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Mathematical Optimisation Techniques for Enterprise Finance: Maximising Returns and Minimising Risk

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Abstract

Enterprises must balance the pursuit of higher investment returns with the need to control financial risk under uncertain market conditions. Mathematical optimisation provides a systematic framework for identifying efficient portfolio allocations based on expected return, volatility, correlation, and covariance. This study applied portfolio optimisation techniques to daily stock-price data from five companies listed on the National Stock Exchange of India between 2015 and 2025. Daily and monthly returns, standard deviations, correlation coefficients, covariance values, annualised portfolio return, volatility, and the Sharpe ratio were calculated. Three strategies were evaluated: an equally weighted portfolio, a minimum-risk portfolio, and a maximum-return portfolio. The equally weighted portfolio generated an annualised return of 19.87% with volatility of 20.80% and a Sharpe ratio of 0.8595. The minimum-risk portfolio achieved a return of 16.91% with lower volatility of 18.55%, allocating the largest weight to Infosys. The maximum-return portfolio produced a return of 32.89% but volatility of 48.72% because the model allocated all capital to Tata Steel. The findings demonstrate that unconstrained return maximisation may create excessive concentration, whereas diversified allocation improves portfolio stability. Mathematical optimisation can therefore support transparent enterprise investment decisions when combined with diversification requirements, exposure limits, and periodic portfolio review.

Keywords: Mathematical optimisation, Enterprise finance, Portfolio optimisation, Risk-return analysis, Mean-variance optimisation, Efficient frontier.

1. Introduction

Enterprise finance is the process that deals with the planning, allocation and control of financial resources to meet the aims of the organisation within a risk level acceptable to the organisation. Financial managers must decide how to invest in capital assets, how much of liquid assets to hold, how to finance these assets, and how these assets should be allocated across different investments in an uncertain world. Such decisions are becoming more complicated as enterprises need to strike a balance between attaining higher returns and capital protection against market fluctuations. Hence, the construction of systematical methods which can help in detecting efficient financial allocation has become a major topic of financial research and managerial practice. A key factor in enterprise financial decision making is the risk–return relationship. The greater the risk, the higher the uncertainty and the higher the potential loss of an investment, and the higher the expected return. On the other hand, conservative investments can minimize financial risks, and may restrict the enterprise in achieving competitive returns.

Modern portfolio theory dictates the guidelines for handling this compromise that involves the mixture of assets that don't necessarily move in the exact same direction. Markowitz (1999) showed that it is necessary to know the variance and covariance in addition to the expected performance of individual assets to make the choice of a portfolio. Tobin (1958) went one step further in expanding the portfolio theory and described how investors might mix risky and riskless assets as per their risk preference. Mathematical optimisation provides a quantifiable objective approach to capturing financial objectives in measurable models of decisions. Optimisation techniques can be used to find the best possible combination of funds available between alternative investments with the help of an objective function and a set of constraints. The goal in portfolio management can be to maximise the expected return on the portfolio or to minimise the variance of the portfolio, or to maximise the risk-adjusted return on the portfolio, or to provide a target return at the lowest possible risk.

Quadratic programming has special relevance as variance of a portfolio is quadratic in the weights of the assets, and investment constraints are often linear. The Sharpe ratio is a useful measure of the performance of a portfolio as well, which compares the excess return to the total risk (Sharpe, 1966). In practice, portfolio optimisation has a number of constraints. Optimised weights are very sensitive to small changes in expected returns and the estimates of the covariances, which can lead to unstable and/or concentrated portfolios. This has been termed the Markowitz optimisation enigma by Michaud (1989), where mathematically optimal portfolios might not necessarily be effective in real investment situations. In a similar vein,

DeMiguel et al. (2009) discovered that in some circumstances, equally weighted portfolios could compete favourably with more complex optimisation procedures, due to the negative impact of estimation errors on their performance. These concerns highlight the need to compare optimised portfolios and a basic equally allocated portfolio.

The practical applicability of optimisation models can be enhanced by financial aspects. Constraints like non-negative investment weights, 100% capital allocation and exposure limits prevent any unrealistic solution and hence help to minimize the concentration. Jagannathan and Ma (2003) demonstrated that under portfolio constraints, the performance could improve because of a reduced impact of covariance-estimation errors. In addition, covariance matrix estimation is basic since the risk of a portfolio, based on these estimates, can be seriously misleading if the estimates are not accurate. When financial data has a large number of assets that are correlated, or few observations, Ledoit and Wolf (2003) suggested using shrinkage-based covariance estimation as a way of stabilizing the estimates. The classical mean variance optimisation has been extended by a number of recent studies using robust, probabilistic, behavioural and multi-objective optimisation. Shadabfar and Cheng (2020) have used Monte Carlo simulation along with the Markowitz model to consider the uncertainty in the portfolio selection process.

Min et al. (2021) proposed a robust mean-risk optimisation approach with machine-learning techniques to enhance the choice of risk-return trade-off parameters. In addition, some of the financial criteria have been combined with others in multi-attribute models to make decisions in portfolio rather than only the expected return and variance (Mendonça et al., 2020). Likewise, optimisation based on prospect theory acknowledges that the financial decision-makers might see gains and losses differently in uncertainty conditions (Li et al., 2022). But few empirical studies use simple to understand mathematical optimisation models with a targeted subset of large Indian companies and explicitly contrast equal-weighted, minimum-risk and maximum-return allocations.

The current study aims to fill this gap by analysing the daily stock-price of the five companies listed in the National Stock Exchange of India from 2015 to 2025. The data set can be used to calculate daily and monthly returns, volatility, correlations, and covariance, portfolio weights, and efficient frontiers. The analysis report consists of equal-weighted portfolio performance, minimum risk allocation, maximum return allocation and annualized risk-return measures. Accordingly, the study aims to apply mathematical optimisation techniques to maximise portfolio returns while minimising financial risk. Specifically, the study seeks to:

1. Evaluate the return and volatility characteristics of the five selected NSE-listed stocks using daily and monthly price data.
2. Examine the diversification potential of the selected stocks through correlation and covariance analysis.
3. Construct and compare equally weighted, minimum-risk, and maximum-return portfolios to identify efficient enterprise investment allocations.

The study can be utilized in enterprise finance by showing that mathematical optimisation models can be used to aid transparent and evidence-based investment decisions. The optimised portfolios should produce different risk-return combinations and diversified allocation will reduce portfolio volatility as compared to a concentrated allocation strategy.

2. Literature Review and Theoretical Framework

2.1 Enterprise Finance and Strategic Financial Planning

Enterprise finance is about acquiring, allocating, using and controlling financial resources to sustain and assist growth and stability of an enterprise. In strategic financial planning, managers have to choose investments which can offer acceptable returns to the enterprise without introducing an excessive amount of risk into the market. The factors of expected profitability, volatility, liquidity, diversification and capital constraints should therefore be taken into account at the same time when making financial decisions. Mathematical models offer a framework for evaluating and comparing investment options and translating strategic financial goals into actual investment decisions. Today, financial planning is more and more often mixed with using traditional statistical methods and computational and machine-learning techniques, but the validity of these methods is dependent on data quality, the assumptions of the models, and how results are interpreted (Wasserbacher and Spindler, 2022).

2.2 Risk–Return Trade-Off in Financial Decisions

The risk–return trade-off is the notion that potentially higher returns are associated with higher risks. Firms must decide if the extra return they are able to get on a risky investment is worth the risk of it. Risk may be expressed in the form of variance or standard deviation, downside risk or conditional loss measures. Reward–risk optimisation goes beyond the basic return maximisation and looks out at the risk exposure by considering the return that's expected from an investment as a result, enterprises do not make investments based on high historical

returns. Reward-risk models could be especially helpful in situations where the returns are either not well-quantifiable or vary with the market conditions (Sehgal and Mehra, 2021).

2.3 Portfolio Diversification and Strategic Allocation

By spreading money around among investments that have different patterns of returns, diversification lowers the risk that any one investment will do poorly, and as a result, decrease reliance on that investment's performance. Its effectiveness is not only depending on the number of securities included, but also on the correlations between the securities. Losses to a security may be partially offset by stable or positive performance of another if the returns of the assets are imperfectly correlated. There is some indication that by picking stocks in one sector, as opposed to the rest of the market, portfolio optimisation can yield different results. While broad-market portfolios offer more diversification, sector-focused portfolios could offer specialised return opportunities with an increased concentration risk (Senthilkumar et al., 2022).

2.4 Capital Asset Pricing and Risk Measurement

Measures must be taken to distinguish potential returns from exposure to market uncertainty for financial risk assessment. The standard deviation indicates the overall return variability of a security and correlation and covariance describe the relationship between securities' returns. These measures help in the estimation of portfolio-risk and developing an efficient investment combination. More sophisticated methods are based on conditional value-at-risk (CVaR), which attempts to measure losses outside a given confidence level. However conditional value-at-risk (CVaR) is different from variance, in that it considers only the most extreme losses and can be written as an optimisation problem appropriate for financial risk management (Rockafellar and Uryasev, 2000).

2.5 Mathematical Optimisation Techniques in Finance

Mathematical optimisation can be used to determine the optimal mix of asset weights to meet a specified financial goal. Linear programming is suitable when the objective and the constraints are linear and quadratic programming is usually used when the variation in a portfolio is taken into account. If the performance ratios or complex risk measures are optimised, it may be necessary to use nonlinear programming. The Black-Litterman model is an important extension of the traditional optimisation as it considers investor expectations instead of market-equilibrium returns, thus delivering less volatile and more diversified portfolio weights than those obtained from a historical-average-only model (Black and

Litterman, 1992). Using simulation techniques, researchers can create many different combinations of a portfolio and compare the returns and risk of each combination. Monte Carlo simulation is especially helpful in visualizing the investment set of feasible investments, and portfolios that are close to the efficient frontier. It also offers an easy way to see how changing the proportions of assets in the portfolio will impact the outcome of the portfolio.

The examination of the stability of the optimal allocation under different return, volatility and correlation assumptions can be done through simulation-based portfolio optimisation and can help to perform sensitivity analysis (Pedersen, 2014). Recently, empirical research has combined technical analysis, clustering and mean-variance optimisation for the enhancement of security selection as well as portfolio allocation. By using the front end of the asset-screening process in conjunction with portfolio models, the number of securities that fall short of the desired performance can be minimized and the meaning of efficient allocations can be better understood. In times of significant market volatility, integrated optimisation methods could be a useful strategy to distinguish between short-term fluctuations in prices and long-term risk-return patterns (Navarro et al., 2023).

2.6 Conceptual Framework of the Study

The proposed conceptual framework assumes the estimated returns, volatility, correlation and covariance are based on historical information of stock prices. The measures are inputs to mathematical optimisation models. The models then construct other portfolios such as equally weighted, minimum risk and maximum return portfolios. They are judged on the basis of return, standard deviation, Sharpe ratio and position on the efficient frontier on an annual basis.

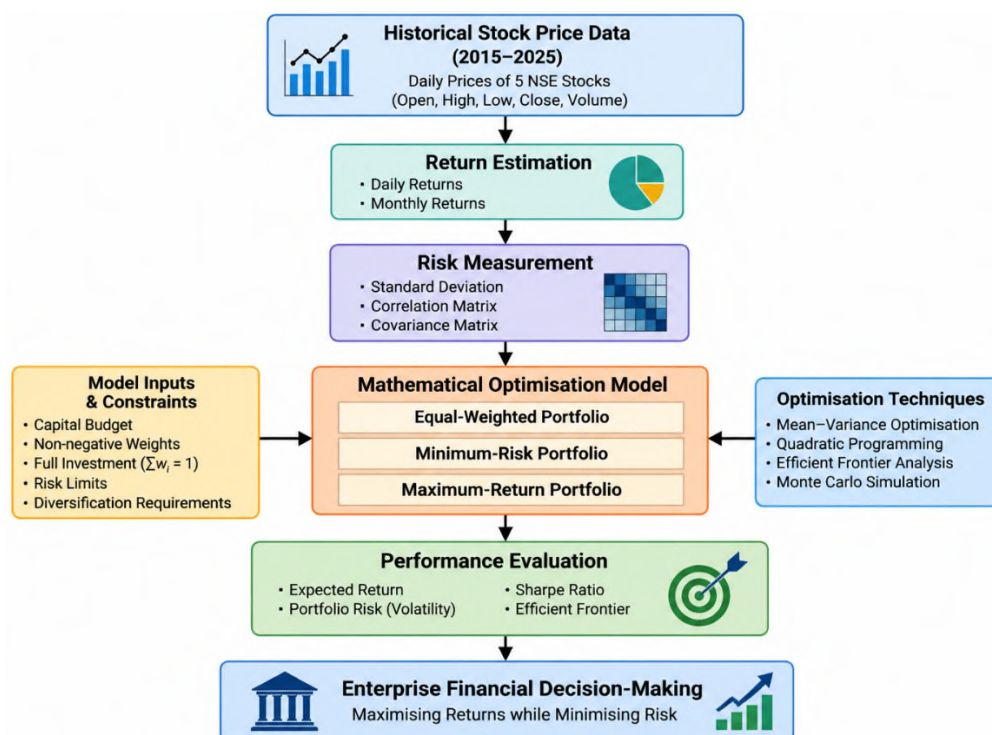


Figure 1. Conceptual framework for mathematical optimisation of enterprise investment portfolios

Source: Authors construct.

3. Research Methodology

3.1 Research Design and Analytical Approach

In this study the researcher decided to use quantitative research design type, secondary data research type to research the application of mathematical optimisation in supporting enterprise investment decisions. The analysis was done using descriptive financial analysis, return and risk estimation, correlation and covariance assessment and portfolio optimisation. Three different portfolio strategies were considered: equally weighted portfolio, a minimum risk portfolio, and a maximum return portfolio. The efficient frontier (EF) was also plotted to show the set of all feasible portfolios that can be constructed with the highest possible level of expected return for a certain level of risk.

3.2 Data Sources and Sample Selection

The study employed data on stock prices of the companies on a daily basis from January 2015 to January 2025. The five companies that were chosen are Maruti Suzuki, ICICI Bank, Infosys, Sun Pharmaceutical Industries, and Tata Steel and are all listed on the National Stock Exchange of India. These companies belonged to different industries giving the opportunity of

diversification. The data was daily data of Opening price, High price, Low price, Close price, and Trading volume. Closing values were chosen as the main basis of calculation of returns as they represent the final market value of the stocks for a trading day. The variables used in the analysis are summarised in Table 1.

Table 1. Definition and measurement of study variables

Variable	Measurement	Analytical purpose
Closing price	Daily market closing value	Return calculation
Daily return	Percentage change in daily closing price	Short-term performance
Monthly return	Percentage change in month-end closing price	Medium-term performance
Mean return	Average periodic return	Expected return estimation
Standard deviation	Dispersion of returns	Individual asset risk
Correlation	Pairwise return association	Diversification analysis
Covariance	Joint movement of returns	Portfolio risk estimation
Portfolio weight	Proportion invested in each stock	Optimisation decision variable
Sharpe ratio	Excess return relative to volatility	Risk-adjusted performance

Source: Authors' estimation

3.3 Definition and Measurement of Financial Variables

The daily returns have been worked out from the successive closing price. The monthly returns were obtained from the month end closing values. The arithmetic mean of periodic returns was used as expected return and standard deviation was used as a measure of risk. The strength and direction of co-movement between the five stocks were determined using correlation coefficient while the covariance matrix was directly applied in the estimation of portfolio variance.

3.4 Expected Return and Risk Estimation

The return of stock (i) at time (t) was calculated as:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

where ($P_{i,t}$) is the closing price at time (t), and ($P_{i,t-1}$) is the previous closing price. The expected portfolio return was estimated as:

$$E(R_p) = \sum_{i=1}^n w_i E(R_i)$$

Portfolio variance was calculated as:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

where (w_i) and (w_j) are portfolio weights and (σ_{ij}) is the covariance between asset returns.

3.5 Development of the Optimisation Model

In the optimisation model, the decision variables were the weights of different assets. An equally weighted portfolio allocated 20% to each of the stocks. The minimum-risk portfolio had the weight combination that resulted in the minimum portfolio variance and the maximum-return portfolio allocated capital to the weight combination that resulted in the maximum expected return.

3.6 Objective Function and Financial Constraints

The minimum-risk objective was expressed as:

$$\min \sigma_p^2$$

The maximum-return objective was:

$$\max E(R_p)$$

Both models were subject to:

$$\begin{aligned} \sum_{i=1}^n w_i &= 1 \\ w_i &\geq 0 \end{aligned}$$

These constraints required full capital allocation and prohibited short selling.

3.7 Optimisation Techniques and Solution Procedure

The estimated mean-return vector and the covariance matrix were used to perform mean–variance optimisation and quadratic programming. Several different, viable, weight combinations were tested to develop the efficient frontier. The optimisation process produced the minimum-risk and maximum-return portfolios, and made comparisons with the equal-weight benchmark. The statistical analysis and portfolio optimisation were implemented using Python, employing libraries such as Pandas for data processing, NumPy for numerical computation, SciPy for optimisation, and Matplotlib for visualisation of the efficient frontier.

3.8 Model Validation, Sensitivity Analysis, and Performance Evaluation

The performance of the models was evaluated using annualised return, annualised volatility, portfolio variance, Sharpe ratio and positioning on the efficient frontier. Sensitivity was explored by considering the different results for the portfolio with different asset-weight structure. Reliability of the results was assessed by whether the optimised portfolios were significantly better than the equally weighted portfolio.

4. Results and Analysis

4.1 Descriptive Analysis of Financial Data

The research was done on the daily stock-price data of five companies from 2015 to 2025, which are Maruti Suzuki, ICICI Bank, Infosys, Sun Pharmaceutical Industries and Tata Steel. The daily closing prices were converted to daily percentage returns and prices at the end of the month were used to calculate monthly percentage returns. The return series exhibited positive and negative fluctuations during the time of study, indicating that all five investment alternatives were subject to market fluctuations. But the size of the changes varied greatly between the stocks. This variation was used to measure diversification and portfolio risk, and a mathematically optimal capital allocation.

4.2 Return and Risk Characteristics of Investment Alternatives

The results of return and volatility showed that there were different risk – return characteristics of the stocks selected. The highest average daily return of 0.001305 and highest average monthly return of 0.026472 was shown by Tata Steel. It also had the highest day-to-day and month-to-month variance, which meant that its return benefit came with significantly higher risk. On the other hand, Infosys had an average monthly return of 0.015038 and the lowest monthly standard deviation of 0.070480. Chevrolet's Sun Pharmaceutical Industries had the weakest monthly average returns of 0.009661, and Maruti Suzuki and ICICI Bank had relatively low return and risk. The principal return and risk measures are summarised below.

Table 2. Return, volatility, and optimal portfolio allocation of the selected stocks

Stock	Average daily return	Average monthly return	Daily standard deviation	Monthly standard deviation	Minimum-risk weight	Maximum-return weight
Maruti Suzuki	0.000670	0.014123	0.017540	0.080917	0.2492	0.0000
ICICI Bank	0.000738	0.015038	0.019802	0.080600	0.1189	0.0000
Infosys	0.000751	0.015038	0.016673	0.070480	0.3473	0.0000
Sun Pharmaceutical	0.000479	0.009661	0.018401	0.082503	0.2616	0.0000
Tata Steel	0.001305	0.026472	0.030691	0.130514	0.0230	1.0000

Source: Authors' estimation

These values show that the asset which made the best return was not the least risky option. Tata Steel provided the best return prospects but also had the most volatile return pattern while Infosys had more stable return pattern.

4.3 Correlation and Covariance Analysis

All the daily-return correlations were positive, but less than 0.50. The highest correlation was obtained between Maruti Suzuki and ICICI Bank which is 0.4188. The least correlation was observed in the case of Infosys and Tata Steel with a coefficient of correlation of 0.1470. These moderate and low correlation suggested that the selected stocks didn't move in the same direction and thus provided good diversification opportunities. Table 3 presents the pairwise correlation coefficients among the daily returns of the five selected NSE-listed companies. The matrix is used to assess the degree of co-movement among the stocks and evaluate their potential contribution to portfolio diversification.

Table 3. Correlation Matrix of Daily Stock Returns

Stock	Maruti Suzuki	ICICI Bank	Infosys	Sun Pharmaceutical	Tata Steel
Maruti Suzuki	1.0000	0.4188	0.2073	0.2141	0.2711
ICICI Bank	0.4188	1.0000	0.2411	0.2268	0.3155
Infosys	0.2073	0.2411	1.0000	0.1708	0.1470
Sun Pharmaceutical	0.2141	0.2268	0.1708	1.0000	0.2003
Tata Steel	0.2711	0.3155	0.1470	0.2003	1.0000

Source: Authors' estimation

The correlations are positive but remain below 0.50, indicating that the selected stocks do not move perfectly together. The lowest correlation is between Infosys and Tata Steel, while the

highest is between Maruti Suzuki and ICICI Bank. This supports the diversification potential of the selected portfolio.

4.4 Results of the Return-Maximisation Model

The return-maximisation model selected Tata Steel as the sole investment. Its estimated annualised return was calculated as:

$$E(R_p) = 1.0000(0.3289) + 0(0.1691) = 0.3289$$

So, the model had a return of 32.89% annually. The annualised volatility, however, for the corresponding period was 48.72% indicating that the maximum-return solution involved significant financial exposure. The result was very aggressive since the model did not have an upper limit on the amount of weight to be put on each asset.

4.5 Results of the Risk-Minimisation Model

The minimum-risk model resulted in a diversified portfolio that consisted of 24.92% Maruti Suzuki, 11.89% ICICI Bank, 34.73% Infosys, 26.16% Sun Pharmaceutical and 2.30% Tata Steel. The weights were the following and they had been consistent with the full-investment constraint:

$$0.2492 + 0.1189 + 0.3473 + 0.2616 + 0.0230 = 1.0000$$

The annualised return came to 16.91% with an annualised volatility of 18.55%. This allocation, given the weightage, was the lowest for Tata Steel as it was the most volatile stock in that month while Infosys was allocated the maximum weightage as it was the least volatile stock.

4.6 Optimal Allocation of Enterprise Financial Resources

The equally weighted portfolio assigned 20% to each stock and generated an annualised return of 19.87%, annualised variance of 0.0432, annualised volatility of 20.80%, and a Sharpe ratio of 0.8595. Table 4 compares the annualised return, volatility, and principal allocation characteristics of the equally weighted, minimum-risk, and maximum-return portfolios. This comparison highlights the trade-off between return maximisation, risk reduction, and portfolio concentration.

Table 4. Comparative Performance of Portfolio Strategies

Portfolio strategy	Annualised return	Annualised volatility	Sharpe ratio	Main allocation feature
Equally weighted portfolio	19.87%	20.80%	0.8595	20% invested in each stock
Minimum-risk portfolio	16.91%	18.55%	Not reported	Highest allocation to Infosys at 34.73%
Maximum-return portfolio	32.89%	48.72%	Not reported	100% allocation to Tata Steel

Source: Authors' estimation

The minimum-risk portfolio provides the lowest volatility, while the maximum-return portfolio generates the highest return but creates extreme concentration and substantially greater risk. The equally weighted portfolio offers a comparatively balanced risk–return position.

Its portfolio return can be expressed as:

$$E(R_p) = \sum_{i=1}^5 0.20E(R_i) = 0.1987$$

The minimum-risk portfolio had an annualised volatility that was 2.25% lower than the equal-weight portfolio, but an annualised return that was 2.96% less. The maximum-return portfolio saw an 13.02 percentage point gain in annualised return while the volatility rose by 27.92 percentage points.

4.7 Efficient Frontier and Risk–Return Scenarios

The efficient frontier demonstrated the range of levels of possible annualised returns and volatility. At the lower left corner was located the minimum risk portfolio and at the upper right corner was located the maximum return portfolio. Portfolios on the frontier were efficient as there was no way to gain a higher return without taking on more risk.

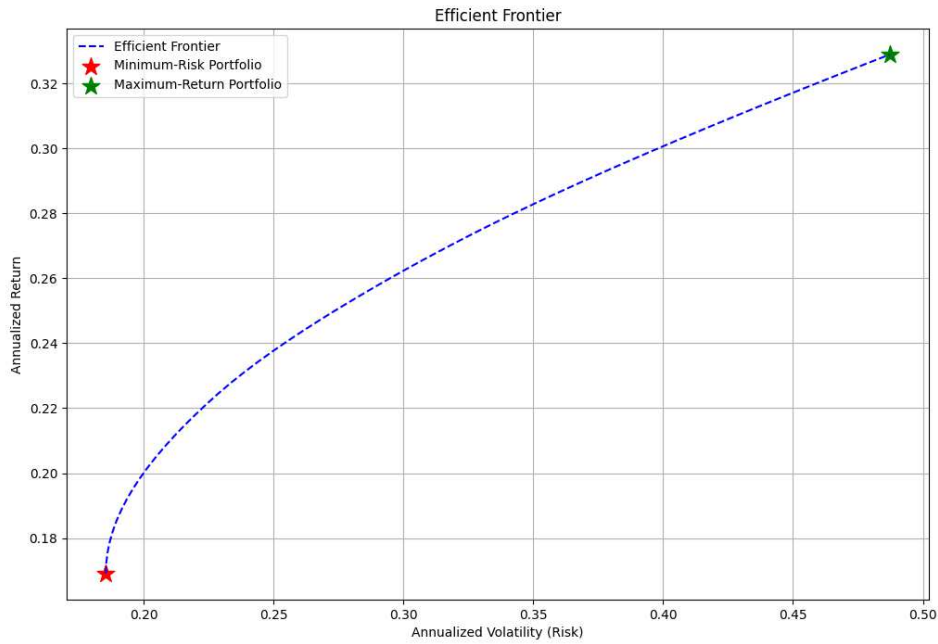


Figure 2. Efficient frontier showing the minimum-risk and maximum-return portfolios

Source: Authors' estimation

4.8 Sensitivity and Robustness Analysis

The reverse results of the three portfolio strategies were used to determine sensitivity to the objective function chosen. The winner of minimising the risk, and therefore diversification, was to be completely concentrated in Tata Steel, for the sake of the maximum return. The equal weight portfolio had an intermediate risk/return ratio, and a reported Sharpe ratio of 0.8595. Overall, the diversification enhanced the stability of the portfolio and the unconstrained return maximisation enhanced the expected return and the volatility exposure of the portfolio.

5. Discussion and Managerial Implications

5.1 Interpretation of the Optimisation Results

The results show that there were significant changes in the outcomes of the portfolios based on the optimisation objective. The equally weighted portfolio had an annualised return of 19.87%, and an annualised volatility of 20.80%, which is a moderate level of return and risk. The minimum-risk portfolio, on the other hand, found a way to decrease the volatility to 18.55% while the annualised return was 16.91%. This finding reinforces the idea that risk reduction is the general rule of having lower expected return. The entire investment was made in Tata Steel and the maximum-return portfolio had an annualised return of 32.89%. The volatility of its allocation was 48.72% over the past year, reflecting the return maximisation

goal which resulted in a concentrated and risky allocation. The outcome implies that, an enterprise without concentration restrictions that maximises historical return may not result in a practically acceptable enterprise portfolio. The minimum-risk one was more diversified with major allocation to Infosys, Sun Pharmaceutical Industries and Maruti Suzuki and only 2.30% of the allocation was made to Tata Steel due to its comparatively high volatility.

5.2 Comparison with Previous Financial Optimisation Studies

The minimum-risk results confirm the fundamental idea of mean–variance portfolio selection that the weights of investments are based on the expected returns, individual risks, and return interdependencies. The current findings suggest that in a portfolio of imperfectly correlated assets it is possible to get a lower overall portfolio volatility even though each asset is individually subject to market risk (Markowitz, 1999). The focused maximum return is also a sign of the optimisation instability discovered in previous financial studies. With the absence of maximum weights on the assets, small differences in the means of expected returns can result in extreme weights on the assets. This vindicates the notion that mathematically optimal portfolios may be hard to implement when making financial decisions on a practical basis (Michaud, 1989). The results of the equal-weight portfolio showed a balance between return and volatility and indicated that simple diversification strategies can still work well even if there is uncertainty in the inputs to the optimisation. So, an equal allocation could be used as a practical benchmark to measure the results of complex optimisation (DeMiguel et al., 2009).

5.3 Effectiveness of the Selected Optimisation Techniques

The optimisation techniques used were effective to create the clearly differentiated portfolio strategies. The quadratic mean-variance optimization was able to find the combination of weights that result in the least variance of the portfolio, while the linear return maximisation model identified the asset with the highest estimated annualized return. The efficient frontier was also able to show how the return was higher when more volatility was taken on. But mathematical optimality is not necessarily a criterion for effectiveness. The maximum return model was not only managerially weak but also technically weak as it invested everything in one stock. Constraints need therefore to mirror risk tolerance, diversification policy, liquidity requirements, and exposure limits; and thus are of a practical nature.

Portfolio restrictions can mitigate the effects of estimation errors, and generate allocations that are more stable and realistic to decision makers (Jagannathan and Ma, 2003).

The covariance matrix was a key component of the risk-minimisation. The variance of the portfolio depends on the risk of each asset as well as the covariance between the assets, so cross asset covariance estimates can significantly impact the optimal asset mix. A more stable method of estimating the covariance of the assets could make future portfolios more reliable, especially if there are a large number of assets to be included in the portfolio and fewer observations available (Ledoit and Wolf, 2003).

5.4 Implications for Investment and Capital Allocation

The find out has direct consequences for enterprise capital allocation. A firm that wishes to preserve its capital might choose to go with the minimum-risk portfolio since it had the lowest annualized volatility and spread their risk among the five stocks. The enterprise that is willing to accept moderate risk and wants to have a moderate balance between growth and stability can look into the equally weighted portfolio which gave a higher return than the minimum-risk strategy and considerably less volatility than the maximum-return strategy. Only very high-risk companies with large risk capacity and the ability to absorb large fluctuations would be able to use the maximum return allocation. If the asset allocation is 100% in Tata Steel then it would be subjected to company risk and sector risk. Hence, the maximum return outcome should be regarded as a maximum return line and not as an automatic recommendation of an investment strategy.

5.5 Implications for Enterprise Risk Management

Diversification is an enterprise risk-management mechanism especially called to attention in the analysis. The correlation of the stocks selected is positive, but with moderate level which means that the portfolio is diversified, but not to the maximum. Enterprises should pay attention to the change in correlation as assets that may be weakly correlated in normal market situations could turn into strongly correlated in financial crises. Risk management should also be based on downside measures and not just variance and standard deviation. The CVaR can help to estimate the size of the losses in case of bad market conditions and can thus be used in conjunction with the traditional mean–variance optimisation (Rockafellar and Uryasev, 2000).

5.6 Strategic and Managerial Recommendations

The choice of an optimisation model by the financial manager should follow the objective of the portfolio. The three different managerial priorities are return maximisation,

volatility minimisation and risk adjusted performance, which result in a different asset allocations. Enterprises should therefore set acceptable levels of risk, the minimum acceptable return and the maximum weight of assets. One possible approach would be to put an absolute cap on the amount of each stock that could be allocated, so as not to have too much concentration of a stock. It is also important to periodically rebalance the portfolio since historical returns, volatilities and correlations may change over time. To investigate the sensitivity of portfolio performance to various market conditions, scenario analysis and Monte Carlo simulation can be used in addition to the use of one set of historical estimate (Shadabfar and Cheng, 2020).

5.7 Study Limitations and Future Research Directions

The study was restricted to five NSE listed companies and the historical data of the stock prices were used for the period 2015 – 2025. This may therefore not show the results of all sectors, asset classes or enterprise investment opportunities. Explicit incorporation of transaction costs, taxes, inflation, liquidity constraints, dividends and rebalancing costs was not made. Future research should be extended to a bigger and more sectorally diversified portfolio, introduce maximum weight constraints, and compare the mean-variance optimisation model with models based on conditional value-at-risk, Black-Litterman, robust optimisation and behavioural portfolio. Machine-learning approaches can also be used to enhance the estimation of expected returns, but these models need to be carefully validated to prevent overfitting models and volatile estimates of expected returns (Min et al., 2021; Wasserbacher and Spindler, 2022).

6. Conclusion

In this study, the mathematical optimisation techniques were used in the data of five companies from the NSE for the period 2015-2025. Analysis of the daily and monthly returns, volatility, correlation, covariance and performance of the portfolio was carried out to evaluate the equally-weighted, minimum-risk, and maximum-return investment strategies. Comparing the results, it was seen that there was a distinct difference between the risk–return characteristics of the selected stocks. The Tata Steel was the most volatile and the highest returning stock. Infosys exhibited less risk and thus the highest weight in the minimum-risk portfolio. An annualised return of 19.87% and a volatility of 20.80% were achieved by the equally weighted portfolio. The minimum-risk portfolio lowered the volatility to 18.55% for a 16.91% return and the maximum-return portfolio maximised the return to 32.89% at the cost

of maximum volatility with a complete investment in Tata Steel. The study was able to successfully analyse the stock level return, risk, diversification using correlation and covariance analysis and create three portfolio strategies. It also illustrated the difference in outcomes of the allocation with respect to different optimisation objectives. The study justifies the applicability of the mean variance optimisation in the field of enterprise finance. It demonstrates that mathematical models can help to make investment allocation more transparent: by quantifying the trade-off between expected return and financial risk. The results support this and show that if we do not put appropriate limits on the maximisation of returns, we can end up with too much concentration and instability. For companies looking to sustainable financial performance, a diversified minimum-risk or balanced allocation would be more suitable. Enterprise investment decisions should thus be made using optimisation models, and risk limits, diversification requirements, and periodic review of the portfolio.

7.0 Funding Declaration

This research did not receive any funding.

7.1 Data Availability Statement

The dataset that supports the findings of this study are available from the corresponding author upon reasonable request, subject to ethical and confidentiality considerations.

7.2 Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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