

Supplementary Material: On the synaptic matrix eigenvalues of sparsely connected neural networks

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The text here contains the tables of the moments displayed in the right most panel of figures 7-12 of main text.

TABLE I. Spectral moments for the **UCE Model** (Real Part). Comparison between empirical data moments and moments derived from the Skew-Student-t fit.

Unbalanced								Balanced			
(lr)3-6	(l)7-10	Case	Source	Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1			Data	-0.0300	0.6730	-16.2833	346.2123	-0.0007	0.1670	0.0050	2.1116
			Fit*	-0.0014	0.0847	-0.0360	0.9478	0.0016	0.0770	-0.0147	0.9443
2			Data	-0.0174	0.4393	-5.7357	85.6042	-0.0006	0.2796	0.0123	0.1624
			Fit*	0.0065	0.2883	0.0106	0.0531	-0.0012	0.2794	-0.0364	0.0103
3			Data	0.0025	0.5360	0.0383	-0.6221	-0.0004	0.5318	0.0150	-0.6403
			Fit*	0.0030	0.6251	-0.0070	-0.2851	0.0032	0.6204	-0.0064	-0.2870
4			Data	0.0379	1.9829	6.9878	111.4295	-0.0002	1.1401	0.0126	-0.8767
			Fit*	-0.0908	1.5305	-0.2019	-0.2255	0.0006	1.4136	-0.0069	-0.2851
5			Data	0.0537	3.1716	9.7897	175.1784	-0.00003	1.4728	0.0120	-0.8783
			Fit*	-0.1006	1.9643	-0.1893	-0.2336	0.0081	1.8326	-0.0152	0.2857

TABLE II. Comparison of empirical spectral moments and those obtained from the Skew-Student-t fit for the imaginary part of the UCE Model.

		Unbalanced				Balanced			
Case	Source	Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.0000	0.1569	0.0000	1.2790	0.0000	0.1616	0.0000	1.5251
	Fit*	-0.0047	0.0730	-0.1216	0.9600	-0.0067	0.0725	-0.1592	0.9877
2	Data	0.0000	0.2692	0.0000	-0.2890	0.0000	0.2729	0.0000	-0.1853
	Fit*	0.0003	0.3025	-0.0201	0.3347	0.0005	0.2879	0.0056	0.1766
3	Data	0.0000	0.5277	0.0000	-0.7796	0.0000	0.5228	0.0000	-0.8106
	Fit*	0.0042	0.6200	-0.0064	-0.2857	0.0036	0.6201	-0.0065	-0.2863
4	Data	0.0000	1.1119	0.0000	-0.9869	0.0000	1.1262	0.0000	-0.9483
	Fit*	0.0033	1.5320	-0.0074	-0.2868	0.0048	1.4168	-0.0060	-0.2852
5	Data	0.0000	1.4348	0.0000	-0.9769	0.0000	1.4563	0.0000	-0.9318
	Fit*	0.0037	1.9702	-0.0066	-0.2855	0.0066	1.8161	-0.0056	-0.2867

TABLE III. Similarly for PCE Model (Real Part). Comparison between empirical data moments and moments derived from the Skew-Student-t.

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.2651	0.9500	-0.2669	0.2851	0.2671	0.9452	-0.2613	0.2753
	Fit*	0.2366	1.0576	-0.6517	1.3858	0.2410	1.0485	-0.6338	1.3652
2	Data	0.2651	0.8789	0.0017	0.5458	0.2649	0.8736	0.0118	0.5315
	Fit*	0.3973	0.8822	0.8944	1.6848	0.3970	0.8740	0.9036	1.7025
3	Data	0.2651	2.7319	-3.9066	25.2408	0.2781	2.5403	-3.6587	22.8752
	Fit*	0.5167	1.1185	0.0131	0.4674	0.5265	1.1334	0.0360	0.5197
4	Data	0.2651	2.9475	0.4395	0.1598	0.2639	2.9537	0.4405	0.1604
	Fit*	0.2170	2.9070	0.2721	-0.2020	0.2115	2.8933	0.2576	-0.2083
5	Data	0.2651	43.2789	11.9146	187.6702	0.1433	26.6483	8.9541	117.3673
	Fit*	-0.2583	8.4958	0.0697	-0.2134	-0.2410	8.4265	0.0866	-0.2065

TABLE IV. Spectral moments for the PCE Model (Imaginary Part).

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	—	—	—	—	—	—	—	—
	Fit*	—	—	—	—	—	—	—	—
2	Data	0.0000	0.0431	0.0000	6.9312	0.0000	0.0432	0.0000	6.8793
	Fit*	0.0066	1.8161	-0.0056	-0.2867	0.0067	0.0001	1.0543	1.9807
3	Data	0.0000	0.4134	0.0000	1.2648	0.0000	0.4213	0.0000	1.2638
	Fit*	-0.0085	0.0986	-0.2201	1.0129	-0.0083	0.1046	-0.2063	1.0088
4	Data	0.0000	1.9053	0.0000	-0.3615	0.0000	1.8986	0.0000	-0.3628
	Fit*	0.0034	1.9372	-0.0234	-0.2878	0.0043	1.9349	-0.0206	-0.2872
5	Data	0.0000	6.8984	0.0000	-0.4838	0.0000	7.0027	0.0000	-0.4836
	Fit*	-0.0166	9.0812	-0.0584	0.9486	-0.0173	9.2447	-0.0585	0.9483

TABLE V. Spectral moments for the ECE Model (Real Part).

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.2651	0.9501	-0.2670	0.2850	0.2671	0.9453	-0.2614	0.2749
	Fit*	0.2355	1.0581	-0.6554	1.3875	0.2406	1.0494	-0.6359	1.3668
2	Data	0.2651	0.8668	0.0970	0.5585	0.2647	0.8624	0.1085	0.5410
	Fit*	0.4078	0.8861	1.1099	2.0919	0.4065	0.8792	1.1181	2.1107
3	Data	0.2651	3.2119	-3.5681	19.4531	0.2757	3.0823	-3.5163	19.3426
	Fit*	0.5269	1.1874	-0.0398	-0.0867	0.5324	1.1851	-0.0157	-0.0932
4	Data	0.2651	3.5915	0.7059	1.0151	0.2640	3.6000	0.7060	1.0152
	Fit*	0.1386	3.1119	0.2168	-0.2327	0.1310	3.1034	0.2007	-0.2385
5	Data	0.2651	50.6817	11.3385	164.4911	0.1559	36.4716	10.2416	143.0511
	Fit*	-0.3274	9.7741	-0.0352	-0.1871	-0.3390	9.9049	-0.0486	-0.1195

TABLE VI. Similarly for the ECE Model (Imaginary Part).

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	—	—	—	—	—	—	—	—
	Fit*	—	—	—	—	—	—	—	—
2	Data	0.0000	0.0844	0.0000	4.9828	0.0000	0.0842	0.0000	4.9961
	Fit*	-0.0090	0.0001	1.1068	2.0849	-0.0091	0.0001	1.1090	2.0897
3	Data	0.0000	0.4854	0.0000	1.1991	0.0000	0.4938	0.0000	1.2008
	Fit*	-0.0107	0.1043	-0.2494	1.0156	-0.0091	0.1177	-0.2151	1.0125
4	Data	0.0000	2.1162	0.0000	-0.3198	0.0000	2.1112	0.0000	-0.3088
	Fit*	0.0014	2.1178	-0.0310	-0.2875	0.0023	2.1170	-0.0269	-0.2879
5	Data	0.0000	7.9278	0.0000	-0.4848	0.0000	8.0549	0.0000	-0.4868
	Fit*	-0.0181	10.4365	-0.0592	0.9480	-0.0186	10.7106	-0.0586	0.9480

TABLE VII. Comparison of empirical spectral moments and those obtained from the Skew-Student-t fit for the real part of the SCE R=5 Model.

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.0000	0.5529	-0.3522	4.9663	0.0010	0.5513	-0.3179	4.8963
	Fit*	0.0549	0.3386	1.1365	2.1486	0.0544	0.3398	1.1369	2.1493
2	Data	0.0000	0.6025	-0.0008	2.6631	0.0004	0.6025	0.0247	2.6202
	Fit*	0.0778	0.5989	1.1477	2.1731	0.0774	0.5970	1.1519	2.1810
3	Data	-0.0000	0.8410	0.3172	0.4873	-0.0010	0.8442	0.3234	0.5079
	Fit*	0.0277	0.8215	0.5338	-0.0215	0.0240	0.8171	0.5264	-0.0269
4	Data	-0.0000	2.3477	0.6821	1.2598	-0.0047	2.3569	0.6689	1.2828
	Fit*	-0.1107	2.0591	0.0727	-0.3072	-0.1014	2.0709	1.0400	-0.2975
5	Data	-0.0000	3.4462	0.8147	1.8143	-0.0064	3.4560	0.7994	1.8289
	Fit*	-0.1885	2.7736	-0.0683	-0.2832	-0.1801	2.7680	-0.0526	-0.2850

TABLE VIII. Similarly for the SCE R=5 Model (Imaginary Part).

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.0000	0.3436	0.0000	1.0688	0.0000	0.3431	0.0000	1.0927
	Fit*	-0.0122	0.0959	-0.2896	1.0419	-0.0119	0.0954	-0.2867	1.0420
2	Data	0.0000	0.5134	0.0000	0.8433	0.0000	0.5120	0.0000	0.8618
	Fit*	-0.0097	0.2185	-0.1897	1.0115	-0.0098	0.2156	-0.1893	1.0096
3	Data	0.0000	0.8465	0.0000	0.6498	0.0000	0.8444	0.0000	0.6602
	Fit*	-0.0093	0.5091	-0.1469	0.9770	-0.0092	0.5064	-0.1471	0.9763
4	Data	0.0000	1.6440	0.0000	0.4886	0.0000	1.6423	0.0000	0.4877
	Fit*	-0.0111	1.3062	-0.1409	0.9727	-0.0105	1.3157	-0.1373	0.9710
5	Data	0.0000	1.9630	0.0000	0.4495	0.0000	1.9621	0.0000	0.4460
	Fit*	-0.0107	1.5917	-0.1412	0.9730	-0.0101	1.6114	-0.1380	0.9725

TABLE IX. Spectral moments for the SCE R=15 Model (Real Part). Comparison between empirical data moments and moments derived from the Skew-Student-t fit.

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	-0.0000	1.3563	-1.0098	7.8096	0.0035	1.3474	-0.8958	7.5749
	Fit*	0.0648	0.8275	0.8989	1.6930	0.0593	0.8172	0.8949	1.6856
2	Data	-0.0000	1.3911	-0.2173	3.0432	0.0020	1.3907	-0.1381	3.0224
	Fit*	0.0851	1.2780	0.8278	1.0759	0.0720	1.2674	0.7933	1.0363
3	Data	0.0000	1.8877	0.3341	0.2431	-0.0009	1.8940	0.3551	0.3004
	Fit*	0.0303	1.9333	0.4753	-0.0646	0.0320	1.9323	0.4795	-0.0613
4	Data	0.0000	5.7292	1.1450	4.1533	-0.0089	5.7148	1.1103	3.9450
	Fit*	-0.1759	4.8298	0.0659	-0.3091	-0.1763	4.8379	0.0742	-0.3070
5	Data	0.0000	8.7785	1.5236	6.1546	-0.0125	8.7290	1.4851	5.9475
	Fit*	-0.2884	6.4102	-0.0486	-0.2873	-0.2882	6.4045	-0.0473	-0.2871

TABLE X. Spectral moments for the SCE R=15 Model (Imaginary Part).

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.0000	0.7243	0.0000	0.9332	0.0000	0.7256	0.0000	0.9580
	Fit*	-0.0108	0.4674	-0.1413	0.9746	-0.0110	0.4672	-0.1430	0.9743
2	Data	0.0000	1.1143	0.0000	0.7430	0.0000	1.1115	0.0000	0.7556
	Fit*	-0.0118	0.9216	-0.1205	0.9602	-0.0108	0.9142	-0.1148	0.9605
3	Data	0.0000	1.8888	0.0000	0.5046	0.0000	1.8781	0.0000	0.5007
	Fit*	-0.0151	1.9557	-0.1076	0.9601	-0.0146	1.9453	-0.1052	0.9563
4	Data	0.0000	3.6136	0.0000	-0.1476	0.0000	3.6120	0.0000	-0.1601
	Fit*	0.0010	3.4923	-0.0320	-0.2884	0.0004	3.4853	-0.0324	-0.2880
5	Data	0.0000	4.3428	0.0000	-0.3592	0.0000	4.3528	0.0000	-0.3647
	Fit*	0.0056	4.4274	-0.0233	-0.2878	0.0049	4.4255	-0.0241	-0.2875

TABLE XI. Spectral moments for the SCE R=60 Model (Real Part), presented in the same manner as in the previous table

Case	Source	Unbalanced				Balanced			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1	Data	0.0000	4.6709	-2.8477	25.4958	0.0141	4.5038	-2.6729	24.8175
	Fit*	0.1359	2.6998	0.7305	1.4257	0.1344	2.6569	0.7513	1.4587
2	Data	0.0000	4.3722	-0.9147	7.8412	0.0090	4.3465	-0.8531	7.6341
	Fit*	0.1104	3.6343	0.4493	0.2205	0.1144	3.7040	0.5081	0.3665
3	Data	0.0000	5.7080	0.1816	-0.2105	-0.0012	5.7506	0.1817	-0.1629
	Fit*	0.0411	6.2544	0.3014	-0.1828	0.0323	6.2211	0.2828	-0.1944
4	Data	0.0000	19.9390	2.3125	15.9698	-0.0289	19.2402	2.1042	14.3655
	Fit*	-0.2384	15.3182	-0.0037	-0.2889	-0.2422	15.3146	-0.0066	-0.2856
5	Data	0.0000	32.1554	3.3085	25.1793	-0.0414	30.5577	3.0684	23.3997
	Fit*	-0.3212	20.1831	-0.0525	-0.2848	-0.3443	20.1868	-0.0685	-0.2814

TABLE XII. Spectral moments for the SCE R=60 Model (Imaginary Part).

(lr)3-6 (l)7-10	Case	Source	Unbalanced				Balanced			
			Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
1		Data	0.0000	2.2815	0.0000	0.3282	0.0000	2.3106	0.0000	0.4610
		Fit*	-0.0150	2.1205	-0.0929	0.9563	-0.0177	2.1311	-0.1149	0.9598
2		Data	0.0000	3.3071	0.0000	0.0950	0.0000	3.3243	0.0000	0.2013
		Fit*	-0.0081	3.6040	-0.0468	0.5594	-0.0102	3.5817	-0.0769	0.6133
3		Data	0.0000	5.6104	0.0000	-0.1513	0.0000	5.5724	0.0000	-0.1463
		Fit*	0.0078	5.5705	-0.0143	-0.2846	0.0073	5.5374	-0.0150	-0.2856
4		Data	0.0000	11.2975	0.0000	-0.8474	0.0000	11.4014	0.0000	-0.8244
		Fit*	0.0171	13.8310	-0.0077	-0.2855	0.0165	13.5914	-0.0103	-0.2861
5		Data	0.0000	14.3491	0.0000	-0.8743	0.0000	14.5317	0.0000	-0.8580
		Fit*	0.0197	18.0111	-0.0078	-0.2854	0.0192	17.6935	-0.0084	-0.2854