

Supporting Information

Voronoi-entropy characterisation of geometric disorder in two-dimensional porous networks

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S1 Network generation

The obstacle positions were drawn at random within the square domain subject to a minimum surface-to-surface separation of $0.09\text{ }\mu\text{m}$ between obstacles, which keeps the pore space connected. For each target configuration, 1,000 independent realisations were generated. For the studies that hold the entropy fixed, only the realisations whose Voronoi cell-area entropy fell within a narrow tolerance of the target value were retained, and the flow was solved on this retained ensemble, as in the other studies.

S2 Entropy estimation and bin-width sensitivity

The Voronoi cell-area Shannon entropy (Eq. 2 of the main text) was computed by scaling the cell areas by their mean $\langle A \rangle$, discretising the distribution of $A/\langle A \rangle$ over the interval $[0, 5]$ into 50 equal bins (width 0.1 in units of $\langle A \rangle$), and normalising the binned frequencies. Because this is a binned estimate of a continuous distribution, the absolute value of H depends on the bin width: with the interval fixed, finer bins raise H and coarser bins lower it. Recomputed on the obstacle coordinates of the five disorder cases for bin counts from 10 to 80, H rises with disorder under every rule and the rank order of the five cases is preserved; only the absolute scale shifts, the fully random case moving from about 0.7 at the coarsest binning to about 2.3 at the finest, with the reported value of 1.92 corresponding to the 50-bin convention above. The absolute value of the entropy is therefore convention-dependent and must be reported together with its bin definition, whereas the increase with disorder and the rank order on which the analysis relies are not.

S3 Lattice-Boltzmann implementation

The single-phase flow was solved with the D2Q9 lattice and a single-relaxation-time collision operator with an explicit-force fluid–solid treatment, as implemented in the Taxila package. Flow was driven by injection at the left boundary, and the tortuosity and Darcy permeability were evaluated once the velocity field reached a steady state. The domain was discretised

on a 256×256 lattice, so the spacing is $\Delta x = 5.68 \mu\text{m}/256 \approx 22.2 \text{ nm}$; the smallest obstacle radius ($0.22 \mu\text{m}$) spans about ten nodes and the minimum throat ($0.09 \mu\text{m}$) about four. The permeability is returned by the solver in lattice units and converted to physical units through $k = k_{\text{lu}}(\Delta x)^2$; the raw lattice-unit values for every case are listed in Table S1. The collision used a single relaxation time $\tau_{\text{LB}} = 1.23$, set by the brine kinematic viscosity. Flow was driven by a fixed inlet velocity (0.003 in lattice units, 0.44 m s^{-1}) with an open outlet and periodic lateral boundaries. At the pore scale the Reynolds number is small ($Re \approx 0.25$ on the obstacle scale, 3.17 on the domain scale), so the flow is in the linear Stokes regime; the lattice Mach number is of order 10^{-3} , which keeps the lattice-Boltzmann compressibility error negligible.

S4 Geometric design of the controlled studies

The three families used to isolate the geometric factors are designed in the same way: for a target combination of porosity, obstacle size and entropy, many random arrangements are generated and the entropy is computed from the Voronoi tessellation, so that each family is characterised not by a single entropy value but by a range that reflects the diversity of accessible configurations. The representative Voronoi diagrams, the cell-area distributions and the entropy ranges for each family are shown in Figs. S1–S3. The filled symbol marks the representative configuration whose Voronoi diagram is shown in panel (a); the lattice-Boltzmann flow in the main text was solved over the full ensemble of configurations at each point.

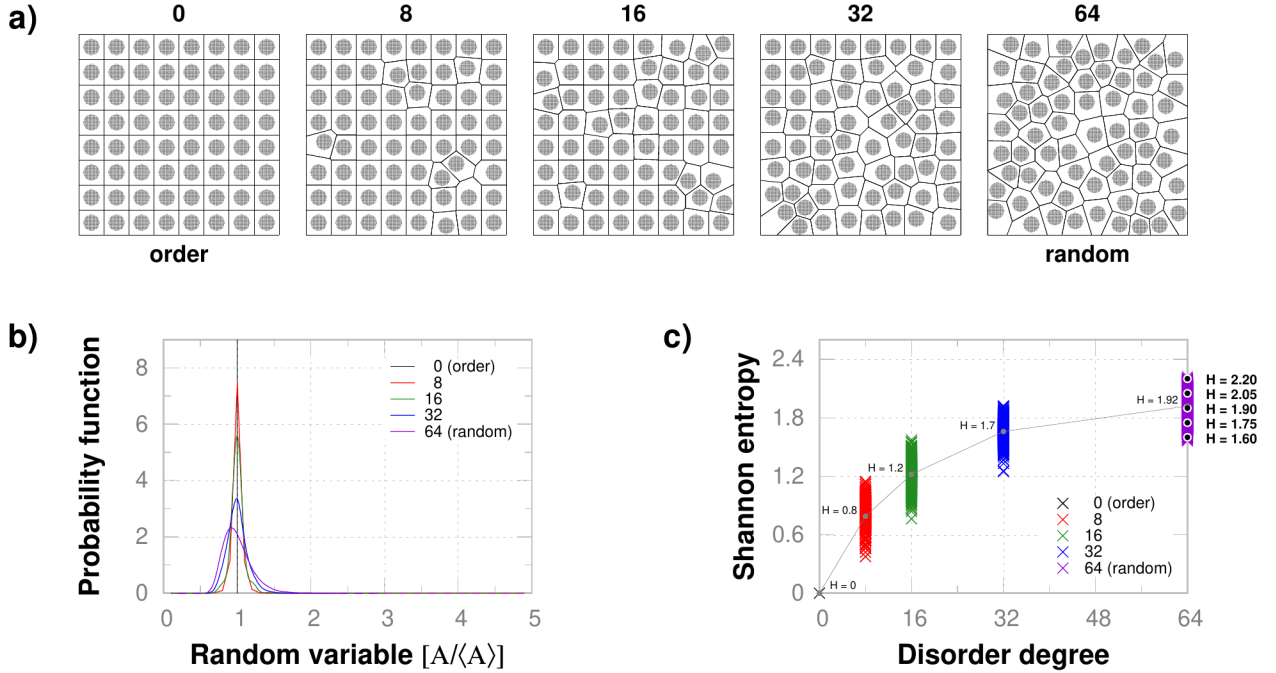


Figure S1: Geometric design of the positional-disorder study, in five steps from an ordered to a fully random arrangement at fixed porosity ($\phi = 0.68$) and obstacle radius ($r = 0.22 \mu\text{m}$). (a) A single representative Voronoi diagram from the 1,000 generated configurations. (b) Cell-area distributions for each disorder level. (c) Shannon entropy at each level, with the spectrum of values from the 1,000 configurations; the labels at the right mark the five entropy targets ($H = 1.60$ to 2.20) drawn from the fully random ensemble for the entropy study.

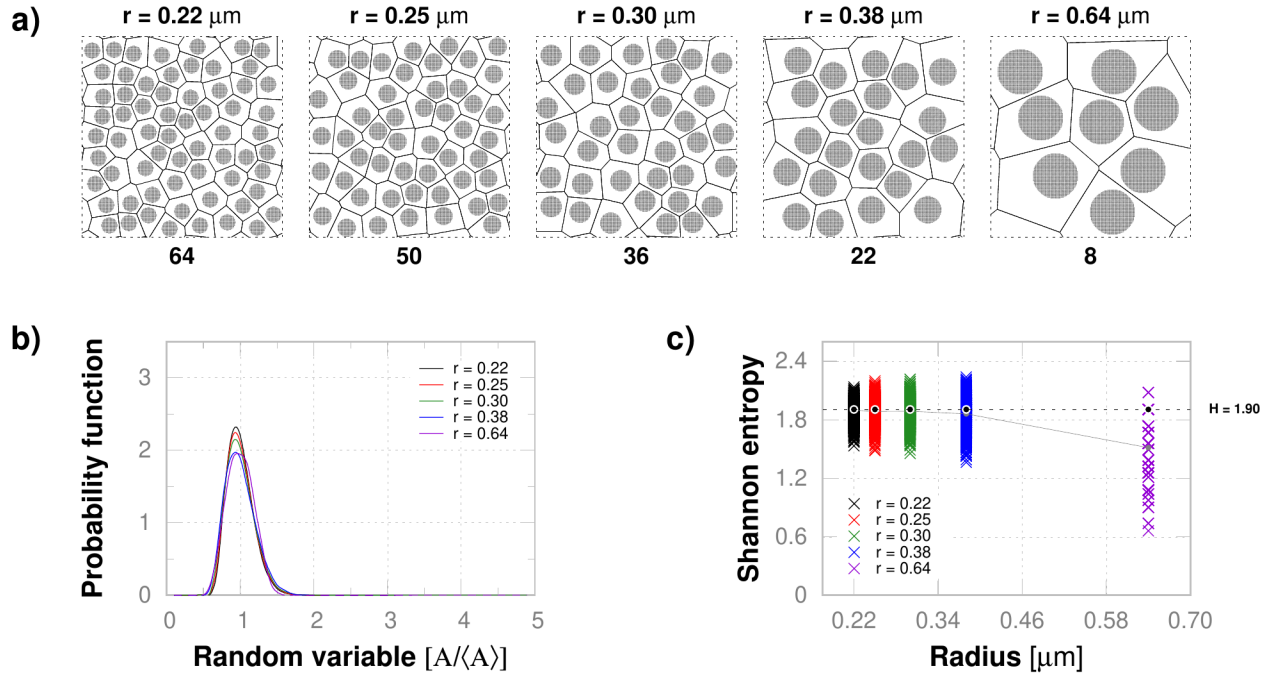


Figure S2: Geometric design of the obstacle-size study: five obstacle radii at fixed porosity ($\phi = 0.68$) and entropy ($H \approx 1.90$), obtained by reducing the obstacle count from 64 to 8. (a) A single representative Voronoi diagram from the 1,000 generated configurations. (b) Cell-area distributions. (c) Shannon entropy with the spectrum of values.

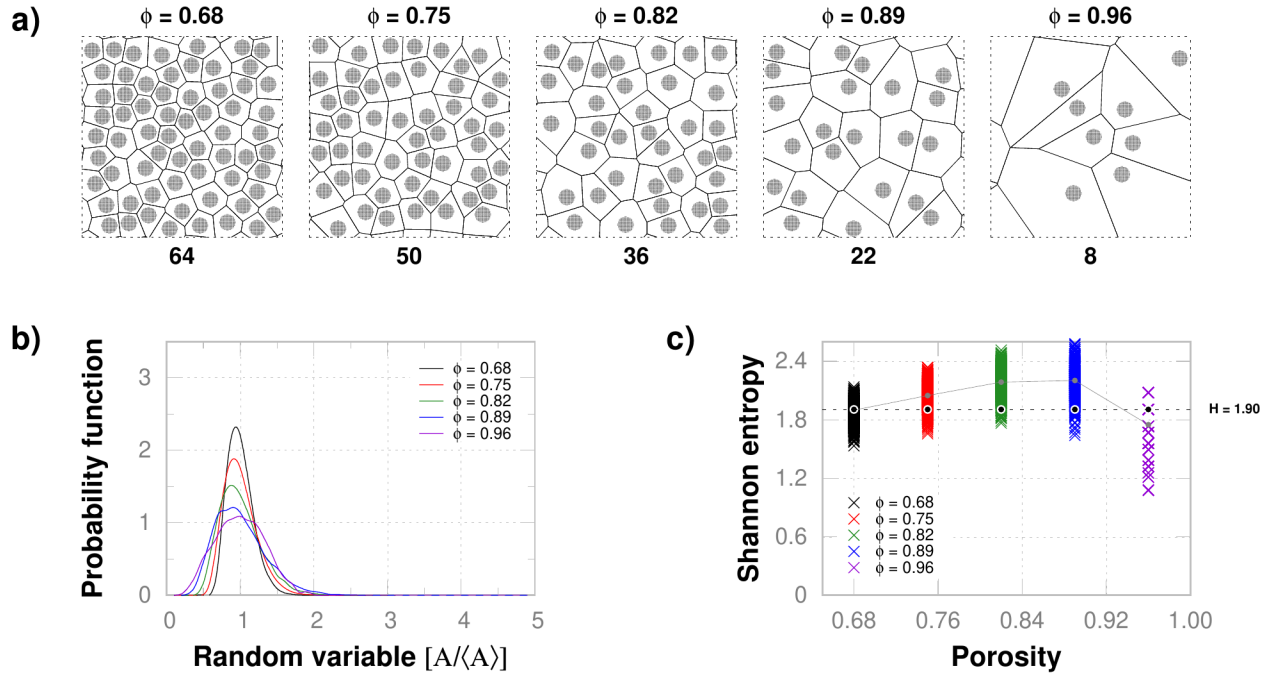


Figure S3: Geometric design of the porosity study: five porosities at fixed obstacle radius ($r = 0.22 \mu\text{m}$) and entropy ($H \approx 1.90$), obtained by reducing the obstacle count from 64 to 8. (a) A single representative Voronoi diagram from the 1,000 generated configurations. (b) Cell-area distributions. (c) Shannon entropy with the spectrum of values.

S5 Transport results for all configurations

Table S1 lists, for every configuration, the hydraulic tortuosity and the Darcy permeability produced by the lattice-Boltzmann solver, given both in lattice units and in physical units. The physical permeability follows from $k = k_{\text{lu}}(\Delta x)^2$ with $\Delta x = 5.68 \mu\text{m}/256$, a factor of 0.4988 per lattice unit in millidarcy.

Table S1: Transport results for each case, averaged over the 1,000 configurations (standard deviations shown as error bars in Figs. 2–5 of the main text): hydraulic tortuosity τ (dimensionless), permeability in lattice units k_{lu} , and the physical permeability k in millidarcy, with $k(\text{mD}) = 0.4988 k_{\text{lu}}$.

Study	Varied parameter	τ	k_{lu} (lu)	k (mD)
Positional disorder	$H = 0.00$ (ordered)	1.037 85	9.517 08	4.75
	$H = 0.81$ (8 random)	1.070 17	9.712 35	4.84
	$H = 1.22$ (16 random)	1.090 69	9.777 95	4.88
	$H = 1.65$ (32 random)	1.135 44	9.799 12	4.89
	$H = 1.92$ (fully random)	1.160 47	10.035 35	5.01
Entropy	$H = 1.60$	1.158 46	9.641 00	4.81
	$H = 1.75$	1.160 03	9.615 21	4.80
	$H = 1.90$	1.163 46	10.123 03	5.05
	$H = 2.05$	1.165 44	10.060 57	5.02
	$H = 2.20$	1.173 95	9.790 55	4.88
Obstacle size	$r = 0.22 \mu\text{m}$	1.152 61	10.144 27	5.06
	$r = 0.25 \mu\text{m}$	1.158 39	13.467 64	6.72
	$r = 0.30 \mu\text{m}$	1.164 66	17.858 09	8.91
	$r = 0.38 \mu\text{m}$	1.165 98	31.160 96	15.54
	$r = 0.64 \mu\text{m}$	1.193 85	69.084 01	34.46
Porosity	$\phi = 0.68$	1.152 61	10.144 27	5.06
	$\phi = 0.75$	1.147 84	21.412 14	10.68
	$\phi = 0.82$	1.122 69	47.622 13	23.75
	$\phi = 0.89$	1.095 69	141.272 28	70.47
	$\phi = 0.96$	1.053 06	746.218 90	372.22

S6 Supporting geometric analysis: pore-throats and the permeability–porosity relation

Note. The analysis in this section is computed from the obstacle coordinates of the generated configurations, using the network-generation and Voronoi-analysis scripts described in the Methods; it supports the main-text statements that the permeability is governed by the pore-throat scale and that positional disorder, at fixed porosity, does not change the typical throat and hence the permeability.

S6.1 Pore-throat definition

The resistance to flow is set by the constrictions between obstacles, which we characterise directly from the obstacle coordinates. Two obstacles define a pore throat when they are neighbours in the Gabriel graph of the centres, a subgraph of the Delaunay triangulation that joins two points only when no third obstacle falls inside the circle drawn on the segment between them as diameter. This proximity rule discards the spurious long links that the bare Delaunay triangulation lays across the diagonals of an ordered array, so a perfectly ordered lattice reduces to a single throat width. The throat width between two neighbouring obstacles i and j is the surface-to-surface gap,

$$w_{ij} = \|c_i - c_j\| - 2r, \quad (1)$$

the centre-to-centre distance minus the two radii, and we summarise the network by the median value w_{med} .

S6.2 Permeability follows the pore-throat scale

Across the porosity, obstacle-size and positional-disorder studies, the Darcy permeability follows the median throat width as a single power law, $k \propto w_{\text{med}}^{2.9}$ (fitted exponent 2.9), with a coefficient of determination of 0.99 on the pooled fifteen configurations (Fig. S4a). The exponent is close to the cubic value expected for a two-dimensional channel, whose conductance grows as the cube of its aperture. Raising the porosity widens the median throat from 0.26 to 1.05 μm , and enlarging the obstacles widens it from 0.26 to 0.47 μm ; in the disorder study, at fixed porosity and obstacle size, the median stays at 0.26 μm , which is why the permeability remains near 5 mD. Disorder reshapes the distribution without moving its centre: the ordered lattice has a single throat width, whereas the random arrangements develop both narrower and wider throats, the widest growing from 0.26 to about 0.66 μm (Fig. S4b). These few wide channels account for the small (about 5%) permeability rise but are too rare to move the typical constriction that sets the bulk conductance.

S6.3 The entropy and the coefficient of variation of the cell areas

The same coordinates permit the comparison, quoted in the Discussion of the main text, between the cell-area entropy and the coefficient of variation (CV) of the Voronoi cell areas, a bin-independent measure of their spread. The CV was evaluated for each level of the positional-disorder series and averaged over the available realisations of that level. It rises monotonically with disorder, through 0, 0.05, 0.08, 0.13 and 0.19, and tracks the hydraulic tortuosity as closely as the entropy does: the coefficients of determination are 0.99 for the CV and 0.95 for the entropy, and the two descriptors are themselves correlated with a Pearson coefficient of 0.98.

S6.4 Same porosity, different permeability

The four studies, plotted together as permeability against porosity (Fig. S5), recover the observation that motivates the main text. At a single porosity, $\phi = 0.68$, the permeability

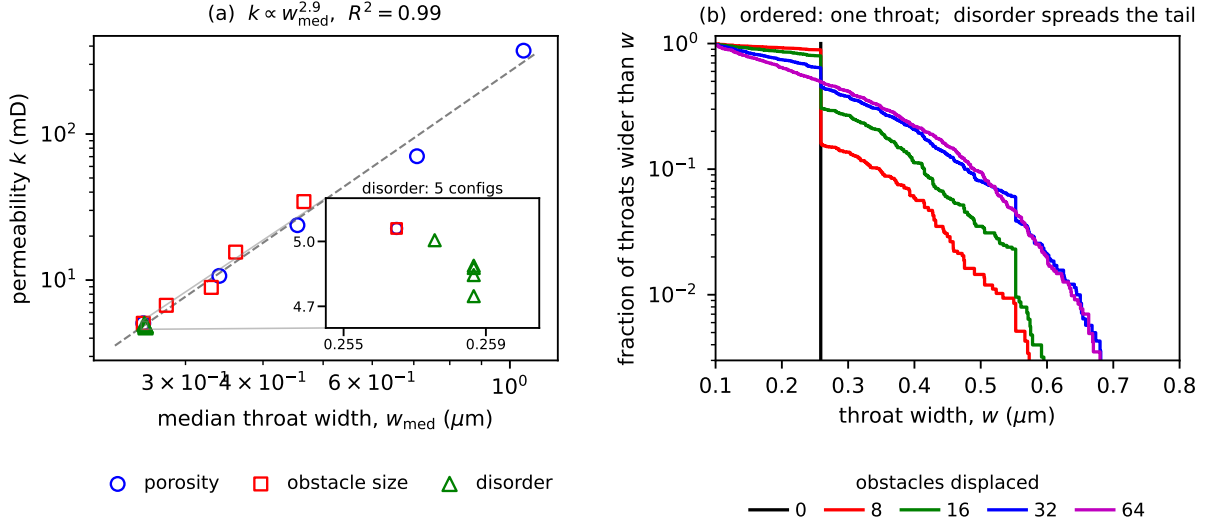


Figure S4: Pore-throat geometry and the permeability. (a) The Darcy permeability follows the median throat width w_{med} as a single power law $k \propto w_{\text{med}}^{2.9}$ across the three studies (dashed; coefficient of determination 0.99); the disorder configurations (triangles) sit at the low-throat end at fixed median (inset). (b) Throat-width survival in the disorder study: the ordered lattice has one throat width near $0.26 \mu\text{m}$, whereas displacing obstacles spreads the tail to about $0.66 \mu\text{m}$ without moving the median.

already spans about sevenfold, from 5.1 to 34 mD: the obstacle-size series raises it as the throats widen, whereas the positional-disorder and entropy series leave it clustered near 5 mD. The fixed-porosity spread is set by the pore-throat scale, not by the degree of positional disorder; its magnitude here, about sevenfold, is smaller than the several decades seen in natural rock, as expected for a planar medium with a single obstacle shape and a narrow range of sizes.

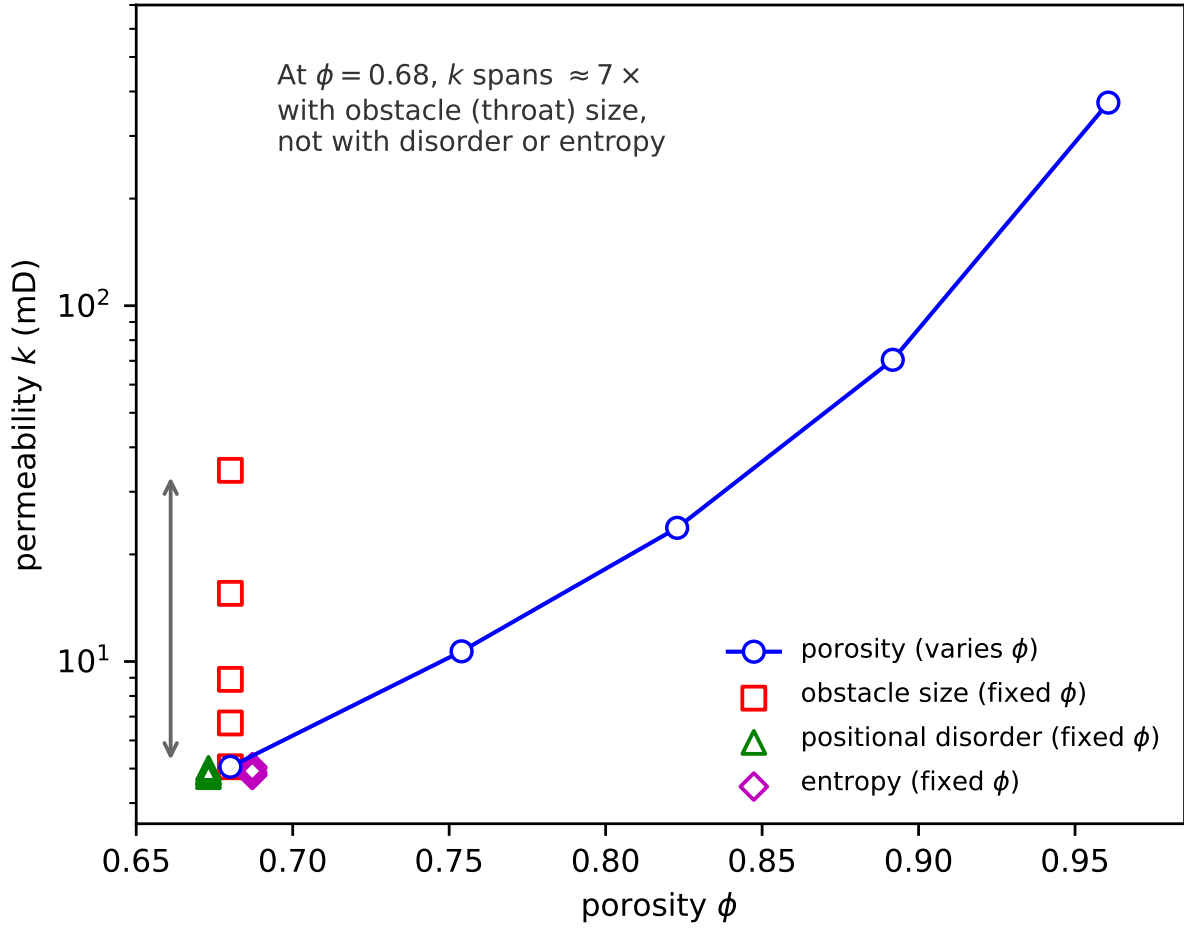


Figure S5: Same porosity, different permeability. Permeability against porosity for the four studies. The porosity series (circles) traces the permeability–porosity trend; at $\phi = 0.68$ the obstacle-size series (squares) spans 5.1 to 34 mD, whereas the positional-disorder (triangles) and entropy (diamonds) series stay clustered near 5 mD. The three fixed-porosity studies are drawn at slightly offset ϕ for clarity.