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Optimization of Maintenance Strategies for Leased Equipment Considering Learning Effects and Spare Parts Inventory

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Abstract: In the leasing equipment maintenance supply chain consisting of lessors and lessees, the divergence in decision objectives and individual interests between the two parties often prevents the alignment of maintenance decisions and the achievement of system-wide optimality. To comprehensively investigate the coordination of maintenance decisions, this study incorporates both learning effects and spare parts inventory into the analytical framework. Decentralized and centralized models are developed, and Uptime Target Bonus (UTB) strategy and Failure Limitation (FL) strategy are proposed to achieve the centralized optimum. The results show that both strategies can effectively coordinate the supply chain. Furthermore, the optimal maintenance decisions and profits of the lessor, the lessee, and the overall supply chain increase with the learning level and lease period. Spare parts inventory enhances operational continuity by accelerating repair response, reducing downtime risks, and mitigating component shortages, thereby affecting the maintenance frequencies and profits of the lessor, lessee, and the entire supply chain, while significantly improving system stability. Since inventory exerts the greatest influence on the lessee's decisions, the lessee can share the gains from improved operational continuity through subsidies that encourage the lessor to maintain appropriate inventory levels. In addition, learning effects reduce maintenance costs, while spare parts inventory enhances operational continuity; together, they further strengthen supply chain coordination.

Keywords: Equipment maintenance; Learning effects; Spare parts inventory; Supply chain coordination

1 Introduction

In recent years, the rise of the Sharing Economy (SE) has reshaped the operational model of the traditional equipment rental industry, encouraging users to shift from pursuing ownership toward more flexible access-based consumption, while placing higher demands on equipment flexibility, efficiency, and reliability (Wang K, Li Y, Yue X, et al.2024). Globally, the equipment rental market has experienced rapid growth. The global machinery rental market was valued at USD 136.12 billion in 2025 and is projected to reach USD 181.04 billion by 2031, driven by the increasing demand for the rental of large-scale equipment, such as construction machinery (Yin J, Fang Y, Zhang H, et al.2023). Given the high acquisition costs of large equipment and the long-term reliability risks and maintenance expenditures borne by operators, an increasing number of enterprises prefer to obtain equipment through rental services rather than

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direct ownership (Huang Y S, Ho J W, Hung J W, et al.2021). Under the rental mode, lessees acquire the right to use equipment by paying rental fees, while lessors are responsible for providing equipment maintenance services (Liu Y, Liu B, Yang H.2024). Since equipment operating conditions directly affect the production continuity of lessees and the maintenance costs and service returns of lessors, developing appropriate maintenance strategies has become a critical issue in the operational management of rental equipment. Maintenance decision-making for rental equipment mainly involves determining the optimal preventive maintenance (PM) frequency and maintenance intervals, which further influence the economic performance of both parties by affecting equipment reliability and maintenance costs. Generally, the maintenance services provided by lessors can be classified into two categories: preventive maintenance (PM) and corrective maintenance (CM) (Zhao T, Chen Z L, Liu X.2026). PM aims to mitigate equipment degradation risks and improve equipment availability, whereas CM is performed to restore equipment functionality after failures occur (Liu B, Pang J, Yang H, et al.2024). CM can be further categorized into perfect repair, minimal repair (MR), and imperfect repair. Among these types, minimal repair represents a specific form of CM, in which the failed equipment is restored only to its pre-failure operating condition without improving its subsequent reliability performance.

However, in the rental mode, preventive maintenance decision-making is not merely an equipment reliability optimization problem, but also a supply chain decision-making problem involving the coordination of interests between lessors and lessees. In this context, lessors are primarily responsible for maintenance costs, whereas lessees are concerned with equipment availability and production continuity, resulting in divergent objectives between the two parties. Lessors tend to reduce maintenance expenditures, while lessees expect higher equipment reliability and availability. Consequently, under decentralized decision-making, maintenance frequency, repair response, and resource allocation may fail to achieve the system-wide optimum. Therefore, coordinating the maintenance behaviors of rental parties and balancing equipment reliability with operational costs have become critical issues in the maintenance management of rental equipment.

It is noteworthy that, during the repeated implementation of rental maintenance activities, lessors continuously accumulate maintenance experience through ongoing PM operations, resulting in significant learning effects in terms of maintenance efficiency, failure diagnosis capability, and resource scheduling capability. Previous studies have demonstrated that the accumulation of maintenance experience and operational data enables firms to effectively reduce equipment downtime and maintenance costs while improving maintenance efficiency. However, the benefits generated by learning effects cannot be automatically transformed into improved supply chain performance. On the one hand, learning effects represent a proprietary capability of lessors, and their realization is constrained by lessees' production plans, equipment utilization strategies, and information feedback. On the other hand, without effective coordination between the two parties, learning benefits may not be appropriately allocated, thereby weakening their contribution to maintenance decision

optimization. Therefore, how to reasonably allocate learning benefits and motivate coordinated behaviors among rental parties remains a critical challenge for achieving maintenance optimization.

In addition to learning effects, spare parts inventory configuration is another critical factor influencing maintenance decisions for rental equipment. In traditional studies, equipment failures are often assumed to require waiting for spare parts replenishment before maintenance can be completed. However, in practical rental operations, lessors usually maintain a certain level of spare parts inventory as an emergency guarantee. The availability of spare parts can shorten failure repair response time, improve equipment availability, and reduce downtime losses for lessees. However, spare parts inventory management also increases lessors' inventory holding and capital occupation costs, while introducing additional risks of spare parts degradation due to inventory accumulation and prolonged storage. Since spare parts supply capability directly affects the execution efficiency of maintenance activities and equipment availability, inventory configuration levels further influence maintenance strategy selection and the allocation of benefits between rental parties. Consequently, rental equipment maintenance has evolved from a single reliability optimization problem into an integrated optimization problem involving "maintenance decision-making, inventory management, and supply chain coordination." Existing studies have rarely considered learning effects and spare parts inventory simultaneously in maintenance processes, and systematic investigations into the decision-making behaviors of rental parties and coordination mechanisms under the joint influence of these two factors remain limited. Therefore, considering the combined impacts of learning effects and spare parts inventory, how to optimize maintenance strategies and coordinate the interests of supply chain members in rental equipment operations represents an important research issue that requires further investigation.

To address the aforementioned issues, this study investigates the coordination problem of rental equipment maintenance supply chains under the joint effects of learning effects and spare parts inventory. Unlike existing studies that mainly focus on maintenance strategy optimization while neglecting the influence of inventory factors, this study incorporates maintenance experience accumulation and spare parts inventory configuration into a unified analytical framework and further explores their interactive effects on maintenance decisions, operational performance, and coordination mechanisms of rental parties. By extending traditional maintenance optimization models, this study simultaneously captures the improvement in maintenance efficiency induced by learning effects and the impact of spare parts availability on equipment reliability and availability, thereby providing a more realistic representation of the full life-cycle operational scenario of rental equipment. The main contributions of this study are summarized as follows:

- (1) This study develops a maintenance decision-making framework for rental equipment by incorporating both learning effects and spare parts inventory. The proposed framework investigates the impacts of maintenance experience accumulation and inventory configuration on maintenance decisions and cost structures.

- (2) This study establishes rental equipment maintenance optimization models

under decentralized and centralized decision-making settings. The game-theoretic relationship between lessors and lessees is analyzed, and penalty mechanisms based on repair time and failure frequency are incorporated to reveal how learning effects and inventory levels influence the optimal maintenance decisions of both parties.

(3) This study compares the optimal decisions under decentralized and centralized settings and designs coordination mechanisms, including failure limitation and uptime target bonus strategies, to align the maintenance decisions of rental parties and improve overall supply chain performance.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the research problem and introduces the underlying assumptions. Section 4 develops the maintenance decision-making model incorporating both learning effects and spare parts inventory. Section 5 presents the coordination strategies and compares their performance. Section 6 validates the effectiveness of the proposed mechanisms through numerical experiments and sensitivity analyses, and further compares the results with those obtained under the scenario without spare parts inventory. Section 7 discusses the managerial implications of the findings. Finally, Section 8 concludes the paper and outlines potential directions for future research.

2 Literature Review

The literature most closely related to this study can be broadly classified into four streams: equipment maintenance optimization, supply chain coordination mechanisms, learning effects, and spare parts inventory management. This section reviews the existing research in these areas.

2.1 Equipment maintenance optimization

Maintenance optimization has been widely applied in various industries, including manufacturing, energy, and transportation. Appropriate maintenance strategies can enhance operational continuity and system stability while reducing losses caused by equipment failures and downtime. Consequently, research on PM optimization has evolved continuously, progressing from early static PM policies to more sophisticated models that incorporate dynamic factors such as equipment utilization, operating stages, and failure conditions, thereby enabling the dynamic optimization of maintenance timing and intensity. For example, Liu et al. (2023) developed a multi-stage PM decision model based on variations in equipment utilization rates. A Ben Mabrouk et al. (2023) proposed a PM optimization model integrating maintenance intervals and reliability thresholds to maximize the profitability of leased equipment during the warranty period. Pei et al. (2023) introduced a group-based PM policy for flexible manufacturing systems, achieving a joint optimization of system-level reliability and maintenance costs. As the complexity of real operations has increased, additional factors such as finite leasing horizons and imperfect maintenance have gradually been incorporated into maintenance models. For instance, Mabrouk B A et al. (2017) proposed a sequential hybrid imperfect maintenance model under a finite leasing period, while Jin et al. (2020) developed a periodic imperfect PM policy considering uncertainty in maintenance duration. Subsequent studies have further integrated maintenance decision-making with multi-objective optimization techniques. Huang et al. (2021), considering

heterogeneous consumers and two-dimensional warranty policies, classified lessees according to their utilization rates and jointly designed differentiated pricing and PM strategies. Yang et al. (2024) employed the NSGA-II algorithm to develop a joint decision model for maintenance thresholds and maintenance frequencies, thereby addressing a multi-objective optimization problem. Although existing studies have extensively investigated the optimization of the lessor's PM policies for profit maximization, the impact of operational experience accumulation, commonly referred to as the learning effect, on maintenance decision-making has not been systematically examined.

2.2 Supply chain coordination mechanisms

As research in this area has progressed, equipment maintenance has evolved from a single-agent decision-making problem to a collaborative optimization problem involving both the lessor and the lessee, thereby exhibiting prominent supply chain characteristics. Against this backdrop, the equipment maintenance supply chain has increasingly attracted scholarly attention as an important analytical framework for investigating the interactions and decision-making behaviors of multiple stakeholders. Existing studies have primarily examined this issue from two perspectives: model development and coordination mechanism design. From the perspective of model development, Zhao et al. (2026) established both game-theoretic and centralized decision-making models for leasing supply chains under demand uncertainty and incomplete information, and further introduced cost-sharing and information-sharing mechanisms to achieve supply chain coordination. Yin et al. (2023) incorporated carbon taxes and consumers' preferences for remanufactured products into a closed-loop supply chain and achieved coordination in the construction machinery industry through a leasing compensation–cost-sharing contract. Research on coordination mechanism design mainly focuses on promoting supply chain cooperation through contractual arrangements, incentive schemes, and operational constraints. Regarding cost-sharing and profit-sharing mechanisms, Gupta et al. (2021) proposed a cost-sharing coordination scheme in which contractors bear a proportion of non-utility costs, thereby mitigating opportunistic behavior and improving incentive compatibility. Xia et al. (2023) developed a revenue-sharing contract under a carbon quota policy to achieve centralized optimization of production and maintenance decisions. Zhou et al. (2025) compared the profit performance under cost-subsidy and failure-limit policies and identified the corresponding optimal centralized decision scheme. In terms of maintenance incentives and failure-related constraints, Tarakci et al. (2006) proposed several maintenance contracts to encourage contractors to adopt maintenance policies that maximize the overall profit of the supply chain. Hezarkhani et al. (2022) introduced an operating-time incentive mechanism and coordinated the profits of both parties through downtime compensation arrangements. Pascual et al. (2013) investigated a reward-based mechanism in which the lessee provides cost subsidies to the lessor in order to improve maintenance service quality. He et al. (2021) proposed a coordination mechanism based on failure limits, which enhances cooperation quality and operational efficiency by clearly defining the boundaries of equipment usage and maintenance responsibilities.

2.3 Learning effects

Although the aforementioned studies have systematically investigated supply chain coordination mechanisms, most of them treat the behavior and operational efficiency of supply chain participants as fixed parameters, overlooking the dynamic improvements that arise from accumulated experience in practice. Consequently, incorporating learning effects into maintenance optimization and supply chain coordination has emerged as an important research direction. The concept of the learning curve was first introduced by Wright (1936) in the context of aircraft manufacturing, laying the foundation for its application in labor-hour forecasting and production management. Early studies primarily focused on the role of learning effects in production planning and scheduling. For example, Keachie et al. (1966) examined the impact of learning effects on lot-sizing decisions in intermittent production systems. Pei et al. (2018) incorporated learning effects into single-machine serial batch scheduling problems to optimize batch formation and job sequencing decisions. Wang et al. (2024) considered dual learning effects in a dual-resource flexible job-shop scheduling environment, highlighting the influence of accumulated experience by both workers and machines on scheduling performance. In the fields of maintenance optimization and warranty design, learning effects are commonly used to characterize the efficiency improvements achieved by maintenance personnel through experience accumulation. Following this perspective, Zhang et al. (2025) investigated a single-machine deteriorating maintenance scheduling problem, incorporating outsourced jobs and processing-time variability to analyze the influence of learning effects on maintenance decisions. Tarakci et al. (2013) integrated learning and forgetting mechanisms into contract design and developed a supply chain profit-maximization model. Vandana et al. (2023) investigated the impacts of learning and forgetting effects on manufacturing processes and carbon emission costs. Based on the Weibull degradation rate, they developed a learning–forgetting–learning production model and demonstrated that learning effects can reduce unit production costs and improve production efficiency. He et al. (2025) combined learning–forgetting dynamics in maintenance activities with energy availability objectives and proposed an innovative warranty strategy that simultaneously improves economic and operational performance. Tarakci (2013) were the first to introduce learning effects into the corrective maintenance process and systematically investigated the combined impacts of repetitive learning and failure-based learning on maintenance activities. Heuser et al. (2023) further distinguished between within-category learning effects and cross-category forgetting effects in scheduling models, providing a more realistic representation of learning behaviors in practical maintenance environments. Zhou et al. (2025) incorporated learning behavior and standby machine allocation into an equipment leasing supply chain and analyzed their impacts on maintenance policies and coordination mechanisms. However, the existing literature has largely examined the effects of learning on individual decision-makers, with limited attention devoted to understanding how learning effects influence overall system performance from a supply chain perspective under multi-party decision-making.

2.4 Spare parts inventory management

Furthermore, with the rapid development of equipment leasing and servitization models, spare parts inventory has become increasingly important in maintenance optimization. Inventory levels directly affect equipment availability, response efficiency, and overall supply chain costs. Spare parts inventory is not only closely related to the lessor's cost control but also crucial for maintaining the lessee's operational continuity. Consequently, it exerts a significant influence on maintenance decisions and overall system performance (Tatham P, Wu Y, Kovács G, et al.2017; Teixeira C, Lopes I, Figueiredo M.2018; Zhang X, Zeng J.2017). To achieve synergistic benefits, several studies have integrated inventory decisions with maintenance policies within a joint optimization framework. Hajej et al. (2013) incorporated leasing and warranty constraints into a manufacturing system and investigated the effects of warranty periods on maintenance and inventory decisions. Askri et al. (2016) developed an integrated model for parallel leased equipment, jointly optimizing production, maintenance, and inventory policies and proposing a coordinated operational strategy. Lu et al. (2019) established a condition-based joint optimization model for maintenance and spare parts inventory management. In addition, demand heterogeneity and maintenance opportunities in operational environments have also been recognized as important determinants of inventory policies. Lolli et al. (2025) investigated spare parts supply issues in equipment maintenance processes by integrating joint replenishment strategies with additive manufacturing. They optimized spare parts inventory decisions under different manufacturing modes and analyzed their impacts on maintenance costs and service levels. Wang et al. (2023) examined joint optimization strategies for equipment maintenance and spare parts inventory under different demand-priority settings. Cao et al. (2023) investigated the interaction between opportunistic maintenance and inventory control and proposed a dynamic spare parts replenishment policy to enhance the coordination between maintenance activities and inventory management. With the advancement of digitalization, intelligent condition monitoring, and predictive maintenance technologies, research on maintenance–inventory coordination has increasingly evolved toward intelligent decision-making. Shokri et al. (2025) jointly modeled equipment health monitoring, failure prediction, and inventory replenishment policies within a digital supply chain environment. Joost et al. (2026) proposed a deep reinforcement learning-based optimization approach for industrial-scale spare parts inventory management. By enabling intelligent inventory decisions, their approach reduced the risk of critical material shortages and provided a new solution for data-driven spare parts inventory management. Reddy et al. (2025) employed machine learning techniques to improve long-term spare parts demand forecasting for end-of-life products, thereby providing data-driven support for procurement and inventory decisions. Similarly, Drent et al. (2024) emphasized the importance of sharing operational and failure-related data to improve inventory forecasting accuracy, providing a representative example of the integration of learning effects and data-driven inventory optimization. Nevertheless, although existing studies have examined the impacts of operating time and usage conditions on equipment deterioration, limited attention has been paid to the interaction between the lessor's learning effects and spare parts inventory management in leasing environments.

In summary, although extensive research has been conducted on equipment maintenance optimization, supply chain coordination, learning effects, and spare parts inventory management, existing studies predominantly adopt a single-agent perspective and typically assume constant maintenance efficiency. Consequently, the interactive effects of learning and spare parts inventory on maintenance decision-making and supply chain performance have not been systematically explored. To fill this research gap, this study contributes to the literature in three aspects. First, the lessor's learning effect and spare parts inventory decisions are incorporated into the PM model, and their impacts on failure dynamics, maintenance efficiency, and system profitability are analyzed. Second, within both cooperative and non-cooperative game settings, the roles of learning effects and inventory levels in shaping the profit distribution between the lessor and the lessee are investigated. Third, coordination mechanisms are developed to improve maintenance service performance and achieve effective coordination between the leasing parties.

3 Problem description

In the leasing supply chain considered in this study, the lessor and the lessee enter into a leasing contract under which the lessor provides the equipment to the lessee for a leasing period of H . During the contract period, the lessor is responsible for providing maintenance services, while the lessee pays rental fees to the lessor in accordance with the terms of the contract. The structure of the leasing supply chain is illustrated in Fig. 1. During the leasing period, the lessor performs N PM actions on the leased equipment. The interval between two consecutive PM actions is denoted by T . Let T_i represent the length of the i th PM interval, which is defined as the time elapsed between the completion of the i th PM action and the commencement of the $i+1$ th PM action, as illustrated in Fig. 2. Meanwhile, whenever a failure occurs, MR is performed. After each MR action, the equipment failure rate is restored to the level immediately prior to failure. Specifically, the PM frequency N , defined as the number of PM activities conducted over the lease period, is treated as the primary decision variable. The corresponding PM interval length T is determined by the lease duration H , PM frequency N , and the PM execution time under the influence of learning effects. It is assumed that the equipment continues to deteriorate gradually during the repair process. Let θ denote the learning level, where $\theta \geq 0$ represents the slope of the learning curve. A larger value of θ indicates a higher learning level, whereas $\theta = 0$ implies the absence of any learning effect. Furthermore, to mitigate the risk of maintenance interruptions caused by spare parts shortages during repair activities, the lessor maintains an appropriate level of spare parts inventory. This inventory ensures the continuity of maintenance operations by reducing the likelihood of stockouts, thereby increasing equipment uptime and generating additional operational benefits. The corresponding decision-making framework is illustrated in Fig. 3.

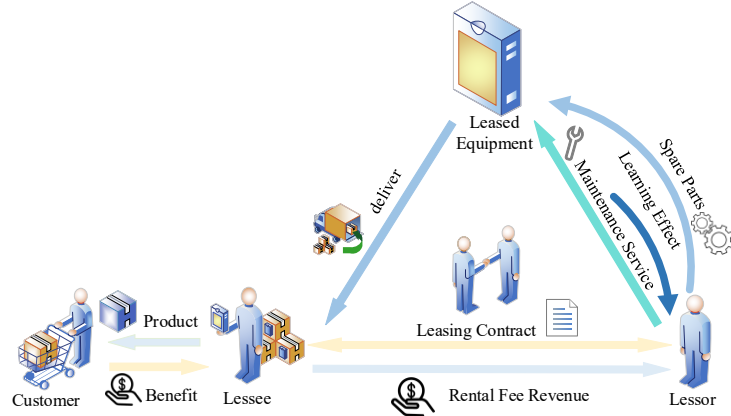


Figure.1 Leasing supply chain process

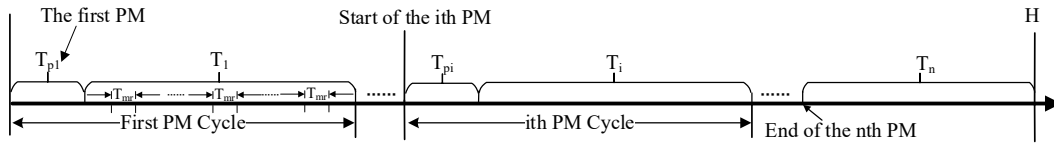


Figure 2 Maintenance Activity Plan Chart

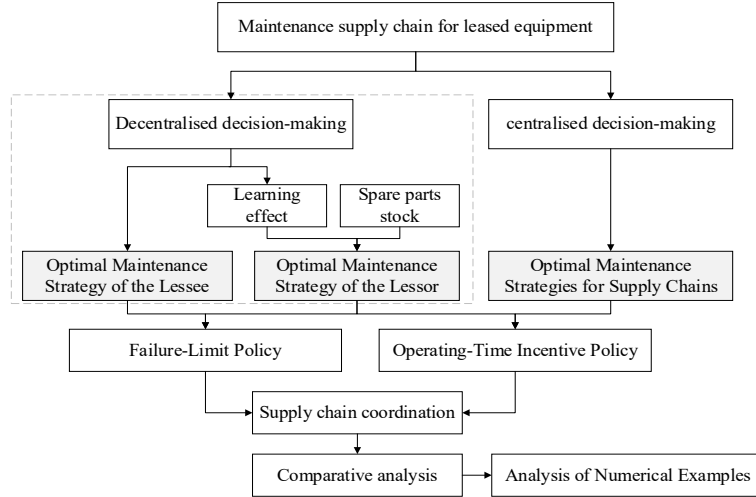


Figure. 3 Decision-making Flowchart

To facilitate model formulation and analysis, the following assumptions are made without affecting the optimal decision-making results:

- (1) The equipment failure rate increases over time during operation.
- (2) PM is assumed to be perfect; therefore, the system failure rate is restored to zero after each PM action.
- (3) The time and cost of PM activities are affected by learning effects, whereas the MR process is assumed to be independent of learning effects.
- (4) After equipment failure, MR is performed to restore the equipment to its previous operating condition. MR does not change the failure rate characteristics or the subsequent degradation process of the equipment.
- (5) The spare parts considered are small- and medium-sized commonly failed components that are used exclusively for MR. These parts are not consumed during PM activities, and their replenishment lead times are assumed to be relatively stable.
- (6) The lessor maintains a certain level of spare parts inventory to prevent

maintenance interruptions caused by stockouts. This improves operational continuity and increases the effective equipment uptime. Let x denote the operational continuity improvement factor, and let C_e represent the marginal benefit per unit increase in operational continuity, which reflects the marginal revenue generated by additional effective operating time resulting from improved equipment availability.

(7) A spare parts cost-sharing mechanism is adopted between the lessor and lessee. The lessee bears all spare parts inventory holding costs incurred by the lessor, while the lessor is responsible for spare parts stocking and equipment maintenance services.

4 Methods

4.1 Failure Rate Function

The intended use and leasing period of the equipment are typically specified in the leasing contract, thereby providing a formal basis for equipment management and responsibility allocation (Liu B, Wang Y, Yang H, et al. 2021). Assume that the leasing period of the equipment is H , and the utilization rate during the leasing period is m . Let t denote the utilization rate per unit time during the leasing period. Accordingly, the total utilization of the equipment over the leasing horizon is mt . The Weibull distribution has been widely applied in maintenance and reliability studies to characterize the failure behavior of equipment and systems. Its main advantage lies in its flexibility, as different failure patterns can be represented by adjusting the shape parameter β and the scale parameter η . Accordingly, the failure rate function can be expressed as $\lambda(t) = \eta\beta(\eta t)^{\beta-1}$. For the convenience of model formulation and analytical derivation, the failure rate function is further transformed into a linear function associated with equipment utilization and operating time, denoted by $\lambda(t) = \varepsilon_1 mt + \varepsilon_2 t$, where the shape parameter and scale parameter are represented by $\beta = 2$ and $\eta = \sqrt{(\varepsilon_1 m + \varepsilon_2)/2}$, respectively, and ε_1 and ε_2 denote the coefficients reflecting the effects of utilization rate and operating time on the failure rate. Furthermore, if the lessee makes an effort to properly operate and protect the leased equipment during the leasing period, both the impact of equipment utilization on the failure rate and the losses caused by equipment failures can be reduced. By incorporating the lessee's effort level, the failure rate function is further extended as $\lambda(t) = (\rho_1 - \rho_2 e)mt + \rho_3 t$, where the shape parameter and scale parameter are denoted by $\beta = 2$ and $\eta = \sqrt{(\rho_1 m - \rho_2 em + \rho_3)/2}$, respectively. In the function, let $\varepsilon_1 = \rho_1 - \rho_2 e$ and $\varepsilon_2 = \rho_3$, where ρ_1 , ρ_2 , and ρ_3 represent the coefficients reflecting the effects of equipment utilization, the lessee's effort level, and operating time on the failure rate, respectively. The definitions of the remaining model parameters are summarized in Table 1. Since equipment failures occur randomly, this study adopts the cumulative failure intensity function to represent the expected number of failures during the lease period. Therefore, the number of equipment failures, denoted by $M(T)$, occurring within the maintenance interval $[0, T]$ can be expressed as follows:

$$M(T) = \int_0^T \lambda(t) dt = \frac{1}{2} T^2 (\rho_1 m - \rho_2 em + \rho_3) \quad (1)$$

Table 1 Explanation of Model Symbols

Symbol	Description
H	leasing period
C_{pm}	preventive maintenance cost

C_{mr}	minimal repair cost
Π_c	total profit of the lessor
Π_m	total profit of the lessee
R	operating benefit per unit time
C	penalty cost
U	rental cost per unit time
P	fixed payment
T_{pm}	preventive maintenance time
T_{mr}	minimal repair time
τ_{mr}	minimal repair time threshold
τ_f	failure threshold during the leasing period
P_{mr}	penalty coefficient for minimum repair time in a single maintenance event
P_f	penalty coefficient per failure
W_i	time required for the i th minimal repair
C_u	unit spare parts cost
C_e	unit incremental benefit of operational continuity
P_k	spare parts subsidy
x	operational continuity improvement factor
C_{pm}^{total}	total PM cost
C_{mr}^{total}	total MR Cost
T_i	PM interval
θ	learning level
N	PM frequency

4.2 Profit functions of both parties

4.2.1 Maintenance Costs under Learning Effects

To simplify the analysis, this study assumes that the learning effect influences only the duration and cost of PM, while having no impact on MR. Let T_{pm} and C_{pm} denote the duration and cost of the first PM action, respectively. As maintenance experience accumulates and the learning effect gradually emerges, subsequent PM activities can be performed more efficiently, resulting in reduced maintenance time and lower maintenance costs under the same operating conditions. Therefore, the cost and duration of the i th PM are assumed to be $C_{pm}i^{-\theta}$ and $T_{pm}i^{-\theta}$, respectively. For simplicity, the learning rate associated with maintenance cost and maintenance time is assumed to be the same. The approximation $\sum_{i=1}^N i^{-\theta} \approx \int_0^N i^{-\theta} di = \frac{N^{1-\theta}}{1-\theta}$ is adopted to characterize the learning effect. This approximation has been shown to be highly accurate for values of θ less than or equal to 0.6, which covers nearly all practical situations (Tarakci H, Tang K, Teyarachakul S. 2013). Although it tends to slightly overestimate the total maintenance cost, the resulting bias has a negligible impact on the conclusions of this study. Accordingly, the total PM cost and total MR cost can be expressed as follows:

$$C_{pm}^{total} = C_{pm} \frac{N^{1-\theta}}{1-\theta} \quad (2)$$

$$C_{mr}^{total} = C_{mr} \sum_{i=1}^N M(T_i) \quad (3)$$

4.2.2 Penalty cost

(1) maintenance downtime penalty

Equipment failures occurring during the lease period generally interrupt production activities, thereby causing capacity losses or economic losses to the lessee. The lessor and the lessee typically agree on a contractual limit for repair duration when signing the lease contract. If the actual repair time exceeds the specified limit, the lessee imposes a penalty on the lessor at a rate of p_{mr} per unit time. Let W_i denote the duration of the i th minimal repair. Let $K(T_i) = \sum_{i=1}^N M(T_i)$, then $1 \leq i \leq \sum_{i=1}^N K(T_i)$.

Accordingly, After N preventive maintenance actions, if the lessor causes delays due to repair durations exceeding the contractual limit, a corresponding penalty cost C_t shall be incurred by the lessor.

$$\begin{aligned} C_t &= p_{mr} \sum_{n=0}^{K(T_i)} \max(0, W_i - \sigma) \\ &= p_{mr} K(T_i) \int_{\sigma}^{\infty} (w - \sigma) g(w) dw \\ &= p_{mr} K(T_i) \int_{\sigma}^{\infty} [1 - G(w)] dw \end{aligned} \quad (4)$$

where p_{mr} is the penalty paid by the lessor to the lessee when the duration of a minimal repair exceeds the prescribed limit, and σ is the time threshold for minimal repair. $G(w)$ and $g(w)$ denote the probability density function and cumulative distribution function of the minimal repair time, respectively. For notational convenience, let $V = \int_{\sigma}^{\infty} [1 - G(w)] dw$; then $C_t = p_{mr} K(T_i) V$.

(2) Penalty for the Total Number of Failures

During the lease period, if frequent failures caused by inadequate equipment quality disrupt the lessee's normal production activities, the lessor is required to pay a corresponding penalty to the lessee for each failure. Let p_f denote the unit penalty imposed for each failure. Then, the penalty cost C_f incurred by the lessor due to equipment failures can be expressed as follows:

$$\begin{aligned} C_f &= p_f \max[0, K(T_i)] = p_f K(T_i) \\ C_f &= \begin{cases} 0, & K(T_i) \leq 0 \\ p_f K(T_i), & K(T_i) > 0 \end{cases} \end{aligned} \quad (5)$$

Based on the two penalty cost components described above, the total penalty cost of the equipment over the lease period H can be expressed as follows:

$$C = C_t + C_f \quad (6)$$

4.2.3 Profit of the Lessor

The total profit of the lessor during the lease period is determined by multiple revenue and cost components, including leasing revenue, total preventive maintenance cost, total repair cost, penalty cost, spare-parts-related cost, spare-parts subsidy, and a fixed payment. Let U denote the leasing revenue per unit of effective operating time. In addition, the lessee pays a fixed fee to the lessor, denoted by P . The constraint ensures that the total time is limited to the entire leasing horizon H . Accordingly, the profit of the lessor can be expressed as follows:

$$\Pi_c = U \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) + P - p_{mr}K(T_i)V - p_fK(T_i) - C_uK(T_i) + P_k \quad (7)$$

$$s.t. \sum_{i=1}^N T_{pm} i^{-\theta} + \sum_{i=1}^N T_{ci} = H \quad (8)$$

4.2.4 Profit of the Lessee

The profit of the lessee is influenced by several factors, including the expected revenue generated during normal operating time, leasing cost, fixed payment, additional revenue from increased operating time, spare-parts subsidy expenditure, and penalty income. During the production process, the revenue generated per unit of operating time is denoted by R , where downtime consists of both PM time and MR time. In addition, the lessee provides a spare-parts subsidy to the lessor to cover the spare-parts-related expenses incurred by the lessor. Accordingly, the profit of the lessee can be expressed as follows:

$$\Pi_m = (R - U) \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - P + p_{mr}K(T_i)V + p_fK(T_i) + xC_eT_{mr}K(T_i) - P_k \quad (9)$$

$$s.t. \sum_{i=1}^N T_{pm} i^{-\theta} + \sum_{i=1}^N T_{mi} = H \quad (10)$$

4.3 Decentralized Decision-Making

Under the decentralized decision-making mode, the lessor and the lessee make decisions independently based on the information and resources available to them. Both parties aim to maximize their own operational efficiency and profits, which may hinder the coordinated optimization of the entire supply chain.

(1) Decision of the Lessor

Lemma 1. For a given N , the optimal PM intervals of the lessor are equal and are denoted by $T_{c_1}^* = T_{c_2}^* = \dots = T_{c_N}^* = \frac{H}{N_c^*} - \frac{N_c^{*-\theta}}{1-\theta} T_{pm}$. The optimal PM frequency of the lessor can be obtained from Eq. (11):

$$f(N_c^*) = \frac{UT_{pm} + C_{pm}}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u} \quad (11)$$

(2) Decision of the Lessee

Lemma 2. The optimal PM interval of the lessee is $T_{m_1}^* = T_{m_2}^* = \dots = T_{m_N}^* = \frac{H}{N_m^*} - \frac{N_m^{*-\theta}}{1-\theta} T_{pm}$.

Under decentralized decision-making, the optimal PM frequency of the lessee can be obtained from Eq. (12):

$$f(N_m^*) = \frac{(R - U)T_{pm}}{(R - U)T_{mr} - p_{mr}V - p_f - xC_eT_{mr}} \quad (12)$$

As indicated by Lemmas 1 and 2, the optimal decisions of the lessor and the lessee are not aligned when each party seeks to maximize its own profit. Under decentralized decision-making, the two parties make decisions independently and pursue their

individual objectives. As a result, the discrepancy between their optimal strategies often prevents coordination, thereby reducing the overall operational efficiency of the supply chain.

4.4 Centralized Decision-Making

Under the centralized decision-making mode, all members of the supply chain act with the objective of maximizing the overall system profit. By coordinating the decisions of the lessor and the lessee, conflicts of interest between the two parties can be effectively mitigated. This not only improves the coordination efficiency and service performance of the supply chain but also leads to a higher overall profit.

The profit of the supply chain under centralized decision-making can be expressed as follows:

$$\Pi_o = R \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) - C_u K(T_i) + x C_e T_{mr} K(T_i) \quad (13)$$

The Lagrangian function associated with the supply chain decision model can be expressed as follows:

$$\begin{aligned} L(T_{oi}, \gamma) = & R \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) - C_u K(T_i) \\ & + x C_e T_{mr} K(T_i) + \gamma_2 \left(\sum_{i=1}^N T_{pm} i^{-\theta} + \sum_{i=1}^N T_{oi} - H \right) \end{aligned} \quad (14)$$

The solution to Eq. (14) yields the optimal decisions for the supply chain. The conditions under which the optimal supply chain profit can be achieved under centralized decision-making are summarized in Lemma 3.

Lemma 3. The optimal maintenance interval of the supply chain is $T_{o_1}^* = T_{o_2}^* = \dots = T_{o_N}^* = \frac{H}{N_o^*} - \frac{N_o^{*-\theta}}{1-\theta} T_{pm}$. The optimal PM frequency of the supply chain can be obtained from Eq. (15):

$$f(N_o^*) = \frac{RT_{pm} + C_{pm}}{RT_{mr} + C_{mr} + C_u - x C_e T_{mr}} \quad (15)$$

Proposition 1. The relationship among the optimal PM frequencies of the lessor, the lessee, and the supply chain under centralized decision-making is as follows:

1. If $f(N_c^*) = f(N_m^*)$, then $N_c^* = N_o^* = N_m^*$.
2. If $f(N_c^*) < f(N_m^*)$, then $N_c^* > N_o^* > N_m^*$.
3. If $f(N_c^*) > f(N_m^*)$, then $N_c^* = N_o^* = N_m^*$.

Proposition 2. For a given PM frequency N , when $0 \leq \theta \leq 1 - \frac{1}{\ln N_c^*}$ holds, the optimal maintenance interval T^* increases with the increase of θ . When $1 - \frac{1}{\ln N_c^*} \leq \theta \leq 1$ holds, the optimal maintenance interval T^* decreases with the increase of θ . The optimal PM frequency increases with the increase of θ .

Corollary 1. In the absence of any coordination mechanism, the lessor and the lessee cannot achieve consistency in their preventive maintenance frequency decisions

due to the inconsistency of their objective functions.

1. The optimal PM frequency of the lessor is given by:

$$f(N_c^*) = \frac{UT_{pm} + C_{pm}}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u}$$

2. The optimal PM frequency of the lessee is given by:

$$f(N_m^*) = \frac{(R-U)T_{pm}}{(R-U)T_{mr} - p_{mr}V - p_f - xC_eT_{mr}}$$

3. The optimal PM frequency of the supply chain is given by:

$$f(N_o^*) = \frac{RT_{pm} + C_{pm}}{RT_{mr} + C_{mr} + C_u - xC_eT_{mr}}$$

In summary, only when the optimal decisions of the lessor and the lessee are consistent with those under centralized decision-making can the overall supply chain profit be maximized. However, as indicated in Corollary 1, the optimal decisions derived from individual profit maximization are often inconsistent between the two parties, leading to inefficiencies at the system level. Therefore, it is necessary to design appropriate coordination mechanisms to achieve incentive compatibility between the two parties, thereby improving the overall performance of the supply chain.

5 Coordination Strategy

In practice, it is uncommon for the lessor and lessee to independently arrive at decisions that coincide with the system-wide optimum. Therefore, appropriate coordination mechanisms are required to improve the overall efficiency of the equipment leasing supply chain. Although coordination strategies can enhance total supply chain profit, several critical issues remain, including how to design incentive-compatible coordination mechanisms that sustain cooperation, how to allocate the additional benefits generated through coordination fairly, and how to ensure the lessor's willingness to participate actively. To address these challenges, two feasible coordination strategies are proposed and analyzed in this study.

(1) Failure Limitation Strategy (FL):

The FL sets a maximum allowable number of failures during the lease period and imposes penalty costs on failures exceeding the predefined threshold, thereby partially internalizing failure risks for the lessor. By aligning the lessor's maintenance incentives with equipment reliability objectives, this mechanism mitigates the conflict of maintenance responsibility between the lessor and lessee, encourages preventive maintenance investment, and improves supply chain coordination.

(2) Uptime Target Bonus Strategy (UTB):

The UTB strategy is a performance-based incentive mechanism that rewards the lessor for achieving a predefined uptime target. Specifically, a bonus is awarded only when the actual equipment uptime exceeds the target level established by the lessee. By directly linking financial incentives to operational performance, this strategy encourages the lessor to enhance maintenance effectiveness and maintain a high level of equipment reliability and availability throughout the leasing period.

5.1 Failure Limitation Strategy

Under the FL strategy, the lessee determines the allowable failure threshold and

the reward associated with each avoided failure. Consequently, the incentive cost incurred by the lessee is denoted by $R_g[\tau_g - K(T)]^+$. Accordingly, the profit functions of the lessor and the lessee can be expressed as follows:

$$\begin{aligned} \Pi_c^L = & U \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) + P - p_{mr}K(T_i)V \\ & - p_f K(T_i) - C_u K(T_i) + P_k + R_g[\tau_g - K(T)]^+ \end{aligned} \quad (16)$$

$$\begin{aligned} \Pi_m^L = & (R - U) \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - P + p_{mr}K(T_i)V + p_f K(T_i) \\ & + xC_e T_{mr} K(T_i) - P_k - R_g[\tau_g - K(T)]^+ \end{aligned} \quad (17)$$

Taking the first-order derivative of Π_c^L with respect to N and setting it equal to zero yields:

$$f(N_c^L) = \frac{UT_{pm} + C_{pm}}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u + R_g} \quad (18)$$

Let $f(N_c^L) = f(N_o^*)$, τ_g and R_g satisfy the following condition:

$$\frac{UT_{pm} + C_{pm}}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u + R_g} = \frac{RT_{pm} + C_{pm}}{RT_{mr} + C_{mr} + C_u - xC_e T_{mr}}$$

Furthermore, when the number of failures exceeds the specified failure threshold, the lessee determines the payment for each excess failure. The additional cost borne by the lessee is denoted by $U_g[K(T) - \tau_g]^+$. Accordingly, the profit functions of the lessor and the lessee are given as follows:

$$\begin{aligned} \Pi_c^L = & U \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) + P - p_{mr}K(T_i)V \\ & - p_f K(T_i) - C_u K(T_i) + P_k + U_g[K(T) - \tau_g]^+ \end{aligned} \quad (19)$$

$$\begin{aligned} \Pi_m^L = & (R - U) \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - P + p_{mr}K(T_i)V + p_f K(T_i) \\ & + xC_e T_{mr} K(T_i) - P_k - U_g[K(T) - \tau_g]^+ \end{aligned} \quad (20)$$

Similarly, it can be obtained that:

$$f(N_c^L) = \frac{UT_{pm} + C_{pm}}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u - U_g} \quad (21)$$

Let $f(N_c^L) = f(N_o^*)$, τ_g and U_g satisfy the following condition:

$$\frac{UT_{pm} + C_{pm}}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u - U_g} = \frac{RT_{pm} + C_{pm}}{RT_{mr} + C_{mr} + C_u - xC_e T_{mr}}$$

At this point, the lessor's profit satisfies condition $\Pi_c \geq \pi$, while the lessee's profit satisfies condition $\Pi_m \geq \Pi_o - \pi$. Accordingly, the fixed payment $P \in [P_1, P_2]$ can be determined as follows:

Proposition 3. The Failure Limitation strategy can achieve supply chain

coordination.

(1) Under condition $f(N_c^*) > f(N_o^*)$, the lessee determines the maximum allowable number of failures τ_g and the reward amount R_g associated with each avoided failure.

$$P_1 = \pi - U \sum_{i=1}^N (T_i - M(T_o^*)T_{mr}) + C_{pm} \frac{N^{1-\theta}}{1-\theta} + C_{mr} \sum_{i=1}^N M(T_o^*) + p_{mr} K(T_o^*)V \\ + p_f K(T_o^*) + C_u K(T_o^*) - P_k - R_g [\tau_g - K(T_o^*)]^+ \quad (22)$$

$$P_2 = (R - U) \sum_{i=1}^N (T_i - M(T_o^*)T_{mr}) + \pi + p_{mr} K(T_o^*)V + p_f K(T_o^*) \\ + xC_e T_{mr} K(T_o^*) - P_k - R_g [\tau_g - K(T_o^*)]^+ - \Pi_o^L(T_o^*, N_c^*) \quad (23)$$

(2) When condition $f(N_c^*) < f(N_o^*)$ is satisfied, the lessee determines the maximum allowable number of failures τ_g and the payment for each excess failure beyond this limit U_g .

$$P_3 = \pi - U \sum_{i=1}^N (T_i - M(T_o^*)T_{mr}) + C_{pm} \frac{N^{1-\theta}}{1-\theta} + C_{mr} \sum_{i=1}^N M(T_o^*) + p_{mr} K(T_o^*)V \\ + p_f K(T_o^*) + C_u K(T_o^*) - P_k - U_g [K(T_o^*) - \tau_g]^+ \quad (24)$$

$$P_4 = (R - U) \sum_{i=1}^N (T_i - M(T_o^*)T_{mr}) + \pi + p_{mr} K(T_o^*)V + p_f K(T_o^*) \\ + xC_e T_{mr} K(T_o^*) - P_k - U_g [K(T_o^*) - \tau_g]^+ - \Pi_o^L(T_o^*, N_c^*) \quad (25)$$

According to Proposition 3, the FL strategy enables supply chain coordination, and the resulting profit can be allocated between the lessor and the lessee through a fixed payment P . Specifically, when condition $P = P_3$ holds, the lessor's optimal profit is $\Pi_c^L(T_c^L, N_c^L)$; when condition $P = P_4$ holds, the lessee's optimal profit is $\Pi_m^L(T_m^L, N_m^L)$.

5.2 Uptime Target Bonus Strategy

Let τ_U denote the uptime level, and let B represent the bonus per unit time awarded when the lessor's uptime exceeds the target level. Let $E(T) = \sum_{i=1}^N (T_i - M(T_i)T_{mr})$, and let the actual equipment uptime be $E(T)$. Accordingly, the incentive cost is given by $B[E(T) - \tau_U]^+$, and the lessor's profit function becomes:

$$\Pi_c^U = U \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) + P - p_{mr} K(T_i)V \\ - p_f K(T_i) - C_u K(T_i) + P_k + B[E(T) - \tau_U]^+ \quad (26)$$

where $[x]^+ = \max\{x, 0\}$. Accordingly, the profit function of the lessee becomes:

$$\Pi_m^U = (R - U) \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - P + p_{mr} K(T_i)V + p_f K(T_i) \\ + xC_e T_{mr} K(T) - P_k - B[E(T) - \tau_U]^+ \quad (27)$$

To achieve supply chain coordination, the lessee determines the parameters B, τ_U and P , such that the lessor chooses $T = T_c^*$ to maximize its profit under the UTB

strategy. By inspection, it can be observed that Eqs. (7) and (26) become identical when $B[E(T) - \tau_U]^+ \leq 0$ holds. However, this condition alone is insufficient to achieve supply chain coordination. Therefore, condition $B[E(T) - \tau_U]^+ > 0$ must also be satisfied. By comparing Eqs. (13) and (26), it follows that when condition $T = T_o^*$ holds, conditions $\tau_U < E(T_o^*)$ and $B = R - U + \frac{D'(T_o^*) + N'(T_o^*)}{E'(T_o^*)}$ must be satisfied simultaneously. Lemma 4 further specifies the feasible ranges of τ_U and P required to achieve supply chain coordination.

Lemma 4. Supply chain coordination can be achieved when conditions $\tau_U \in [\tau_1, \tau_2]$ and $P \in [P_5, P_6]$ are satisfied, where:

$$\tau_1 = E(T_o^*) + \frac{UE(T_o^*) - A(T_o^*) - \pi}{B} \quad (28)$$

$$\tau_2 = E(T_o^*) + \frac{UE(T_o^*) - UE(T_c^*) + A(T_c^*) - A(T_o^*)}{B} \quad (29)$$

$$P_5 = \pi - UE(T_o^*) + A(T_o^*) - P_k - B[E(T_o^*) - \tau_U] \quad (30)$$

$$P_6 = \pi + (R - U)E(T_o^*) + D(T_o^*) + N(T_o^*) - P_k - B[E(T_o^*) - \tau_U] - \Pi_o^U(T_c^*, N_c^*) \quad (31)$$

5.3 Comparative Analysis of the Failure Limitation and Uptime Target Bonus Strategies

Although both the FL strategy and the UTB strategy are capable of achieving effective supply chain coordination, differences in their parameter settings may exert distinct influences on the decision-making behavior of the lessor and the lessee. Therefore, it is necessary to further investigate the impacts of these coordination mechanisms on the decisions and profits of both parties. To this end, a comparative analysis of the two coordination strategies is conducted as follows.

Let Π_c^L and Π_c^U denote the lessor's profits under the FL strategy and the UTB strategy, respectively. Furthermore, let $P_L \in [P_1, P_2]$ and $P_U \in [P_3, P_6]$ represent the fixed payments made by the lessee to the lessor under the FL and UTB strategies, respectively. The difference between the lessor's profits under the two coordination strategies is given by:

$$\Pi_c^L - \Pi_c^U = (P_L - P_U) + R_g[\tau_g - K(T)]^+ - B[E(T) - \tau_U]^+ \quad (32)$$

When $\Pi_c^L = \Pi_c^U$ is satisfied, $P_L - P_U = B[E(T) - \tau_U]^+ - R_g[\tau_g - K(T)]^+$ follows.

Let $A_1 = B[E(T) - \tau_U]^+ - R_g[\tau_g - K(T)]^+$

Similarly, let Π_m^L and Π_m^U denote the lessee's profits under the FL strategy and the UTB strategy, respectively. The difference in the lessee's profit between the two strategies is given by:

$$\Pi_m^L - \Pi_m^U = (P_B - P_U) - R_g[\tau_g - K(T)]^+ + B[E(T) - \tau_U]^+ \quad (33)$$

Next, the preferences of the lessor and the lessee regarding the two coordination strategies are examined. If both the UTB strategy and the FL strategy are considered effective in coordinating the supply chain, the lessor and the lessee must determine their

preferred strategy from these two alternatives.

(1) If $P_L - P_U = A_1$, then $\Pi_c^L = \Pi_c^U$ and $\Pi_m^L = \Pi_m^U$. In this case, both the lessor and the lessee are indifferent between the two strategies.

(2) If $P_L - P_U > A_1$, then $\Pi_c^L > \Pi_c^U$ and $\Pi_m^L < \Pi_m^U$. Under this condition, the lessor prefers the FL strategy, whereas the lessee prefers the UTB strategy.

(3) If $P_L - P_U < A_1$, then $\Pi_c^L < \Pi_c^U$ and $\Pi_m^L > \Pi_m^U$. Under this condition, the lessor prefers the UTB strategy, whereas the lessee prefers the FL strategy.

6 Numerical analysis and discussions

6.1 Numerical Example

It is assumed that the repair time follows an exponential distribution (Huang et al.2019). Let $G(w)=1-e^{-w/t}$, where t denotes the mean repair time of a MR. Furthermore, assume that $\sigma=0.3$. The corresponding parameter settings are summarized in Table 2.

Table 2 Initial Values of Parameters

U	C_{pm}	H	T_{mr}	ρ_2	m	p_f	x
1000	500	300	0.3	0.001	100	60	0.3
R	C_{mr}	T_{pm}	ρ_1	ρ_3	p_{mr}	C_u	C_e
1500	70	1	0.004	0.6	40	200	800

Note: Time is measured in days, and profit is measured in yuan (CNY).

First, under the condition of $\theta=0$, the lessor's optimal PM frequency is determined to be 110 according to the expression of $f(N_c^*)$, which corresponds to an optimal PM interval of approximately 1.73. By substituting the optimal solution into the corresponding expressions of $f(N_m^*)$ and $f(N_o^*)$, it can be verified that the relationship $f(N_m^*) > f(N_o^*) > f(N_c^*)$ holds. Given that $f(N)$ decreases as N increases, $N_m < N_o < N_c$ can be readily derived. Moreover, when the learning effect of the lessor is taken into account, Figure 4 shows that the lessor's optimal PM frequency increases with the learning level. This observation further validates the analytical result established in Proposition 2. When $\theta=0.1$, the lessor's optimal PM frequency rises from 110 to 140, substantially exceeding that in the absence of learning. This result can be attributed to the fact that learning continuously improves maintenance efficiency by reducing both the time and cost required for PM activities. Consequently, preventive maintenance becomes increasingly attractive to the lessor. Specifically, the lessor is able to conduct PM more frequently without significantly compromising equipment uptime. The increased PM frequency enhances equipment reliability, lowers the probability of failures, and decreases the number of MR, thereby yielding higher maintenance benefits at a lower time-loss cost. As the learning level continues to improve, the time and cost associated with PM are further reduced. The decreases in T_{pm} and C_{pm} strengthen the economic advantage of PM, providing additional incentives for the lessor to increase the PM frequency. Under decentralized decision-making, when $\theta=0.1$, the optimal PM frequencies of the lessor and the lessee are 140 and 35, respectively, whereas the optimal PM frequency for the integrated supply chain under centralized decision-making is 123. At this learning level, the profits of the lessor, the lessee, and the supply chain amount to 100,263, 113,851, and 191,985, respectively. Figure 5 illustrates that, as the learning level increases, the optimal PM frequency rises continuously, while the total maintenance cost declines, resulting in profit growth for all parties. From the

lessee's perspective, reductions in PM duration and failure occurrence increase effective operating time and consequently enhance operational benefits. From the lessor's perspective, learning improves maintenance efficiency by lowering both PM costs and maintenance time, thereby increasing profitability. At the supply chain level, the combined effects of extended operating time and reduced maintenance expenditure generate higher overall profits. These results indicate that improvements in the learning level create mutual benefits for both supply chain members and the system as a whole.

Moreover, as the learning level increases, the optimal PM frequencies of the lessor and the supply chain exhibit similar upward trends, whereas the lessee's optimal PM frequency remains significantly lower, with the gap between them continuously widening. This phenomenon arises from the inherent differences in the decision-making objectives among the three parties. When the lessor balances maintenance costs against reliability gains, its optimal decision is highly aligned with the system-wide objective of profit maximization. In contrast, the lessee is more concerned with its own operational benefits and rental costs, and therefore tends to reduce PM frequency in order to minimize maintenance-related time consumption and cost burden. As a result, when learning reduces the relative cost of PM, the optimal PM frequencies of both the lessor and the supply chain increase monotonically, whereas the lessee, due to its weaker dependence on PM for performance improvement, experiences a relatively slower growth in optimal PM frequency, thereby further widening the gap between its decisions and those of the other two parties.

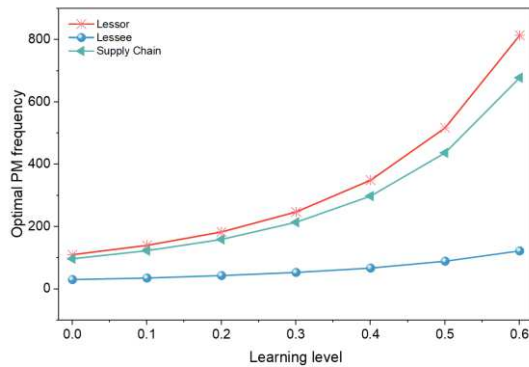


Figure.4 The influence of learning level on the optimal PM frequency

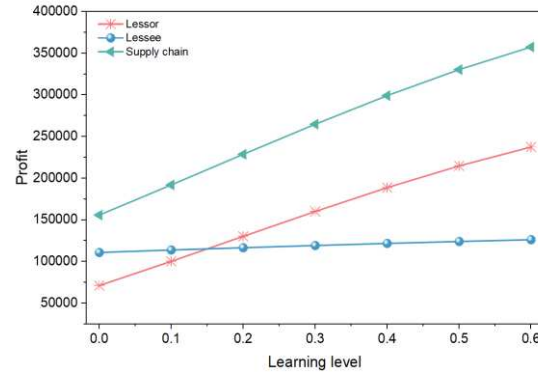


Figure.5 The influence of learning level on expected profit

Subsequently, by fixing the lessor's learning level at $\theta = 0.1$, a comparative analysis was performed to evaluate the impacts of spare parts inventory on the optimal decisions and profits of the lessor, the lessee, and the entire supply chain. The corresponding results are presented in Table 3.

Table 3 Comparison of Decisions and Profits with/without Spare Parts

	θ	N_c	Π_c	N_m	Π_m	N_o	Π_o
Without Spare Parts Inventory	0.1	129	100087	113	83053	125	189000
With Spare Parts Inventory	0.1	140	100263	35	113851	123	191985

As shown in Table 3, the introduction of spare parts leads to significant changes in the maintenance strategies and profit structures of supply chain members. First, spare parts availability improves the continuity of repair operations and reduces the risk of

downtime caused by spare parts shortages, thereby enhancing equipment operational stability. Under this condition, the optimal PM frequencies of the lessee and the overall supply chain both decrease. The reduction in maintenance intensity not only lowers maintenance costs but also extends effective operating time, resulting in significantly higher profits for both the lessee and the entire supply chain compared with the scenario without spare parts. In contrast, the optimal PM frequency of the lessor increases after the introduction of spare parts. Since spare parts constraints are alleviated, the lessor is more inclined to increase PM frequency to improve equipment condition. However, higher PM frequency increases the lessor's maintenance expenditures, while spare parts inventory introduces additional holding costs. Although spare parts inventory improves lessee benefits by enhancing equipment operational continuity and reducing downtime losses, its configuration costs are mainly borne by the lessor, resulting in a certain mismatch between cost responsibility and benefit allocation. Therefore, a spare parts subsidy mechanism provided by the lessee to the lessor is introduced to internalize the positive externality generated by spare parts inventory and motivate the lessor to optimize spare parts configuration, thereby improving overall supply chain performance. The results indicate that, with an appropriate subsidy level, the lessor's profit can be restored and further exceed the level achieved without considering spare parts. Meanwhile, the profits of both the lessee and the entire supply chain are further improved, achieving a mutually beneficial outcome among supply chain members.

Second, within the supply chain, the lessor tends to adopt a high PM frequency to ensure equipment reliability and maximize its own profit, whereas the lessee, who bears the maintenance costs, is inclined to reduce PM frequency. This creates a “downward pull” effect, leading to an overall maintenance level that is lower than the supply chain optimum, thereby preventing the full realization of the benefits brought by learning. To address this divergence, this study proposes two coordination strategies: the FL strategy and the UTB strategy. The former imposes a constraint on the number of allowable failures, thereby guiding the lessor to improve maintenance practices and optimize maintenance decisions. The latter introduces a performance-based incentive mechanism linked to equipment operating time, motivating the lessor to enhance maintenance efficiency and reduce maintenance costs, while simultaneously promoting supply chain coordination. A comparative analysis of the characteristics of these two strategies is presented in Table 3.

Table 4 Comparison of Failure Limitation Strategy and Uptime Target Bonus Strategy

Characteristic	Failure Limitation Strategy	Uptime Target Bonus Strategy
Supply chain coordination	Always	Always
Incentive for the lessor to improve repair time	No	Yes
Incentive for the lessor to reduce repair costs	No	Yes
Incentives for the Lessee to Reduce Failure Rate	Yes	Yes
Coordination under improved repair time	Sometimes	Always
Coordination under improved repair cost	No	Usually

We first examine the FL strategy. Under this mechanism, the lessee incurs an additional payment for each failure that exceeds the prescribed failure threshold. The failure limit is set at 130, and the penalty for each excess failure is assumed to be 50. To guarantee the lessor's willingness to participate in the coordination scheme, a reservation profit of 110,000 is imposed, ensuring that the lessor's coordinated profit is no less than its reservation level. According to Proposition 3 provides the condition for achieving supply chain coordination, namely $P \in [P_3, P_4]$. Based on this condition, the lessor's expected profit under the FL strategy is calculated as 112,152. This indicates that the participation constraints of both parties are satisfied. Under this strategy, the feasible range of the fixed payment is derived as $P \in [11539, 14199]$.

We next investigate the UTB strategy. Let $B = R - U + \frac{D'(T_o^*) + N'(T_o^*)}{E'(T_o^*)} = 782$.

Based on Lemma 5, the corresponding bounds $\tau_1 = 159.1$ and $\tau_2 = 171.5$ can be obtained. Consequently, supply chain coordination can be achieved provided that τ_U falls within this interval. For illustrative purposes, τ_U is set to 171.5, from which $P_5 = 9729$ and $P_6 = 12389$ are determined. When $P = 9729$ holds, the lessor's expected profit reaches 112,182, exceeding its reservation profit of 110,000. Therefore, the participation constraint of the lessor is satisfied. After selecting τ_U , the fixed payment P can be chosen within the range specified in Lemma 5 to achieve both supply chain coordination and profit allocation between the two parties.

6.2 Sensitivity Analysis

(1) Utilization rate

In modeling the failure rate, this study considers the utilization rate, operating time, and the lessee's effort level. In this subsection, a sensitivity analysis is conducted on the utilization rate parameter, and the results are presented in Table 5.

Table.5 The Impact of Usage Rate on the Optimal PM Frequency and Profit

m	Optimal PM Frequency			Profit		
	N_c	N_m	N_o	Π_c	Π_m	Π_o
60	135	34	119	100117	113837	191815
80	137	34.5	121	100219	113846	191944
100	140	35	123	100263	113851	191985
120	142	36	125	100222	113846	191942
140	144	36.4	127	100128	113839	191819
160	146	37	129	99984	113823	191621
180	148	38	131	99791	113783	191350

As the equipment utilization rate increases, the optimal PM frequencies of the lessor, the lessee, and the entire supply chain all exhibit a gradual upward trend. This indicates that when the operating intensity increases and the failure risk rises, all parties need to increase PM investments to maintain equipment availability. The profit results

(see Table 5) show that the profits of the lessor and the lessee remain relatively stable in the low- to medium-utilization range, but decrease significantly in the high-utilization regime. This can be explained as follows. For the lessor, although it benefits from the increase in utilization at the initial stage, the accelerated growth of the failure rate at higher utilization levels leads to a simultaneous increase in both PM and MR requirements, resulting in a sharp rise in maintenance costs that erodes its profit margin. For the lessee, higher utilization reduces equipment availability, shortens failure intervals, and restricts effective operating time, thereby increasing opportunity losses and leading to a more pronounced decline in profit. The overall supply chain profit exhibits an inverted U-shaped pattern and reaches its maximum around $m = 100$. This variation is driven by the trade-off between revenue expansion and cost escalation associated with increasing utilization. At the initial stage, higher utilization enhances effective operating intensity, thereby increasing operational revenue. However, as utilization continues to increase, the failure rate rises significantly, leading to higher PM and MR demands. Consequently, maintenance frequency and total operational costs increase rapidly. Eventually, the growth rate of costs exceeds that of revenue, shifting the system from a revenue-dominated regime to a cost-dominated regime, which results in the observed inverted U-shaped profit pattern.

(2) Lease duration

In leasing contracts, the lease duration is a critical factor that influences the decision-making of all parties. Variations in lease duration lead to corresponding adjustments in the strategies of the lessor, the lessee, and the entire supply chain. As shown in Figure 6, extending the lease duration has a consistently positive impact on all parties. Although the optimal PM frequency rises as the lease duration increases, the additional maintenance activities do not adversely affect the profits of the lessor, the lessee, or the overall supply chain. For the lessor, the additional rental income generated from a longer lease duration outweighs the increased PM costs, leading to a continuously increasing profit trend. For the lessee, the extended effective operating time generates higher production benefits, which are sufficient to offset the time losses caused by PM and MR, thereby also improving its profit. The supply chain profit increases most significantly, driven by simultaneous improvements in both parties' profits and enhanced resource utilization efficiency. Overall, extending the lease duration leads to a win-win outcome by simultaneously increasing maintenance demand and improving resource utilization efficiency, thereby achieving coordinated profit growth and system-level optimization.

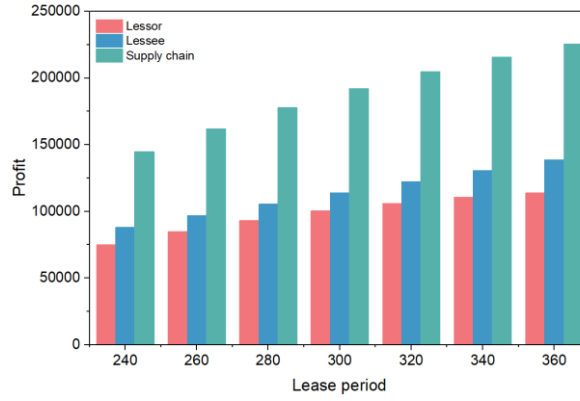


Figure.6 The Impact of Lease Period on Profit

(3) PM time

Next, we investigate the impact of PM time on the optimal PM frequency and profit levels. As shown in Table 6, as PM time increases, the optimal PM frequencies of the lessor, the lessee, and the entire supply chain all exhibit a decreasing trend, indicating that longer PM duration reduces the level of preventive maintenance effort across all parties. In terms of profit performance, all three parties exhibit an inverted U-shaped pattern with respect to PM time, reaching a relatively optimal level at $T_{pm} = 1$.

Furthermore, the supply chain profit is more sensitive to variations in PM time, whereas the lessee's profit responds more moderately, reflecting a more direct system-level response to changes in the maintenance time structure. The underlying reason is as follows. When PM time is relatively short, although each maintenance activity is time-efficient, the higher PM frequency required to maintain reliability reduces effective operating time, thereby constraining profit improvement. As PM time increases moderately, a more favorable trade-off emerges between maintenance frequency and per-maintenance duration, which helps reduce failure risk and improve system operational efficiency, leading to higher profits. However, when PM time becomes excessively long, the increased downtime associated with each maintenance activity reduces maintenance efficiency and consequently weakens overall system performance. Overall, the impact of PM time on system performance is highly nonlinear, primarily driven by the trade-off between maintenance frequency and time-related losses.

Table.6 The Impact of PM Time on the Optimal PM Frequency and Profit

T_{pm}	Optimal PM Frequency			Profit		
	N_c	N_m	N_o	Π_c	Π_m	Π_o
0.7	161	43	143	97484	113380	188387
0.8	153	40	136	99109	113660	190379
0.9	146	37	129	99984	113823	191621
1	140	35	123	100263	113851	191985
1.1	134	33	118	100042	113801	191716
1.2	128	32	113	99249	113742	190849
1.3	124	31	108	98359	113656	189298

(4) operational continuity coefficient

Figures 7 and 8 illustrate the impacts of improved operational continuity on the

PM strategies and profits of the lessor, the lessee, and the entire supply chain. As the lessor enhances operational continuity by increasing spare parts inventory, repair interruptions caused by part shortages are significantly reduced, thereby improving equipment availability. The lessee's PM frequency decreases significantly with increases in x , indicating that higher operational continuity reduces the need for the lessee to rely on frequent PM activities to compensate for potential repair interruptions. In contrast, the lessor's PM frequency remains unchanged throughout the entire range of x . This is because the lessor's maintenance decisions are primarily determined by its own maintenance responsibilities and cost structure rather than by changes in equipment availability. At the supply chain level, the PM frequency exhibits only a slight decline. Although improved availability reduces the need for PM, the additional costs associated with spare parts deployment limit the extent of adjustments in maintenance decisions. In terms of profit performance, the lessee's profit increases continuously as x rises, suggesting that the additional effective operating time generated by enhanced operational continuity can be directly translated into higher returns. The lessor's profit remains nearly unchanged because improved continuity does not generate additional rental income. The overall supply chain profit exhibits a slight upward trend, indicating that the system-level benefits derived from enhanced operational continuity slightly outweigh the additional investment in spare parts inventory, ultimately resulting in a limited but positive net gain.

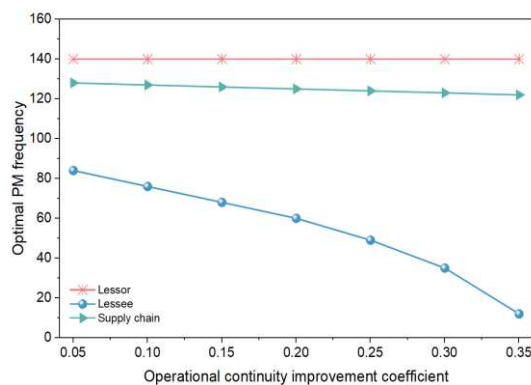


Figure 7 The Impact of Maintenance Continuity Enhancement Coefficient on the Optimal PM Frequency

(5) Spare parts-related cost

Table 7 reports the impact of increasing spare parts-related costs on the optimal PM frequency and profit levels. The lessee neither bears maintenance costs nor is affected by fluctuations in spare parts prices; therefore, its optimal PM frequency and profit remain unchanged. In contrast, the lessor's PM frequency increases slightly as costs rise, reflecting a strategic tendency to strengthen preventive maintenance under higher spare parts cost pressure, so as to reduce potential repair requirements and maintain operational continuity. At the supply chain level, the optimal PM frequency decreases slightly, indicating that under higher cost conditions, the marginal benefit of increasing PM is insufficient to offset the additional cost, leading to a more conservative maintenance strategy. In terms of profit performance, both the lessor and the supply

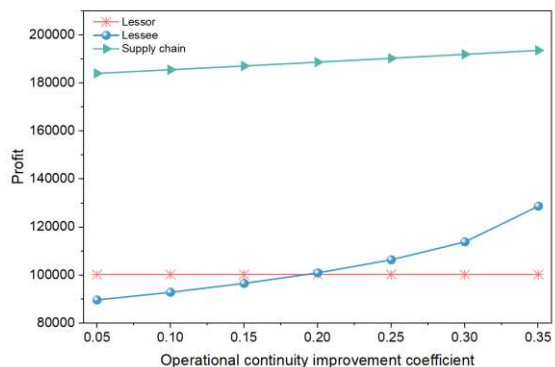


Figure 8 The Impact of the Maintenance Continuity Enhancement Coefficient on Expected Profit

chain experience a decline in profit as spare parts costs increase, with the supply chain exhibiting the largest reduction, highlighting its greater sensitivity to spare parts-related costs. Overall, rising spare parts costs not only compress profit margins but also induce heterogeneous adjustments in PM strategies across different stakeholders, driven by their distinct roles and payoff structures.

Table 7 The Impact of Spare Parts-related Costs on the Optimal PM Frequency and Profit

C_u	Optimal PM Frequency			Profit		
	N_c	N_m	N_o	Π_c	Π_m	Π_o
100	129	35	113	112087	113851	206688
150	134	35	118	105942	113851	199089
200	140	35	123	100263	113851	191985
250	145	35	127	94938	113851	185314
300	149	35	132	89942	113851	179028
350	154	35	136	85237	113851	173086

7 Discussion

The findings of this study provide the following managerial implications for supply chain practice.

First, firms should incorporate learning effects into long-term maintenance decision-making systems rather than treating them merely as passive outcomes of maintenance cost reduction. As maintenance experience accumulates, the time and cost required for PM gradually decrease, enabling firms to adjust maintenance frequencies while maintaining equipment reliability and achieve more efficient allocation of maintenance resources. Therefore, for long-term operating rental equipment, firms should emphasize the accumulation and utilization of maintenance experience. By establishing mechanisms for maintenance data accumulation, experience feedback, and cross-cycle knowledge sharing, firms can continuously leverage learning effects to improve maintenance efficiency and enhance equipment lifecycle management capabilities.

Second, spare parts inventory configuration should be jointly optimized with maintenance strategies rather than being determined independently. The results demonstrate that an appropriate level of spare parts inventory can reduce repair waiting time and improve equipment availability, whereas excessive inventory levels may increase operational costs. Therefore, for high-value rental equipment with high failure risks, firms should dynamically adjust spare parts inventory strategies by considering equipment reliability levels, maintenance demand characteristics, and downtime loss costs. This enables a balance between maintenance responsiveness and inventory investment.

Furthermore, different coordination strategies have distinct applicability conditions in improving maintenance efficiency and overall system performance. For failure-sensitive equipment, the failure limitation strategy can be adopted to constrain the allowable failure level, thereby encouraging lessors to increase maintenance investment and enhance equipment reliability. When equipment utilization is high or downtime costs are substantial, the introduction of an uptime target bonus strategy can

more effectively motivate lessors to increase PM frequency. Therefore, managers should select appropriate coordination mechanisms based on equipment characteristics, operational contexts, and the stability of cooperative relationships, while improving supply chain efficiency and maintaining the feasibility of incentive implementation and controllability of associated costs.

Finally, firms should promote the transition from fixed-interval maintenance toward dynamic maintenance strategies. Since learning levels, equipment utilization intensity, failure characteristics, and inventory conditions jointly influence optimal maintenance decisions, traditional static maintenance policies may be insufficient to adapt to complex operating environments. Firms can leverage equipment operating data and condition information to dynamically adjust maintenance frequency, spare parts configuration, and coordination mechanisms, thereby achieving the synergistic improvement of equipment reliability, economic performance, and overall supply chain performance in rental operations.

8 Conclusions and further research

This study develops a failure rate function based on the Weibull distribution by capturing the evolution of equipment failure behavior during operation, and incorporates tenant effort level, equipment utilization rate, and operating time into a unified analytical framework. On this basis, the lessor's learning effects are embedded into both PM duration and cost, while spare parts inventory is introduced to characterize its impact on maintenance responsiveness and repair efficiency. A supply chain profit model is further constructed by incorporating penalty mechanisms based on operating time and failure frequency. The results indicate that both learning effects and spare parts inventory exert significant influences on the optimal PM frequency, maintenance efficiency, and cost structure, and that a clear coupling relationship exists between them. Improvements in the learning level can reduce PM duration and maintenance costs, and these benefits are more effectively translated into higher equipment availability when spare parts are sufficiently supplied. However, under conditions of spare parts shortage, even if learning effects improve maintenance efficiency, repair delays may still weaken their overall effectiveness. Similarly, increases in spare parts inventory can only be fully exploited under a sufficiently high learning level, enabling faster maintenance response. A further comparative analysis shows that, with the inclusion of spare parts, the profits of the lessor, lessee, and the overall supply chain are all improved relative to the case without inventory, indicating that well-designed spare parts allocation not only accelerates maintenance response but also enhances system stability and profitability.

Under decentralized decision-making, the lessor and lessee independently optimize their decisions based on their respective profit and cost objectives, which often leads to inconsistent maintenance strategies, incentive misalignment, and reduced system efficiency. Appropriate coordination mechanisms can effectively alleviate such conflicts. Accordingly, this study proposes two coordination strategies. The first is a failure-limit strategy, which restricts the lessee's operational behavior by imposing an upper bound on the allowable number of failures, thereby reshaping the maintenance incentive structure and improving equipment reliability. The second is a run-time

incentive strategy, which sets a threshold for normal operating time and provides incentives to the lessor once the threshold is reached, thereby strengthening maintenance effort. With properly designed threshold and reward levels, this mechanism can achieve outcomes close to those under centralized decision-making. Overall, the failure-limit strategy is more suitable for failure-sensitive equipment, whereas the run-time incentive strategy performs better in high-utilization or high-downtime-cost scenarios. Therefore, these two strategies are selected as coordination instruments not only because they act on two key decision dimensions—tenant usage behavior and lessor maintenance effort—but also because they are highly compatible with learning effects and spare parts mechanisms, thereby effectively mitigating incentive distortions under decentralized decision-making and improving overall system performance.

Future research may further extend this framework in several directions. First, the game structure between the two parties can be extended to a multi-agent supply chain to investigate coordination mechanisms and profit allocation with multiple lessors. Second, risk and uncertainty factors such as demand fluctuations, failure rate variability, and market or policy changes can be incorporated into the model. In the context of sustainability and digitalization, the introduction of green maintenance indicators and data-driven predictive maintenance approaches may further enhance supply chain efficiency and coordination performance. Finally, differences in equipment life cycles can be considered to explore maintenance strategies and coordination mechanisms under scenarios such as second-hand equipment and multi-period leasing. These extensions would provide new insights for both theoretical development and practical applications of maintenance management in equipment leasing supply chains.

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Author contributions

C. Z. and L. L. contributed to the study design, model development, model solution, and analysis. J. C. contributed to the writing, review, and revision of the manuscript.

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Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Appendix

Proof of Lemma 1

To solve the decentralized decision-making model of the lessor, for any given N , the Lagrangian function of the lessor's decision model is given by:

$$L(T_{ci}, \gamma) = U \sum_{i=1}^N (T_i - M(T_i)T_{mr}) - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} \sum_{i=1}^N M(T_i) + P - p_{mr}K(T_i)V - p_fK(T_i) \\ - C_uK(T_i) + P_k + \gamma \left(\sum_{i=1}^N T_{pm}i^{-\theta} + \sum_{i=1}^N T_{ci} - H \right)$$

Taking the first-order and second-order partial derivatives of $L(T_{ci}, \gamma)$ with respect to T_i , respectively, we obtain:

$$\frac{\partial L}{\partial T_{ci}} = U - U\lambda(T_i)T_{mr} - C_{mr}\lambda(T_i) - p_{mr}\lambda(T_i)V - p_f\lambda(T_i) - C_u\lambda(T_i) + \gamma$$

$$\frac{\partial^2 L}{\partial T_{ci}^2} = -(UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u)\lambda'(T_i) < 0$$

Therefore, $L(T_{ci}, \gamma)$ is a concave function with respect to T_i . When $\lambda(T_i) = \frac{U + \gamma}{UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u}$, the first-order derivative is given by $\frac{\partial L}{\partial T_{ci}} = 0$. Since $\lambda(T_i)$ increases with T_i , It follows that:

$$T_{c_1}^* = T_{c_2}^* = \dots = T_{c_N}^* = \frac{H}{N_c^*} - \frac{N_c^{*-\theta}}{1-\theta} T_{pm}$$

Substituting this expression into Equation (7), we obtain:

$$\Pi_c = U \left[H - \frac{N^{1-\theta}}{1-\theta} T_{pm} - T_{mr} NM \left(\frac{H}{N} - \frac{N^{-\theta}}{1-\theta} T_{pm} \right) \right] - C_{pm} \frac{N^{1-\theta}}{1-\theta} - C_{mr} NM \left(\frac{H}{N} - \frac{N^{-\theta}}{1-\theta} T_{pm} \right) \\ + P - p_{mr} V NM \left(\frac{H}{N} - \frac{N^{-\theta}}{1-\theta} T_{pm} \right) - p_f NM \left(\frac{H}{N} - \frac{N^{-\theta}}{1-\theta} T_{pm} \right) - C_u NM \left(\frac{H}{N} - \frac{N^{-\theta}}{1-\theta} T_{pm} \right)$$

Next, taking the first-order and second-order derivatives of the $L(T_{ci}, \gamma)$ with respect to N , we obtain:

$$\frac{\partial L}{\partial N} = -(UT_{pm} + C_{pm}) + (UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u)f(N)$$

$$\frac{\partial^2 L}{\partial N^2} = (UT_{mr} + C_{mr} + p_{mr}V + p_f + C_u)f'(N) < 0$$

Therefore, $L(T_{ci}, \gamma)$ is a concave function with respect to N . Here:

$$f(N) = \left(HN^{\theta-1} - \frac{\theta T_{pm}}{1-\theta} \right) \lambda \left(\frac{H}{N} - \frac{N^{-\theta} T_{pm}}{1-\theta} \right) - N^\theta M \left(\frac{H}{N} - \frac{N^{-\theta} T_{pm}}{1-\theta} \right)$$

Setting the first-order derivative $\frac{\partial L}{\partial N} = 0$ equal to zero yields the optimal solution

$$N_c^*.$$

Proof of Lemma 2: Similar to the proof of Lemma 1.

Proof of Lemma 3: Similar to the proof of Lemma 1.

Proof of Proposition 2

Based on

$$T_c^* = \frac{H}{N_c^*} - \frac{N_c^{*-\theta}}{1-\theta} T_{pm}$$

the first-order derivative of T_c^* with respect to θ is given by

$$\frac{\partial T_c^*}{\partial \theta} = \frac{N_c^*[1-(1-\theta)\ln N_c^*]}{(1-\theta)^2}$$

It follows that

$$0 \leq \theta \leq 1 - \frac{1}{\ln N_c^*}, \frac{\partial T_c^*}{\partial \theta} > 0$$

And

$$1 - \frac{1}{\ln N_c^*} \leq \theta \leq 1, \frac{\partial T_c^*}{\partial \theta} < 0$$

Proof of Lemma 4

Under the UTB Strategy, the lessor chose T^* or T_c^* . Let:

$$A(T) = C_{pm} \frac{N^{1-\theta}}{1-\theta} + C_{mr} \sum_{i=1}^N M(T_i) + p_{mr}VK(T_i) + p_fK(T_i) + C_uK(T_i)$$

$$N(T) = xC_eT_{mr}K(T_i), \quad D(T) = (p_{mr}V + p_f)K(T_i),$$

$$F(T) = C_uK(T_i), \quad E(T) = \sum_{i=1}^N (T_i - M(T_i)T_{mr})$$

If the lessor chose T^* under the UTB Strategy, its profit should be no less than that obtained under the decentralized decision-making scenario with T_c^* , then

$$\Pi_c^U(T^*) \geq \Pi_c(T_c^*).$$

If the bonus scheme takes effect, then $E(T^*) > \tau_U$, which yields

$$B[E(T^*) - \tau_U] \geq UE(T_c^*) - UE(T^*) + A(T^*) - A(T_c^*)$$

Accordingly, an upper bound for τ_U can be obtained as

$$\tau_U \leq E(T^*) + \frac{UE(T^*) - UE(T_c^*) + A(T_c^*) - A(T^*)}{B}$$

Let π denote the reservation profit of the lessor. Then, the participation constraint requires $\Pi_c^U(T^*) \geq \pi$, Substituting the profit function into the above inequality gives

$$UE(T^*) - A(T^*) + P + P_k + B[E(T^*) - \tau_U] \geq \pi$$

from which the lower bound of P can be derived as

$$P \geq \pi - UE(T^*) + A(T^*) - P_k - B[E(T^*) - \tau_U]$$

Since P must be non-negative, the lower bound of τ_U is obtained as

$$\tau_U \geq E(T^*) + \frac{UE(T^*) - A(T^*) + P_k - \pi}{B}$$

Moreover, the lessee's expected profit under the running-time incentive strategy must be no less than that under the conventional strategy, then $\Pi_m^U(T^*) \geq \Pi_o^U - \pi$, Substituting the corresponding profit expressions yields

$$(R - U)E(T^*) + D(T^*) + N(T^*) - P_k - P - B[E(T^*) - \tau_U] \geq RE(T_c^*) - A(T_c^*) + D(T_c^*) + N(T_c^*) - \pi$$

Therefore, the upper bound of P can be obtained as

$$P \leq \pi + (R - U)E(T^*) - RE(T_c^*) + D(T^*) + N(T^*) - P_k + A(T_c^*) - N(T_c^*) - D(T_c^*) - B[E(T^*) - \tau_U]$$

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