Star-forming galaxies dominate the diffuse, isotropic γ-ray background: supplementary information

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ABSTRACT

1 Source count distribution

We first discuss the method by which we construct Figure 3 of the main text. This involves two parts: a Monte Carlo simulation to estimate the number count of SFGs in the redshift interval $z < 0.1$ that is poorly sampled by CANDELS, and a statistical calculation of the uncertainties in the source counts induced by the fact that CANDELS samples a small field of view, while *Fermi* can only detect SFGs over a portion of the sky due to the Galactic plane and other foreground sources.

1.1 Monte Carlo estimation

The sample of CANDELS galaxies we utilise for the derivation of the γ-ray background is sensitive to cosmic variance at low redshift due to the small solid angle of the survey field. Unfortunately, this is also where the sources with the highest observed fluxes will be located. To verify that our model for γ-ray emission is compatible with the observed number of such bright sources, we simulate a population of low-redshift ($z < 0.1$) sources using a Monte Carlo scheme. The process for a single Monte Carlo realisation is as follows.

The first step is to produce a sample of SFGs. To do so, we draw galaxies from the observed distribution of star formation rates in the local Universe [1, 2]. For each SFG drawn, we also draw an associated redshift in the range $z = 0 - 0.1$, with probability proportional to the co-moving volume element. We continue drawing galaxies until the total star formation rate of the population we have drawn matches the integrated star formation rate within the volume $z = 0 - 0.1$ as determined from the cosmic star formation history [3]. The second step is to assign γ-ray luminosities for these galaxies based on our model for the CANDELS galaxies. For this purpose, we apply our model to predict the photon luminosity integrated over the 1 - 100 GeV band (i.e., the number of photons per unit time emitted in this energy range) for all CANDELS galaxies with $z < 1.5$, and fit a power law relationship between this luminosity and the star formation rate; we neglect $\gamma\gamma$ opacity in this calculation, since this effect is unimportant for the galaxies at $z < 0.1$ and the energy range < 100 GeV that we are simulating. We then assign each of our SFGs a γ-ray photon luminosity using this powerlaw fit, and in conjunction with the redshift, an observed photon flux S.

At this point we have a sample of γ-ray photon fluxes S for simulated $z < 0.1$ SFGs, which we can place in bins of S to construct a synthetic prediction for $S^2(dN/dS)$. We carry out 13,000 Monte Carlo trials of this type, and in each bin of S record the mean and the 68% and 90% probability intervals, which we show as the blue points and bands in Figure 3.

1.2 Confidence interval calculation

Calculation of confidence intervals on $S^2(dN/dS)$ for the observed sources (both the *Fermi*-observed SFGs and our model-predicted CANDELS SFGs) is non-trivial, because both surveys cover a fraction $f < 1$ of the sky, and the number of sources per bin for at least some bins of S is small, so we cannot compute the uncertainty by assuming that we are in the large N limit. To perform the calculation, we assume that the SFG population follows a Poisson distribution on the sky (i.e., we are in the cosmological isotropic limit), so if the entire sky contains $N_{\rm tot}$ SFGs within some bin of photon flux, the probability that $N_{\rm obs}$ will be found within the observable region can be written

$$P\left(N_{\rm obs}|N_{\rm tot}\right) = \frac{\left(fN_{\rm tot}\right)^{N_{\rm obs}} e^{-fN_{\rm tot}}}{N_{\rm obs}!}. \tag{1}$$

We wish to solve the inverse problem, i.e., given an observed number $N_{\rm obs}$, what is the probability distribution of $N_{\rm tot}$? The answer is given by Bayes's Theorem, which requires

$$P\left(N_{\rm tot}|N_{\rm obs}\right) = P\left(N_{\rm obs}|N_{\rm tot}\right)\frac{P\left(N_{\rm tot}\right)}{P\left(N_{\rm obs}\right)} \tag{2}$$

where $P\left(N_{\rm tot}|N_{\rm obs}\right)$ is the posterior probability, $P\left(N_{\rm tot}\right)$ is the prior probability, and $P\left(N_{\rm obs}\right)$ is a normalisation factor. We adopt a flat prior $P\left(N_{\rm tot}\right) \propto 1$, so we can then write

$$P\left(N_{\rm tot}|N_{\rm obs}\right) = \mathscr{N}\, N_{\rm tot}^{N_{\rm obs}} e^{-fN_{\rm tot}}, \tag{3}$$

where $\mathscr{N}$ is a normalisation constant. For $e^{-f} < 1$, which is always the case since $0 < f \leq 1$, the value of $\mathscr{N}$ required to guarantee that $\sum_{N_{\rm tot}} P\left(N_{\rm tot}|N_{\rm obs}\right) = 1$ is

$$\mathscr{N} = \frac{1}{\mathrm{Li}_{-N_{\rm obs}}\left(e^{-f}\right)}, \tag{4}$$

where $\mathrm{Li}_s\left(z\right)$ is the polylogarithm of order s.

To compute the confidence interval we require the cumulative distribution function. In the discrete case this is given by calculating the probability that $N_{\rm tot} < N$, which is

$$P\left(N_{\rm tot} < N\right) = \mathscr{N} \sum_{n=0}^{N-1} n^{N_{\rm obs}} e^{-fn} \tag{5}$$

$$= 1 - \mathscr{N} \sum_{n=N}^{\infty} n^{N_{\rm obs}} e^{-fn} \tag{6}$$

$$= 1 - \mathscr{N} \sum_{i=0}^{\infty} \left(i+N\right)^{N_{\rm obs}} e^{-f(i+N)} \tag{7}$$

$$= 1 - \frac{e^{-fN}\, \Phi\left(e^{-f}, -N_{\rm obs}, N\right)}{\mathrm{Li}_{-N_{\rm obs}}\left(e^{-f}\right)} \tag{8}$$

where $\Phi(z,s,a)$ is the Lerch Phi function (sometimes also referred to as the Lerch Zeta function). To obtain a particular percentile p in the range 0 to 1, we simply use the continuous forms of the polylogarithm and the Lerch Phi functions, set $p = P\left(N_{\rm tot} < N\right)$ and invert the problem numerically to find the appropriate value for N; for the purposes of Figure 3, we are interested in the 90% confidence interval, so we take $p = 0.05$ and $p = 0.95$. For the special case $N_{\rm obs} = 0$, the result simplifies to

$$p = 1 - e^{-fN}\left(\frac{1}{\mathrm{Li}_0\left(e^{-f}\right)} + 1\right), \tag{9}$$

which we can invert numerically for $p = 0.9$ to obtain the 90% confidence upper limit.

In order to use the result we have just derived, we require a value for f. For the CANDELS data points, this is straightforward: our data come from the GOODS-S field, which has an area of 173 arcmin^2, corresponding to $f = 1.16 \times 10^{-6}$. Assigning a value of f to the *Fermi* data is more complex: *Fermi* LAT surveys the entire sky, but it cannot detect faint sources, such as SFGs, that are too close to the Galactic plane because they are hidden by the Galactic diffuse foreground. As a result, the effective survey area depends at least somewhat on the flux and spectral shape of the target SFG – brighter and harder sources can be detected closer to the plane than fainter and softer ones. Capturing this effect in detail would require extensive testing of the *Fermi* reduction pipeline using artificial sources, which is beyond the scope of this work. For the purposes of computing the confidence intervals shown in Figure 3, we ignore this complexity, and roughly estimate that SFGs are undetectable within 15° degrees of the Galactic plane, which corresponds to approximately $f = 0.7$.

2 Neutrinos

In addition to the γ-rays produced in π^0 decay, the decay of $\pi^{\pm}$ produces leptons. Neutrinos are of particular interest as they propagate largely unhindered from the source to the observer. Our goal here is to compute the all-species neutrino flux due to SFGs, so that we may compare to the astrophysical neutrino background measured by IceCube [4].

The relationship between the γ-ray and neutrino spectra is approximately given by $E_\nu^2 F_\nu\left(E_\nu = E_\gamma/2\right) = (3/2) E_\gamma^2 F_\gamma\left(E_\gamma\right)$ [5]. However, we compute the neutrino flux from the charged pion decay in our sample galaxies using the more detailed method in Ref. [6–8][1]. The rate at which CR collisions with the ISM produce pions of energy E_π is given by

$$\frac{d\dot{N}_\pi}{dE_\pi}(E_\pi) = \frac{n_\mathrm{H}\, c}{K_\pi}\, \beta\, \sigma_\mathrm{pp}(E)\, \frac{d\dot{N}}{dE}(E)\, f_\mathrm{cal}(E), \tag{10}$$

where n_H is the ISM density, c the speed of light, β is the CR velocity divided by c, and

$$E = \frac{E_\pi}{K_\pi} + m_\mathrm{p}c^2. \tag{11}$$

Here $K_\pi = 0.17$ the fraction of energy transferred from the CR to the pion and m_p is the proton rest mass, so $E_\pi/K_\pi + m_\mathrm{p}c^2$ is then just the total energy E of the CR that produces a pion of energy E_π.

Charged pion decay produes neutrinos in two steps: the initial decay of the pion creates a muon and a muon neutrino, and then the muon decays, yielding an electron, an electron neutrino, and a second muon neutrino (where we do not distinguish between particles and anti-particles). The all-flavour neutrino spectrum is then a sum over the energy distributions of all three neutrinos produced in this chain, given by

$$\frac{d\dot{N}_\nu}{dE_\nu}(E_\nu) = 2\int_0^1 \left(f_{\nu_\mathrm{e}}(x) + f_{\nu_\mu^{(2)}}(x)\right) \frac{d\dot{N}_\pi}{dE_\pi}\left(\frac{E_\nu}{x}\right) \frac{dx}{x} + \frac{2}{\lambda}\int_0^\lambda \frac{d\dot{N}_\pi}{dE_\pi}\left(\frac{E_\nu}{x}\right) \frac{dx}{x}, \tag{12}$$

where $\lambda = 1 - \left(m_\mu/m_\pi\right)^2$, $x = E_\nu/E_\pi$, the second integral accounts for the muon neutrinos produced in the initial charged pion decay, and the functions f_{ν_e} and $f_{\nu_\mu^{(2)}}$ describe the energy distributions for the electron and muon neutrinos produced by decay of the secondary muon, respectively; we take them from Eqns. 40 and 36 of Ref. [6, 7]. The ratio of neutrino flavours at the source is $\left(\nu_e : \nu_\mu : \nu_\tau\right) = (1:2:0)$. However, neutrino oscillations will bring this to an even $\left(\nu_e : \nu_\mu : \nu_\tau\right) = (1:1:1)$ for an observer at Earth.

Equation 12 is the analogue to Equation 1 of the main text for γ-rays, and we can compute the resulting specific neutrino flux for each galaxy from Equation 2 of the main text simply by replacing $d\dot{N}_\gamma/dE_\gamma$ with $d\dot{N}_\nu/dE_\nu$ and setting the opacities $\tau_{\gamma\gamma} = \tau_\mathrm{EBL} = 0$. We use this to calculate a predicted neutrino flux from each CANDELS galaxy, and we sum to compute the neutrino background due to SFGs using Equation 3 of the main text, exactly as we do for the γ-ray background. We plot the resulting predicted neutrino spectrum in Extended Data Figure 4.

We find that our model predicts that SFGs produce a neutrino flux that is $\approx 15\%$ of the astrophysical neutrino background, as measured by IceCube [4], for a CR spectral cutoff energy of $E_\mathrm{cut} = 100$ PeV. However, while the choice of E_cut has no significant effect on the γ-ray spectrum (as explained in the main text), it does matter for the neutrino spectrum due to the high energies of the astrophysical neutrinos observable by IceCube. Consequently, we find that SFGs produce $\ll 15\%$ of the observed neutrino background if we adopt a smaller value of E_cut [9]. To illustrate this, in Extended Data Figure 4 we show two calculations: one with our fiducial $E_\mathrm{cut} = 100$ PeV, and one with a smaller $E_\mathrm{cut} = 1$ PeV. The cutoffs in the neutrino spectrum

[1] We caution readers, a number of recent publications calculate the neutrino spectrum using an incorrect formula for the parameterisation function $g_{\nu_e}(x)$ given in Ref. [6]. An Erratum has been published in Ref. [7]. Use of the incorrect formula leads to overestimation of the neutrino emission by a factor of ~ 2.

shown in Extended Data Figure 4 are a direct result of the adopted value of E_{cut}. We also note that the normalisation of our predicted neutrino spectrum is sensitive to bright, hard neutrino sources at low redshift, which dominate at high energy but are poorly sampled by the small CANDELS field of view. This suggests that it would be worthwhile in the future to repeat this analysis using a survey of SFGs that is wider but shallower than CANDELS.

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