansion by compressing the current layer. This effect is most pronounced near the poles, where the vertical component of gravity is dominant, as shown in Figure 8 (d). Physically, this term captures the compression of the magnetic field toward regions of higher gravitational potential. The inward pull counteracts outward magnetic pressure, subtly reshaping the effective magnetic field near the stellar surface and emphasizing the role of gravity in stabilizing and modulating the surface magnetic structure.

The final contributing term to the effective surface magnetic field from Equation (20) represents the centrifugal force. This term directly embodies the outward force from rotation, scaling as $\Omega^2(R + \xi_0)^2 \sin^2 \theta / \cot \theta'$. It is strongest at the equator as shown in Figure 8 (e), where centrifugal forces are maximal, potentially inflating the magnetic field and driving outflows or equatorial enhancements. For millisecond pulsars (high Ω), this term could dominate, leading to distorted surface field geometries and increased magnetic surface activity, such as enhanced radio emissions or particle. The equatorial enhancement aligns with results from global magnetospheric simulations, which show that rotation not only strengthens magnetic activity near the equator but also contributes to observable pulsar phenomena such as beam asymmetries, torque modulation, and surface heating [37, 38].

Comparing the first plot (a) with the subsequent plots (b–f) in Figure 8 clearly illustrates how rotation-driven processes perturb the preexisting unperturbed magnetic field structure. The global field B_0 in plot (a) is a dipole-quadrupole configuration, representing the large-scale magnetic topology anchored in the crust. However, the inclusion of rotation-induced terms introduces systematic local perturbations to this global structure. The combined effect of these rotationally induced terms produces the effective surface magnetic field shown in Figure 8(f). This plot indicates that neutron star rotation introduces local perturbations that modulate the surface magnetic strength and structure, thereby linking rotational dynamics to observable surface magnetic activity in neutron stars.

Canonical neutron star models often assume a global dipole magnetic field inferred from spin-down measurements. However, observations and studies [29] increasingly reveal that the rotation of a neutron star generates a low-order multipole. In our study, we found that rotation-induced electron currents within a thin surface shell induce toroidal perturbations that impact the local effective surface magnetic field.

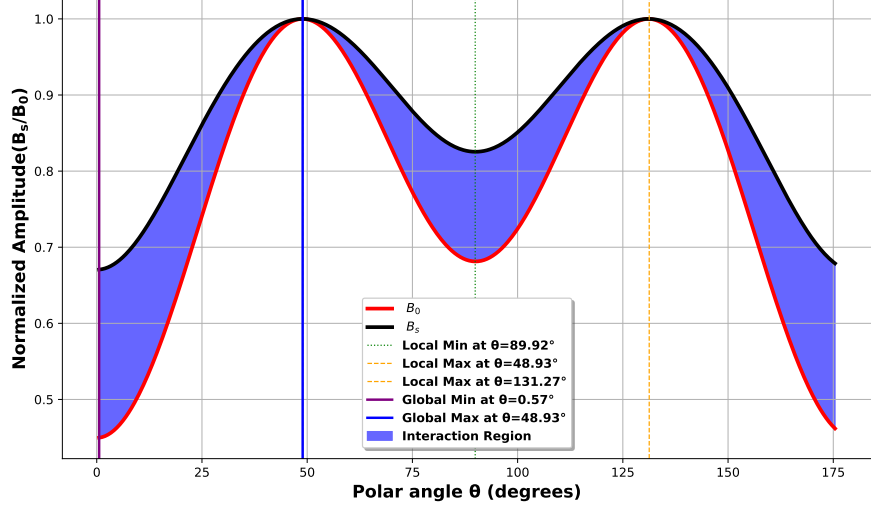


Fig. 9 Comparison of the effective surface magnetic field (B_s) with the unperturbed field (B_0) as a function of polar angle.

Figure 9 presents a comparison between the effective surface magnetic field (B_s) and the pre-existing unperturbed field (B_0) as a function of polar angle. The profile of B_s/B_0 shows a pronounced latitude-dependent variation in magnetic field strength, revealing the strong influence of rotation on the magnetic topology of the neutron star's surface. The global maximum at $\theta \approx 48.93^\circ$ indicates that the magnetic intensity peaks at mid-latitudes rather than at the poles, resulting from the constructive interaction between centrifugal expansion and the unperturbed dipole-quadrupole magnetic geometry. Local minimum near the equator ($\sim 89.92^\circ$) reflects destructive interference between rotation-induced currents and centrifugal effects. The weakest field occurs near the poles ($\sim 0.57^\circ$), where the centrifugal contribution vanishes. The interaction region between the two local maxima (approximately $48.93^\circ < \theta < 131.27^\circ$) represents the area with the steepest gradients and strongest interaction among competing surface forces, where oppositely directed field lines and induced toroidal components can reconnect, converting magnetic energy into thermal and radiative energy. Such reconnection-driven activity provides a plausible surface mechanism for the generation of magnetar-like bursts, X-ray flares, and localized magnetic spots observed in rotating neutron stars, consistent with observations [39, 40].

4 Conclusions

This study developed a self-consistent analytical model within an ideal MHD framework to evaluate the impact of uniform neutron star rotation on local electron dynamics and surface magnetic field configurations within the thin surface layer of a

neutron star. Our findings show that rotation modifies the surface field via two primary pathways: the generation of localized toroidal magnetic loops from rotationally deflected radial currents, and the reconfiguration of the surface force balance.

The resulting effective magnetic field exhibits a highly non-dipolar, anisotropic layout where the peak intensity is shifted away from the poles to mid-latitudes, and a distinct interaction region forms across the equatorial belt. This structural rearrangement provides possible mechanism to create localized gradients capable of driving magnetic reconnection, providing a physical baseline for modelling observed surface anomalies, hotspots, and high-energy transient bursts in active pulsars and magnetars.

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Statements and Declarations

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Competing Interests

The authors declare that they have no competing financial or personal interests that could influence the work reported in this paper.

Author Contributions

TA: Conceptualization, Analytical Derivation, Numerical Coding.

AB: Supervision, Reviewed analytical formulation.

YM: Literature review; Manuscript Drafting.

Data Availability

The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request. The Python scripts used to perform numerical integration and generate figures are also available.

Ethics Approval

Not applicable. This article does not contain any studies involving human participants or live vertebrate animals.

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