

# Molybdenum Scarcity in Copper-Main-Product Mines: Continuous-Time Theory and Evidence from Chile

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## Research Article

**Keywords:** copper, joint production, marginal yield, mine depletion, molybdenum, scarcity rent

**Posted Date:** August 27th, 2026

**DOI:** <https://doi.org/10.21203/rs.3.rs-10690259/v1>

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**Additional Declarations:** No competing interests reported.

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# Abstract

Molybdenum is commonly recovered from porphyry copper-molybdenum and copper-molybdenum-gold deposits in which copper generally pays most common mining and processing costs. This paper develops a continuous-time framework that extends joint-resource scarcity analysis to heterogeneous ore domains. It separates domain-specific stock rents from common-capacity rent and derives the exact links between the marginal efficient Mo/Cu yield, current mine-stream exposure, cumulative mine-life exposure, and company or national portfolio exposure. The paper also proposes the base-normalized Relative Scarcity Index from Marginal Yields (RSI-MY), defined as the initial marginal Mo/Cu yield divided by its current value. An RSI-MY above one indicates that molybdenum has become scarcer relative to copper through the marginal-yield channel; a value below one indicates an improvement in marginal molybdenum yield and therefore lower relative scarcity through that channel. A solved two-domain benchmark and a discrete Bellman approximation verify the analytical results. Chilean production data illustrate the aggregation mechanisms, while Los Pelambres Q1 2024 and Q1 2025 grades and recoveries yield an RSI-MY decline from 1.000 to 0.894. This means that the marginal-yield component of molybdenum scarcity fell by 10.6% relative to copper; it does not establish that absolute molybdenum scarcity fell, because the active ore rent remains unobserved. The framework identifies a new measurable bridge between heterogeneous mine depletion and observed companion-metal supply.

**JEL classification:** D24; L72; Q31; Q32; Q38

## 1. Introduction

Molybdenum occupies an unusual place in mineral supply. It is indispensable in alloy steels, stainless steels, cast irons, superalloys, lubricants, catalysts, and chemical applications, but much of its mine supply comes from large porphyry Cu-Mo and Cu-Mo-Au systems. These deposits are important because geological processes co-concentrated molybdenite with copper mineralization. The two elements are not brought together by the mine or plant. In many operations, however, copper is the main product and its revenue pays most common access, extraction, comminution, concentration, tailings, water, and site-infrastructure costs. Once the common ore has been mined and milled, product-specific separation and recovery decisions determine how much contained molybdenum becomes saleable output. The geological association explains why copper and molybdenum occur together, while copper's economic dominance explains why the shared mine-plant system is developed and operated (John and Taylor 2016; Sillitoe 2010).

Chile makes the problem economically material rather than merely conceptual. In the internally consistent COCHILCO/World Metal Statistics series used here (Chilean Copper Commission 2026b), Chile produced 43.4 kt of molybdenum in 2025, about 16.7% of the reported world total of 260.5 kt, ranking second after China. Over 2015–2025, its share ranged from 15.1% in 2024 to 22.1% in 2018. Other sources use different coverage and revisions: the U.S. Geological Survey placed Chile third in 2024 at approximately 15% of world production (US Geological Survey 2026), while the International

Molybdenum Association (IMOA) reports a higher global total (International Molybdenum Association 2026). The paper therefore preserves source consistency rather than combining incompatible series. Figure 1 introduces Chile's current market position and its production and share trends relative to the world from 2015 to 2025.

Chile's long record contains the variation a useful scarcity theory must explain. Molybdenum output expanded strongly from 2000 to the middle of the 2010s, contracted materially between 2015 and 2024 while copper output changed far less, and rebounded in 2025. At individual operations, copper grade, molybdenum grade, recovery, hardness, throughput, and mine phase can move in different directions. A mine can therefore produce less copper and more molybdenum, or maintain copper while molybdenum declines. These observations reject a fixed-coefficient account of companion supply, but they do not by themselves measure economic scarcity.

Scarcity is the paper's central object. In the opportunity-cost tradition, in-situ scarcity is not synonymous with declining production, lower grade, a reserve-to-production ratio, or a short-run market deficit. It is the present value of the current and future sacrifices caused by removing one additional unit from a finite stock. Hotelling (1931) formalized this opportunity cost for a homogeneous resource. Pindyck (1982) showed that resources produced jointly from a composite ore share an ore-level scarcity mechanism and that the rent of one constituent depends on the efficient marginal amount of ore needed to obtain an additional unit. Campbell (1985) supplied a complementary industry insight: a metal is a by-product when its value does not change the common production decision, a co-product when it does, and a main product when it can support that decision alone.

The unresolved mining problem is heterogeneity. Copper and molybdenum grades and recoveries are not fixed proportions throughout a porphyry deposit. They can reflect different mineralizing pulses, alteration domains, vein generations, depth intervals, hardness classes, and process responses (Ayuso et al. 2010; John and Taylor 2016; Sillitoe 2010; Romero et al. 2011; Cernuschi et al. 2023). Joint block-support simulation is used because Cu and Mo must be represented as a multivariate distribution rather than a fixed ratio (Hosseini et al. 2017). The mine also chooses a space-time path through the geology. As emphasized in Guzmán (2019), the economic mine is neither a purely temporal homogeneous stock nor a purely spatial block model. Its decisions are jointly spatial and intertemporal. Consequently, the next efficient tonne can have a different Mo/Cu yield from both the previous tonne and the historical average.

This paper develops an exact continuous-time bridge from that heterogeneous marginal ore unit to observable production. It asks four questions. First, how should stock scarcity be represented when a deposit generates a field of domain-specific opportunity costs? Second, under what conditions can the active ore rent be expressed in copper and molybdenum units? Third, how does marginal efficient yield appear in current mine output, cumulative mine-life production, and company or national portfolio exposure? Fourth, what can Chilean public data identify about the mechanism, and which scarcity components remain latent without mine-route costs, schedules, and shadow values?

The analysis uses a solvable two-domain benchmark rather than an expansive model that is never implemented. The copper-only and joint copper-molybdenum problems are solved separately. Domain costates define an active marginal stock rent, while the common throughput constraint produces a distinct capacity rent. A local perturbation derives product-specific in-situ scarcity without allocating capacity quasi-rent to geological scarcity. Four exposure measures are then connected by exact identities. The continuous solution is verified by a discrete Bellman approximation. Chilean national data are used as portfolio growth accounting, and a public Los Pelambres route comparison provides partial identification of relative scarcity from grades and recoveries. The empirical work is therefore linked to the theory at the level actually observed. It is not presented as a structural estimate of an unobserved rent.

The remainder of the paper is organized as follows. Section 2 locates the novelty boundary in the exhaustible-resource, joint-production, grade-selection, accessibility, and porphyry-geology literatures. Section 3 defines and solves the continuous-time heterogeneous mine. Section 4 derives product-specific scarcity and the transition from marginal yield to flow, cumulative, and portfolio exposure. Section 5 presents the solved benchmark, comparative statics, and numerical verification. Section 6 establishes Chile's world-market relevance and develops national growth accounting and route-level partial identification. Section 7 draws implications for molybdenum availability. Section 8 concludes and identifies limitations and future research. The Appendix presents the discrete-time approximation and additional derivations.

## 2. Joint-Production Scarcity and the Novelty Boundary

Mineral scarcity has several meanings that should not be used interchangeably. In-situ economic scarcity is the opportunity cost of extracting an additional unit from a finite stock. Physical availability concerns material that can be discovered, accessed, processed, and recovered. Market tightness concerns prices, inventories, utilization, conversion capacity, and production-use balance. Criticality adds geographic concentration, governance, substitutability, recycling, and strategic exposure. The classic scarcity literature explains why rent, price, grade, cost, reserves, and physical abundance can move differently (Fisher 1977; Brown and Field 1978; Krautkraemer 1998; Livernois 2009; Tilton and Lagos 2007; Watson and Eggert 2021). This paper concerns in-situ scarcity and the way it is translated, imperfectly, into observed mine supply.

Mining economics began to relax the homogeneous-stock abstraction early. Carlisle (1954) separated the extraction rate from the recovery level of a mineral fund. Conrad and Hool (1984) and Cairns (1986) showed that ore quality and economically defined reserves can be endogenous. Grade-selection models demonstrate that prices and expectations can change the quality of ore processed, not merely total extraction, and that the direction can depend on geology, uncertainty, and capacity (Krautkraemer 1988; Slade 1988; Farrow and Krautkraemer 1989; Shinkuma and Nishiyama 2000; Cairns and Shinkuma 2003). Recent system-dynamics work makes continuous cut-off-grade adjustment explicit (Rakhsha et al. 2026), while geological-information models show how data acquisition can change mining

investment under uncertainty (Cáceres and Emery 2026). Exploration and heterogeneous-deposit models make the resource base informationally and spatially contingent (Cairns and Van Quyen 1998). Capacity choice adds another opportunity cost because the mine-plant system limits the speed at which ore can become saleable output (Cairns 2001; Guzmán 2026). These strands provide essential components, but they do not derive the exact map from a heterogeneous marginal ore unit to current, cumulative, and portfolio co-product exposure.

Pindyck (1982) is the closest exhaustible-resource benchmark. Composite ore is extracted and separated into constituent resources that can be sold, stored, partly recovered, or discarded. The ore has a transfer price and a scarcity rent. Individual constituent prices need not obey separate Hotelling paths. For each constituent, in-situ rent is the ore rent multiplied by the efficient marginal ore requirement for one additional unit. This result is deeper than a fixed recovery coefficient because Pindyck already permits variable and incomplete separation. The extension developed here is operational heterogeneity: the active ore domain, capacity rent, and efficient marginal yields change through the mine life.

Campbell (1985) provides the industry supply taxonomy. A strict by-product does not affect the profit-maximizing common output. Once incremental recovery cost is covered, short-run supply is available only up to a ceiling created by main-product production. A co-product affects the common decision. Campbell's empirical analysis also found no systematic copper-molybdenum price co-movement in its historical sample, consistent with distinct demand drivers. Tilton and Guzmán (2016) applies the same decision-based classification and combines main-product and by-product sources when constructing long-run supply. The paper extends this logic by defining product status at a particular control margin—entry, capacity, sequence, or recovery—rather than assigning one permanent label.

Recent companion-metal research makes clear that geological presence is not accessible supply. Nassar et al. (2015) document widespread by-product dependence. Greffe et al. (2024) distinguish available, potentially accessible, and accessible ratios at different supply-chain stages. Menezes Cunha et al. (2026) similarly separate geospheric occurrence from anthropogenic accessibility. Afflerbach et al. (2014), Jordan (2017, 2018), and Redlinger and Eggert (2016) show that joint production affects price behavior, volatility, and own- and cross-price responses. Fikru and Awuah-Offei (2022) and Fikru and Romani (2025) model recovery and joint processing decisions. Bigerna and Campbell (2025) examine main-product constraints on critical-metal availability, Ryter et al. (2025) build multicommodity mine supply surfaces, Rousseau et al. (2026) propose an intrinsic-inelasticity metric, and Brainard (2024) traces rhenium through a specific copper-molybdenum recovery chain. These contributions are cited here for the particular supply margin or accessibility concept they establish, rather than because of the journal in which they appeared. The present contribution is not the claim that companion supply is inelastic. It identifies the mine-level scarcity and aggregation objects that determine where and why that inelasticity arises.

The novelty is therefore the integration of six steps. First, a heterogeneous mine generates domain-specific stock rents rather than one undifferentiated scalar rent. Second, the active marginal rent is

separated from common-capacity quasi-rent. Third, a conditioned marginal-cost perturbation converts that rent into product-specific scarcity units. Fourth, marginal, current-flow, cumulative, and portfolio Mo/Cu exposure are defined separately and linked by exact equations. Fifth, the paper proposes the base-normalized RSI-MY to isolate changes in relative in-situ scarcity generated by the marginal-yield channel. Sixth, those objects are partially identified using Chilean physical and production data without substituting a national ratio for an unobserved rent. Figure 2 summarizes the theoretical architecture.

### 3. A Continuous-Time Heterogeneous Mine

Consider a mine with two accessible ore domains indexed by  $j \in \{1,2\}$ . A domain can represent a mine phase, alteration domain, block group, or another support at which saleable yields and common costs are approximately constant. Let  $S_j(t)$  denote remaining ore in tonnes, and let  $q_j(t)$  denote the extraction and processing rate in tonnes per year. Removing ore from domain  $j$  changes its stock according to

$$\dot{S}_j(t) = -q_j(t), \quad S_j(0) = S_{j0} > 0, \quad j \in \{1,2\}. \quad (1)$$

Installed common mine-plant capacity is  $K$  tonnes of ore per year. The benchmark contains one route, no stockpile, and no mandatory blend. The instantaneous controls must therefore satisfy

$$q_j(t) \geq 0, \quad q_1(t) + q_2(t) \leq K. \quad (2)$$

A tonne from domain  $j$  yields  $y_{Cj} > 0$  tonnes of saleable copper and  $y_{Mj} \geq 0$  tonnes of saleable molybdenum after the maintained recovery and payability configuration. Product prices are  $p_C$  and  $p_M$ , and common variable cost is  $c_j$  dollars per tonne of ore. For problem  $a \in \{C, J\}$ , where  $C$  denotes the copper-only counterfactual and  $J$  the joint problem, following the notation in Guzmán (2019), define instantaneous operating profit per tonne of ore as

$$\pi_j^a = p_C y_{Cj} + \mathbf{1}_{\{a=J\}} p_M y_{Mj} - c_j. \quad (3)$$

The indicator removes molybdenum value before the copper-only continuation values are calculated. This separation is essential: a copper-only priority cannot use shadow values already influenced by molybdenum.

Let  $r > 0$  be the continuous discount rate and let  $T^a$  be the endogenous exhaustion date under problem  $a$ . The value function for remaining stocks  $S = (S_1, S_2)$  is

$$V^a(S) = \max_{\{q_j(t)\}} \int_0^{T^a} e^{-rt} \sum_{j=1}^2 q_j(t) \pi_j^a dt, \quad (4)$$

subject to Eqs. (1) and (2), exhaustion, and non-negativity. In a fixed-access regime, the stationary current-value Hamilton-Jacobi-Bellman equation is

$$rV^a(S) = \max_{q_1, q_2 \geq 0, q_1 + q_2 \leq K} \sum_{j=1}^2 q_j [\pi_j^a - \lambda_j^a(S)], \quad \lambda_j^a(S) = \frac{\partial V^a}{\partial S_j}. \quad (5)$$

The costate  $\lambda_j^a$  is the stock rent on one additional tonne remaining in domain  $j$ . Let  $\mu^a \geq 0$  be the current rent on common capacity. The Kuhn-Tucker condition for each accessible domain is

$$\pi_j^a - \lambda_j^a - \mu^a \leq 0, \quad q_j(\pi_j^a - \lambda_j^a - \mu^a) = 0, \quad \mu^a(K - q_1 - q_2) = 0. \quad (6)$$

Equation (6) separates stock scarcity from short-run bottleneck value. An active interior domain has netback equal to its stock rent plus the capacity rent. An excluded domain has weakly lower opportunity-adjusted netback. If both domains are processed simultaneously without a forced blend, their adjusted values must be equal. This formulation also clarifies why a heterogeneous deposit contains a rent field  $\{\lambda_1, \lambda_2\}$  rather than one universal stock rent.

A closed-form benchmark follows under linear netbacks, positive margins, no ties, and full-capacity operation. Suppose  $\pi_1^a > \pi_2^a$ , so discounting makes it optimal to exhaust domain 1 before domain 2. Define  $\tau_j = S_{j0}/K$ . The exact value is

$$V^a = \frac{K}{r} [\pi_1^a(1 - e^{-r\tau_1}) + e^{-r\tau_1}\pi_2^a(1 - e^{-r\tau_2})]. \quad (7)$$

While domain 1 is active, let  $u_1(t)$  be its remaining operating time. Its stock rent and the common-capacity rent are

$$\lambda_1^a(t) = e^{-ru_1(t)} [\pi_1^a - \pi_2^a(1 - e^{-r\tau_2})], \quad \mu^a(t) = \pi_1^a - \lambda_1^a(t). \quad (8)$$

After domain 1 is exhausted, let  $u_2(t)$  be the remaining operating time of domain 2. The corresponding rents are

$$\lambda_2^a(t) = \pi_2^a e^{-ru_2(t)}, \quad \mu^a(t) = \pi_2^a - \lambda_2^a(t). \quad (9)$$

Let  $j^*(t)$  denote the active marginal domain in the joint problem. The active marginal stock rent is defined by

$$\lambda^*(t) = \lambda_{j^*(t)}^J(t). \quad (10)$$

The rent can change discontinuously at a domain switch because the marginal ore unit changes. This is an economically meaningful transition, not a violation of intertemporal optimality. Smooth rent-growth statements apply within a fixed marginal regime; they need not hold across a change in ore domain, route, or recovery configuration.

Molybdenum can change the ore order even when total throughput remains at capacity. To illustrate this decision margin, consider a copper-rich domain 1 and a molybdenum-rich domain 2, with  $y_{M2} > y_{M1}$ . Holding continuation values fixed for the local comparison, the molybdenum price that equalizes current opportunity-adjusted netbacks is

$$p_M^{sw} = \frac{p_C(y_{C1} - y_{C2}) - (c_1 - c_2) - (\lambda_1^J - \lambda_2^J)}{y_{M2} - y_{M1}}. \quad (11)$$

A larger copper-yield advantage in domain 1 raises the molybdenum price needed to switch to domain 2. A higher relative common cost or a lower continuation value in domain 2 lowers the threshold. Below the threshold, molybdenum can remain a by-product at the sequence margin; above it, the optimal feed order changes and molybdenum becomes a co-product at that margin.

## 4. Product-Specific Scarcity and Observable Exposure

The active marginal ore unit yields  $y_C^m(t)$  tonnes of saleable copper and  $y_M^m(t)$  tonnes of saleable molybdenum. The marginal efficient relative yield is defined as

$$\kappa^m(t) = \frac{y_M^m(t)}{y_C^m(t)}. \quad (12)$$

This is the physical-economic conversion relevant for the next efficient ore tonne. It is not generally observed in annual production data. For one physically matched mine stream, let  $Y_C(t)$  and  $Y_M(t)$  denote current saleable outputs. Contemporaneous flow exposure is

$$\kappa^f(t) = \frac{Y_M(t)}{Y_C(t)} = \frac{\sum_j q_j(t) y_{Mj}}{\sum_j q_j(t) y_{Cj}}. \quad (13)$$

If exactly one domain is active and the recovery configuration is fixed,  $\kappa^f = \kappa^m$ . With blending or simultaneous extraction,  $\kappa^f$  is a copper-output-weighted average of the active domain ratios and therefore lies between their minimum and maximum. A reporting-unit ratio may differ further when inventories, filtration, shipping, or route aggregation separate production from contemporaneous feed.

Let cumulative saleable production through time  $t$  be  $Q_i(t) = \int_0^t Y_i(s) ds$ . Cumulative mine-life exposure is

$$\kappa^{cum}(t) = \frac{Q_M(t)}{Q_C(t)} = \frac{\int_0^t Y_M(s) ds}{\int_0^t Y_C(s) ds}. \quad (14)$$

For positive cumulative copper production, applying the quotient rule to Eq. (14) gives the exact adjustment equation:

$$\dot{\kappa}^{cum}(t) = \frac{Y_C(t)}{Q_C(t)} [\kappa^f(t) - \kappa^{cum}(t)]. \quad (15)$$

Equation (15) shows that cumulative exposure moves toward current flow exposure at a speed determined by current copper output relative to cumulative copper production. If current flow exposure is non-increasing,  $\kappa^f(t) \leq \kappa^{cum}(t) \leq \kappa^f(0+)$  and cumulative exposure is non-increasing. In that inequality, zero-plus denotes the right-hand limit immediately after production begins. It is not a typographical error or an addition sign. The right-hand limit is used because cumulative exposure is undefined before any copper has been produced. The cumulative measure need not equal the final marginal yield at finite exhaustion.

A company or country contains several mines or reporting units. Let  $Y_{C,m}(t)$  be copper output from unit  $m$ , let  $\kappa_m^f(t)$  be its matched-stream exposure, and define the copper weight  $w_m(t) = Y_{C,m}(t) / \sum_n Y_{C,n}(t)$ . Portfolio exposure is

$$\kappa^P(t) = \frac{\sum_m Y_{M,m}(t)}{\sum_m Y_{C,m}(t)} = \sum_m w_m(t) \kappa_m^f(t). \quad (16)$$

Its continuous-time change separates within-mine and mine-mix effects:

$$\dot{\kappa}^P(t) = \sum_m w_m(t) \dot{\kappa}_m^f(t) + \sum_m \dot{w}_m(t) [\kappa_m^f(t) - \kappa^P(t)]. \quad (17)$$

The first term measures changes within operating units. The second captures entry, closure, expansion, disruptions, and changing copper weights. Portfolio exposure can therefore rise even while exposure is declining at several mature mines.

The scarcity conversion requires an explicit marginal-cost perturbation. Consider one additional saleable unit of product  $i \in \{C, M\}$  obtained through the active marginal ore unit. Let  $MC_i^{tech}$  include the common cash cost and the net value of the other product, but exclude stock scarcity and the opportunity cost of binding common capacity. Stationarity implies

$$p_i - MC_i^{tech}(t) = \frac{\lambda^*(t) + \mu^J(t)}{y_i^m(t)}. \quad (18)$$

Full marginal cost must include the capacity opportunity cost  $\mu^J/y_i^m$ . Define  $MC_i^{full} = MC_i^{tech} + \mu^J/y_i^m$ . Product-specific in-situ scarcity is then

$$h_i^S(t) = p_i - MC_i^{full}(t) = \frac{\lambda^*(t)}{y_i^m(t)}. \quad (19)$$

Equation (19) is the heterogeneous-mine counterpart of Pindyck (1982)'s constituent-rent formula. The two  $h_i^S$  values are alternative units of the same active stock rent. They are not independent rents to be added. The result requires a common active marginal ore unit and consistent accounting of common and product-specific costs. It must be expanded when stockpiles, intermediates, separate downstream bottlenecks, or different marginal ore units carry additional shadow values.

When copper and molybdenum share the active marginal unit, their relative in-situ scarcity is

$$\frac{h_M^S(t)}{h_C^S(t)} = \frac{y_C^m(t)}{y_M^m(t)} = \frac{1}{\kappa^m(t)}. \quad (20)$$

This yields two distinct scarcity channels. Absolute molybdenum scarcity changes with both the active ore rent and marginal molybdenum yield, whereas relative scarcity between the products depends only on their marginal yield ratio:

$$\frac{d \ln h_M^S(t)}{dt} = \frac{d \ln \lambda^*(t)}{dt} - \frac{d \ln y_M^m(t)}{dt}, \quad \frac{d}{dt} \ln \left( \frac{h_M^S(t)}{h_C^S(t)} \right) = - \frac{d \ln \kappa^m(t)}{dt}. \quad (21)$$

Pindyck (1982) derives the level of constituent rent but does not provide a base-date-normalized pairwise indicator of the marginal-yield channel. The broader indicator literature uses different objects: crustal abundance (Arvidsson et al. 2020), by-product accessibility ratios (Grefe et al. 2024), and multidimensional criticality (Schicho and Tercero Espinoza 2026). A targeted review did not identify the

exact mine-level normalization below. The paper therefore proposes the Relative Scarcity Index from Marginal Yields (RSI-MY) as an operational corollary of Eq. (20), not as a replacement for physical scarcity or criticality indicators. With a chosen base date, the index is defined as follows:

$$RSI_{M/C}(t; t_0) = \frac{h_M^S(t)/h_C^S(t)}{h_M^S(t_0)/h_C^S(t_0)} = \frac{\kappa^m(t_0)}{\kappa^m(t)}. \quad (22)$$

The RSI-MY equals one at the base date. A value above one indicates that the marginal common-ore requirement for an additional efficient unit of molybdenum has increased relative to copper. A value below one indicates the opposite. Because the active stock rent cancels from the product ratio, the index isolates scarcity transmitted through changing marginal yield without requiring an estimate of the rent level. Absolute molybdenum scarcity remains only partially identified unless the active ore rent, or a defensible bound on it, is available. The index is used below in Figures 3, 4, and 8 and in the benchmark and Los Pelambres numerical results.

Campbell's product classification can be made decision-specific. For an optimized continuous control  $x^*(p_M)$ —such as common output, capacity, or recovery—define the local molybdenum control elasticity as

$$E_M^x = \frac{p_M}{x^*} \frac{\partial x^*}{\partial p_M}. \quad (23)$$

Molybdenum is a by-product at decision margin  $x$  when the local response is zero and a co-product when it is nonzero. A discrete sequence decision uses the threshold in Eq. (11) or an observed change in the optimized policy. The same metal can consequently be a by-product at common throughput, a co-product at sequence or recovery, and a main product at a dedicated molybdenum mine.

Finally, distinguish technical potential from economic supply. For current state  $S$  and a specified copper-output basis  $\bar{Y}_C$ , the technical maximum feasible Mo/Cu intensity is

$$\kappa_{tech}^{max}(S, \bar{Y}_C) = \frac{1}{\bar{Y}_C} \max_{q \in \mathcal{F}(S)} \left\{ \sum_j q_j y_{Mj} : \sum_j q_j y_{Cj} = \bar{Y}_C \right\}. \quad (24)$$

The economic frontier adds recovery costs, fixed circuit investment, payability, and displaced ore value. The gap among  $\kappa^m$ ,  $\kappa^f$ , and  $\kappa_{tech}^{max}$  can therefore reveal different binding margins and should not be collapsed into one “by-product ratio.”

## 5. Solved Benchmark, Comparative Statics, and Numerical Verification

The benchmark uses Los Pelambres Q1 2024-Q1 2025 grade-recovery combinations to anchor the scale of the two saleable-yield vectors, while ore stocks, common costs, and the domain sequence remain transparent illustrative inputs. It is therefore a Chile-informed analytical benchmark, not a calibration of a named mine plan. Domain 1 contains 320 Mt and domain 2 contains 640 Mt. Common capacity is 64 Mt/y, producing five- and ten-year domain durations, and the discount rate is 8%. To avoid combining observed grade-recovery pairs with unrelated illustrative prices, the stationary benchmark uses the arithmetic mean of the official Q1 2024 and Q1 2025 quarterly averages: copper at US\$4.03/lb (US\$8,895/t) and molybdenum oxide at US\$20.13/lb (US\$44,387/t). The underlying quarterly averages are US\$3.83/lb Cu and US\$19.84/lb Mo in Q1 2024, and US\$4.24/lb Cu and US\$20.43/lb Mo in Q1 2025 (Chilean Copper Commission 2026c). A common price vector is used because the analytical benchmark is stationary.

Table 1 reports the principal inputs. The saleable-yield entries equal head grade multiplied by metallurgical recovery. They are expressed as percent of ore for readability.

Table 1  
Inputs to the solved continuous-time benchmark

| Parameter                 | Domain 1 | Domain 2 | Status and unit                                 |
|---------------------------|----------|----------|-------------------------------------------------|
| Initial ore stock         | 320      | 640      | Illustrative, Mt ore                            |
| Saleable copper yield     | 0.4680   | 0.4939   | Chile-informed, % of ore                        |
| Saleable molybdenum yield | 0.01440  | 0.01358  | Chile-informed, % of ore                        |
| Common variable cost      | 27       | 31       | Illustrative, US\$/t ore                        |
| Copper price              | 8,895    | 8,895    | Official pooled Q1 2024-Q1 2025 mean, US\$/t Cu |
| Molybdenum oxide price    | 44,387   | 44,387   | Official pooled Q1 2024-Q1 2025 mean, US\$/t Mo |
| Full-capacity duration    | 5        | 10       | Derived, years                                  |
| Common capacity           | 64       | 64       | Chile-informed scale, Mt/y                      |

The joint exact present value is US\$11.143 billion, compared with US\$7.677 billion in the copper-only counterfactual. The values are not intended as a project valuation. Their role is to make the ore ordering and rent decomposition internally consistent under the period-anchored price vector. Figure 3 shows the exact continuous-time solution. Marginal and flow exposure coincide because only one domain is active at a time. Cumulative exposure adjusts with delay. The active stock rent increases within each domain

but changes at the five-year switch; the capacity rent declines as exhaustion approaches. Normalized product-specific scarcity grows through both the stock-rent and yield channels, while RSI-MY isolates the marginal-yield component.

The marginal Mo/Cu yield falls from 30.77 to 27.50 kg Mo/t Cu at the domain switch. Relative molybdenum scarcity therefore rises from 32.49 to 36.36 tonnes Cu-equivalent per tonne Mo-equivalent, and RSI-MY rises from 1.000 to 1.119. Because the active stock rent also changes, absolute molybdenum scarcity does not move by the inverse-yield ratio alone. This distinction is the central economic reason to report both product-specific scarcity and RSI-MY.

Table 2 summarizes the solution and the discrete verification.

Table 2  
Continuous solution and numerical verification

| Result                                                | Value              |
|-------------------------------------------------------|--------------------|
| Joint continuous value                                | US\$11.143 billion |
| Copper-only continuous value                          | US\$7.677 billion  |
| Domain switch time                                    | 5.000 years        |
| Initial marginal Mo/Cu yield                          | 30.774 kg/t Cu     |
| Post-switch marginal Mo/Cu yield                      | 27.502 kg/t Cu     |
| Post-switch RSI-MY increase                           | + 11.896%          |
| Static sequence-switch price in separate illustration | US\$25.61/lb       |
| Discrete value error, annual step                     | 4.053%             |
| Discrete value error, 1/16-year step                  | 0.250%             |

Comparative statics clarify the roles of recovery and stock rent. Raising molybdenum recovery in the post-switch domain increases marginal Mo/Cu yield and can offset the composition-driven increase in RSI-MY. It does not necessarily lower absolute scarcity if the active stock rent rises more rapidly. Figure 4(a) reports the RSI-MY recovery sensitivity. Figure 4(b) applies the partial-identification equation developed below to Los Pelambres: the absolute scarcity change varies with the unobserved rent ratio, while RSI-MY is identified from the grade-recovery yield ratio.

The separate sequence illustration gives domain 2 a modestly higher molybdenum yield than domain 1 while preserving domain 1's copper advantage. Eq. (11) then gives a switching threshold of approximately US\$25.61/lb under the period-anchored copper price, the illustrative costs, and the maintained continuation-value assumption. This is not a molybdenum price forecast. It demonstrates that product status is attached to a decision margin: a product can remain a by-product at common throughput while becoming a co-product at the sequence margin.

The Bellman approximation processes the same capacity quantum and uses the same netbacks as the continuous benchmark. The relative value error falls from 4.05% with annual periods to 0.25% with sixteenth-year periods, while the policy switch remains at year five. Figure 5 reports both convergence and the separate price threshold. Online Resource 1 also tests stock conservation, capacity feasibility, the rent identities, all four exposure definitions, and the Chilean calculations.

## 6. Chile in the World Molybdenum Market and Partial Identification

Chile's importance is established by both scale and source structure. In the consistent COCHILCO/World Metal Statistics series, it supplied 43.4 kt in 2025, or 16.7% of the reported world total, and ranked second. Its production is overwhelmingly associated with copper-main-product porphyry operations rather than a large domestic segment of primary molybdenum mines. Chile is therefore a high-value setting for studying scarcity transmitted through the copper mine sequence. The expanded 2015–2025 comparison shows that Chilean output and world output did not move proportionally: Chile's share rose above 22% in 2018, fell to 15.1% in 2024, and recovered in 2025. At the same time, the source comparison warns against false precision. IMOA reports 672.6 million lb, or about 305.1 kt, of world production in 2025, materially above the 260.5 kt in the COCHILCO/World Metal Statistics table (International Molybdenum Association 2026). The paper uses each series only within its own internally consistent boundary.

National Chilean production from the official historical series (Chilean Copper Commission 2026a) supplies long-run context but not a direct scarcity measure. Let  $C^{CL}(t)$  and  $M^{CL}(t)$  be national copper and molybdenum mine output, and define national portfolio exposure

$\kappa_{CL}^P(t) = M^{CL}(t) / C^{CL}(t)$ . The exact accounting identity is

$$M^{CL}(t) = C^{CL}(t) \kappa_{CL}^P(t), \quad \frac{d \ln M^{CL}(t)}{dt} = \frac{d \ln C^{CL}(t)}{dt} + \frac{d \ln \kappa_{CL}^P(t)}{dt}.$$

25

The identity is descriptive, not causal. National copper includes operations and routes without molybdenum recovery.  $\kappa_{CL}^P$  combines within-mine changes with mine mix.

From 2000 to 2015, Chilean copper output rose 25.4%, molybdenum output 58.4%, and portfolio exposure 26.3%. From 2015 to 2024, copper declined 4.6%, molybdenum 26.8%, and exposure 23.3%. In 2025, copper declined a further 2.1%, while molybdenum rebounded 12.7% and exposure 15.1%. Over 2000–2025, all three remained above their 2000 levels. Figure 6 combines the national paths with a complete 2015–2025 comparison of actual world production, actual Chilean production, and Chile's world-market share. The evidence rejects both a deterministic depletion narrative and a fixed Mo/Cu coefficient.

The operation-level reporting panel is informative but imperfect. For Andina, Collahuasi, El Teniente, Los Pelambres, and Salvador, copper and molybdenum are reported under the same operation or division, but annual route-level feed, grades, and recoveries are not consistently aligned. Their ratios are therefore reporting-unit exposure, not geological yields. Descriptive log slopes over positive observations are approximately  $-10.5\%/y$  at Andina,  $-3.6\%$  at Collahuasi,  $-2.2\%$  at El Teniente,  $+2.1\%$  at Los Pelambres, and  $-18.7\%$  at Salvador. The exact midpoint decomposition from 2016 to 2024 attributes a 2.42 kg Mo/t Cu decline to 1.88 kg/t within operations and 0.54 kg/t mine mix. Extending the endpoint to 2025 reverses the aggregate sign because Los Pelambres receives a large copper weight and its exposure increases sharply. Figure 7 demonstrates why portfolio evidence must be decomposed and tested across endpoints.

A stronger link to the theory is possible for Los Pelambres because its Q1 production report discloses daily ore throughput, copper and molybdenum grades, recoveries, and outputs (Antofagasta plc 2025). Let route-level head grade be  $g_i$ , recovery  $R_i$ , and saleable yield  $y_i = g_i R_i$  for  $i \in \{C, M\}$ . The physically implied relative yield is  $\kappa^{feed} = y_M/y_C$ . Reported flow exposure is  $\kappa^f = M/C$ . The two can differ because quarterly production can reflect filtration, inventories, shipping, payability, or timing not captured by contemporaneous head-grade averages.

Table 3  
Los Pelambres Q1 partial identification

| Variable                           | Q1 2024 | Q1 2025 | Change or interpretation                     |
|------------------------------------|---------|---------|----------------------------------------------|
| Daily ore throughput, kt/d         | 177.1   | 173.5   | -2.0%                                        |
| Cu grade, %                        | 0.560   | 0.530   | Lower Cu head grade                          |
| Cu recovery, %                     | 88.2    | 88.3    | Approximately stable                         |
| Mo grade, %                        | 0.016   | 0.019   | Higher Mo head grade                         |
| Mo recovery, %                     | 84.9    | 75.8    | Lower Mo recovery                            |
| Feed-based, kg Mo/t Cu             | 27.50   | 30.77   | + 11.9%                                      |
| Reported flow exposure, kg Mo/t Cu | 39.78   | 32.90   | Timing/route mismatch is material            |
| Relative scarcity, t Cu/t Mo       | 36.36   | 32.49   | -10.6%, conditional on common marginal route |
| RSI-MY, Q1 2024 = 1                | 1.000   | 0.894   | -10.6%; route-identified relative scarcity   |

The feed-based relative yield rises because the higher molybdenum grade more than offsets the lower molybdenum recovery and lower copper grade. Under the common-marginal-unit assumptions, the route-implied molybdenum scarcity relative to copper falls by 10.6%. RSI-MY therefore moves from 1.000 in Q1 2024 to 0.894 in Q1 2025. This is a scarcity result rather than simply a production ratio because it

follows from the inverse marginal yield in Eq. (20). Figure 8 separates the underlying grades and recoveries from reported output exposure and presents RSI-MY explicitly.

Absolute molybdenum scarcity remains partially identified because the active stock rent is not public. Across the two observations, Eq. (19) implies

$$\frac{h_{M,2025}^S}{h_{M,2024}^S} = \frac{\lambda_{2025}^*}{\lambda_{2024}^*} \frac{y_{M,2024}^m}{y_{M,2025}^m}.$$

26

The disclosed yield data imply  $y_{M,2025}^m/y_{M,2024}^m \approx 1.060$ . Absolute molybdenum scarcity therefore rises only if the active ore rent increases by more than about 6.0%. It falls if the rent increase is smaller. Public grade-recovery data identify the relative-scarcity movement but not the absolute level or change in  $h_M^S$ . This result clarifies what a genuine structural calibration would require: common ore costs, opportunity-adjusted sequence, remaining stocks, capacity rents, and consistent route-level output.

Recent operating disclosures reinforce the need to keep the physical margins separate. Antofagasta reported that full-year 2025 copper production at Los Pelambres was 8% below 2024 while molybdenum output rose 48%, attributing the divergence to lower throughput, harder ore and lower copper grades alongside higher by-product grades (Antofagasta plc 2026). At Collahuasi, record fourth-quarter throughput partly mitigated lower feed grades and recovery pressure (Anglo American plc 2026). These observations do not reveal the active stock rent, but they support the model's premise that throughput, Cu grade, Mo grade, recovery and mine phase need not co-move.

The route comparison also explains why national production can rebound during long-run maturity. A higher-Mo phase can increase  $\kappa^m$  even when copper grade or throughput declines. Conversely, recovery loss or a shift toward low-Mo ore can increase relative scarcity without a large change in copper production. The national series disciplines the scale and timing of the problem; the route data identify one component of the scarcity conversion; neither substitutes for the unobserved rent field.

## 7. Implications for Molybdenum Availability

The framework changes the information required for molybdenum-supply analysis. Copper output is a platform variable rather than a complete supply equation. Molybdenum recovered from a copper-main-product route equals common ore throughput multiplied by molybdenum grade, recovery, payability, and route availability. The relative scarcity of the next efficient tonne depends on the corresponding copper and molybdenum yields, while absolute scarcity additionally depends on the active ore rent.

Three implications follow. First, aggregation can hide the marginal process. A decline in marginal efficient Mo/Cu yield raises RSI-MY immediately, but it appears in current-flow exposure only when a single unblended stream is observed. Cumulative exposure adjusts with a stock-dependent lag, and

portfolio exposure can reverse because mine weights change. Analysts should therefore avoid treating a national Mo/Cu ratio as either a geological coefficient or RSI-MY.

Second, the binding decision margin determines the supply response. When molybdenum does not affect common output, it is a by-product at that margin and a price increase may first raise recovery utilization or the capacity rent. It becomes a co-product at sequence when it changes the active domain, at recovery when it changes circuit operation or investment, and at capacity when it changes the mine-plant scale. This decision-specific taxonomy operationalizes Campbell's insight and helps reconcile low aggregate own-price elasticity with material physical changes inside a mine.

Third, scarcity, tightness, and criticality require different indicators. The stock-rent equations measure in-situ scarcity. Market tightness depends additionally on demand, inventories, conversion capacity, and primary molybdenum mines. Criticality depends on source concentration, substitution, recycling, and geopolitical exposure. A quarterly production-use deficit is not direct evidence of rising in-situ rent, while a deteriorating marginal yield can be masked temporarily by inventories or new primary-mine output.

Chile's recent experience illustrates all three points. Its share of world supply makes operation-level changes globally relevant. The 2015–2024 decline in portfolio exposure indicates that copper output alone was a poor predictor of molybdenum output, but the 2025 rebound shows that the path is not mechanically downward. Los Pelambres demonstrates that a higher molybdenum grade can lower relative molybdenum scarcity even when copper grade declines. A forward-looking assessment should therefore monitor scheduled Cu and Mo grades, recovery, hardness-dependent throughput, remaining mine life, circuit availability, and the evolving mix between copper-linked and primary molybdenum supply.

## 8. Conclusions

This paper develops a continuous-time mineral-economics theory of molybdenum scarcity in copper-main-product mines and links it to Chilean evidence at the level public data can support. A heterogeneous deposit generates a field of domain-specific stock rents. The rent of the active marginal domain, separated from the quasi-rent on common capacity, is converted into copper and molybdenum scarcity through marginal efficient yields. This produces a precise distinction between absolute scarcity, which depends on both the active rent and product yield, and relative scarcity, which depends on the marginal Mo/Cu yield ratio.

The paper also derives the map from the unobserved marginal process to observed production. Contemporaneous matched-stream exposure equals marginal yield only under restrictive single-stream conditions. Cumulative exposure adjusts toward current flow with an exact lag equation. Portfolio exposure adds a mine-mix term that can obscure or reverse within-mine changes. These results explain why national copper output and national Mo/Cu exposure are useful disciplines but cannot be substituted for the scarcity rent of the next efficient ore tonne.

Chile provides both global relevance and empirical variation. In the consistent series used here, the country supplied 16.7% of world molybdenum mine production in 2025. Its national record shows expansion, contraction, and rebound rather than a deterministic depletion path. Public Los Pelambres data permit a more direct partial identification: the grade-recovery Mo/Cu yield increased between Q1 2024 and Q1 2025, so RSI-MY declined from 1.000 to 0.894 under the common-marginal-route conditions. Absolute molybdenum scarcity remains unobserved unless the change in the active ore rent can be estimated or bounded. This is the precise boundary between production accounting and scarcity measurement.

The framework contributes to mineral economics by extending Pindyck (1982)'s homogeneous composite-ore scarcity conversion to a heterogeneous mine with domain-specific rents and common capacity, by converting Campbell's product taxonomy into decision-specific control responses, by deriving the exposure aggregation needed to connect mine planning with country evidence, and by proposing RSI-MY as a transparent normalization of the marginal-yield scarcity channel. The result is a lower-dimensional economic language for use alongside block-model optimization and multicommodity market simulation.

The analysis has limitations that define the next research agenda. The solved benchmark has two accessible domains, constant within-domain yields, one common route, and deterministic prices. A full application should add precedence, blends, stockpiles, hardness-dependent throughput, multiple destinations, recovery trade-offs, and uncertainty. The Chilean panel remains imperfectly route matched, and the Q1 Los Pelambres comparison does not reveal costs or mine-plan shadow values. The highest-value empirical extension is a mine-route panel combining ore tonnes, Cu and Mo grades, recoveries, payability, circuit events, remaining mine phases, and opportunity-adjusted costs. That evidence would permit structural estimation of the active rent and a direct test of whether the marginal-yield framework predicts the direction and timing of molybdenum supply.

## Declarations

## Author Contribution

J.I.G did all the work since it is the unique author.

## Data Availability

Data are available under request.

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Figures

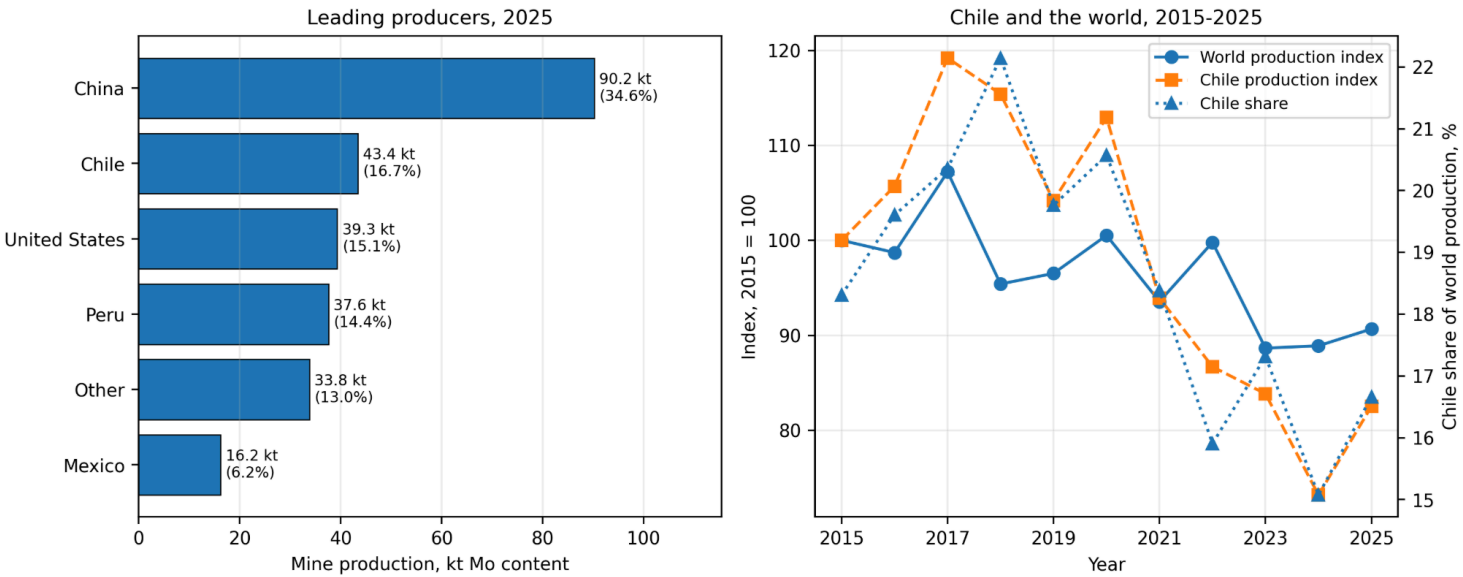


Figure 1

Chile in the world molybdenum market. Panel (a) reports 2025 mine production and shares from the COCHILCO/World Metal Statistics series. Panel (b) uses the same statistical boundary to compare Chilean and world production indices and Chile's share from 2015 to 2025. Source: author calculations from COCHILCO/World Metal Statistics.

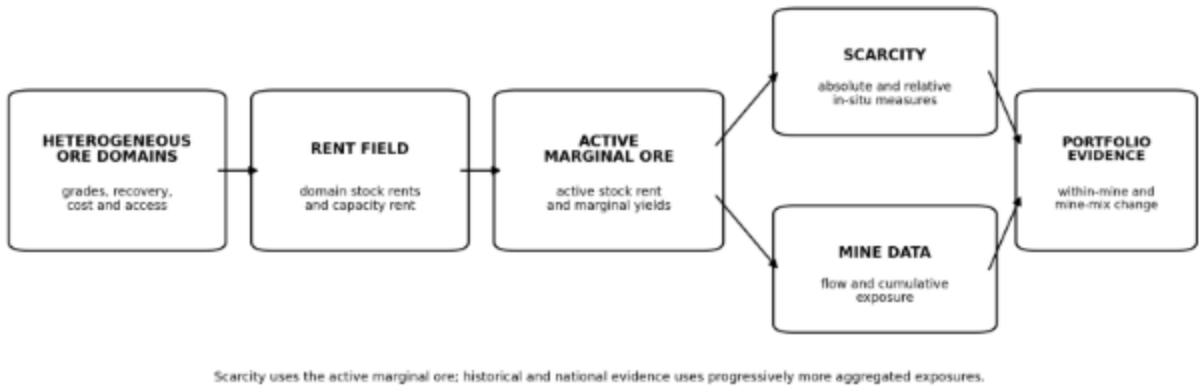


Figure 2

Scarcity and exposure architecture. Heterogeneous ore creates a field of stock rents and a common-capacity rent. The active marginal ore determines product-specific scarcity, while mine and portfolio data observe progressively more aggregated exposure measures.

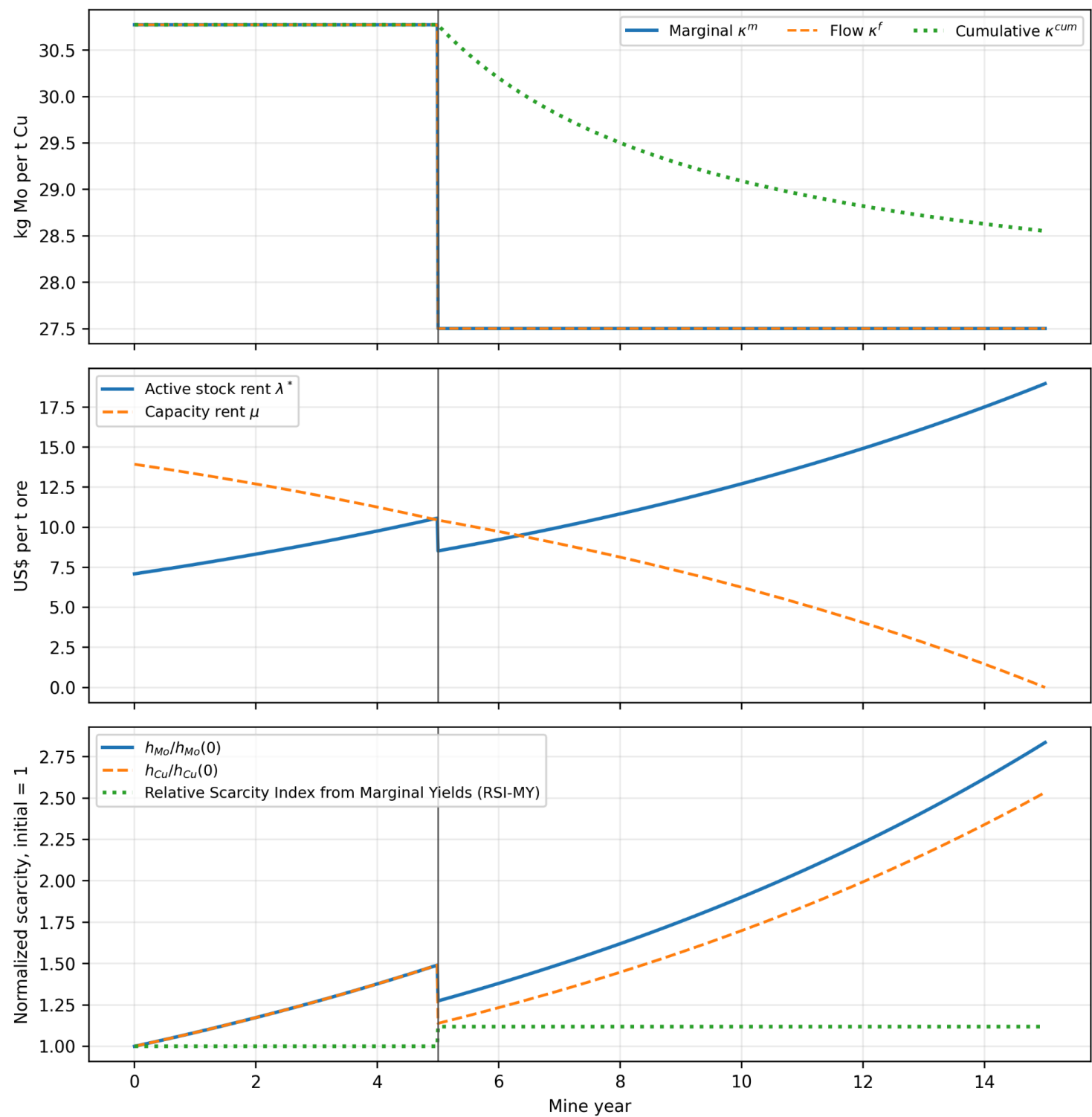


Figure 3

Solved continuous-time benchmark. The panels report marginal, flow, and cumulative exposure. The active stock and common-capacity rents; normalized copper and molybdenum scarcity; and the Relative Scarcity Index from Marginal Yields (RSI-MY). The vertical line marks the domain switch.

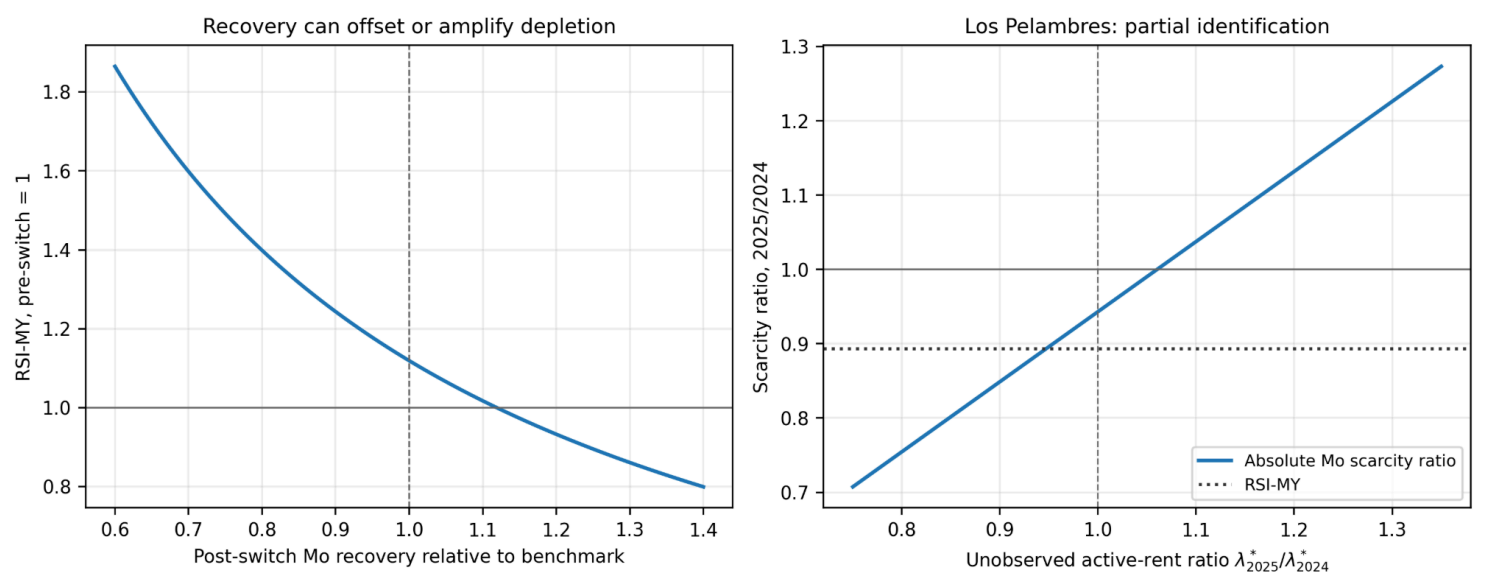


Figure 4

Scarcity comparative statics and partial identification. Panel (a) shows how post-switch molybdenum recovery changes RSI-MY in the benchmark. Panel (b) shows the Los Pelambres absolute scarcity ratio over alternative active-rent ratios and the route-identified RSI-MY.

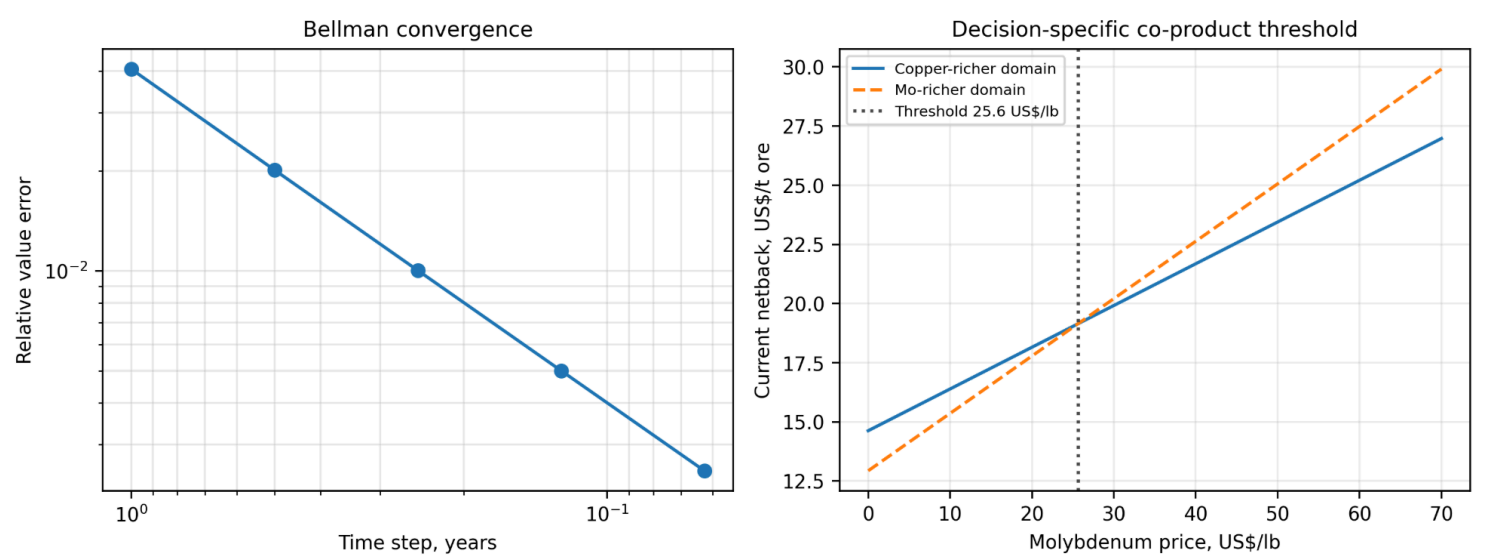


Figure 5

Numerical verification and a decision-specific co-product threshold. Panel (a) reports convergence of the discrete Bellman value to the exact continuous value. Panel (b) reports the netback threshold at which a molybdenum-richer domain overtakes the copper-richer domain under the period-anchored copper price.

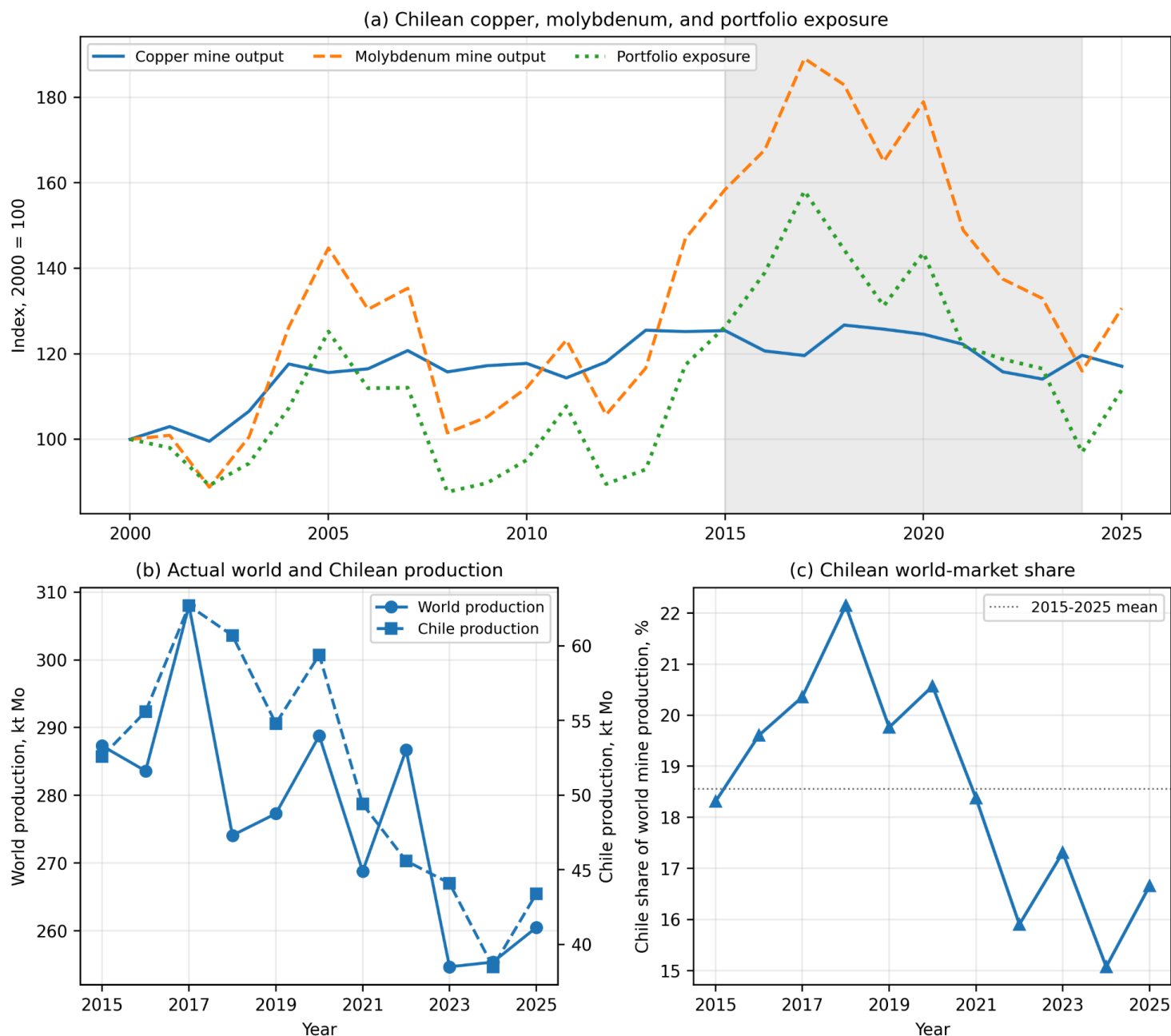


**Figure 6**

Chilean molybdenum production, portfolio exposure, and world share. Panel (a) reports Chilean copper, molybdenum, and portfolio-exposure indices from 2000 to 2025. Panel (b) reports actual world and Chilean molybdenum mine production from 2015 to 2025 on separate production axes. Panel (c) reports Chile's share of the same world series. Source: author calculations from COCHILCO and COCHILCO/World Metal Statistics.

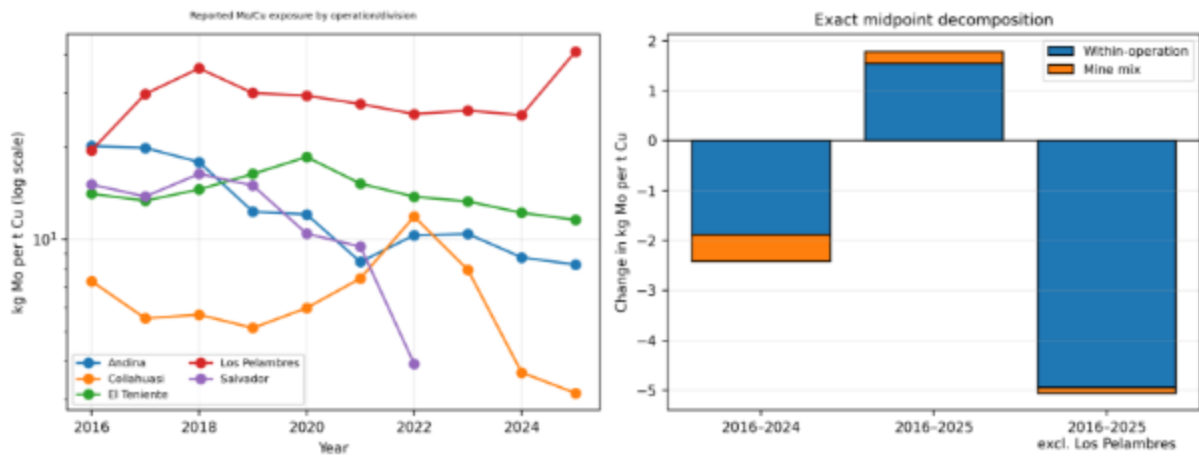


**Figure 7**

Chilean reporting-unit exposure and portfolio decomposition. Panel (a) reports same-operation or same-division Mo/Cu ratios on a logarithmic scale. Panel (b) separates within-operation and mine-mix components under alternative endpoints. Source: author calculations from COCHILCO.

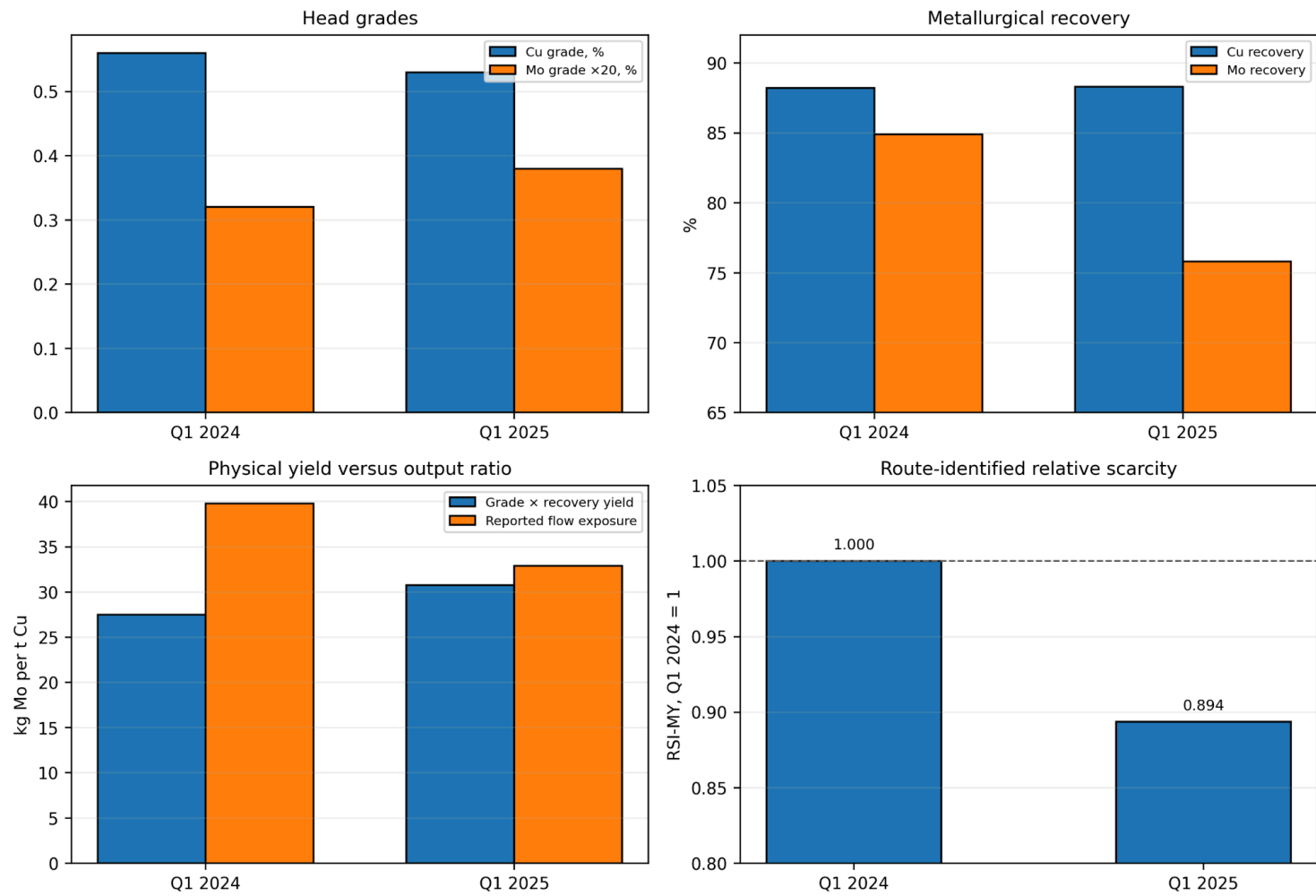


**Figure 8**

Los Pelambres partial identification and RSI-MY. Panels (a) and (b) compare disclosed head grades and metallurgical recoveries. Panel (c) compares the grade-recovery relative yield with reported output exposure. Panel (d) reports the route-identified Relative Scarcity Index from Marginal Yields, normalized to one in Q1 2024. Molybdenum grade is rescaled only for graphical comparability in panel (a). Source: author calculations from Antofagasta plc Q1 production reports.

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Appendix.docx](#)