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Hybrid LMANN Framework for Predicting Bioconvective Buongiorno Casson Nanofluid Flow over a Riga Plate

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Abstract:

A hybrid Levenberg–Marquardt artificial neural network (LMANN) approach is developed to investigate and predict bioconvective Buongiorno Casson nanofluid flow over a Riga plate. The coupled transport model is formulated through the governing equations for continuity, momentum, microrotation, energy, nanoparticle concentration, and motile microorganism density. By employing appropriate similarity transformations, the governing nonlinear partial differential equations are converted into a system of nonlinear ordinary differential equations. The resulting boundary-value problem is solved numerically using the MATLAB bvp5c solver and the shooting technique. The obtained numerical solutions are utilized as reference data for training, testing, and validation of the proposed LMANN model. The developed network accurately predicts the nonlinear behavior of the velocity, microrotation, temperature, nanoparticle concentration, and microorganism profiles. The close agreement between the numerical and neural-network predictions confirms the reliability and efficiency of the proposed hybrid methodology. The results establish LMANN as a promising alternative computational framework for modeling and predicting complex bioconvective nanofluid transport over Riga plates.

Keywords: Buongiorno nanofluid, Bioconvection, Riga plate, Motile microorganisms, Microrotation, Levenberg–Marquardt artificial neural network

I. Introduction:

Non-Newtonian fluids have attracted research interest owing to their rheological behavior and widespread in engineering, manufacturing, and chemical industries. Unlike Newtonian fluids, their viscosity varies with applied shear rate, enabling them to describe the flow attributes of practical materials, such as molten polymers, lubricants, drilling muds, food products, and biological suspensions. Consequently, non-Newtonian fluid models have become indispensable for analyzing transport phenomena in coating processes, wire drawing, glass-fiber manufacturing.

Among the available constitutive models, the Casson fluid model got noteworthy attention on account of its competence to typify yield-stress fluids encountered in medical applications. Kameswaran et al. [1] investigated the flow by incorporating Casson model and demonstrated that the Casson parameter considerably influences the flow attributes. More recently, Zheng et al. [2] employed the LMANN technique to predict radiative Casson fluid flow with viscous dissipation and chemical reaction effects, illustrating the potential of intelligent computational techniques for highly nonlinear transport problems. Similarly, Maddina et al. [3] reported Casson fluid flow over a rotating non-uniform surface and confirmed their capability to accurately predict complex transport behavior. Collectively, these investigations establish the Casson fluid model as an important framework for studying nonlinear transport processes in thermal engineering, biomedical systems, and data-driven computational analysis.

Another important non-Newtonian model is the Williamson fluid. Its rheological characteristics provide a realistic description of coupled momentum, heat, and mass transfer under magnetohydrodynamic conditions. Jalili et al. [4] and Shafiq et al. [5] demonstrated that the Williamson fluid formulation accurately predicts nonlinear transport behavior under the combined influence of magnetic fields, thermal effects, and other governing physical parameters, confirming its suitability for advanced numerical investigations of non-Newtonian flow systems. Sherani et al. [6] reported the flow past a nonlinear surface of Williamson fluid using Buongiorno model.

The flow due to stretching surfaces is a key area of boundary-layer research because of its direct relevance thermal processing. Stretching surfaces arise in polymer sheet production, glass-fiber manufacturing, metal spinning, continuous casting, crystal growth, and electronic

cooling. Sharma [7] demonstrated the engineering significance of exponentially stretching sheets for Darcy–Forchheimer nanofluid flow. Furthermore, Jameel et al. [8] reported that flow parameters substantially influence Maxwell nanofluid transport over stretching surfaces. Recently, a comparative analysis was presented by Saleem et al. [9] considering Williamson–Casson model and flow geometry is exponentially stretching surface. The slip Darcy–Forchheimer flow caused by a nonlinear stretching sheet was explored by Rassol et al. [10]. Further, Divya et al. [11] published their research work to analyse the flow over a rigid plate using incorporating Tiwari and Das [12] nanofluid model. Bilal et al. [13] presented a study on nanofluid flow across a rigid plate adopting artificial neural network model. They found that skin friction is influenced due to rigid surface. These studies collectively confirm that stretching surface configurations provide a reliable platform for investigating nonlinear transport mechanisms relevant to modern industrial technologies.

Nanofluids have emerged as efficient heat-transfer media because the dispersion of nanoparticles into conventional base fluids significantly improves thermal conductivity and transport performance. The pioneering work of Choi [14] initiated extensive research on nanofluid technology, whereas the Buongiorno [15] model identified Brownian diffusion and thermophoresis as the dominant slip mechanisms governing nanoparticle transport. Buongiorno's model says that metallic nanoparticles strongly improve the thermal performance of ordinary fluids, making nanofluids attractive for modern cooling technologies. Building on this concept, Ullah et al. [16] demonstrated that Brownian diffusion and thermophoresis strongly sway nanoparticle concentration and heat-transfer. Ajam et al. [17] used Buongiorno model to investigate the basic flow characteristics of a nanofluid past a stretching sheet. Similarly, Al-Kouz and Owhaib [18] incorporated the Buongiorno model into rotating Casson nanofluid flow and noticed enhanced thermal transport due to the combined influence of Brownian motion and thermophoretic migration. Furthermore, Alebraheem and Ramzan [19] investigated Casson nanofluids containing gyrotactic microorganisms using the Cattaneo–Christov heat flux model, while Awwad et al. [20] emphasized that Brownian diffusion and thermophoresis play decisive roles in improving thermal performance in buoyancy-driven nanofluid transport.

The incorporation of activation energy and motile microorganisms has further expanded the capability of MHD nanofluid models by simultaneously accounting for chemically reactive transport and bioconvection. Activation energy governs reaction kinetics and concentration transport, whereas gyrotactic microorganisms improve nanoparticle suspension through

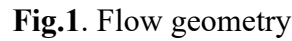
bioconvection. Faisal et al. [21] addressed the flow over orthogonal disks in the presence of activation energy (AE) and revealed that AE depreciated concentration distribution. Nihaal et al. [22] and Siva Sankari et al. [23] demonstrated that activation energy significantly affects concentration and heat-transfer characteristics in nonlinear MHD nanofluid flows. Likewise, Chaudhry et al. [24] and reported that motile microorganisms regulate microorganism distribution and bioconvective transport.

Alongside these physical developments, artificial intelligence has emerged as a powerful computational tool for nonlinear transport analysis. Among various machine-learning techniques, the Levenberg–Marquardt Artificial Neural Network (LMANN) has gained considerable attention because of its ability to establish accurate nonlinear relationships between governing parameters and transport variables without repeatedly solving the governing equations. Kumar et al. [25] and Kumar et al. [26] demonstrated that LMANN accurately predicts velocity, temperature, and concentration distributions in complex MHD transport problems. Likewise, Anjum et al. [27] and Barakat et al. [28] reported excellent convergence, predictive accuracy, and computational efficiency of the Levenberg–Marquardt algorithm, while Chrimatopoulos et al. [29] emphasized its applicability to engineering systems involving electrically conducting fluids.

Although substantial progress has been achieved in modeling non-Newtonian nanofluid flows, previous investigations have generally focused on individual physical mechanisms or specific computational approaches. The combined influence of Casson/Williamson rheology, stretching-surface transport, activation energy, motile microorganisms, and intelligent LMANN-based prediction remains comparatively less explored for the present flow configuration. Motivated by this gap, the present study develops a comprehensive computational framework to investigate the coupled transport phenomena and employs the Levenberg–Marquardt artificial neural network to obtain accurate, robust, and computationally efficient predictions for nonlinear MHD heat and mass transfer characteristics.

II. Modelling of the Problem

We consider the Casson–Williamson 2D Darcy–Forchheimer flow past an exponentially stretchable plate. The x-axis is perpendicular to the stretching direction, and the y-axis is parallel to it.


$$W = -pI + \tau_w$$

Where $\mathbf{Z}$ denotes the first Rivlin–Ericksen tensor, $\Gamma_0 > 0$ represents the time constant, μ_0 and μ_∞ are the limiting viscosity at zero and infinite shear rates, respectively, $\mathbf{W}$ is the extra stress tensor, $\bar{A}$ also represents a tensor and $\pi = \text{trace}(\mathbf{S}^2) = \text{trace}(\nabla V + (\nabla V)^T)^2$.

$$\tau_w = \left(\frac{\mu_0}{1 - \Gamma_0 \bar{A}} \right) S \rightarrow \mu_0 (1 + \Gamma_0 \bar{A}) S \quad (2)$$

The shear stress tensor of the Casson fluid model is incorporated as

$$\tau_{ij} = \begin{cases} 2 \left(\mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c, \\ 2 \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c, \end{cases} \quad (3)$$

Where, τ_{ij} denotes Shear stress tensor (Pa), e_{ij} denotes Rate of deformation tensor, μ_B denotes Plastic dynamic viscosity of Casson fluid (Pa·s), p_y denotes Yield stress of the fluid (Pa), π denotes Product of deformation tensor components (second invariant), π_c denotes Critical value of π (yield condition threshold).

According to the standard Boussinesq approximations, the thermal conductivity and viscosity are considered to be temperature-dependent, whereas the mass diffusivity is assumed to be a concentration function. As a result, the Williamson–Casson boundary layers approximation for the velocity, temperature, and concentrations are:

$$l \frac{\partial l}{\partial x} + m \frac{\partial m}{\partial y} = 0 \quad (4)$$

$$l \frac{\partial l}{\partial x} + m \frac{\partial l}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 l}{\partial y^2} - \frac{v}{k} \left(1 + \frac{1}{\beta} \right) l + \sqrt{2} v \Gamma_0 \frac{\partial l}{\partial y} \frac{\partial^2 l}{\partial y^2} - \frac{F'}{\sqrt{R}} l^2 + \frac{\pi J_0 M_0}{8 \rho_f} \exp \left(-\frac{\pi}{a} y \right) + g \left[\frac{(1 - C_\infty) \beta_1 (T - T_\infty) - (\rho_p - \rho_f) \beta_2 (L - L_\infty)}{-(\rho_m - \rho_p) \beta_3 (N - N_\infty)} \right] \quad (5)$$

$$l \frac{\partial T}{\partial x} + m \frac{\partial T}{\partial y} + \lambda_E \Omega_E = \left(\frac{1}{\rho C_p} \right) \left(\kappa + \frac{16}{3} \frac{\sigma^* T_\infty^3}{\kappa^*} \right) + \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial L}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (6)$$

$$l \frac{\partial L}{\partial x} + m \frac{\partial L}{\partial y} = D_B \frac{\partial^2 L}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - K_r (L - L_\infty) \left(\frac{T}{T_\infty} \right)^n \exp \left(-\frac{E_a}{WT} \right) \quad (7)$$

$$l \frac{\partial N}{\partial x} + m \frac{\partial N}{\partial y} = D_m \frac{\partial^2 N}{\partial y^2} - \frac{b^* W c}{(L_w - L_\infty)} \frac{\partial}{\partial y} \left[N \frac{\partial L}{\partial y} \right] \quad (8)$$

$$\text{Where } \Omega_E = l \frac{\partial l}{\partial x} \frac{\partial T}{\partial x} + m \frac{\partial m}{\partial y} \frac{\partial T}{\partial y} + l \frac{\partial m}{\partial x} \frac{\partial T}{\partial y} + m \frac{\partial l}{\partial y} \frac{\partial T}{\partial x} + 2lm \frac{\partial^2 T}{\partial x \partial y} + l^2 \frac{\partial^2 T}{\partial x^2} + m^2 \frac{\partial^2 T}{\partial y^2};$$

$$\tau = \frac{(\rho C_p)_p}{(\rho C_p)_f} \text{ is the heat-capacity ratio.}$$

The appropriate BCs are

$$\text{At } y = 0 \Rightarrow l = l_w(x) = l_0 e^{\frac{x}{l_1}}, m = -m_w(x) = m_0 e^{\frac{x}{2l_1}}, T = T_w(x), L = L_w(x), N = N_w(x), \quad (9)$$

$$\text{As } y \rightarrow \infty \Rightarrow l \rightarrow 0, T \rightarrow T_\infty, L \rightarrow L_\infty, N \rightarrow N_\infty, \quad (10)$$

where l and m indicate the velocity components in x and y directions of the fluid. The l_0 and m_0 are the reference velocities and $m_w(x)$ represents the suction/injection phenomena. $m_w(x) > 0$ represents the suction velocity and $m_w(x) < 0$ represents the injection velocity.

The permeability, drag and Riga filed are $k = k_0 e^{\frac{x}{l_1}}$ and $F' = \frac{C_b}{x\sqrt{k}}$.

$$\begin{aligned} l &= l_0 e^{\frac{x}{l_1}} f'(\xi); \quad m = -\sqrt{\frac{l_0 v}{2l_1}} e^{\frac{x}{2l_1}} [f(\xi) + \xi f'(\xi)]; \quad \xi = y \sqrt{\frac{l_0}{2vl_1}} e^{\frac{x}{2l_1}}; \quad T = T_\infty + (T_w - T_\infty) e^{\frac{x}{2l_1}} \theta(\xi); \\ L &= L_\infty + (L_w - L_\infty) e^{\frac{x}{2l_1}} \phi(\xi); \quad N = N_\infty + (N_w - N_\infty) e^{\frac{x}{2l_1}} \chi(\xi); \quad \theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty}; \quad \phi(\xi) = \frac{L - L_\infty}{L_w - L_\infty}; \\ \chi(\xi) &= \frac{N - N_\infty}{N_w - N_\infty}; \end{aligned} \quad (11)$$

Equation (4) is fulfilled by using these transformations, as are the following

Eqs. (5)–(8) with BCs (9) and (10)

$$\left(1 + \lambda f''\right) \left(1 + \frac{1}{\beta}\right) f''' - \left(1 + \frac{1}{\beta}\right) D_a f' + f f'' - 2f'^2 - F_c f'^2 \quad (12)$$

$$+ M \exp(-m_1 \xi) + \lambda_1 [\theta - Nr\phi - Rb\chi] = 0$$

$$\theta''(1 + Rc) + Pr[\theta' f - \theta f'] - Pr \Gamma_E [f^2 \theta'' - 3ff' \theta' + 3f'^2 \theta - ff'' \theta'] + Nb Pr \theta' \phi' + Nt Pr \theta'^2 = 0 \quad (13)$$

$$\phi'' + \frac{Nt}{Nb} \theta'' + Le[f\phi' - f'\phi] - Le\sigma_m [1 + \Gamma\theta]^n e^{\left[-\frac{E}{1+\Gamma\theta}\right]} \phi = 0 \quad (14)$$

$$\chi'' + Lb(f\chi' - f'\chi) - Pe[(\chi + \delta)\phi'' + \chi'\phi'] = 0 \quad (15)$$

with the prime representation having differentiation with respect to ξ

$$f(0) = S, f'(0) = 1, \theta(0) = 1, \phi(0) = 1, \chi(0) = 1 \quad (16)$$

$$\text{As } \xi \rightarrow \infty \Rightarrow f'(\xi) \rightarrow 0, \theta(\xi) \rightarrow 0, \phi(\xi) \rightarrow 0, \chi(\xi) \rightarrow 0 \quad (17)$$

Here prime is $\frac{d}{d\xi}$.

The dimension less factors are:

$$\lambda = \Gamma_0 l_w(x) \sqrt{\frac{l_0 e^{\frac{x}{l_1}}}{\nu l_1}} \text{ (Williamson parameter),}$$

$$\beta = \frac{\mu_B \sqrt{2\pi_c}}{p_y} \text{ (Casson parameter),}$$

$$D_a = \frac{\nu l_1}{l_0 k_0} \text{ (Darcy parameter),}$$

$$Fc = \frac{l_1 C_b}{x \sqrt{k}} \text{ (Forchheimer number),}$$

$$M = \frac{\pi J_0 M_0 l_1}{4 \rho_f l_0^2 e^{\frac{x}{l_1}}} \text{ (Modified Hartmann number),}$$

$$m_1 = \frac{\pi}{a e^{\frac{x}{2l_1}}} \sqrt{\frac{2\mu_f l_1}{l_0 \rho_f}} = \frac{\pi}{a e^{\frac{x}{2l_1}}} \sqrt{\frac{2\nu_f l_1}{l_0}} \text{ (modified Hartmann number or Riga plate parameter),}$$

$$\lambda_1 = \frac{2l_1 g \beta_1 (1 - L_\infty)(T_w - T_\infty)}{l_0^2 e^{\frac{x}{l_1}}} \text{ (mixed convection parameter),}$$

$$Nr = \frac{(\rho_p - \rho_f) \beta_2 (L_w - L_\infty)}{\rho_f \beta_1 (1 - L_\infty)(T_w - T_\infty)} \text{ (buoyancy-ratio parameter),}$$

$$Rb = \frac{\beta_3 (\rho_m - \rho_f)(N_w - N_\infty)}{\rho_f \beta_1 (1 - L_\infty)(T_w - T_\infty)} \text{ (bioconvection Rayleigh parameter),}$$

$$Pr = \frac{\nu}{\alpha} = \frac{(\mu C_p)_f}{k_f} \text{ (Prandtl number),}$$

$$Rc = \frac{16}{3} \frac{\sigma^* T_\infty^3}{kk^*} \text{ (Thermal radiation parameter),}$$

$$Nb = \frac{D_B (L_w - L_\infty)(\rho C_p)_p}{\nu_f (\rho C_p)_f} \text{ (Brownian motion parameter),}$$

$$Nt = \frac{D_T (\rho C_p)_p (T_w - T_\infty)}{v_f (\rho C_p)_f T_\infty} \text{ (Thermophoresis parameter),}$$

$$\Gamma_E = \frac{\lambda_E l_0 e^{\frac{x}{l_1}}}{2l_1} \text{ (thermal relaxation parameter),}$$

$$Le = \frac{v_f}{D_B} \text{ (Lewis number),}$$

$$\sigma_m = \frac{2l_1 k_r^2}{l_0 e^{\frac{x}{l_1}}} \text{ (reaction factor),}$$

$$\Gamma = \frac{T_w - T_\infty}{T_\infty} \text{ (temperature difference),}$$

$$E = \frac{E_a}{wT_\infty} \text{ (activation energy),}$$

$$Lb = \frac{v_f}{D_m} \text{ (bioconvection Lewis number),}$$

$$Pe = \frac{b^* Wc}{D_m} \text{ (Peclet),}$$

$$\delta = \frac{N_\infty}{N_w - N_\infty} \text{ (microorganism-difference parameter),}$$

$$S = m_0 \sqrt{\frac{2l_1}{l_0 v}} \text{ (suction/injection parameter).}$$

The physical quantities after transformation as dimensionless quantities are:

$$\sqrt{2\text{Re}_x} Cf_x = \left(1 + \frac{1}{\beta}\right) \left[f'' + \frac{\lambda}{2} f'^2 \right]_{\xi=0} \quad (18)$$

$$\frac{Nu_x}{\sqrt{\text{Re}_x}} = -(1 + Rc) [\theta']_{\xi=0} \quad (19)$$

$$\frac{Sh_x}{\sqrt{Re_x}} = -[\phi']_{\xi=0} \quad (20)$$

$$\frac{Nn_x}{\sqrt{Re_x}} = -[\chi']_{\xi=0} \quad (21)$$

III. Discussion of results

The numerical and LMANN-based prediction results obtained from the proposed mathematical model are presented and discussed in this section. A comprehensive parametric analysis is conducted to examine the effects of the governing dimensionless parameters on the velocity, temperature, nanoparticle concentration, and motile microorganism distributions. To clearly identify the influence of each parameter, only one parameter is varied while all remaining parameters are held constant. The resulting profiles are presented in Figs. (2–55) and interpreted based on the underlying physical mechanisms governing momentum, heat, mass, and microorganism transport. In addition, the predictive capability of the proposed Hybrid LMANN framework is evaluated through its network architecture, training performance, regression analysis, fitness and error curves, error histogram, and training-state plots. These results demonstrate the effectiveness, convergence, robustness, and prediction accuracy of the proposed LMANN model in capturing the complex nonlinear behavior of bioconvective Buongiorno Casson nanofluid flow over a Riga plate.

Figs. (2–9) depict the effects of the Williamson parameter, mixed convection parameter, modified Hartmann number (Riga plate parameter), buoyancy-ratio parameter, bioconvection Rayleigh parameter, Darcy parameter, Forchheimer number, and Casson parameter on the velocity distribution. The velocity profile is significantly enhanced by increasing the mixed convection parameter because stronger buoyancy forces accelerate the fluid. In contrast, the remaining parameters reduce the velocity by increasing the overall flow resistance. Specifically, the Williamson parameter raises the effective viscosity, the modified Hartmann number intensifies the electromagnetic braking force, the Darcy and Forchheimer parameters increase porous and inertial resistance, the buoyancy-ratio and bioconvection Rayleigh parameters strengthen opposing buoyancy effects, and the Casson parameter increases the fluid yield stress, collectively leading to a reduction in the momentum boundary layer thickness.

The temperature distribution is affected by the Brownian motion parameter, thermophoresis parameter, Prandtl number, thermal radiation parameter, and suction/injection parameter, as shown in Figs. (10–14). The temperature profile is found to rise with greater values of the

Brownian motion, thermophoresis, and thermal radiation parameters while decreasing with rising Prandtl number and suction parameter. Physically, a higher Prandtl number corresponds to lower thermal diffusivity, which weakens heat diffusion and reduces the thermal boundary layer thickness. Similarly, suction extracts the heated fluid from the surface, thereby suppressing the temperature field. In contrast, Brownian motion enhances random nanoparticle movement, promoting energy transport within the fluid, while thermophoresis drives nanoparticles from the heated surface toward cooler regions, increasing the thermal energy of the fluid. Furthermore, thermal radiation supplies additional heat to the flow, resulting in an increase in the fluid temperature and a thicker thermal boundary layer.

Figs. (15–17) illustrate the effects of the activation energy, temperature difference parameter, and Lewis number on the nanoparticle concentration profile, whereas Figs. (18–19) present the influence of the Peclet number and bioconvection Lewis number on the motile microorganism distribution. It is evident that increasing the activation energy enhances the concentration profile by weakening the chemical reaction rate, thereby reducing nanoparticle consumption. On the other hand, because of increased thermal diffusion and decreased mass diffusivity, respectively, greater values of the temperature difference parameter and Lewis number result in a decrease in concentration. Additionally, by decreasing microbe diffusion and enhancing convective transport, raising the Peclet number and bioconvection Lewis number inhibits the dispersion of motile microorganisms.

Figs. (20–24) illustrate the proposed LMANN framework and its predictive performance for the bioconvective Casson nanofluid flow over a Riga plate. The multilayer LMANN architecture effectively learns the nonlinear relationships between the governing parameters and the corresponding velocity, temperature, nanoparticle concentration, and motile microorganism density profiles. Physically, the hidden layer captures the coupled effects of momentum, heat, mass, and microorganism transport, enabling accurate prediction of the complex flow behavior. For increasing values of the Casson parameter (β), the velocity decreases because the higher yield stress increases the fluid resistance, whereas the temperature, nanoparticle concentration, and motile microorganism density increase due to weakened convective heat and mass transport, resulting in greater thermal energy and species accumulation within the boundary layer. The excellent agreement between the LMANN predictions and the numerical results, together with the negligible fitting errors (of the order of 10^{-3}), demonstrates the high accuracy, robustness, and reliability of the proposed LMANN framework.

Table 1 presents the training progress of the proposed LMANN model for the Casson parameter (β). The network converged after 710 epochs within 19 s, achieving a low-performance error of 1.47×10^{-6} and a small gradient of 3.94×10^{-6} . The training stopped after 20 validation checks, confirming stable convergence, excellent learning capability, and high prediction accuracy. Fig. 25 illustrates the best validation performance of the LMANN model for the Casson parameter (β). The training, validation, and testing errors converge rapidly to a minimum mean squared error of 1.47×10^{-6} at 710 epochs, demonstrating stable convergence, excellent generalization, and high prediction accuracy with negligible overfitting. Fig. 26 illustrates the error histogram of the LMANN model for the Casson parameter (β). Most prediction errors are clustered around zero, indicating excellent agreement with the numerical results. The narrow error distribution confirms the high prediction accuracy, robustness, and reliability of the proposed LMANN model. Fig. 27 illustrates the neural network regression performance of the LMANN model for the Casson parameter (β). The predicted outputs closely match the target values and are nearly aligned with the reference line ($Y = T$), indicating excellent regression accuracy. The strong agreement between the predicted and target data demonstrates the robustness, reliability, and high predictive capability of the proposed LMANN model. Fig. 28 presents the training state of the LMANN model for the Casson parameter (β). The rapid reduction in the gradient, adaptive adjustment of the learning parameter (μ), and termination after 20 validation checks indicate stable convergence and effective prevention of overfitting. These results demonstrate the robustness and high predictive capability of the proposed LMANN framework.

Fig. 29 presents the LMANN architecture developed for the Williamson parameter (λ). The multilayer network learns the nonlinear mapping between the governing parameters and the flow responses, while the hidden layer captures the coupled transport mechanisms of momentum, heat, mass, and microorganisms. This architecture provides accurate, stable, and reliable prediction of the Williamson nanofluid flow characteristics. Figs. (30–33) present the LMANN-predicted velocity, temperature, nanoparticle concentration, and motile microorganism density profiles, together with their corresponding fitting errors, for different values of the Williamson parameter (λ). It is observed that increasing λ suppresses the velocity due to the enhanced non-Newtonian elastic effects, which increase the fluid resistance and reduce the momentum boundary layer thickness. In contrast, the temperature, nanoparticle concentration, and motile microorganism density profiles increase because the reduced fluid motion weakens convective heat and mass transport, promoting the accumulation of thermal

energy, nanoparticles, and microorganisms within the boundary layer. The LMANN predictions closely match the numerical results, while the fitting errors remain very small (of the order of 10^{-3}), confirming the high accuracy, robustness, and reliability of the proposed LMANN framework.

Table 2 and Figs. (34–37) summarize the training performance and predictive capability of the proposed LMANN model for the Williamson parameter (λ). The network completed 1000 epochs in 11 s, achieving a low-performance error of 1.29×10^{-7} with a small gradient of 5.82×10^{-7} , indicating efficient learning and stable convergence. The best validation curve shows that the training, validation, and testing errors converge smoothly with excellent generalization and no evidence of overfitting. The error histogram reveals that most prediction errors are concentrated around the zero-error line, confirming the high accuracy of the model. Likewise, the neural network regression plots demonstrate an excellent agreement between the predicted and target values, with the data points closely aligned along the reference line ($Y = T$). Furthermore, the training-state plots exhibit a steadily decreasing gradient, adaptive adjustment of the learning parameter (μ), and stable validation performance throughout the training process. Overall, these results verify the robustness, convergence, and high predictive accuracy of the proposed LMANN framework for modeling the Williamson parameter.

Figs. (38–41) illustrate the LMANN architecture together with the predicted velocity, temperature, and nanoparticle concentration profiles and their corresponding fitting errors for different values of the thermal relaxation parameter (Γ_E). The multilayer LMANN effectively captures the nonlinear relationship between the governing parameters and the flow characteristics. It is observed that increasing Γ_E suppresses both the velocity and temperature profiles because the thermal relaxation effect delays heat propagation and weakens thermal diffusion. In contrast, the nanoparticle concentration increases due to the reduction in thermal energy transport, which weakens thermophoretic transport and promotes nanoparticle accumulation within the boundary layer. The LMANN predictions closely match the numerical solutions, while the fitting errors remain very small (of the order of 10^{-3}), demonstrating the high accuracy, robustness, and reliability of the proposed LMANN framework. Fig. 42 shows the LMANN-predicted motile microorganism density profile and the corresponding fitting error for different values of the thermal relaxation parameter (Γ_E). Increasing Γ_E enhances the microorganism density due to weaker convective transport. The excellent agreement between the LMANN and numerical results, together with the negligible fitting errors (of the order of 10^{-3}), confirms the accuracy and reliability of the proposed LMANN framework.

Table 3 summarizes the training progress of the proposed LMANN model for the thermal relaxation parameter (Γ_E). The network achieved a very low-performance error and gradient, indicating efficient learning and stable convergence. The training process demonstrates the model's ability to accurately capture the nonlinear relationship between the thermal relaxation parameter and the flow characteristics, ensuring robust and reliable predictions. Fig. 43 presents the best validation performance of the LMANN model for the thermal relaxation parameter (Γ_E). The smooth convergence of the training, validation, and testing curves to a low mean squared error indicates stable convergence, excellent generalization, and high prediction accuracy. Fig. 44 illustrates the error histogram of the LMANN model for the thermal relaxation parameter (Γ_E). The prediction errors are clustered around zero, demonstrating excellent fitting accuracy and strong agreement with the numerical results. Fig. 45 illustrates the neural network regression performance of the LMANN model for the thermal relaxation parameter (Γ_E). The predicted values closely match the target values, indicating excellent regression accuracy, strong generalization, and reliable predictive performance. Fig. 46 presents the training state of the LMANN model for the thermal relaxation parameter (Γ_E). The steadily decreasing gradient, adaptive adjustment of the learning parameter (μ), and stable validation checks demonstrate efficient convergence, effective learning, and high prediction accuracy.

Fig. 47 illustrates the proposed LMANN architecture for predicting the flow characteristics corresponding to the activation energy parameter (E). The multilayer network effectively learns the nonlinear relationship between the governing parameters and the velocity, temperature, nanoparticle concentration, and motile microorganism density profiles. Physically, the hidden layer captures the coupled effects of momentum, heat, mass, and microorganism transport, enabling accurate prediction of the complex transport behavior. The proposed LMANN architecture demonstrates robust learning capability and reliable prediction of the activation energy effects on the bioconvective nanofluid flow over the Riga plate. Figs. (48–51) present the LMANN-predicted velocity, temperature, nanoparticle concentration, and motile microorganism density profiles, together with their corresponding fitting errors, for different values of the activation energy parameter (E). It is observed that increasing E enhances the velocity, temperature, concentration, and motile microorganism density. Physically, higher activation energy suppresses the chemical reaction rate, reducing species consumption and energy depletion, which promotes thermal energy, nanoparticle, and microorganism accumulation within the boundary layer while weakening the resistive effects on the flow. The

LMANN predictions exhibit excellent agreement with the numerical results, and the fitting errors remain negligible (of the order of 10^{-3}), confirming the high accuracy, robustness, and reliability of the proposed LMANN framework.

Table 4 and Figs. (52–55) summarize the training performance and predictive capability of the proposed LMANN model for the activation energy parameter (E). The network achieved a very low-performance error and gradient, indicating efficient learning and stable convergence. The best validation results show smooth convergence of the training, validation, and testing curves with excellent generalization and no evidence of overfitting. The error histogram reveals that most prediction errors are concentrated around the zero-error line, confirming the high accuracy of the model. Similarly, the neural network regression plots demonstrate excellent agreement between the predicted and target values, with the data points closely aligned along the reference line ($Y = T$). Furthermore, the training-state plots exhibit a steadily decreasing gradient, adaptive adjustment of the learning parameter (μ), and stable validation performance throughout the training process. Overall, these results verify the robustness, convergence, and high predictive accuracy of the proposed LMANN framework for modeling the activation energy parameter (E).

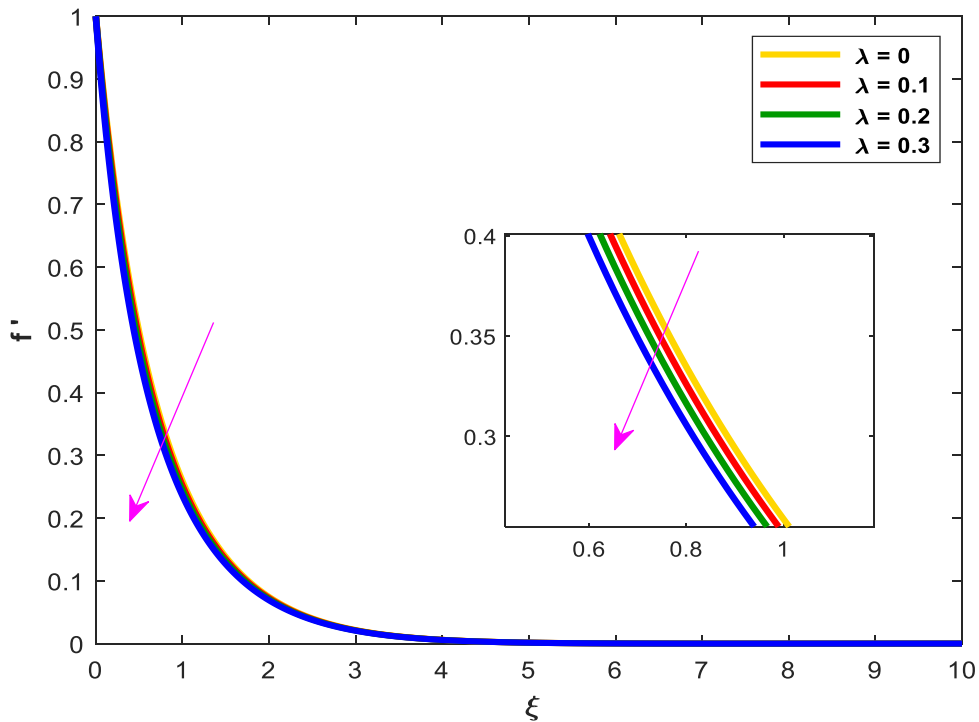


Figure.2. velocity verses λ

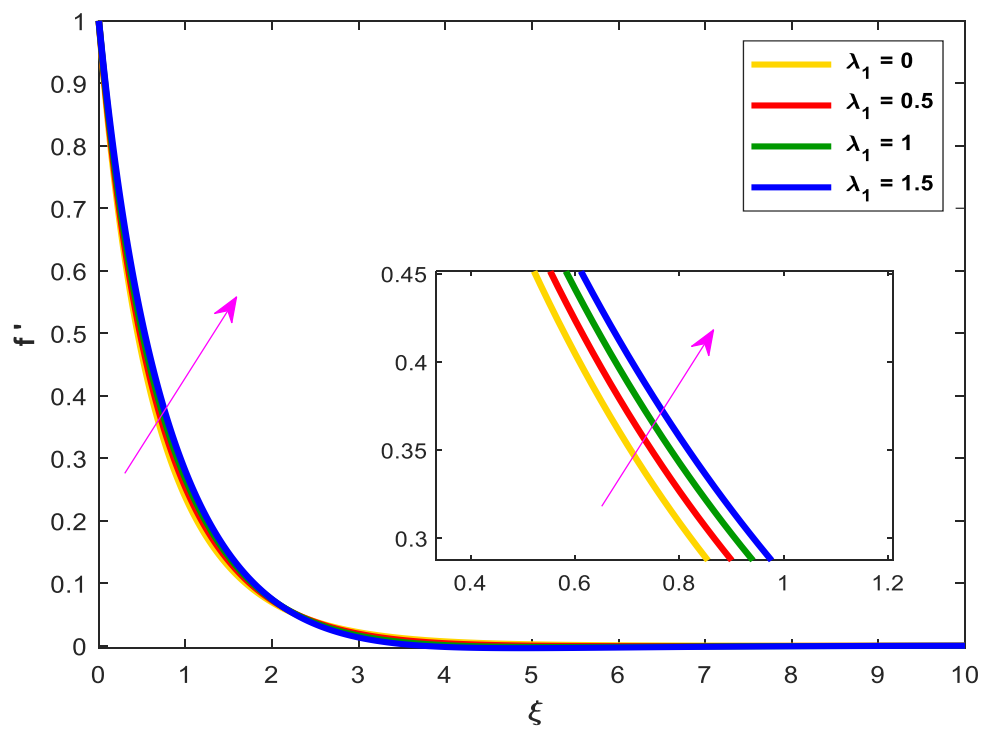


Figure.3. velocity verses λ_1

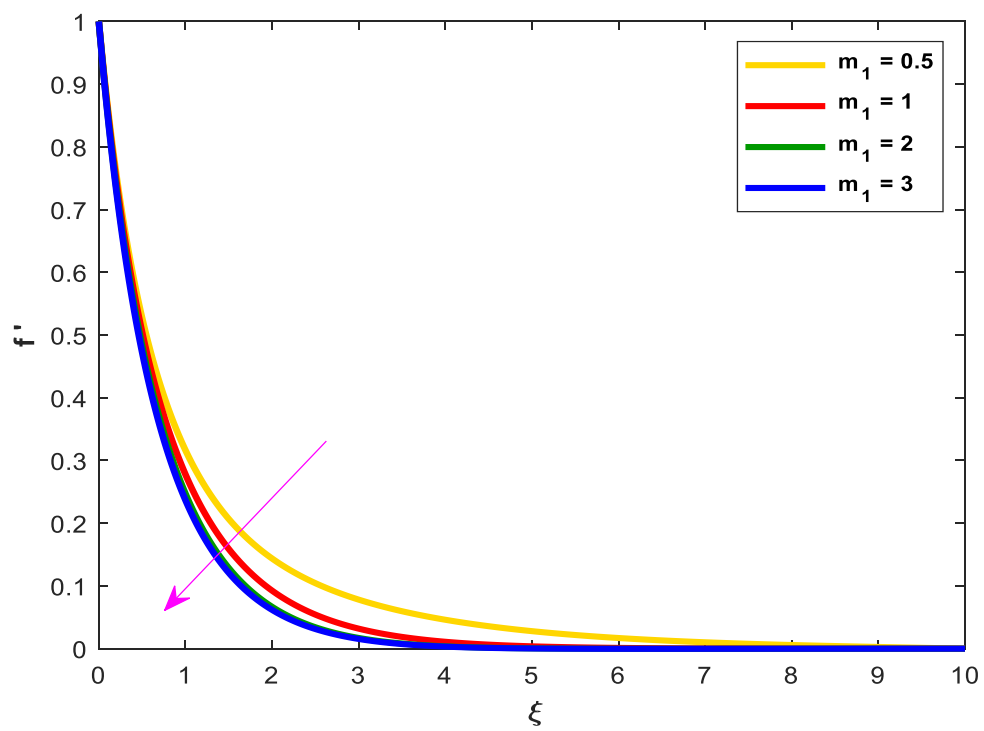


Figure.4. velocity verses m_1

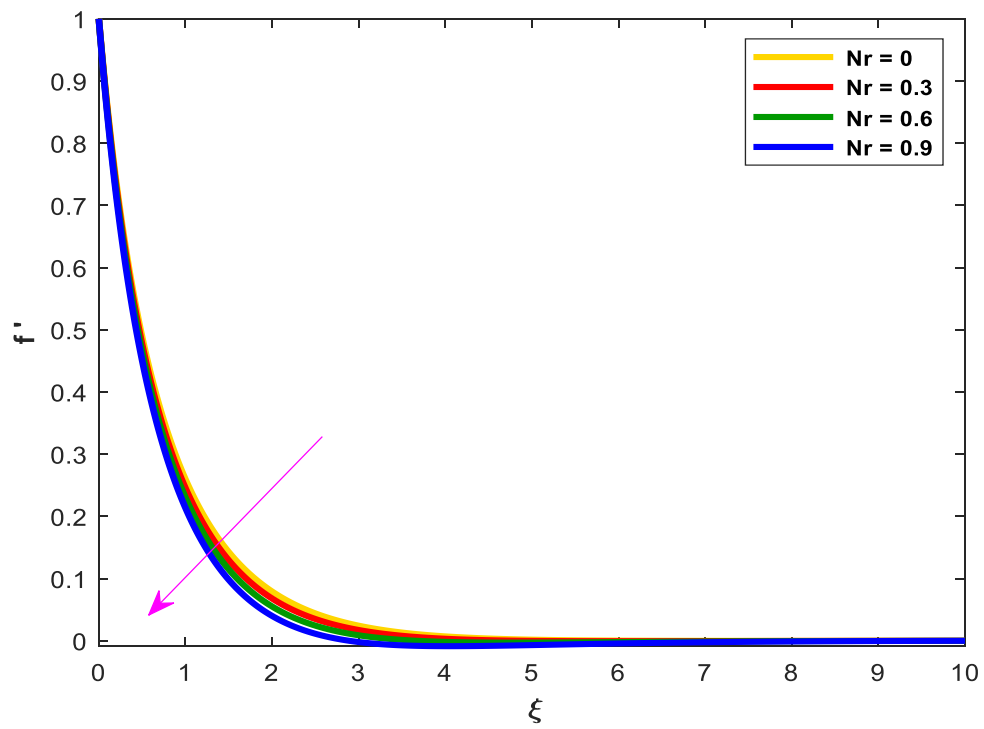


Figure.5. velocity verses Nr

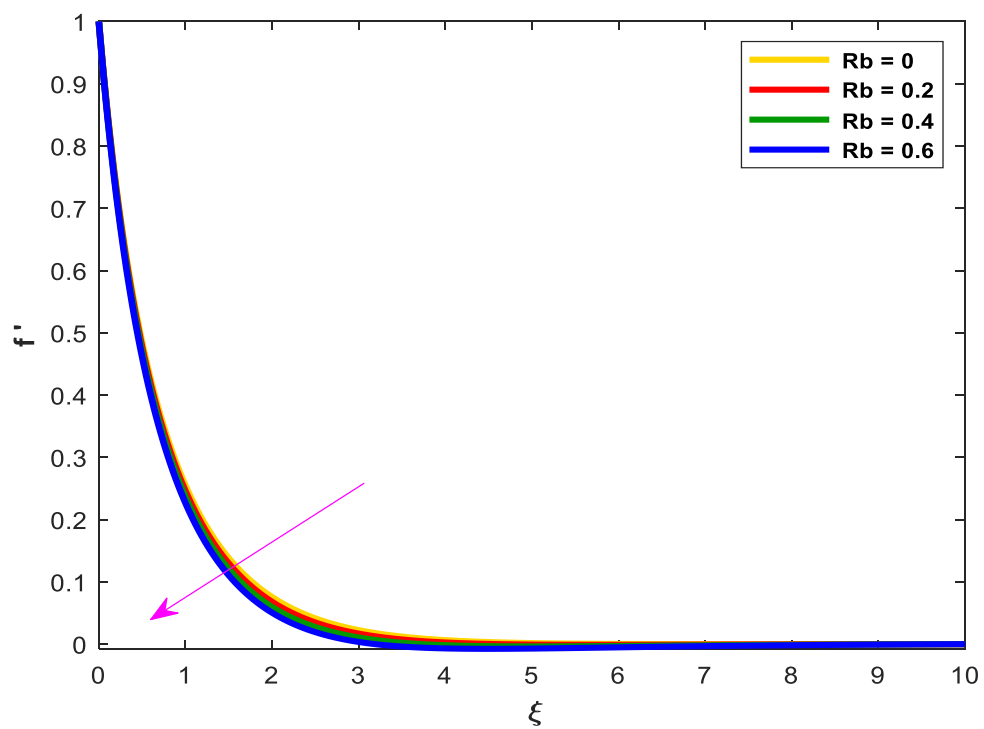


Figure.6. velocity verses Rb

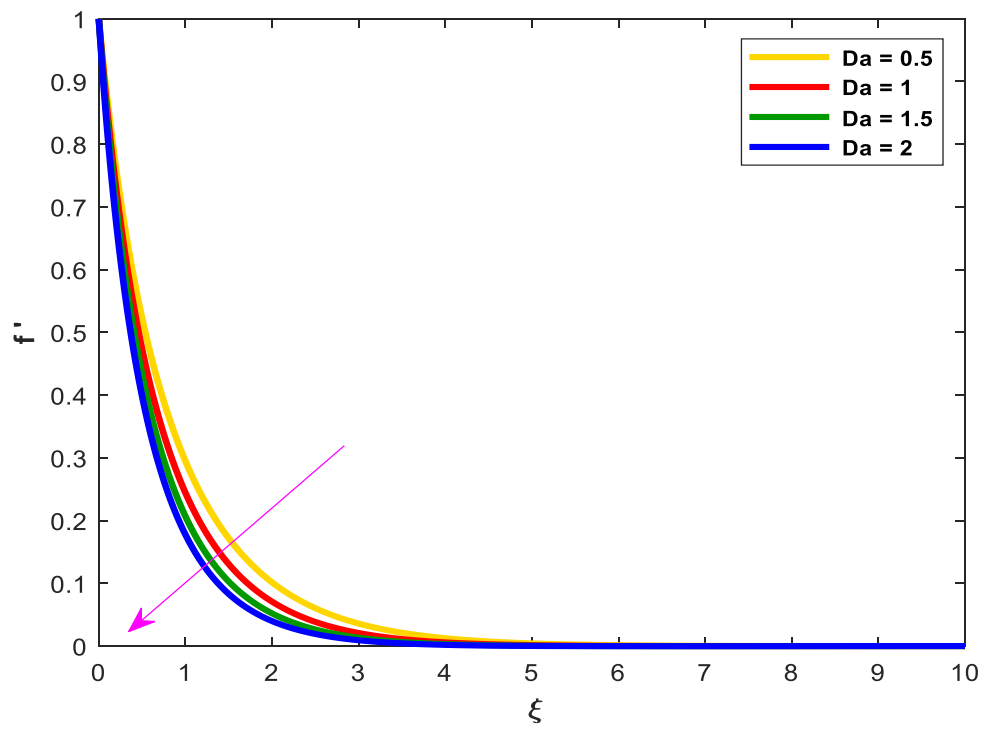


Figure.7. velocity verses Da

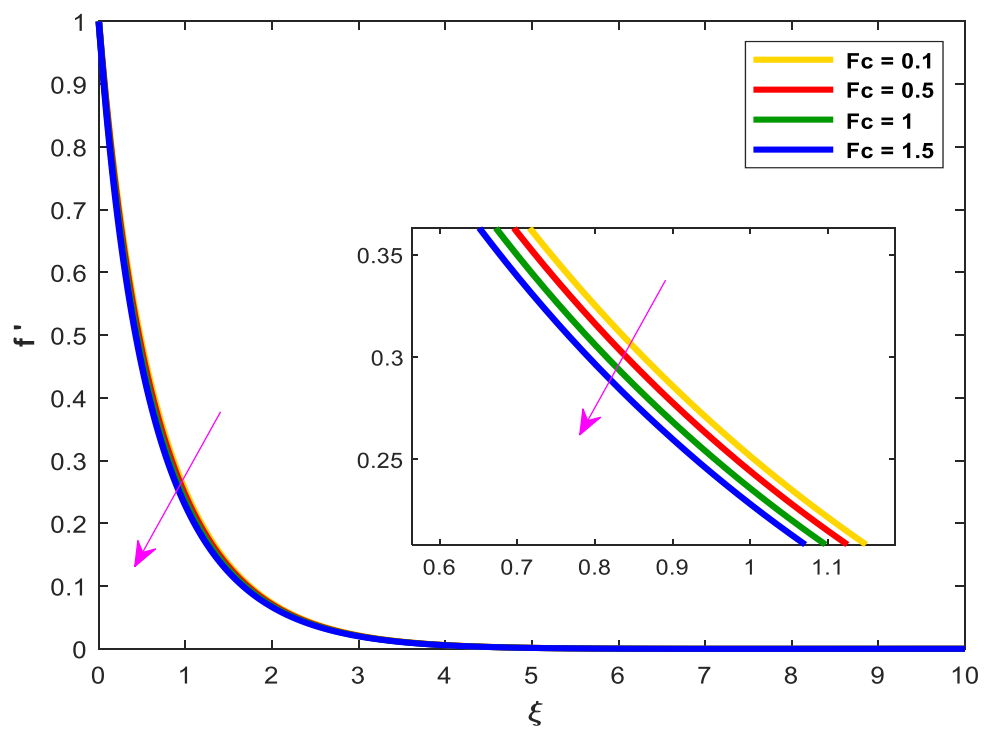


Figure.8. velocity verses Fc

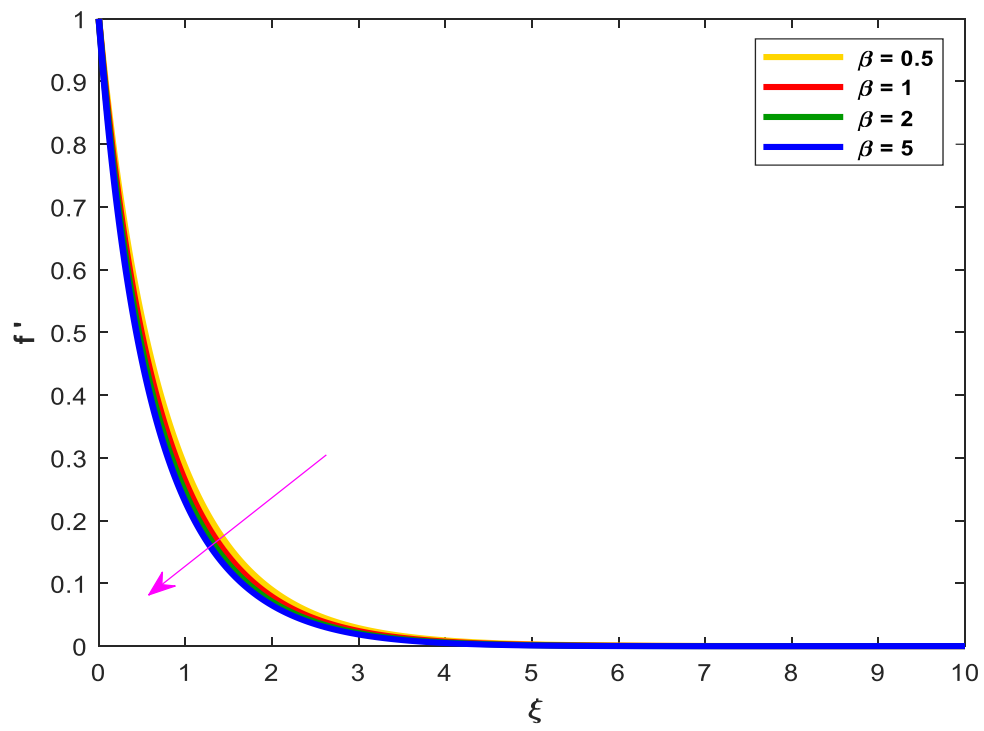


Figure.9. velocity verses β

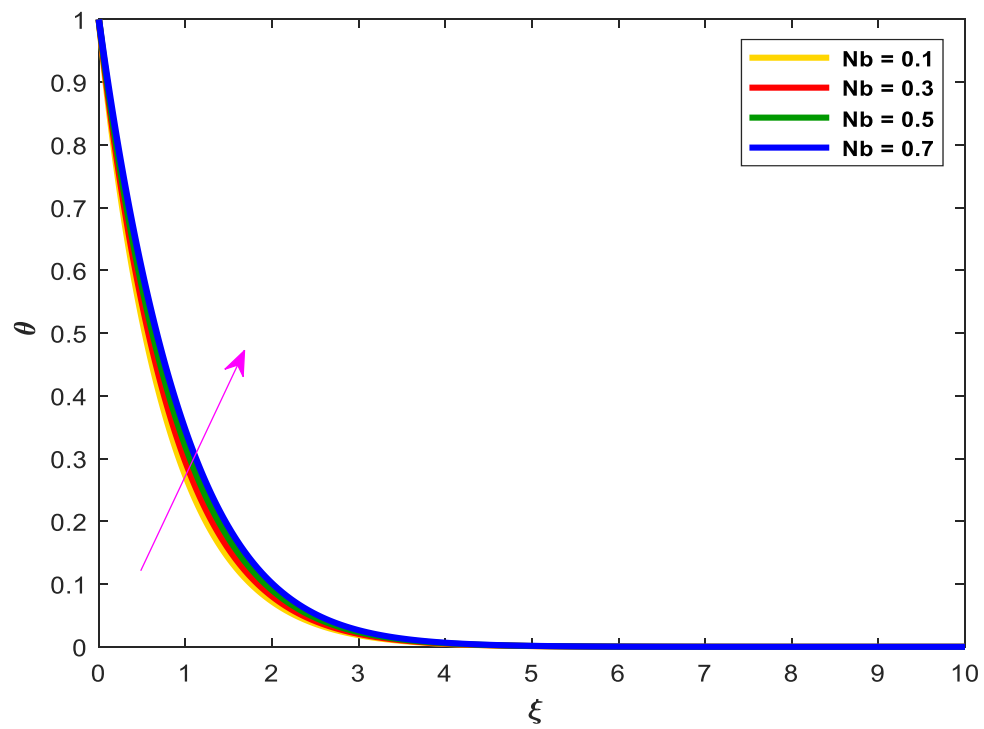


Figure.10. temperature verses Nb

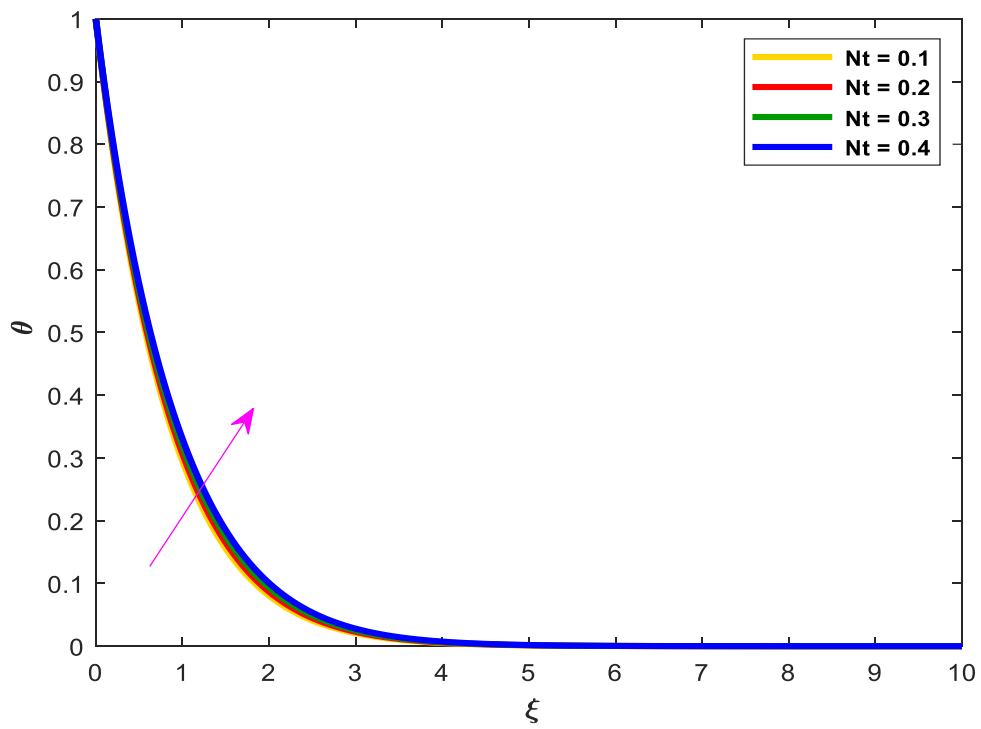


Figure.11. temperature verses Nt

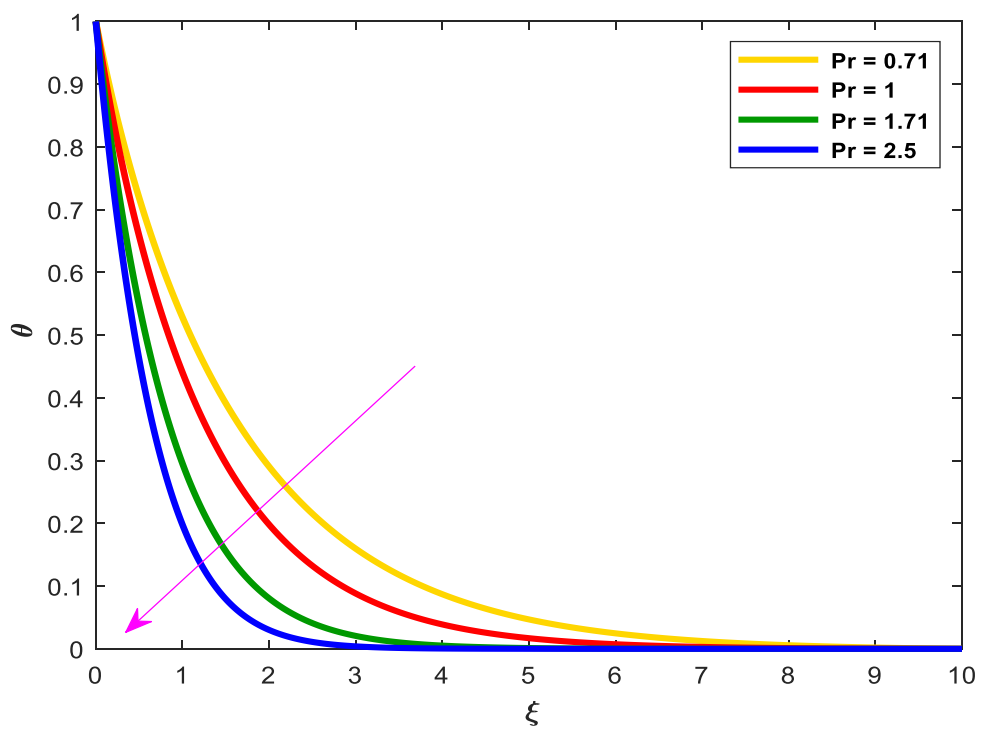


Figure.12. temperature verses Pr

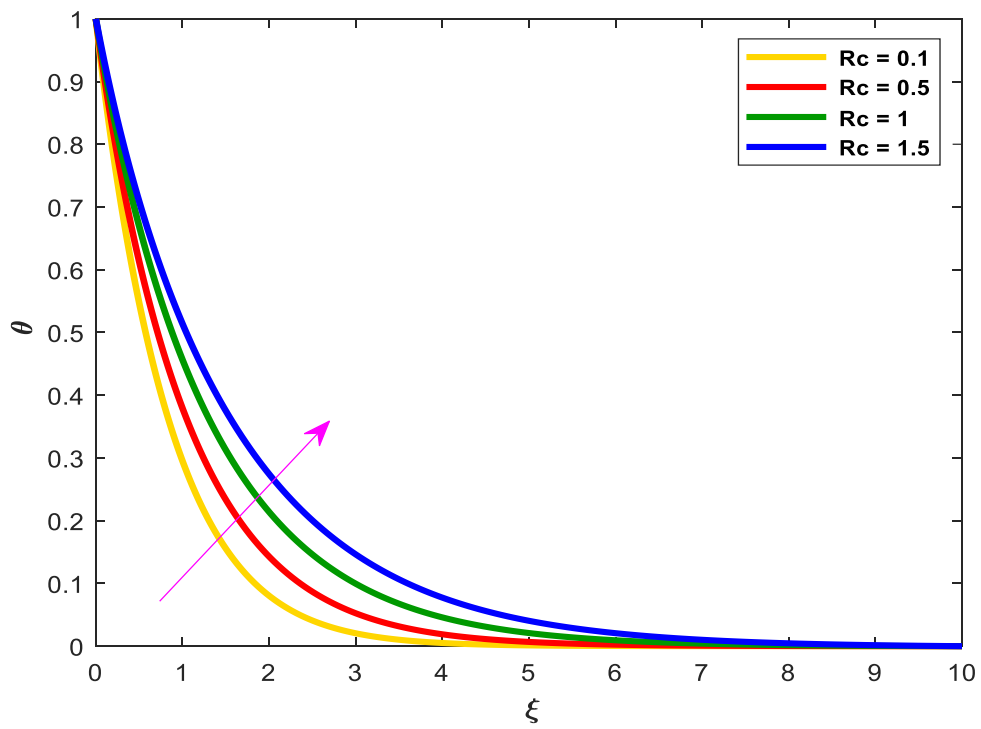


Figure.13. temperature verses Rc

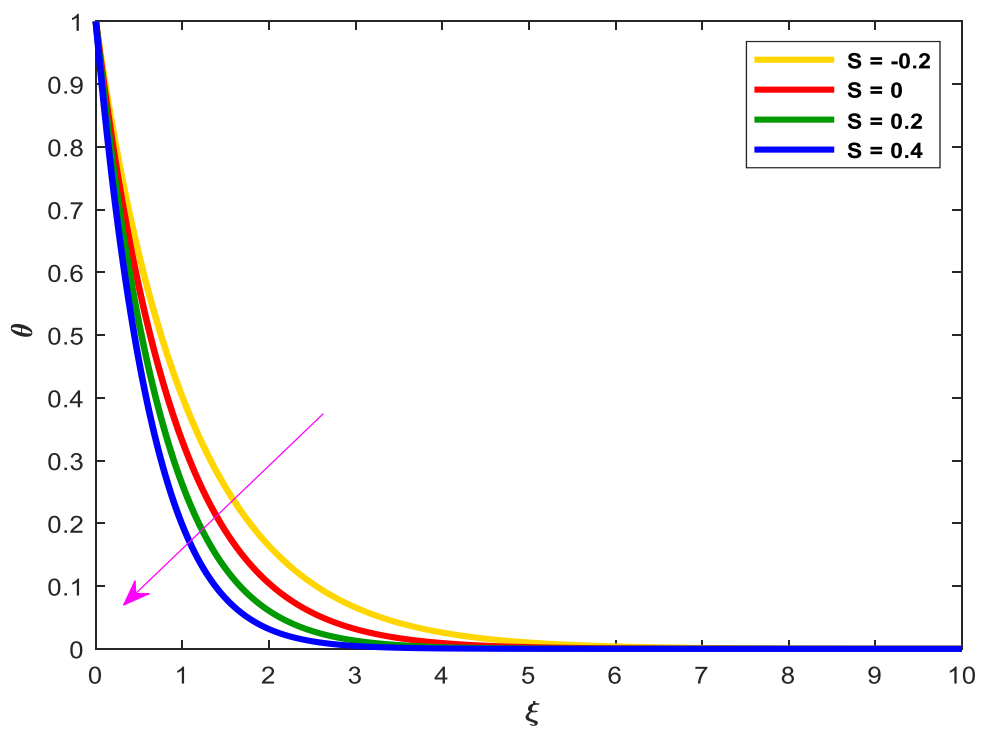


Figure.14. temperature verses S

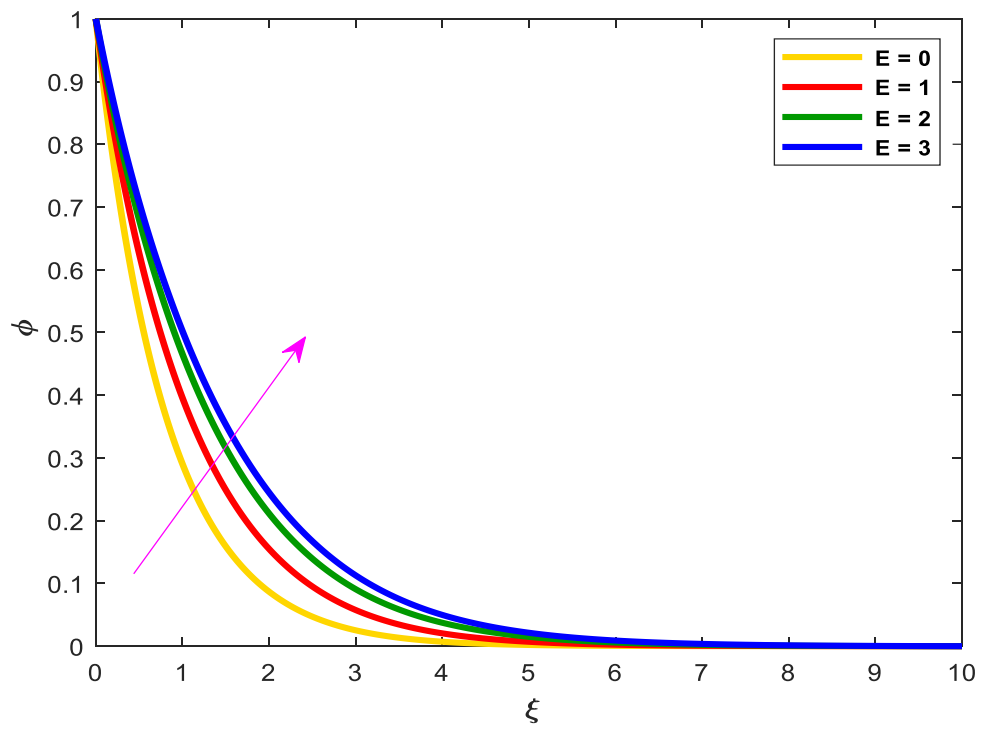


Figure.15. Concentration verses E

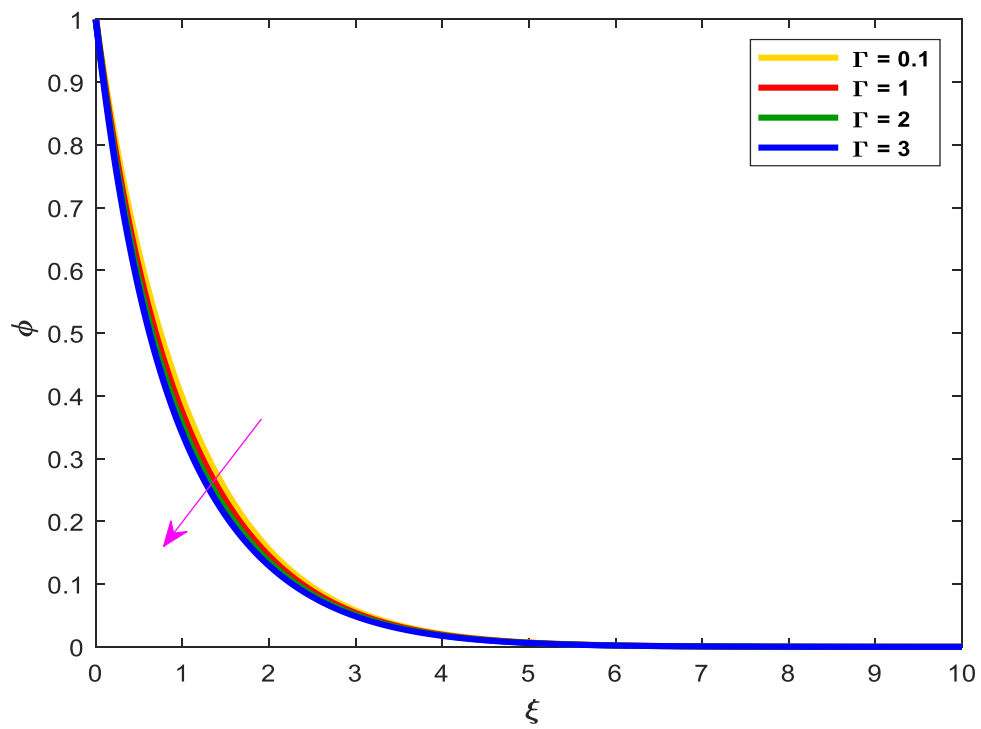


Figure.16. Concentration verses Γ

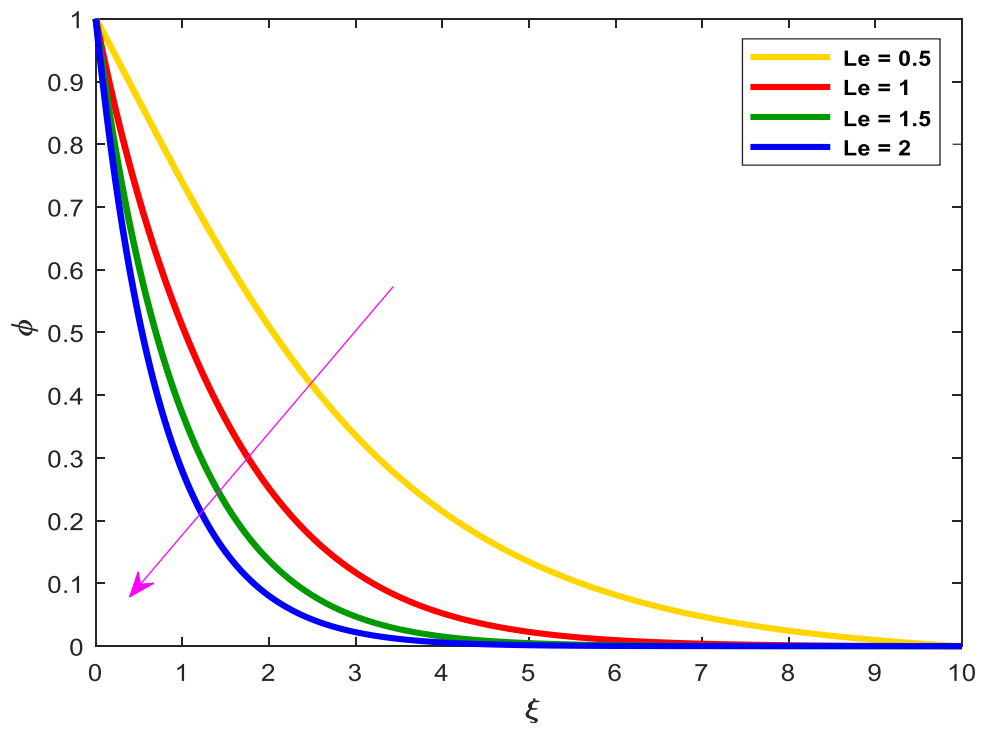


Figure.17. Concentration verses Le

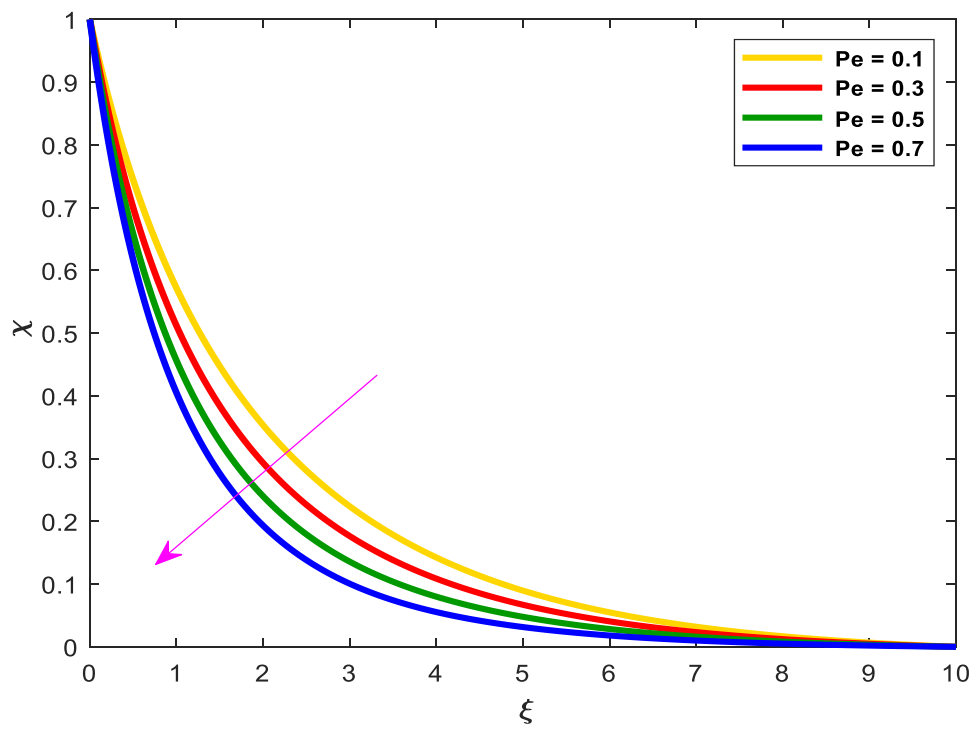


Figure.18. Motile-organism verses Pe

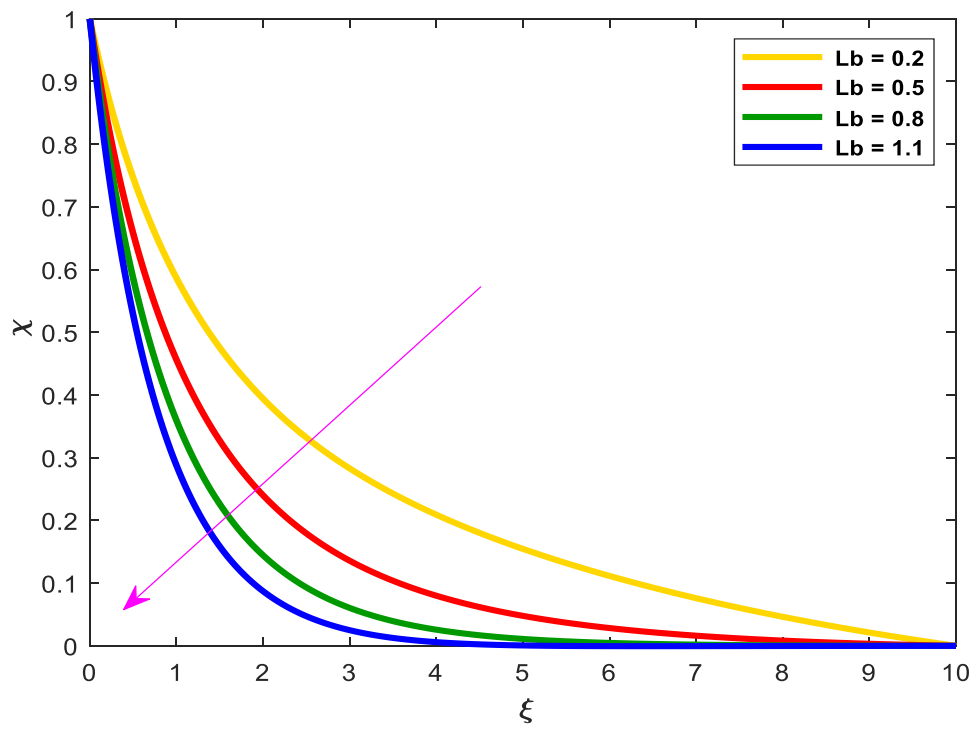


Figure.19. Motile-organism verses Lb

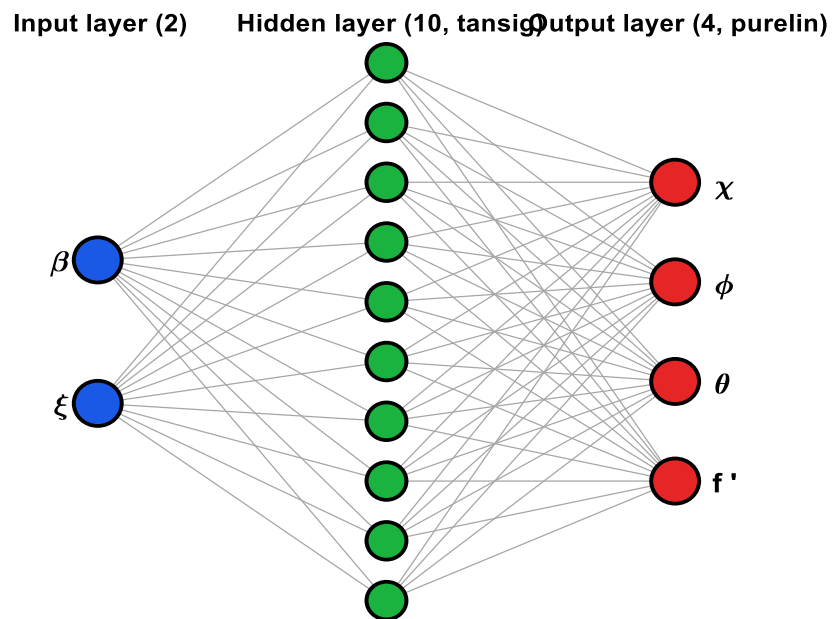


Figure.20: Casson parameter (β) LMANN Architecture

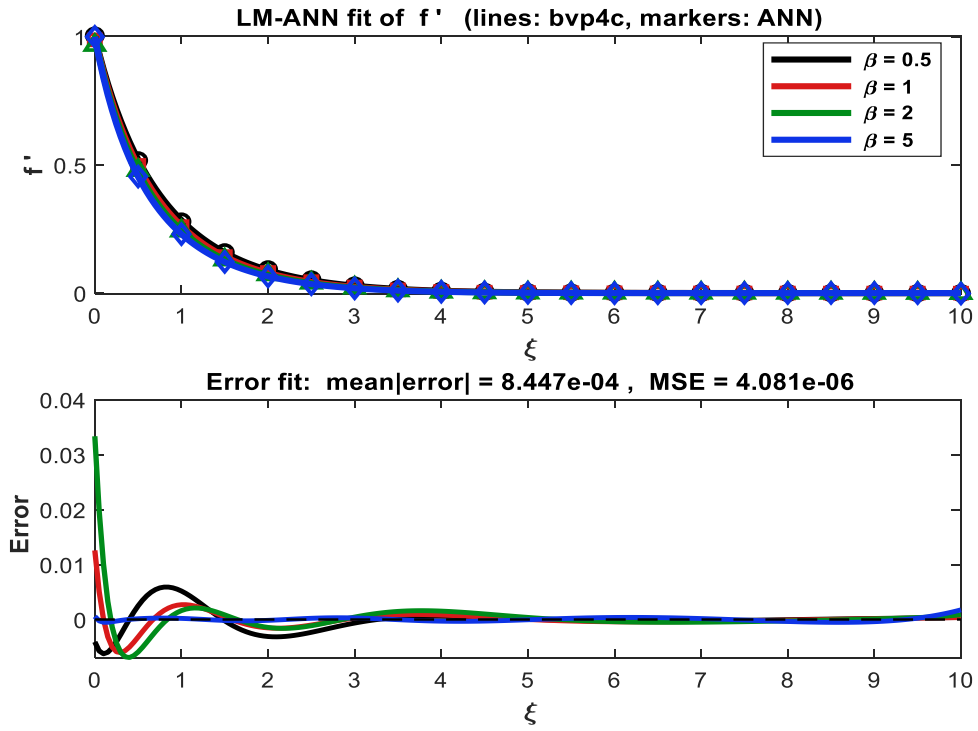


Figure.21: Casson parameter (β) velocity profile fit and error

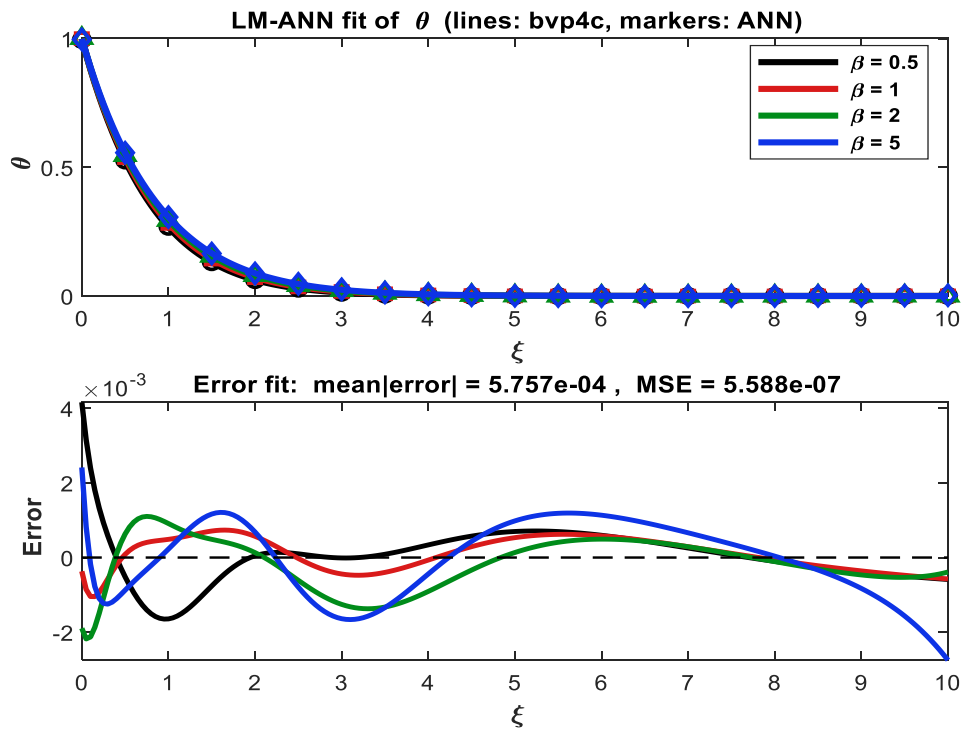


Figure.22: Casson parameter (β) temperature profile fit and error

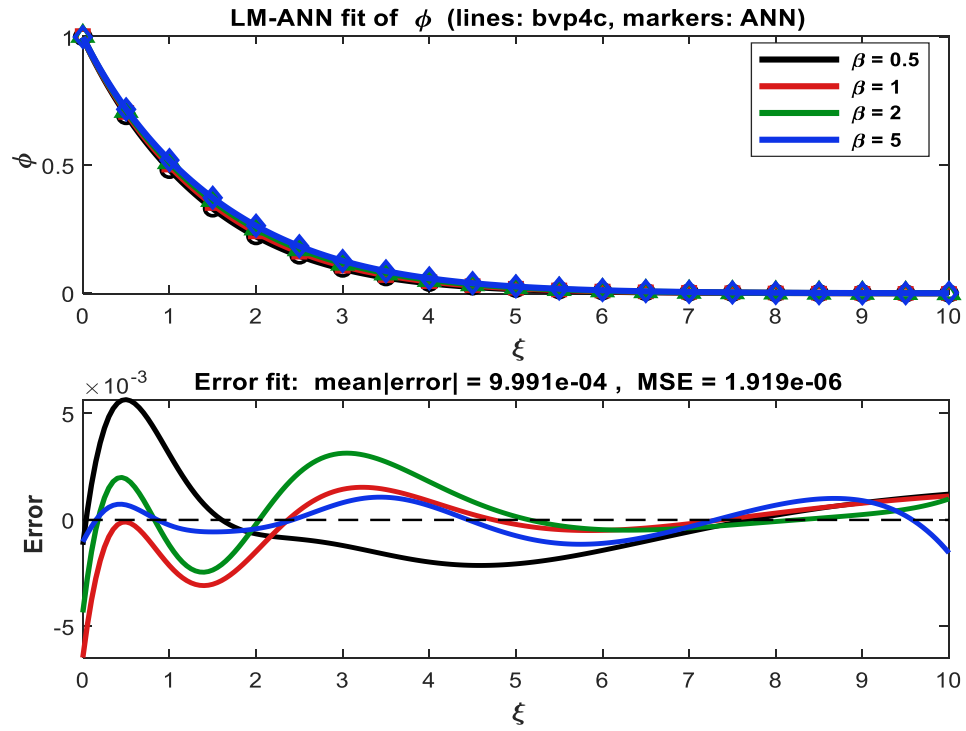


Figure.23: Casson parameter (β) concentration profile fit and error

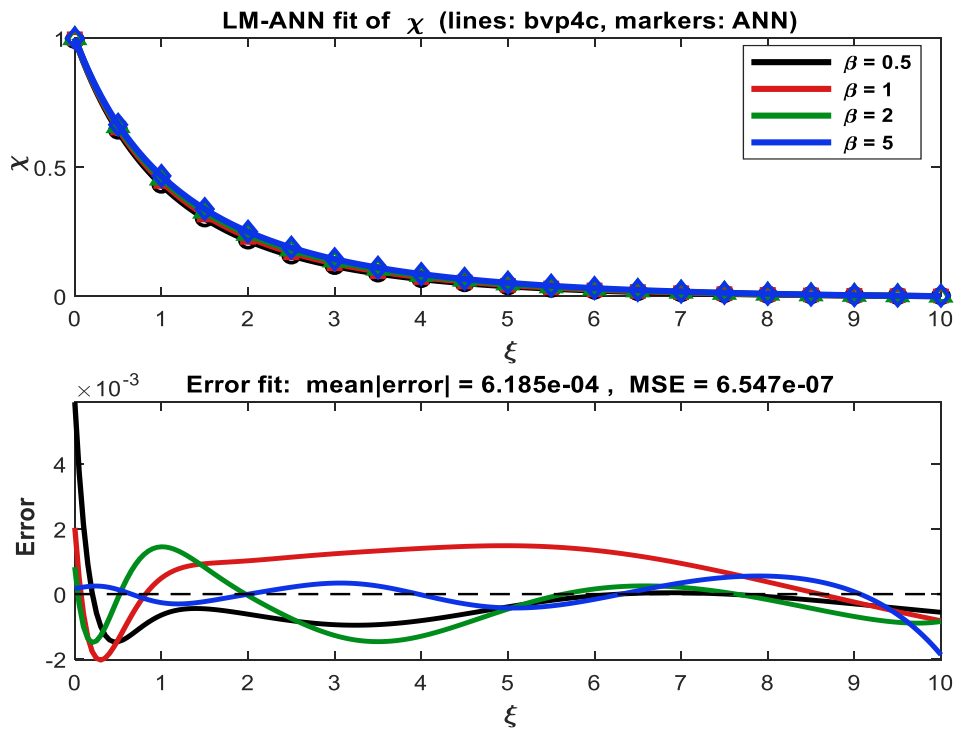


Figure.24: Casson parameter (β) motile microorganism density profile fit and error

Table1: Training Progress of parameter Casson parameter (β)

Training Progress				
Unit	Initial Value	Stopped Value	Target Value	
Epoch	0	710	1000	
Elapsed Time	-	00:00:19	-	
Performance	0.876	1.47e-06	1e-10	
Gradient	0.897	3.94e-06	1e-07	
Mu	0.001	1e-09	1e+10	
Validation Checks	0	20	20	

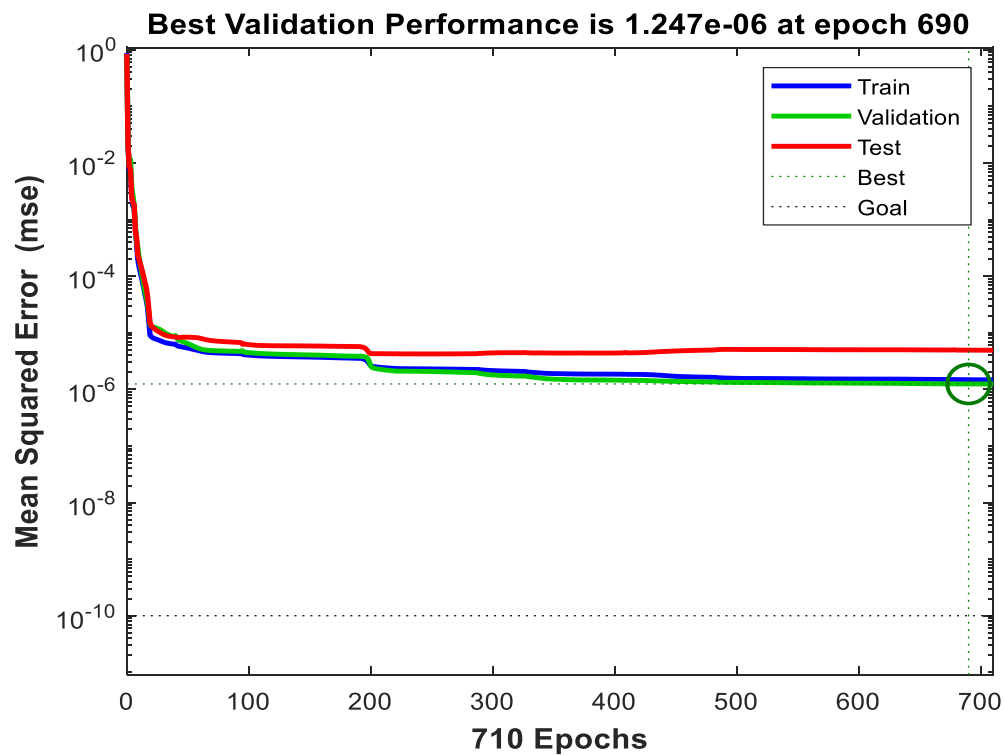


Figure.25: Casson parameter (β) best validation

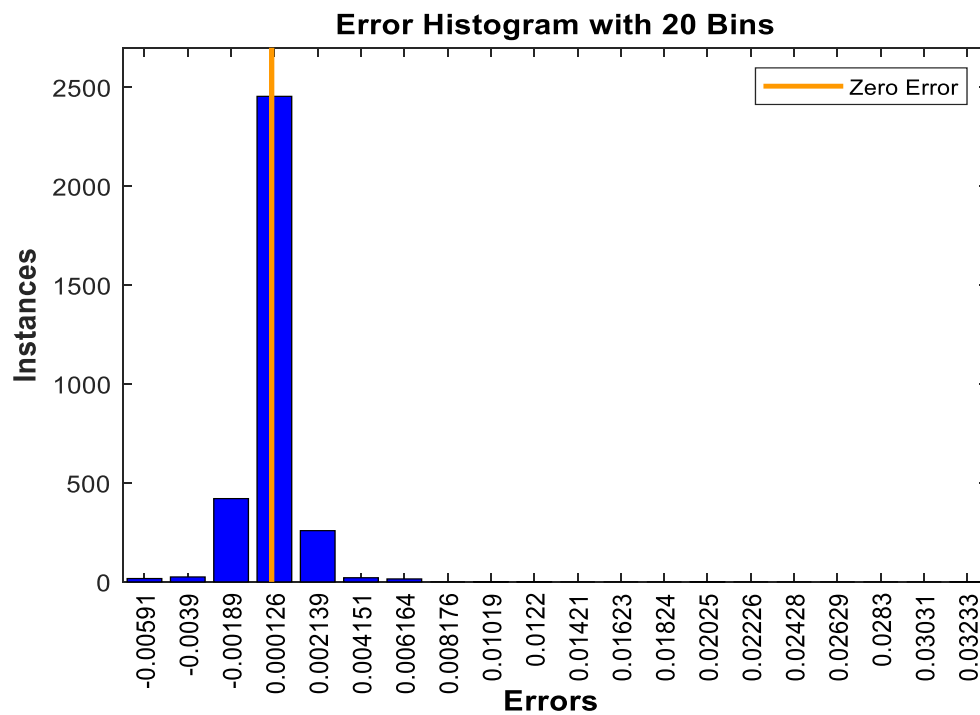


Figure.26: Casson parameter (β) histogram

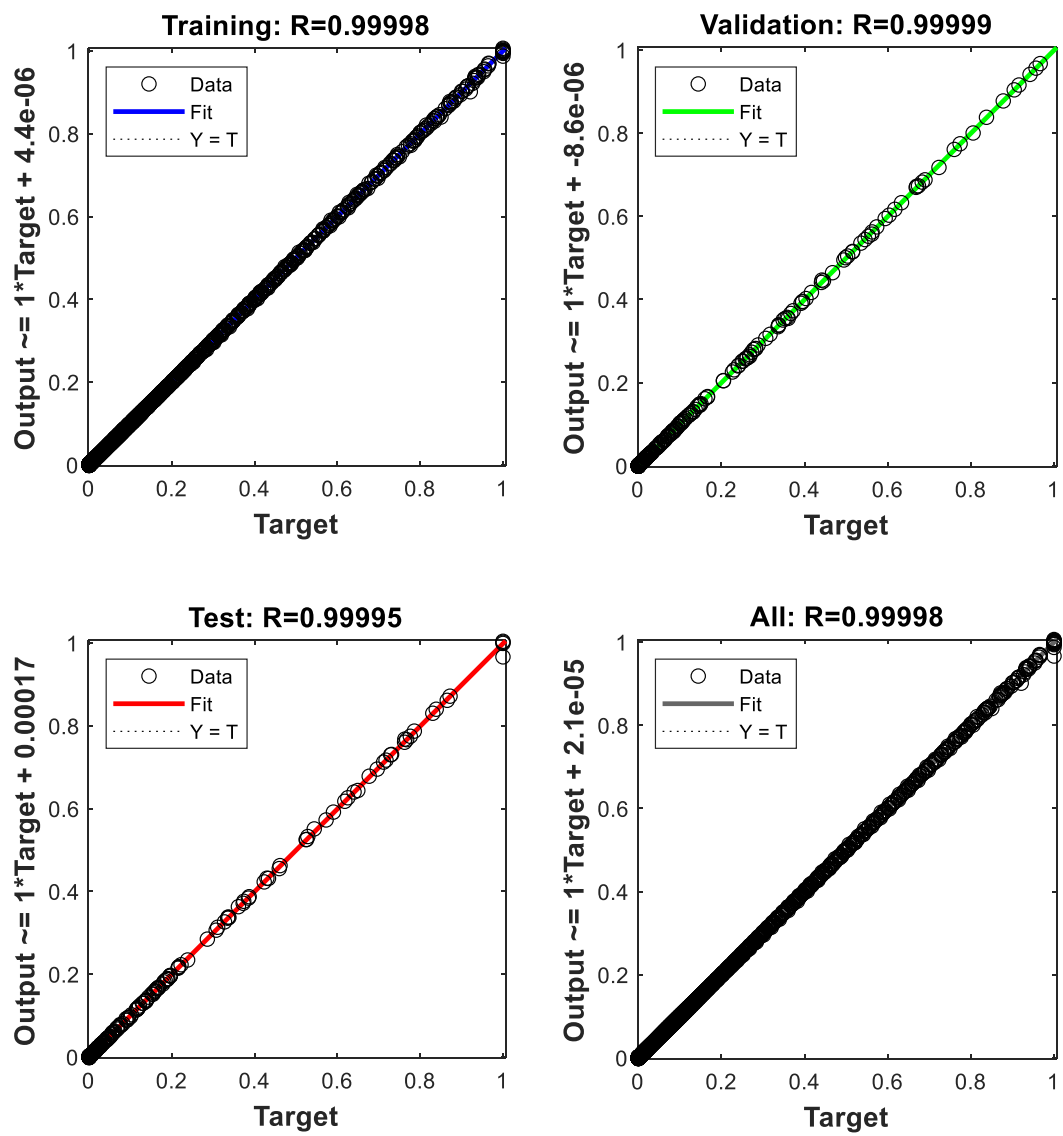


Figure.27: Casson parameter (β) Neural Network Training Regression

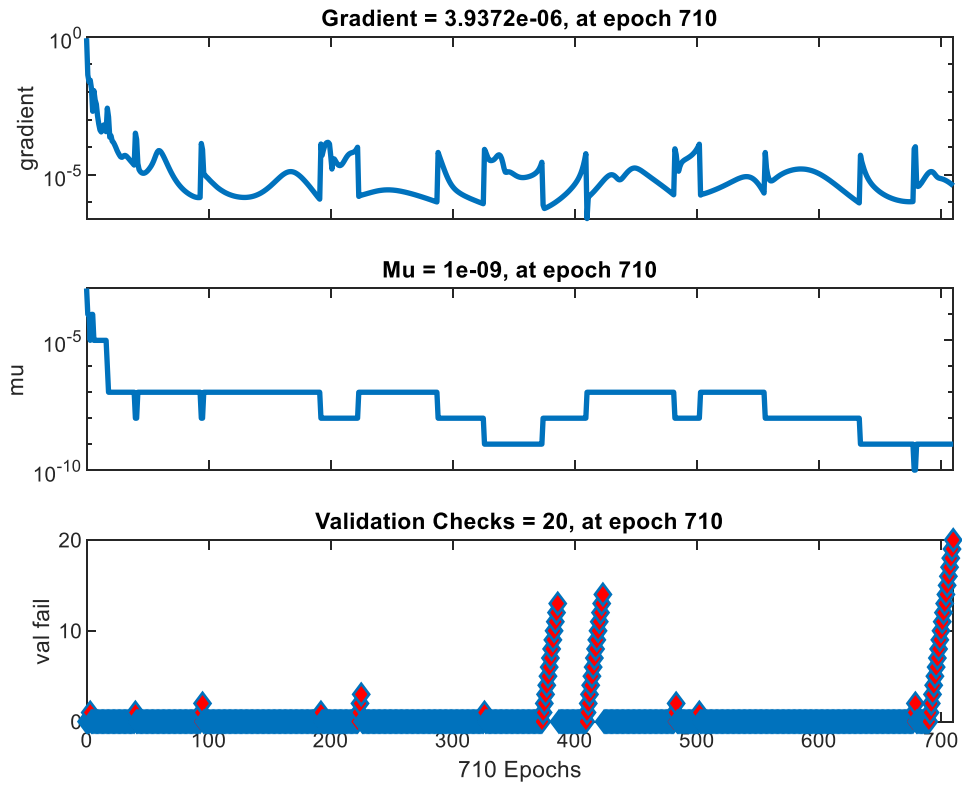


Figure.28: Casson parameter (β) training state

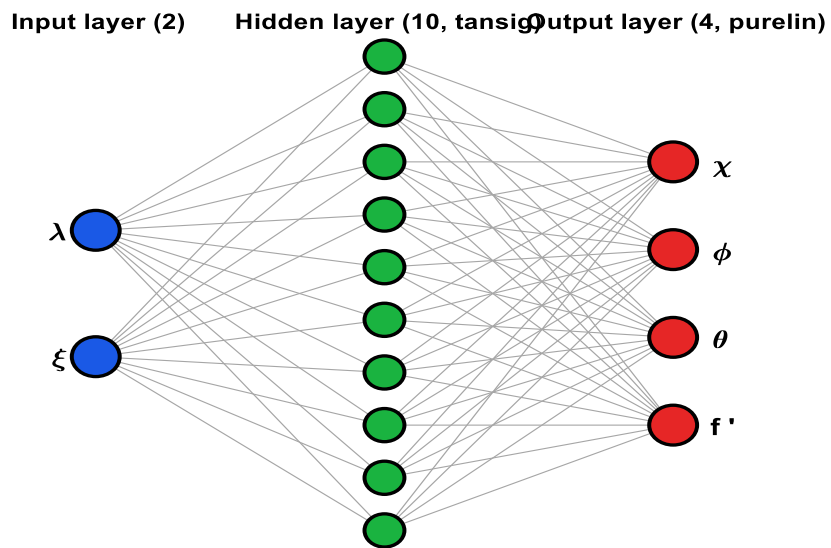


Figure.29: Williamson parameter (λ) LMANN Architecture

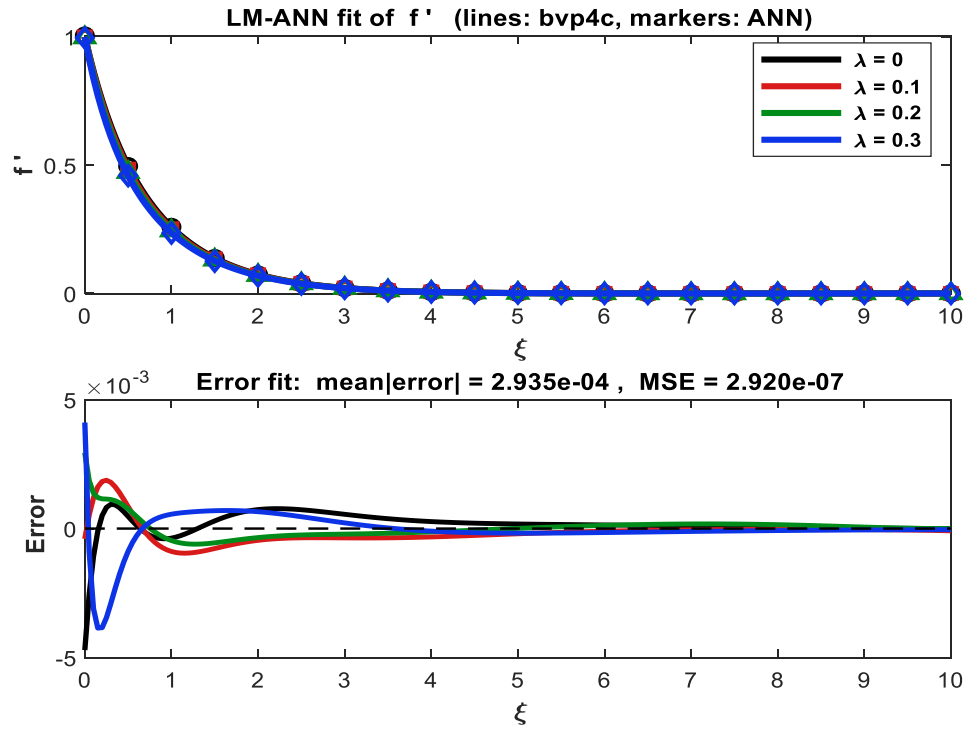


Figure.30: Williamson parameter (λ) velocity profile fit and error

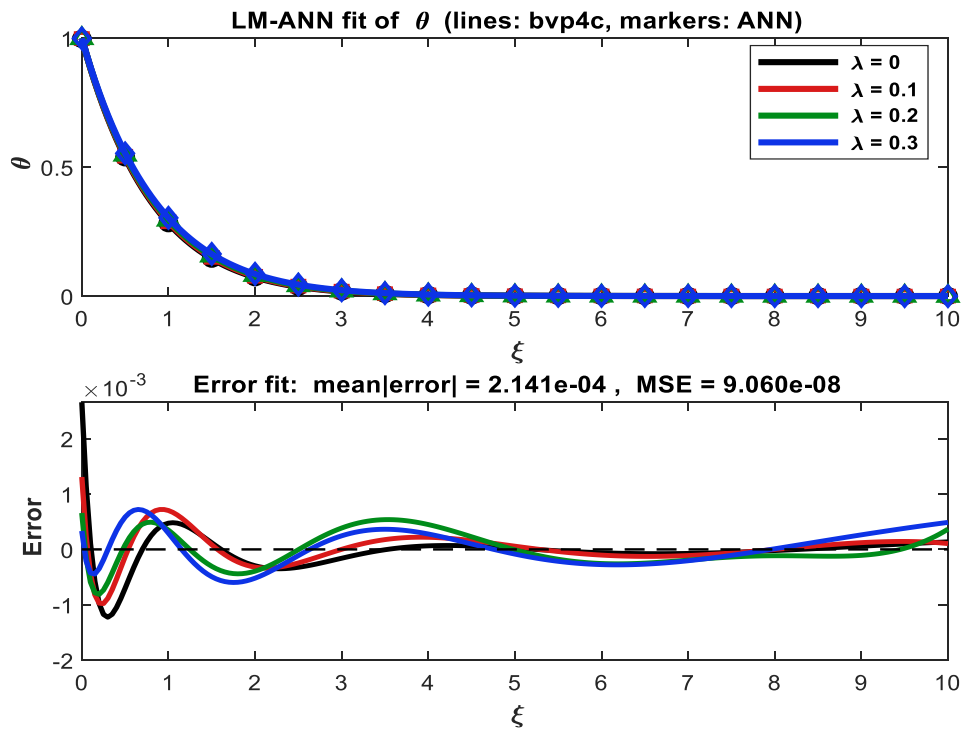


Figure.31: Williamson parameter (λ) temperature profile fit and error

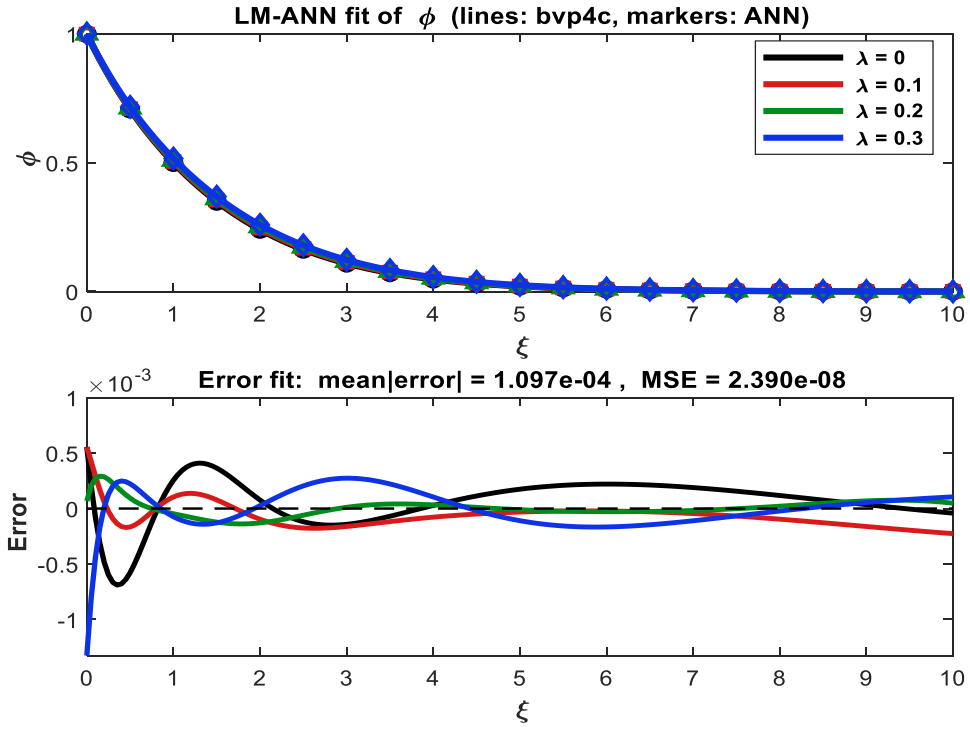


Figure.32: Williamson parameter (λ) concentration profile fit and error

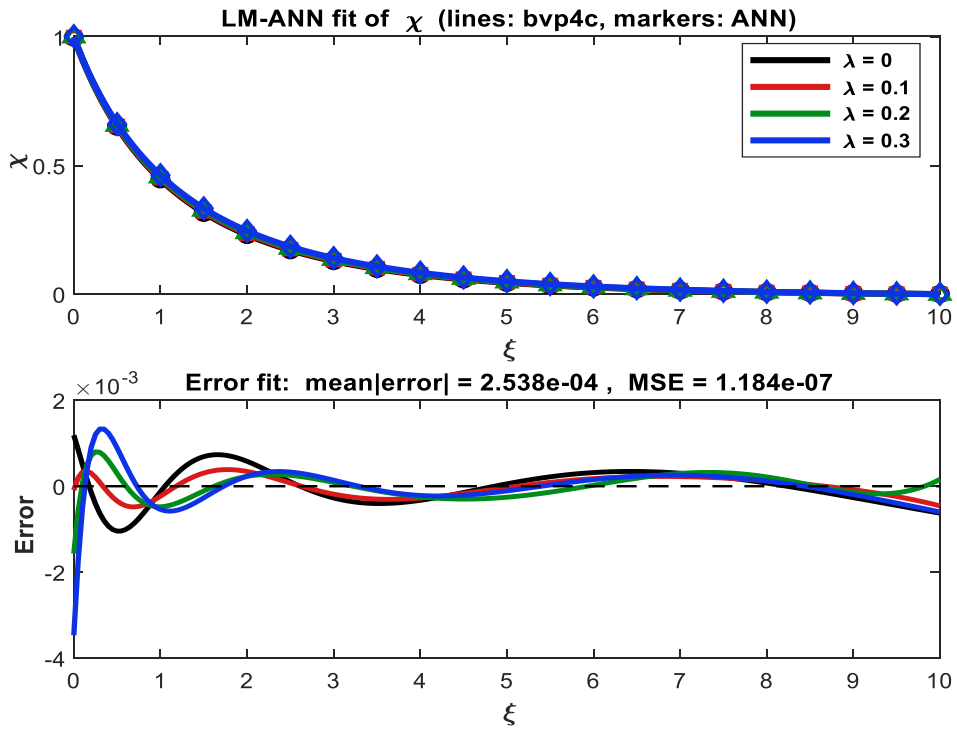


Figure.33: Williamson parameter (λ) motile microorganism density profile fit and error

Table2: Training Progress of parameter Williamson parameter (λ)

Training Progress				
Unit	Initial Value	Stopped Value	Target Value	
Epoch	0	1000	1000	
Elapsed Time	-	00:00:11	-	
Performance	0.992	1.29e-07	1e-10	
Gradient	0.914	5.82e-07	1e-07	
Mu	0.001	1e-08	1e+10	
Validation Checks	0	0	20	

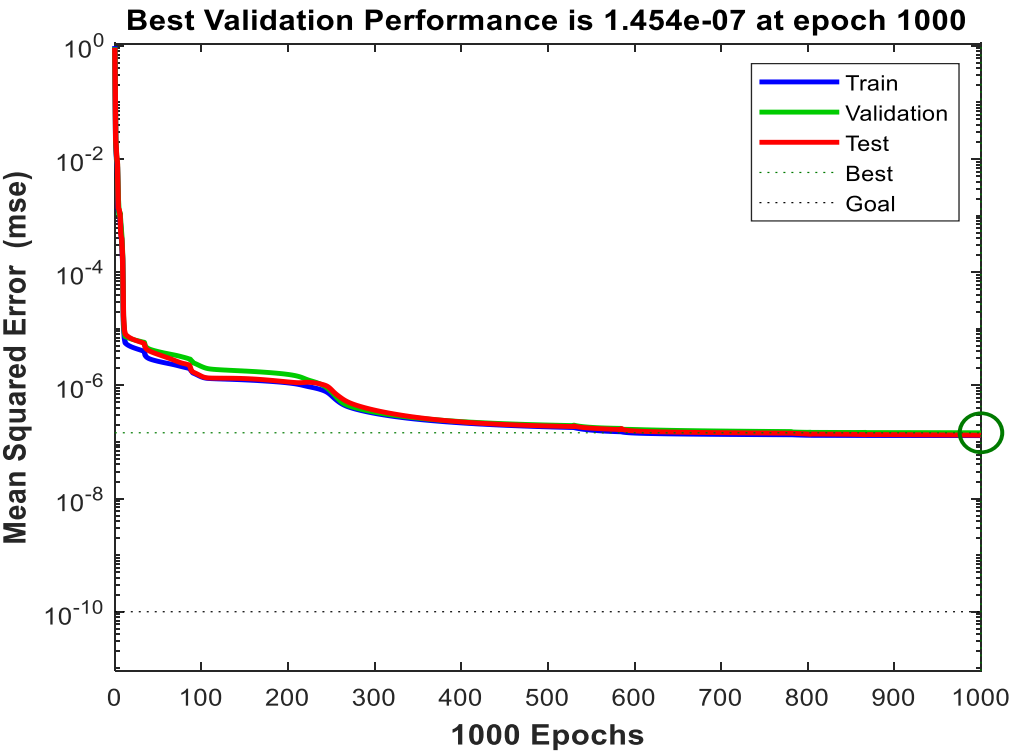


Figure.34: Williamson parameter (λ) best validation

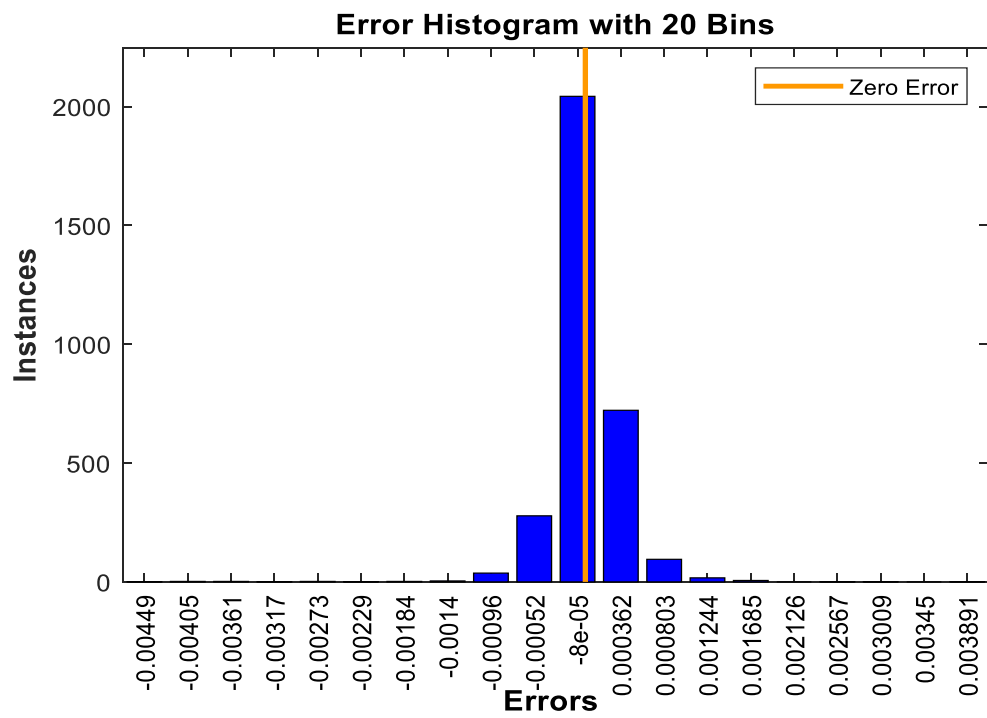


Figure.35: Williamson parameter (λ) histogram

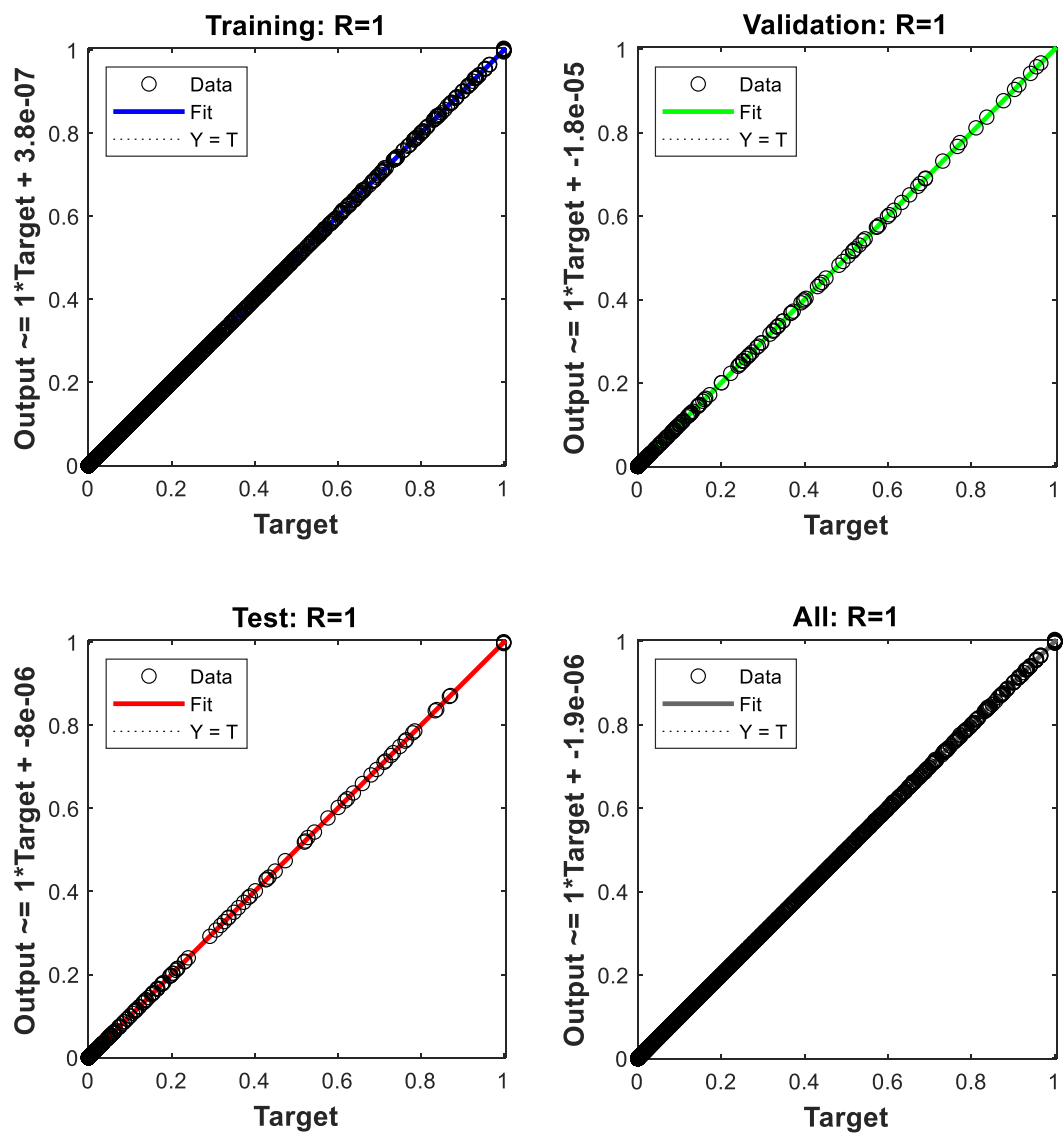


Figure.36: Williamson parameter (λ) Neural Network Training Regression

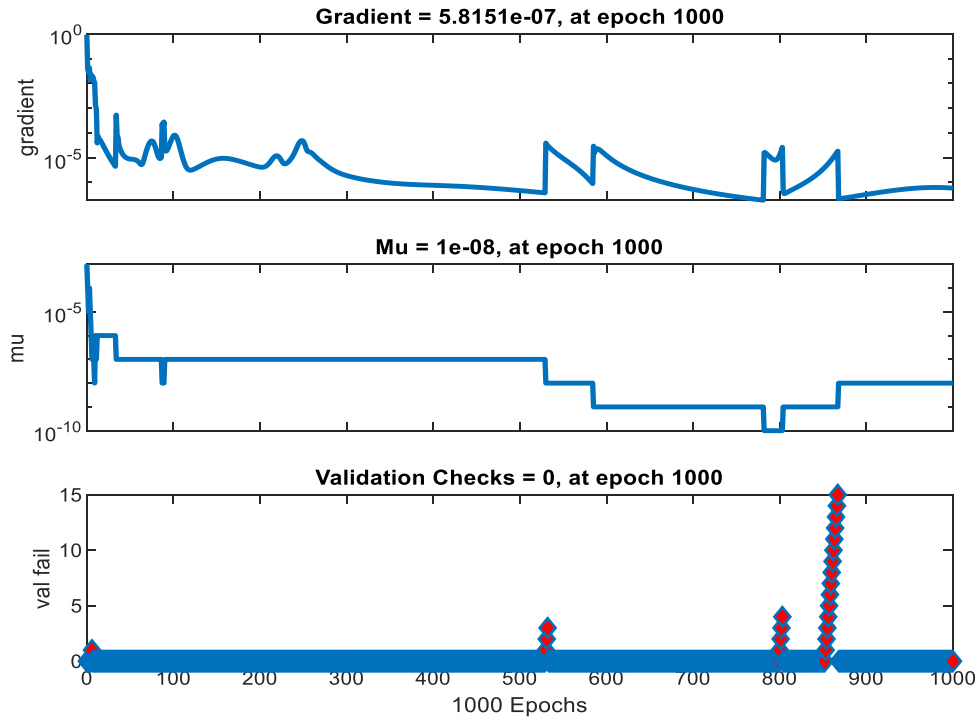


Figure.37: Williamson parameter (λ) training state

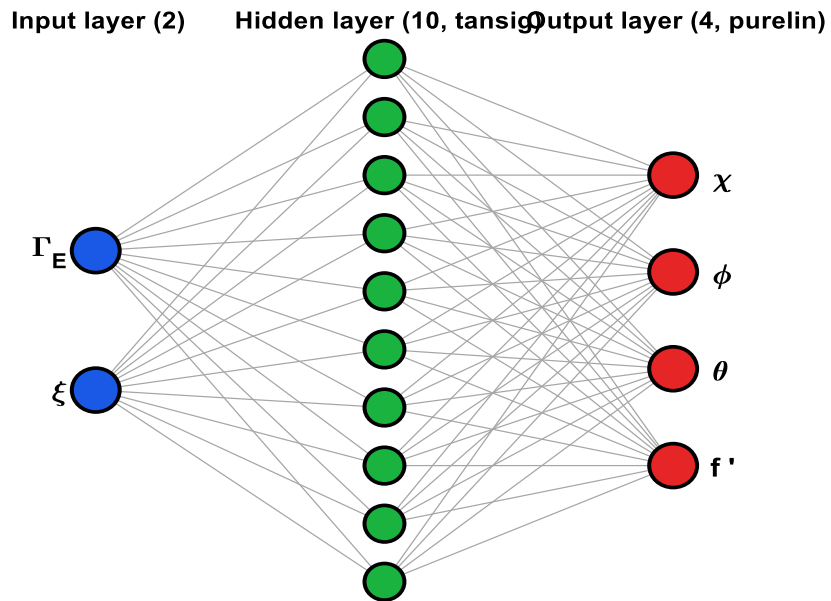


Figure.38: Thermal relaxation parameter (Γ_E) LMANN Architecture

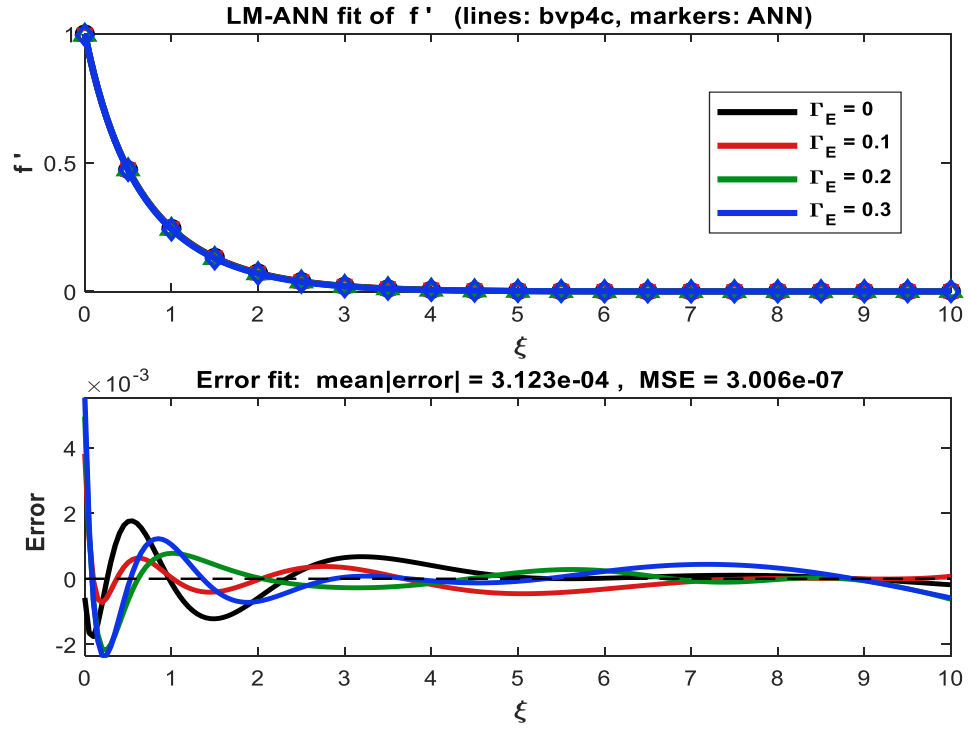


Figure.39: Thermal relaxation parameter (Γ_E) velocity profile fit and error

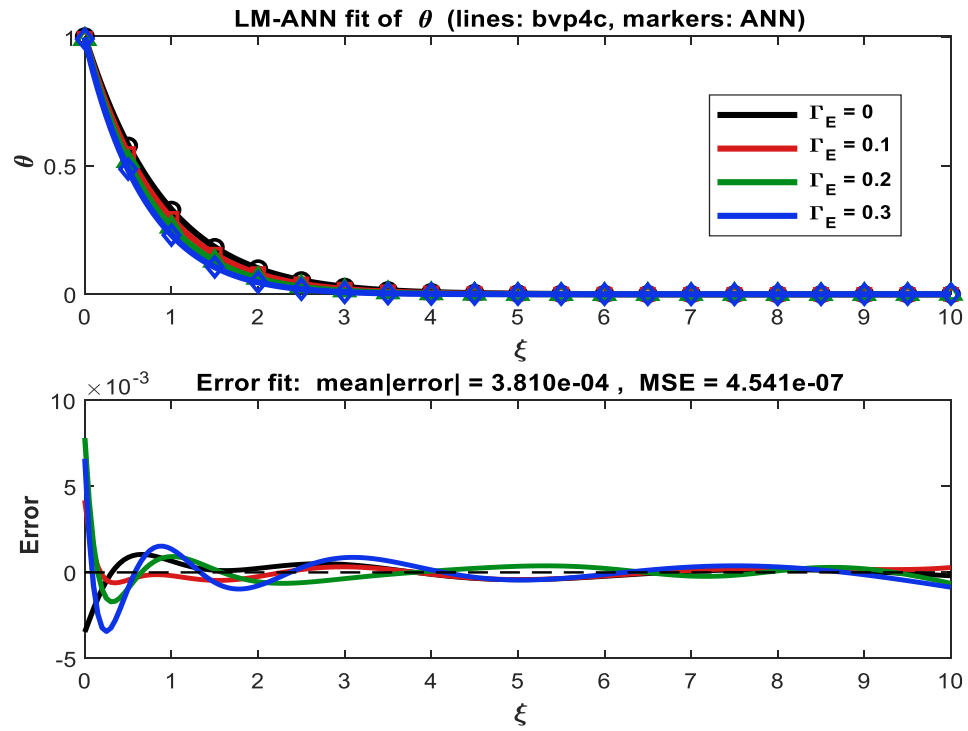


Figure.40: Thermal relaxation parameter (Γ_E) temperature profile fit and error

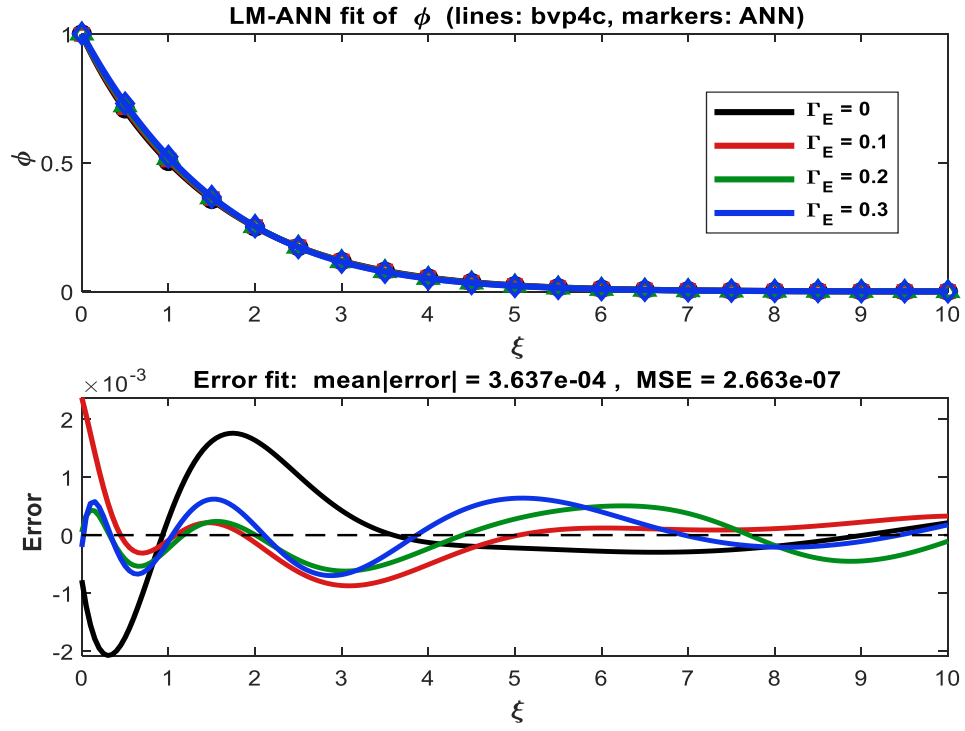


Figure.41: Thermal relaxation parameter (Γ_E) concentration profile fit and error

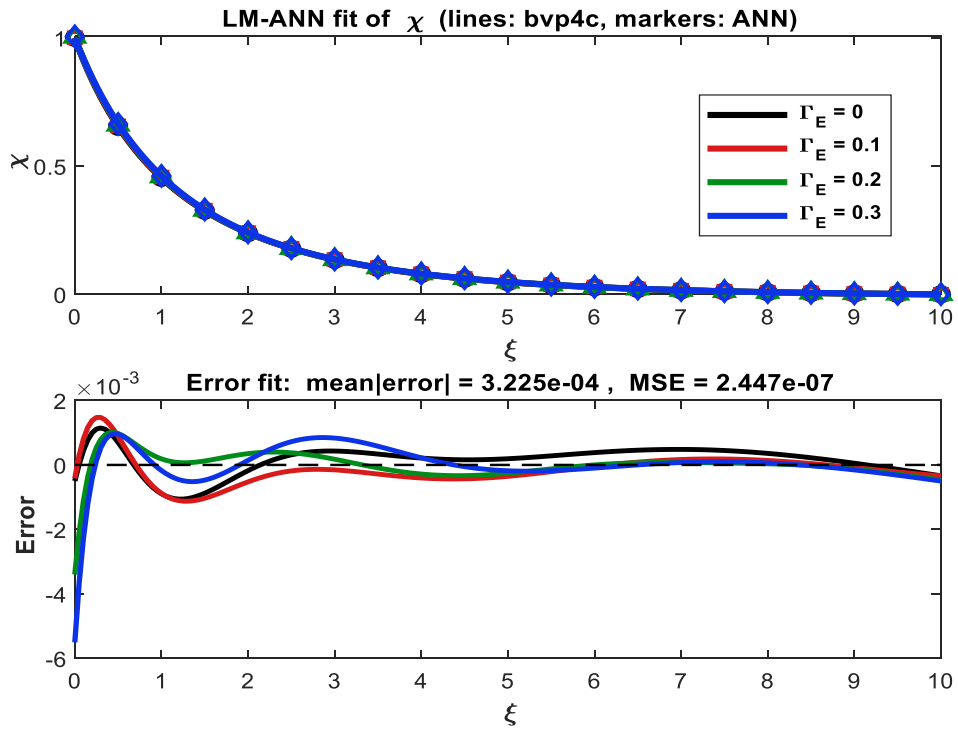


Figure.42: Thermal relaxation parameter (Γ_E) motile microorganism density profile fit and error

Table3: Training Progress of parameter Thermal relaxation parameter (Γ_E)

Training Progress				
Unit	Initial Value	Stopped Value	Target Value	
Epoch	0	369	1000	▲
Elapsed Time	-	00:00:20	-	
Performance	0.992	2.85e-07	1e-10	
Gradient	0.914	9.59e-06	1e-07	
Mu	0.001	1e-09	1e+10	
Validation Checks	0	20	20	▼

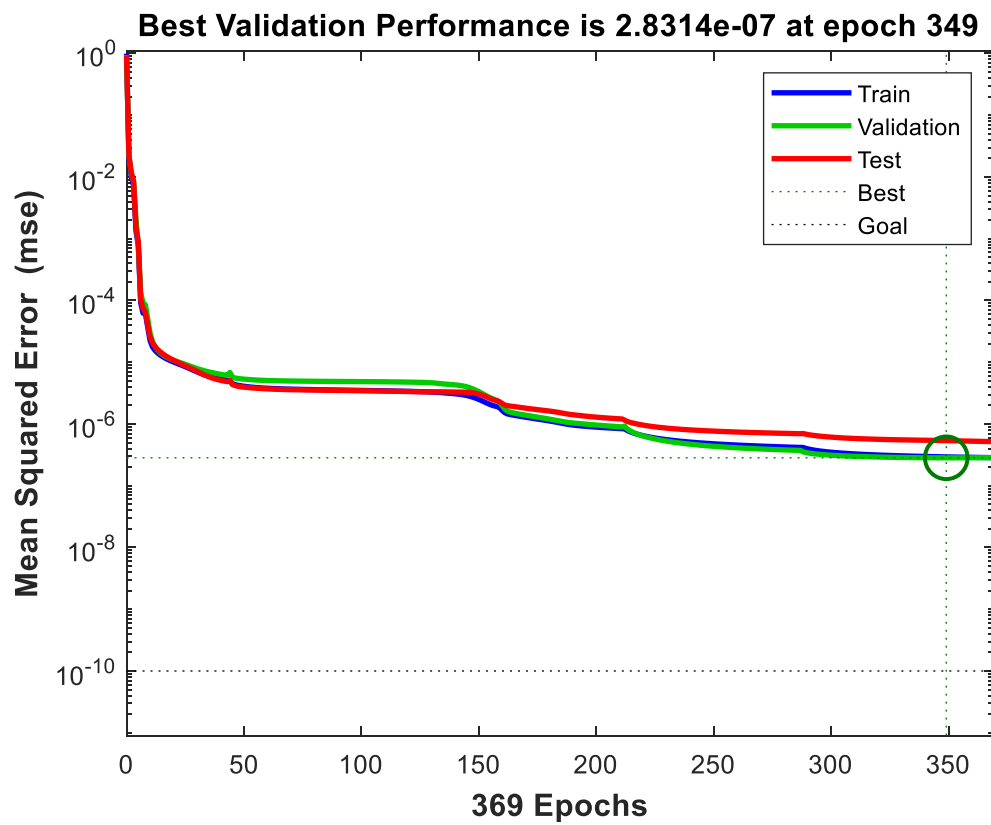


Figure.43: Thermal relaxation parameter (Γ_E) best validation

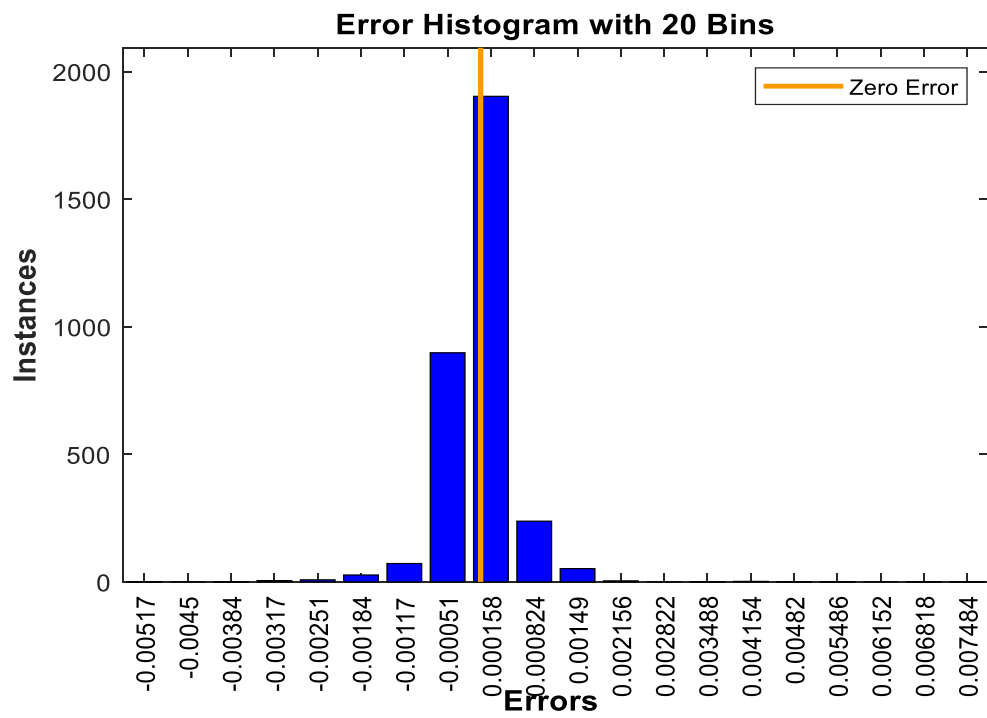


Figure.44: Thermal relaxation parameter (Γ_E) histogram

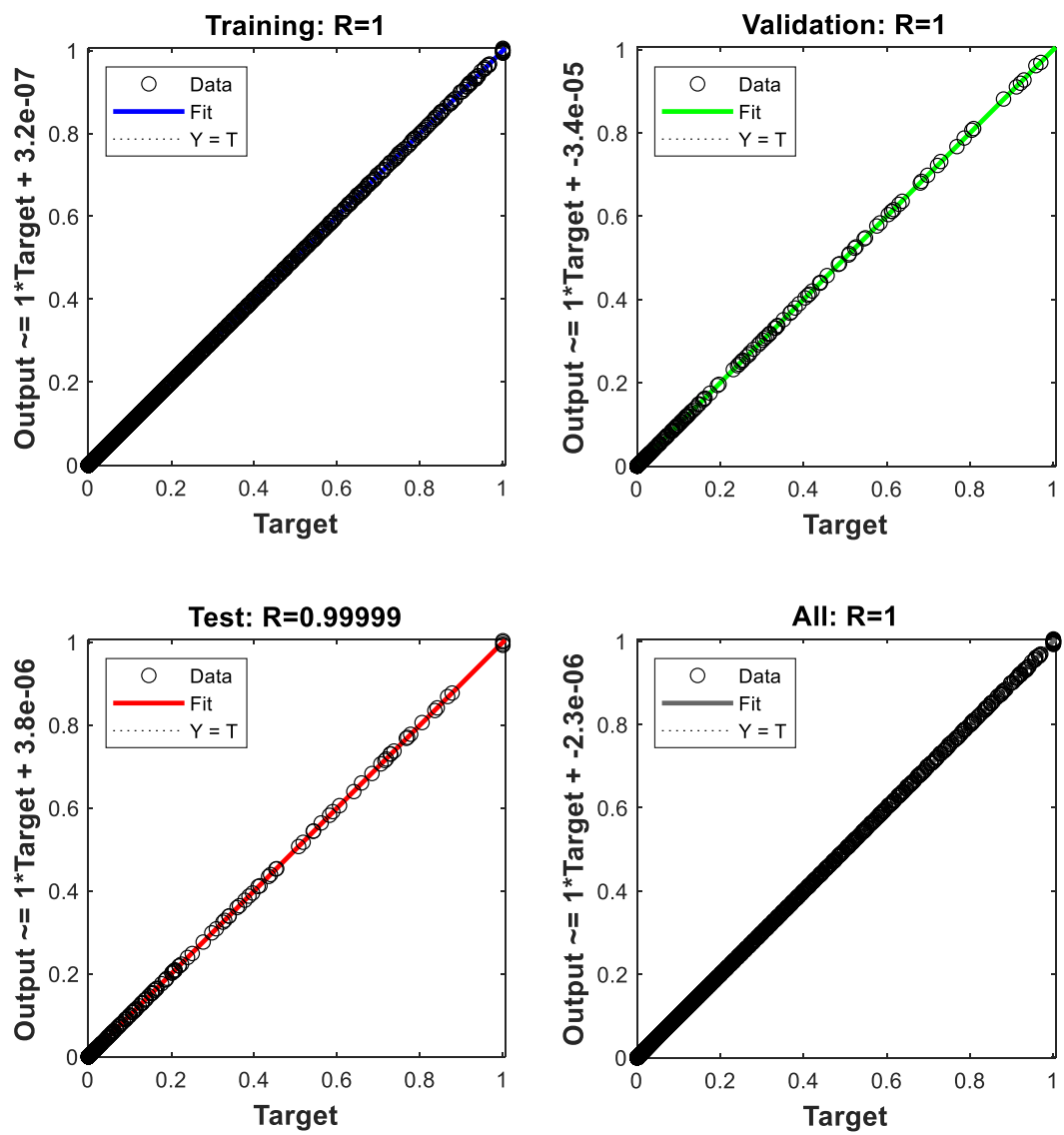


Figure.45: Thermal relaxation parameter (Γ_E) Neural Network Training Regression

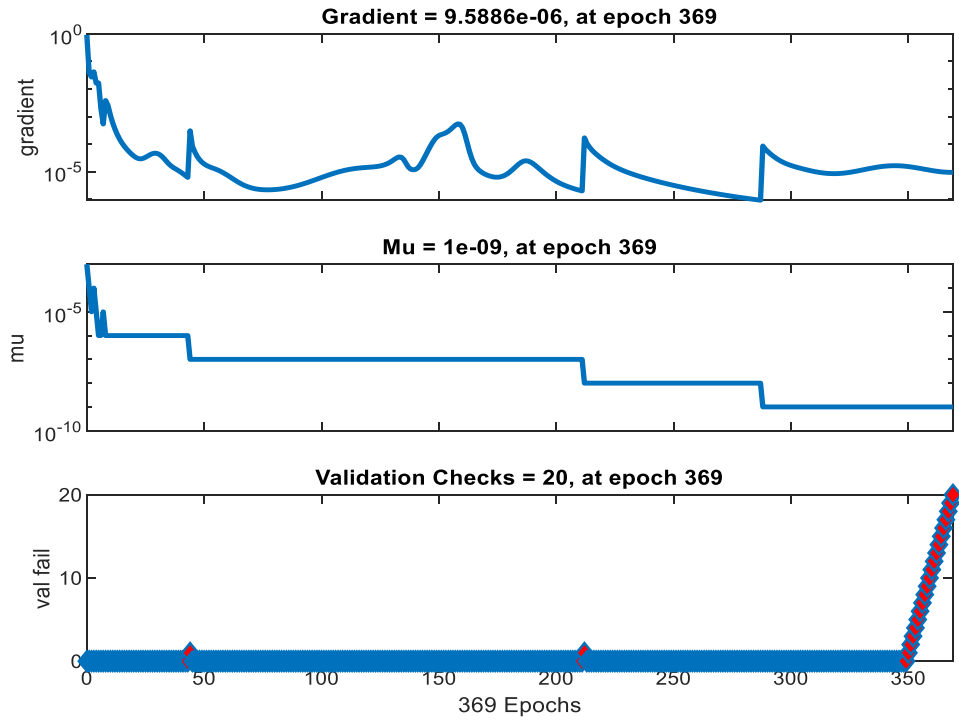


Figure.46: Thermal relaxation parameter (Γ_E) training state

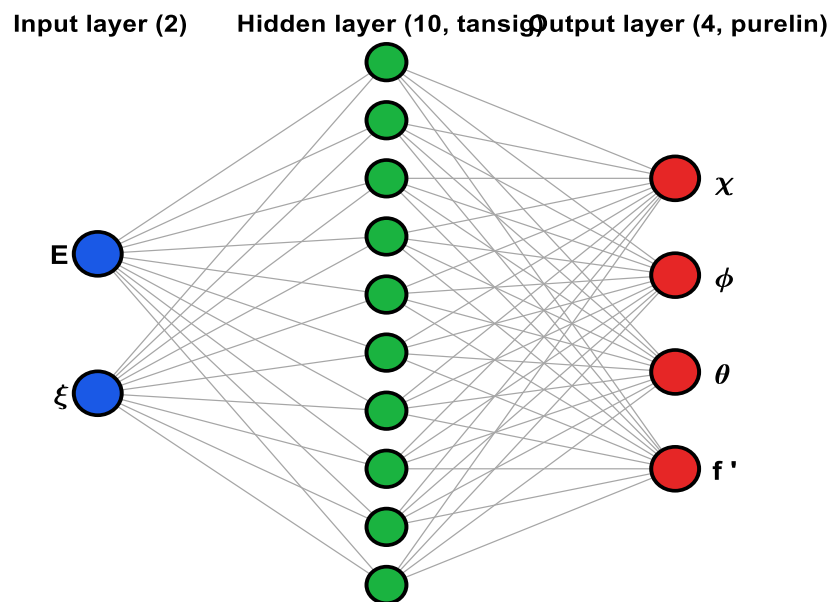


Figure.47: Activation energy parameter (E) LMANN Architecture

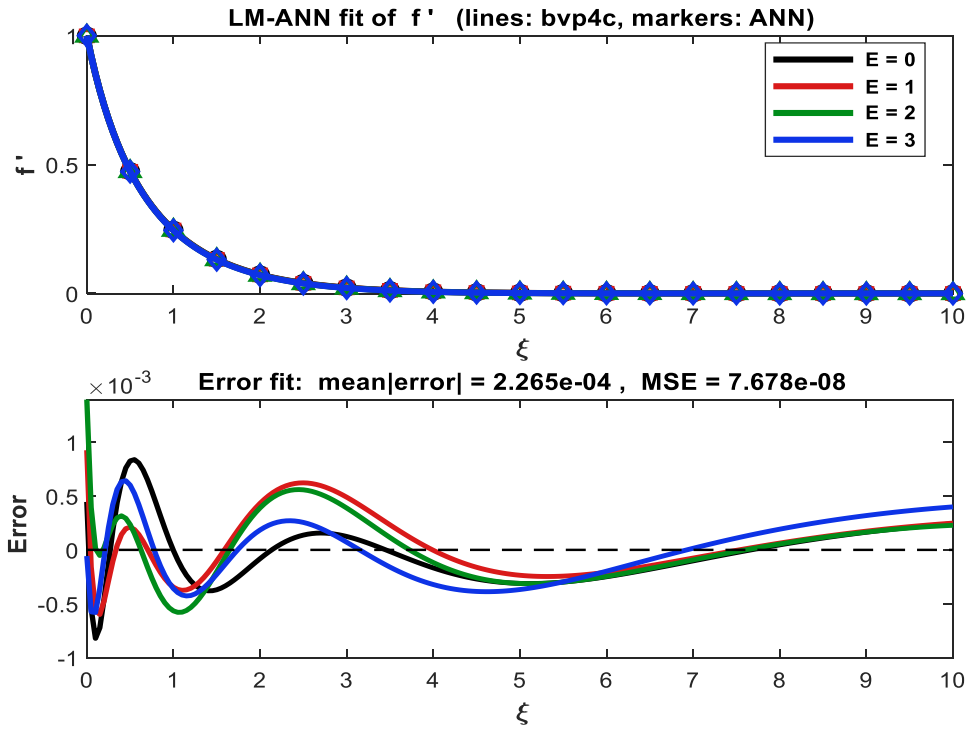


Figure.48: Activation energy parameter (E) velocity profile fit and error

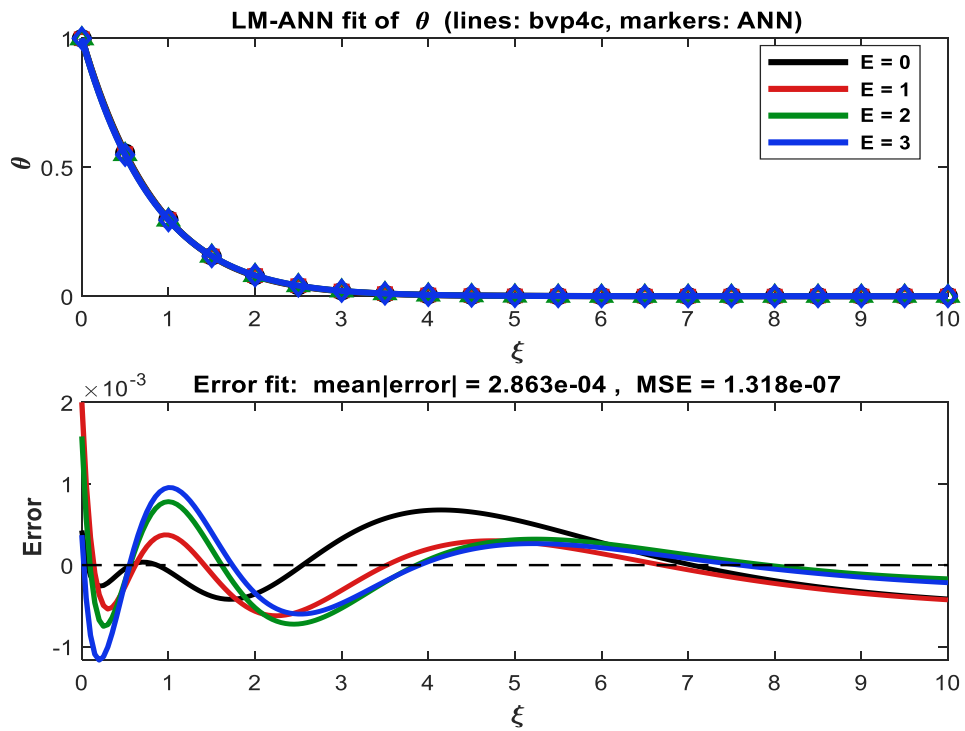


Figure.49: Activation energy parameter (E) temperature profile fit and error

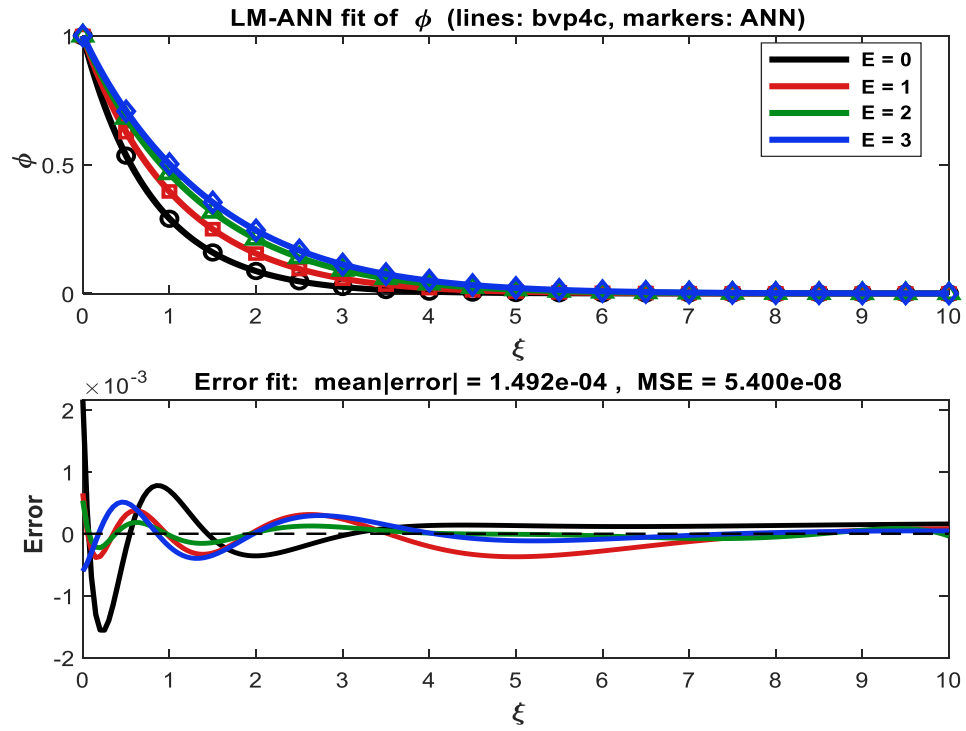


Figure.50: Activation energy parameter (E) concentration profile fit and error

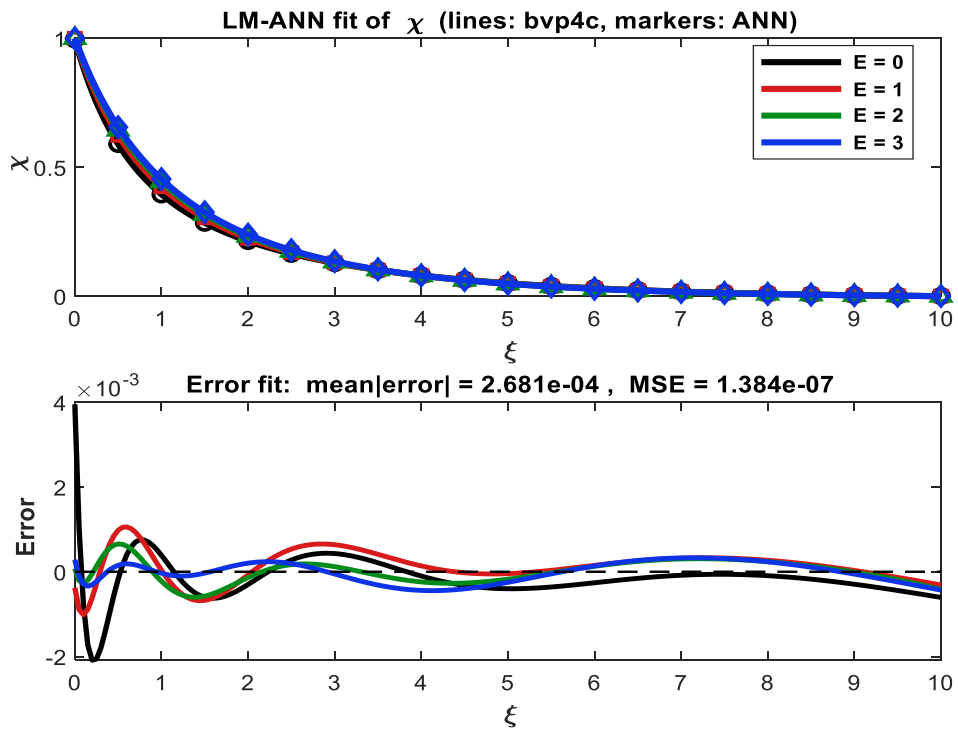


Figure.51: Activation energy parameter (E) motile microorganism density profile fit and error

Table4: Training Progress of parameter Activation energy parameter (E)

Training Progress				
Unit	Initial Value	Stopped Value	Target Value	
Epoch	0	1000	1000	▲
Elapsed Time	-	00:00:07	-	
Performance	0.997	9.88e-08	1e-10	
Gradient	0.917	1.72e-06	1e-07	
Mu	0.001	1e-10	1e+10	
Validation Checks	0	0	20	▼

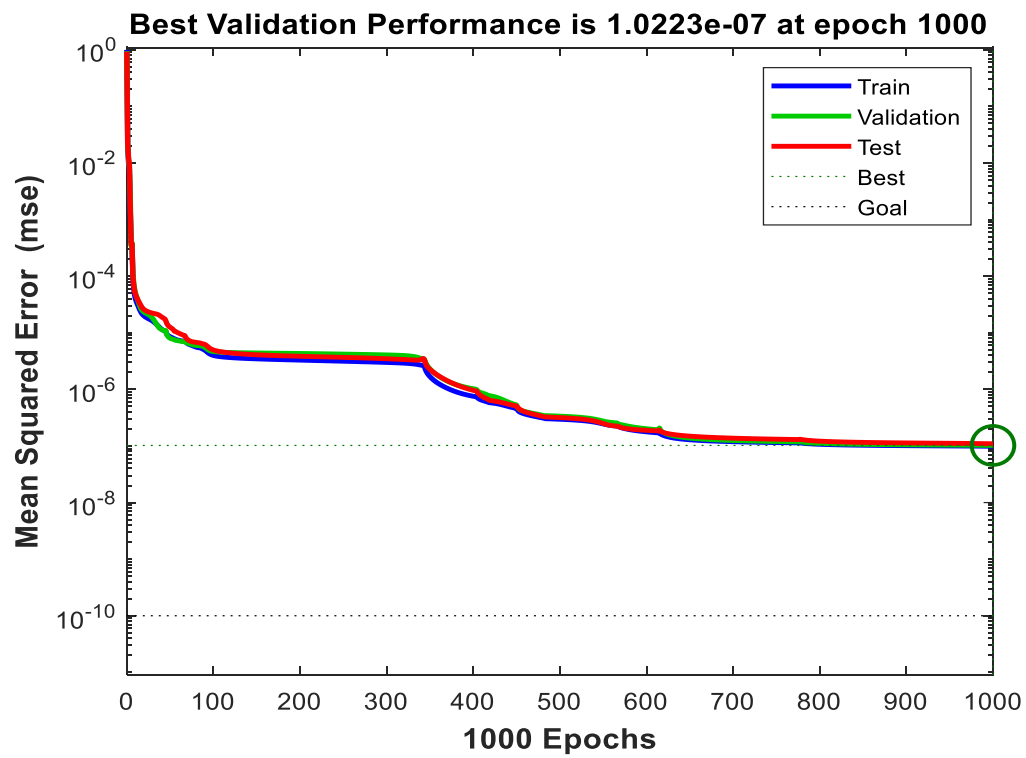


Figure.52: Activation energy parameter (E) best validation

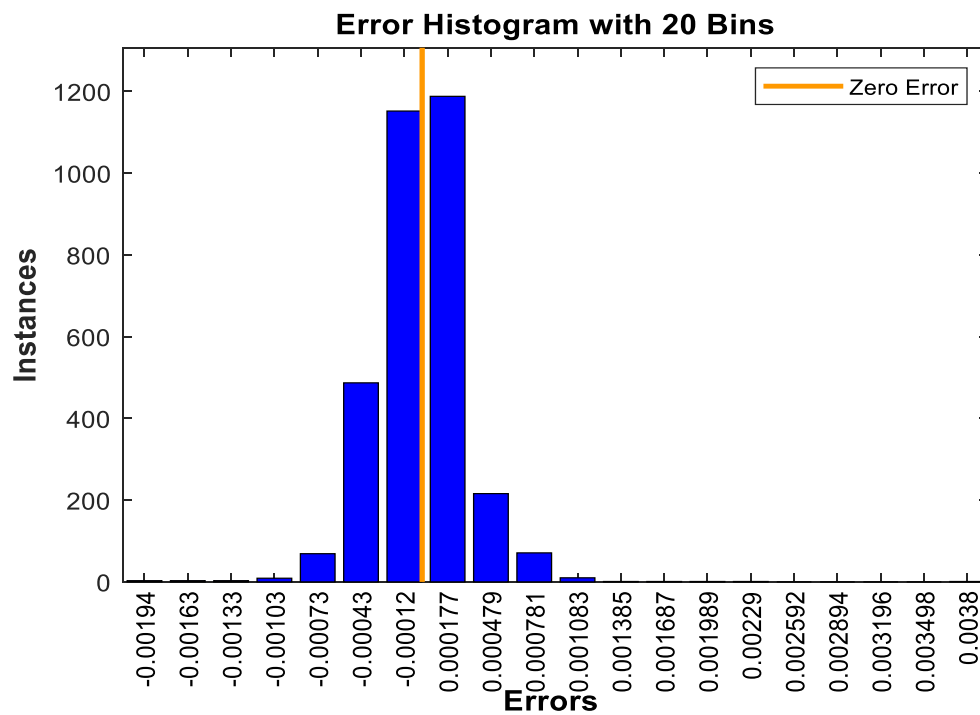


Figure.53: Activation energy parameter (E) histogram

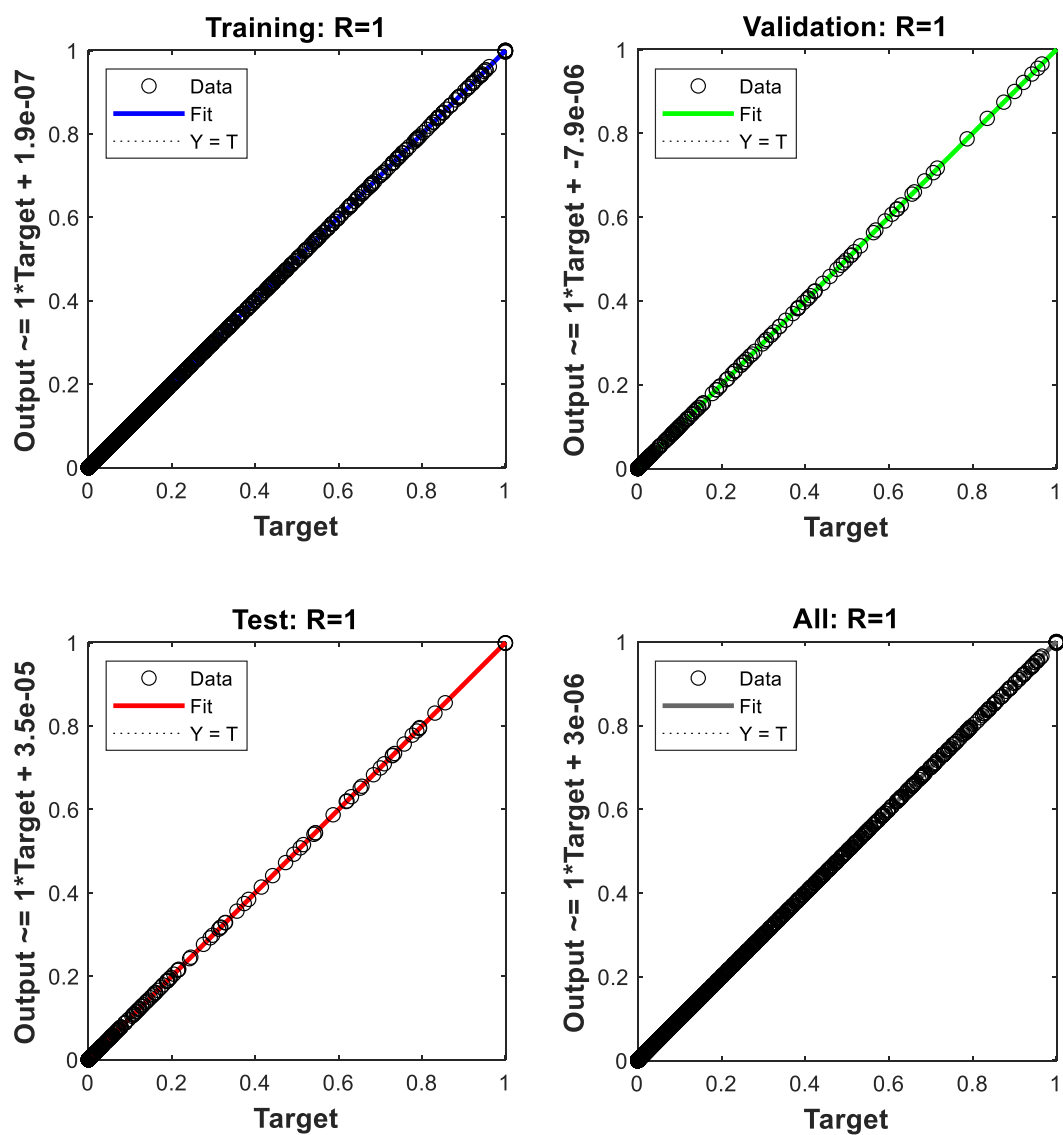


Figure.54: Activation energy parameter (E) Neural Network Training Regression

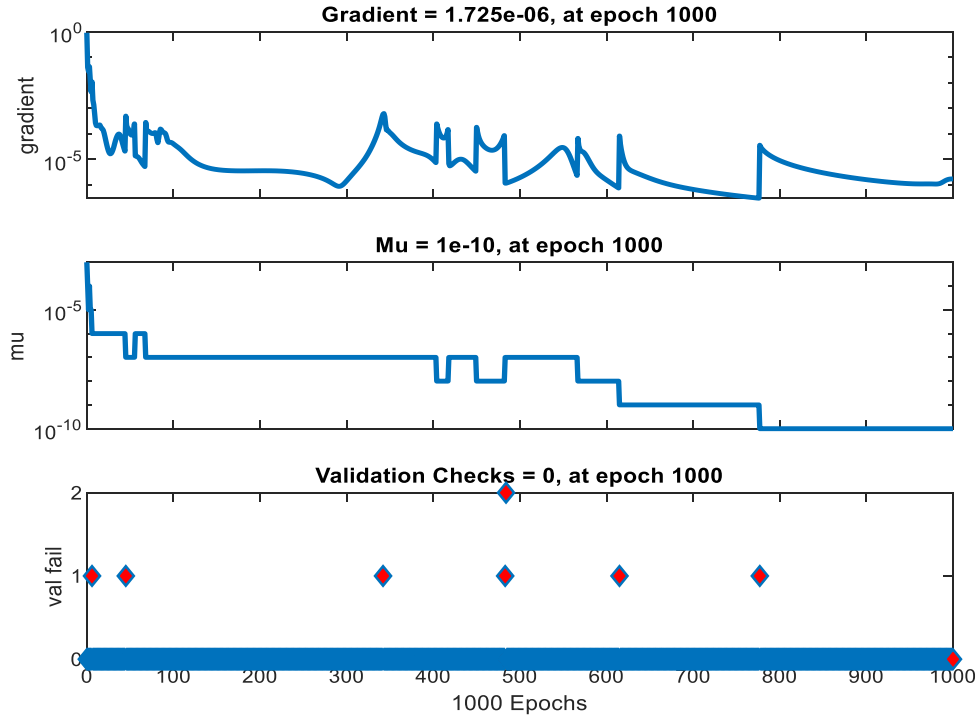


Figure.55: Activation energy parameter (E) training state

Table 5 shows the influence of the governing physical parameters on the local skin friction coefficient $C_f(Re_x)^{1/2}$. It is observed that the magnitude of the skin friction coefficient decreases with increasing Casson parameter (β), Williamson parameter (λ), modified Hartmann number (M), Darcy number (D_a), Riga plate parameter (m_1), and mixed convection parameter (λ_1). This behavior is attributed to the reduction in the wall velocity gradient caused by non-Newtonian fluid characteristics, electromagnetic forcing, enhanced porous medium permeability, and buoyancy-assisted flow. In contrast, increasing the Forchheimer inertia parameter (Fc) increases the magnitude of the skin friction coefficient due to the additional inertial resistance offered by the porous medium. Similarly, higher values of the buoyancy ratio parameter (N_r) and the bioconvection Rayleigh number (Rb) strengthen the combined thermal, solutal, and microorganism buoyancy effects, resulting in an increase in the wall shear stress. Furthermore, the suction parameter (S) enhances the skin friction coefficient by thinning the momentum boundary layer and increasing the velocity gradient at the wall. Overall, the results demonstrate that the local skin friction coefficient is strongly governed by the combined influence of fluid rheology, electromagnetic effects, porous medium properties, buoyancy forces, and wall suction, providing valuable insight into the momentum transport characteristics of the present flow model.

Table 6 presents the variation of the local Nusselt number $Nu(Re_x)^{-1/2}$ for different thermal parameters. The results reveal that the local heat transfer rate increases with increasing thermal radiation parameter (Rc), Prandtl number (Pr), and thermal relaxation parameter (Γ_E). An increase in the thermal radiation parameter enhances thermal energy transport, leading to a steeper temperature gradient at the wall and consequently a higher Nusselt number. Similarly, higher values of the Prandtl number reduce thermal diffusivity, resulting in a thinner thermal boundary layer and improved heat transfer. The thermal relaxation parameter also enhances the wall heat transfer by accounting for finite heat propagation effects. Conversely, increasing the Brownian motion parameter (Nb) and the thermophoresis parameter (Nt) decreases the local Nusselt number due to the thickening of the thermal boundary layer caused by intensified nanoparticle diffusion and thermophoretic transport. Overall, the results demonstrate that the heat transfer characteristics are significantly governed by thermal radiation, fluid thermal properties, relaxation effects, and nanoparticle transport mechanisms.

Table 7 presents the variation of the local Sherwood number $Sh(Re_x)^{-1/2}$ for different mass transfer parameters. The results show that the local mass transfer rate decreases with increasing thermophoresis parameter (Nt), while it increases with increasing Brownian motion parameter (Nb) due to enhanced nanoparticle diffusion. The Lewis number (Le) also enhances the Sherwood number by reducing mass diffusivity and increasing the concentration gradient at the wall. Similarly, an increase in the temperature difference parameter (Γ) improves the local mass transfer rate. However, increasing the activation energy parameter (E) decreases the Sherwood number because higher activation energy weakens the concentration gradient near the surface. Overall, the results indicate that the local mass transfer characteristics are strongly influenced by nanoparticle transport, temperature difference, molecular diffusion, and activation energy.

Table 8 presents the variation of the local motile microorganism density number $Nh(Re_x)^{-1/2}$ for different bioconvection parameters. The results reveal that the local motile microorganism density number increases with increasing bioconvection Lewis number (Lb), Péclet number (Pe), and microorganism-difference parameter (δ). An increase in Lb reduces microorganism diffusivity, leading to a steeper microorganism concentration gradient at the wall. Similarly, higher values of Pe enhance convective transport of microorganisms, thereby increasing the motile microorganism density number. Furthermore, increasing the microorganism-difference parameter (δ) strengthens the microorganism concentration gradient

near the surface, resulting in an enhanced local motile microorganism density number. Overall, the results indicate that microorganism transport is significantly influenced by bioconvection diffusion, convective transport, and microorganism concentration differences.

Table 5

Variations in skin friction for various parameter values

[illegible]

Table 6

Variations in the rate of heat transfer for various parameter values

Rc	Pr	Γ_E	Nb	Nt	$Nu(Re_x)^{-\frac{1}{2}}$
0.10					1.335492
0.50					1.513348
1.00					1.686486
	0.71				0.773830
	1.00				0.964277
	1.71				1.335492
		0.00			1.191333
		0.10			1.335492
		0.20			1.479146
			0.10		1.460683
			0.30		1.335492
			0.50		1.218695
				0.10	1.335492
				0.20	1.300801
				0.30	1.267877

Table 7

Variations in the rate of mass transfer for various parameter values

Nt	Nb	Le	Γ	E	$Sh(Re_x)^{-\frac{1}{2}}$
0.10					0.692395
0.20					0.421014
0.30					0.164847
	0.10				0.011887
	0.30				0.692395
	0.50				0.827298
		0.50			0.252502
		1.00			0.692395
		1.50			1.038028
			0.10		0.955680
			1.00		1.061040
			2.00		1.138921
				0.00	1.272959
				1.00	0.955680
				2.00	0.788487

Table 8

Variations in the motile density number for various parameter values

Lb	Pe	δ	$Nh(Re_x)^{-\frac{1}{2}}$
0.20			0.656810
0.50			0.913700
0.80			1.135983
	0.10		0.638132
	0.30		0.774909
	0.50		0.913700
		0.10	0.886463
		0.30	0.940935
		0.50	0.995399

IV. Conclusions

The present study investigates the bioconvective Buongiorno Casson nanofluid flow over a Riga plate by incorporating gravitational effects, activation energy, and Cattaneo–Christov heat flux. The governing nonlinear equations are solved numerically, and the predictive capability of the proposed LMANN framework is evaluated through comprehensive training, validation, and regression analyses. The major findings are summarized as follows:

- The Riga plate effectively controls the momentum boundary layer through electromagnetic forcing, while the Casson and Williamson parameters increase fluid resistance and suppress the velocity distribution.
- The mixed convection parameter enhances the fluid velocity, whereas the Darcy, Forchheimer, buoyancy ratio, and bioconvection Rayleigh parameters reduce the flow due to increased drag and opposing buoyancy effects.
- The Brownian motion, thermophoresis, and thermal radiation parameters increase the fluid temperature, whereas the Prandtl number, thermal relaxation parameter, and suction reduce the thermal boundary layer thickness.
- Increasing the activation energy parameter enhances the nanoparticle concentration and motile microorganism density by reducing the chemical reaction rate and promoting species accumulation within the boundary layer.
- Higher values of the Lewis number, temperature difference parameter, Peclet number, and bioconvection Lewis number suppress the concentration and microorganism distributions because of reduced mass diffusivity and stronger microorganism transport.

- The Cattaneo–Christov heat flux model captures finite thermal relaxation effects, providing a more realistic description of heat transport than the classical Fourier heat conduction model.
- The interaction of gravitational buoyancy, nanoparticle transport, and bioconvection significantly influences the coupled momentum, heat, mass, and microorganism transport mechanisms, emphasizing the importance of multiphysics modeling.
- The proposed LMANN model accurately predicts the velocity, temperature, concentration, and motile microorganism profiles with negligible fitting errors (approximately 10^{-3}), showing excellent agreement with the numerical solutions.
- The training progress, validation, regression, histogram, and training-state analyses demonstrate stable convergence, excellent generalization, and robust predictive performance of the LMANN framework without evidence of overfitting.
- The proposed hybrid numerical–LMANN framework provides an efficient and reliable computational tool for analyzing complex nonlinear bioconvective nanofluid flows and has potential applications in electronic cooling, biomedical engineering, chemical processing, renewable energy systems, and advanced thermal management technologies.

Conflict of Interest Statement: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement:

The data are, however, available from the corresponding author, [Initials], upon reasonable request.

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Contribution of author

K. R. Sekhar: Conceptualization, Methodology, Software, Writing – original draft.

Janke V Ramana Reddy: Visualization, Investigation, Supervision,

Gurram Dharmaiah: Supervision, Formal analysis, Methodology, Software.

Seepana Praveenkumar: Formal analysis, Software, Investigation.

K.V. Nagaraja: Supervision, Validation.

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