

Multi-objective Portfolio Optimization Based on the Fama–French Three-Factor Model and the Black–Litterman Framework

Shili Dang

20241011@gdufe.edu.cn

Guangdong University of Finance & Economics

Yang Liu

Guangdong University of Finance & Economics

Research Article

Keywords: mean absolute semi-deviation, entropy, Fama–French three-factor model, Black–Litterman model, ideal-point method

Posted Date: August 28th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10707431/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Title page

Multi-objective Portfolio Optimization Based on the Fama– French Three-Factor Model and the Black–Litterman Framework

Shili Dang

School of Mathematics and Statistics, Guangdong University of Finance &
Economics, Guangzhou 510320. P.R. China. **E-mail:** 20241011@gdufe.edu.cn

Yang Liu

School of Finance, Guangdong University of Finance & Economics, Guangzhou
510320. P.R. China. **E-mail:** liuyang@student.gdufe.edu.cn

Corresponding: Shili Dang* (E-mail: 20241011@gdufe.edu.cn)

- Ethical Statement: no unethical activities were performed.
- Funding: This work was supported by the Guangdong Provincial Philosophy and Social Science Planning Project (Youth Project) (Grant No. 21400-KYB02026007) and the Scientific Research Project of Guangdong Provincial Department of Education (Youth Innovative Talents Project) (Grant No. 21400-KYC02025004).
- Conflict of Interest: None
- Informed consent: we (all the co-authors) are informed about this research work and show are consent for publication.
- Acknowledgement: we thank all co-author

Multi-objective Portfolio Optimization Based on the Fama–French Three-Factor Model and the Black–Litterman Framework

Abstract: As financial-market volatility increases, constructing portfolios that balance return, risk, and diversification has become a major challenge in investment management. Most existing portfolio optimization models focus primarily on return and risk but pay insufficient attention to return-estimation error, downside risk, portfolio diversification, and practical investment constraints, which may limit their robustness in real-world applications. To address these issues, this study proposes a multi-objective portfolio optimization model that integrates the Fama–French three-factor framework with the Black–Litterman model. Specifically, the systematic risk exposures of individual assets are first estimated using the three-factor model and used to construct prior expected returns in the Black–Litterman Bayesian updating process, thereby obtaining posterior expected returns. Downside risk is measured by mean absolute semi-deviation, and portfolio diversification is quantified by entropy. Transaction costs and upper and lower bounds on risky-asset weights are further incorporated, and the resulting three-objective optimization problem is solved using the ideal-point method. The empirical results show that the proposed model outperforms the benchmark strategies both in sample and out of sample, generating higher returns, lower downside risk, and greater diversification.

Keywords: mean absolute semi-deviation; entropy; Fama–French three-factor model; Black–Litterman model; ideal-point method

1 Introduction

Against the backdrop of rapidly evolving global financial markets and rising uncertainty, how to construct investment portfolios that maintain robust performance in volatile environments has become a major research topic in the field of asset management. In 1952, Markowitz [1] proposed the mean-variance model, which formalized the trade-off between return and risk, laying the foundation for modern portfolio theory. However, variance, as a measure of risk, includes both positive and negative deviations from the mean in its calculation, thereby treating “favorable deviations” as risk as well. Furthermore, the model is based on the assumption that returns follow a normal distribution, whereas actual financial data often exhibit heavy-tailed behavior, which limits the ability of variance to capture downside risk. To address these issues, Speranz [2] proposed the mean–semi-absolute deviation model, which focuses specifically on downside deviations, which are of primary concern to investors. Subsequently, Zhang Peng et al. [3] extended this framework by considering the mean absolute semi-deviation under short-selling constraints under restricted

short-selling conditions; Vercher and Bermúdez [4] further developed a multi-objective portfolio optimization method based on the half absolute deviation. These studies suggest that the half absolute deviation, as a measure of downside risk, effectively enhances the model's interpretability and its ability to align with actual investor preferences.

Nevertheless, classical mean–variance and mean absolute deviation models are highly sensitive to estimation errors in expected returns and covariance matrices, often resulting in extreme portfolio weights and unstable out-of-sample performance. To mitigate the impact of estimation biases, Black and Litterman [5] proposed the Black-Litterman(BL) model, which provides a Bayesian framework for combining market-equilibrium returns with investors' subjective views, thereby making estimates of expected returns smoother and more robust. Subsequently, many studies have extended the BL model. For example, Song Zhengyang et al. [6] constructed investor views using various forecasting algorithms and ensemble methods and combined them with the BL model to enhance portfolio performance; Yao Haixiang et al. [7] utilized adaptive boosting algorithms to generate sector views for optimal asset allocation; Sun et al. [8] introduced time-varying covariance into the BL model, and empirical evidence showed that it outperformed traditional methods; Meng Yong [9] constructed an industry sentiment matrix based on social media sentiment; Xu Meiping et al. [10] utilized a vector autoregressive error correction model and a multidimensional biased t-distribution prior to construct a BL model with a t-Copula structure. While these studies have expanded the methods used to construct investor views from various perspectives, they generally still face the challenges of over-reliance on empirical data and a lack of systematic theoretical foundations for generating investor views.

When the BL model relies solely on market-equilibrium returns or subjective views, its conceptual framework lacks a unified theoretical foundation. The Capital Asset Pricing Model (CAPM) provides a possible theoretical basis for addressing this issue. Traditionally, the CAPM describes sources of return using market risk factors; while this approach is simple, it fails to adequately explain empirical anomalies such as the size effect and the value effect [12]. The Fama–French [13] three-factor model introduces two additional factors—size (SMB) and value (HML)—to the CAPM framework, thereby providing a more comprehensive characterization of the systematic determinants of asset returns: market risk, size, and value. The model has a well-established theoretical and empirical foundation. Through a structured regression framework, the factor model can reliably derive quantitative measures of factor risk premia, thereby providing an objective and economically meaningful basis for constructing views within the BL model. Motivated by this perspective, an increasing number of studies in recent years have incorporated the three-factor model into expected-return estimation to enhance explanatory power and predictive capability in portfolio allocation. For example, Ko et al. [14] incorporated factor structure into the

view-building process of the BL model, which not only theoretically strengthened the economic significance of the views but also empirically reduced estimation bias, thereby improving the robustness and performance of the portfolio. Zhang Peng et al. [15] used the least squares method to fit the relationship between asset returns and common factors—such as the market factor, size factor, and book-to-market factor—in the Fama-French three-factor model, and constructed a three-factor mean-variance portfolio model based on a backpropagation (BP) neural network. Yang Zunhan [16], building on the CAPM and Fama–French three-factor models, tested the validity of the CAPM and the Fama-French three-factor model in China’s A-share market.

Although factor models improve the theoretical basis of expected-return estimation, relying solely on return forecasts does not fully address the practical requirements of portfolio optimization. On the one hand, investors not only seek a balance between return and risk but also place greater emphasis on the degree of portfolio diversification to mitigate the concentration risk associated with concentrated positions. Entropy, as an effective metric for measuring portfolio diversification, captures the evenness of portfolio weights and may improve portfolio robustness under adverse market conditions [17]. Many scholars have also incorporated entropy as an objective or constraint into portfolio models. For example, Sun Rong and Yin Hao [18] introduced coherent fuzzy numbers to describe the returns of risky assets, used an adaptive parameter k to characterize investors’ varying return expectations, and constructed a coherent fuzzy mean-variance-proportion entropy model. Zeng Yongquan et al. [19] introduced an entropy function to enhance portfolio diversification, constructing three uncertain mean-absolute deviation portfolio decision models with different entropy constraints. On the other hand, real-world financial markets are generally subject to practical constraints such as transaction costs, borrowing constraints on risk-free assets, and upper and lower bounds on risky-asset weights [20][21]. If these practical constraints are ignored, the optimization results are often difficult to implement and may even fall outside the investor’s feasible investment set.

In summary, this study constructs a multi-objective portfolio model that integrates the Fama–French three-factor model with the Black–Litterman framework and incorporates return, downside risk, diversification, and practical investment constraints. Specifically, the Fama–French three-factor model is first estimated using the sample stocks to obtain their exposures to the market, size, and value factors. The expected returns estimated from the factor model are then combined with market-equilibrium returns within the Black–Litterman Bayesian updating framework to obtain FF–BL posterior expected returns. These posterior expected returns are used as the return input for portfolio optimization, while downside risk is measured using mean absolute semi-deviation and portfolio diversification is quantified using entropy. After incorporating transaction costs and portfolio-weight constraints, a three-objective portfolio optimization model is constructed and

solved using the ideal-point method. Using 30 constituent stocks of the CSI 300 Index as the sample, this study conducts an empirical analysis based on daily data from January 1, 2024, to June 30, 2025, and compares the proposed model with the equal-weighted 1/N strategy in terms of return, risk, and diversification.

The main contributions of this study are threefold: **First**, by embedding the Fama–French three-factor model into the Black–Litterman framework, we provide a unified theoretical basis for constructing investor views through structured factor pricing. This effectively mitigates the subjectivity in view construction and estimation errors in expected returns associated with the traditional BL model, thereby enhancing the scientific rigor and stability of return forecasts. **Second**, we construct a three-objective optimization model—mean, semi-absolute deviation, and entropy—that simultaneously balances return maximization, downside risk minimization, and greater portfolio diversification, better aligning with the multi-objective preferences of investors in real markets. **Third**, we incorporate real-world investment constraints such as transaction costs and portfolio-weight constraints into the model, making the optimization results more implementable in practice. Through multi-dimensional performance metrics and robustness tests, we demonstrate that the proposed model comprehensively outperforms traditional strategies in terms of returns, risk-adjusted performance, downside-risk control, and portfolio robustness. The proposed framework provides theoretical and methodological support for institutional investment, wealth management, and asset allocation decisions in China’s capital markets.

The remainder of this paper is organized as follows. Section 2 presents the Fama–French three-factor model, the Black–Litterman model, mean absolute semi-deviation, and entropy. Section 3 describes the construction of the FF–BL mean absolute semi-deviation–entropy portfolio model. Section 4 introduces the ideal-point method used to solve the multi-objective optimization problem. Section 5 presents and discusses the empirical results, including model fitting, out-of-sample performance comparisons, and robustness tests. Section 6 concludes the paper and discusses directions for future research.

2 Preliminaries

This study combines the Fama–French three-factor model [13] with the Black–Litterman model [5] to forecast stock returns and construct a multi-objective portfolio optimization model. The relevant definitions and concepts are introduced below.

Definition 2.1 [11][23] In the original CAPM model, an asset’s excess return is modeled as a function of a single market factor:

$$r_i = R_i - R_f = a_i + b_i(R_m - R_f) \quad (1)$$

The Fama–French three-factor model extends the CAPM by adding two factors—size and value—to the market risk premium, in order to improve the model’s ability to explain asset returns. Its multi-factor regression model can be expressed as:

$$r_{it} - r_{ft} = \alpha_i + \beta_i MKT_t + s_i SMB_t + h_i HML_t + \varepsilon_{it} \quad (2)$$

Where r_{it} represents the return on risky asset i in period t , r_{ft} denotes the risk-free rate of return in period

t , β_i , s_i and h_i denote the factor loadings for the i -th risky asset, α_i is the intercept term, and ε_{it} is the error term.

Definition 2.2 [15][23] This study employs the standard 2×3 sorting procedure by Fama and French to calculate the size factor (SMB) and the book-to-market ratio factor (HML). The specific calculation process is as follows: Specifically, **At each period t** , stocks are first divided into three groups—high (H), medium (M), and low (L)—based on their book-to-market ratios in that period; Subsequently, within each book-to-market ratio group, **stocks are first sorted into three groups according to their book-to-market ratios: the small (S) and big (B) groups**. Through this two-stage grouping mechanism, **all sample stocks are classified into six portfolios**, denoted as SH, SM, SL, BH, BM, BL and BM.

Market factor (MKT):

$$MKT_t = R_{m,t} - R_{f,t} \quad (3)$$

Here, $R_{m,t}$ denotes the market-weighted return of the market portfolio for all stocks in period t , and $R_{f,t}$ denotes the risk-free rate.

Size factor (SMB):

$$SMB_t = \frac{(SH_t + SM_t + SL_t)}{3} - \frac{(BH_t + BM_t + BL_t)}{3} \quad (4)$$

Value factor (HML):

$$HML_t = \frac{(SH_t + BH_t)}{2} - \frac{(SL_t + BL_t)}{2} \quad (5)$$

Here, SH_t , SM_t , SL_t , BH_t , BL_t represent the market-capitalization-weighted portfolio returns for each group of stocks in period t .

Definition 2.3 Following Markowitz [1], the classical mean–variance model is formulated as the following optimization problem:

$$\begin{aligned} \min_{\omega} \quad & \frac{\lambda}{2} \omega^T \Sigma \omega - \omega^T \mu, \\ \text{s.t.} \quad & \omega^T \mathbf{1} = 1 \end{aligned} \quad (6)$$

Here, $\omega \in \mathbb{R}^N$ denotes the portfolio weight vector, $\Sigma \in \mathbb{R}^{N \times N}$ denotes the **covariance matrix of asset returns**, $\mu \in \mathbb{R}^N$ denotes the expected return vector, and $\lambda \in \mathbb{R}$ denotes the investor's risk aversion coefficient. **The analytical solution obtained using Merton's method is given as follows** [24]:

$$\omega^* = \frac{1^T \Sigma^{-1} \mu}{1^T \Sigma^{-1} \mu} \cdot \frac{\Sigma^{-1} \mu}{1^T \Sigma^{-1} \mu} + \left(1 - \frac{1^T \Sigma^{-1} \mu}{\lambda}\right) \cdot \frac{\Sigma^{-1} \mathbf{1}}{1^T \Sigma^{-1} \mathbf{1}} \quad (7)$$

Here, ω^* denotes the **optimal portfolio-weight vector**.

Definition 2.4 [5][25] The Black-Litterman model uses Bayes' theorem to **combines market-equilibrium returns with investor views** to calculate expected returns and the covariance matrix. Regarding market views, the prior distribution of expected returns is:

$$r \sim N(\mu, \Sigma) \quad (8)$$

Where μ is the equilibrium expected return and Σ is the covariance matrix of historical returns. In the Black-Litterman model, μ can be expressed in terms of probability and follows a normal distribution.

$$\mu \sim N(\pi, \tau \Sigma) \quad (9)$$

In this context, π is μ ; since the estimate of $\tau \Sigma$ is uncertain, and $\tau = 1/T$ (where T represents the sample size), the formula can be transformed into:

$$\mu \sim N\left(\pi, \frac{\Sigma}{T}\right) \quad (10)$$

π can be calculated using the Capital Asset Pricing Model (CAPM) theory and the inverse optimization formula, expressed as follows:

$$\pi = 2\lambda \sum x_{eq} \quad (11)$$

Here, x_{eq} represents the market weight of the portfolio, and λ is the average market risk aversion coefficient, which can be derived from the following formula:

$$\lambda = \frac{E(r) - r_f}{\sigma^2} \quad (12)$$

Here, $E(R_i)$ denotes the expected market return, $\sigma^2 = x'Gx$ represents the variance of the market return, and r_f is the risk-free rate.

From a subjective perspective, suppose an expert generates K independent opinions about N assets and expresses these opinions as a $K \times N$ "opinion selection matrix P ." Then these opinions follow a normal distribution:

$$V = P\mu \sim N(v, \Omega) \quad (13)$$

Here, v is the vector of expected returns for the views, and Ω is the covariance matrix, representing expert opinions and uncertainty, respectively.

According to Definition 2.1, the Fama-French model can be estimated as follows:

$$\Pi_{FF} = R_f + \beta_{MKT} \cdot E[MKT] + \beta_{SMB} \cdot E[SMB] + \beta_{HML} \cdot E[HML] \quad (14)$$

Using Bayes' theorem, we can derive the posterior distribution of the expected return:

$$r \sim N(\mu_{BL}, \Sigma_{BL}) \quad (15)$$

In which μ_{BL} is expressed as follows:

$$\mu_{BL} = [(\tau \Sigma)^{-1} + P^T \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi_{FF} + P^T \Omega^{-1} Q] \quad (16)$$

3 Multi-objective Portfolio Model

3.1 Problem Description

Suppose an investor invests in n risky assets in the stock market. For convenience, let R_i denote the random return on risky asset i , and let r_i denote the expected value of R_i , let $r_i = E(R_i)$; let x_i be the investment proportion in risky asset i , where $x = (x_1, x_2, \dots, x_n)'$, and $x_1 + x_2 + \dots + x_n = 1$; The covariance matrix $\Omega = (\sigma_{ij})_{n \times n}$, where $\sigma_{ij} = \text{COV}(R_i, R_j)$, $i, j = 1, 2, \dots, n$; x_{0i} is the initial allocation to risky asset i ; r_N is the net return on portfolio x ; u_i is the upper bound constraint on x_i ; l_i is the lower bound constraint on x_i ; c_i is the transaction cost per unit of risky asset i ; In this paper, $(\cdot)$ denotes the transpose of a matrix; the expected return of the portfolio is $r_p = r_1 x_1 + \dots + r_n x_n$, and the variance is $\sigma_p^2 = x' \Omega x$.

3.2 Portfolio Returns and Risk

Assuming that the expected return on risk asset i in period t is $r_{i,t}$, and applying the Fama-French-Black-Litterman model, the expected return is given by:

$$\mu_{BL} = [(\tau \Sigma)^{-1} + P^T \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi_{FF} + P^T \Omega^{-1} Q] \quad (17)$$

Where: $\Pi_{FF} = R_f + \beta_{MKT} \cdot E[MKT] + \beta_{SMB} \cdot E[SMB] + \beta_{HML} \cdot E[HML]$, P is the investor opinion matrix, Q is the opinion return vector, Ω is the opinion uncertainty matrix, τ is the confidence parameter, and Σ is the covariance matrix of returns.

Traditional mean-variance models use variance as a measure of risk, but variance treats positive and negative deviations equally. To more accurately measure downside risk, this paper introduces semi-absolute deviation [2] as a measure of risk:

$$SAD = E[| \min(0, R_p - E[R_p]) |] \quad (18)$$

Here, R_p represents the portfolio return.

Let r_{pt} be the return on the portfolio in period t , and $\bar{r}_p$ be the average return on the portfolio; then the semi-absolute deviation is estimated as:

$$SAD = \frac{1}{T} \sum_{t=1}^T |\min(0, r_{pt} - \bar{r}_p)| \quad (19)$$

Let the transaction costs for portfolio x be a V-shaped function, and let the initial investment weights be $x=(x_{01}, x_{02}, \dots, x_{0n})'$. Then, the total transaction costs for portfolio x are:

$$C = \sum_{i=1}^n c_i |x_i - x_{0i}|$$

(20)

Therefore, the net return on portfolio x is:

$$r_N = \sum_{i=1}^n \mu_{BL} x_i - \sum_{i=1}^n c_i |x_i - x_{0i}| \quad (21)$$

In addition, this paper introduces entropy to measure the dispersion of portfolio x , which can be expressed as:

$$H(x) = - \sum_{i=1}^n x_i \ln x_i \quad (22)$$

In particular, when $x_i=0$, the portfolio entropy $H(x)=0$, indicating that all investors have invested in risk-free assets; when $x_i=1/n$, the portfolio entropy $H(x)=\ln n$, indicating that investors have invested in equal proportions.

3.3 FF-BL Mean-Semi-Absolute Deviation-Entropy Portfolio Model

Given the practical constraints, the FF-BL portfolio model based on mean-absolute deviation-entropy is constructed as follows:

$$\begin{aligned} \max \quad & f_1(x) = \sum_{i=1}^n \mu_{BL} x_i - \sum_{i=1}^n c_i |x_i - x_{0i}| \\ \min \quad & f_2(x) = \frac{1}{T} \sum_{t=1}^T |\min(0, \sum_{i=1}^n r_{it} x_i - \bar{r}_p)| \\ \max \quad & f_3(x) = - \sum_{i=1}^n x_i \ln x_i \\ \text{s.t.} \quad & \begin{cases} \sum_{i=1}^n x_i = 1 & (a) \\ l_i \leq x_i \leq u_i, \quad i, j = 1, 2, \dots, n & (b) \end{cases} \end{aligned} \quad (23)$$

In particular, constraint (a) specifies that the sum of the investment proportions in risk-free assets equals 1; constraint (b) specifies the upper and lower bounds for the investment proportion of each risk asset.

Since the mean-absolute deviation-entropy investment model is a multi-objective optimization problem, this paper will adopt the ideal-point method [24] to solve it, transforming Model (23) into:

$$\begin{aligned}
\min f_1(x) &= -\left(\sum_{i=1}^n \mu_{BL} x_i - \sum_{i=1}^n c_i |x_i - x_{0i}|\right) \\
\min f_2(x) &= \frac{1}{T} \sum_{i=1}^T |\min(0, \sum_{i=1}^n r_{it} x_i - \bar{r}_p)| \\
\min f_3(x) &= \sum_{i=1}^n x_i \ln x_i \\
s.t. &\begin{cases} \sum_{i=1}^n x_i = 1 \\ l_i \leq x_i \leq u_i, \quad i, j = 1, 2, \dots, n \end{cases}
\end{aligned} \tag{24}$$

4 Ideal Point Method

4.1 Basic Theory [24]

First, find the minimum value of each function $f_k(x)$ on the constraint set X , denoted as f_k^* , where $k=1, 2, \dots, m$. Then, use f_k^* as the ideal reference value for each $f_k(x)$ to find a solution on the constraint set X where each objective function is close to its ideal value.

Suppose that x^{k*} and f_k^* are the minimum point and minimum value, respectively, of $f_k(x)$ on the constrained set X , $f_k^* = f_k(x^{k*}) = \min_{x \in X} f_k(x)$, where $k = 1, 2, \dots, m$. Then $f^* = (f_1^*, \dots, f_m^*)^T$ is called the ideal point of the composite function $f(x)$.

By selecting a specific norm $\|\cdot\|$ for R^m , To solve a multi-objective minimization problem, (Model 1) can be transformed into $U(f(x)) = \|f(x) - f^*\|$ or the evaluation function $U(\varphi(x)) = \|\varphi(x) - \varphi^*\|$ for the normalized objective function $\varphi(x)$ to solve the minimization problem over the constraint set X :

$$\min_{x \in X} U(\varphi(x)) = \min_{x \in X} \|\varphi(x) - \varphi^*\| \tag{25}$$

In particular, the partial functions $\varphi(x) = (\varphi_1(x), \dots, \varphi_m(x))^T$ correspond to the normalized functions $f_k(x)$ and $\varphi^* = (\varphi_1^*, \dots, \varphi_m^*)^T$ is an ideal point of $\varphi(x)$.

4.2 Specific steps

Step 1: Find the minimum point x^{k*} , the minimum value f_k^* , and the maximum value f_k^Δ .

$$f_k^* = f_k(x^{k*}) = \min_{x \in X} f_k(x), \quad f_k^\Delta = \max_{x \in X} f_k(x), k=1, \dots, m. \tag{26}$$

Step 2: Check each local minimum.

- (1) If $x^* = x^{1*} = \dots = x^{m*}$
- (2) If there exist $i, j \in m$, such that $x^{i*} \neq x^{j*}$, then proceed to Step 3.

Step 3: Normalize the objective functions.

Select the interval $[0, 1]$ as the normalization interval, compute the normalized objective functions $\varphi_k(x)$ using Equation (26), and obtain the normalized composite function $\varphi(x) = (\varphi_1(x), \dots, \varphi_m(x))^T$.

$$\varphi_k(x) = \frac{f_k(x) - f_k^*}{f_k^\Delta - f_k^*} \tag{27}$$

From $\varphi_k^* = \varphi_k(x^{k*}) = \min_{x \in X} \varphi_k(x) = 0$, we know that the ideal points of $\varphi(x)$ are

$$\varphi^* = (0, 0, \dots, 0)^T.$$

Step 4: Construct the ideal evaluation function.

Select a specific distance $\|\cdot\|$ from R^m , and construct an ideal evaluation function (Equation 28) based on the normalized composite objective function $\varphi(x)$ and the ideal point $\varphi^* = (0, 0, \dots, 0)^T$ as shown below:

$$U(\varphi(x)) = \|\varphi(x) - \varphi^*\| = \|\varphi(x)\|. \quad (28)$$

Then, use an appropriate numerical optimization method to solve the following minimization problem and obtain the optimal solution.

$$\min_{x \in X} \|\varphi(x)\| \quad (29)$$

5 Empirical Analysis

5.1 Data Selection and Parameter Settings

This study selects 30 representative stocks from the CSI 300 Index to form the stock universe. The stock codes are as follows: 002050, 002407, 300014, 300033, 300059, 300072, 300124, 300136, 300207, 300255, 300274, 300346, 300394, 300418, 600030, 600031, 600048, 600196, 600276, 600519, 600588, 600887, 601166, 601186, 601318, 601601, 601688, 601857, 601888, 603026. This paper collects daily closing prices from January 1, 2024, to June 30, 2025, as the sample data, which is sourced from the CSMAR database. Set the daily return on the risk-free asset to $r_f = 0.000119$, Short selling of risk assets is not permitted. The lower and upper bounds for the investment ratio in risk assets are $l_i = 0$ and $u_i = 0.3$, respectively. The cost of stock trading is $c_i = 0.003$.

5.2 The Fama-French Three-Factor Model

(1) Regression results for the Fama–French three-factor model

This study estimates the Fama–French three-factor model using ordinary least squares (OLS) regression, The estimated coefficients are then used, to quantify the exposure of each stock, size, and value factors. The intercept term captures the component of excess returns not explained by the three factors. **The regression results are reported in Table 1:**

<i>code</i>	α_i	β_i	s_i	h_i	<i>code</i>	α_i	β_i	s_i	h_i
002050	−0.0004	0.5435	0.6795	−0.7400	600031	0.0004	0.7139	0.1505	0.0786
002407	−0.0008	0.8421	0.9817	0.1603	600048	−0.0007	1.0588	0.2609	0.3375
300014	0.0000	1.3830	1.1030	0.1383	600196	−0.0003	0.6985	0.4938	0.1025
300033	0.0011	1.6769	0.5114	−0.7762	600276	0.0000	0.7589	0.1129	−0.1455
300059	0.0008	1.5267	0.2954	−0.6226	600519	−0.0011	0.9419	−0.3465	−0.2021
300072	0.0003	1.2734	1.3837	0.5825	600588	−0.0008	0.7488	1.1559	−0.0379
300124	−0.0004	1.1450	0.3745	−0.2622	600887	−0.0003	0.8296	−0.2185	−0.0776
300136	−0.0002	1.1716	1.3565	0.1108	601166	0.0010	0.6708	−0.0395	0.2538
300207	0.0007	1.2979	1.3286	0.3282	601186	−0.0001	1.0230	0.4156	0.6686
300255	0.0042	0.9127	1.6190	0.0058	601318	0.0005	1.0472	0.0181	0.1505
300274	−0.0008	1.1881	0.8221	0.0747	601601	0.0008	1.2651	0.0340	0.2761
300346	0.0003	0.8229	1.1918	−0.1329	601688	0.0002	1.0057	0.3723	0.0293
300394	0.0000	0.9104	1.0160	−0.8661	601857	0.0002	0.9729	−0.1665	0.4709
300418	0.0000	1.0825	1.8116	0.2778	601888	−0.0013	1.1870	0.3558	0.0150
600030	0.0004	0.9343	0.1661	−0.2870	603026	−0.0006	0.8930	1.0014	0.2034

(2) Goodness-of-fit and validity assessment of the Three-Factor Model

To assess the goodness of fit of the Fama-French three-factor model, **this study uses the**

mean absolute error, mean squared error, root mean square error, and coefficient of determination as evaluation metrics [20]. The specific calculation formulas are as follows:

$$MAE = \frac{1}{n} \sum_{t=1}^n |R_{i,t} - \hat{R}_{i,t}| \quad (30)$$

$$MSE = \frac{1}{n} \sum_{t=1}^n (R_{i,t} - \hat{R}_{i,t})^2 \quad (31)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (R_{i,t} - \hat{R}_{i,t})^2} \quad (32)$$

$$R^2 = 1 - \frac{\sum_{t=1}^n (R_{i,t} - \hat{R}_{i,t})^2}{\sum_{t=1}^n (R_{i,t} - \bar{R}_i)^2} \quad (33)$$

The results are summarized in Table 2:

code	R ²	MAE	MSE	RMSE	code	R ²	MAE	MSE	RMSE
002050	0.5244	0.0158	0.0005	0.022	600031	0.3382	0.0108	0.0002	0.0146
002407	0.5434	0.0124	0.0003	0.0179	600048	0.3865	0.0135	0.0004	0.019
300014	0.6522	0.015	0.0004	0.0199	600196	0.59	0.0079	0.0001	0.0107
300033	0.6246	0.0177	0.0007	0.0263	600276	0.3718	0.0118	0.0003	0.016
300059	0.6873	0.0124	0.0004	0.0192	600519	0.7182	0.0062	0.0001	0.0085
300072	0.3785	0.0218	0.0011	0.0339	600588	0.4915	0.0159	0.0005	0.0227
300124	0.5547	0.0133	0.0003	0.0184	600887	0.4938	0.0084	0.0001	0.0116
300136	0.6141	0.0152	0.0005	0.0221	601166	0.4387	0.0077	0.0001	0.0104
300207	0.7129	0.0119	0.0003	0.0174	601186	0.6752	0.0076	0.0001	0.0110
300255	0.3295	0.0271	0.0018	0.042	601318	0.6882	0.0071	0.0001	0.0096
300274	0.3984	0.0178	0.0008	0.0275	601601	0.5963	0.0105	0.0002	0.0141
300346	0.5006	0.0169	0.0006	0.0244	601688	0.7217	0.0072	0.0001	0.0101
300394	0.4029	0.0263	0.0015	0.039	601857	0.5091	0.0103	0.0002	0.0138
300418	0.5237	0.0191	0.0009	0.0298	601888	0.5895	0.0104	0.0002	0.0156
600030	0.6200	0.0085	0.0002	0.0128	603026	0.4117	0.0167	0.0006	0.0239

Based on the calculations of the above error metrics, this paper finds that for the full sample, the mean absolute error (MAE) is 0.0134, the mean squared error (MSE) is 0.0004, the mean root mean square error (RMSE) is 0.0195, and the mean coefficient of determination (R²) is 0.5363. This indicates that the overall regression error is relatively low, and the Fama-French three-factor model has substantial explanatory power for stock returns.

5.3 FF-BL Model Predictions and Evaluation of Prediction Results

This paper employs the FF-BL model to perform a Bayesian adjustment of the return for period $t + 1$. The core logic involves using the market equilibrium return implied by the CAPM as the prior distribution and treating the predictions from the Fama-French three-factor model as a perspective reflecting factor pricing logic. After weighting and integrating these elements using Bayes' theorem, the final posterior expected return vector μ_{BL} is obtained. The specific results are as follows:

Code	Forecast Return	Variance	Lower-Half Variance	Standard Deviation	Lower-Half Standard Deviation
002050	0.001107	0.001019	0.000435	0.031917	0.020848

002407	0.002209	0.000701	0.000291	0.026475	0.017062
300014	0.002194	0.001138	0.000425	0.033732	0.020616
300033	0.004375	0.001848	0.000613	0.042994	0.024766
300059	0.003926	0.001186	0.000392	0.034433	0.019788
300072	0.005090	0.001853	0.000780	0.043043	0.027921
300124	0.000877	0.000761	0.000301	0.027591	0.017351
300136	0.003183	0.001267	0.000555	0.035594	0.023555
300207	0.002911	0.001055	0.000430	0.032485	0.020747
300255	0.007952	0.002642	0.000981	0.051399	0.031317
300274	0.003202	0.001263	0.000654	0.035535	0.025573
300346	0.002598	0.001197	0.000503	0.034598	0.022430
300394	0.002194	0.002549	0.001261	0.050489	0.035515
300418	0.002466	0.001875	0.000715	0.043298	0.026740
600030	0.002316	0.000432	0.000172	0.020780	0.013126
600031	-0.000615	0.000323	0.000144	0.017965	0.011983
600048	0.000355	0.000593	0.000237	0.024343	0.015394
600196	0.000638	0.000282	0.000123	0.016778	0.011103
600276	-0.000820	0.000408	0.000173	0.020192	0.013161
600519	-0.001081	0.000256	0.000098	0.015985	0.009918
600588	0.001854	0.001018	0.000436	0.031911	0.020879
600887	-0.001626	0.000268	0.000106	0.016375	0.010304
601166	0.000915	0.000193	0.000096	0.013908	0.009807
601186	0.000907	0.000375	0.000149	0.019354	0.012194
601318	0.001247	0.000298	0.000130	0.017252	0.011391
601601	0.002070	0.000496	0.000226	0.022264	0.015029
601688	0.002266	0.000367	0.000160	0.019158	0.012651
601857	0.001004	0.000388	0.000190	0.019703	0.013766
601888	0.000313	0.000593	0.000211	0.024351	0.014532
603026	0.003220	0.000972	0.000410	0.031174	0.020242

Based on the results in Table 3, we find that the FF-BL model is capable of generating differentiated expected returns based on the differences in factor exposure among individual stocks. Furthermore, the lower-half variance for each stock in the table is generally lower than its total variance, indicating that the model effectively captures the downside risk when modeling risk.

5.4 Analysis of Portfolio Net Asset Value Performance

To verify the effectiveness of the FF-BL mean-absolute deviation-entropy model proposed in this paper, we conducted a backtest comparing it with the traditional 1/N model on out-of-sample data, drawing on existing research ^[26], and obtained the following results:

(1) Cumulative returns and excess returns

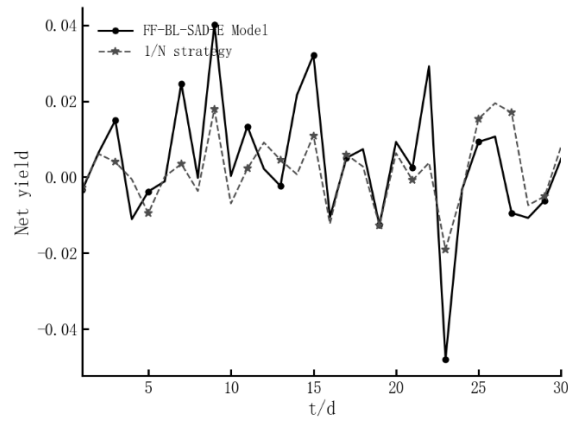


Figure 2: Daily net returns of the FF-BL mean-absolute deviation-entropy model versus the 1/N model

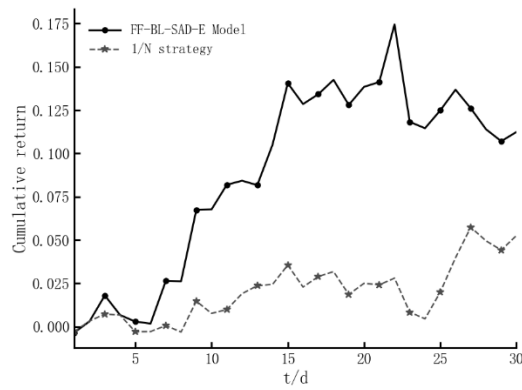


Figure 3: Cumulative net return of the FF-BL mean-semi-absolute deviation-entropy model versus the 1/N model

Figures 2 and 3 present the out-of-sample daily net returns and cumulative returns of the FF-BL model and the equal-weighted 1/N strategy. The FF-BL model exhibits greater variation in daily net returns during the out-of-sample period, with several observations exceeding 2% and occasionally approaching 4%, whereas the 1/N strategy fluctuates mainly between -1% and 2%. Although the FF-BL model experiences a marked drawdown around day 23, it generates higher returns on most trading days. The cumulative-return paths also diverge substantially: the FF-BL model quickly outperforms the 1/N strategy during the early out-of-sample period and reaches a peak cumulative return of approximately 17.5%. After the subsequent drawdown, its cumulative return remains above 11%, whereas the 1/N strategy records only a limited positive gain.

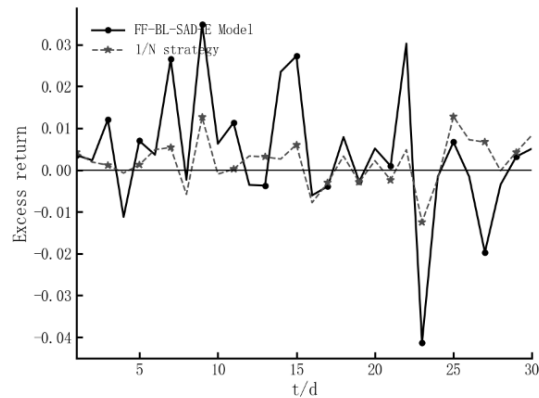


Figure 4: Excess Returns of the FF-BL Mean-Absolute Deviation-Entropy Model and the 1/N Model

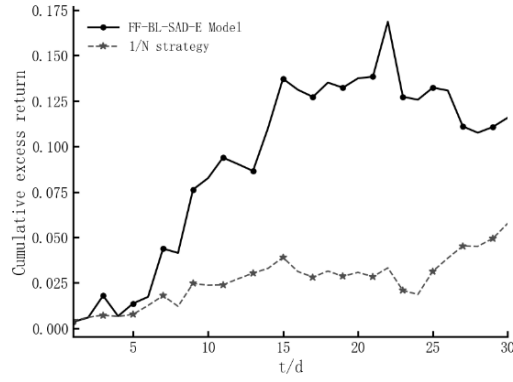


Figure 5: Cumulative excess returns of the FF-BL-semi-absolute deviation-entropy model versus the 1/N model

Figures 4 and 5 show the daily and cumulative excess returns of the FF-BL model and the equal-weighted 1/N strategy. The FF-BL model generates positive excess returns during most periods and reaches substantially higher peaks. In contrast, the excess returns of the 1/N strategy fluctuate around zero and are negative on some dates, suggesting that the strategy does not consistently exceed the benchmark expectation. The cumulative excess return of the FF-BL model exceeds 11% over the 30-day out-of-sample period, indicating stronger excess-return accumulation than the 1/N strategy.

(2) Analysis of the Sharp Ratio and the Sortino Ratio

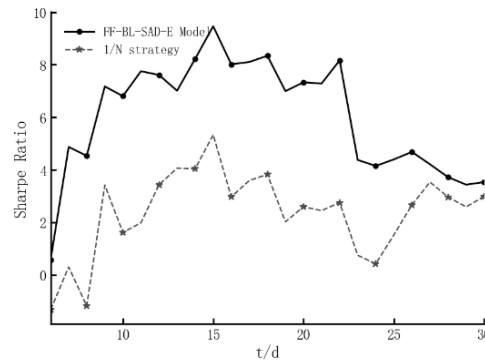


Figure 6: Sharpe Ratios of the FF-BL-Semi-Absolute Deviation-Entropy Model and the 1/N Model

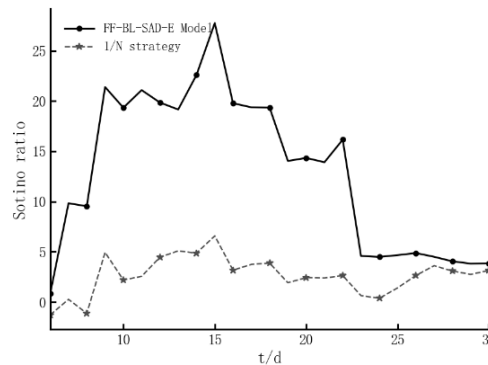


Figure 7: Sortino ratios for the FF-BL-semi-absolute deviation-entropy model and the 1/N model

Figure 6 shows that the Sharpe ratio of the FF-BL model increases rapidly and remains at a relatively high level, reaching a peak around day 15. Although it subsequently declines, it remains above 3.5, indicating relatively strong risk-adjusted performance. By contrast, the Sharpe ratio of the 1/N strategy is initially negative and subsequently fluctuates between 0 and 5, suggesting less stable risk-adjusted performance. Overall, the FF-BL model consistently achieves a higher Sharpe ratio than the 1/N strategy during most of the out-of-sample period.

Figure 7 shows that the Sortino ratio of the FF-BL model peaks above 27 and remains substantially higher than that of the 1/N strategy during most of the out-of-sample period. Because the Sortino ratio accounts primarily for downside risk, a higher value indicates better downside-risk-adjusted performance. By contrast, the Sortino ratio of the 1/N strategy remains below 5 for most of the period and is negative during the early stage. Overall, the larger advantage of the FF-BL model in the Sortino ratio than in the Sharpe ratio suggests stronger downside-risk-adjusted performance.

5.5 Out-of-sample Performance Evaluation of Investment Portfolios

Following Wang Shuai and Wang Jianzhou [27] and Hou Shengjie [28], this study uses net return, the Sharpe ratio (SR), the skewness-adjusted Sharpe ratio (ASR), the mean absolute deviation ratio (MADR), the Sortino-Satchell ratio (SSR), and the Farinelli-Tibiletti ratio (FTR) to evaluate portfolio performance. The FTR is characterized by two parameters, u and v , and different parameter combinations represent different investor risk preferences: $\{u = 0.5, v = 2\}$ represents defensive investors, $\{u = 1.5, v = 2\}$ represents conservative investors, and $\{u = 1, v = 1\}$ represents moderate investors. To evaluate the out-of-sample performance of the FF-BL model, this study compares it with the equal-weighted 1/N strategy and the CSI 300 Index.

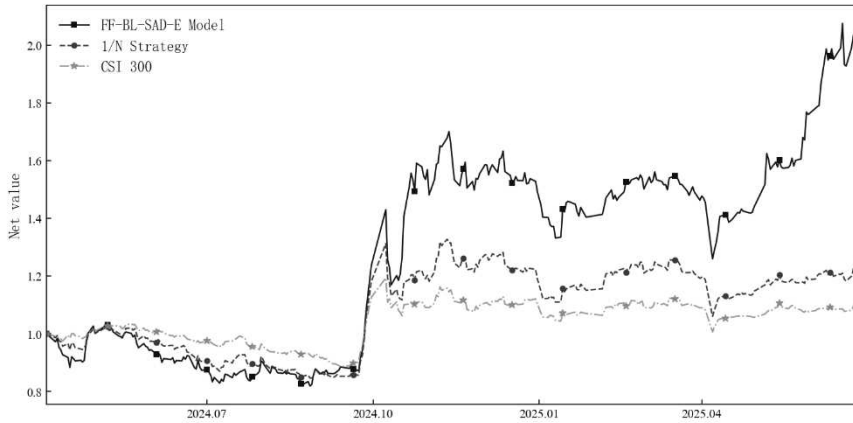


Figure 8: Net Value Curve of the Portfolio Model

Figure 8 shows that, against the backdrop of a market rebound from previous lows, the net asset value paths of the different strategies follow similar broad patterns but diverge materially in magnitude. The FF-BL model captures returns effectively during the rebound phase and around trend reversals, and its net asset value remains above those of the benchmark strategies. Its advantage partly arises from the use of mean absolute semi-deviation as a downside-risk measure, which penalizes unfavorable deviations without penalizing positive deviations from the mean. By contrast, the CSI 300 Index exhibits comparatively limited growth over the sample period. This limitation is associated with its passive, market-capitalization-weighted structure and the absence

of an active portfolio-adjustment mechanism. Unlike the proposed model, the index does not explicitly optimize diversification through an entropy objective or constraint. These results suggest that the proposed model may improve the risk–return trade-off relative to passive index investing and equal-weighted diversification.

Table 4 Performance Evaluation Results of the Portfolio Model

<i>model</i>	<i>SR</i>	<i>ASR</i>	<i>MADR</i>	<i>SSR</i>	<i>FTR(0.5,2)</i>	<i>FTR(1.5,2)</i>	<i>FTR(1,1)</i>
<i>FF-BL</i>	0.0972	0.0985	0.1545	0.1717	0.2503	1.0025	1.3679
<i>CSI 300</i>	0.0251	0.0252	0.0574	0.0530	0.1760	0.8138	1.1219
<i>1/N</i>	0.0431	0.0433	0.0799	0.0793	0.1986	0.8822	1.1741

Table 4 shows that the FF–BL model performs better than both the CSI 300 Index and the equal-weighted 1/N strategy on all seven reported metrics, including SR, SSR, and FTR. The FTR values are higher under all three parameter settings, suggesting that the model performs relatively well across different investor risk preferences. In particular, the Sharpe ratio of the FF–BL model is more than twice that of the 1/N strategy and is also substantially higher than that of the CSI 300 Index. The model also achieves a higher Sortino–Satchell ratio, indicating better downside-risk-adjusted performance. These results suggest that the proposed framework improves upon naïve equal-weight diversification and passive index investing.

5.6 Robustness Testing

In portfolio construction, the choice of rebalancing frequency and rolling-window length may affect the resulting portfolio performance. To examine the sensitivity of the FF–BL mean absolute semi-deviation–entropy model to these two parameters, this section conducts a robustness analysis. The results are reported in Table 5.

Table 5 Robustness Testing of the FF-BL (Half-Absolute Deviation) Entropy Model

<i>parameter</i>	<i>SR</i>	<i>ASR</i>	<i>MADR</i>	<i>SSR</i>	<i>FTR(0.5, 2)</i>	<i>FTR(1.5, 2)</i>	<i>FTR(1, 1)</i>
<i>T=10, W=100</i>	0.1176	0.1194	0.1879	0.2086	0.2618	1.0281	1.4661
<i>T=15, W=100</i>	0.1151	0.1169	0.1842	0.2038	0.2599	1.0220	1.4548
<i>T=20, W=100</i>	0.1168	0.1186	0.1868	0.2070	0.2600	1.0255	1.4630
<i>T=25, W=100</i>	0.1172	0.1190	0.1872	0.2081	0.2621	1.0290	1.4637
<i>T=30, W=100</i>	0.1150	0.1169	0.1839	0.2038	0.2597	1.0231	1.4540
<i>T=10, W=50</i>	0.0863	0.0874	0.1375	0.1529	0.2417	0.9942	1.3214
<i>T=10, W=100</i>	0.1176	0.1194	0.1879	0.2086	0.2618	1.0281	1.4661
<i>T=10, W=252</i>	0.1389	0.1351	0.2247	0.2120	0.2529	0.8515	1.5772

Table 5 reports the performance of the model under two scenarios. In the first scenario, the rolling-window length W is fixed at 100, while the rebalancing period T is set to 10, 15, 20, 25, and 30. In the second scenario, T is fixed at 10, while W is set to 50, 100, and 252. When W is fixed at 100, the performance metrics vary only modestly across different values of T and remain relatively stable. The Sharpe ratio ranges from 0.1150 to 0.1176, suggesting that the model is relatively insensitive to the rebalancing frequency within the tested range. A longer rebalancing period may also help reduce transaction costs, although this effect should be quantified separately. When T is fixed at 10, increasing W from 50 to 100 improves the reported performance metrics, while $W = 252$ produces the highest Sharpe ratio among the tested window lengths. These results suggest that a longer historical window may produce more stable parameter estimates by reducing the influence

of short-term market noise. Overall, the reported results remain relatively stable over the tested parameter ranges, providing support for the robustness of the proposed portfolio model. However, the sensitivity analysis alone does not prove the absence of overfitting; a separate walk-forward or additional out-of-sample test would provide stronger evidence.

6 Conclusion

This study investigates the application of the Fama–French three-factor framework and the Black–Litterman model to portfolio management. It aims to address the sensitivity of traditional mean–variance models to parameter-estimation errors and their limited ability to capture asymmetric risk. First, the Fama–French three-factor model is used to construct factor-based views, which are then incorporated into the Black–Litterman framework. This approach mitigates the limitations of traditional Black–Litterman models, including the difficulty of constructing theoretically grounded views and the instability of posterior expected-return estimates. Second, a portfolio optimization model based on mean absolute semi-deviation and entropy is developed. Mean absolute semi-deviation is used instead of variance to characterize downside risk, while entropy is incorporated to promote portfolio diversification. Finally, the performance of the proposed multi-objective model is evaluated using constituent stocks of the CSI 300 Index.

The empirical results show that the FF–BL mean absolute semi-deviation–entropy model outperforms the CSI 300 Index and the equal-weighted 1/N strategy in terms of cumulative returns, SR, ASR, MADR, SSR, and FTR under the tested risk-preference settings. These results suggest that the proposed model achieves stronger risk-adjusted performance and greater resilience under adverse market conditions. The robustness tests further show that the model maintains relatively stable performance across different rebalancing periods and rolling-window lengths. In particular, longer historical windows produce more stable performance in the reported experiments, possibly because they reduce the influence of short-term market noise.

References:

- [1] Markowitz, H. “Portfolio Selection” [J]. *Journal of Finance*, 1952, 7(1): 77–91.
- [2] Speranza, M. G. “Linear Programming Models for Portfolio Optimization” [J]. *The Journal of Finance*, 1993, 14(1): 107–123.
- [3] Zhang, P., Zhang, Z. W., & Yue, C. Y. A mean–absolute semi-deviation portfolio model with short-selling constraints and a study of its rotation algorithm [J]. *Chinese Journal of Management Science*, 2006, (2): 7–11.
- [4] Vercher, E., & Bermúdez, J. D. Portfolio Optimization Using a Credibility Mean–Absolute Semi-Deviation Model [J]. *Expert Systems with Applications*, 2015, 42(20): 7121–7131.
- [5] Black F, Litterman R. Global Portfolio Optimization [J]. *Financial Analysts Journal*, 1992, 48(5), 28–43.
- [6] Song Zhengyang, Zhou Zhongbao, Yu Le’an, et al. A Study on Portfolio Optimization Strategies Based on Hybrid Ensemble Forecasting Algorithms and the Black-Litterman Model [J/OL]. *Chinese Journal of Management Science*, 1–12 [2025-11-26]
- [7] Yao Haixiang, Li Xiaoxin, Fang Yong. Asset Allocation Based on the AdaBoost Ensemble Algorithm and the Black-Litterman Model [J]. *Theory and Practice of Systems Engineering*, 2023, 43(11): 3182–3196.

- [8] Sun Y, Wu Y, De G. A Novel Black-Litterman Model with Time-Varying Covariance for Optimal Asset Allocation of Pension Funds [J]. *Mathematics*, 2023, 11(6): 1476.
- [9] Meng Yong, Ren Meng, Zhao Xin. A Study on Black-Litterman Model Allocation of Sectoral Assets: Based on Social Media Sentiment Text Mining Algorithms [J]. *Research on Quantitative Economics and Technical Economics*, 2022, 39(01): 154-173.
- [10] Xu Meiping, Yu Jian, Du Quanying. Dynamic Sectoral Tactical Asset Allocation Based on VECM and BLCOP Models [J]. *Mathematical Statistics and Management*, 2021, 40(02): 310-318.
- [11] Sharp W F. Capital asset prices: A theory of market equilibrium under conditions of risk[J]. *The Journal of Finance*, 1964, 19(3): 425-442.
- [12] Bollerslev, T. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 1986, 31(3), 307–327.
- [13] Fama E F, French K R. The cross-section of expected stock returns[J]. *The Journal of Finance*, 1992, 47(2): 427-465
- [14] Ko H, Son B, Lee J. A novel integration of the Fama–French and Black–Litterman models to enhance portfolio management [J]. *Journal of International Financial Markets, Institutions and Money*, 2024, 91.
- [15] Zhang Peng, Zhu Suihong, Cui Shulin. Three-Factor Mean-Variance Portfolio Optimization Based on BP Neural Networks [J]. *Practice and Understanding of Mathematics*, 2025, 5(01): 26–40.
- [16] Yang Zunhan. Testing the CAPM Model and the Fama-French Three-Factor Model in China's A-Share Market [J]. *Journal of Economic Research*, 2022, (08): 117-119.
- [17] Dang Shili, Huang Meiyu, Zhang Peng. Mean-CVaR Interval Portfolio Optimization with Entropy Constraints [J]. *Fuzzy Systems and Mathematics*, 2024, 38(05): 153-168.
- [18] Sun Rong, Yin Hao. A Coherent Fuzzy Mean-Variance-Proportional Entropy Portfolio Model [J/OL]. *Systems Science and Mathematics*, 1-18 [2025-11-20].
- [19] Zeng Yongquan, Zhang Peng, Wang Xiaohui. Uncertain M-AD Portfolio Decision-Making with Different Entropy Constraints [J]. *Fuzzy Systems and Mathematics*, 2025, 39(01): 155-169.
- [20] Zhang Peng, Dang Shili, Huang Meiyu, et al. Two-Step M-SV Portfolio Optimization Based on Machine Learning for Predicting Stock Returns [J]. *Chinese Journal of Management Science*, 2023, 31(12): 96–106.
- [21] Derya T, Kelce G M, Atalay D K. Modeling Portfolio Selection Under Intuitionistic Fuzzy Environments [J]. *Mathematics*, 2025, 13(20): 3303–3303.
- [22] Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, 27(3), 379–423.
- [23] Fama E F, French K R. Common risk factors in the returns on stocks and bonds[J]. *Journal of Financial Economics*, 1993, 33(1):3-56.
- [24] Deng X, Geng F. A Novel Mean–Variance-Entropy Portfolio with Two-Parameter Coherent Triangular Intuitionistic Fuzzy Number. *Computational Economics* [J]. 2025, 66: 3081-3130.
- [25] Deng X, Chen S. Fuzzy portfolio with a novel power membership function based on GARCH and Black–Litterman model. *International Journal of Fuzzy Systems*, 2025, 27(1), 267–298.
- [26] Zhang Peng, Mo Shiyin, Cao Qing. Research on Multi-objective Portfolio Optimization Based on Machine Learning [J]. *Journal of South China Normal University (Natural Science Edition)*, 2024, 56(04):100–110.
- [27] Hou Shengjie, Guan Zhongcheng, Dong Xuefan. A Multi-objective Portfolio Model Based on

- Entropy and CVaR and Its Empirical Study [J]. Systems Science and Mathematics, 2021, 41(03): 640-652.
- [28] Wang Shuai, Wang Jianzhou. A Study on M-CVaR-Higher-Order Moment Portfolio Based on Complex Multidimensional Scenario Generation [J]. Journal of Statistical Research, 2024, 41(12): 136-150.