

Second-Order Sliding Mode Control of a Quadrotor UAV Based on a Multi-Channel Decomposition Model

Halima Boussadia

halima.boussadia@univ-usto.dz

USTO-MB University

Abir HASNAOUI

Ecole Supérieur en Génie Electrique et Energétique d'Oran

Kadda Zemalache Meguenni

USTO-MB University

Abdelkrim Meche

USTO-MB University

Nabil Boughanmi

USTO-MB University

Research Article

Keywords: Quadrotor UAV, Second Order Sliding Mode Control, Chattering

Posted Date: August 24th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10639177/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Second-Order Sliding Mode Control of a Quadrotor UAV Based on a Multi-Channel Decomposition Model

Halima Boussadia^{1,*}, Abir HASNAOUI², Kadda Zemalache Meguenni¹, Abdelkrim Meche¹ and Nabil Boughanmi³

¹ Department of Automatic, USTO-MB University, Oran, Algeria

² Ecole Supérieure en Génie Electrique et Energétique d'Oran, Oran, Algeria.

³ Department of Electronic, USTO-MB University, Oran, Algeria

* Correspondence: halima.boussadia@univ-usto.dz

Abstract: In quadrotor unmanned aerial vehicle (UAV) systems, chattering is a well-known disadvantage of sliding mode control (SMC). Moreover, quadrotors are frequently subjected to aerodynamic perturbations such as wind gusts, which can drastically degrade flight performance if not properly compensated. To solve these problems, this work proposes a novel control strategy that combines a second-order sliding mode control system with a multi-channel decomposition of the quad-rotor dynamics.

In order to develop specialized controllers for each motion channel, the quadrotor model is first decomposed into independent subsystems. After that, a super-twisting algorithm is used to ensure continuous control action while preserving the robustness properties of the second order sliding mode control. The proposed method is designed to effectively suppress both chattering phenomenon and disturbance forces acting on the dynamic of the quadrotor. Numerical simulations under various reference trajectories and external disturbances are used to validate the control strategy's efficacy. The results show fast convergence, strong robustness, and better performance compared to classical sliding mode control.

Keywords: Quadrotor UAV, Second Order Sliding Mode Control, Chattering.

1 Introduction

The evolution of unmanned aerial vehicle (UAV) systems has accelerated in recent years. The quadrotor UAVs have become an important research area because of their large domain of applications [1-2].

The quadrotor UAV features a symmetrical airframe, four rotors integrated with onboard data processing, and high-performance propellers. These characteristics have made it a subject of great interest in recent research efforts [3]. UAVs have played an important role in a wide range of applications, including military operations, thanks to their notable advantages over manned aerial vehicles, such as lower cost and greater maneuverability [4]. As highlighted by [5] and [6], UAV uses include picture and video mapping, construction, medical services, parcel delivery, search and rescue missions, wireless communication, aerial surveillance, and exploration of inaccessible locations. Furthermore, they are used in precision agriculture as well as in monitoring oil rigs and power lines [7].

Control design for quadrotor UAV systems has been a topic of active research in recent years due to their important potential applications. This system is a type of powered aircraft that does not carry a human pilot, and can fly autonomously or be remotely controlled. The first reported quadrotor helicopter was built in 1907 by the Breguet Brothers [8]. However, there was no means of control provided to the pilot other than a throttle for the engine to change the rotor speed, and the stability of the machine was found to be very poor. The machine was subsequently tethered so that it could move only vertically upward. In recent years, UAV technology has experienced a rapid growth in popularity. This has been driven by advancements in computing and positioning technologies as well as the benefits UAVs could offer us. Moreover, UAV field seems to step out of the exclusivity of the military applications, and a lot of potential civil applications have emerged. Examples of applications include fire ground surveillance, agriculture and film production, management of the large infrastructures, recognition and surveillance vehicles, and search and rescue operations. For these reasons, the automatic control of a quadrotor UAV, the path tracking problem and attitude stabilization, has attracted the attention of many researches in the past few years [8].

The X4 flyer is an engine of 68cm*68cm of total size and not exceeds 2Kg in mass. It is designed in a cross form and made of carbon fiber. Each tip of the cross has a rotor including an electric brushless motor, a speed controller and two-blade ducted propeller. The operating principle of the X4 can be presented thus: two rotors turn clockwise, and the two other rotors turn counterclockwise to maintain the total equilibrium in yaw motion.

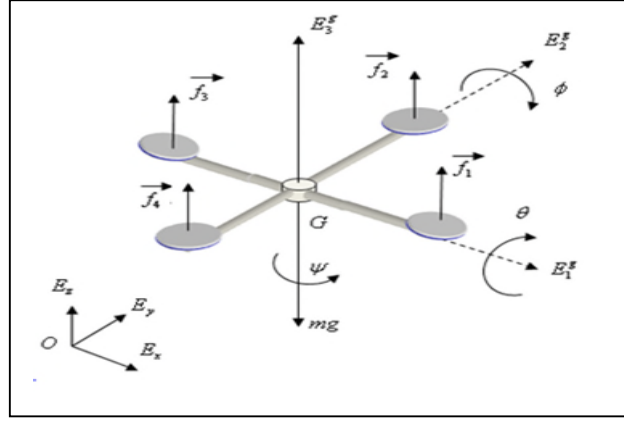


Fig. 1 Each figure should.

Many strategies have been proposed in literature to stabilize the four rotors and to generate its trajectory, such as design due to its distinct advantages. However, it suffers from a disadvantage called the "chattering" phenomenon [9], which represents a major obstacle to the practical implementation of SMC in engineering applications [10]. To eliminate this phenomenon, researchers have carried various studies [11–13]. In previous methods, the discontinuous components were replaced with continuous functions, which minimized chattering. However, this solution often came at the cost of significantly diminished control performance [14]. A variable structure terminal guidance law incorporating an adaptive fast-reaching law and a filter method based on a sliding mode observer was proposed in [15]. Wu et al. proposed the use of slack matrices feedback linearization control, visual control, backstepping control and sliding-mode control.

Sliding Mode Control (SMC), as a variable structure control technique, is particularly well-suited for quadrotor UAV system along with a delay-dependent sufficient condition to ensure that the sliding mode dynamics are generalized, quadratically stable, and robustly passive [16, 17]. Moreover, Wu et al. designed an integral sliding surface function along with an observer to estimate the system states, and it was demonstrated that these states could be reached in finite time [18, 19]. An integral-type sliding surface function is developed to establish the sliding mode dynamics, which can be modeled as a switched stochastic system subject to external disturbances and uncertainties [20]. However, these control methods are based on the classical variable structure control theory, which is not well suited for quadrotor UAV systems due to the presence of the chattering phenomenon. Therefore, it is important to explore advanced sliding mode control strategies that not only ensure strong robustness but also enable continuous control action. Shtessel et al. proposed a second-order sliding mode variable structure control algorithm, which demonstrated strong robustness and effectively eliminated the chattering phenomenon [21–23]. However, the high-order sliding mode control algorithm has certain limitations, such as the complexity of algorithm. To overcome this limitation, higher-order sliding mode (HOSM) techniques have been introduced. In particular, second-order sliding mode control algorithms, such as the super-twisting algorithm. When compared to classical SMC, these techniques have shown better results in reducing chattering and improving tracking accuracy even under severe disturbance forces. However, because of the high interaction between system dynamics, its application to quadrotor systems often results in increased design complexity. By dividing system dynamics into independent subsystems, model decomposition approaches provide an appealing solution in this situation by simplifying controller design, and developing different controllers for each motion channel, which reduces complexity and improves implementation performances. This paper proposes a novel control strategy that combines a multi-channel decomposition of the quadrotor dynamics with a second-order sliding mode control scheme based on the super-twisting algorithm. The main objective is to reduce the complexity of controller design while ensuring high robustness and eliminating the chattering phenomenon.

This paper addresses the chattering phenomenon in quadrotor UAV tracking systems. To tackle this problem, the quadrotor UAV motion is decomposed into five-channel subsystems, and a second-order sliding mode control, specifically the super-twisting algorithm is proposed, which has fast convergence and strong robustness.

The rest of this paper is organized in five sections, the second section relates the modeling of the X4, and a five-channel decomposition model is proposed. The following section presents the design of the five-channel controllers. Section 5 evaluates the control performance through simulations. Finally, Section 6 concludes the paper.

2 Dynamic Model of a Quadrotor

This section presents the dynamic model of the X4 quadrotor drone. The model is developed using the Lagrange-Euler approach, while considering the following simplifying assumptions:

- The carbon fiber assumed to be rigid.
- The drone is symmetrical, implying a diagonal inertia matrix.
- The thrust and drag forces generated by the motors are proportional to the square of the angular velocity.
- The center of gravity coincides with the origin of the body-fixed frame.

The mathematical model of X4 quadrotor drone is defined by both the translation and rotation equations. The translation motion is modeled by the following equations:

$$m\ddot{x} = -S_\theta u_3 \quad (1.a)$$

$$m\ddot{y} = C_\phi S_\theta u_3 \quad (1.b)$$

$$m\ddot{z} = C_\phi C_\theta u_3 - mg \quad (1.c)$$

Where $\xi = (x, y, z) \in \mathbb{R}^3$ represents the position of the helicopter's center of mass, $\eta = (\theta, \phi, \psi) \in \mathbb{S}^3$ are the three Euler angles (roll, pitch, and yaw) representing the orientation of the helicopter in the body-fixed frame, m represents the mass, and u_3 is the thrust generated by the four motors.

And, the rotation motion is modeled by:

$$\ddot{\phi} = \frac{1}{I_{yy}C_\phi C_\theta} (\tau_\phi + I_{yy}S_\phi C_\theta \dot{\phi}^2 + I_{yy}S_\theta C_\phi \dot{\theta} \dot{\phi}) \quad (2.a)$$

$$\ddot{\theta} = \frac{1}{I_{xx}C_\phi} (\tau_\theta + I_{xx}S_\phi \dot{\phi} \dot{\theta}) \quad (2.b)$$

$$\ddot{\psi} = \frac{\tau_\psi}{I_{zz}} \quad (2.c)$$

Where $I = \text{diag}[I_{xx}, I_{yy}, I_{zz}]$ is the inertia matrix, and $(\tau_\phi, \tau_\theta, \tau_\psi)$ are the generalized torques.

Since the translation subsystem is completely independent of the rotation subsystem, the model presented in Eq. (1) and Eq. (2) is reduced to a simplified form, also called decoupled dynamic.

The decoupled dynamic model of rotation can be written as:

$$\begin{cases} m\ddot{x} = -S_\theta u_3 \\ m\ddot{y} = C_\theta S_\phi u_3 \\ m\ddot{z} = C_\phi C_\theta u_3 - mg \\ \ddot{\theta} = u_4 \\ \ddot{\phi} = u_5 \\ \ddot{\psi} = u_6 \end{cases} \quad (3)$$

The vectors u_3, u_4, u_5 and u_6 combines the principal non conservative forces applied to the engine airframe including forces generated by the motors and drag terms.

The resulting model is nonlinear, characterized by a linear translation motion, and a nonlinear rotation motion.

2.1 Decomposition of Mathematical Model

In this session, the quadrotor dynamics are decomposed into multiple subsystems, each associated with a specific motion channel. This decomposition allows the design of independent controllers for each channel, significantly reducing the complexity of the control problem.

Each subsystem can be expressed in the following nonlinear form:

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \dot{x}_{i+1} = f_i(x_i, x_{i+1}) + h_i U_j \end{cases} \quad (4)$$

where:

x_i represents the state vector of the i^{th} subsystem,

U_i is the corresponding control input.

136 2.1.1 Vertical Motion Subsystem

137 The vertical motion (altitude) is governed by:

$$\ddot{z} = \frac{1}{m} (C_\phi C_\theta u_3) - g \quad (5)$$

138 We define the state variables: $x_1 = z$ and $x_2 = \dot{z}$

139 The state-space representation becomes:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{\cos \theta \cos \phi}{m} U_3 - g \end{cases} \quad (6)$$

140 2.1.2 Horizontal Motion Subsystems

141 The horizontal motions along the x and y axes are described by:

$$m\ddot{x} = -S_\theta u_3 \quad (7)$$

$$m\ddot{y} = C_\theta S_\phi u_3 \quad (8)$$

142 Define the state variables: $x_3 = x$, $x_4 = \dot{x}$, $x_5 = y$, and $x_6 = \dot{y}$

143 Thus, the state-space form is:

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{-\sin \theta}{m} U_3 \end{cases} \quad (9)$$

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{\cos \theta \sin \phi}{m} U_3 \end{cases} \quad (10)$$

144 2.1.3 Attitude Subsystems

145 The rotational dynamics are decomposed into three independent channels corresponding to roll, pitch, and yaw mo-
146 tions.

147 2.1.3.1 Channel Subsystem of Roll angle

148 The channel subsystem of Roll angle is given as follows:

$$\begin{cases} \dot{x}_7 = x_8 \\ \dot{x}_8 = U_4 \end{cases} \quad (11)$$

149 2.1.3.2 Channel Subsystem of Pitch angle

150 The channel subsystem of pitch angle is described by:

$$\begin{cases} \dot{x}_9 = x_{10} \\ \dot{x}_{10} = U_5 \end{cases} \quad (12)$$

151 2.1.3.3 Channel Subsystem of Yaw angle

152 The channel subsystem of Yaw angle is expressed as:

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = U_6 \end{cases} \quad (13)$$

153 3 Sliding mode control

154 Sliding Mode Control (SMC) is a control strategy derived from Variable Structure Control (VSC) [3]. The main objec-
155 tive of SMC is devised two stages. In the first stage, a discontinuous control law is designed to enforce the error vector
156 toward the sliding surface. In this phase, the control action switches between the two sides of the sliding surface. In the
157 second stage, once the error vector reaches and remains on the sliding surface, the system follows the dynamics de-
158 fined by the sliding surface equations. This component of the controller is referred to as the equivalent control [6].

159 The sliding surface equation determines the dynamics of the system, it is defined as:

$$S = \dot{e} + \lambda e \quad (15)$$

160 Where λ is a positive tuning parameter.

161 The control law $U(t)$ is composed by two components: a continuous component $u_{eq}(t)$, and a discontinuous compo-
162 nent $u_d(t)$. Thus, it can be expressed as follows:

$$U(t) = u_{eq}(t) + u_d(t) \quad (16)$$

163 The first component depends on the controlled variable and the reference value. The second component includes a
164 nonlinear term containing a switching element.

$$u_d(t) = k_d \text{sign}(S(t)) \quad (17)$$

165 k_d is a tuning parameter design.

166 The chattering phenomenon [6,7] can be reduced by represented Eq. (17) as follows:

$$u_d(t) = k_d \text{sat}(S(t)) \quad (18)$$

167 3.1 Sliding mode control of the translation subsystem

168 The altitude, can be controlled by the SMC controller. With through the equation of the following movement z .

$$m\ddot{z} = C_\theta C_\phi u_3 - mg \quad (19)$$

169 The surface is deduced from the [27, 28] here the degrees of the sliding surface ^{r} equal to 2 so that one obtain:

$$S(z) = \dot{e}_z + \lambda_z e_z \quad (20)$$

170 Where,

$$e_z = z_{ref} - z \quad (21)$$

$$\dot{e}_z = \dot{z}_{ref} - \dot{z} \quad (22)$$

$$\ddot{e}_z = \ddot{z}_{ref} - \ddot{z} \quad (23)$$

171 As result the surface derivative is:

$$\dot{S}(z) = \ddot{e}_z + \lambda_z \dot{e}_z \quad (24)$$

172 When the sliding mode occurs, the surface $S(z)$ became null also its derivative. That gives the control:

$$u_{eq} = \frac{m}{C_\theta C_\phi} \left(\dot{z}_{ref} + g + \lambda_z (\dot{z}_{ref} - \dot{z}) \right) \quad (25)$$

173 During de convergence mode we have to satisfy the condition $S(z)\dot{S}(z) < 0$ by choosing:

$$u_d = -k_d \text{sat}(S(z)) \quad (26)$$

174 So that, the output command of the altitude is given as follows:

$$u_3 = \frac{m}{C_\theta C_\phi} \left(\dot{z}_{ref} + g + \lambda_z (\dot{z}_{ref} - \dot{z}) \right) - k_z \text{sat}(s(z)) \quad (27)$$

175 From the model presented in Eq. (3),we can see that the motion through the axes x and y depends on u_3 . In fact u_3 is
176 the total thrust vector oriented to obtain the desired linear motion. If we considered $u_x = S_\theta$ and $u_y = C_\theta S_\phi$ the orien-
177 tations of u_3 responsible for the motion through x and y axis respectively, we can then extract the roll and pitch angle
178 necessary to compute the control u_x and u_y .

$$u_x = \frac{-m}{u_3} \left(\dot{x}_{ref} + \lambda_x (\dot{x}_{ref} - \dot{x}) \right) - k_x \text{sat}(s(x)) \quad (28)$$

$$u_y = \frac{-m}{u_3} \left(\dot{y}_{ref} + \lambda_y (\dot{y}_{ref} - \dot{y}) \right) - k_y \text{sat}(s(y)) \quad (29)$$

179 3.2 Sliding Mode Control of the Rotations Subsystem

180 The SMC is applied to roll angle, in such a way to obtain simple surface. The degrees of the sliding surface r equal
181 to 2 so that one obtain:

$$S(\Phi) = \dot{e}_\Phi + \lambda_\Phi e_\Phi \quad (30)$$

182 Where,

$$e_\Phi = \Phi_{ref} - \Phi \quad (31)$$

$$\dot{e}_\Phi = \dot{\Phi}_{ref} - \dot{\Phi} \quad (32)$$

$$\ddot{e}_z = \ddot{\Phi}_{ref} - \ddot{\Phi} \quad (33)$$

183 As result the surface derivative is:

$$\dot{S}(\Phi) = \dot{e}_\phi + \lambda_\phi \dot{e}_\phi \quad (34)$$

184 When the sliding mode occurs, the surface $S(\Phi)$ became null as well as its derivative. That gives the control:

$$u_{eq} = (\ddot{\Phi}_{ref} - \lambda_\phi (\dot{\Phi}_{ref} - \dot{\Phi})) \quad (35)$$

185 During the convergence mode we have to satisfy the condition $S(\Phi)\dot{S}(\Phi) < 0$ by choosing:

$$u_d = -k_\phi \text{sat}(S(\Phi)) \quad (36)$$

186 This leads to a sliding mode controller for Φ control given by:

$$u_5 = (\ddot{\Phi}_{ref} - \lambda_\phi (\dot{\Phi}_{ref} - \dot{\Phi})) - k_\phi \text{sat}(S(\Phi)) \quad (37)$$

187 The same steps are followed to extracted

$$u_4 = (\ddot{\theta}_{ref} + \lambda_\theta (\dot{\theta}_{ref} - \dot{\theta})) - k_\theta \text{sat}(s(\theta)) \quad (38)$$

$$u_6 = (\ddot{\psi}_{ref} - \lambda_\psi (\dot{\psi}_{ref} - \dot{\psi})) - k_\psi \text{sat}(s(\psi)) \quad (39)$$

188 4 Higher-order sliding control

189 To reduce or avoid the chattering phenomenon, High-Order Sliding Mode (HOSM) can be used. They generalize the
190 basic sliding mode concept by acting on higher-order time derivatives of the system deviation from the constraint, in-
191 stead of influencing only the first derivative of the deviation as in standard sliding mode [34]. HOSM have the ad-
192 vantage of completely eliminating the chattering effect while preserving the main advantages of the original approach,
193 such as robustness. Several algorithms exist to implement HOSM. In this work, we focus on a specific algorithm, called
194 super-twisting algorithm.

195 The principle of higher-order sliding mode control is to force the system to evolve on a sliding surface defined by an-
196 other sliding variable $S(x, t)$ with a relative degree "r". If r is greater than 1, a control law of the form $U = -K \text{sign}(s)$ is no
197 longer acceptable. Therefore, an alternative control structure must be found. A simple solution is to consider the
198 first $(r - 1)$ derivatives of the sliding variable in the control design, as described in [24].

$$\bar{S} = S^{r-1} + \dots + c_2 S^2 + c_1 S^1 + c_0 \quad (40)$$

199 This gives the command:

$$U = -K \text{sign}(\bar{S}) = -K \text{sign}(S^{r-1} + \dots + c_2 S^2 + c_1 S^1 + c_0 S) \quad (41)$$

200 With $c_{r-1}, \dots, c_1, c_0$: are coefficients.

201 The supertwisting algorithm [25, 26] is one of the simplest and most widely used among second-order sliding mode
202 (SOSM) algorithms. There are two ways to apply the twisting algorithm [25, 26]: either to the main system dynamics of
203 relative degree two, or to the main dynamics of relative degree one with an integrator introduced in series with the
204 plant). For principal dynamics of relative degree two, it can be formulated as follows: let the plant (or the plant plus ac-
205 tuator) be described by the following differential equations, which include both principal and parasitic dynamics.

206 This algorithm can therefore be regarded as a nonlinear generalization of a proportional-integral (PI) control strategy. If
207 the physical system is not of relative degree two and S is in fact derived from an output-based sliding surface, the ap-
208 proach can be generalized to a nonlinear proportional-integral-derivative (PID) control strategy.

209 The control algorithm is given by the control law [27]:

$$u(t) = -\alpha |S|^q \text{sign}(S_i) + \int -\beta \text{sign}(S_i) \quad (42)$$

210 The second-order sliding mode control (SOSMC) technique based on the super-twisting algorithm is developed for the
211 quadrotor UAV. The design relies on the multi-channel decomposition introduced in Section 2.

212 The control design procedure is detailed for the vertical subsystem only. The same methodology can be extended to the
213 horizontal and attitude subsystems.

214

215

216 4.1 Controller Design of vertical Subsystem (Altitude Control)

217 The dynamic of the vertical motion is defined by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{\cos \theta \cos \phi}{m} U_3 - g \end{cases} \quad (43)$$

218 The sliding surface is chosen as:

$$S(z) = \left(\frac{d}{dt} + \lambda_z \right) e(z) \quad (44)$$

219 where the error and its derivative are given by:

$$\begin{cases} e(z) = z_d - z \\ \dot{e}(z) = \dot{z}_d - \dot{z} \end{cases} \quad (45)$$

220 Substituting these expressions into the sliding surface yields:

$$S(z) = \lambda_z (z_d - z) + \dot{z}_d - \dot{z} \quad (46)$$

221 The time derivative of the sliding surface is then expressed as:

$$\dot{S}(z) = \lambda_z (\dot{z}_d - \dot{z}) + \ddot{z}_d - \ddot{z} \quad (47)$$

222 By substituting the system dynamics into the above equation, the derivative of the sliding surface becomes:

$$\dot{S}(z) = \lambda_z (\dot{z}_d - \dot{z}) + \ddot{z}_d - \frac{\cos \theta \cos \phi}{m} U_3 + g \quad (48)$$

223 It can be observed that the control law explicitly appears in the expression of $\dot{S}(z)$, which leads us to introduce the super-twisting law.

$$u_3 = - \frac{m}{\cos \theta \cos \phi} [-g - \ddot{z}_d - \lambda_z (\dot{z}_d - \dot{z}) + \dot{S}(z)] \quad (49)$$

225 The super-twisting control is chosen as in (45):

$$W_{st,3}(S_z) = \dot{S}(z) = -\alpha |S|^{1/2} \text{sign}(S_z) + \int -\beta \text{sign}(S_z) \quad (50)$$

226 Therefore,

$$u_3 = \frac{m}{\cos \theta \cos \phi} [g + \ddot{z}_d + \lambda_z (\dot{z}_d - \dot{z}) + \alpha |S|^{1/2} \text{sign}(S_z) + \int \beta \text{sign}(S_z)] \quad (51)$$

227 The other subsystems are developed in a similar manner, and their control laws are obtained following the same procedure.

$$u_1 = \frac{m}{U_3} [-\ddot{x}_d - \lambda_x (\dot{x}_d - \dot{x}) - \alpha |S|^{1/2} \text{sign}(S_x) + \int \beta \text{sign}(S_x)] \quad (52)$$

$$u_2 = \frac{m}{U_3} [\ddot{y}_d + \lambda_y (\dot{y}_d - \dot{y}) + \alpha |S|^{1/2} \text{sign}(S_y) + \int \beta \text{sign}(S_y)] \quad (53)$$

$$u_5 = [\ddot{\Phi}_d + \lambda_\Phi (\dot{\Phi}_d - \dot{\Phi}) - \alpha |S|^{1/2} \text{sign}(S_\Phi) + \int \beta \text{sign}(S_\Phi)] \quad (54)$$

$$u_4 = [\ddot{\Theta}_d + \lambda_\Theta (\dot{\Theta}_d - \dot{\Theta}) - \alpha |S|^{1/2} \text{sign}(S_\Theta) + \int \beta \text{sign}(S_\Theta)] \quad (55)$$

$$u_6 = [\ddot{\Psi}_d + \lambda_\Psi (\dot{\Psi}_d - \dot{\Psi}) - \alpha |S|^{1/2} \text{sign}(S_\Psi) + \int \beta \text{sign}(S_\Psi)] \quad (56)$$

229 5 Simulation Results

230 For simulation tests, a quadrotor with a mass of 2 kg is considered, the initial values of position and angle of the
231 quadrotor are [0,0,0]m and [0,0,0]rad. In addition, the controller parameters are listed in Table1.

232 **Table1** Controller parameters.

Variables	Values	Variables	Values
λ_1	1	α_5	6
β_1	0.1	λ_7	40
α_1	30	β_7	12
λ_3	2.5	α_7	80
β_3	10	λ_8	0.5
α_3	4	β_8	3
λ_5	3	α_8	0.1
β_5	20		

233 At first, we work with a straight-line trajectory along the z, x, and y axes. The drone must fly in the z direction (vertical
234 flight), followed by motion along the x axis, and then along the y axis.

235 The reference trajectory for the vertical flight is defined by the following equation:

$$z^r(t) = h_d \frac{t^5}{t^5 + (T_f^1 - t)^5} \quad (57)$$

With h_d is the desired altitude of 10m, and T_f^1 is the final time. The final value of one flight phase is the initial value of the subsequent motion, the reference trajectory along x can be written as follows:

$$x^r(t) = h_d \frac{(t - T_f^1)^5}{(t - T_f^2)^5 + (T_f^1 - (t - T_f^2))^5} \quad (58)$$

The motion along y can be expressed by the following formula:

$$y^r(t) = h_d \frac{(t - T_f^2)^5}{(t - T_f^3)^5 + (T_f^2 - (t - T_f^3))^5} \quad (59)$$

Where;

T_f^2 : The final time of the motion along x .

T_f^3 : The final time of the motion along y .

The simulations results are shown in Figs. 2–3.

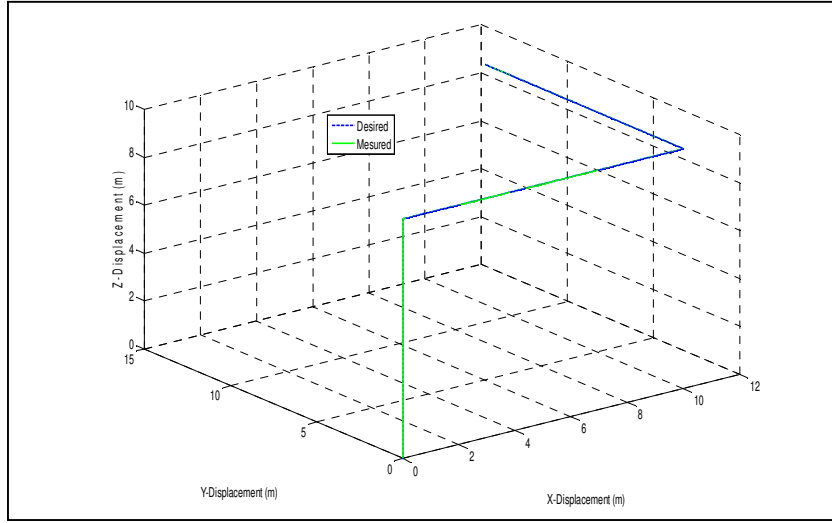


Fig.2 The 3D displacement of the drone.

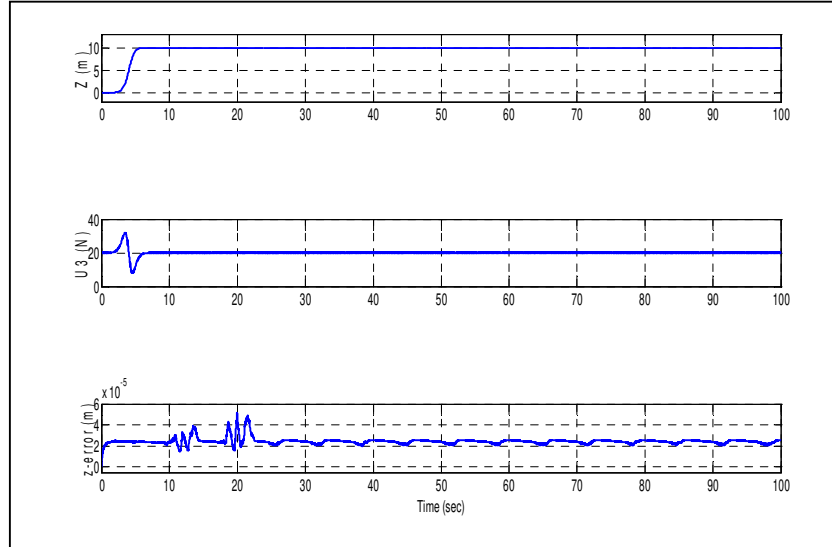


Fig.3 Vertical flight, control input U_3 , and the error e_z .

Figure 2 illustrates the reference trajectories for flights along the x and y directions, as well as the vertical flight along z. The drone performs a vertical motion of 10 m, followed by a motion along the x axis over a distance of 10 m, and then a motion along the y axis over the same distance.

Figure 3 presents the vertical flight response, the control input U_3 , and the tracking error e_z obtained using the Second-Order Sliding Mode Controller (Super-Twisting Algorithm). The drone attains the desired altitude with a rapid response. It is also shown in this figure that the displacement error according to the direction z quickly converges to zero. Note

that the control input $U_3 = mg$ at the equilibrium state is always verified without chattering. These results demonstrate that the proposed controller ensures smooth control action and solve the problem of the chattering phenomenon.

5.1 Trajectory Tracking Results

Two trajectories are retained in the simulation study, namely a semi-circular trajectory and a spiral trajectory. These trajectories were selected because they offer increasing levels of complexity and therefore allow a progressive evaluation of the controller performance.

The first tracking test concerns the semi-circular trajectory. We work with trajectories to perform maneuvers such as turns. A turn is represented by a circular arc with radius α and angle $\alpha(t)$ (the turning angle). The time dependence of α implies that we can plan how the turn is executed. In this way, it is possible to schedule acceleration or deceleration during a turn. Similarly, the angle α , which directly affects the yaw behavior, can take values between zero (a sharp angle) for a change in direction, and 2π for navigating around an intersection.

We propose to link z and x through the following equations.

$$\begin{cases} x_d(t) = -\rho \cos(\alpha(t)) + \rho \\ z_d = \rho \sin(\alpha(t)) + h_d \end{cases} \quad (60)$$

And for the reference transition between x and y , we take:

$$\begin{cases} x_d(t) = \rho \sin(\alpha(t)) + x_d(T_f^2) \\ y_d(t) = -\rho \cos(\alpha(t)) + \rho \end{cases} \quad (61)$$

To ensure continuity of the time derivatives of $x_d(t)$ and $y_d(t)$, that is, in terms of velocity, we have:

$$\alpha(t) = \frac{\alpha_d(t - t_f)^5}{(t - t_f)^5 + (t_a - t)^5} \quad (62)$$

Numerically, $\rho = 2m$, $\alpha_d = k_\alpha \pi/2$ ($k_\alpha = 1.2$), $t_a = 6 \text{ sec}$, $t_f = 8 \text{ sec}$ and $x_d(T_f^2) = y_d(T_f^3) = h_d + \rho = 10 \text{ m}$.

The corresponding results are shown in Figure 4.

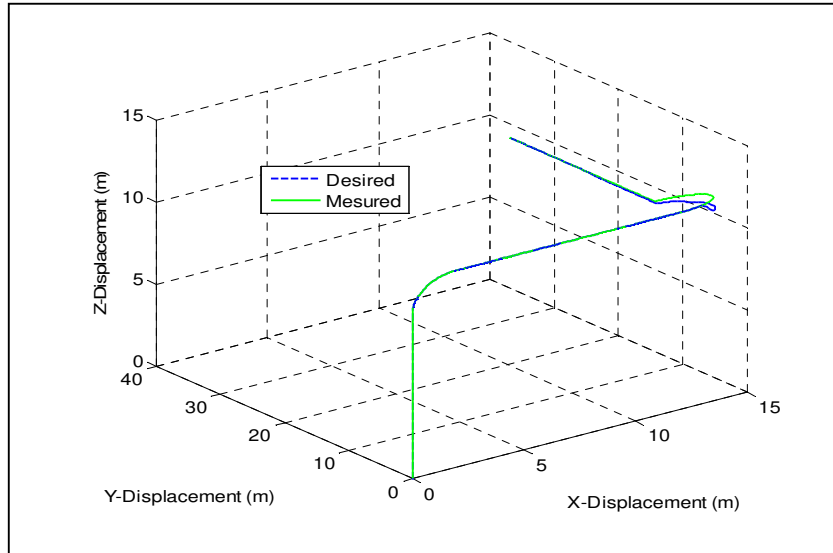


Fig.4 3D Tracking of a Semi-Circular Trajectory.

Figure 4 shows the comparison between the desired trajectory and the measured trajectory. We observe that the measured trajectory follows the desired trajectory very closely during the whole motion. The two curves almost overlap, which shows that the controller provides accurate trajectory tracking. Small tracking errors can be observed at some points, especially during changes in direction, but these errors remain very small.

The second test corresponds to the spiral trajectory, which is more demanding since it requires the quadrotor to simultaneously track a horizontal circular motion and a continuously increasing altitude. This trajectory is defined by:

$$\begin{cases} x_d(t) = R\cos(\omega t) \\ y_d(t) = R\sin(\omega t) \\ z(t) = 0.2t \end{cases} \quad (63)$$

Where R denotes the trajectory radius and ω is the angular speed.

Numerically, $R = 1 \text{ m}$ and $\omega = 0.5 \text{ rad/s}$.

The corresponding results are shown in Figure 5.

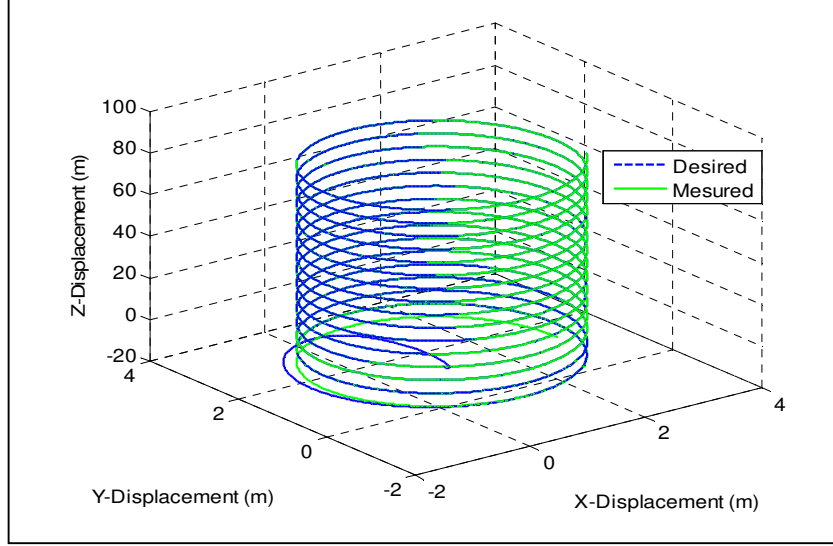


Fig.5 Tracking performance for a spiral reference trajectory.

Compared with the circular case, the spiral trajectory introduces an additional challenge because the vertical motion must be coordinated with the horizontal displacement. Nevertheless, the simulation results show that the quadrotor remains capable of following the prescribed path with satisfactory overall accuracy. Small tracking deviations may appear during the transient phase, but the general shape of the spiral is correctly reproduced. This figure shows the effectiveness of the proposed method.

5.2 Controllers robustness

One of the primary motivations for implementing the second Order Sliding Mode Control is its theoretical invariance to matched uncertainties and its inherent robustness against external disturbances. In real-world outdoor applications, quadrotors are frequently subjected to aerodynamic perturbations such as wind gusts, which can drastically degrade flight performance if not properly compensated. According to the literature, these environmental disturbances are typically modeled as additive time-varying forces and aerodynamic drag torques acting on the dynamic of the quadrotor. This drag force can be calculated as follows:

$$f = \frac{1}{2} C_x \rho A V^2 \quad (64)$$

where:

f : drag force [N];

V : relative velocity [m/s];

A : characteristic frontal area of the drone structure [m²];

ρ : air density [kg/m³];

C_x : drag coefficient.

Numerically, $\rho = 1.22 \text{ kg/m}^3$, $A = 0.031 \text{ m}^2$, $C_x = 0.5$.

Figures 6-8 present the simulation results in the presence of different drag forces.

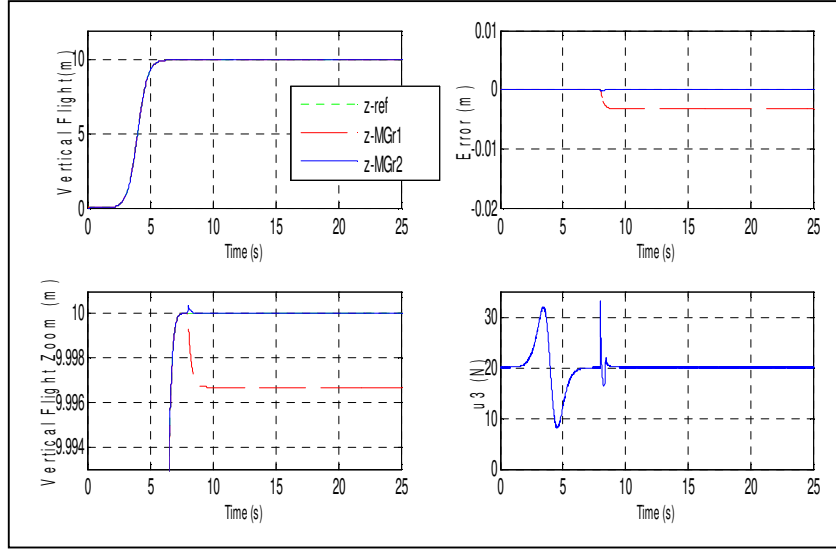


Fig.6 Impact of a Drag Force (3.2 N).

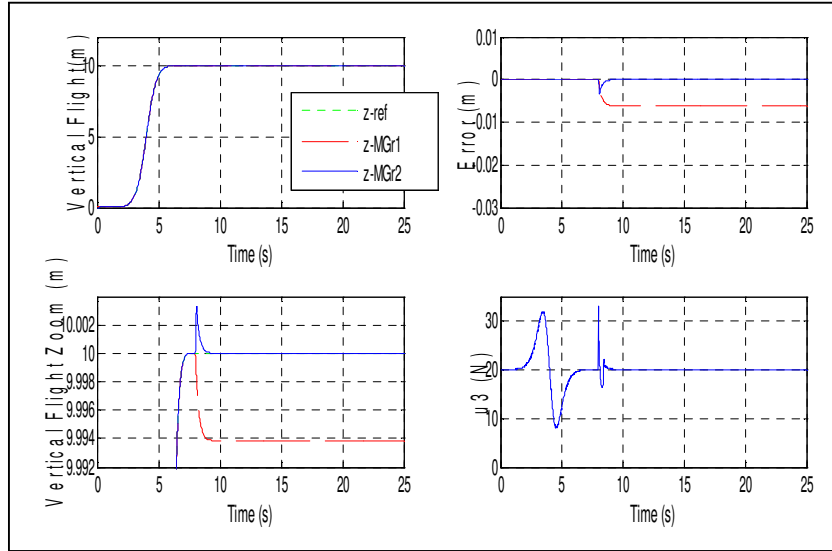


Fig.7 Impact of a Drag Force (6 N).

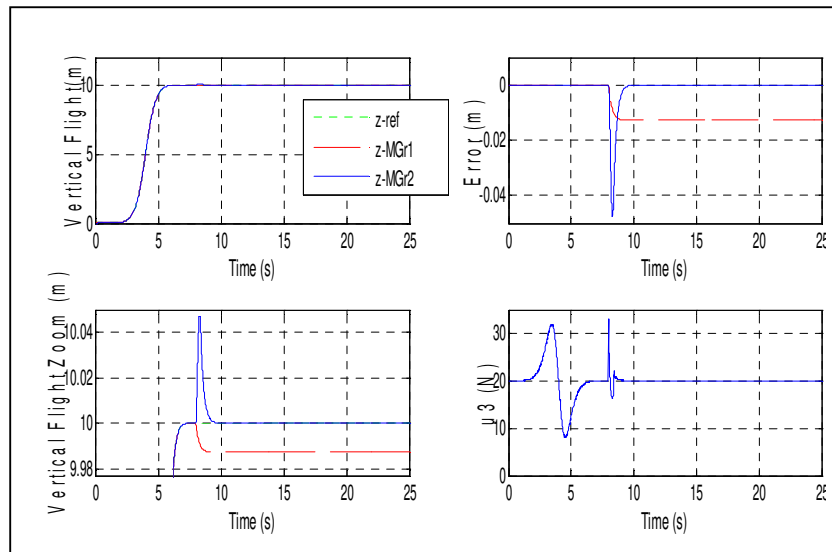


Fig.8 Impact of a Drag Force (12 N).

Figures 6-8 show the trajectory tracking performance under different drag forces applied at the time $t = 8$ sec and compare the second order Sliding Mode Control with the Classical Sliding Mode Control. We observe that the second order sliding mode control drives the drone back to its desired equilibrium position, small tracking errors appear due to the increased drag force disturbance. Moreover, the control input converges to its equilibrium value $U_3 = mg$. These results confirm that the proposed control strategy remains robust even under severe disturbance forces .

6 Conclusion

This work presented a robust control method for quadrotors based on a second-order sliding mode control approach combined with multi-channel decomposition. Firstly, the quadrotor dynamic was divided into independent motion channels. Then, the second order sliding mode control (Super-Twisting Algorithm) was designed to reduce the chattering phenomenon and suppress the disturbance forces acting on the dynamic of the quadrotor.

The simulation results under different reference trajectories and external aerodynamic disturbances showed that the proposed method presents rapid convergence and accurate trajectory tracking compared with the results obtained using classical sliding mode control.

The obtained results demonstrate that the proposed method is suitable for robust quadrotor control in the presence of disturbances.

Acknowledgements

The authors thank the anonymous referees for their insightful comments and suggestions, which greatly improved the quality of the work.

Author Contributions

All authors contributed to the study conception and design.

Funding

The authors did not receive support from any organization for the submitted work.

Data Availability

The authors confirm that the data supporting the findings of this study are available within the article.

Declarations

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. Zihan Song , Peng Dong and Yinshuai Sun (2026) Collision-Free End-Effector Pose Control of an Unmanned Aerial Redundant Manipulator in Confined Spaces Using Null-Space and Impedance Method. Aerospace Systems. <https://doi.org/10.1007/s42401-026-00480-3>.
2. Al-Khafaji, A.J.D., Al-Haddad, L.A (2026) Parametric aerodynamic characterization of tail geometry variations in fixed-wing UAVs. Aerospace Systems. <https://doi.org/10.1007/s42401-025-00406-5>.
3. Legowo, A., Sulaeman, E. and Rosli, D. (2019) Review on system identification for quadrotor unmanned aerial vehicle (UAV)'. Advances in Science and Engineering Technology International Conferences (ASET). IEEE, pp. 1–8.
4. Thamer Al-Hsnawy and Ali Al-Ghanimi (2024) A review of control methods for quadrotor UAV. Kufa Journal of Engineering. <https://doi.org/10.30572/2018/KJE/150408>.
5. Utsav, A., Abhishek, A., Suraj, P., & Badhai, R. K. (2021) An IoT based UAV network for military applications. Sixth International Conference on Wireless Communications, Signal Processing and Networking (WiSPNET) (pp. 122-125). IEEE.
6. Jeler, G.E. (2019) Military and civilian applications of UAV systems. International scientific conference strategies XXI. The Complex and Dynamic Nature of the Security Environment-Volume 1. Carol I National Defence University Publishing House, pp. 379–386.
7. Mohsan, S. A. H., Khan, M. A., Noor, F., Ullah, I., & Alsharif, M. H. (2022) Towards the unmanned aerial vehicles (UAVs): A comprehensive review. Drones. <https://doi.org/10.3390/drones6060147>.
8. T. Madani and A. Benallegue (2006) Backstepping control for quadrotor helicopter. Proceeding of the IEEE International Conference on Intelligent Robots and Systems, Beijing, China, pp. 3255-3260, 8: 102.

9. Boussadia, H.; Si Mohammed, A.M.; Boughanmi, N.; Meche, A (2022) Sliding Mode Control Based on Backstepping Approach for Microsatellite Attitude Pointing. Eng. Proc. [https://doi.org/ 10.3390/engproc2022014024](https://doi.org/10.3390/engproc2022014024).
10. S. Bouabdallah, P. Murrieri, and R. Siegwart (2004) Design and Control of an Indoor Micro Quadrotor. Proceedings of the 2004 IEEE International Conference on Robotics & Automation. New Orleans, LA, pp. 4393-4398.
11. H. Bouadi, M. Bouchoucha and M. Tadjine (2008) Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor. International Journal of Applied Mathematics and Computer Sciences, Vol. 4 No. 1.
12. J. Colorado, A. Barrientos, A. Martinez, B. Lafaverge, and J. Valente (2010) Mini-quadrotor Attitude Control based on Hybrid Backstepping and Frenet-Serret Theory. IEEE International Conference on Robotics and Automation (ICRA).
13. Zachary T. Dydek, Anuradha M. Annaswamy and Eugene Lavretsky (2010) Adaptive Control of Quadrotor UAVs in the Presence of Actuator Uncertainties. American Institute of Aeronautics and Astronautics (AIAA), Atlanta, Georgia. 10.2514/6.2010-3416.
14. Zhou Fang, Zhang Zhi, Liang Jun and Wang Jian (2008) Feedback Linearization and Continuous Sliding Mode Control for a Quadrotor UAV. Proceedings of the 27th Chinese Control Conference July, Kunming, Yunnan, China, 16–18.
15. DongBin Lee, Timothy C. Burg, Darren M. Dawson, Dule Shu, Bin Xian and Enver Tatlicioglu (2009) Robust Tracking Control of an Underactuated Quadrotor Aerial-Robot Based on a Parametric Uncertain Model. Clemson University CRB, Technical report.
16. Alaeddin Bani Milhim, Youmin Zhang and Camille-Alain Rabbath (2010) Gain Scheduling Based PID Controller for Fault Tolerant Control of a Quad- Rotor UAV. Conference: AIAA Infotech@Aerospace. <https://doi.org/10.2514/6.2010-3530>.
17. Alexis K. Nikolakopoulos, G. Tzes (2010) Constrained optimal attitude control of a quadrotor helicopter subject to wind-gusts: Experimental studies. American Control Conference (ACC), pp 4451-4455.
18. Guilherme V. Raffo, Manuel G. Ortega and Francisco R. Rubio (2010) An integral predictive/nonlinear H_∞ control structure for a quadrotor helicopter. Automatica. 10.1016/j.automatica.2009.10.018.
19. K. M. Zemalache, M. Tahar, M. R. Benhadria (2010) Robust Control of an Under-Actuated X4-Flyer Using Integral Sliding Mode Controller. International Conference on Electrical Engineering, Electronics and Automatics, ICEEA'10, Bejaia, Algeria, 2-3 November.
20. K. M. Zemalache, L. Beji and H. Maaref (2007) Two Inertial Models of X4-Flyers Dynamics, Motion Planning and Control. Journal of the Integrated Computer-Aided Engineering (ICAIE). 10.3233/ICA-2007-14201.
21. V. I. Utkin (1993) Sliding Mode control design principles and applications to electric drives. IEEE Trans. Ind. Electron. doi:10.1109/41.184818 .
22. W. S. Levine (1996) The control handbook, CRC Press.
23. S.V. Emelyanov (1967) Variable structure control systems, Moscow Nauka.
24. A. Levant (1993) Sliding order and sliding accuracy in sliding mode control. International Journal of Control. <https://doi.org/10.1080/00207179308923053>.
25. S. V. Emelyanov, S. K. Korovin, L. V. Levantovsky (1986) Second order sliding modes in controlling uncertain systems. Soviet Physics Doklady; 24(4):63–68.
26. A. Levant (1993) Sliding order and sliding accuracy in sliding mode control. International Journal of Control. <https://doi.org/10.1080/00207179308923053>
27. L. Fridman and A. Levant (2002) Higher order sliding mode modes.