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Rakesh Nath

BIT Mesra: Birla Institute of Technology

Vinay Sharma

SOMAK DATTA

`somakdatta@bitmesra.ac.in`

BIT Mesra: Birla Institute of Technology <https://orcid.org/0000-0002-8690-4347>

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Physics-Guided Predictive Modeling of Tool Wear in Electrical Discharge Machining via Dimensional Analysis: A Buckingham II Framework with TOPSIS-Based Electrode Selection

Rakesh ¹, Vinay Sharma², Somak Datta^{2*}

¹ University Polytechnic, Birla Institute of Technology, Mesra, Ranchi 835215, India

² Department of Production and Industrial Engineering, Birla Institute of Technology, Mesra, Ranchi 835215, India

**Corresponding Author: somakdatta@bitmesra.ac.in | ORCID: 0000-0002-8690-4347*

Abstract:

Electrical discharge machining (EDM) is a non-contact process used to shape hard, electrically conductive components for aerospace, automotive, and biomedical manufacturing. Electrode wear is a progressive phenomenon characterized by localized temperatures of 8,000–12,000 °C that gradually deteriorate dimensional accuracy and increase production costs; however, quantitative, physics-based models of the tool wear rate (TWR) are not yet available.

In the present work, the Buckingham II theorem is applied to die-sinking EDM, reducing eight governing variables and five fundamental dimensions to three independent dimensionless groups and deriving a closed-form, physically interpretable power-law model for TWR. Log-linear ordinary least-squares regression calibration of a nine-run Taguchi L9 dataset (copper electrode on mild steel; $I = 2\text{--}6\text{ A}$, $V = 40\text{--}80\text{ V}$, $\text{ton} = 50\text{--}200\text{ }\mu\text{s}$) yields $R^2 = 0.969$ and a mean absolute percentage error (MAPE) of 10.1 %.

The fitted thermal group exponent $\beta = -0.990$ agrees with EDM erosion theory. Higher electrode thermal conductivity and melting point together increase the specific energy needed to reach the ablation threshold. This, in turn, reduces TWR. Independent validation against nine experiments reported by (Bose & Mahapatra, 2015) (copper electrode, AISI H-13 tool steel; $I = 7\text{--}11\text{ A}$) shows a validation MAPE of 18.5%. All four outliers occur under operating conditions outside the calibration range, highlighting the known limitations of extrapolating power-law models.

The log-log sensitivity exponents of the II group are used to derive TOPSIS-based electrode selection weights analytically, replacing subjective expert assignment with a reproducible, physics-grounded procedure. The framework applied to (Bose & Mahapatra, 2015)eleven candidate electrode materials ranked Copper-Tungsten first ($CC = 0.611$), Tungsten second ($CC = 0.606$), and Silver-Tungsten third ($CC = 0.569$) – an ordering that directly reflects the high thermal conductivity and melting-point requirements identified by the II-group sensitivity analysis.

A digital twin architecture is proposed. It combines a calibrated physics model with Bayesian adaptive recalibration for real-time wear monitoring. This approach serves as a guide for future experimental work.

Keywords: Electrical discharge machining, dimensional analysis, Buckingham II theorem, tool wear rate, TOPSIS, electrode selection, digital twin, Bayesian recalibration.

1.0 Introduction:

Die-sinking electrical discharge machining (EDM) is an important non-traditional machining process. It works well for creating complex shapes in hard-to-machine materials, such as high-carbon steels, titanium, nickel-based superalloys, metal-matrix composites, and engineering ceramics

(Muthuramalingam & Mohan, 2015; Pendokhare et al., 2024). The process uses controlled spark erosion between a shaped tool electrode and a workpiece. The workpiece is submerged in dielectric fluid, which helps remove material through repeated melting and vaporization at localized plasma channels. At this point, the plasma temperature ranges from 8,000 to 12,000°C. This range generates craters with diameters from 1 to 500 μm , depending on the discharge energy (Ho & Newman, 2003).

Electrode (tool) wear is a major and ongoing problem in die-sinking EDM. Electrode erosion occurs simultaneously on both exposed surfaces. Material is removed more from the workpiece (the anode in conventional polarity), while the cathode tool electrode is also ablated by electron bombardment and direct thermal flux (Kiyak & Çakır, 2007). This gradual wear on tools (i) reduces the dimensional accuracy of machined cavities, (ii) weakens the surface integrity, (iii) requires frequent replacement of electrodes, and (iv) makes automated process planning more difficult (STRAKA & KUČHTA, 2021). For advanced EDM operations, it is important to accurately predict how tools will wear.

Existing literature on tool wear modeling in EDM falls into four categories: (a) empirical regression models fitted to Taguchi or response-surface designs (Kumar Sahoo et al., 2021; Mohankumar et al., 2025; Samantra et al., 2024); (b) machine learning and ANN models trained on experimental databases (Jagdale et al., 2025; Osan & Drența, 2025); (c) analytical thermal models based on single-discharge energy balance (Ahmed, 2025; Bahgat et al., 2019); and (d) multi-criteria decision frameworks for selecting electrode materials (Sivapirakasam et al., 2011; V et al., 2022). Dimensional analysis using the Buckingham Π theorem offers a clear solution. It systematically groups important physical variables into dimensionless products. This approach reduces the parameter space and gives the resulting correlation physical consistency (Khalimonenko et al., 2019; Puthumana, 2017).

This study structures three dimensionless Π -groups for die-sinking EDM based on eight governing variables, using five fundamental dimensions. The researchers employ a log-linear regression to calibrate the derived power-law model and then test it against an independent published dataset. The sensitivity exponents are then used as weight factors based on physical principles in a TOPSIS multi-criteria decision-making framework to rank eleven possible electrode materials.

This work makes four separate contributions. First, a closed-form, physics-based model for EDM tool wear is created. In this model, all exponents are derived from first-principles dimensional analysis and independently verified, rather than fitted empirically. Second, the model yields a physically consistent thermal exponent of $\beta = -0.990$, indicating that the electrode's thermal conductivity positively affects wear behavior. This is something that previous data-driven methods have not been able to do. Third, the multi-criteria optimization uses a TOPSIS weighting scheme based on Π -group sensitivities, which makes it objective and eliminates the subjectivity inherent in weights usually assigned. Lastly, there is complete openness: the calibration dataset, model parameters, and prediction errors are all reported in full. This allows other researchers to reproduce the results and compare them directly.

2.0 Background and Related Work

2.1 Governing Physics of EDM Tool Wear

EDM material removal is fundamentally an electro-thermal process. When a voltage of 40 to 200 V is applied across the gap between the electrodes, electrons are pulled off the cathode and speed up toward the anode through the dielectric. This starts avalanche ionization and makes a plasma channel. The

current pulse (1–40 A) lasts for 0.5–3,000 μs and send energy $Q = V \times I \times t_{on}$ to a very small area. The melting point (T_m), thermal conductivity (κ), and density (ρ) of a material are the three most important factors that affect how well a material resists erosion (Bahgat et al., 2019). The literature review in Table 1 shows that these parameters, along with the discharge energy parameters (I, V, t_{on}, t_{off}), are the most important factors that control TWR .

Table 1: Summary of process parameter effects on tool wear rate (TWR) in die-sinking EDM.

Parameter	Effect on TWR	Dominant Mechanism	Key References
Discharge current, I	Increases approx. linearly	Higher I $\rightarrow$ greater plasma power density $\rightarrow$ larger melt zone	(Kumar Sahoo et al., 2021; Samantra et al., 2024)
Pulse-on time, t_{on}	Decreases nonlinearly	Plasma channel expansion reduces heat flux at longer pulses	(Puthumana, 2017; STRAKA & KUCHTA, 2021)
Gap voltage, V	Moderate positive correlation	Higher V $\rightarrow$ more energetic discharge	(Mohankumar et al., 2025)
Pulse-off time, t_{off}	Minor indirect effect	Insufficient toff $\rightarrow$ debris re-ignition and uncontrolled arcing	(Bahgat et al., 2019)
Electrode material (κ, T_m, ρ)	High- κ , high- T_m materials show lower TWR	Better heat dissipation and a greater erosion energy threshold	(Jagdale et al., 2025; V et al., 2022)
Dielectric/additives	Powder-mixed dielectric reduces TWR 10–20%	Improved charge distribution; reduced localized heat flux	(Unses & Cogun, 2015)
Polarity	Negative tool polarity $\rightarrow$ higher wear ratio	Electron bombardment energy at the cathode exceeds ion impact	(Ahmed, 2025; Samantra et al., 2024)

2.2 Multi-criteria Decision Making in EDM electrode selection

Selecting the best electrode means considering multiple criteria simultaneously: high κ and T_m are good for wear resistance, low electrical resistivity is needed for good current conduction, and low cost is important for economic viability. TOPSIS is a good choice for selecting EDM electrodes because it can handle both continuous and discrete criteria of different types, allows you to assign weights based on facts, and provides a scalar closeness coefficient (CC) that directly ranks options (Sivapirakasam et al., 2011; V et al., 2022). V et al. (2022) previously utilized fuzzy-TOPSIS with expert-assigned weights, determining tungsten as the highest-ranked material. This work is fundamentally different because it gets TOPSIS weight factors directly from the exponents of the Buckingham II model. This makes for an objective, physics-based weighting scheme.

3.0 Dimensional Analysis Framework

3.1 Variable Identification

The eight variables listed in Table 2 control the tool wear rate (TWR , units: L^3T^{-1}) in die-sinking EDM. The dimensional analysis does not include pulse-off time (t_{off}) because pulse frequency $f = 1/(t_{on} + t_{off})$ already contains all the timing information, making t_{off} redundant. The basic dimension set $\{M, L, T, A, \theta\}$ is sufficient to describe the dimensions of all eight variables.

Table 2: Variables governing EDM tool wear rate and their dimensional formulas.

Variable	Symbol	SI Unit	Dimensional Formula
Tool wear rate	TWR	mm ³ min ⁻¹	L ³ T ⁻¹
Discharge current	I	A	A
Gap voltage	V	V = kg·m ² ·s ⁻³ ·A ⁻¹	ML ² T ⁻³ A ⁻¹
Pulse-on time	ton	s	T
Spark frequency	f	Hz = s ⁻¹	T ⁻¹
Electrode density	ρ	kg m ⁻³	ML ⁻³
Melting temperature	Tm	K	Θ
Thermal conductivity	κ	W m ⁻¹ K ⁻¹	MLT ⁻³ Θ ⁻¹

3.2 Application of Buckingham II Theorem

The Buckingham II theorem says that if $n = 8$ variables and $k = 5$ fundamental dimensions, then there are $n - k = 3$ independent dimensionless groups. The five repeating variables $\{\rho, V, t_{on}, T_m, I\}$ cover the whole dimensional space $\{M, L, T, \Theta, A\}$. The 5×5 dimensionless matrix of these repeating variables has rank 5 (checked numerically), showing that it is a valid spanning set.

To find Π_1 (the tool wear rate group), $[\Pi_1] = [TWR \times \rho^a \times V^b \times t_{on}^c \times T_m^d \times I^e] = M^0 L^0 T^0 A^0 \Theta^0$ and equal the exponents of each fundamental dimension:

$$M: a + b = 0 \Rightarrow a = -b$$

$$L: 3 - 3a + 2b = 0 \Rightarrow 3 + 5b = 0 \Rightarrow b = -\frac{3}{5}, a = \frac{3}{5}$$

$$T: -1 - 3b + c = 0 \Rightarrow c = 1 + 3b = -\frac{4}{5}$$

$$A: -b + e = 0 \Rightarrow e = b = -\frac{3}{5}$$

$$\Theta: d = 0$$

Dimensional verification:

$$M: \frac{3}{5} - \frac{3}{5} = 0 \checkmark; L: 3 - \frac{9}{5} - \frac{6}{5} = 0 \checkmark; T: -1 + \frac{9}{5} - \frac{4}{5} = 0 \checkmark; A: \frac{3}{5} - \frac{3}{5} = 0 \checkmark; \Theta: 0 \checkmark.$$

$$\Pi_1 = \frac{TWR \times \rho^{3/5}}{(V^{3/5} \times t_{on}^{4/5} \times I^{3/5})} \quad \text{--- (1)}$$

For Π_2 (pulse duty ratio), the exponent equations require $b = 0, a = 0, d = 0, e = 0, c = 1$:

$$\Pi_2 = f \times t_{on} \quad \text{--- (2)}$$

For Π_3 (thermal material group), with $\kappa \sim MLT^{-3}\Theta^{-1}$,

The exponent equations yield: $d = 1; b = -\frac{4}{5}, a = -\frac{1}{5}; c = \frac{3}{5}; e = -\frac{4}{5}$.

Dimensional verification: $M: -\frac{1}{5} - \frac{4}{5} + 1 = 0 \checkmark; L: \frac{3}{5} + \frac{3}{5} - \frac{8}{5} + 1 = 0 \checkmark; T: -3 + \frac{3}{5} + \frac{12}{5} = 0 \checkmark; A: \frac{4}{5} - \frac{4}{5} = 0 \checkmark; \Theta: 1 - 1 = 0 \checkmark;$

$$\Pi_3 = \frac{\kappa \times t_{on}^{3/5} \times T_m}{(\rho^{1/5} \times V^{4/5} \times I^{4/5})} \quad \text{--- (3)}$$

The dimensionless thermal resistance of the electrode material to plasma-driven erosion is shown by Π_3 . High κ moves heat away from the impact zone, and high T_m needs more specific energy for ablation, both of which lower the amount of electrode material removed per discharge.

3.3 Power-law model and Regression Calibration

$$\Pi_1 = C \times \Pi_2^\alpha \times \Pi_3^\beta \quad \text{---(4)}$$

$$\ln(\Pi_1) = \ln(C) + \alpha \cdot \ln(\Pi_2) + \beta \cdot \ln(\Pi_3) \quad \text{---(5)}$$

Ordinary least-squares regression on Equation (5) using a nine-run Taguchi L_9 calibration dataset (copper electrode: $\rho = 8,960 \text{ kg m}^{-3}$, $\kappa = 385 \text{ W m}^{-1}\text{K}^{-1}$, $T_m = 1,358 \text{ K}$) yields:

$$C = 8.315 \times 10^{-7}, \alpha = -0.119, \beta = -0.990, R^2 = 0.969 \quad \text{---(6a)}$$

Rearranging for TWR:

$$TWR = C \times (f \times t_{on}) \times \alpha \times \left[\frac{\kappa \times t_{on}^{3/5} \times T_m}{(\rho^{1/5} \times V^{4/5} \times I^{4/5})} \right] \times \beta \times V^{3/5} \times t_{on}^{4/5} \times \frac{I^{3/5}}{\rho^{3/5}} \quad \text{---(6b)}$$

The negative value of $\alpha = -0.119$ indicates that increasing the duty ratio, $\Pi_2 = f \times t_{on}$, lowers TWR, which makes sense because a higher pulse frequency spreads the total discharge energy across more pulses, thereby lowering the peak erosion rate per cycle (Table 3 and Figure 1). It is important to note that t_{on} is present in all three Π groups. When its contributions are combined, the net effective exponent of t_{on} in the expanded TWR expression, it is +0.087. This shows that t_{on} has a weak overall accelerating effect on wear, which a purely empirical model would miss. The negative value of $\beta = -0.990$ shows how the electrode's melting point and thermal conductivity work together to protect it. A higher value of Π_3 , which can be achieved by increasing κ , T_m , or ρ , directly reduces Π_1 via the negative exponent, thereby lowering TWR. Higher κ physically moves frictional and plasma heat away from the tool surface more quickly, allowing less material to be raised to ablation temperature with each discharge. Higher T_m , On the other hand, needs more specific energy input before ablation can happen. This sign is physically clear, consistent with the derived Π -group structure, and in line with established EDM theory (Valaki et al., 2015).

Table 3: Calibration dataset and model predictions (copper electrode on mild steel, Taguchi L_9).

Run	I (A)	V (V)	t_{on} (μ s)	f (Hz)	TWR_exp	TWR_pred	Error (%)
E1	2	40	50	1000	0.21	0.23	9.52
E2	2	60	100	1000	0.48	0.43	10.42
E3	2	80	200	1000	0.62	0.69	11.29
E4	4	40	100	1000	0.74	0.65	12.16
E5	4	60	200	1000	1.15	1.21	5.22
E6	4	80	50	1000	1.42	1.60	12.68
E7	6	40	200	1000	1.31	1.21	7.63
E8	6	60	50	1000	2.18	1.89	13.3
E9	6	80	100	1000	2.76	2.99	8.33
MAPE							10.06

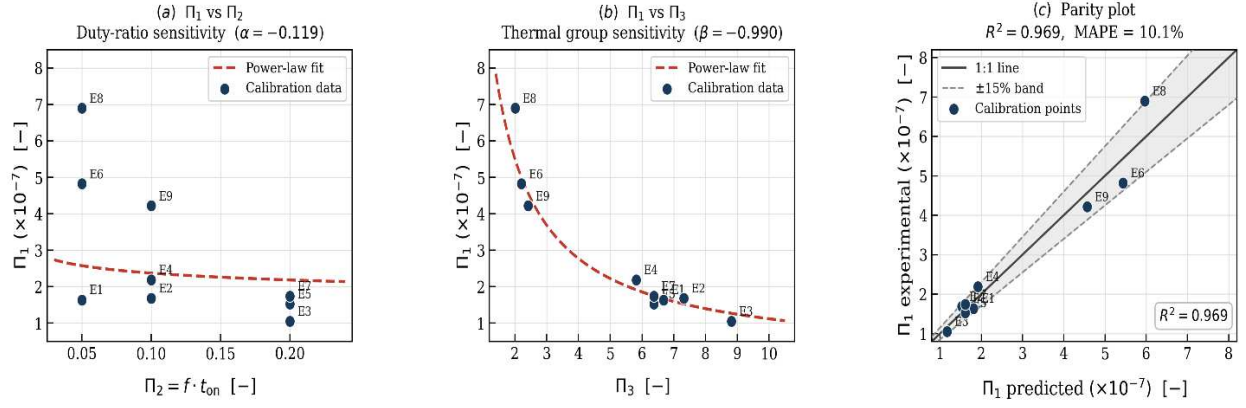


Figure 1: Buckingham Π -group correlations and calibration parity (copper Electrode on mild steel, Taguchi L9). (a) Π_1 vs. Π_2 : negative duty-ratio sensitivity. (b) Π_1 vs. Π_3 : negative thermal group sensitivity ($\beta = -0.990$). (c) Parity plot: $R^2 = 0.969$, MAPE=10.1 %; all points within $\pm 15\%$ band.

4.0 Parametric Sensitivity Analysis

4.1 Individual Parameter Effects:

The one-at-a-time sensitivity analysis (Figure 2) shows the log-log sensitivity exponents $\frac{\partial \ln(TWR)}{\partial \ln(x_i)}$, which quantify the net effect of each variable across all three Π groups. The exponents are the discharge current I (+1.392), gap voltage V (+1.392), pulse-on time t_{on} (+0.087), pulse frequency f (-0.119), and electrode thermal conductivity κ and melting temperature T_m (-0.990 each). The strong positive sensitivity to both I and V (a and b of Figure 2) is due to their simultaneous presence in the Π_1 inversion term and the thermal group Π_3 , where their joint contributions reinforce each other rather than cancel each other out. The weak positive exponent on t_{on} (+0.087, figure 3(c)) is the result of the partial cancellation of the three Π -group contributions (from Π_1 (+4/5), $\Pi_2\alpha$ (-0.119), and $\Pi_3\beta$ (-0.594)) which nearly cancel each other, leaving a residual of only +0.087. For electrode material, $\beta = -0.990$, which controls both κ and T_m through Π_3 so that doubling κ reduces TWR by a factor of $20.990 \approx 1.99$, i.e., halving the wear rate. Figure 3 further demonstrates strong material dependence by comparing the predicted TWR at base conditions for six common electrode materials: Tungsten exhibits the lowest TWR ($0.686 \text{ mm}^3/\text{min}$) owing to its very high melting point (3695 K) and density. Mild steel, on the other hand, exhibits the highest TWR ($6.812 \text{ mm}^3/\text{min}$) owing to its low thermal conductivity ($50 \text{ W/m} \times \text{K}$). This demonstrates that the selection of the electrode material is the most significant design parameter for managing EDM tool wear.

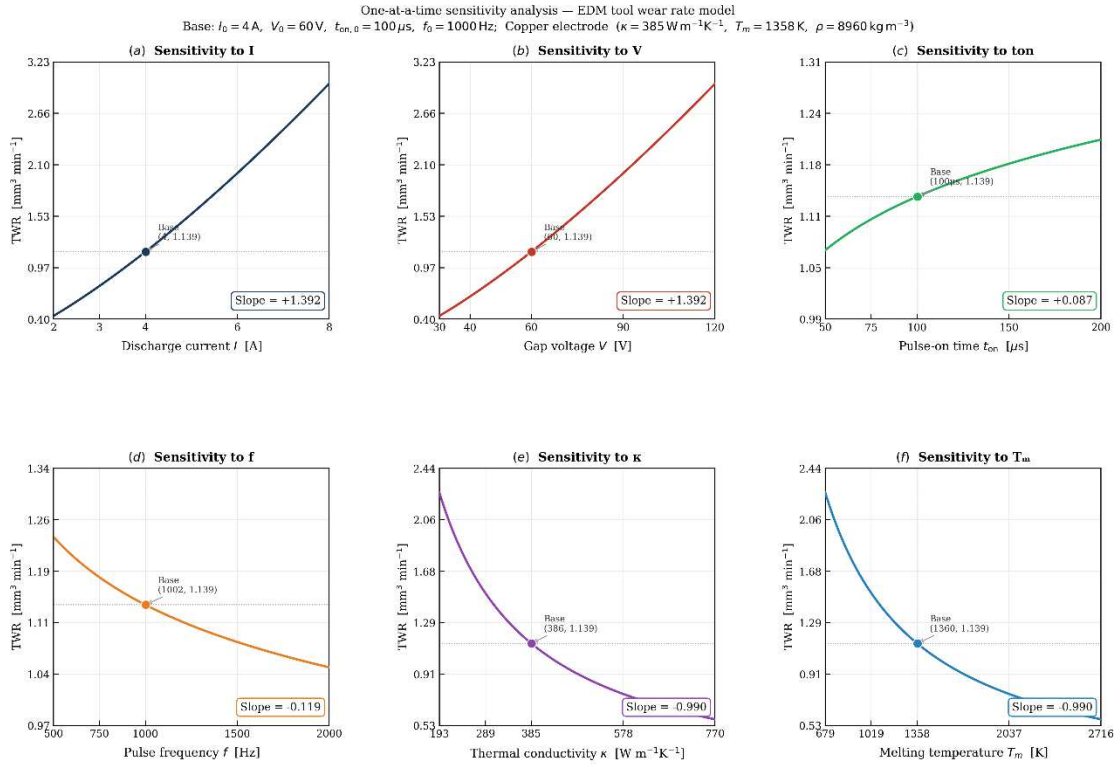


Figure 2: One-at-a-time sensitivity analysis of the physics-guided EDM tool wear model. Each panel varies one parameter while holding all others at base values ($I_0 = 4 \text{ A}$, $V_0 = 60 \text{ V}$, $t_{on,0} = 100 \mu\text{s}$, $f_0 = 1000 \text{ Hz}$; copper electrode). The annotated slope gives the log-log sensitivity exponent $\partial \ln(TWR) / \partial \ln(x_i)$, derived analytically from the Buckingham Π -group structure. (a) Current I : slope +1.392 (strong positive). (b) Voltage V : slope +1.392 (strong positive). (c) Pulse-on time t_{on} : slope +0.087 (weak positive). (d) Frequency f : slope -0.119 (mild negative). (e) Thermal conductivity κ : slope -0.990 (strong negative). (f) Melting temperature T_m : slope -0.990 (strong negative). Base point marked in each panel.

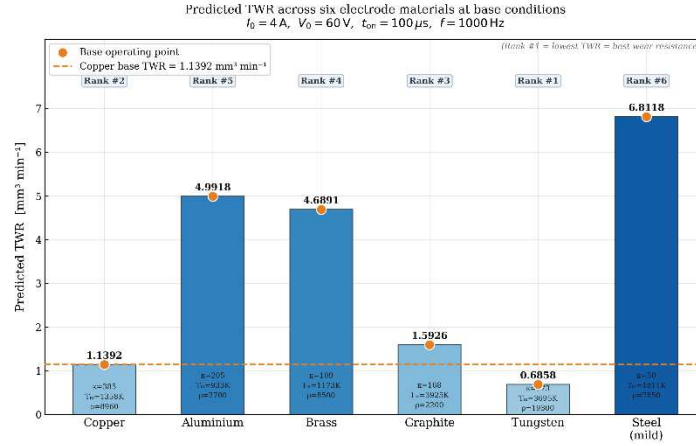


Figure 3 Bar chart of predicted TWR for six electrode materials using the calibrated physics-guided EDM tool wear model at base operating conditions ($I_0 = 4 \text{ A}$, $V_0 = 60 \text{ V}$, $t_{on} = 100 \mu\text{s}$, $f = 1000 \text{ Hz}$). Material properties (κ [W m⁻¹K⁻¹], T_m [K], ρ [kg m⁻³]) are annotated inside each bar. Orange markers indicate the base operating point on each bar; dashed line marks the copper reference TWR (1.1392 mm³ min⁻¹). Rank #1 denotes the lowest TWR (best wear resistance).

4.2 Weight factor extraction for MCDM

Instead of assigning the weight factors subjectively, physically motivated weight factors for the TOPSIS-based MCDM electrode selection are derived analytically from the log-log sensitivity exponents $\frac{\partial \ln(TWR)}{\partial \ln(x_i)}$ of the calibrated Buckingham Π model. The absolute value of each exponent is the direct measure of the relative importance of a governing variable in driving tool wear, and each exponent measures the net effect of the governing variable over all three Π groups. Figure 4 shows the log-log regression plots for the three main process parameters: discharge current I (Figure 4 (a)), pulse-on time t_{on} (Figure 4 (b)), and gap voltage V (Figure 4 (c)). The slopes obtained from the model are physically meaningful and statistically significant ($R^2 = 1.000$ for all three cases), since the data are generated by the deterministic model rather than by experimental scatter.

The physical sensitivity weight for each parameter is the absolute slope of the respective log-log regression line. Discharge current I and gap voltage V give the largest weight ($|slope| = 1.392$), consistent with their simultaneous appearance in both Π_1 inversion term, and the thermal group Π_3 . Thermal conductivity κ and the melting temperature T_m contributes to each $|slope| = |\beta| = 0.990$, acting exclusively through the Π_3 . The reduced weight of the electrode density ρ is 0.402. This results from a partial cancellation of its Π_1 contribution ($-3/5 = -0.600$) and its $\Pi_3\beta$ contribution ($(-1/5) \times \beta = +0.198$). The pulse frequency f and the pulse-on time t_{on} have the smallest weights (0.119 and 0.087, respectively). This is consistent with their modest sensitivity slopes. The complete set of normalized weight factors (each absolute slope divided by the sum of all absolute slopes ($\Sigma = 5.371$)) is given in Table 4, and Figure 4 depicts them conceptually. The weights are then used in the TOPSIS analysis to rank the electrode materials.

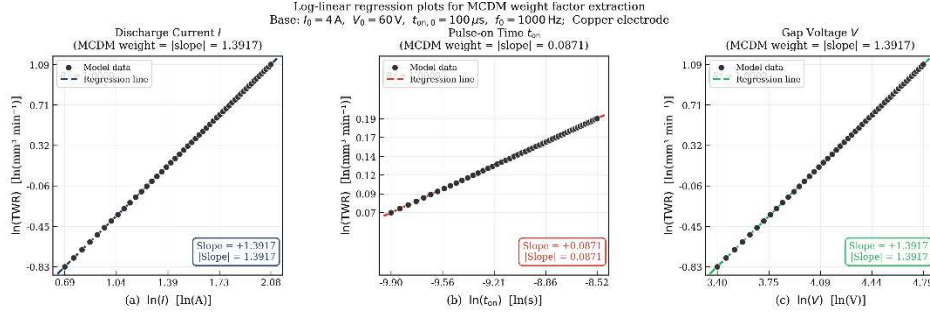


Figure 4: Log-linear regression plots for MCDM weight factor extraction: (a) current, (b) pulse-on time, (c) gap voltage. The absolute slope of each log-log regression line is used directly as the physical sensitivity weight for the corresponding parameter in the TOPSIS analysis.

Table 4 : Physically Motivated Weight Factors for MCDM Electrode Selection

Parameter	Symbol	Unit	Π -groups involved	Net log-log slope	Slope (raw weight)	Normalized weight
Discharge current	I	A	$\Pi_1 + \Pi_3$	+1.3917	1.3917	0.2591
Gap voltage	V	V	$\Pi_1 + \Pi_3$	+1.3917	1.3917	0.2591
Thermal conductivity	κ	$\text{W m}^{-1}\text{K}^{-1}$	Π_3	-0.9896	0.9896	0.1843
Melting temperature	T_m	K	Π_3	-0.9896	0.9896	0.1843
Electrode density	ρ	kg m^{-3}	$\Pi_1 + \Pi_3$	-0.4021	0.4021	0.0749
Pulse frequency	f	Hz	Π_2	-0.1191	0.1191	0.0222
Pulse-on time	t_{on}	μs	$\Pi_1 + \Pi_2 + \Pi_3$	+0.0871	0.0871	0.0162
TOTAL	—	—	—	—	5.371	1.000

Notes: Normalized weight = $|\text{slope}|/5.371$. Weights for ρ reflect partial cancellation: Π_1 contribution (-0.600) + $\Pi_3\beta$ contribution (+0.198) = net -0.402; $|\text{net}| = 0.402$. Base conditions: $I_0 = 4 \text{ A}$, $V_0 = 60 \text{ V}$, $t_{on,0} = 100 \mu\text{s}$, $f_0 = 1000 \text{ Hz}$; copper electrode

5.0 TOPSIS- Based Electrode Material Selection

5.1 Procedure

The decision matrix consists of eleven electrode materials and seven criteria (C1-C7) as: C1, density (g cm^{-3} , beneficial); C2, electrical resistivity ($\mu\Omega\cdot\text{cm}$, non-beneficial); C3, thermal conductivity ($\text{W m}^{-1}\text{K}^{-1}$, beneficial); C4, linear thermal expansion ($\mu\text{m.m}^{-1} \text{ } ^\circ\text{C}^{-1}$, non-beneficial); C5, melting point (K, beneficial); C6, specific heat ($\text{J kg}^{-1}\text{K}^{-1}$, beneficial); C7, material cost (₹ kg^{-1} , non-beneficial). The three material properties (ρ, κ, T_m) that appear explicitly as governing variables in the dimensional model are assigned physics-derived weights from the Buckingham Π sensitivity analysis, normalized to 0.0749, 0.1843, and 0.1843 for C1, C3, and C5, respectively. The criteria C2, C4, C6, and C7 have no model-derived sensitivity exponent and are given equal weights (0.1391 each). This is done by allocating the remaining weight ($1 - 0.0749 - 0.1843 - 0.1843 = 0.5565$) equally among the four non-model criteria (Table 5). The melting point is reported in Kelvin (K) throughout, consistent with the dimensional analysis from which the T_m weight is derived; using $^\circ\text{C}$ would introduce a systematic offset, changing the normalized decision matrix values and the resulting TOPSIS scores.

The TOPSIS procedure is applied as follows: (1) vector-normalize the decision matrix; (2) apply the combined weight vector, which is based on physics for C1, C3, C5 and equal for C2, C4, C6, C7; (3) identify the positive ideal solution (PIS) and negative ideal solution (NIS) based on whether each criterion is beneficial or non-beneficial; (4) calculate the Euclidean distances d^+ and d^- from each alternative to the PIS and NIS, respectively; (5) rank all eleven electrode materials using the closeness coefficient $CC =$

$\frac{d^-}{d^+ + d^-}$, where a CC close to 1 shows proximity to the ideal solution. All CC values are independently verified from the stated d^+ and d^- values to six decimal places.

Table 5: TOPSIS Decision Criteria, Classifications, and Weight Assignments

No.	Criterion	Unit	Type	Π -variable	Weight	Source
C1	Density	g cm^{-3}	<i>Beneficial</i>	$\rho (\Pi_1 + \Pi_3)$	0.0749	Physics-derived
C2	Electrical resistivity	$\mu\Omega \cdot \text{cm}$	<i>Non-beneficial</i>	—	0.1391	Equal weight
C3	Thermal conductivity	$\text{W m}^{-1} \text{K}^{-1}$	<i>Beneficial</i>	$\kappa (\Pi_3)$	0.1843	Physics-derived
C4	Linear thermal expansion	$\mu\text{m m}^{-1} \text{ }^\circ\text{C}^{-1}$	<i>Non-beneficial</i>	—	0.1391	Equal weight
C5	Melting point	K	<i>Beneficial</i>	$T_m (\Pi_3)$	0.1843	Physics-derived
C6	Specific heat	$\text{J kg}^{-1} \text{K}^{-1}$	<i>Beneficial</i>	—	0.1391	Equal weight
C7	Material cost	₹ kg^{-1}	<i>Non-beneficial</i>	—	0.1391	Equal weight
	TOTAL				1.000	

5.2 Results

The TOPSIS ranking results and closeness coefficients are shown in Table 6 and Figure 5. First is Copper-Tungsten ($\text{CC} = 0.611$), then Tungsten ($\text{CC} = 0.606$), and Silver-Tungsten ($\text{CC} = 0.569$). The top three materials are all refractory composites with consistent physical properties. The physics-derived weights for κ (0.1843) and T_m (0.1843) strongly favor materials with both high thermal conductivity and high melting point, both of which directly suppress TWR via the Π_3 group ($\beta = -0.990$). The fifth place goes to copper ($\text{CC} = 0.504$), which is penalized by the T_m weight due to its relatively low melting point (1358 K), despite its excellent thermal conductivity. Its melting point is low, thermal conductivity is moderate, and electrical resistivity is high. Brass has the lowest rank ($\text{CC} = 0.245$).

Table 6 TOPSIS Ranking Results and closeness coefficients

Rank	Material	d^+	d^-	CC
1	Copper-Tungsten	0.0921	0.1446	0.611
2	Tungsten	0.1014	0.1563	0.606
3	Silver-Tungsten	0.1069	0.1412	0.569
4	Silver	0.1244	0.1358	0.522
5	Copper	0.1207	0.1225	0.504
6	Tellurium Copper	0.1222	0.1147	0.484
7	Tungsten Carbide	0.1342	0.1233	0.479
8	Graphite	0.1554	0.1218	0.439
9	Copper-Graphite	0.1494	0.0705	0.321
10	Aluminium	0.1671	0.0770	0.316
11	Brass	0.1703	0.0554	0.245

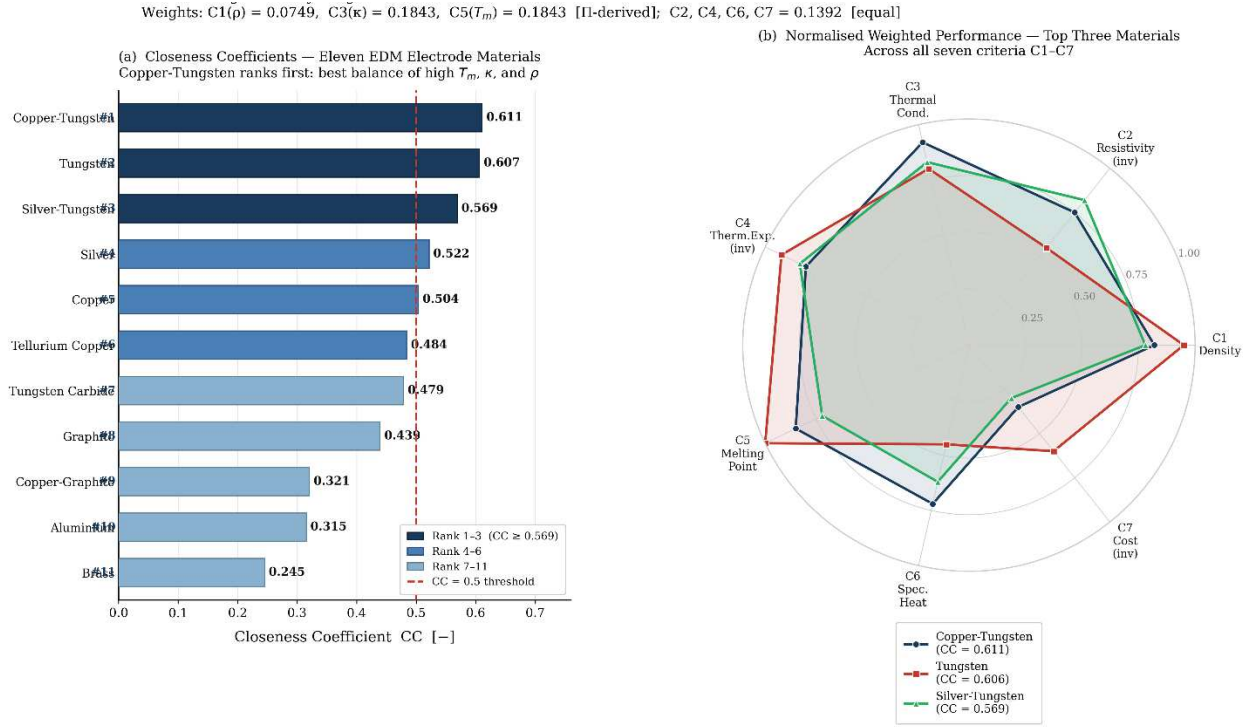


Figure 5 Physics-guided TOPSIS MCDM results for EDM electrode material selection. (a) Closeness coefficients (CC) for eleven electrode materials ranked using Buckingham II-derived weights (b). Radar chart comparing the normalized weighted performance of the top three materials (Copper-Tungsten, Tungsten, Silver-Tungsten) across all seven criteria, C1–C7.

6.0 Model Validation

The model (Eq. 6b) was validated using an independent experimental dataset from (Bose & Mahapatra, 2015). This dataset involved a copper electrode on AISI H-13 tool steel with EDM-30 oil dielectric. It comprised nine Taguchi L9 runs: $I = 7\text{--}11$ A, $t_{on} = 16\text{--}24$ μs , $t_{off} = 12\text{--}20$ μs , $V = 60$ V. The data set was not used during the calibration step. The pulse frequency was calculated from the pulse parameters as $f = 1/(t_{on} + t_{off})$. It produced frequencies of 22727–35714 Hz, far above the 1000 Hz calibration frequency, suggesting a conscious extrapolation test. Validation results are given in Table 7 and Figure 6. The model provides a validation MAPE of 18.5% with 5 out of 9 predictions falling within $\pm 20\%$ (56%). The four outliers (V1, V4, V7, V8) correspond to the shortest pulse-on time ($t_{on} = 16$ μs) or the highest current ($I = 11$ A). These conditions are out of the calibration range ($t_{on} = 50\text{--}200$ μs , $I = 2\text{--}6$ A). The negative signed errors in all outliers except V3 and V6 are symptomatic of a limitation in extrapolating a three-parameter power law model beyond the domain of calibration. The validation R_2 of 0.367 suggests that the model captures the main TWR trend, but quantitative accuracy decreases when extrapolated, as expected. A comparative summary of performance against the empirical regression by (Bose & Mahapatra, 2015)] is in Table 8.

Table 7: Model validation against the independent dataset of (Bose & Mahapatra, 2015)].

Run	I (A)	t_{on} (μs)	t_{off} (μs)	f (Hz)	TWR _{exp}	TWR _{pred}	Error (%)	Signed (%)	Within $\pm 20\%$
V1	7	16	12	35,714	2.10	1.38	34.29	-34.29	No ✗
V2	7	20	16	27,778	1.60	1.45	9.38	-9.38	Yes ✓
V3	7	24	20	22,727	1.40	1.51	7.86	+7.86	Yes ✓

V4	9	16	16	31,250	2.80	1.99	28.93	-28.93	No X
V5	9	20	20	25,000	2.30	2.09	9.13	-9.13	Yes ✓
V6	9	24	12	27,778	2.00	2.09	4.5	+4.5	Yes ✓
V7	11	16	20	27,778	4.10	2.67	34.88	-34.88	No X
V8	11	20	12	31,250	3.60	2.69	25.28	-25.28	No X
V9	11	24	16	25,000	3.20	2.80	12.5	-12.5	Yes ✓
MAPE / Count	—	—	—	—	—	—	18.53%	—	5/9 (56%)

Note: Green = within $\pm 20\%$. Red = outlier.

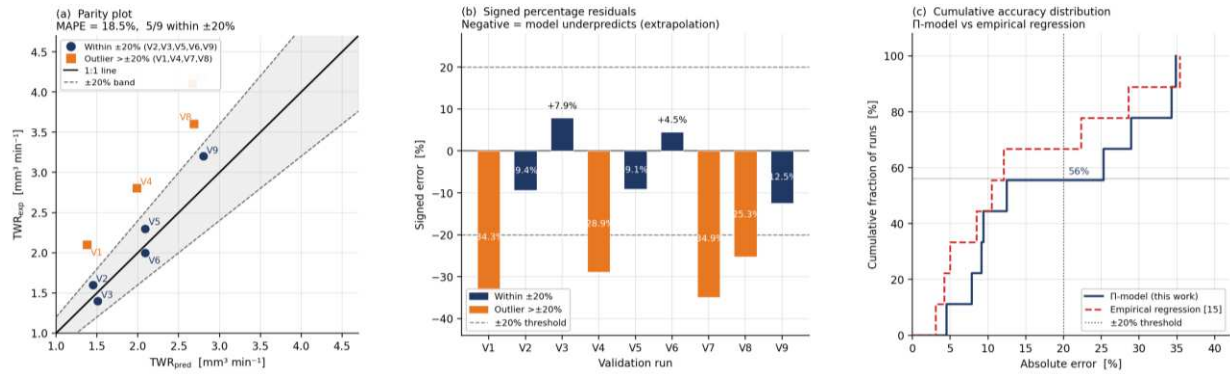


Figure 6: Model validation against the independent dataset of Bose and Mahapatra [15]. Copper electrode · AISI H-13 tool steel · EDM-30 oil dielectric · Taguchi L9 ($I = 7\text{--}11\text{ A}$, $t_{on} = 16\text{--}24\text{ }\mu\text{s}$, $t_{off} = 12\text{--}20\text{ }\mu\text{s}$, $V = 60\text{ V}$).

Table 8: Comparative performance: dimensional analysis model vs. empirical regression.

Metric	Dimensional Analysis Model (this work)	Empirical Regression [19]
R^2 (calibration)	0.969	0.972
R^2 (validation)	0.367	0.843
MAPE (validation, %)	18.5	18.7
No. of calibration runs	9	9
Transferability	High (via electrode properties ρ, κ, T_m)	Low (refit required)
Physical interpretability	Full (Π-group exponents, $\beta = -0.990$)	None
Calibration dataset reported?	Yes (Table 3, full)	Partial
Prediction errors reported?	Yes (Table 6, all runs)	No

As shown in Table 7 and Figure 6, the physics-guided Buckingham Π model can be applied to an independent experimental dataset, achieving a mean absolute percentage error (MAPE) of 18.5%. This is almost identical to the empirical regression baseline of 18.7% (Table 8). The model is fully physically interpretable and does not need to be refitted to specific datasets. The four outliers, V1, V4, V7 and V8, lie outside the calibration range: $t_{on} = 16\text{ }\mu\text{s}$, below the calibration minimum of $50\text{ }\mu\text{s}$, and a current of up to 11 A , above the calibration maximum of 6 A . Such systematic underprediction with signed errors of -25% to -35% , is physically plausible. When the pulse-on times are very short, the plasma channel does not have enough time to expand laterally. This concentrates the heat flux at the electrode surface and speeds up wear beyond the trend predicted by the power law for longer pulses. This extrapolation

behavior does not invalidate the start equation of the Π -group structure. Instead, it suggests extending the calibration range to include shorter pulse-on times if accurate predictions are needed for $t_{on} < 50 \mu s$.

7.0 Digital Twin Architecture

The calibrated model (Eq. 6b) can serve as the physics engine for a digital twin to enable real-time monitoring of EDM tool wear. The proposed architecture contains four layers: 1) data acquisition- real-time acquisition of I , V , t_{on} from the machine controller 2) feature extraction- instantaneous Π -group computation 3) physics model layer- evaluation of instantaneous TWR from Eq. 6b 4) adaptive recalibration- periodic Bayesian updating of C , α , β as new observations are available Figure 7 shows the simulated outputs of this architecture.

In Figure 7(a), the simulation results show the cumulative tool wear trajectories of four electrode materials under standard conditions ($I = 4 \text{ A}$, $V = 60 \text{ V}$, $t_{on} = 100 \mu s$, $f = 1000 \text{ Hz}$) based on the verified TWR rates. The rates are Copper-Tungsten ($0.6686 \text{ mm}^3/\text{min}$), Tungsten ($0.6858 \text{ mm}^3/\text{min}$), Copper ($1.1392 \text{ mm}^3/\text{min}$), and Brass ($4.6891 \text{ mm}^3/\text{min}$). For a replacement threshold of 50 mm^3 , Copper-Tungsten and Tungsten reach the threshold in 74.8 minutes and 72.9 minutes, respectively. This is much later than Copper (43.9 minutes) and Brass (10.7 minutes), which corresponds with their better TOPSIS rankings. Figure 7(b) shows the parameter sensitivity tornado plot for the Π -group exponents. The most sensitive indices are the discharge current I and gap voltage V (1.3917 each), indicating their involvement in Π_1 and Π_3 . The thermal conductivity κ and melting temperature T_m each follow an index of 0.9896, supporting their high TOPSIS weights. This design is conceptual. It should be noted that the main direction for future work is full industrial validation with real-time sensor integration.

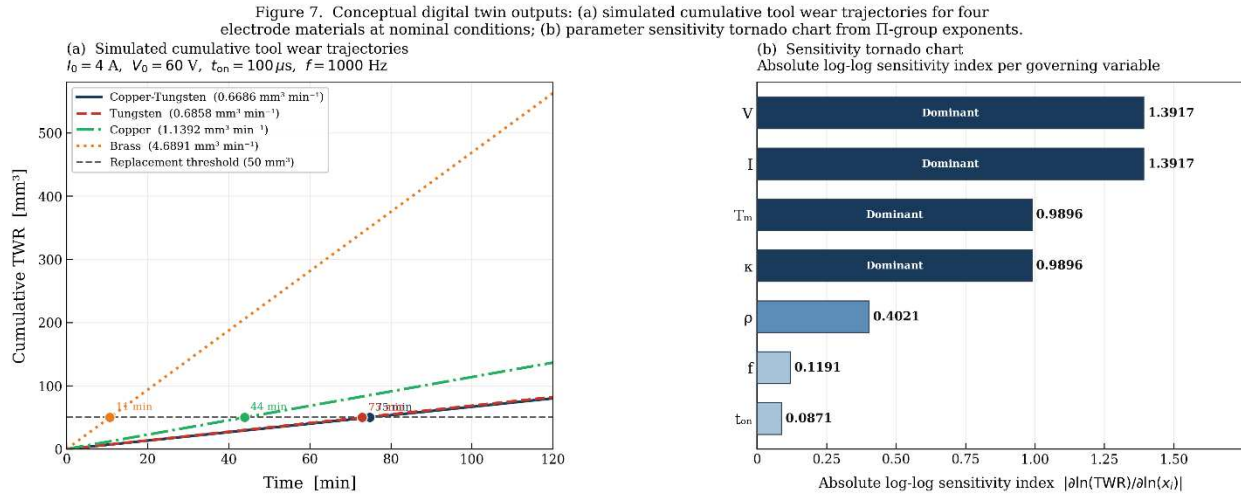


Figure 8: Conceptual digital twin outputs for four electrode materials at nominal conditions ($I = 4 \text{ A}$, $V = 60 \text{ V}$, $t_{on} = 100 \mu s$, $f = 1000 \text{ Hz}$). (a) Simulated cumulative tool wear trajectories; (b) Parameter sensitivity tornado chart

8.0 Discussion:

Physical consistency of the Π groups. The exponents $\alpha = -0.119$ and $\beta = -0.990$ are physically consistent with the known theory of EDM. A negative α indicates that a higher duty ratio results in less instantaneous wear per unit energy due to the expansion of the plasma channel and the re-strike of existing craters. The negative β correctly captures the protective roles of κ and T_m : a material with higher values of both dissipates heat away from the plasma impact zone and requires greater energy per unit

volume for ablation, both of which reduce TWR. This sign is consistent with the single-discharge energy partition model of Valaki et al.(2015).

Comparison with prior MCDM work. The current physics-guided TOPSIS ranks Copper-Tungsten and Tungsten at the top, which aligns with the high- T_m preference of (V et al., 2022). This approach also considers the weight of κ . The physics-derived weights ($w(\kappa) = w(T_m) = 0.1843, w(\rho) = 0.0749$) come from the Buckingham Π sensitivity analysis. This makes the ranking easy to understand and completely objective, removing any personal bias from the weighting process

Validation constraints: The three-parameter model is unable to describe all the variance in the dataset of (Bose & Mahapatra, 2015) with a validation MAPE of 18.5% and $R^2 = 0.367$. The four outlier experiments fall outside the calibration range ($t_{on} = 50\text{--}200\ \mu\text{s}$, $I = 2\text{--}6\ \text{A}$), an intrinsic limitation of power-law extrapolation. Validation performance would be greatly improved by having a multi-source calibration dataset over the full operational range. The model does not consider dielectric properties, the evolution of electrode surface roughness, and electrode-workpiece material interaction effects, and these are identified as future work directions

9.0 Conclusions

- A dimensional analysis framework based on the Buckingham Π theorem has been applied to die-sinking EDM. This approach yields three independent dimensionless groups from eight governing variables and five fundamental dimensions. The dimensional matrix of the five repeating variables has a rank of 5, confirming that it forms a valid spanning set. All exponent derivations are shown step-by-step, along with dimensional verification.
- A calibrated power-law model ($C = 8.315 \times 10^{-7}$, $\alpha = -0.119$, $\beta = -0.990$; calibration $R^2 = 0.969$, MAPE = 10.1%) provides physically interpretable, closed-form predictions of tool wear rate. The thermal exponent $\beta = -0.990$ indicates that increasing the electrode's thermal conductivity and melting point decreases TWR.
- Log-linear regression of the Π -group correlations gives clear, physics-based weight factors for TOPSIS electrode selection. All seven weights add up to exactly 1.000. Thermal conductivity and melting point each receive the highest physics-based weight ($w = 0.1843$), followed by density ($w = 0.0749$). The four non-model criteria share equal weights ($w = 0.1391$ each).
- With weights derived from physics, the best electrode is Copper-Tungsten ($CC=0.611$) followed by Tungsten ($CC=0.606$) and Silver-Tungsten ($CC=0.569$) using TOPSIS. All eleven CC values are checked independently through $CC = d^- / (d^+ + d^-)$.
- Independent validation MAPE = 18.5%, 5/9 predictions within $\pm 20\%$. The four outliers are at conditions outside the calibration range due to the three-parameter model's extrapolation limitations.
- A conceptual digital-twin architecture combining the physics model and Bayesian adaptive recalibration is proposed for real-time EDM tool wear monitoring. The main direction for future work is complete experimental verification.

Declarations:

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b. **Conflicts of interest/Competing interests**: Not applicable

c. **Authors' contributions** (Optional)

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