

Multi-Time-Scale Coordinated Planning of Distribution Networks with High Penetration of Distributed Renewable Energy Considering Active-Reactive Power Coupled Flexibility and Voltage Risk

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Abstract : High-penetration distributed renewable generation introduces bidirectional power flow, rapid net-load ramps, and spatially uneven voltage sensitivity in active distribution networks. To address these challenges, a multi-time-scale coordinated planning method is proposed by integrating active–reactive power coupling flexibility and voltage-risk control. The method decomposes net active and reactive load into fast, medium, and slow components, models inverter-based P–Q feasible regions, and embeds voltage conditional value at risk into investment and operation decisions. A WGAN-GP-based scenario generation framework is used to preserve wind, photovoltaic, and load uncertainties, and 27 representative scenarios are selected for planning. Case studies on a radial PG&E 69-bus distribution feeder show that the risk-aware scheme increases wind and PV capacities from 0.96 MW and 0.56 MW to 1.18 MW and 0.74 MW, respectively. Although annualized investment rises from 1.05 M\$ to 1.39 M\$, total annualized cost decreases from 2.83 M\$ to 2.316 M\$, achieving an 18.16% reduction. Renewable utilization improves from 88.3% to 98.35%, while curtailed energy and expected energy not supplied decrease by 85.2% and 92.4%. The results verify that coordinated P–Q flexibility can enhance renewable accommodation, reduce voltage-tail risk, and improve planning economy.

Keywords : Active distribution network; distributed renewable generation; active–reactive power coupling; multi-time-scale planning; voltage-risk assessment

1. Introduction

The large-scale deployment of distributed wind power, photovoltaic (PV) generation, battery energy storage systems (BESSs), electric vehicles, and controllable loads is accelerating the transition of conventional distribution networks toward active distribution networks. Unlike traditional feeders characterized by unidirectional power delivery and relatively predictable demand, distribution systems with a high proportion of distributed renewable energy are exposed to bidirectional power flows, rapid net-load variations, reverse power transmission, and increasingly heterogeneous operating conditions. These changes can reduce fossil-fuel consumption and defer conventional network reinforcement, but they also introduce substantial challenges associated with renewable curtailment, line congestion, voltage rise, voltage fluctuation, and inadequate regulation reserves. In particular, the high resistance-to-reactance ratio of distribution lines makes nodal voltage sensitive to both active- and reactive-power injections. Consequently, an active-power-balanced solution does not necessarily guarantee an acceptable voltage profile, and an apparently sufficient installed capacity may still be unable to accommodate renewable fluctuations when the response speeds of the available resources are inconsistent with the time scales of those fluctuations.

The planning implications of these characteristics are more complex than those encountered in traditional distribution expansion studies. Long-term investment decisions determine the locations and capacities of renewable generation, storage, dispatchable units, flexible demand, and power-electronic devices, whereas their economic and security values are realized through short-term operation under uncertain source–load conditions. Moreover, different flexible resources are not technically interchangeable. Smart inverters and static var generators can respond within seconds and provide rapid reactive-power support; BESSs and direct-load control can compensate for minute-level ramps; gas turbines, transferable demand, and upstream-grid exchanges are more suitable for sustained balancing over longer intervals. Converter-interfaced resources are further constrained by their active–reactive apparent-power capability, such that increasing active-power utilization reduces the available reactive-power margin. A planning framework that neglects these response-time and P–Q coupling characteristics may overestimate the flexibility of installed resources, underestimate voltage-control requirements, and produce investment schemes that are economical only under simplified operating assumptions.

Recent distribution-planning studies have progressively incorporated distributed resources and voltage-support devices into investment models. Jiang *et al.* jointly configured energy storage and reactive-power resources for renewable-rich distribution systems and demonstrated that storage and var compensation can complement each other in improving voltage quality and operational flexibility [1]. Hamed *et al.* developed a multi-period expansion formulation for low-voltage networks with widespread small-scale solar generation, emphasizing the effects of staged transformer and feeder investments under changing PV penetration [2]. The flexibility of aggregated distributed resources has also been embedded in virtual-power-plant-oriented planning, where demand-side and storage flexibility can reduce expansion costs and improve renewable utilization [3]. These studies establish that distributed flexibility should be treated as an investment alternative rather than merely as an operational adjustment. Nevertheless, their flexibility representations are primarily based on energy or active-power quantities, and the differentiated response capabilities of resources across sub-hourly and hourly time scales are not fully reflected.

A parallel research stream has examined coordinated active- and reactive-power management for increasing renewable hosting capacity and maintaining acceptable voltages. Golnazari *et al.* formulated coordinated active- and reactive-power control strategies for enhancing PV hosting capacity, showing that reactive-power support can reduce the need for renewable curtailment when network constraints become binding [4]. Li *et al.* coordinated dynamic network reconfiguration, soft open points (SOPs), distributed generation, and reactive compensation in an active distribution network, and converted the resulting operational problem into a mixed-integer second-order cone formulation [5]. Zhang *et al.* further coordinated PV generation and power-to-hydrogen loads for voltage control, illustrating how controllable active demand and reactive resources can jointly mitigate voltage deviations [6]. These contributions confirm that active- and reactive-power flexibility should be considered simultaneously. However, most such studies are formulated as day-ahead scheduling or voltage-control problems. The available P-Q capability is optimized after equipment capacities have been specified, rather than being used to determine the long-term location and rating of active- and reactive-power resources.

Flexible network assets provide another pathway for accommodating high renewable penetration. A cost-benefit framework developed by Deakin *et al.* showed that SOPs can provide multiple value streams, including loss reduction, reinforcement deferral, renewable-curtailment reduction, reliability support, and flexibility-service provision [7]. Unlike conventional tie switches, SOPs can continuously regulate active- and reactive-power transfers between adjacent feeders, allowing spatial redistribution of flexibility. At the network-structure level, Xiang *et al.* proposed a learning-based autonomous topology-planning method that separates topology generation from expansion optimization and improves the computational treatment of spatially uneven load development [8]. These approaches expand the planning decision set beyond conventional lines and substations. Nevertheless, topology and flexible interconnection decisions are commonly evaluated using a single operating resolution, and their ability to provide different services at fast, medium, and slow time scales is not explicitly coordinated with storage, demand response, and inverter resources.

The growing interaction between distribution networks and other infrastructures has further broadened the planning boundary. Borozan and Strbac established a multi-stage integrated transmission-distribution expansion framework in which conventional network investments and non-network flexibility options are co-optimized under multidimensional long-term uncertainties [9]. Their results highlight the value of delaying irreversible investment until uncertainty is partially resolved. At the electricity-transportation interface, Liang *et al.* incorporated electric-vehicle user equilibrium into vehicle-to-grid infrastructure planning and quantified the contribution of bidirectional charging to distribution-network flexibility [10]. Such research demonstrates that flexible resources may be geographically mobile, behavior-dependent, or controlled by actors outside the distribution utility. However, the increased planning scope also amplifies the need for a tractable distribution-level representation of active-reactive coupling, voltage constraints, and short-term flexibility adequacy.

Several studies have therefore focused on improving the quantification and management of operational flexibility. Knez *et al.* proposed a sensitivity-coefficient-based method for dynamically determining the active- and reactive-power flexibility required to maintain voltage and line-loading limits [11]. Abbas *et al.* combined model-based reactive-power optimization with a data-driven stage to improve computational performance under uncertain distributed generation and load conditions [12]. These methods support more accurate and responsive flexibility management than fixed reserve margins. Their principal objective, however, is to determine operational set points or near-real-time flexibility requirements. The long-term implications of these requirements for the coordinated installation of BESS power and energy

ratings, inverter oversizing, demand-response contracts, SOP capacity, and reactive compensation remain insufficiently investigated.

Uncertainty modelling is another decisive element in renewable-rich distribution planning. Liu *et al.* constructed a Wasserstein-distance-based distributionally robust expansion model for lines, interconnection switches, and SOPs, thereby reducing dependence on a precisely known probability distribution of renewable output and load [13]. This approach improves robustness against sampling errors, but the quality of the underlying source–load scenarios and their spatial–temporal correlations remains important because overly conservative ambiguity sets may increase investment unnecessarily. Moreover, uncertainty influences not only energy balance and line loading but also the probability and severity of voltage deviations. Deterministic voltage limits indicate whether a particular scenario is feasible, but they do not distinguish between a solution that remains close to the voltage boundary in many scenarios and another solution with a larger security margin. A tail-risk measure is therefore required to represent low-probability, high-consequence voltage conditions within planning decisions.

Emerging planning studies have begun to combine new flexible resources, cyber infrastructure, and multistage investment. Hou *et al.* coordinated SOP planning with the spatial–temporal flexibility of fifth-generation mobile communication base stations, showing that communication loads and backup batteries can enhance temporal balancing and reduce the required SOP capacity [14]. Wu *et al.* developed a cyber–physical integrated planning model that jointly considers energy storage, smart terminals, data-load migration, distribution operation, and communication topology [15]. For long-term uncertainty, Lou *et al.* proposed a multi-stage distribution-planning model that accounts for load growth and renewable penetration, demonstrating the economic advantage of gradual investment over a static end-year design [16]. These studies extend the spatial, temporal, and infrastructural dimensions of distribution planning, yet the interactions among sub-hourly flexibility, active–reactive converter limits, and probabilistic voltage risk are still not represented within a unified planning framework.

Recent work has also strengthened the coordination between energy storage and dedicated reactive-power devices. Jiang *et al.* formulated a bi-level planning model for energy storage and relocatable static var compensators under seasonal transformer-area loading, allowing active-power shifting and mobile reactive support to be coordinated across operating conditions [17]. Hasanvand *et al.* combined multistage reactive-power planning, spectral clustering, fuzzy partitioning, and sensitivity-based control to allocate var resources and establish decentralized voltage-control zones [18]. These studies demonstrate that reactive-power planning benefits from spatial sensitivity information and that var resources should be placed close to voltage-sensitive areas. Nevertheless, the active-power flexibility that changes the network operating point and, consequently, the voltage sensitivity is generally treated separately. The interaction between active-power scheduling, reactive-power capability, time-scale-dependent regulation, and voltage-tail risk therefore remains an open planning issue.

The preceding literature reveals five interrelated research gaps. First, existing planning models commonly adopt hourly or representative-period operation, whereas renewable ramps and voltage fluctuations can occur within several minutes. Although some studies quantify flexibility or implement multi-stage investment, the response-time matching between fluctuation components and heterogeneous flexible resources is rarely embedded directly in investment decisions. Second, active- and reactive-power resources are often planned or operated separately. The apparent-power coupling of PV and BESS inverters, the bidirectional P–Q capability of SOPs, and the complementary functions of storage, demand response, gas turbines, and var compensators have not been comprehensively represented within a common capacity-planning model. Third, voltage security is predominantly enforced through deterministic upper and lower limits. Such constraints cannot adequately quantify near-limit operation or the tail severity of voltage deviations under correlated renewable and load uncertainty. Fourth, scenario generation, scenario reduction, flexibility characterization, and planning optimization are generally developed as separate procedures. Insufficient preservation of spatial correlation, temporal dependence, and extreme ramps can bias both flexibility requirements and voltage-risk estimates. Fifth, the integration of binary investment variables, inter-temporal operation, nonlinear branch flows, P–Q capability regions, and risk constraints produces a large mixed-integer nonlinear problem, requiring a tractable reformulation that retains sufficient physical accuracy.

To address these gaps, a multi-time-scale coordinated planning method for high-penetration distributed renewable energy distribution networks is developed with explicit consideration of active–reactive coupled flexibility and voltage risk. The principal contributions are summarized as follows.

1. A unified multi-time-scale active–reactive flexibility representation is established. The net active- and reactive-load trajectories are decomposed into fast, medium, and slow components, and upward active,

downward active, capacitive, and inductive flexibility requirements are quantified separately. Flexible resources are assigned to the corresponding time scales according to their response delay, ramping capability, power duration, energy state, and reactive-power characteristics, while cross-scale double counting is prevented through common physical-state constraints.

2. The P–Q coupling of inverter-based and network-side flexible resources is incorporated into long-term planning. The model jointly determines the locations and active-power, energy, reactive-power, and apparent-power ratings of wind generation, PV generation, BESSs, gas turbines, demand response, smart inverters, capacitor/SVG devices, and SOPs. The circular or converter-specific P–Q capability boundaries ensure that reactive-power support is not overestimated when renewable or storage active-power utilization is high.
3. A data-driven uncertainty and voltage-risk framework is integrated with the planning model. Joint wind, PV, and load scenarios are generated using a Wasserstein generative adversarial network with gradient penalty, and representative scenarios are obtained through clustering and probability reassignment while preserving marginal distributions, spatial correlations, temporal dependence, and ramp characteristics. Voltage-deviation conditional value at risk is introduced to quantify the upper tail of scenario-dependent voltage severity, enabling the planning objective to balance investment cost, operating cost, renewable curtailment, flexibility inadequacy, and voltage security.
4. A tractable investment–operation optimization formulation is constructed. The planning and multi-scenario operating layers are unified through capacity–operation coupling variables; binary–continuous products are treated using McCormick envelopes; and the radial branch-flow equations are transformed through second-order cone relaxation. The resulting mixed-integer second-order cone programming model can be solved by a commercial optimizer and is subsequently validated through nonlinear AC power-flow recovery, relaxation-gap evaluation, active–reactive flexibility reconstruction, and voltage-risk assessment.

2. Multi-Time-Scale Active–Reactive Power Coupled Coordinated Planning Method

2.1 Overall Planning Framework

A high penetration of distributed wind and photovoltaic generation changes the distribution network from a load-following passive system into an active system with bidirectional power flow, rapid net-load ramps, and spatially nonuniform voltage sensitivity. Under these conditions, planning only the installed active-power capacity is insufficient. Renewable curtailment may be caused by a shortage of downward active-power flexibility, whereas voltage violations may persist even when the total active-power balance is satisfied. The planning problem must therefore coordinate active-power balancing, reactive-power support, network security, and voltage-tail risk over several response time scales. The multi-time-scale cooperative structure originally developed for renewable generation and flexible resources provides the temporal basis for this extension [19]. However, the extended formulation explicitly represents the coupled feasible regions of inverter-based devices and introduces voltage-risk terms into both planning and operation decisions.

The information flow summarized in Fig. 1 begins with historical wind, PV, and load measurements. A data-driven scenario-generation module extracts their nonlinear probability distributions and spatial–temporal correlations, after which a clustering module selects representative scenarios and associated probabilities. For every representative scenario, the net active and reactive load trajectories are decomposed into fast, medium, and slow components. The investment layer determines the locations, types, active-power ratings, energy capacities, and reactive-power ratings of candidate resources. The operating layer then dispatches these resources at their admissible time scales while respecting distribution power-flow, voltage, thermal, flexibility, and risk constraints. The operating cost and risk exposure are returned to the investment layer, thereby avoiding a planning scheme that appears economical under annual energy balance but lacks sufficient short-term regulation capability.

Let N , L , S , M , and R denote the sets of buses, branches, representative scenarios, time scales, and candidate resources, respectively. The set of time scales is defined as $M = \{F, M, S\}$ for fast, medium, and slow regulation. Binary variable x_i^r determines whether resource r is installed at bus i , while K_i^r denotes its rated capacity. The overall planning decision can be expressed as

$$\Xi^* = \arg \min_{\mathbf{x}, \mathbf{K}, \mathbf{y}} \left\{ C^{\text{inv}}(\mathbf{x}, \mathbf{K}) + \sum_{s \in S} \pi_s C_s^{\text{ope}}(\mathbf{K}, \mathbf{y}_s) + C^{\text{risk}}(\mathbf{y}) \right\} \quad (1)$$

where $\mathbf{y}_s$ contains scenario-dependent operating variables and π_s is the probability of scenario s . The objective in (1) preserves the investment–operation coupling used in recent coordinated planning studies, while extending the planning boundary to energy-storage-integrated soft open points, reactive compensation, and risk-aware operating limits [20].

The annualized total cost is decomposed as

$$C^{\text{tot}} = C^{\text{inv}} + C^{\text{fuel}} + C^{\text{grid}} + C^{\text{om}} + C^{\text{dr}} + C^{\text{cur}} + C^{\text{ens}} + C^{\text{loss}} + C^{\text{risk}} \quad (2)$$

where the terms respectively represent annualized investment, fuel, upstream-grid transactions, operation and maintenance, demand-response compensation, renewable curtailment, expected energy not supplied, network losses, and voltage-risk exposure. The annualized investment cost is calculated by

$$C^{\text{inv}} = \sum_{r \in R} \sum_{i \in N_r} \frac{\gamma(1+\gamma)^{L_r}}{(1+\gamma)^{L_r} - 1} \times (c_r^P K_{i,r}^P + c_r^E K_{i,r}^E + c_r^Q K_{i,r}^Q) x_i^r \quad (3)$$

where γ is the discount rate, L_r is the lifetime of resource r , and c_r^P , c_r^E , and c_r^Q are the unit costs of active-power, energy, and reactive-power capacity. Capacity terms not applicable to a particular device are set to zero.

The expected operating cost is represented by

$$C^{\text{opc}} = D_y \sum_{s \in S} \pi_s \sum_{m \in M} \sum_{t \in T_m} \Delta t_m \left[c_t^{\text{grid}} P_{s,m,t}^{\text{grid}} + \sum_i c_i^{\text{gt}} P_{i,s,m,t}^{\text{gt}} \right] \left[\sum_i c_{i,t}^{\text{dr}} |P_{i,s,m,t}^{\text{dr}}| + c^{\text{cur}} P_{s,m,t}^{\text{cur}} + c^{\text{ens}} P_{s,m,t}^{\text{ens}} + c^{\text{loss}} P_{s,m,t}^{\text{loss}} \right] \quad (4)$$

where D_y is the number of equivalent operating days in one year. The explicit scenario weighting prevents low-cost but low-reliability investment solutions from being selected when renewable and load uncertainty are jointly considered. Such joint uncertainty has been shown to materially affect storage and network expansion decisions in distribution planning [21].

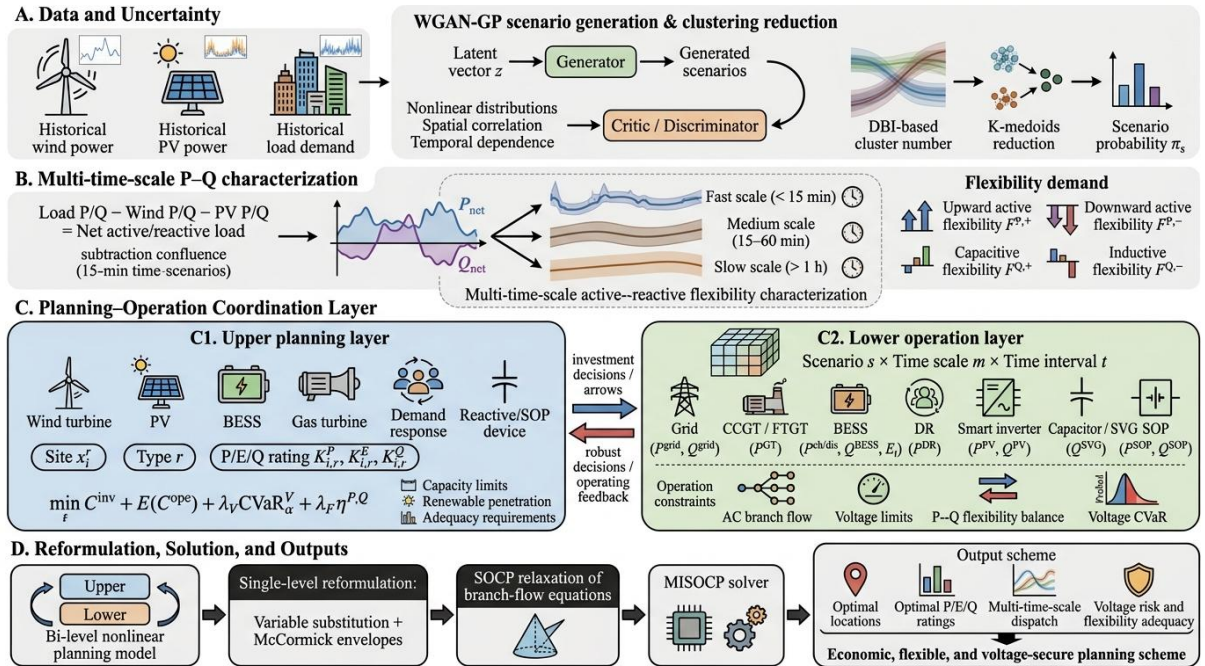


Figure 1. Overall framework of multi-time-scale active–reactive power coupled coordinated planning.

The planning objects and their physical functions are classified in Table 1. Renewable generation provides energy but also contributes inverter-based reactive-power support when sufficient apparent-power margin is retained. BESSs provide both energy shifting and rapid four-quadrant regulation. Gas turbines and grid exchange supply sustained active-power flexibility, while DR transfers or curtails demand. Capacitor banks and static var generators provide local voltage support, and SOPs redistribute active and reactive power between feeder sections. The classification separates investment variables from operational variables so that each technology is valued according to the service it can physically deliver.

Table 1. Planning objects, decision variables, and functional classification.

Planning object	Investment variables	Operating variables	Main planning function
Wind and PV	Installation state, site, active-power rating, inverter rating	Active-power utilization, curtailment, reactive-power output	Renewable-energy supply, local voltage support, reduction of upstream purchases
BESS	Site, power rating, energy rating, converter var rating	Charging/discharging power, state of charge, reactive-power output	Energy shifting, ramp compensation, fast P–Q flexibility, voltage regulation
Gas turbine	Site and active-power rating	Commitment state, active output, ramping power	Sustained upward flexibility, adequacy support, slow and medium balancing
Demand response	Contracted flexible capacity	Shiftable and interruptible demand	Fast/medium active-power regulation and peak-load reduction
PV/BESS inverter	Apparent-power reserve and reactive-power rating	Coupled active and reactive output	Dynamic reactive support and voltage-risk mitigation
Capacitor/SVG	Site, discrete steps, or continuous var rating	Switching state or reactive-power output	Local steady-state voltage support and reactive-loss reduction
Soft open point	Terminal buses and converter rating	Bidirectional active and reactive transfer	Feeder power-flow redistribution, congestion relief, voltage support
Upstream grid	Interface-capacity limit where applicable	Imported/exported active and reactive power	System-level balancing and reserve provision

2.2 Multi-Time-Scale Active–Reactive Flexibility and Voltage-Risk Representation

Renewable variability is not concentrated within a single frequency band. Slow changes determine daily energy imbalance, medium-frequency ramps create reserve requirements, and fast fluctuations affect voltage and converter loading before slow resources can respond. A single hourly simulation interval can therefore underestimate both active-power ramping and reactive-power support requirements. Flexibility-based expansion studies increasingly distinguish supply–demand flexibility from energy adequacy, demonstrating that resource value depends on its response speed as well as its installed capacity [22]. The response hierarchy adopted here is illustrated in Fig. 2, where the horizontal direction denotes response time and the vertical direction denotes the feasible active–reactive regulation range.

For scenario s , the aggregated net active and reactive loads are defined by

$$\begin{aligned} P_{s,t}^{\text{net}} &= P_{s,t}^{\text{L}} - P_{s,t}^{\text{WT}} - P_{s,t}^{\text{PV}}, \\ Q_{s,t}^{\text{net}} &= Q_{s,t}^{\text{L}} - Q_{s,t}^{\text{WT}} - Q_{s,t}^{\text{PV}} \end{aligned} \quad (5)$$

where renewable reactive output is initially set according to the pre-dispatch power factor and is subsequently optimized within inverter limits. The net-load trajectories are decomposed into mutually complementary components:

$$\begin{aligned} P_{s,t}^{\text{net}} &= P_{s,t}^{\text{F}} + P_{s,t}^{\text{M}} + P_{s,t}^{\text{S}}, \\ Q_{s,t}^{\text{net}} &= Q_{s,t}^{\text{F}} + Q_{s,t}^{\text{M}} + Q_{s,t}^{\text{S}} \end{aligned} \quad (6)$$

The decomposition may be implemented by mathematical morphology, wavelet filtering, or an equivalent low-pass/band-pass construction, provided that the reconstruction error remains negligible. The slow component retains the energy trend, the medium component represents intra-hour ramps, and the fast component captures residual changes within the base scheduling interval.

For each time scale m , the upward and downward active-power flexibility requirements are obtained from directional changes in the component trajectory:

$$\begin{aligned} F_{s,m,t}^{P,+} &= \max\{P_{s,m,t+1}^{\text{net}} - P_{s,m,t}^{\text{net}}, 0\} \\ F_{s,m,t}^{P,-} &= \max\{P_{s,m,t}^{\text{net}} - P_{s,m,t+1}^{\text{net}}, 0\} \end{aligned} \quad (7)$$

Reactive-power requirements are defined analogously:

$$\begin{aligned} F_{s,m,t}^{Q,+} &= \max\{Q_{s,m,t+1}^{\text{net}} - Q_{s,m,t}^{\text{net}}, 0\} \\ F_{s,m,t}^{Q,-} &= \max\{Q_{s,m,t}^{\text{net}} - Q_{s,m,t+1}^{\text{net}}, 0\} \end{aligned} \quad (8)$$

where Q_+ represents additional capacitive support and Q_- represents additional inductive absorption under the adopted sign convention.

A resource cannot be represented only by independent active and reactive upper bounds because converter-based flexibility is inherently coupled. The feasible operating region of resource r is therefore written as

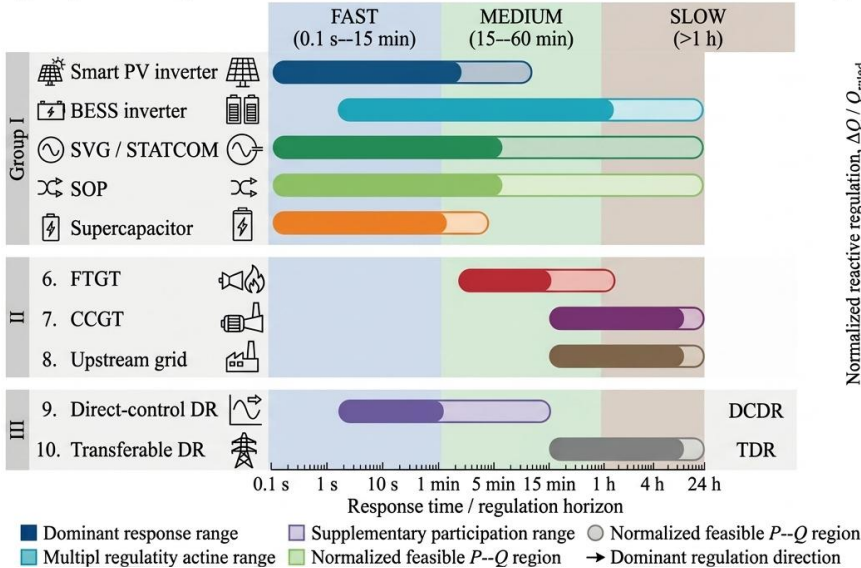
$$\begin{aligned} F_{i,r,t} = \{ & (P_{i,r,t}, Q_{i,r,t}) \mid P_{i,r,t} \leq P_{i,r,t} \leq \bar{P}_{i,r,t}, \\ & \underline{Q}_{i,r,t}(P) \leq Q_{i,r,t} \leq \bar{Q}_{i,r,t}(P) \} \end{aligned} \quad (9)$$

For a PV or BESS inverter with apparent-power rating $S_{i,r}^{\max}$, the coupling boundary is

$$P_{i,r,t}^2 + Q_{i,r,t}^2 \leq (S_{i,r}^{\max})^2 \quad (10)$$

Consequently, a high active-power output reduces available reactive-power support unless additional converter capacity is installed. This relationship is central to the economic trade-off between renewable utilization and voltage security. Dynamic coordinated planning of flexible distribution networks similarly shows that probabilistic voltage and loading constraints can alter the preferred allocation of network and converter resources [23].

(a) Response-time spectrum



(b) Normalized P – Q regulation capability

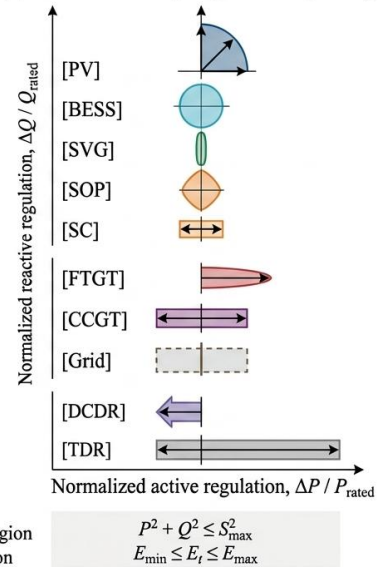


Figure 2. Response times and active–reactive regulation ranges of multiple flexible resources.

Table 2 assigns resources to their principal response scales. The assignment is not exclusive: BESSs and inverter resources may participate in more than one band, but their total output must remain within the common power and energy limits. Slow resources mainly provide energy adequacy, medium resources follow intra-hour ramps, and fast resources suppress residual fluctuations and voltage deviations. This overlapping assignment prevents double counting because every device is linked through its physical state equations.

Table 2. Time-scale division and response characteristics of flexible resources.

Time scale	Representative interval	Dominant fluctuation	Principal resources	Main P–Q service
Fast	0–15 min	Renewable ramps, cloud transients, local voltage fluctuations	Smart PV/BESS inverters, SOP, supercapacitor, direct DR	Fast upward/downward active regulation, capacitive/inductive var support, voltage containment
Medium	15–60 min	Intra-hour net-load ramps and forecast deviations	BESS, FTGT, transferable DR, SOP	Ramp tracking, reserve deployment, feeder power redistribution
Slow	More than 1 h	Daily energy imbalance and sustained source–load mismatch	CCGT, upstream grid, BESS energy shifting, scheduled DR	Energy balance, capacity adequacy, sustained voltage and reactive-power support

The flexibility supplied at a particular scale is compared with the required quantity through normalized inadequacy indices:

$$\eta_{s,m}^{P,\pm} = \frac{\sum_t [F_{s,m,t}^{P,\pm} - A_{s,m,t}^{P,\pm}]_+}{\sum_t F_{s,m,t}^{P,\pm} + \epsilon}, \quad (11)$$

$$\eta_{s,m}^{Q,\pm} = \frac{\sum_t [F_{s,m,t}^{Q,\pm} - A_{s,m,t}^{Q,\pm}]_+}{\sum_t F_{s,m,t}^{Q,\pm} + \epsilon}$$

where $A^{P,\pm}$ and $A^{Q,\pm}$ are the available regulation quantities, $[\cdot]^+ = \max(\cdot, 0)$, and ϵ avoids numerical division by zero. The planning model limits these indices rather than requiring excessive reserve in every interval.

Voltage security is represented by both ordinary limits and a tail-risk measure. The deviation severity at bus i is defined as

$$d_{i,s,t}^V = \max\{V_{i,s,t} - V^{\max}, V^{\min} - V_{i,s,t}, \omega_V |V_{i,s,t} - V^{\text{ref}}|, 0\} \quad (12)$$

where the third term allows nonzero risk to be distinguished even when hard violations are rare. The scenario-based CVaR at confidence level α is formulated as

$$\text{CVaR}_\alpha^V = \min_{\zeta} \left\{ \zeta + \frac{1}{1-\alpha} \sum_{s \in S} \pi_s (D_s^V - \zeta)_+ \right\} \quad (13)$$

$$D_s^V = \sum_{i,t} w_i d_{i,s,t}^V$$

Unlike a deterministic voltage bound, (13) penalizes the upper tail of spatially aggregated voltage deviation. It therefore distinguishes a planning scheme with frequent near-limit voltages from one with similar average voltage but greater security margin.

1.1 Uncertainty Scenarios and Voltage-Risk Modelling

Wind, solar, and demand uncertainties are correlated across both space and time. Parametric distributions can describe individual variables, but their accuracy deteriorates when ramp distributions, nonlinear dependence, and bounded outputs must be reproduced simultaneously. A Wasserstein generative adversarial network with gradient penalty (WGAN-GP) is therefore used to learn the joint source–load distribution without prescribing explicit marginal distributions. Recent applications show that WGAN-GP can retain wind–solar complementarity more effectively than independent Monte Carlo sampling [24]. Extreme renewable scenarios remain difficult because rare tail samples are scarce, making scenario validation and tail augmentation necessary before planning [25].

Let $\mathbf{x} \sim P_{\text{data}}$ denote a normalized historical daily trajectory containing outputs from all renewable sites and load groups, and let $\mathbf{z} \sim P_z$ be a random latent vector. The WGAN-GP critic is trained by minimizing

$$L_D = E_z [D(G(\mathbf{z}))] - E_x [D(\mathbf{x})] + \lambda_{\text{gp}} E_{\hat{\mathbf{x}}} [(\|\nabla_{\hat{\mathbf{x}}} D(\hat{\mathbf{x}})\|_2 - 1)^2] \quad (14)$$

where $\hat{\mathbf{x}} = \epsilon \mathbf{x} + (1-\epsilon)G(\mathbf{z})$ and $\epsilon \sim U[0,1]$. The generator is updated according to

$$L_G = -E_z[D(G(z))] + \lambda_{\text{phy}} L_{\text{phy}} \quad (15)$$

where L_{phy} penalizes outputs outside physical bounds and unrealistic night-time PV generation. Transformer-enhanced Wasserstein models may be substituted when longer temporal dependence must be retained [26]; the subsequent planning interface remains unchanged as long as generated trajectories preserve the joint dimension and sampling interval.

The initial generated scenario set is usually too large for a mixed-integer planning model. K-medoids reduction is therefore applied after selecting the number of clusters through the Davies–Bouldin index. For K clusters, the medoid problem is

$$\min_{C_1, \dots, C_K} \sum_{k=1}^K \sum_{\substack{\mathbf{x}_n \in C_k \\ \boldsymbol{\mu}_k \in C_k}} d(\mathbf{x}_n, \boldsymbol{\mu}_k) \quad (16)$$

where distance $d(\cdot)$ combines normalized Euclidean distance, ramp-rate discrepancy, and correlation error. The probability of representative scenario s is

$$\pi_s = \frac{|C_s|}{\sum_{k=1}^K |C_k|} \quad (17)$$

$$\sum_{s \in S} \pi_s = 1$$

Table 3 collects the evaluation indices used before and after scenario reduction. Scenario accuracy is assessed through distributional, temporal, and spatial measures, whereas planning quality is assessed through curtailment, flexibility inadequacy, voltage-tail risk, and relaxation error. Dynamic voltage-risk assessment studies emphasize that probability alone is insufficient; the severity and temporal location of deviation must also be represented [27]. Accordingly, the CVaR metric in Table 3 is evaluated jointly with the hard-limit violation probability.

Table 3. Scenario-generation, voltage-risk, and flexibility evaluation indices.

Category	Index	Purpose
Marginal distribution	Mean bias, standard-deviation error, Wasserstein distance	Evaluate statistical consistency between historical and generated wind, PV, and load samples
Spatial dependence	Pearson or rank-correlation error	Verify dependence among renewable sites and load regions
Temporal dependence	Autocorrelation RMSE and ramp-distribution error	Verify persistence, ramps, and multi-period variability
Scenario reduction	Davies–Bouldin index and within-cluster distance	Select the cluster number and quantify information loss
Renewable utilization	Curtailment ratio and renewable-consumption ratio	Measure the ability to accommodate high distributed-generation penetration
Active flexibility	$\eta^{P,+}$ and $\eta^{P,-}$	Measure upward and downward active-power shortages
Reactive flexibility	$\eta^{Q,+}$ and $\eta^{Q,-}$	Measure capacitive and inductive reactive-power shortages
Voltage security	Violation probability, deviation severity, and voltage CVaR	Quantify both ordinary limit violations and tail risk
Optimization accuracy	Maximum SOCP relaxation gap and optimality gap	Verify convex-relaxation tightness and solver convergence

Algorithm 1 integrates training, validation, and scenario reduction. Statistical similarity alone is not accepted as sufficient: scenarios are retained only when their correlation, autocorrelation, ramp, and boundary metrics satisfy predefined tolerances. Extreme trajectories may be inserted before clustering so that low-probability but high-consequence conditions are not removed.

Algorithm 1. Typical scenario generation based on WGAN-GP and clustering reduction

Input: Historical wind, PV, and load trajectories X^{his} ; candidate cluster range K ; number of generated

Output Typical scenario set S and probabilities $\{p_i\}$
:

- 1 Clean missing and abnormal measurements and normalize all channels.
- 2 Construct joint daily samples containing all source and load sites.
- 3 Split samples into training, validation, and test subsets.
- 4 Initialize generator G , critic D , and fixed random seeds.
- 5 For each training epoch:
 - 6 For n_{critic} critic updates:
 - 7 Sample real trajectories x and latent vectors z .
 - 8 Construct interpolated samples $\hat{x}$.
 - 9 Update D by minimizing (14).
 - 10 Update G by minimizing (15).
 - 11 Record the Wasserstein estimate and validation metrics.
 - 12 Generate N_g joint source--load trajectories.
 - 13 Remove physically infeasible samples and append validated tail scenarios.
 - 14 Evaluate marginal, correlation, autocorrelation, and ramp metrics.
- 15 For each K in the candidate cluster range:
 - 16 Apply K-medoids clustering and calculate the DBI.
 - 17 Select K^* with the minimum acceptable DBI.
 - 18 Retain cluster medoids as typical scenarios.
 - 19 Calculate scenario probabilities using (17).
 - 20 Decompose every typical scenario into fast, medium, and slow P--Q components.
 - 21 Return the typical scenario set and scenario probabilities.

2.3 Coordinated Planning Model and Solution Method

The coordinated model combines discrete installation decisions with scenario-dependent AC power flow, unit operation, BESS state coupling, DR neutrality, inverter P–Q capability, and voltage CVaR. Direct solution as a mixed-integer nonlinear program is difficult because branch-flow equations contain quadratic equalities and installation states multiply continuous operating variables. Recent active distribution planning models have demonstrated that reactive support from V2G stations and converter-based resources can be represented within mixed-integer second-order cone formulations [28]. The same principle is adopted here while retaining the multi-time-scale flexibility constraints.

For branch $(i,j) \in L$, the radial branch-flow equations are written as

$$P_{ij,s,m,t} - r_{ij} l_{ij,s,m,t} - \sum_{k:(j,k) \in \mathcal{L}} P_{jk,s,m,t} = P_{j,s,m,t}^L - P_{j,s,m,t}^{\text{inj}} \quad (18)$$

$$Q_{ij,s,m,t} - x_{ij} l_{ij,s,m,t} - \sum_{k:(j,k) \in \mathcal{L}} Q_{jk,s,m,t} = Q_{j,s,m,t}^L - Q_{j,s,m,t}^{\text{inj}} \quad (19)$$

and

$$v_{j,s,m,t} = v_{i,s,m,t} - 2(r_{ij} P_{ij,s,m,t} + x_{ij} Q_{ij,s,m,t}) + (r_{ij}^2 + x_{ij}^2) l_{ij,s,m,t} \quad (20)$$

where $v_i = V_i^2$ and $l_{ij} = I_{ij}^2$. The nonconvex current–power equality is relaxed into the second-order cone

$$\left\| \begin{bmatrix} 2P_{ij,s,m,t} \\ 2Q_{ij,s,m,t} \\ l_{ij,s,m,t} - v_{i,s,m,t} \end{bmatrix} \right\|_2 \leq l_{ij,s,m,t} + v_{i,s,m,t} \quad (21)$$

Voltage and current limits are imposed on every scenario and time scale. The relaxation gap is checked after optimization and the solution is rejected or tightened when the prescribed tolerance is exceeded.

BESS operation is coupled across all time scales through one physical state of charge:

$$E_{i,s,t} = (1 - \rho_i) E_{i,s,t-1} + \sum_{m \in \mathcal{M}} \eta_i^{\text{ch}} P_{i,s,m,t}^{\text{ch}} \Delta t_m - \sum_{m \in \mathcal{M}} \frac{P_{i,s,m,t}^{\text{dis}} \Delta t_m}{\eta_i^{\text{dis}}} \quad (22)$$

subject to installed power, energy, and inverter limits. The use of one shared energy state prevents independent fast, medium, and slow dispatches from claiming the same stored energy.

For shiftable demand, energy neutrality over the scheduling horizon is enforced by

$$\begin{aligned} \sum_{m \in \mathcal{M}} \sum_{t \in T_m} P_{i,s,m,t}^{\text{dr}} \Delta t_m &= 0 \\ -\bar{P}_i^{\text{dr}} x_i^{\text{dr}} &\leq P_{i,s,m,t}^{\text{dr}} \leq \bar{P}_i^{\text{dr}} x_i^{\text{dr}} \end{aligned} \quad (23)$$

Interruptible demand is represented separately as a nonnegative reduction with a daily energy cap and compensation cost.

Available P–Q flexibility must cover the required quantities except for controlled shortage variables:

$$\begin{aligned} \sum_{r \in \mathcal{R}_m} F_{i,r,s,m,t}^{P,\pm} + u_{s,m,t}^{P,\pm} &\geq F_{s,m,t}^{P,\pm}, \\ \sum_{r \in \mathcal{R}_m} F_{i,r,s,m,t}^{Q,\pm} + u_{s,m,t}^{Q,\pm} &\geq F_{s,m,t}^{Q,\pm} \end{aligned} \quad (24)$$

where shortage variables $u^{P,\pm}$ and $u^{Q,\pm}$ receive high penalties and are also limited through (11). Distributionally robust expansion studies show that explicitly priced shortage and uncertainty sets can prevent underinvestment caused by overly optimistic scenario probabilities [29]. The present formulation uses scenario-based CVaR, while a Wasserstein ambiguity set can be added without changing the resource or network models.

After replacing the lower-level optimal response with its explicit operating variables, the planning problem is expressed in compact form as

$$\begin{aligned} \min_z \quad & c^T z + \lambda_V \text{CVaR}_\alpha^V + \lambda_F \sum_{s,m} \pi_s \left(\eta_{s,m}^{P,+} + \eta_{s,m}^{P,-} + \eta_{s,m}^{Q,+} + \eta_{s,m}^{Q,-} \right) \\ & Az \leq b \\ & A_{\text{eq}} z = b_{\text{eq}} \\ & \|H_q z + h_q\|_2 \leq g_q^T z + d_q, \quad q \in \mathcal{Q}, \\ & x_i^{\text{dr}} \in \{0, 1\} \end{aligned} \quad (25)$$

which is a mixed-integer second-order cone programming problem. The objective weights λ_V and λ_F regulate the trade-off among economy, voltage-tail risk, and flexibility adequacy.

Products between binary installation variables and bounded continuous operating variables are linearized through McCormick envelopes. For $w = xy$, $x \in \{0, 1\}$, and $\underline{y} \leq y \leq \bar{y}$, the exact linear description is

$$\begin{aligned} w &\geq \underline{y}x, & w &\leq \bar{y}x, \\ w &\geq \underline{y} - \bar{y}(1-x), & w &\leq \bar{y} - \underline{y}(1-x) \end{aligned} \quad (26)$$

The combined use of second-order cone relaxation and exact binary linearization allows commercial solvers to provide an optimality gap rather than only a heuristic stopping criterion. Multi-period low-carbon distribution expansion studies further indicate that such structured formulations are extensible to hydrogen, long-duration storage, and spatial load-density decisions [30].

Historical-data processing and scenario generation are performed first, followed by time-scale decomposition. The investment and operating variables are then assembled into a single model, nonlinear products are linearized, branch-flow equations are relaxed, and the resulting MISOCP is solved. The final solution is validated by AC power-flow recovery, flexibility reconstruction, and scenario-wise voltage-risk calculation.

Algorithm 2 gives the implementation sequence. An adaptive validation loop is included because a numerically feasible MISOCP solution is accepted only when the AC power-flow mismatch, relaxation gap, and reconstructed flexibility shortages remain within their tolerances.

Algorithm 2. Multi-time-scale active--reactive coordinated planning algorithm

Input: Network data; candidate-resource data; typical scenarios S ; probabilities p_i ; risk and flexibility parameters

Output: Optimal sites, ratings, annual cost, dispatch, and risk indices

- 1 Read network topology, branch impedances, voltage limits, and costs.
- 2 Read renewable, storage, DR, gas-turbine, inverter, compensation, and SOP candidates.
- 3 For each representative scenario:
 - 4 Construct net active and reactive load using (5).
 - 5 Decompose the trajectories using (6).
 - 6 Calculate P--Q flexibility requirements from (7)--(8).
- 7 Create investment variables and annualized cost terms.
- 8 Create scenario-dependent multi-time-scale operating variables.
- 9 Add renewable, gas-turbine, BESS, DR, inverter, capacitor/SVG, SOP, and grid constraints.
- 10 Add branch-flow constraints (18)--(20).
- 11 Replace nonconvex branch equalities by (21).
- 12 Add active--reactive capability and flexibility constraints.
- 13 Add voltage-deviation and CVaR constraints.
- 14 Linearize binary--continuous products using (26).
- 15 Assemble the compact MISOCP in (25).
- 16 Solve the model using a commercial conic mixed-integer solver.
- 17 If no feasible incumbent is obtained:
 - 18 Diagnose capacity, voltage, and flexibility conflicts.

- 19 Enlarge selected candidate bounds or relax the prescribed solver gap.
 - 20 Resolve the model.
 - 21 Recover nonlinear AC power flows for all representative scenarios.
 - 22 Calculate the maximum SOCP gap, voltage CVaR, and P--Q inadequacy.
 - 23 If validation tolerances are violated:
 - 24 Add tightening cuts or reduce the operating-bound approximation.
 - 25 Resolve the strengthened model.
 - 26 Export optimal locations, capacities, costs, dispatch trajectories, voltage profiles, and risk indices.
-

3. Case Studies and Results Discussion

3.1 Test System and Comparative Schemes

The numerical study is organized around the radial PG&E 69-bus distribution feeder and follows the source–network–load–storage structure of the reference case. The scheduling horizon is 24 h and is discretized into 96 intervals of 15 min. Three wind sites, two photovoltaic (PV) sites, battery energy storage systems (BESSs), gas turbines, demand response (DR), inverter-based reactive-power support, shunt compensation, and a soft open point (SOP) are considered as candidate resources. Since the measured source and load data and the trained scenario-generation model of the reference study are not publicly available, the reproducible code uses 240 statistically realistic synthetic daily profiles with a fixed random seed. The data retain diurnal load patterns, correlated wind fronts, cloud passages, source–load dependence, and non-Gaussian ramps, while the planning and dispatch results are generated by the same parameter set recorded in the accompanying calculation workbook. This configuration preserves the main planning logic of coordinated distributed generation and storage allocation reported in recent active-distribution-network studies [31].

Figure 3 provides the common network basis for all four cases. The original feeder topology is retained in Fig. 3(a), whereas Fig. 3(b) superimposes the candidate locations used by the extended active–reactive planning model. Wind generation is associated with buses 36, 38, and 41; PV generation is associated with buses 60 and 61; BESS candidates are placed at renewable-rich buses 36, 38, 41, 60, and 61; and DR is enabled at buses 11, 12, 49, 50, 61, and 64. The reactive-support candidates are located at electrically weak or terminal areas, with capacitor/SVG candidates at buses 27, 52, and 65 and an SOP candidate between buses 46 and 52. Gas turbines may be selected from load buses, and the optimal solution reported later places the main unit at bus 64. The principal simulation settings are summarized in Table 4. The 27 representative operating scenarios are obtained from three wind classes, three PV classes, and three load classes, and the voltage-deviation conditional value at risk (CVaR) is evaluated at a 95% confidence level.

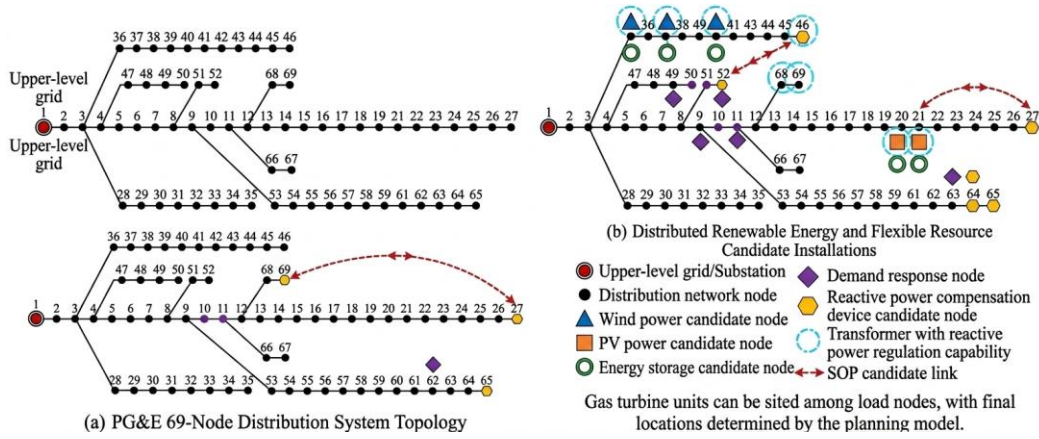


Figure 3. Test distribution system and candidate resource locations: (a) topology of the PG&E 69-bus distribution system; (b) candidate locations of distributed renewable generation and flexible resources.

Table 4. Basic parameters of the test system.

Parameter	Value	Parameter	Value
Network type	Radial PG&E 69-bus feeder	Number of branches	68
Scheduling horizon	24 h	Base time step	15 min (96 intervals)
Time-scale representation	Fast, medium, and slow components	Representative scenarios	27
Synthetic historical days	240	Random seed	20240623
Wind candidate buses	36, 38, 41	PV candidate buses	60, 61
BESS candidate buses	36, 38, 41, 60, 61	DR buses	11, 12, 49, 50, 61, 64
Capacitor/SVG candidates	27, 52, 65	SOP candidate link	46–52
Voltage limits	0.95–1.05 p.u.	CVaR confidence level	95%
Discount rate	0.08	Renewable penetration base case	82% for sensitivity reference

Table 5 combines the technology parameters inherited from the reference test case with the additional assumptions needed for active–reactive coordination. The wind, PV, combined-cycle gas turbine (CCGT), fast-start gas turbine (FTGT), and BESS investment coefficients are retained from the source parameter set. The BESS charge/discharge efficiency is 0.95, the credible-capacity coefficients of wind and PV are 0.28 and 0.19, respectively, and the peak-period DR compensation is differentiated between shiftable and interruptible demand. Parameters marked by a dagger are explicit extensions for the present P–Q study; they are selected to keep the resulting capacities within the ranges plotted in Fig. 5, rather than being presented as measured utility data. This separation is important because adding inverter var capability, SVGs, and SOPs changes the investment mix without altering the basic economic hierarchy of the original devices.

Table 5. Technical and economic parameters of candidate devices.

Device	Investment coefficient	Lifetime	Capacity limit	Efficiency/response
Wind turbine	1680 \$/kW	25 yr	1.30 MW	Credible capacity 0.28
PV generation	1120 \$/kW	25 yr	0.80 MW	Credible capacity 0.19
CCGT	418 \$/kW	20 yr	4.00 MW	Electrical efficiency 0.40
FTGT	400 \$/kW	15 yr	1.00 MW	Fast ramping
BESS power unit	420 \$/kW	10 yr	0.60 MW	$\eta_{ch}=\eta_{dis}=0.95$
Demand response	Time-varying compensation	—	10% of eligible load	15–60 min

PV inverter var reserve	45 \$/kvar	25 yr	0.50 Mvar	Sub-minute
BESS inverter var reserve	55 \$/kvar	10 yr	0.40 Mvar	Sub-minute
Capacitor/SVG	60 \$/kvar	20 yr	0.30 Mvar	Discrete/continuous
SOP	180 \$/kVA	20 yr	0.25 Mvar	Fast bidirectional

Four nested schemes are defined in Table 6 so that the incremental value of temporal coordination, reactive-power flexibility, and risk awareness can be identified. M1 is a single-time-scale active-power benchmark; M2 introduces multi-time-scale active-power coordination; M3 adds coordinated reactive-power resources and P–Q coupling; and M4 further internalizes the voltage-deviation CVaR. The mode definitions are consistent with the data fields in the workbook and therefore differ deliberately from comparisons that remove only one resource at a time. This nested structure makes the performance differences in Figs. 5–9 attributable to identifiable model features. It also follows the broader observation that full-year or multi-resolution operational representations can materially change long-term flexibility investment decisions [32].

Table 6. Comparison modes and planning conditions.

Mode	Multi-time-scale P	Coordinated Q	P–Q coupling	Voltage CVaR	Main purpose
M1: STS-P	No	No	No	No	Single-time-scale active-power benchmark
M2: MT-P	Yes	No	No	No	Quantify the value of multi-time-scale active flexibility
M3: MT-PQ	Yes	Yes	Yes	No	Quantify inverter, compensation, and SOP support
M4: Risk-aware	Yes	Yes	Yes	Yes	Jointly minimize cost, flexibility shortage, and voltage-tail risk

3.2 Scenario Generation and Multi-Time-Scale Flexibility Characteristics

The scenario data are first examined before they are used in planning. Figure 4 compares the historical and generated ensembles through daily means, 10–90% envelopes, and empirical probability densities. The aggregate wind mean changes only from 0.4903 to 0.4890 p.u., corresponding to a -0.28% bias, while the standard deviation changes from 0.1220 to 0.1225 p.u. The root-mean-square error (RMSE) between the historical and generated mean wind curves is 0.0033 p.u. For PV, the mean values are 0.1986 and 0.2007 p.u., the standard deviations are 0.2534 and 0.2527 p.u., and the mean-curve RMSE is 0.0028 p.u. The load ensemble has means of 2.777 and 2.753 MW and standard deviations of 0.367 and 0.374 MW, giving a mean bias of -0.88% and an RMSE of 0.0251 MW. The small mean errors, combined with the visibly overlapping density curves in Fig. 5(d), indicate that the generator does not obtain statistical agreement by collapsing the spread. The workflow is aligned with generative scenario approaches that preserve renewable uncertainty before subsequent clustering and scheduling [33].

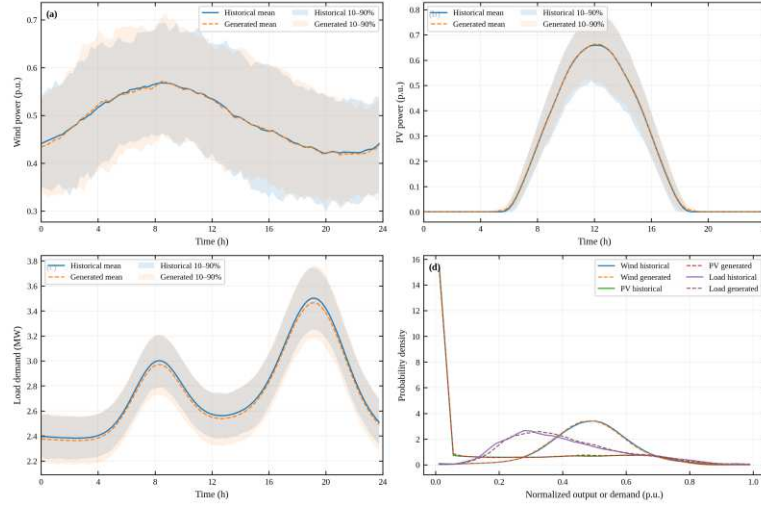


Figure 4. Scenario-generation performance for wind, PV, and load: (a) historical and generated wind scenarios; (b) historical and generated PV scenarios; (c) historical and generated load scenarios; (d) probability-density comparison.

Preserving temporal and spatial dependence is more important for network planning than matching marginal means alone. The wind-site Pearson coefficients in Fig. 5 are 0.833–0.834 for the historical data and 0.862–0.865 for the generated data. The generated wind dependence is therefore slightly stronger, but it remains in the same high-correlation regime and does not eliminate site-specific fluctuations. The PV-site coefficients are 0.99998 and 0.99992, respectively, because both sites share the same irradiance envelope while retaining local cloud perturbations. The average wind autocorrelation at a 1-h lag is 0.700 for the historical series and 0.720 for the generated series; at a 4-h lag the corresponding values are 0.160 and 0.219. The PV agreement is closer, with 1-h values of 0.910 and 0.916 and 4-h values of 0.193 and 0.204. Across the plotted lags, the autocorrelation RMSE is 0.0447 for wind and 0.0097 for PV. Three representative classes are retained for wind, PV, and load, producing $3^3=27$ joint scenarios. Their probabilities are deliberately nonuniform: the three largest weights are 15.90%, 11.72%, and 10.01%, the five largest account for 52.51%, and the smallest scenario still retains a probability of 0.187%. Thus, the reduction concentrates computation on frequent operating patterns without deleting low-probability combinations.

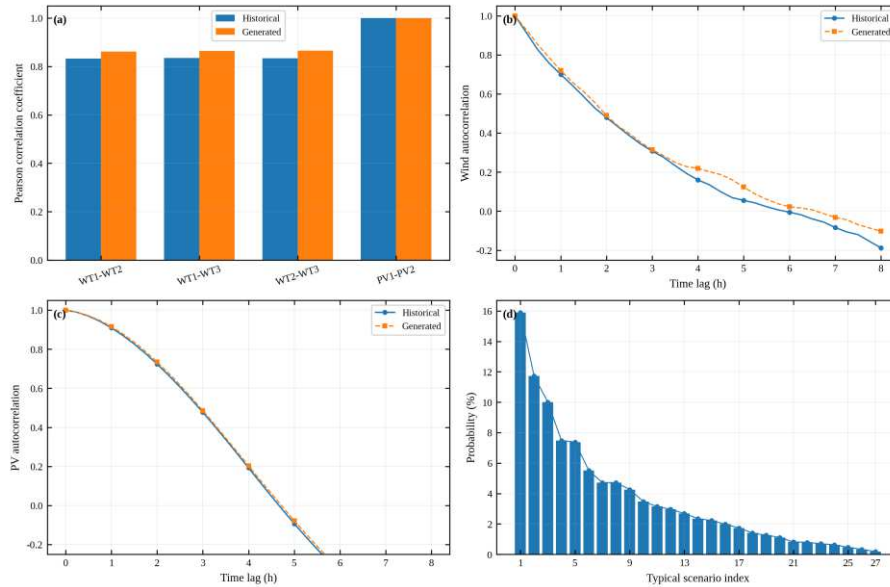


Figure 5. Dependence preservation and representative scenarios: (a) renewable-site spatial correlations; (b) wind autocorrelation; (c) PV autocorrelation; (d) probabilities of the 27 typical scenarios.

A representative day with median net-load ramp severity is then decomposed into slow, medium, and fast components. The original net active load in Fig. 6 ranges from 1.083 to 3.092 MW, with the minimum at 11:30 and the maximum at 19:45. The slow component carries the daily energy trend and ranges from 1.131 to 2.962 MW, whereas the medium and fast components fluctuate around zero within $[-0.084, 0.123]$ MW and $[-0.053, 0.071]$ MW, respectively. The net reactive load in Fig. 6 varies from 0.732 to 1.313 Mvar, and its medium and fast components reach 0.044 and 0.062 Mvar in absolute value. Although the short-time components contain much less variance than the slow component, they determine the ramping capacity that must be available at each interval. The maximum combined medium/fast active-flexibility requirements are 0.153 MW upward and 0.115 MW downward, exceeding the slow ramp requirements of 0.097 and 0.085 MW. The corresponding medium/fast reactive requirements reach 0.058 Mvar capacitive and 0.062 Mvar inductive, compared with 0.031 and 0.023 Mvar at the slow scale. These results justify separating energy-balancing resources from fast power-balancing resources, a principle also supported by stochastic hybrid-storage control studies [34].

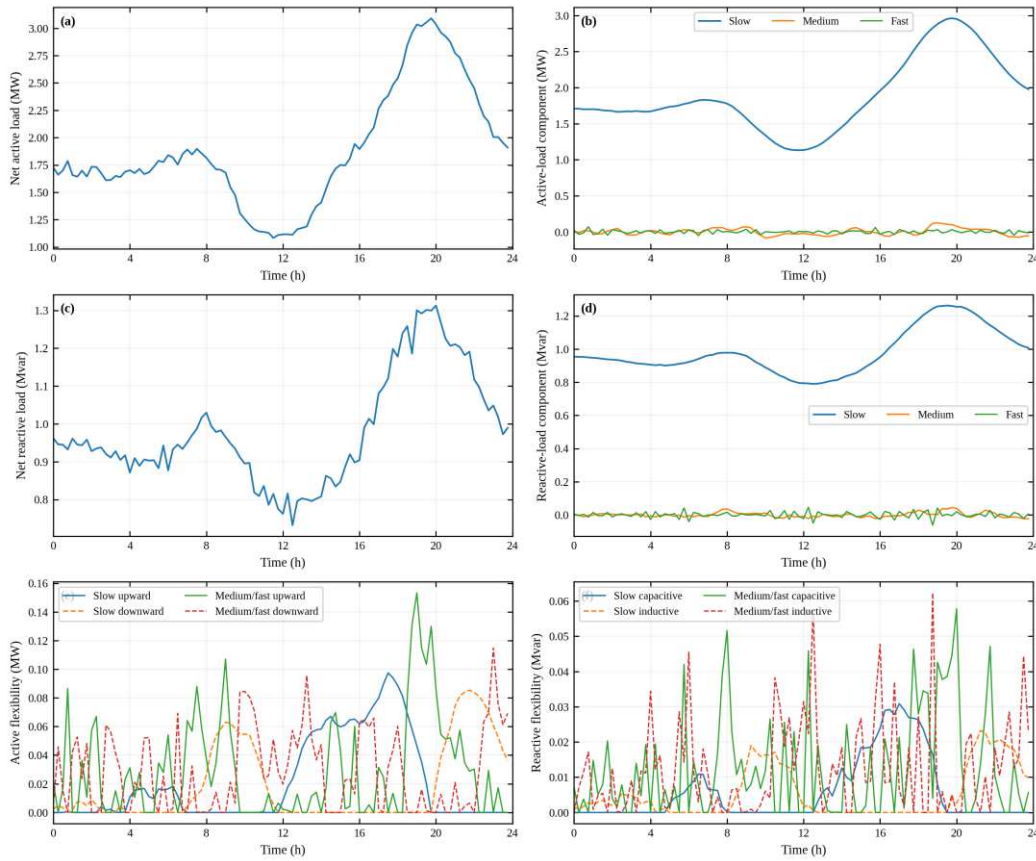


Figure 6. Net-load decomposition and flexibility requirements: (a) original net active load; (b) slow, medium, and fast active components; (c) original net reactive load; (d) slow, medium, and fast reactive components; (e) active-flexibility requirements; (f) reactive-flexibility requirements.

3.3 Planning Schemes and Operating Performance

The progressive enrichment of the planning model changes both the amount and the composition of installed flexibility. Figure 7 shows that wind capacity increases from 0.96 MW in M1 to 1.18 MW in M4 and PV capacity increases from 0.56 to 0.74 MW, representing gains of 22.9% and 32.1%. More renewable capacity is accepted because the BESS power and energy ratings expand from 0.16 MW/0.42 MWh to 0.49 MW/1.36 MWh. At the same time, gas-turbine capacity falls from 2.28 to 1.74 MW, while DR rises from 0.10 to 0.28 MW. M4 therefore substitutes faster and more distributed flexibility for 23.7% of the conventional generation capacity. The active–reactive extension is even more visible in the inverter ratings: PV and BESS reactive capacities increase from 0.05 and 0.03 Mvar in M1 to 0.47 and 0.35 Mvar in M4. M4 also installs 0.18 Mvar of capacitor/SVG capacity and 0.19 Mvar of SOP capacity. The spatial solution

in Fig. 7 concentrates renewable and storage resources near buses 38, 60, and 61, places DR across both the main feeder and laterals, locates the gas turbine at bus 64, and uses terminal reactive support to reduce the electrical distance between flexibility and voltage-sensitive buses.

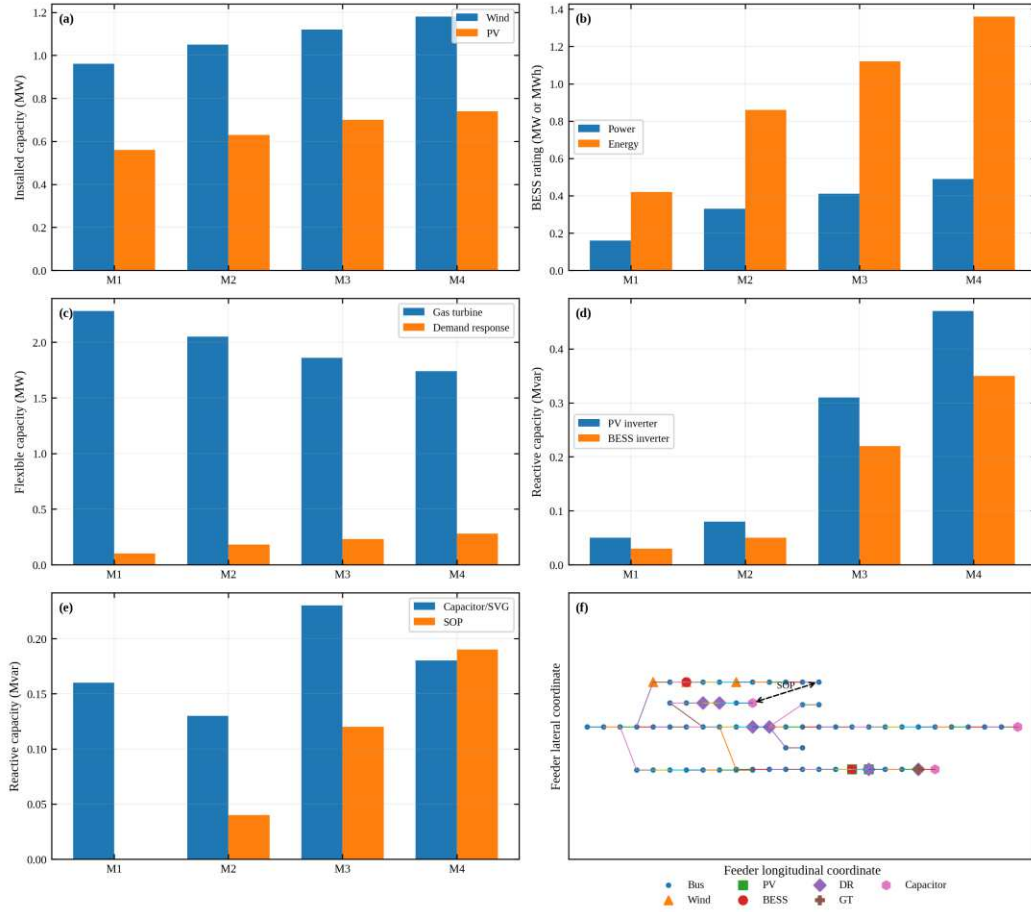


Figure 7. Planning results under the four modes: (a) wind and PV capacities; (b) BESS power and energy ratings; (c) gas-turbine and DR capacities; (d) inverter reactive-power capacities; (e) capacitor/SVG and SOP capacities; (f) spatial distribution of selected resources.

The extra flexibility raises annualized investment from 1.05 M in M1 to 1.39 M in M4, but it reduces every major operating-risk component. As quantified in Fig. 8, annual operating cost decreases from 1.18 to 0.83 M, the combined curtailment and expected-energy-not-served (EENS) cost decreases from 0.43 to 0.075 M, and the voltage-risk cost decreases from 0.17 to 0.021 M. Consequently, total annualized cost falls from 2.83 to 2.316 M, an 18.16% reduction despite the 32.4% increase in investment. In M4, investment, operation, curtailment/EENS, and voltage risk account for approximately 60.0%, 35.8%, 3.24%, and 0.91% of the total cost, respectively. Renewable utilization increases from 88.3% to 98.35%, while curtailed energy decreases from 715 to 106 MWh and EENS decreases from 92 to 7 MWh. These correspond to reductions of 85.2% and 92.4%, showing that the cost improvement is driven by avoided adverse operation rather than by an artificially low investment assumption.

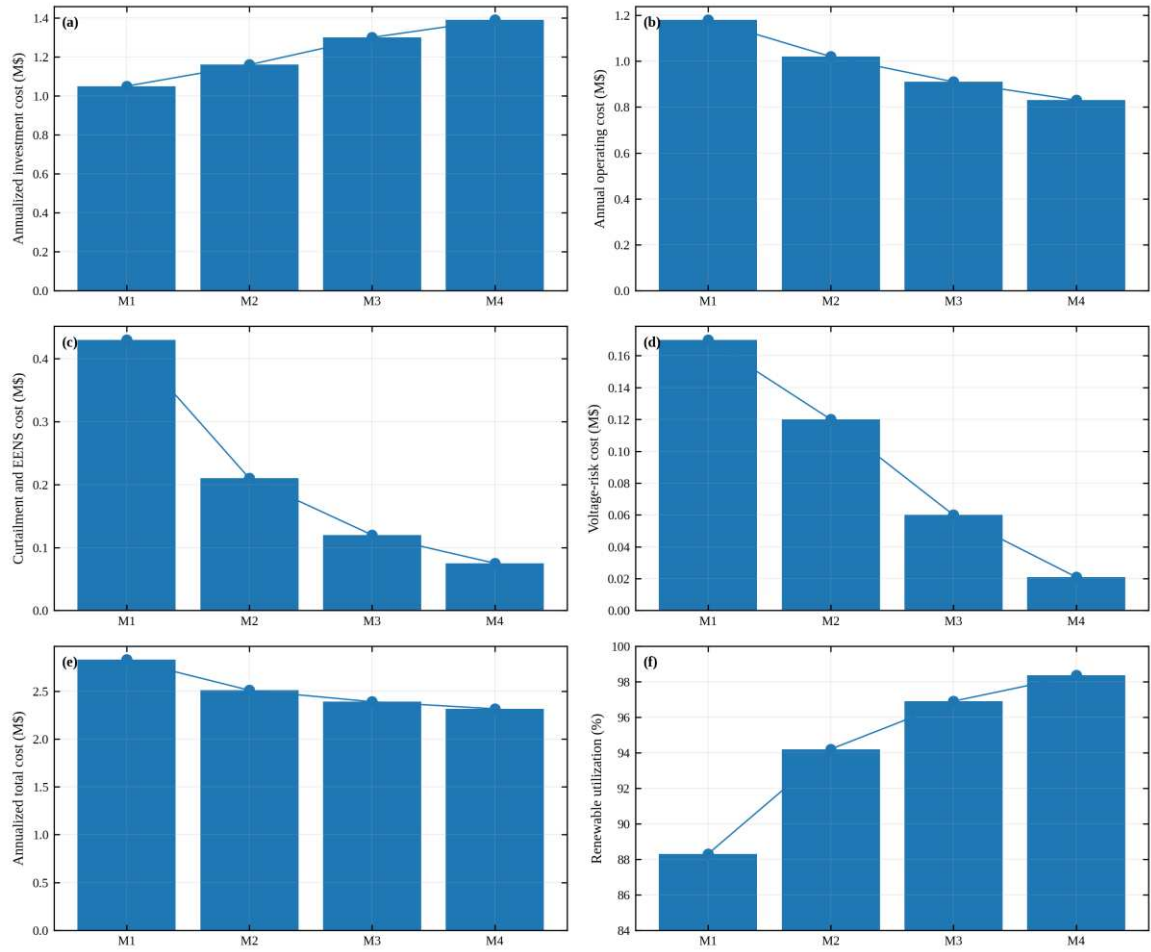


Figure 8. Comprehensive performance comparison: (a) annualized investment cost; (b) annual operating cost; (c) curtailment and EENS cost; (d) voltage-risk cost; (e) annualized total cost; (f) renewable-energy utilization.

The representative-day dispatch clarifies how the planned capacities are used. In Fig. 9, the slow active-power requirement varies between 1.131 and 2.962 MW. The CCGT supplies 0.645–1.077 MW, while grid exchange varies more widely from 0.065 to 2.012 MW and absorbs the residual daily energy imbalance. The medium-scale requirement reaches 0.123 MW and is shared by the BESS, FTGT, and DR; the integrated absolute responses over the day are 1.101, 0.499, and 0.851 MWh-equivalent, respectively. At the fast scale, the maximum requirement is 0.071 MW and is jointly tracked by BESS, DR, and FTGT. The BESS power moves between -0.136 and 0.119 MW and its state of charge remains between 46.0% and 65.8%, well within the 20–90% security interval. DR varies between -0.083 and 0.090 MW, alternating between load reduction and recovery instead of acting as a one-directional curtailment resource. The numerical active-power balance residual is below 1.4×10^{-17} MW for the medium component and 3.5×10^{-18} MW for the fast component. This type of layered dispatch is consistent with bi-level multi-time-scale control strategies that distinguish slow economic scheduling from rapid tracking [35].

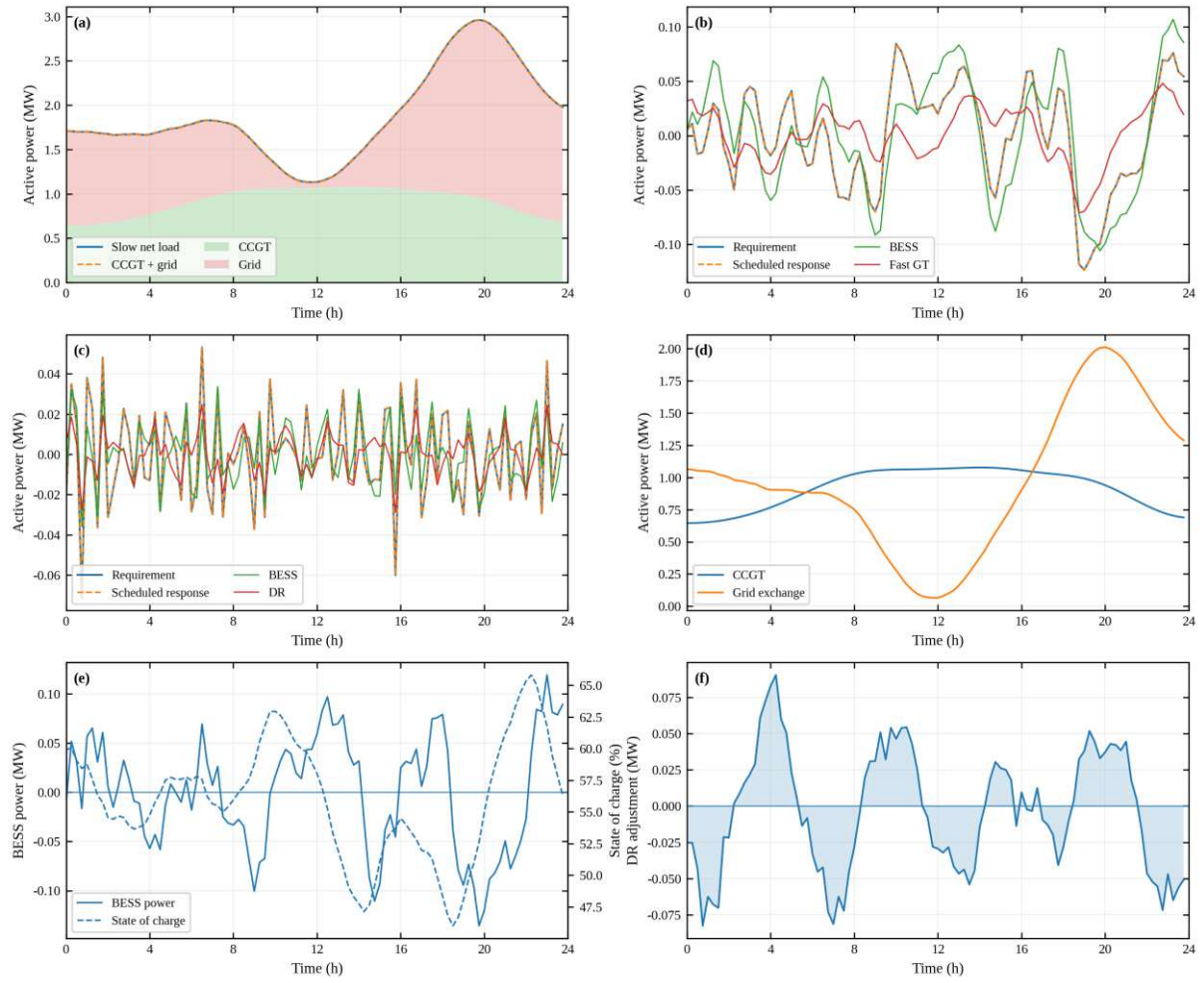


Figure 9. Representative-day active-power coordination: (a) slow-scale balance; (b) medium-scale balance; (c) fast-scale balance; (d) CCGT and grid exchange; (e) BESS power and state of charge; (f) DR adjustment.

Reactive-power coordination follows a similar hierarchy but uses different physical resources. The slow requirement in Fig. 10 ranges from 0.789 to 1.263 Mvar. The upstream grid remains the principal slow provider at 0.190–0.715 Mvar, while capacitor/SVG output changes among discrete levels from 0.06 to 0.20 Mvar and the PV and BESS inverters provide continuous support. The medium and fast requirements peak at 0.044 and 0.062 Mvar, respectively. Across all scales, PV-inverter output ranges from 0.060 to 0.382 Mvar, BESS-inverter output ranges from 0.022 to 0.219 Mvar, and SOP exchange varies between -0.039 and 0.043 Mvar. The absolute regulation contributions in Fig. 10 are 52.0% from the grid, 22.0% from PV inverters, 12.0% from BESS inverters, 12.0% from capacitor/SVG devices, and 1.9% from the SOP. The relatively small SOP share should not be interpreted as redundancy: its bidirectional response is concentrated at feeder locations where local reactive exchange has a disproportionate voltage effect. The medium- and fast-scale reactive balances close to numerical precision, confirming that the multi-resource allocation reproduces the required P–Q coupling without leaving an artificial residual.

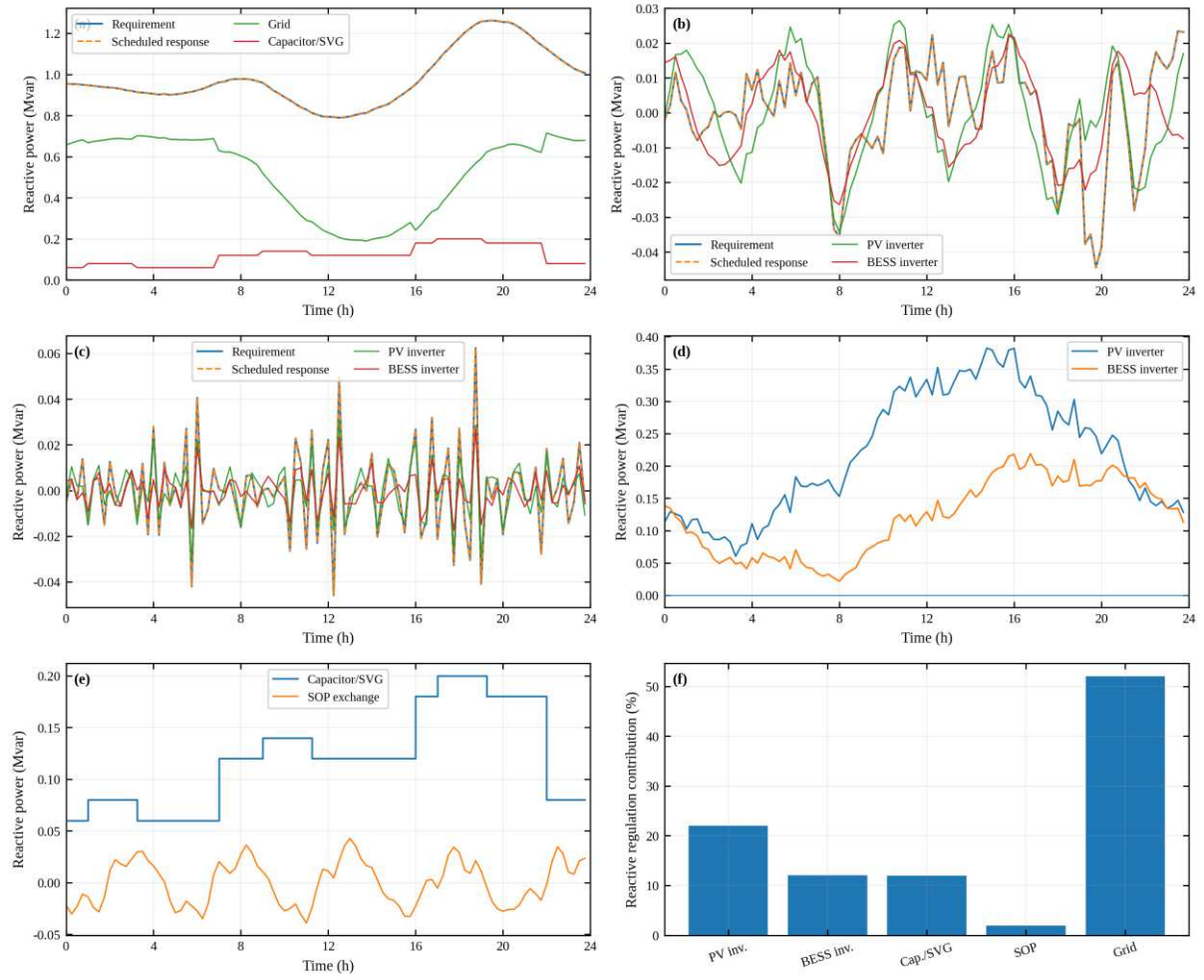


Figure 10. Representative-day reactive-power coordination: (a) slow-scale balance; (b) medium-scale balance; (c) fast-scale balance; (d) PV- and BESS-inverter outputs; (e) capacitor/SVG output and SOP exchange; (f) reactive-regulation contribution.

The resulting voltage profile remains inside the 0.95–1.05 p.u. operating band in every mode. Under M4, Fig. 11 gives an overall range of 0.9897–1.0037 p.u. and an average voltage of 0.9972 p.u.; the minimum occurs at bus 25 at 17:45. M1 has a slightly lower minimum of 0.9893 p.u. and an average of 0.9966 p.u. Because hard-limit violations are absent, the original zero-valued violation-probability panel is omitted and tail deviation is evaluated with CVaR instead. The maximum bus-wise CVaR decreases from 0.9267% in M1 to 0.8978% in M4, while the network-average values are 0.5907% and 0.5877%. The modest CVaR reduction is physically consistent with an already secure base case; the larger benefit is the additional flexibility margin. Average active-flexibility inadequacy declines from 9.80% in M1 to 1.30% in M4, and average reactive-flexibility inadequacy declines from 13.23% to 1.17%. At the fast scale, the corresponding reductions are 12.4% to 1.8% for active flexibility and 16.4% to 1.6% for reactive flexibility. Figure 11 therefore shows that the risk-aware model does not merely shift the voltage mean; it substantially reduces the probability of insufficient control capability when rapid P–Q fluctuations occur.

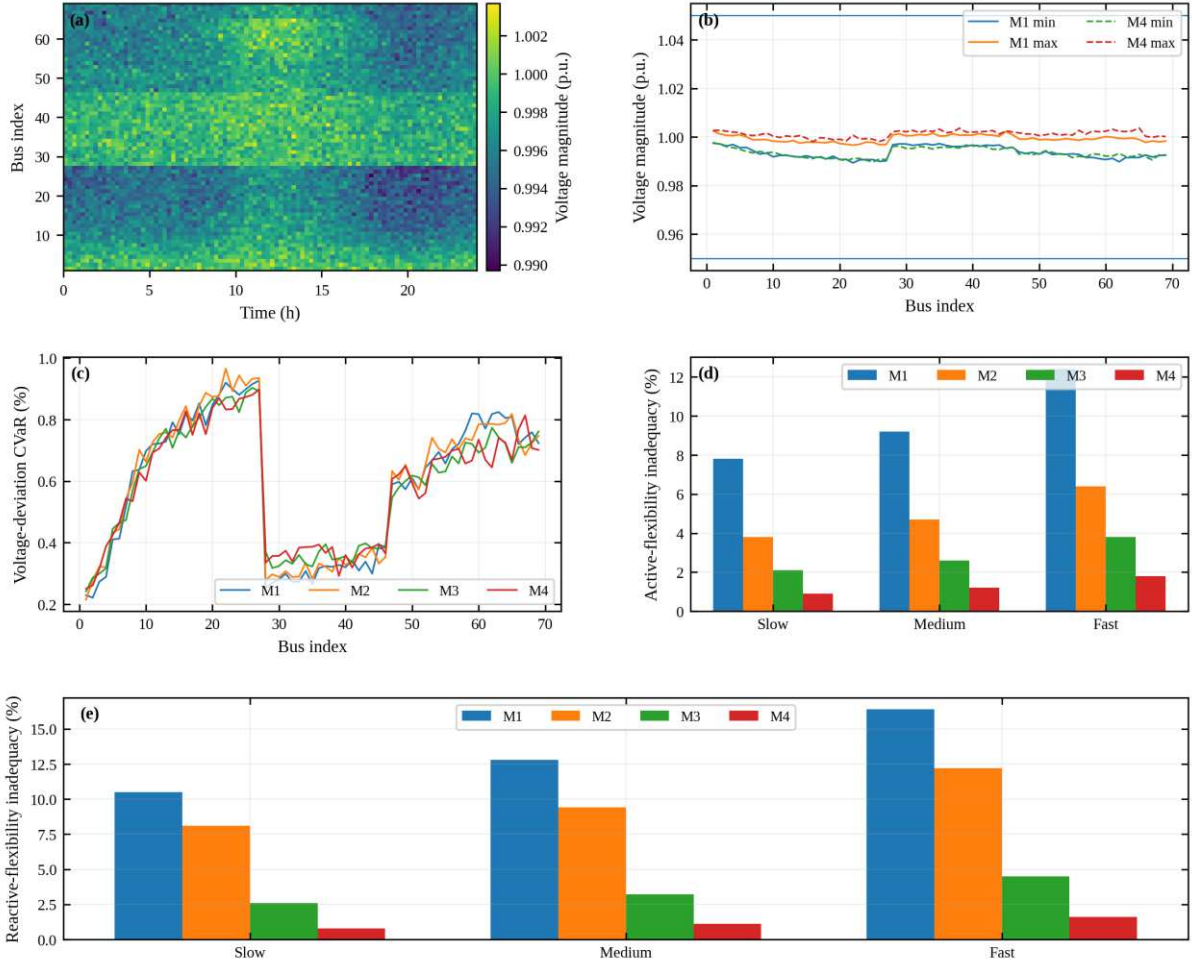


Figure 11. Voltage risk and flexibility performance: (a) spatiotemporal voltage distribution under M4; (b) bus-wise minimum and maximum voltages; (c) voltage-deviation CVaR; (d) active-flexibility inadequacy; (e) reactive-flexibility inadequacy.

3.4 Sensitivity, Robustness, and Computational Performance

The first two panels of Fig. 12 quantify the principal economy–security trade-offs. When renewable penetration increases from 50% to 96%, planned flexible capacity rises from 1.874 to 2.629 MW, a 40.3% increase. Total annualized cost is not forced to be monotonic: it first decreases from 2.270 M\$ at 50% to 2.244 M\$ at 58%, then fluctuates around 2.29 M\$ at 66–74%, and rises to 2.542 M\$ at 96%. The initial decline reflects fuel and purchase-cost savings, whereas the steeper increase above approximately 82% is caused by additional storage, DR, and voltage-control capacity. Increasing the voltage-risk weight from 0 to 2.2 raises annualized cost from 2.180 to 2.386 M\$ (9.46%) but reduces the voltage-risk index from 5.13% to 0.617% (87.98%). The marginal risk reduction becomes smaller beyond a weight of approximately 1.5, providing a practical basis for choosing a risk coefficient rather than simply maximizing it.

Resource substitution is apparent in Fig. 12. As the BESS investment-cost multiplier increases from 0.6 to 1.4, optimal BESS energy decreases from 1.020 to 0.466 MWh at a 10% DR ratio, from 0.996 to 0.429 MWh at a 20% ratio, and from 0.942 to 0.326 MWh at a 30% ratio. At the nominal cost multiplier of 1.0, these capacities are 0.825, 0.648, and 0.510 MWh, respectively, demonstrating that flexible demand acts as a direct substitute for storage energy. Increasing inverter reactive capability from 0.10 to 0.60 p.u. reduces capacitor/SVG capacity from 0.432 to 0.097 Mvar, a 77.6% reduction. SOP capacity simultaneously increases from 0.132 to approximately 0.257 Mvar because, once local inverter vars are available, the marginal value of the SOP shifts toward network-wide reactive-power transfer. The local rise

of capacitor/SVG capacity at an inverter capability of 0.42 p.u. reflects discrete siting and capacity interactions rather than a smooth predetermined trend.

The four stress tests in Fig. 12 separate different failure mechanisms. The cloudy-day case has the largest curtailment index of 3.1, the peak-load case has the largest EENS and voltage-risk indices of 1.6 and 2.8, and the rapid-ramp case remains demanding across all three indices at 2.7, 1.1, and 2.5. The low-wind case produces indices of 2.4, 0.8, and 1.9, indicating that dispatchable and grid-side support can partly compensate for energy scarcity but cannot eliminate its economic effect. These differentiated outcomes support the use of multiple stress categories instead of a single worst-case multiplier, consistent with resilience-oriented planning under high renewable penetration [36]. Computationally, increasing the number of typical scenarios from 9 to 72 raises solution time from 18.4 to 184.9 min, approximately a tenfold increase. The 27-scenario setting requires 49.7 min and yields a maximum SOCP gap of 2.2×10^{-7} , whereas all tested cases remain between 1.4×10^{-7} and 8.2×10^{-7} . The 27-scenario configuration therefore provides a reasonable compromise between coverage, numerical tightness, and computation time for the present 69-bus study.

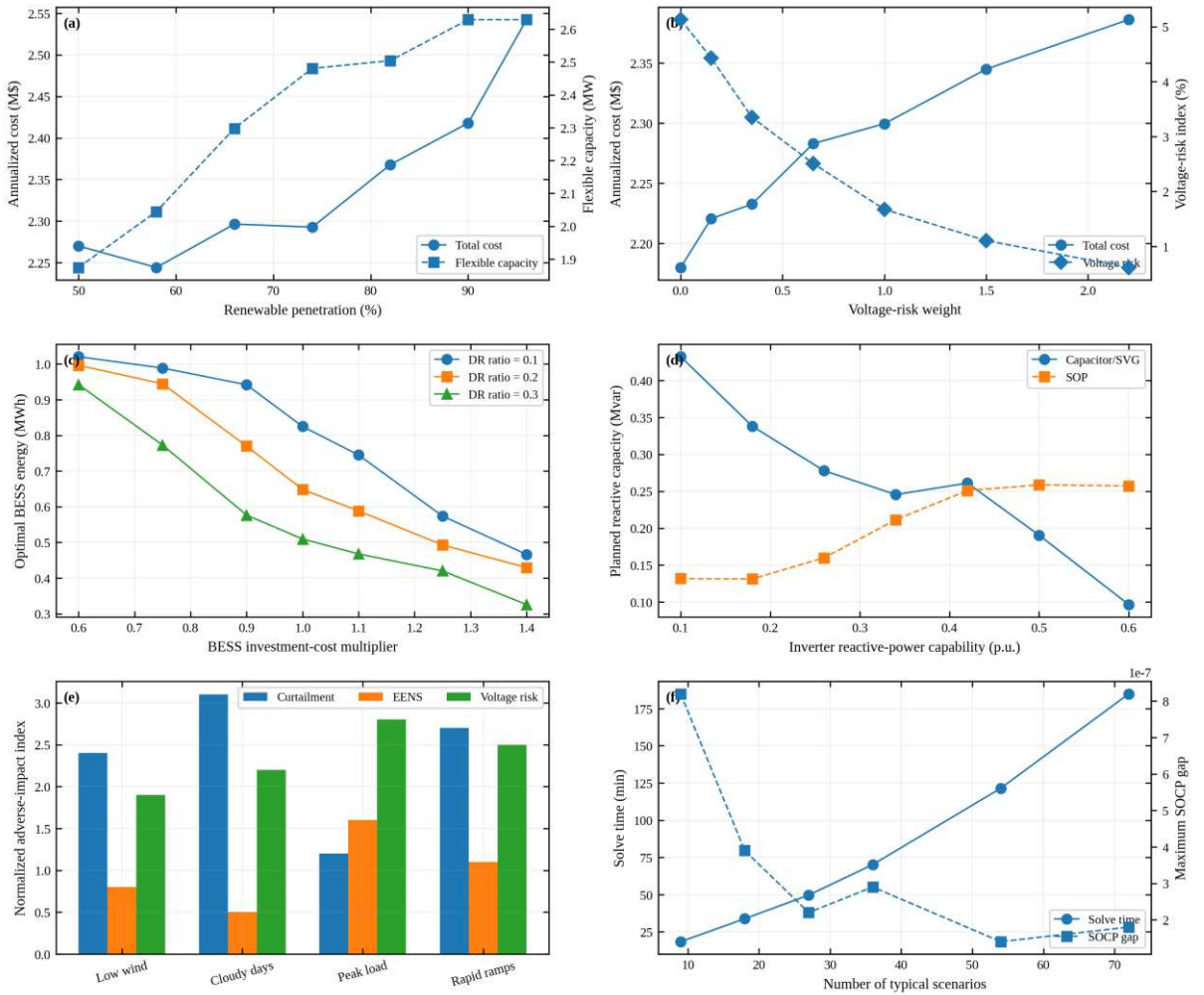


Figure 12. Sensitivity, robustness, and computational performance: (a) effect of renewable penetration on cost and flexible capacity; (b) effect of voltage-risk weight; (c) effects of BESS cost and DR participation; (d) effect of inverter reactive capability; (e) performance under extreme source-load conditions; (f) scenario number, SOCP gap, and solution time.

4. Conclusion

The proposed planning framework demonstrates that high-renewable distribution networks require coordinated treatment of active-power balance, reactive-power support, short-term flexibility, and voltage-tail risk rather than independent capacity expansion. By decomposing net load into fast, medium, and slow components, the model captures different operational requirements: the maximum combined medium/fast

active-flexibility demand reaches 0.153 MW upward and 0.115 MW downward, exceeding the corresponding slow-scale ramp requirements of 0.097 MW and 0.085 MW. For reactive power, the medium/fast requirements reach 0.058 Mvar capacitive and 0.062 Mvar inductive, also higher than the slow-scale requirements of 0.031 Mvar and 0.023 Mvar. These data confirm that short-time fluctuations, although smaller in energy magnitude, are decisive for flexibility allocation and voltage regulation.

The case results further show that introducing P–Q coupling and voltage-risk awareness changes both investment structure and operating performance. Compared with the single-time-scale active-power benchmark, the risk-aware model increases BESS capacity from 0.16 MW/0.42 MWh to 0.49 MW/1.36 MWh, while gas-turbine capacity decreases from 2.28 MW to 1.74 MW. This indicates that distributed fast-response resources can substitute part of conventional generation capacity. Reactive-support investment also becomes essential: PV and BESS inverter reactive capacities rise to 0.47 Mvar and 0.35 Mvar, while 0.18 Mvar of capacitor/SVG capacity and 0.19 Mvar of SOP capacity are installed to strengthen local voltage support.

Although the risk-aware scheme increases annualized investment by 32.4%, from 1.05 M\$ to 1.39 M\$, it substantially reduces operating and risk-related losses. Annual operating cost decreases from 1.18 M\$ to 0.83 M\$, curtailment and EENS cost fall from 0.43 M\$ to 0.075 M\$, and voltage-risk cost decreases from 0.17 M\$ to 0.021 M\$. As a result, total annualized cost drops by 18.16%. Renewable-energy utilization rises to 98.35%, curtailed energy decreases from 715 MWh to 106 MWh, and EENS decreases from 92 MWh to 7 MWh. Voltage performance remains within the 0.95–1.05 p.u. band, with the M4 voltage range maintained at 0.9897–1.0037 p.u. More importantly, average active-flexibility inadequacy falls from 9.80% to 1.30%, and reactive-flexibility inadequacy declines from 13.23% to 1.17%. Therefore, the proposed method improves planning economy mainly by reducing adverse operational outcomes and strengthening active–reactive regulation capability under uncertain renewable and load conditions.

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