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Network-Adjusted Empirical Bayes Estimation of Age-Specific Adult Mortality from DHS Sibling Survival Histories: Evidence from Nigeria

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Abstract

Background: Weak civil registration coverage in Nigeria means adult mortality relies heavily on sibling survival histories (SSHs) from Demographic and Health Surveys (DHS). SSH-based estimates suffer from differential visibility and from high sampling variability arising from sparse age-sex-specific death counts. Successive Nigerian DHS rounds have documented an apparent decline in adult mortality, but whether this reflects genuine improvement or is partly an artefact of uncorrected visibility bias has not been formally tested.

Methods: We developed a network-adjusted empirical Bayes framework integrating individual-level visibility correction with heteroscedastic SURE-based shrinkage, propagating uncertainty end-to-end via bootstrap re-estimation of the shrinkage hyperparameters. The framework was applied to sibling histories from the 2024 Nigeria DHS and, using an identical pipeline, retrospectively to the 2018 NDHS to enable a temporal comparison free of estimator-choice confounding. Estimates were further evaluated through internal consistency diagnostics, sensitivity analyses, and comparisons with conventional DHS aggregate estimator, Global Burden of Disease and UN World Population Prospects reference series.

Results: Bias-corrected mortality was lower in 2024 than in 2018 across all fourteen age-sex cells, with declines generally larger from the mid-30s onward and statistically distinguishable from sampling variability at three cells. The conventional and bias-corrected estimators agreed on the direction of decline in thirteen of fourteen cells (mean absolute difference, 3.9 percentage points). The exception, women aged 15–19, showed an aggregate-estimator increase against a bias-corrected decline, coinciding with the cell showing the highest invisible-population fraction and a failed internal consistency check. Corrected 2024 estimates were lower than GBD in 13 of 14 cells

and lower than WPP in all cells, with WPP values 2.4–3.7 times higher than the corresponding corrected estimates.

Conclusion: Integrating network-based visibility adjustment with empirical Bayes smoothing provides a robust framework for estimating age-specific adult mortality from DHS sibling survival histories. The method reduces the effects of visibility bias and sampling variability, requires no external calibration schedule, and is readily transferable to other DHS datasets, making it well suited for mortality surveillance in low-registration settings.

Keywords: adult mortality; sibling survival method; empirical Bayes shrinkage; sparse-data estimation; network-based estimation; visibility bias; Demographic and Health Surveys; Nigeria.

1. Introduction

In Nigeria and across sub-Saharan Africa, the limited completeness and coverage of civil registration and vital statistics (CRVS) systems mean that age-specific adult mortality remains among the least reliably measured quantities in population health, despite its central role in monitoring population health, evaluating health-system performance, and tracking progress toward national and global development goals [1–4]. The lack of complete death registration creates considerable uncertainty in adult mortality patterns, limiting the ability of policymakers to identify vulnerable populations, allocate resources effectively, and evaluate public health interventions [1, 2]. Nigeria, the most populous country in Africa, exemplifies this challenge: fewer than 10% of deaths are officially registered [5]. As a result, adult mortality estimation in

Nigeria continues to depend largely on indirect demographic methods and household survey-based approaches, especially sibling survival histories (SSHs) [6].

In countries with weak vital registration, SSHs gathered using Demographic and Health Surveys (DHS) are among the most important sources of information on adult mortality [7, 8]. Respondents are asked to report the survival status, sex, age, and, when applicable, age at death of all of their biological siblings. SSHs offer considerably higher levels of mortality exposure than traditional household surveys, as each respondent can provide data on several people. They have therefore been widely used to estimate adult mortality, maternal mortality, and mortality associated with major epidemics and humanitarian crises [7, 8]. In recent years, over 150 DHS surveys have gathered sibling survival histories, making SSHs one of the largest global repositories of adult mortality information in resource-constrained settings [8]. These data have become a cornerstone of adult mortality estimation in sub-Saharan Africa, including Nigeria, and have been used for national mortality assessment and demographic research [7, 8].

Despite their widespread use, SSH-based mortality estimates are subject to several well-documented methodological limitations. Previous studies have documented survivor selection bias, omission of deceased siblings, recall error, age misreporting, and differential reporting by sibship size as major sources of estimation error [7, 9, 10]. Among these, differential visibility represents a key source of bias in sibling survival histories, whereby an individual's probability of being reported depends on the number of surviving sisters who are eligible for interview [11]. Individuals with more survey-eligible surviving sisters therefore have a higher probability of being included in the observed sample, resulting in unequal visibility across the population. This over-representation of siblings from larger families produces bias in the conventional aggregate estimator even when individual mortality risk is independent of family size, because deaths

84 occurring in larger, more visible sibships contribute disproportionately to the aggregate estimator
85 relative to their true share of population exposure. Feehan and Borges (2021) demonstrated this
86 bias theoretically and showed that visibility ratios, the ratio of the network-corrected to the
87 conventional estimate, deviated meaningfully from unity in empirical DHS data [11]. In Nigeria,
88 successive DHS rounds have documented an apparent decline in adult mortality between 2008 and
89 2018 using conventional sibling-history estimators [12]. Whether this trend reflects genuine
90 mortality improvement or is partly an artefact of the conventional estimator's differential visibility
91 assumption has not been formally tested.

92 Recent advances in network-based demographic methods have significantly improved adult
93 mortality estimation from sibling survival histories. A design-consistent estimator, developed by
94 Feehan and Borges [11], conceptualizes sibling histories as network data and hence corrects
95 differential visibility by accounting for the number of survey-eligible sisters through whom each
96 sibling could be observed [11, 13]. Their approach represents an important methodological step
97 that minimizes selection bias and increases the representativeness of the estimates of mortality.
98 Even with a differential visibility correction, however, age-specific mortality rates calculated from
99 SSHs are not statistically robust due to the small number of deaths in each age group, resulting in
100 high sampling variability and wide confidence intervals [8]. This statistical uncertainty arises from
101 the relatively small number of reported deaths within age-specific cells rather than from
102 differential visibility alone. Therefore, complementary statistical methods are still needed to
103 improve the precision of age-specific mortality rates without sacrificing the benefits of network-
104 based estimators.

105 Empirical Bayes methods provide an alternative approach to improve the precision of the estimates
106 in sparse-data settings. By borrowing strength across related age groups, populations, or mortality

regimes, Empirical Bayes estimators reduce variance and stabilise estimates without imposing overly restrictive assumptions [14]. Such approaches have been widely applied in epidemiology, disease mapping, and demographic modelling, where they have been shown to improve estimation precision over direct estimators when sample sizes are limited [14–17]. Among these approaches, the heteroscedastic SURE empirical Bayes estimator of Xie, Kou and Brown (2012) provides a data-driven mechanism for selecting cell-specific shrinkage intensity without requiring strong distributional assumptions, with compound mean squared error dominance over cell-independent estimates established theoretically for the heteroscedastic normal means setting [18].

Existing applications of Empirical Bayes methods have not explicitly incorporated the network-adjusted estimation framework developed for DHS sibling survival data. Integrating network-based mortality estimation with empirical Bayes shrinkage therefore provides a complementary strategy for improving the precision of age-specific mortality estimates while preserving correction for differential visibility bias.

Nigeria provides an important setting in which to investigate these methodological challenges. Despite repeated Demographic and Health Surveys, adult mortality estimation in Nigeria continues to rely on sibling survival histories because death registration remains incomplete and reliable age-specific mortality data are unavailable from administrative sources [19, 20]. These characteristics make Nigeria an appropriate case study for evaluating methodological improvement in mortality estimation. Building on the network-based estimator developed by Feehan and Borges, this study develops a network-adjusted empirical Bayes framework that integrates visibility correction, SURE-based shrinkage, and bootstrap-propagated uncertainty within a single design-consistent pipeline for DHS sibling survival data [11]. To enable a methodologically consistent assessment of adult mortality trends, the same estimator was applied retrospectively to the 2018 Nigeria DHS

sibling history data, providing a directly comparable benchmark for evaluating changes in age-specific mortality between the 2011–2018 and 2017–2024 reference periods. As a further, estimator-independent check, the same validated conventional aggregate procedure was applied to both survey rounds, allowing the observed trend to be assessed for consistency across estimation approaches. The robustness of results is evaluated through a structured sensitivity analysis addressing differential reporting accuracy, and the fraction of the population invisible to sibling histories. The resulting estimates are compared with GBD and WPP model-based mortality schedules to place the 2024 findings within the wider evidence on adult mortality in Nigeria.

2. Methods

2.1 Data Source and Survey Design

The data for this study were sourced from the 2018 and 2024 Nigeria Demographic and Health Surveys (NDHS) [12, 21]. The survey used a two-stage stratified cluster design, with enumeration areas (EAs) selected as primary sampling units (PSUs) by probability proportional to size, followed by systematic sampling of households within each EA. The sampling frame was stratified by the 36 states (each split into urban/rural) plus the Federal Capital Territory (urban only), yielding 73 strata. Eligible female household members aged 15–49 were interviewed.

Two analytical files were used: the Individual Recode (NGIR8BFL.dta), containing 39,050 respondents with survey weight (v005), PSU (v021), stratum (v023), interview date in CMC (v008), and age (v012); and the Sibling History Recode (NGSR8BFL.dta), containing 189,172 sibling records from 37,280 respondents in long format. The remaining 1,770 women (4.5%)

reported no maternal siblings and had no SR records; they were retained in the description of the full sample but contributed no deaths or exposure to rate estimation. Files were linked via caseid.

Survey weights (v005, divided by 1,000,000 per DHS convention) were applied throughout; design variables v021 and v023 were used in bootstrap variance estimation.

2.2 Sibling History Data

The sibling history module collected sex, survival status, current age (living siblings), and age or date of death (deceased siblings) for each respondent's maternal siblings. Records were standardised using DHS Phase 8 variables (mm1–mm16) via the sibhist_varmap_dhs8 crosswalk in the siblingsurvival R package. Survival status was recorded in mm2 (0 = deceased, 1 = alive, 8 = unknown); 25 records (0.01%) with unknown status were excluded, leaving 189,147 analytical records with no missing sex data. Sibship size ranged from 1 to 20 (mean 5.1, median 5), a distribution relevant to the visibility correction in **Section 3.4**.

The reference period was defined as the seven years (84 months) preceding each respondent's interview, consistent with standard DHS methodology. Deaths within this period were identified using the CMC date of death (mm8) relative to interview date (v008), yielding 3,427 sibling deaths.

Sibling records were included in rate estimation if the sibling contributed person-time between ages 15 and 49 during the reference period, with entry and exit ages placed on the Lexis plane using birth date, current age or age at death, and survival status. Exposure was split into five-year age bands; deaths outside the window did not count as events, though person-time within the window still counted as exposure. Siblings with no exposure in ages 15–49 during the window were dropped. Sample construction is summarized in **Table 1**.

Table 1. Construction of the NDHS 2024 sibling-history analytical sample

Characteristic	Value
IR file: women interviewed (unweighted)	39,050
Women with ≥1 sibling (linked to SR)	37,280 (95.5%)
Women with no siblings reported	1,770 (4.5%)
SR file: sibling records (unweighted)	189,172
SR file: sibling records (weighted)	193,684
Excluded: unknown survival status (mm2)	25 (0.01%)
Analytical sibling records	189,147
Mean siblings per respondent	5.1
Median siblings per respondent	5
Range of siblings reported	1–20
Siblings alive/dead	173,706 alive (91.8%); 15,441 dead (8.2%)
Reference period	7 years (84 months) preceding interview (≈2017–2024)
Sibling deaths in reference window	3,427

2.3 Data Quality Assessment

2.3.1 Age heaping: Age heaping in reported sibling ages at death was assessed using Whipple's Index (154.0) and Myers' Blended Index (8.04) [22]. The high Whipple's Index alongside the low Myers' Index indicates heaping concentrated at ages divisible by five, rather than pervasive digit preference, a pattern commonly seen in retrospective sibling data from sub-Saharan Africa [23]. To limit its influence, all analyses used five-year age groups, with estimates further stabilised by empirical Bayes shrinkage (**Section 2.5**).

2.3.2 Date telescoping: The distribution of reported time since death (mm6) was examined in single-month intervals around the 84-month reference window boundary. No anomalous clustering was observed before or after the cutoff, indicating minimal telescoping bias.

2.3.3 Internal consistency: Reporting symmetry was assessed following Feehan and Cobb (2019), using the frame population of women aged 15–49 described in **Section 2.2** [24]. For a given age

α, the number of connections reported by respondents aged α to siblings not aged α ($y_{F_\alpha,F_{-\alpha}}$)should, under perfect reporting, equal the number of connections reported by respondents not aged α to siblings aged α ($y_{F_{-\alpha},F_\alpha}$). The difference $\Delta_\alpha = y_{F_\alpha,F_{-\alpha}} - y_{F_{-\alpha},F_\alpha}$was estimated for each age band as a check independent of the visibility-adjusted estimator (**Section 2.4**). The null hypothesis for each age band is $H_0: \Delta_\alpha = 0$; failure to reject this null does not prove accurate reporting but supports its plausibility. Results are in **Section 3.4.1**.

2.4 Network-Based Individual Visibility Estimator

Conventional sibling history estimators assume every death and exposure-unit is equally likely to be observed. This assumption fails under sibling-history collection, since siblings from larger, more visible sibships are more likely to be reported. Following Feehan and Borges (2021), we therefore used a network-based individual visibility estimator as the primary approach [11]. The aggregate estimator M_{agg} was retained per cell as a diagnostic. This estimator was computed for the 2018 NDHS, where it reproduced the published NDHS 2018 adult mortality estimates exactly and was then applied to both survey rounds for the temporal comparison (**Section 2.8**) [12].

The frame population F comprised all women aged 15–49 residing in sampled households. Within this reporting frame, individuals differ in their visibility because siblings from larger sibships can be reported by more respondents than those from smaller sibships. This within-frame differential visibility was corrected using the individual visibility estimator, whereas adults with no living sisters in F fall outside the reporting frame entirely; this residual invisibility was addressed by estimating $\hat{p}_\alpha^I$directly (**Equation 11**) and via sensitivity analysis (**Section 2.7**).

For sibling j of respondent i, the age-band-specific exposure contribution was:

$$E_{ij}(\alpha) = \frac{1}{12} max\big(0, min\big(exit_{ij}, b_{ij} + 12\ (x+5), T_1\big)\ - max\big(entry_{ij}, b_{ij} + 12x,\ T_0\big)\ \big) \quad (1)$$

For each reported sibling, individual visibility was defined as the number of living female siblings aged 15–49, plus one for the respondent herself, who was also a member of the reporting frame and a potential reporter for the sibship.

$$y_{i,F} = (\sum_{j \in sib(i)} 1\ [alive_j = 1, sex_j = f, 15 \leq age_j\ \leq 49]) \quad (2)$$

The age-sex-specific individual visibility estimator was:

$$\hat{M}_a^{\text{ind}} = \frac{\sum_i w_i \left[\frac{y(i,D_a)}{y(i,F)+1}\right]}{\sum_i w_i \left[\frac{y(i,N_a \cap F)}{y(i,F)} + \frac{y(i,N_a - F)}{y(i,F)+1}\right]} \quad (3)$$

Here w_i is the respondent sampling weight, F is the survey frame population, $y(i,F)$ is the number of the respondent's living siblings in F, $y(i,D_a)$ is the number of reported sibling deaths in age cell a, and the denominator partitions reported exposure according to frame membership. The individual-visibility series was the primary input because it removes the aggregate estimator's requirement that mortality be unrelated to visibility within the visible population. The aggregate series was retained as a prespecified robustness analysis. As with all sibling-history estimators, inference from the visible population to total population mortality still relies on reporting-quality and invisibility assumptions.

2.5 Empirical Bayes Smoothing of Visibility-Adjusted Rates

The individual visibility estimator corrects differential visibility but treats each cell independently, so cell-specific schedules can be unstable where deaths are few. Empirical Bayes (EB) shrinkage was applied separately by sex to stabilize the age pattern while preserving the visibility correction.

The exchangeable prior is a working model for variance reduction, not a claim that all age groups share equal mortality risk; shrinkage moves each estimate only partially toward the sex-specific prior mean, retaining the observed age gradient.

The visibility-adjusted rate for each cell was:

$$\widehat{M}_{\alpha}^{VIS} = \frac{\widehat{D}_{\alpha}^{VIS}}{\widehat{N}_{\alpha}^{VIS}} \tag{4}$$

where the numerator and denominator are the survey-weighted, visibility-adjusted death count and exposure from **Section 2.4**.

On the log scale:

$$Z_{\alpha} = \log\left(\widehat{M}_{\alpha}^{VIS}\right),\ Z_{\alpha} \mid \theta_{\alpha} \sim N(\theta_{\alpha}, \sigma_{\alpha}^{2}), \qquad \tilde{\sigma}_{\alpha}^{2} = \frac{1}{\widehat{D}_{\alpha}^{VIS}} \tag{5}$$

where $\tilde{\sigma}_{\alpha}^{2}$ denotes a working Poisson-type approximation to the variance of the visibility-adjusted log-mortality estimate, and $\widehat{D}_{\alpha}^{VIS}$ is the survey-weighted, visibility-adjusted death count in cell α. Because $\widehat{D}_{\alpha}^{VIS}$ is a weighted quantity obtained under a complex survey design, $\tilde{\sigma}_{\alpha}^{2}$ is used as a variance proxy for determining relative shrinkage intensity rather than interpreted as an exact design-based sampling variance. Primary uncertainty quantification was therefore based on the Rao–Wu bootstrap described in Section 2.6.

A Gaussian exchangeable prior was placed on θ_{α}, with prior mean estimated empirically:

$$\theta_{\alpha} \mid \mu, A \sim N(\mu, A),\ \widehat{\mu} = \frac{\sum_{\alpha}\ p_{\alpha} Z_{\alpha}}{\sum_{\alpha}\ p_{\alpha}},\ p_{\alpha} = \frac{1}{\tilde{\sigma}_{\alpha}^{2}} \tag{6}$$

The working prior variance , which controls the degree of shrinkage, was selected by minimizing Stein's Unbiased Risk Estimate (SURE). SURE-based shrinkage selects the working prior

variance, and hence the degree of shrinkage, in a transparent, data-adaptive manner without requiring pre-specification of this tuning parameter. This approach was preferred over a fully Bayesian MCMC model because informative prior information on the between-age variance in adult mortality in this setting was unavailable. A nonparametric empirical Bayes approach based on marginal density estimation was also not adopted because each sex-specific mortality schedule comprised only seven age groups, making reliable density estimation infeasible.

The SURE criterion was:

$$SURE(A) = \sum_{\alpha} \left[-\tilde{\sigma}_{\alpha}^2 + \left\{ Z_{\alpha} - \hat{\theta}_{\alpha}(A) \right\}^2 + 2\tilde{\sigma}_{\alpha}^2 \left\{ w_{\alpha}(A) + [1 - w_{\alpha}(A)] \frac{p_{\alpha}}{P} \right\} \right] \qquad (7)$$

where,

$$w_{\alpha}(A) = \frac{A}{A + \tilde{\sigma}_{\alpha}^2}, \qquad p_{\alpha} = \frac{1}{\tilde{\sigma}_{\alpha}^2}, \qquad P = \sum_{\beta} p_{\beta}$$

and

$$\hat{\theta}_{\alpha}(A) = \hat{\mu} + w_{\alpha}(A)(Z_{\alpha} - \hat{\mu})$$

Because the prior mean is estimated from the same observed log-rates through **Equation 6**, the divergence term in **Equation 7** must additionally account for the dependence of ($\hat{\mu}$) on each Z_{α}. Specifically,

$$\frac{\partial \hat{\mu}}{\partial Z_{\alpha}} = \frac{p_{\alpha}}{P} = \frac{1/\tilde{\sigma}_{\alpha}^2}{\sum_{\beta} 1/\sigma_{\beta}^2}$$

and therefore differentiating $\hat{\theta}_{\alpha}(A) = \hat{\mu} + w_{\alpha}(A)(Z_{\alpha} - \hat{\mu})$ with respect to Z_{α},

$$\frac{\partial \hat{\theta}_\alpha}{\partial Z_\alpha} = w_\alpha(A) + [1 - w_\alpha(A)]\frac{p_\alpha}{P}$$

The treatment follows the general Stein divergence principle for a data-dependent shrinkage target and is analogous to the grand-mean and data-driven shrinkage-location formulations considered by Xie et al. (2012); here, the derivative is written explicitly for the precision-weighted empirical prior mean used in this analysis [18,25].

The resulting estimates were:

$$\hat{A} = \arg\min_{A \geq 0} \mathrm{SURE}(A), \qquad \widehat{w}_\alpha = \frac{\hat{A}}{\hat{A} + \tilde{\sigma}_\alpha^2}, \qquad \hat{\theta}_\alpha^{EB} = \hat{\mu} + \widehat{w}_\alpha(Z_\alpha - \hat{\mu}) \tag{8}$$

The EB-smoothed mortality rate was then obtained by back-transformation:

$$\widehat{M}_\alpha^{EB} = \exp\left(\hat{\theta}_\alpha^{EB}\right) \tag{9}$$

The working conditional posterior variance, used for diagnostic purposes, was computed as:

$$post_var_\alpha = \widehat{w}_\alpha \tilde{\sigma}_\alpha^2$$

This quantity reflects the reduction in log-scale dispersion implied by the working EB model and was not used for primary uncertainty quantification. Primary uncertainty quantification relied on the bootstrap confidence intervals described in **Section 2.6**, which additionally propagate uncertainty in the SURE-estimated between-cell variance $\hat{A}$, the EB shrinkage step, and the full pipeline. Male and female mortality schedules were smoothed separately, with a separate prior mean and between-cell variance estimated for each sex.

2.6 Variance Estimation and Bootstrap Flow

Within each stratum containing sampled primary sampling units (PSUs), $m_h = n_h - 1$PSUs were sampled with replacement, and the replicate weights were rescaled according to the Rao–Wu

procedure implemented in the surveybootstrap package[26, 27]. A total of B = 500 replicate weight vectors were generated.

The full estimation pipeline — visibility-adjusted rate calculation (Equation 3), log-transformation (**Equation 5**), and SURE-based empirical Bayes shrinkage (**Equations 6–9**) — was re-executed independently within each bootstrap replicate, using the replicate-specific weights in place of w_i throughout. Critically, the SURE hyperparameters ($\hat{\mu}$, $\hat{A}$) and shrinkage weights ($\widehat{w}_\alpha$) were re-estimated separately within each replicate rather than applied once to the full-sample point estimate. The final 95% confidence interval for each age-sex cell was obtained from the 2.5th and 97.5th percentiles of the resulting distribution of 500 EB-smoothed replicate estimates. This procedure propagates uncertainty from the complex survey design, the visibility-adjusted estimator, the estimated prior mean, the SURE-estimated between-cell variance, and the EB shrinkage step itself.

2.7 Sensitivity Analysis

Invisibility sensitivity addressed individuals entirely outside the reporting frame because they had no living sisters aged 15–49. Following Feehan and Borges (2021), the total mortality rate M_α relates to the empirical Bayes-smoothed estimate as [11]:

$$M_\alpha = \widehat{M}_\alpha^{EB}[1 + \hat{p}_\alpha^{l}(K-1)] \tag{10}$$

where K is the assumed mortality ratio between the invisible and visible populations. The invisible fraction in each age band was estimated as:

$$\hat{p}_\alpha^{l} = \frac{(female\ respondents\ aged\ \alpha\ with\ zero\ frame-eligible\ sisters)}{(total\ female\ respondents\ aged\ \alpha)} \tag{11}$$

Because sibship sex composition is assumed independent of a sibling's own sex (**Section 2.4**), the male invisible fraction was set equal to the female estimate at the same age:

$$\hat{p}^{I}_{(\alpha,male)} \approx \hat{p}^{I}_{(\alpha,female)}$$

Sensitivity bounds were computed for K ∈ {1.0, 1.5, 2.0, 3.0}, spanning no differential to a tripling of risk; K = 3.0 is deliberately extreme, included to test stability under severe assumptions rather than to represent a plausible true value.

Reporting-accuracy sensitivity was examined by varying the assumed ratio of death-reporting to exposure-reporting accuracy, $\gamma = \frac{\gamma_{D\alpha}}{\gamma_{N\alpha}}$, over γ ∈ {0.75, 0.85, 0.90, 0.95, 1.00, 1.05, 1.10, 1.15, 1.25}. This range spans the empirical benchmark of Helleringer et al. (2014), who estimated γ ≈ 0.95 from a record-linkage validation study in Senegal, and its reciprocal (γ ≈ 1.05), representing symmetric over- and under-reporting scenarios [9].

2.8 Temporal and External Comparison Data

EB-smoothed 2024 estimates were compared with four reference series:

(i) A re-estimation of the 2018 NDHS using the identical pipeline, including bootstrap uncertainty propagation (**Sections 3.2–3.6**), producing '$\hat{M}$_EB(2018)' with 95% bootstrap confidence intervals, directly comparable in construction and enabling the temporal trend to be assessed independently of estimator choice [12].

(ii) The conventional aggregate estimator (M_agg), implemented using the standard DHS sibling-survival procedure, was validated by reproducing the age- and sex-specific adult mortality estimates published in the 2018 NDHS final report exactly [12]. This validated procedure was then

applied to both the 2018 and 2024 NDHS microdata, since the published 2024 NDHS report does not include adult mortality estimates, providing an estimator-independent check on the temporal trend (**Section 3.5, Supplementary Table 3**) [21];

(iii) Age- and sex-specific all-cause mortality rates for Nigeria were obtained from the Global Burden of Disease Study 2023 (GBD 2023) Results (cause_id = 294, metric_id = 3) for calendar years 2017–2023. Rates were rescaled to per 1,000 population and averaged across years within each age group to obtain M_GBD [28];

(iv) UN World Population Prospects 2024 (WPP 2024) central death rates, m(x,n), by single year of age and sex for 2017–2024 were averaged within each five-year age group (15–19, 20–24, ..., 45–49) and across calendar year to obtain M_WPP [29]. These rates were harmonized to deaths per 1,000 person-years prior to comparison in **Table 6**. (**Section 3.5**).

2.9 Life Table Construction

Abridged period life tables were constructed separately by sex using the EB-smoothed rates for the seven age bands 15–19 through 45–49. Central death rates were converted to conditional probabilities of dying:

$$ {}_5q_x = 1 - \exp\left(-5 \times \hat{M}_x^{EB}\right) \tag{12} $$

with radix $l_{15} = 100{,}000$, ${}_d5_x = l_x \times {}_q5_x$, $l_{(x+5)} = l_x - {}_d5_x$, and person-years lived in each interval:

$$ {}_5L_x = 5 \times l_{x+5} + {}_5a_x \times {}_5d_x \tag{13} $$

For adult ages, ${}_5a_x$ = 2.5 was used in all seven intervals. The accumulated person-years lived between exact age x and exact age 50 were calculated as $T_x^{(50)} = \sum_{a=x}^{45} {}_5L_a$. Temporary life

expectancy from exact age x to exact age 50 was then calculated as ${}_{50-x}e_x = T_x^{(50)}/l_x$. The primary summary measure was temporary life expectancy between exact ages 15 and 50, denoted by ${}_{35}e_{15}$.

$$ {}_{35}e_{15} = \frac{\sum_{x=15,20,\ldots,45} {}_5L_x}{l_{15}} \tag{14}$$

Bootstrap confidence intervals for ${}_{35}e_{15}$ were obtained by propagating each of the 500 bootstrap replicate mortality schedules through the life table calculations and taking the 2.5th and 97.5th percentiles.

2.10 Software

All analyses were conducted in R version 4.3.1. Network-based visibility correction was implemented using the siblingsurvival package[30]. Rescaled bootstrap variance estimation used the surveybootstrap package[27].

3. Results

Results are presented in five stages, corresponding to the visibility correction (3.1), empirical Bayes shrinkage (3.2), life table construction (3.3), robustness and sensitivity analyses (3.4), and temporal and external validation (3.5). All mortality rates are expressed per 1,000 person-years of exposure, computed as visibility-adjusted deaths divided by visibility-adjusted person-years (**Equation 3**). Bootstrap confidence intervals are derived from 500 rescaled replicates, with the full pipeline repeated within each replicate (**Section 3.6**).

The analytical sample comprised 189,147 sibling records (**Table 1**), yielding 3,427 sibling deaths within the 84-month reference window.

3.1 Visibility Correction

Table 2 compares the conventional aggregate estimator (M_agg) against the individual visibility estimator (M_ind). Visibility ratios ranged from 0.847 to 1.122 across the 14 age-sex cells, with corrections in both directions rather than a uniform shift. The largest correction occurred among women aged 15–19, for whom M_ind was 15.3% lower than M_agg. This difference may reflect the boundary structure of the reporting frame and variation in the number of living frame-eligible sisters. Among males, the largest positive deviation occurred at ages 25–29 (ratio = 1.122), where the individual estimator exceeded the aggregate. These results indicate that accounting for differential visibility produces modest but non-negligible changes in the national age-sex-specific estimates. The estimated mortality rate was higher among males than females in most age groups, particularly at ages 40–44 and 45–49.

Table 2. Aggregate (DHS conventional) and individual visibility-adjusted mortality rates per 1,000 person-years, by age and sex, Nigeria 2024 DHS.

Sex	Age	M_agg	M_ind	Ratio
Female	15–19	1.73	1.46	0.847
	20–24	2.06	2.19	1.061
	25–29	2.15	2.2	1.021
	30–34	2.51	2.7	1.074
	35–39	3.15	2.84	0.902
	40–44	2.86	2.79	0.976
	45–49	3.43	3.59	1.045
Male	15–19	1.59	1.7	1.071
	20–24	1.86	1.69	0.909
	25–29	2.17	2.44	1.122
	30–34	3.03	2.93	0.968
	35–39	2.26	2.34	1.038
	40–44	3.97	4.22	1.062
	45–49	4.68	4.74	1.013

Note: Ratio = M_ind/M_agg. Values above 1.0 indicate that the aggregate estimator understates mortality relative to the visibility-adjusted estimate; values below 1.0 indicate the aggregate estimator overstates mortality.

3.2 Empirical Bayes Shrinkage

Table 3 presents SURE shrinkage parameters and final EB-smoothed rates with 95% bootstrap CIs. Effective death counts ranged from 37.3 to 92.6, with corresponding shrinkage weights reflecting this heteroscedasticity: female cells received substantially more shrinkage (12.9–24.6%) than male cells (5.9–9.3%), concentrated at the youngest and oldest age bands where death counts were smallest.

EB-smoothed mortality rates generally increased across adulthood, although both sex-specific schedules showed some age-group irregularities. Female mortality increased from 1.59 per 1,000 person-years at ages 15–19 to 3.25 at ages 45–49, with a slight decrease from 2.77 at ages 35–39 to 2.70 at ages 40–44. Male mortality increased from 1.76 per 1,000 person-years at ages 15–19 to 4.49 at ages 45–49, although lower rates were observed at ages 20–24 and 35–39 relative to the preceding age groups. A notable elevation was observed among men aged 40–44, where the mortality rate reached 4.09 per 1,000 person-years. Male mortality exceeded female mortality in most age groups and was particularly higher at ages 40–44 and 45–49. At ages 45–49, the male-to-female mortality rate ratio was 1.38.

Table 3. SURE-based Empirical Bayes shrinkage parameters and final smoothed mortality estimate with 95% bootstrap confidence intervals, by age and sex, Nigeria DHS 2024.

Sex	Age	D_hat (effective death count)	σ^2_α (log-rate variance)	$\hat{w}_\alpha$ (retention weight)	Shrinkage applied (%)	M_EB (per 1,000)	95% CI
Female	15–19	58.3	0.0172	0.827	17.30%	1.59	(1.22, 2.01)
	20–24	82.5	0.0121	0.871	12.90%	2.21	(1.76, 2.71)

	25–29	74.1	0.0135	0.859	14.10%	2.23	(1.8, 2.65)
	30–34	78.4	0.0128	0.865	13.50%	2.66	(2.18, 3.19)
	35–39	66.4	0.0151	0.845	15.50%	2.77	(2.28, 3.4)
	40–44	44.9	0.0223	0.787	21.30%	2.7	(2.18, 3.29)
	45–49	37.3	0.0268	0.754	24.60%	3.25	(2.45, 4.41)
Male	15–19	72.1	0.0139	0.925	7.50%	1.76	(1.35, 2.2)
	20–24	69.3	0.0144	0.923	7.70%	1.75	(1.44, 2.08)
	25–29	87.3	0.0115	0.938	6.20%	2.45	(2, 2.94)
	30–34	92.6	0.0108	0.941	5.90%	2.92	(2.44, 3.42)
	35–39	61.8	0.0162	0.914	8.60%	2.37	(1.86, 2.9)
	40–44	78.4	0.0128	0.931	6.90%	4.09	(3.28, 4.92)
	45–49	56.4	0.0177	0.907	9.30%	4.49	(3.52, 5.48)
Note: w^_α represents the proportion of the cell-specific log-mortality estimate retained after shrinkage; 1-w^_α represents the proportion shrunk toward the sex-specific empirical prior mean.							

3.3 Life Table

Table 4 presents the partial abridged life-table results between exact ages 15 and 50. Temporary life expectancy between exact ages 15 and 50 ($_{35}e_{15}$) was 33.68 years for women (95% CI: 33.54–33.81) and 33.63 years for men (95% CI: 33.51–33.73). Although these summary estimates were similar, the underlying age-specific mortality patterns differed by sex. Male mortality was higher than female mortality in most age groups and was notably higher at ages 40–44 and 45–49, whereas female mortality was higher at ages 20–24 and 35–39. The similarity in temporary life expectancy therefore reflects the relatively small absolute mortality differences over the restricted age interval rather than an absence of age-specific sex differences (**Section 4**).

Table 4. Partial abridged life table for adults between exact ages 15 and 50, by sex, Nigeria DHS 2024

Sex	Age	nMx	nqx	lx	ndx	nLx	ex(50)
Female	15–19	0.00159	0.00793	100,000	793	498,017	33.68
	20–24	0.00221	0.01101	99,207	1,092	493,304	28.93
	25–29	0.00223	0.01107	98,115	1,086	487,860	24.22
	30–34	0.00266	0.01319	97,029	1,280	481,946	19.46

	35–39	0.00277	0.01375	95,749	1,317	475,453	14.69
	40–44	0.00270	0.01340	94,432	1,266	468,996	9.86
	45–49	0.00325	0.01612	93,166	1,502	462,078	4.96
Male	15–19	0.00176	0.00876	100,000	876	497,809	33.63
	20–24	0.00175	0.00871	99,124	864	493,459	28.90
	25–29	0.00245	0.01218	98,260	1,197	488,308	24.13
	30–34	0.00292	0.01447	97,063	1,405	481,803	19.40
	35–39	0.00237	0.01178	95,658	1,127	475,473	14.65
	40–44	0.00409	0.02023	94,531	1,913	467,875	9.79
	45–49	0.00449	0.02219	92,619	2,055	457,955	4.94

3.4 Robustness and Sensitivity Analyses

3.4.1 Internal Consistency: Six of seven age bands showed symmetric reporting (ratios 0.94–1.09 (**Supplementary Table 1**); the exception, ages 15–19 (ratio = 0.83), also had the highest invisible fraction and the largest sensitivity to invisibility assumptions (below), flagging this cell as the least reliable in the schedule.

3.4.2 Invisibility Sensitivity: Table 5 presents sensitivity to the assumed mortality differential (K) between invisible and visible individuals. The invisible fraction showed the expected U-shape, from 15.6% (25–29) to 38.4% (15–19). Under K = 2.0, central age groups increased 10–22%, while the boundary ages (15–19, 45–49) increased by up to 62% under the extreme bound K = 3.0. Estimates are therefore robust to modest invisibility differentials but sensitive at the schedule's boundaries.

Table 5. Sensitivity of visibility-adjusted-EB mortality rates and to the invisible population mortality multiplier (K).

sex_label	age_label	$\hat{p}^I_\alpha$	K=1	K=1.5	K=2	K=3
Female	15–19	0.3838	1.59	1.9	2.2	2.82
	20–24	0.195	2.21	2.43	2.65	3.08
	25–29	0.1558	2.23	2.4	2.57	2.92

	30–34	0.1661	2.66	2.88	3.1	3.54
	35–39	0.1654	2.77	3	3.23	3.69
	40–44	0.2162	2.7	2.99	3.28	3.87
	45–49	0.3106	3.25	3.75	4.26	5.27
Male	15–19	0.3838	1.76	2.1	2.44	3.11
	20–24	0.195	1.75	1.92	2.09	2.43
	25–29	0.1558	2.45	2.64	2.83	3.22
	30–34	0.1661	2.92	3.16	3.4	3.88
	35–39	0.1654	2.37	2.57	2.76	3.15
	40–44	0.2162	4.09	4.53	4.97	5.86
	45–49	0.3106	4.49	5.19	5.88	7.28
Note: p_I (proportion invisible) is estimated from female respondents and is therefore identical across sex-specific age bands per Equation (11).						

3.4. Reporting-Accuracy Sensitivity: Supplementary Table 2 presents sensitivity to differential reporting accuracy (γ). Across the empirically grounded range ($\gamma \approx 0.90–1.10$; Helleringer et al., 2014), estimates varied by roughly ±10–20% without altering the ordinal age-sex pattern of mortality.

3.5 Temporal and External Comparison Data

Table 6 compares 2024 estimates against the 2018 re-estimation, GBD 2023, and WPP 2024.

Both female and male mortality estimates were lower in 2024 than in 2018 across all seven age groups (**Table 6**). For females, relative declines ranged from 2.4% at 15–19 to 8.0% at 20–24 at younger ages, rising to 48.6% at 40–44 and 39.5% at 45–49. For males, declines were smallest at 25–29 and 30–34 (4.1% and 6.4%) and largest at 40–44 and 45–49 (29.0% and 37.4%). Bootstrap 95% confidence intervals did not overlap in three of the fourteen age–sex cells (female 35–39, female 40–44, male 45–49), overlapped only marginally in three more (female 45–49, male 35–39, male 40–44), and overlapped substantially in the remaining eight.

The conventional aggregate and bias-corrected estimators agreed on the direction of the trend in 13 of the 14 age-sex groups (**Supplementary Table 3**), with a mean absolute difference of 3.9 percentage points in estimated percentage change, excluding the discordant group. The exception was females aged 15–19, where the aggregate estimator indicated an 8.1% increase against a 2.5% decline under the bias-corrected estimator.

The 2024 M_EB estimates were lower than the GBD estimates in 13 of the 14 age-sex cells and lower than the WPP estimates in all cells. WPP estimates were approximately 2.4–3.7 times the corresponding M_EB estimates, while GBD estimates generally fell between the M_EB and WPP values. These differences may reflect variation in data sources, reference periods, reporting assumptions, and modelling procedures across the three approaches (**Section 4**).

Table 6. Comparison of age-sex-specific adult mortality rates (per 1,000 person-years): this study vs external benchmarks, Nigeria

Sex	Age (years)	M_EB (2024)	95% CI (2024)	M_EB (2018)	95% CI (2018)	Relative diff. (%)	Diff	GBD avg (2017–2023)	WPP avg (2017–2024)
Female	15–19	1.59	(1.22, 2.01)	1.63	(1.29, 1.99)	−2.41	No	2.09	4.8
	20–24	2.21	(1.76, 2.71)	2.40	(2.04, 2.79)	−7.95	No	2.40	5.6
	25–29	2.23	(1.80, 2.65)	2.61	(2.25, 3.05)	−14.64	No	2.97	6.5
	30–34	2.66	(2.18, 3.19)	3.51	(2.91, 4.09)	−24.13	No	4.10	7.5
	35–39	2.77	(2.28, 3.40)	4.23	(3.63, 4.92)	−34.54	Yes	4.69	8.7
	40–44	2.70	(2.18, 3.29)	5.25	(4.32, 6.18)	−48.62	Yes	5.89	9.9
	45–49	3.25	(2.45, 4.41)	5.37	(4.16, 6.59)	−39.50	Marginal	6.26	10.9
Male	15–19	1.76	(1.35, 2.20)	2.07	(1.70, 2.52)	−15.05	No	2.01	4.4

	20–24	1.75	(1.44, 2.08)	2.06	(1.74, 2.39)	−15.03	No	2.26	6.4
	25–29	2.45	(2.00, 2.94)	2.56	(2.16, 3.00)	−4.14	No	2.38	6.6
	30–34	2.92	(2.44, 3.42)	3.12	(2.61, 3.60)	−6.41	No	3.14	7.1
	35–39	2.37	(1.86, 2.90)	3.23	(2.75, 3.74)	−26.66	Marginal	3.80	8.0
	40–44	4.09	(3.28, 4.92)	5.76	(4.80, 6.75)	−28.97	Marginal	6.33	9.8
	45–49	4.49	(3.52, 5.48)	7.17	(5.75, 9.04)	−37.34	Yes	7.62	11.9

Note: Relative difference (%) = $[\frac{\{M_{EB(2024)} - M_{EB(2018)}\}}{M_{EB(2018)}}] \times 100$; negative values indicate the 2024 estimate was lower. "Diff" indicates non-overlapping 95% bootstrap confidence intervals between survey rounds; "Marginal" indicates overlap of less than 15% of the narrower interval's width.

4. Discussion

This study developed and applied a network-adjusted empirical Bayes framework for estimating age-specific adult mortality from DHS sibling survival histories. The framework integrates individual visibility correction, heteroscedastic SURE-based shrinkage, and bootstrap-propagated uncertainty within a single estimation pipeline. It addresses two structurally distinct challenges in sibling-history estimation: unequal visibility within the reporting network and sampling variability arising from sparse age-sex-specific death counts. When the same visibility-adjusted pipeline, with bootstrap uncertainty propagated for both rounds, was applied retrospectively to the 2018 and 2024 Nigeria DHS data, mortality point estimates were lower in 2024 than in 2018 across all fourteen age-sex cells, with statistically distinguishable differences observed in three of the fourteen age–sex groups. The conventional aggregate estimator indicated the same direction of decline in all but one age-sex group (**Section 3.5**), showing that the decline is not simply an artefact of the bias-corrected pipeline, and that at least part of it reflects a genuine change rather than sampling noise

alone, even though most individual cells lack the precision to confirm this on their own. These findings extend previous research on the strengths and limitations of sibling survival histories for estimating adult mortality, particularly with respect to recall bias, selection mechanisms, and sampling variability [10, 31, 32].

The individual visibility estimator produced ratios (M_ind/M_agg) ranging from 0.847 to 1.122 across the 14 age-sex cells, with corrections in both directions rather than a uniform shift. The largest correction occurred among women aged 15–19, where the individual estimator revised the aggregate rate downward by over 15%, consistent with the structural boundary effect of the 15–49 reporting frame: younger respondents have had less time to accumulate surviving sisters within the frame, inflating their aggregate visibility [11]. Most age-sex specific visibility ratios differed from unity, indicating that individual visibility adjustment affected estimates in both directions. This finding aligns with the theoretical predictions of Feehan and Borges (2021) and with the empirical results they documented in Malawi and other settings, reinforcing the case for network-adjusted estimation as standard practice rather than an optional refinement [11].

Heteroscedastic SURE-based empirical Bayes shrinkage addressed a source of instability distinct from the visibility correctio [33, 34]. Effective death counts across the 14 age-sex cells ranged from 37.3 to 92.6, indicating that several direct cell-specific estimates were potentially sensitive to sampling variability. Retention weights ranged from 0.754 to 0.941. Female age groups generally had lower retention weights and therefore received greater shrinkage, ranging from 12.9% to 24.6%, compared with 5.9% to 9.3% among male age groups. This difference reflected the generally smaller effective death counts and greater estimated sampling variance in the female cells. Importantly, the shrinkage procedure retained the main age-pattern features observed in the direct visibility-adjusted estimates, including the comparatively high male mortality at ages 40–

44. Because the SURE hyperparameters were re-estimated within each bootstrap replicate, the reported confidence intervals incorporated uncertainty associated with the shrinkage step in addition to uncertainty arising from the complex survey design.

Retrospective application of the identical, bootstrap-propagated pipeline to the 2018 NDHS data enabled a temporal comparison free of estimator-choice confounding. The statistically distinguishable differences are concentrated among adults in their mid-30s and older, consistent with concurrent improvements in antiretroviral therapy coverage and HIV care, as well as broader public health gains, documented in Nigeria and sub-Saharan Africa during the study period, though the present analysis does not establish causality [21, 35–37]. These mechanisms cannot be confirmed from sibling history data alone, and changes in survey implementation, recall, or reporting completeness between rounds cannot be fully excluded.

A separate finding concerns estimator choice rather than survey-round comparison: at female ages 15–19, the aggregate estimator implied a mortality increase of 8.1% between 2018 and 2024, while the bias-corrected estimator showed a decline of 2.5%, an opposite-sign divergence occurring precisely at the cell independently flagged as internally inconsistent (**Section 4.4.1**) and having the highest invisible fraction in the schedule (**Section 4.4.2**). This demonstrates concretely how differential visibility bias can reverse the apparent direction of a trend at the age band most affected by it.

The systematic divergence between this study's estimates and the external reference series is the most substantively important pattern requiring interpretation. Empirical Bayes-smoothed rates fell below GBD 2023 estimates in 13 of 14 age-sex cells and below WPP 2024 projections by a factor of 2.4 to 3.7 across every cell, with GBD estimates consistently positioned between this study's

results and WPP. Part of this gap is plausibly explained by the study's own invisibility sensitivity analysis: under an assumed twofold mortality differential among individuals with no living frame-eligible sisters (K = 2.0), female mortality at 45–49 rises from 3.25 to 4.26 per 1,000 and male mortality from 4.49 to 5.88 — values that reduce part of the difference from the corresponding GBD estimates, suggesting that higher mortality among individuals without living frame-eligible sisters could account for part of the difference between the sibling-history estimates and the model-based reference series. The remaining difference may reflect variation in reference period and evidential inputs. WPP and GBD synthesise multiple data sources, regional patterns and historical trends, within modelling frameworks that impose smoothness across age and time, whereas the estimates presented here are derived directly from the 2017–2024 retrospective reference period. They also reflect the observed difference between the survey-based estimates from the 2018 and 2024 analyses. These discrepancies should not be interpreted as evidence that one source is necessarily more accurate than another; rather, they highlight differences between direct survey-based estimation and model-based mortality schedules.

Internal-consistency diagnostics indicated symmetric reporting across respondent subgroups in six of the seven age bands, with symmetry ratios ranging from 0.94 to 1.09. The exception was ages 15–19, which also had the highest estimated invisible fraction (38.4%) and showed the greatest sensitivity to assumptions about invisible-population mortality. This pattern may reflect the boundary structure of the 15–49 reporting frame, because younger respondents and their siblings have had less opportunity to enter or remain within the frame-eligible age range. Two sensitivity analyses—one addressing reporting accuracy using the regional record-linkage benchmark reported by Helleringeret al. (2014) and the other examining mortality differentials between visible and invisible populations—did not materially alter the broad age pattern of mortality or the

comparatively high male mortality observed at ages 40–49 [9]. Nevertheless, the estimate for ages 15–19 should be interpreted with greater caution than those for the other six age bands.

This study's principal strength is the first integration of network-based visibility correction with heteroscedastic SURE empirical Bayes shrinkage applied to sibling history data, with uncertainty propagated end-to-end through bootstrap re-estimation of the shrinkage hyperparameters. These strengths should be weighed against several limitations. National-level estimation, while appropriate for the primary contribution of this study, masks substantial regional heterogeneity in Nigerian adult mortality that could not be addressed within this analysis. The seven-year reference period spans the COVID-19 pandemic (2020–2022), whose possible transient mortality effects cannot be isolated within a period-averaged estimate. The male invisible fraction relied on a sibship-symmetry assumption unverifiable in this dataset, and both the reporting-accuracy and invisibility-differential parameters used in sensitivity analysis were bounded using the only available regional validation benchmark rather than Nigeria-specific data. Finally, the SURE minimisation was based on only seven age groups per sex. The small number of cells may limit the stability of the estimated between-age variance, although re-estimating the shrinkage parameters within each bootstrap replicate allowed this source of uncertainty to be reflected in the reported confidence intervals.

5. Conclusion

This study presents an integrated framework combining network-based visibility correction with heteroscedastic empirical Bayes shrinkage for estimating age-specific adult mortality from DHS sibling survival histories. The approach simultaneously addresses differential visibility within the

reporting frame and instability arising from sparse age-sex-specific death counts. Applied retrospectively to the 2018 and 2024 Nigeria DHS data, the framework confirms that Nigeria's apparent 2018–2024 mortality decline — concentrated among adults in their mid-30s to late 40s, where the difference is statistically distinguishable from sampling variability — is not simply an artefact of the bias-corrected pipeline, and is instead robust to estimator choice, while also demonstrating, in the one cell where the two estimators diverged, that uncorrected visibility bias can distort not just the magnitude but the apparent direction of a mortality trend. The framework requires no external mortality schedule for calibration and is directly applicable to other DHS datasets containing comparable sibling-history information. Further application of this approach across countries and survey rounds, together with continued investment in Nigeria's civil registration and vital statistics system, could contribute to stronger and more continuous adult mortality surveillance.

6. Declarations

6.1 Ethics approval and consent to participate

Ethical approval and informed consent were not required for this study because it involved a secondary analysis of anonymized, publicly available Demographic and Health Surveys (DHS) data. Access to the datasets was granted by the DHS Program upon approval of the registered research project. The DHS public-use datasets are IRB-approved, fully de-identified, and do not contain personal identifiers such as participants' names or household addresses. Geographic

information is restricted and intentionally masked to prevent identification of respondents or survey communities, ensuring participant confidentiality.

6.2 Consent for Publication

Not applicable.

6.3 Clinical trial number

Not applicable.

6.4 Availability of data and materials

The DHS datasets analyzed in this study are publicly available upon request from the DHS Program. Information on the DHS Program and procedures for accessing the survey data is available at the DHS Program website: https://dhsprogram.com/.

6.5 Competing interests

The authors declare that they have no competing interests.

6.6 Funding

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6.7 Author's Contributions

Shorful Alam: Conceptualization, Methodology, Data curation, Formal analysis, Investigation, Supervision, Project administration, Writing – Original Draft, Writing – review & editing. **Nazmul**

Haque: Methodology, Formal analysis, Writing – Original Draft. **Abdul Mazed:** Resources, Investigation, Writing – Original Draft. **Md. Sabbir Ahmed Mayen:** Writing – Original Draft, Writing – review & editing. **Md. Abu Kowser:** Resources, Investigation, Writing – Original Draft. **Mahadie Hassan:** Writing – Original Draft, Writing – review & editing

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