

Cluster-Guided Topological Semantic Spectra: A Numerically Testable Framework for Local Geometric Semantics in Complex Engineering CAD Scenes

GuoJun Pan

18858146@qq.com

Independent Researcher <https://orcid.org/0009-0002-5750-6523>

Research Article

Keywords: engineering CAD, local geometric semantics, graph Laplacian, spectral geometry, clustering, persistent homology, semantic reconstruction, primitive graph, numerical validation

Posted Date: August 18th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10685082/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: The authors declare no competing interests.

Cluster-Guided Topological Semantic Spectra: A Numerically Testable Framework for Local Geometric Semantics in Complex Engineering CAD Scenes

Guojun Pan

August 2026

Abstract

Recovering engineering semantics from complex three-dimensional CAD scenes is often treated as primitive classification, object detection, or relation-graph recognition. These formulations are insufficient for industrial geometry because local shape and semantic identity are not in one-to-one correspondence: a cylinder may be a pipe, a handrail member, a support rod, a flange slice, or part of equipment; a ring-like elbow motif may be a valve handwheel, a guardrail corner, a lifting eye, or a decorative frame. This paper proposes a cluster-guided topological semantic spectrum for local geometric semantics. The central claim is not that geometry directly determines semantic labels, but that semantic recovery becomes numerically testable when three mechanisms are coupled: local geometric-semantic primitives define the measurable state, unsupervised clustering defines auditable analysis windows, and graph spectra with persistence define topology-sensitive fingerprints inside each window. We formalize hard feasibility gates, primitive embeddings, local assembly graphs, graph Laplacian spectra, persistent connectivity, and posterior semantic naming. For the pipe subproblem, we give a concrete port-graph Laplacian and prove that the multiplicity of its zero eigenvalue equals the number of hard-feasible connected pipe components. We then show how clustering and local spectra close the gap exposed by a negative case: ring-like geometry alone falsely confuses guardrail corners with handwheel-like structures. A real CAD scene with 7880 primitives is used as a numerical audit: all parsed primitives are embedded by type, size, pose, normalized position, local density, and neighborhood type histograms; Ward clustering assigns them to 14 structure clusters; the complete assignment is written back to a colored PHG file for visual inspection. The paper therefore provides both theoretical closure, through graph-spectrum invariants and hard feasibility, and numerical closure, through reproducible clustering, scene-level coverage, negative controls, and explicit claim boundaries.

Keywords: engineering CAD; local geometric semantics; graph Laplacian; spectral geometry; clustering; persistent homology; semantic reconstruction; primitive graph; numerical validation

1 Introduction

Semantic reconstruction of industrial CAD scenes is a prerequisite for digital twins, engineering asset management, reverse engineering, robot inspection, design automation, and machine-readable engineering knowledge. A typical scene consists of thousands or millions of geometric primitives: cylinders, elbows, boxes, cones, hemispheres, meshes, and simplified equipment shells. The desired output, however, is not a list of primitive labels. Engineers need higher-level objects and relations: pipe runs, branches, supports, platforms, valves, nozzles, equipment interfaces, and residual unresolved geometry.

The main difficulty is that engineering semantics is contextual. A cylinder is not a pipe by itself; it becomes a pipe segment only when it satisfies endpoint compatibility, radius continuity, tangent continuity, and long-range connection with other flow-carrying primitives. A box is not

necessarily a support; it may be a beam, a simplified housing, a plate, or a residual part. A circular arrangement of thin elbows is not necessarily a handwheel; it may be a guardrail turn or another ring-like construction. Therefore, semantic reconstruction cannot be reduced to direct Euclidean-shape classification.

This paper develops a framework in which local geometric semantics, clustering, and spectral methods are integrated into one testable pipeline. The idea is simple but restrictive:

Do not name semantics from a single geometric motif. First cluster primitives into auditable local structure classes; then compute topological spectra inside these local windows; only then assign semantic names when the resulting fingerprints are numerically separable and pass negative controls.

The framework is inspired by Fourier analysis but is not a Fourier transform in the classical linear sense. In signal processing, a complex signal is analyzed by choosing a representation domain, applying a window or basis, and studying frequency content. In engineering CAD, primitive embeddings provide the measurable state space; clustering provides local windows; graph Laplacian spectra and persistent connectivity provide topological frequency content. The resulting semantic spectrum is nonlinear, graph-based, constrained by hard feasibility, and defined over CAD assemblies rather than scalar fields.

1.1 Contributions

This paper makes the following contributions.

1. It formulates local geometric semantics as a measurable state-recovery problem over CAD primitives with type, pose, parameters, and local frames.
2. It introduces cluster-guided local spectral analysis: unsupervised primitive clustering is used to define analysis windows before semantic naming.
3. It defines local assembly graphs and topological spectral fingerprints using graph Laplacians, spectral gaps, zero modes, and persistent connectivity.
4. It gives a concrete port-graph spectral operator for the pipe subproblem and proves that its zero-eigenvalue multiplicity equals the number of hard-feasible connected pipe components.
5. It provides a numerical audit on a real CAD scene containing 7880 parsed primitives. All primitives are assigned to 14 structure clusters and written back to a colored PHG scene.
6. It includes a negative-control lesson: a ring-like elbow motif alone is insufficient evidence for a handwheel-like semantic label because guardrail corners can produce the same local geometric motif.
7. It states explicit evidence levels and claim boundaries so that the framework remains falsifiable rather than purely interpretive.

2 Related Work

2.1 Spectral Geometry and Graph Spectra

Spectral geometry studies geometric structure through eigenvalues and eigenfunctions of Laplace-type operators. Classical results such as Weyl’s law relate eigenvalue growth to volume and dimension. In geometry processing, Laplace spectra and heat kernels have been used as shape fingerprints and multiscale descriptors [4, 5]. On graphs, the Laplacian

$$L = D - W, \quad D_{ii} = \sum_j W_{ij} \tag{1}$$

encodes connectivity, diffusion, and low-frequency modes. A standard theorem states that the multiplicity of the zero eigenvalue of L equals the number of connected components of the graph. This theorem gives direct engineering meaning to graph spectra when the graph edges correspond to hard-feasible CAD relations.

2.2 Persistent Topology

Persistent homology studies topological features under a varying filtration parameter [7, 8, 9]. For a weighted graph, one may define

$$G(\tau) = (V, \{(i, j) \mid w_{ij} \leq \tau\}). \quad (2)$$

Long-lived connected components are stable under tolerance variation; short-lived components often indicate noise, accidental proximity, or weak evidence. This is well matched to engineering CAD, where tolerances and modeling simplifications are unavoidable.

2.3 Geometric Deep Learning and Clustering

Graph neural networks and geometric deep learning provide powerful tools for learning from non-Euclidean data [10]. The present work is compatible with such models but does not require them as a first step. The key design choice is to separate unsupervised structural grouping from semantic naming. This makes intermediate outputs inspectable: clusters, local graphs, spectra, persistence curves, and colored PHG scenes can be audited before labels are introduced.

3 Problem Definition

Let an engineering CAD scene be a finite set of primitives

$$S = \{e_i\}_{i=1}^N. \quad (3)$$

Each primitive is represented as

$$e_i = (k_i, p_i, C_i, \theta_i), \quad (4)$$

where k_i is the primitive type, $p_i \in \mathbb{R}^3$ is its placement point, $C_i \in SO(3)$ is a local coordinate frame, and θ_i contains geometric parameters such as radii, lengths, angles, or box dimensions.

The target is not an isolated primitive label map $f(e_i)$. The target is a higher-level semantic assembly graph

$$Y = \{m_j, r_{jl}\}, \quad (5)$$

where m_j denotes an assembly-level motif and r_{jl} denotes engineering relations between motifs.

3.1 Hard Feasible Set

We distinguish hard feasibility from soft scoring. Let

$$A = \{S \mid G_{\text{hard}}(S) = 0\} \quad (6)$$

be the feasible set induced by geometric validity, coordinate-frame validity, endpoint compatibility, type compatibility, contact validity, and scene consistency. Relations outside A are rejected before optimization or semantic naming:

$$S \notin A \Rightarrow S \text{ is not a semantic candidate.} \quad (7)$$

Invalid geometry is not a high-energy legal state; it is outside the candidate space.

4 Cluster-Guided Local Spectral Semantics

4.1 Primitive Embedding

For each primitive e_i , define a geometry-neighborhood embedding

$$x_i = \phi_g(e_i) = [o(k_i), s(\theta_i), p_i^{\text{norm}}, a(C_i), n_i(r), h_i(r)]. \quad (8)$$

Here $o(k_i)$ is a one-hot primitive type vector, $s(\theta_i)$ contains scale, approximate volume, aspect ratio, and type-specific geometric parameters, p_i^{norm} is a scene-bounding-box normalized position, $a(C_i)$ stores local-axis orientation features, $n_i(r)$ is the number of neighboring primitives within radius r , and $h_i(r)$ is a histogram of neighboring primitive types. No semantic label is used in x_i .

The standardized feature matrix X is clustered:

$$\{C_1, \dots, C_K\} = \text{Cluster}(X). \quad (9)$$

The clusters are structural analysis windows, not final semantic labels.

4.2 Local Assembly Graphs

For a cluster or spatially expanded cluster C_k^+ , define a local assembly graph

$$G_k = (V_k, E_k, W_k), \quad V_k = \{e_i \mid e_i \in C_k^+\}. \quad (10)$$

The edge weight between primitives is a compatibility score:

$$W_{ij} = \alpha_d \kappa_d(e_i, e_j) + \alpha_a \kappa_a(e_i, e_j) + \alpha_s \kappa_s(e_i, e_j) + \alpha_c \kappa_c(e_i, e_j) + \alpha_t \kappa_t(e_i, e_j), \quad (11)$$

where κ_d measures spatial proximity, κ_a axial or tangential consistency, κ_s scale or radius compatibility, κ_c contact, containment, or endpoint relation, and κ_t primitive-type compatibility. Hard-infeasible edges are removed.

Define the normalized graph Laplacian

$$L_k = I - D_k^{-1/2} W_k D_k^{-1/2}. \quad (12)$$

The local spectral fingerprint is

$$\psi_k = [\lambda_0, \lambda_1, \dots, \lambda_q, \gamma_k, \beta_0(\tau), \ell_k], \quad (13)$$

where λ_i are low-frequency eigenvalues, γ_k is a spectral gap, $\beta_0(\tau)$ is the connected-component curve under filtration, and ℓ_k summarizes persistence lifetimes.

4.3 Joint Representation

The final structure representation is

$$z_k = [\text{pool}\{x_i \mid e_i \in C_k^+\}, \psi_k]. \quad (14)$$

Semantic naming is posterior:

$$y_k = f_{\text{sem}}(z_k). \quad (15)$$

The function f_{sem} may be manual audit, nearest-prototype matching, a shallow classifier, or a graph neural network. If z_k cannot be separated from competing structure classes under perturbation and negative controls, the correct output is an unnamed structure cluster.

5 A Concrete Spectral Operator for Pipe Connectivity

To make the framework mathematically grounded, we specify a pipe-system subproblem. For axial primitives, define ports with positions x_a , radii r_a , and tangents t_a . The mismatch between ports a, b is

$$\delta_{ab} = \left(\frac{\|x_a - x_b\|}{\epsilon_d} \right)^2 + \left(\frac{r_a - r_b}{\epsilon_r} \right)^2 + \left(\frac{\arccos(|t_a \cdot t_b|)}{\epsilon_\theta} \right)^2. \quad (16)$$

The edge weight is

$$w_{ab} = \begin{cases} \exp(-\delta_{ab}), & \delta_{ab} \leq \tau_0 \text{ and types are compatible,} \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

This gives the port graph

$$G_{\text{pipe}} = (P, E_{\text{pipe}}, w) \quad (18)$$

and its weighted Laplacian

$$L_{\text{pipe}} = D - W. \quad (19)$$

Proposition 1. Let G_{pipe} be a hard-feasible weighted port graph whose retained edges have positive weights. The multiplicity of the zero eigenvalue of L_{pipe} equals the number of connected components of G_{pipe} .

Proof. For any vector f ,

$$f^\top L_{\text{pipe}} f = \frac{1}{2} \sum_{a,b} W_{ab} (f_a - f_b)^2 \geq 0. \quad (20)$$

The quadratic form is zero if and only if $f_a = f_b$ on every positive-weight edge. Hence the nullspace consists of vectors constant on each connected component and independent across components. Its dimension is the number of connected components. $\square$

This proposition closes one important theoretical loop: for the pipe subproblem, the zero spectrum is not merely a descriptor; it is a computable invariant with direct engineering interpretation.

6 Algorithm

Algorithm 1 Cluster-Guided Topological Semantic Spectrum

Require: CAD primitive scene $S = \{e_i\}_{i=1}^N$

Ensure: structure clusters, local spectra, audit files, optional semantic tokens

- 1: Parse primitive type, pose, local frame, and parameters for each e_i .
- 2: Reject invalid primitives and invalid hard relations.
- 3: Compute primitive embedding $x_i = \phi_g(e_i)$.
- 4: Standardize $X = [x_1, \dots, x_N]^\top$.
- 5: Cluster X into structural windows $\{C_k\}_{k=1}^K$.
- 6: **for** each cluster or expanded local window C_k^+ **do**
- 7: Construct local assembly graph $G_k = (V_k, E_k, W_k)$.
- 8: Compute L_k , low-frequency spectrum, spectral gap, and persistence.
- 9: Store $z_k = [\text{pool}(x_i), \psi_k]$.

10: **end for**

11: Apply semantic naming only if z_k is separable and passes controls.

12: Write cluster IDs and/or accepted semantic domains back to CAD/PHG.

7 Numerical Validation

7.1 Scene and Primitive Statistics

We evaluate the primitive-clustering stage on a converted engineering PHG scene. The parsed scene contains $N = 7880$ primitives. Table 1 lists the primitive distribution.

Table 1: Primitive counts in the real CAD scene.

Primitive type	Count
Cylinder	4585
Box	1117
Elbow/wantou	800
Hemisphere	690
Cone	391
Mesh	297
Total	7880

Each primitive is embedded by type, size, approximate volume, aspect ratio, pose axes, normalized scene position, neighbor count, nearest-neighbor scale, and neighborhood type histogram. The neighborhood radius is $r = 1.25$. Ward hierarchical clustering is applied with $K = 14$.

7.2 Cluster Results

All 7880 parsed primitives are assigned cluster IDs, giving scene-level coverage

$$R_{\text{cover}} = \frac{7880}{7880} = 1.0. \quad (21)$$

The cluster-size distribution is shown in Table 2.

Table 2: Unsupervised primitive-cluster sizes. The numbers are not semantic labels; they are structure-cluster IDs.

Cluster	1	2	3	4	5	6	7
Count	383	573	236	1469	960	10	716
Cluster	8	9	10	11	12	13	14
Count	105	1341	675	237	700	244	231

The cluster assignment is written back to a PHG file using a distinct color for each cluster. This is important because the numerical output can be inspected in the original 3D scene rather than only in feature space.

7.3 Negative Audit: Why Direct Geometry Is Insufficient

A geometry-only rule was tested on ring-like elbow structures. It selected large-radius thin elbows sharing a center, radius, and plane normal. This rule generated eight handwheel-like candidates. Visual and contextual audit showed that these candidates were guardrail or handrail corner structures rather than confirmed valve handwheels. Their neighborhoods contained many repeated thin cylinders, cap-like hemispheres, small connector plates, and upper/lower parallel ring layers.

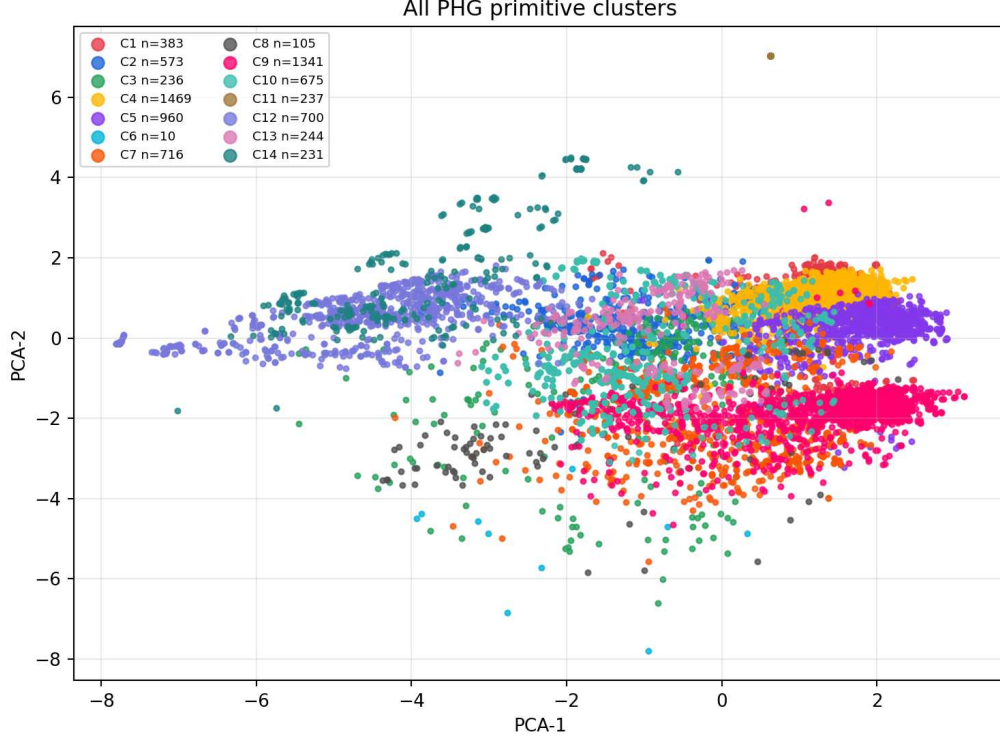


Figure 1: PCA projection of standardized primitive features after unsupervised clustering. The plot is a numerical audit artifact: it visualizes separability in the embedding used before any semantic naming.

This negative case is central:

$$\text{geometric motif similarity} \not\Rightarrow \text{semantic equivalence.} \quad (22)$$

It falsifies the naive claim that a local ring motif is a sufficient semantic detector. It supports the revised framework: geometry generates candidates; clustering defines local structure classes; local spectra and context determine whether semantic naming is justified.

7.4 Numerical Closure Criteria

The method is considered numerically testable because it produces measurable outputs and failure modes:

1. **Coverage:** fraction of parsed primitives assigned to clusters.
2. **Cluster reproducibility:** adjusted Rand index or normalized mutual information under perturbations of features, neighborhood radius, and clustering parameters.
3. **Spectral stability:** stability of low-frequency eigenvalues, spectral gaps, and $\beta_0(\tau)$ under threshold variation.
4. **Negative-control rejection:** semantic outputs must fail when local frames, radii, or admissible relations are randomized.
5. **Scene-level auditability:** cluster or domain labels must be written back to the CAD/PHG scene.
6. **Baseline comparison:** compare against distance-only clustering, type-only clustering, and direct hand-written motif rules.

8 Discussion

8.1 How the Three Components Close the Loop

The proposed framework combines three components that play different roles.

- Local geometric semantics defines the measurable primitives, local frames, ports, contact relations, and candidate assembly graph.
- Clustering provides a data-driven windowing mechanism. It prevents premature semantic naming and exposes repeated structural classes in large scenes.
- Spectral methods provide topology-sensitive invariants inside the windows: zero modes, connected components, spectral gaps, and persistence lifetimes.

Theoretical closure comes from graph-spectrum theorems and hard feasibility. Numerical closure comes from reproducible embeddings, cluster assignments, negative controls, and scene-level visualization. The two closures meet when a semantic claim is accepted only if its local structure class and local topological spectrum remain stable under controlled perturbations.

8.2 Fourier Analogy

The framework is analogous to Fourier analysis only in a restricted sense. A classical Fourier transform decomposes a signal into linear frequency bases. Here, a CAD scene is first converted into primitives and local relations; clustering defines analysis windows; graph spectra decompose local assembly graphs into topology-sensitive modes. Thus:

Signal analysis	Engineering semantic spectrum
Raw signal	CAD primitive scene
Window function	Unsupervised local structure cluster
Frequency basis	Graph Laplacian eigenmodes
Low frequency	Main assembly organization
High frequency	local fragments, small parts, noise, residuals
Filtering	hard feasibility and persistence
Inverse model	semantic token to engineering geometry

9 Limitations and Claim Boundaries

This paper does not claim that all engineering semantics can be recovered from spectra alone. Its claim is narrower: cluster-guided local spectral analysis provides a numerically testable representation in which semantic recovery can be audited and falsified.

Current limitations include:

1. Mesh internals still require mesh segmentation or additional context.
2. Industry-specific CAD conventions may require domain-specific hard gates and compatibility functions.
3. Large factory-scale models require blockwise graph construction and cross-block coupling.
4. Semantic naming requires external validation through annotated data, prototype libraries, or expert audit.
5. The framework is not a substitute for CAD kernels, physical simulation, or engineering standards.

We use the following evidence levels:

Level	Meaning
0	Conceptual analogy only
1	Formal representation and compatibility
2	Toy example or single-case audit
3	Reproducible numerical experiment with controls
4	External dataset or engineering validation

The full semantic recovery task remains Level 3 in this paper: reproducible numerical structure clustering and negative-control evidence are provided, but broad external validation is not yet claimed.

10 Conclusion

This paper presented a cluster-guided topological semantic spectrum for complex engineering CAD scenes. The method integrates local geometric semantics, unsupervised clustering, and spectral topology into one pipeline. Local geometric semantics defines measurable primitive states and hard-feasible relations; clustering defines auditable local windows; graph spectra and persistence define topology-sensitive fingerprints; semantic naming is deferred until the joint representation is numerically separable and passes controls.

The theoretical part is closed by a concrete port-graph operator for pipe connectivity and the zero-spectrum connected-component theorem. The numerical part is closed by a real-scene audit: 7880 CAD primitives are embedded, clustered into 14 structure classes, and written back to a colored PHG file for inspection. A negative audit shows why direct geometry-only semantic rules fail: ring-like elbow motifs can correspond to guardrail structures rather than handwheels. Therefore, the central conclusion is bounded but strong:

Engineering semantic reconstruction should not be treated as direct mapping from Euclidean geometry to labels. It should be treated as a numerically testable hierarchy from primitive geometry to local structure clusters, from clusters to topological spectral fingerprints, and from stable fingerprints to posterior semantic names.

References

- [1] H. Weyl. Ueber die asymptotische Verteilung der Eigenwerte. *Nachrichten von der Gesellschaft der Wissenschaften zu Goettingen*, 1911.
- [2] W. V. D. Hodge. *The Theory and Applications of Harmonic Integrals*. Cambridge University Press, 1941.
- [3] M. Belkin and P. Niyogi. Laplacian eigenmaps and spectral techniques for embedding and clustering. *Advances in Neural Information Processing Systems*, 2001.
- [4] M. Reuter, F.-E. Wolter, and N. Peinecke. Laplace-spectra as fingerprints for shape matching. *Proceedings of the ACM Symposium on Solid and Physical Modeling*, 2006.
- [5] J. Sun, M. Ovsjanikov, and L. Guibas. A concise and provably informative multi-scale signature based on heat diffusion. *Computer Graphics Forum*, 2009.
- [6] A. Singer and H.-T. Wu. Vector diffusion maps and the connection Laplacian. *Communications on Pure and Applied Mathematics*, 2012.

- [7] H. Edelsbrunner, D. Letscher, and A. Zomorodian. Topological persistence and simplification. *Discrete & Computational Geometry*, 2002.
- [8] G. Carlsson. Topology and data. *Bulletin of the American Mathematical Society*, 2009.
- [9] A. Zomorodian and G. Carlsson. Computing persistent homology. *Discrete & Computational Geometry*, 2005.
- [10] M. M. Bronstein, J. Bruna, T. Cohen, and P. Velickovic. Geometric deep learning: grids, groups, graphs, geodesics, and gauges. *arXiv preprint arXiv:2104.13478*, 2021.
- [11] C. M. Hoffmann. *Geometric and Solid Modeling: An Introduction*. Morgan Kaufmann, 1989.
- [12] J. J. Shah and M. Mantyla. *Parametric and Feature-Based CAD/CAM*. Wiley, 1995.
- [13] M. Botsch, L. Kobbelt, M. Pauly, P. Alliez, and B. Levy. *Polygon Mesh Processing*. AK Peters, 2010.
- [14] A. Borrmann, M. Konig, C. Koch, and J. Beetz, editors. *Building Information Modeling: Technology Foundations and Industry Practice*. Springer, 2018.