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Computational Dynamics of Endogenous Pension Returns: Market Feedback, Cohort Risk, and Fiscal Backstops

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Abstract

Large funded-pension reforms are usually evaluated under exogenous asset returns. This paper develops a quarterly computational system-dynamics model in which predictable pension contributions and withdrawals alter the market prices used to value pension assets. The model integrates pension accounting, endogenous price formation, price-insensitive ownership, cohort-specific retirement outcomes, and a finite fiscal backstop. It is implemented for 2026–2080 and evaluated through nested model variants, alternative functional forms, common-shock Monte Carlo simulations, Latin-hypercube sensitivity analysis, and regime maps. Under elastic market absorption, the funded expansion raises the 2080 replacement-rate index from about 45% to 58.3%. With the benchmark flow-price feedback, the index reaches 50.2%, and the cohort retiring in 2052 receives roughly 30% less funded income than under elastic absorption. A state backstop reduces the initial drawdown but shifts adjustment into public financing and exit risk; in the benchmark stochastic experiment, all 500 paths reach the financing trigger. Across a broader prespecified parameter sample, the trigger occurs in 57% of configurations, indicating a regime-dependent mechanism rather than a universal prediction. Wider diversification, gradual rollout, flexible payouts, countercyclical buffers, and a larger price-sensitive investor base weaken the feedback. The contribution is a reproducible computational framework for identifying the market-capacity conditions under which pension reform can endogenize its own returns.

JEL classification: C63, D84, G12, G18, G41, H55, J32

Keywords: computational economics; system dynamics; endogenous asset returns; pension funds; Monte Carlo simulation; fiscal backstops

1 Introduction

Funded-pension reforms are usually evaluated under exogenous asset returns. That assumption is convenient for welfare and accounting analysis, but it can fail when policy creates large, persistent, and valuation-insensitive flows relative to the market that receives them. In that case, the reform may affect the prices and returns used to value its own assets. This paper treats that possibility as a computational economic problem: a dynamic system in which policy flows, market structure, retirement payouts, and public intervention interact over time.

Two literatures motivate the model. Pension and demographic models analyze aging, contribution rules, portfolios, and intergenerational welfare, but generally take market returns as given (Abel, 2001; Poterba, 2001, 2004; Börsch-Supan, Ludwig and Winter, 2006). Research on inelastic markets shows that aggregate flows can move prices when limited capital trades against them (Gabaix and Koijen, 2022). The unresolved computational question is how this price channel feeds back into pension

accounting, cohort outcomes, and fiscal support when contribution and payout rules are themselves state dependent.

The paper develops a quarterly nonlinear system-dynamics model for 2026–2080. Contributions, pension assets, payouts, market prices, latent fundamental value, price-insensitive ownership, demographic change, and a finite state backstop are integrated in one stock-flow system. A share of pension flows enters a modeled receiving market and the remainder earns an exogenous world return. Germany supplies the rollout timetable and broad calibration anchors; it does not identify the flow-price parameters or turn the simulations into a national forecast.

The analysis addresses three computational questions:

1. Scale: Under what combinations of contribution size, market concentration, and price-discovery strength do pension flows materially affect the returns earned by the pension system?
2. Transmission: How do accumulation-phase purchases, endogenous ownership, and payout rigidity generate valuation gaps, drawdowns, and retirement-cohort losses?
3. Design: How do buffers, flexible payouts, diversification, and a capacity-constrained public backstop change the timing and distribution of adjustment?

The computational contribution is threefold. First, the model closes the loop from policy-driven cash flow to market price, measured pension wealth, and future cash flow. Second, it links market-wide states to synthetic retirement cohorts, making path-dependent distributional effects observable. Third, it embeds public purchases, holdings, financing, credibility, and exit rules in the same simulation rather than imposing support as an external shock.

The analysis is a conditional mechanism study, not a causal estimate. The benchmark is evaluated against nested models, alternative equations, shared-shock deterministic comparisons, 500-run Monte Carlo experiments, a prespecified Latin-hypercube parameter sample, regime maps, and time-step checks. Accounting identities, limiting cases, and fixed random seeds support reproducibility. These tests separate results generated by the model's feedback architecture from those tied to one threshold or calibration.

The experiments show a sharp market-capacity margin. With elastic absorption, the expanded funded pillar raises the 2080 replacement-rate index from about 45% to 58.3%. Under the benchmark endogenous-return calibration, the index reaches 50.2%; the cohort retiring in 2052 receives about 30% less funded income than under elastic absorption. The loss results from buying during the accumulation-phase overvaluation and selling into a later drawdown, not from equity exposure alone.

A finite backstop changes the timing of losses more than the final exposure. It limits the first drawdown but transfers risk to gross public financing, state holdings, credibility, and exit. All 500 benchmark stochastic paths reach the financing trigger, whereas 57% of the broader 400-configuration parameter sample do so. These are model frequencies rather than real-world probabilities; they locate a nonlinear regime boundary.

Preventive rules perform better in the model than a permanent rescue promise. Gradual rollout, wider diversification, a countercyclical buffer, flexible payouts, and sufficient price-sensitive ownership weaken the adverse feedback. The policy implication is therefore conditional: funded saving is not inherently destabilizing, but its scale, destination, payout rules, and backstop design cannot be assessed independently.

Section 2 defines the institutional motivation and market boundary. Section 3 positions the contribution in the computational and economic literature. Section 4 specifies the model, numerical implementation, experiments, and verification tests. Section 5 reports the computational results. Section 6 interprets the mechanisms and design implications, and Sections 7 and 8 discuss limitations and conclusions.

2 Institutional motivation and market boundary

2.1 Scale and rule-based pension saving

The United States shows the scale and rule-based nature of a mature funded pension system. Retirement assets totaled $47.6 trillion in March 2026, including $13.8 trillion in defined-contribution plans and $18.2 trillion in individual retirement accounts (Investment Company Institute, 2026). Payroll contributions, default options, and target-date funds follow preset rules rather than current valuations. This feature, not the US system as a whole, motivates the model's price-insensitive flow channel.

Two other features matter. Stock ownership is broad but concentrated: 62% of US adults reported owning stock in 2025, while the wealthiest 10% held about 87% of directly reported equities and mutual-fund shares (Gallup, 2025; Board of Governors of the Federal Reserve System, 2026). Automatic contributions and age-based rebalancing can also make pension flows less sensitive to price. The model represents this second feature with the inelastic-market mechanism of Gabaix and Koijen (2022). It does not infer price effects from wealth concentration.

The S&P 500 is concentrated as well: its ten largest positions made up 36.4% of the index on 30 June 2026 (S&P Dow Jones Indices, 2026). Broad index funds reduce company-specific risk, but they still expose savers to a small group of large firms. This supports focusing on the market that receives the pension flows. It is not a separate model input.

2.2 German policy anchors: timing, not identification

Germany adopted or proposed several pension changes in 2025–26. They must be kept separate: one is enacted law, one is a government draft, and one is a commission recommendation.

Rentenpaket 2025. This law has applied since 1 January 2026. It keeps the statutory pension level at no less than 48% through 2031. After 2031, current adjustment factors may lower the level unless new legislation changes the path.

Frühstart-Rente. The July 2026 ministerial draft is not yet law. It proposes a €10 monthly federal contribution for eligible children, introduced one birth cohort at a time. If a family does not open an individual contract, the Bundesbank would invest the money in a globally diversified equity portfolio. The gradual rollout supports the model's staged contribution path. The global mandate supports the diversification test.

Subsidized private retirement provision. The Altersvorsorgereformgesetz applies to new contracts from January 2027. It allows subsidized accounts to hold shares, funds, and ETFs without a mandatory capital guarantee and includes self-employed workers. The reform expands funded saving, but it does not create the large mandatory pillar used in the model's upper scenarios.

Alterssicherungskommission. Its June 2026 recommendations are not law. They include a mandatory funded contribution of 2% of wages, individual accounts with central investment, a post-tax replacement-rate target, a retirement age linked to longevity, and wider statutory coverage. The 2% proposal informs the scale discussion, but the final policy design remains open.

Together, these measures suggest a gradual funded addition to Germany's pay-as-you-go system, not its replacement. They help set a plausible timing and scale for contributions. They do not reveal how strongly flows move prices, which market receives the money, or whether S3 or S4 would exist. Those are model assumptions tested through comparisons and sensitivity analysis.

2.3 Comparative institutions and public support

Sweden and Australia show that funded pensions can be organized in different ways. Sweden uses a mandatory premium contribution and a state default fund. Australia has compulsory superannuation assets of about A$4.3 trillion, roughly 160% of GDP (Swedish Pensions Agency, 2026; APRA, 2025). These cases show that large, automatic systems are feasible. They do not identify the market-feedback values used in this model.

Norway's Government Pension Fund Global offers a different example. Its foreign-investment rule largely prevents state purchases from directly lifting Norwegian share prices. The fund is also financed by oil revenue rather than pension contributions. Norway therefore shows how investing across countries can weaken the feedback studied here.

Japan is more useful for understanding exit. The Bank of Japan built large ETF holdings, stopped new purchases in 2024, and adopted a slow sale plan in 2025. At the announced pace, the sale would take more than a century. The aim is to avoid large price effects and losses (Charoenwong, Morck and Wiwattanakantang, 2021; Bank of Japan, 2025). Japan shows that entering a market can be much faster than leaving it. It does not validate the fiscal limit used in this model.

Section 5.3 tests the same exit problem. Counting sale proceeds can delay the financing trigger, but it increases total trading and does not prevent the trigger in the benchmark runs.

Research on the US 'policy put' motivates the credibility channel. Expected protection can encourage more risk-taking before a crisis (Miller, Weller and Zhang, 2002). Market falls have often been followed by easier US monetary policy, but much of that response reflects a weaker economy rather than direct support for share prices (Cieslak and Vissing-Jorgensen, 2021). The model therefore treats a policy put as an investor expectation, not as a rule followed by central banks.

The Federal Reserve's 2020 corporate-credit facilities are a direct but different example. Publicly backed vehicles bought corporate bonds and bond ETFs. Such action may increase the belief that authorities will intervene in a severe crisis, but the model does not assume that every rescue creates this belief.

3 Related computational and economic literature

Pension models, aging, and asset prices. Earlier research finds that aging alone is unlikely to cause a sudden market collapse. Households adjust gradually, inheritances absorb some sales, and international investment spreads risk (Abel, 2001; Poterba, 2001, 2004; GAO, 2006; Börsch-Supan, Ludwig and Winter, 2006). Standard overlapping-generation and pension-accounting models are strong at studying welfare and long-run change, but usually keep market prices outside the pension model. This paper asks what changes when pension flows affect prices, payouts adjust slowly, and public support responds to losses.

Inelastic markets and rule-based investing. Gabaix and Koijen (2022) estimate that an inflow equal to 1% of market value can raise total market value by about 5%. Fixed allocations and passive funds can make demand less responsive to price (Anadu et al., 2020; Haddad, Huebner and Loualiche, 2025). Passive ownership can also reduce company-specific information in prices (Sammon, 2024). Momentum and panic add buying and selling that depend on recent market conditions (De Long et al., 1990; Greenwood and Shleifer, 2014; Barberis et al., 2015). The model combines these ideas: regular pension purchases can raise prices when too little capital trades against them.

Price information and the real economy. Prices carry information only when investors spend resources to assess value (Grossman and Stiglitz, 1980; Bai, Philippon and Savov, 2016). The model's price-

alignment index has a narrower meaning: it measures how far price is from the model's unobserved fundamental value. It is not an empirical measure of how informative prices are. Prices may also affect investment and financing decisions (Bond, Edmans and Goldstein, 2012). Equation (4) includes sales by existing holders. It does not include new share issuance, dilution, or issuance proceeds. Equation (8) adds a separate assumed channel from distorted prices to weaker fundamentals. Neither channel is an estimated causal effect.

Government support and moral hazard. A policy put is the belief that authorities will limit severe market losses. This belief can encourage risk-taking before a crisis (Miller, Weller and Zhang, 2002). US evidence is consistent with investors changing such beliefs, but not with a fixed central-bank target for share prices (Cieslak and Vissing-Jorgensen, 2021). Bank of Japan ETF purchases are a closer case of direct equity support. They supported prices but had limited effects on real investment (Charoenwong, Morck and Wiwattanakantang, 2021). The model adds two elements: pension dependence creates political pressure to intervene, and limited public capacity creates a stopping point.

Institutional design. Collective buffers and gradual benefit changes can share investment risk across retirement cohorts better than separate individual accounts (Gollier, 2008; Cui, de Jong and Ponds, 2011). But buffers are politically vulnerable: governments may use them for higher benefits, lower contributions, or other spending. Section 6.2 therefore treats governance as part of the economic mechanism, not as a minor implementation issue.

Computational method. System dynamics tracks stocks, flows, delays, and feedback rules (Forrester, 1961; Sterman, 2000). It is appropriate here because pension assets and demography evolve slowly, whereas prices, panic, and intervention can change quickly. The quarterly implementation combines deterministic scenario analysis, Monte Carlo simulation, Latin-hypercube parameter sampling, regime maps, nested model variants, and alternative functional forms. The contribution is not the method alone, but the reproducible integration of pension accounts, endogenous price formation, retirement cohorts, and finite public support in one testable computational framework (Barlas, 1996; Castellacci, 2018).

4 Computational model

4.1 Scope, state variables, and market boundary

The computational model represents one aging economy and one receiving equity market. A share h of funded assets enters that market; the remainder earns an exogenous world return. The label 'receiving market' is used instead of 'domestic' because geography is determined jointly by M_0 and h. The model is implemented in Python and advanced quarterly from 2026 to 2080 with an ordered Euler scheme. Monetary stocks are measured in constant-2026 EUR trillion, annual flows in EUR trillion per year, population in millions, and prices and shares as dimensionless indices. Table 2 lists the notation. German data and policy provide broad anchors only; the simulations compare mechanisms and are not forecasts.

4.2 Model architecture

The model has four parts. First, policy and population determine contributions and payouts. Second, these cash flows and investment returns change pension assets. Third, the share invested in the receiving market combines with shocks, momentum, state trades, and sales by existing holders to move price relative to fundamental value. Fourth, a large pension system and a market fall can activate public support, which changes state financing, holdings, and investor beliefs. The replacement-rate index is a result, not another feedback variable. Figures 1 and 2 show this sequence.

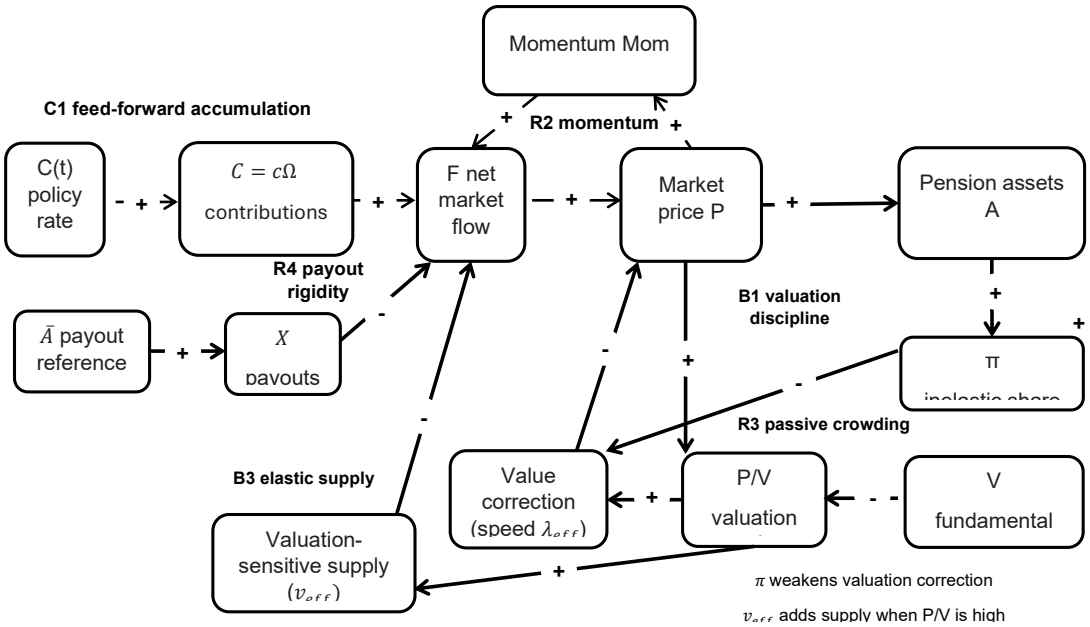
The pension part separates current assets A from the smoothed payout reference $\bar{A}$. Contributions and returns raise A; benefits and panic withdrawals reduce it. Payouts depend on $\bar{A}$, which adjusts more slowly than A. This represents benefit commitments that cannot fall immediately after a market shock. The delay creates the payout-rigidity channel tested later.

Next comes the market block. The home or receiving-market share h maps the pension cash flow C^A - X - qA into the modeled market. Equation (4) adds the shock state S, momentum Mom, the signed state trade G, and valuation-sensitive secondary selling ν_{eff} to obtain net market flow F. Because market value is the derived quantity $M = M_0P$, the ratio F/M converts money flow into a price-growth impulse. Equation (5) combines that impulse with valuation correction to update P. The correction speed λ_eff in equation (6) weakens when the inelastic ownership share π rises; equation (7) links π to the pension system's ownership share. Equation (8) evolves V and applies a reduced-form productivity drag only after overvaluation exceeds its threshold. Thus P is the traded price, V is the model's latent fundamental benchmark, F is a flow, and M is a calculated market value, not four interchangeable stocks.

The policy part uses pension-system size and market drawdown to trigger state purchases or sales. These trades enter both the market and the public account. The facility closes when it reaches its financing trigger, and a chosen exit rule then applies. S4 uses a countercyclical buffer instead. Its rule can see the model's unobserved fundamental value, so S4 only proves a mechanism. A real rule would need a public valuation measure, decision delays, and safety margins.

4.3 Feedback structure

Pension accumulation, price formation, and market structure



State backstop, credibility channels, and fiscal capacity

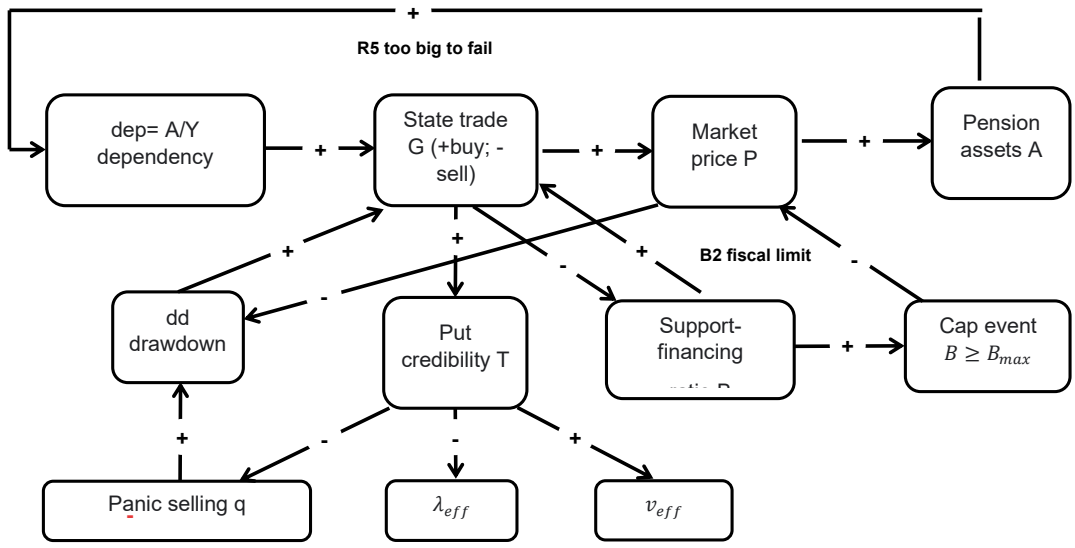


Fig. 1 Equation-aligned causal map. Every arrow carries a polarity: "+" means that a higher cause raises the effect, all else equal; "-" means that it lowers the effect. All causal links are solid black arrows. Colour carries no additional meaning. C1 is a feed-forward accumulation channel because c(t) is policy-set; R and B denote reinforcing and balancing feedbacks

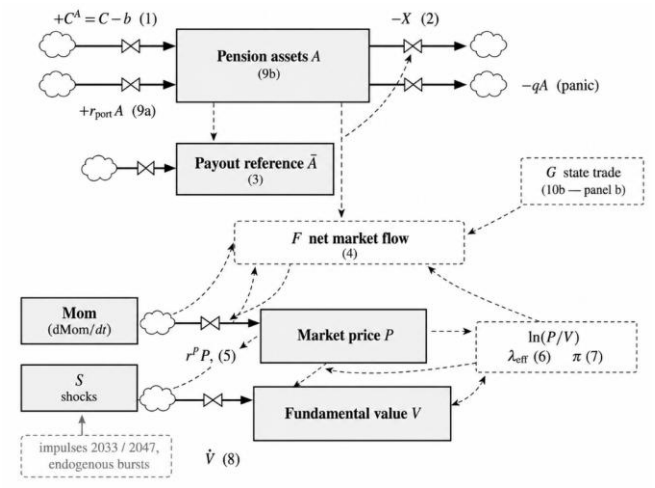
Figure 1 separates the policy-driven accumulation channel from four reinforcing and three balancing feedback mechanisms. In panel (a), valuation discipline (B1), passive crowding (R3), and secondary-

market elastic supply (B3) are distinct mechanisms: λ_{eff} governs correction toward V, whereas ν_{eff} governs valuation-sensitive selling by existing holders. In panel (b), credibility rises only after a successful exit and then affects q, λ_{eff} and ν_{eff} with the signs shown.

- **C1 Contributions add buying pressure.** A higher policy contribution rate c(t) raises contributions C in equation (1). The invested amount C^A increases pension assets in equation (9b) and enters market flow F in equation (4). Higher flow raises price P through equation (5), which lifts reported pension assets through investment returns. This is a one-way policy channel because returns do not change c(t).
- **R2 Momentum can reinforce price changes.** When P rises, the measured return r^p improves and momentum Mom becomes positive. Equation (4) turns Mom into more market flow, and equation (5) turns that flow into further price growth. The same mechanism works in reverse when prices fall. Appendix gives the smoothing rule for Mom.
- **B1 Valuation discipline, expensive markets attract sellers.** The ratio P/V compares the market price with the model's latent fundamental benchmark. When P rises above V, equation (5) pulls price back through λ_{eff} and equation (4) adds valuation-sensitive secondary selling through ν_{eff}. These mechanisms stabilize the market but become weaker when price-insensitive ownership or put credibility rises (equation 6). Primary issuance and dilution are not modeled.
- **R3 Passive crowding, price-insensitive ownership can weaken correction.** As pension assets accumulate in the modeled market, equation (7) raises the ownership measure own and the inelastic share π. Equation (6) then reduces λ_{eff}. Overvaluation can persist for longer, lifting measured portfolio returns and A through equation (9b), which in turn can raise modeled ownership further. This loop operates mainly when P exceeds V.
- **R4 Slow payout adjustment can worsen a downturn.** The smoothed reference $\bar{A}$ falls more slowly than current assets A. After a price fall, A drops at once but payouts X adjust gradually. Pension cash flow therefore weakens, and equations (4) and (5) pass the extra selling pressure back to price P.
- **R5 A large pension system can create pressure for state support.** Equation (10a) measures pension assets relative to one year of GDP. When this ratio and the market drawdown cross their thresholds, equation (10b) starts state trade G. Purchases support price P and enter the state account. A successful exit raises credibility T, which then affects panic and price correction.
- **B2 Fiscal trigger, every intervention uses part of finite support capacity.** State purchases raise B through equation (11a). When the quarterly state crosses B_max, equation (11b) closes the support facility and applies the selected resolution rule. B_{max} is therefore an event trigger in the discrete implementation, not a continuously enforced ceiling; the final quarterly observation can lie above 12%.
- **B3 Elastic secondary supply, high valuations attract more selling by existing holders.** When P/V exceeds the threshold in equation (4), ν_{eff} subtracts from F and restrains further price increases. A credible state buyer can strengthen this response because holders are more willing to sell into the bid. The channel can increase turnover and state holdings without an equally large or lasting price effect. It does not represent newly issued shares.

4.4 Model equations and numerical implementation

Pension balance, market-flow aggregation, and price/fundamental dynamics (Panel A):



State trigger, support financing, holdings, credibility and S4 buffer (Panel B):

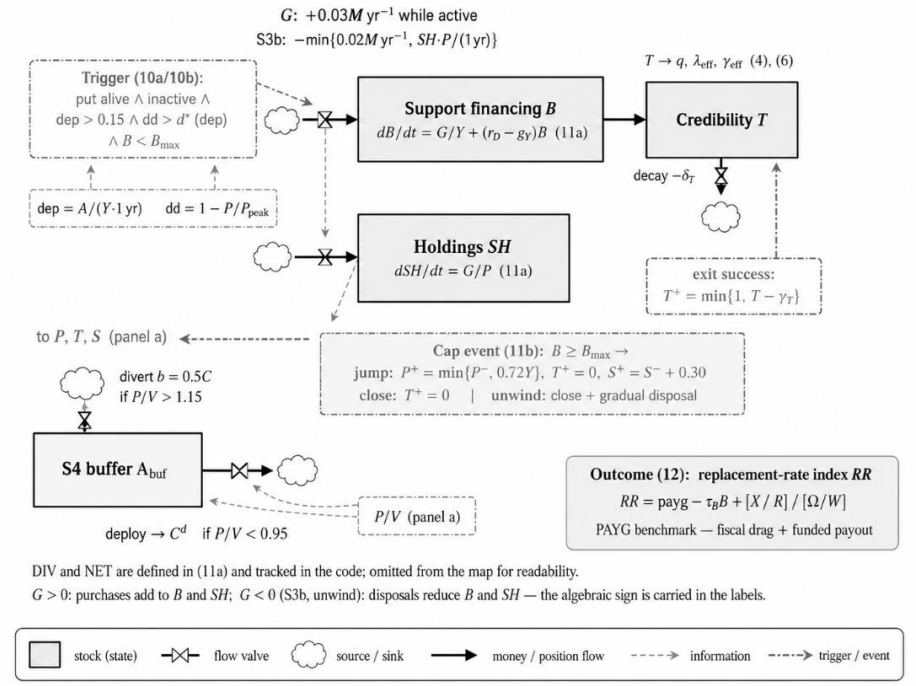


Fig. 2 Equation-aligned stock-and-flow map. Filled rectangles are accumulated states or state-ledger blocks. Solid arrows carry money or positions; dashed gray arrows carry information or calculated inputs; orange dash-dot arrows represent triggers or discrete events

Figure 2 follows the equations in the order just introduced. Pension assets A receive C^A and $r_{port}A$ and lose X and qA under equations (1)-(3) and (9). Mom, S, the pension-flow term, the signed state trade G, and valuation-sensitive secondary supply enter F in equation (4). F/M and the valuation gaps determine P in equation (5); equations (6)-(7) determine correction strength, and equation (8) determines V. The policy panel shows the trigger G, B, SH, T and the resolution event under equations

(10)–(11). Equation (12), added as a separate outcome box, combines PAYG adequacy, the fiscal drag and the funded payout relative to wages; it is not another stock. G > 0 denotes purchases and G < 0 denotes disposals. DIV and NET remain defined in equation (11a) but are omitted from the map for readability. The domestic market value is the derived quantity M(t) = M_0P(t), with P(2026) = 1 and M_0 = Y(2026)·1 yr = EUR 4.6 trillion. W, R, Y, and Ω are exogenous drivers. The S4 P/V rule is a latent-signal proof of concept and would require an observable proxy in practice.

Monetary stocks are measured in EUR trillion and annual monetary flows in EUR trillion per year. P, V, shares, indicators, and log gaps are dimensionless. A rate multiplying a stock has units 1/year; a monetary flow divided by M also has units 1/year. This unit convention is used consistently in equations (1)-(12). Subscript n indexes the quarterly implementation step (Δt = 0.25 yr), so P_n is the price at step n and A_{n+1} the asset stock one step later; superscripts on returns (r^P, r^{home}, r^{port}, r^w) are type labels, not exponents. The diagrams drop the step index and write the same objects as r^P, r^{port}, etc.

Equation (1) - Contributions:

$$C(t) = c(t)\,\Omega(t)$$

Equation (1) defines the annual contribution flow as C = cΩ. It depends on the policy contribution rate and the wage bill, not on contemporaneous asset returns. This predetermined-flow assumption is required to study how a staged pension reform interacts with market absorption. The rate c is dimensionless, Ω and C are annual flows, and C has units of EUR trillion per year. Contributions enter pension assets directly and reach the modeled market only through the allocated flow C^A.

Equation (2) - Payouts:

$$X(t) = min\{\omega(t)\,\bar{A}(t), x_{cap}\,A(t)\},\ \ \omega(t) = \omega_{max}\ \ m(t)[R(t)/(24\,million\,persons)]$$

Equation (2) applies the maturity- and demography-dependent withdrawal rate ω to the smoothed asset reference Ā, subject to an annual cap of 25% of current assets. Smoothing represents pooled benefit commitments that do not adjust immediately to market prices; the cap preserves accounting feasibility in extreme paths. Because ω has units 1/year, ωĀ is an annual monetary flow. The min operator selects the smaller of the commitment-based payout and the feasible payout ceiling.

Equation (3) - The smoothed reference:

$$d\bar{A}/dt = [ln\,2/(3\,yr)]\,[A(t) - \bar{A}(t)]$$

Equation (3) gives Ā a three-year half-life. After a negative asset-price shock, A falls immediately while Ā adjusts gradually, temporarily increasing payout pressure X/A. This state variable represents institutional payout inertia and creates the forced-sale channel examined in loop R4; it is not intended as an estimate of a particular pension contract.

Equation (4) – Net market flow:

$$F = h[C^A - X - qA] - \kappa SM + G + \chi\,Mom\,M - \nu_{eff}[ln(P/V) - \theta_{iss}]_+ M,\ \ \nu_{eff} = \nu_{iss}\{1 + a_{iss}\,T\mathbb{1}\,[put\ alive]\}$$

Equation (4) aggregates net secondary-market flow: allocated pension contributions add demand; payouts, redemptions, and risk-off sales subtract it; momentum, valuation-sensitive sales by existing holders, and the signed state trade G complete the identity. Primary issuance, dilution, and issuance proceeds are not modeled. Accordingly, ν_eff is a reduced-form secondary-selling response above the valuation threshold, amplified by put credibility while the backstop is active. Without an active backstop, credibility is zero and ν_eff returns to its baseline value.

Equation (5) - The price:

$$r^{P}(t) \equiv (1/P)dP/dt = g + \mu F/M + \lambda_{eff}\, ln(V/P) + \lambda_{deep}[ln(V/P) - \delta_{deep}]_{+}$$

Equation (5) decomposes the annual price return into fundamental growth, flow-price impact μF/M, valuation correction toward V, and a deep-undervaluation correction. The benchmark μ = 5 is a behavioral calibration to the aggregate inelastic-markets evidence, not a Germany-specific estimate. The sign of μF/M follows net flow F, whereas ln(V/P) lowers returns above fundamental value and raises them below it. The additional deep-value term activates only beyond its stated threshold and prevents extreme shocks from producing an economically uninterpretable price spiral.

Equation (6) - How fast price discovery works:

$$\lambda_{eff} = \lambda_0(1 - \pi)\{1 - \psi T\, \mathbb{1}[P > V]\mathbb{1}[put\ alive]\}$$

Equation (6) reduces the baseline correction speed λ_0 by the price-insensitive ownership share π. Put credibility weakens correction only when P > V and the facility is active. This deliberate asymmetry isolates the hypothesized support-credibility channel: it delays the correction of overvaluation without suppressing deep-value demand in undervalued states. The indicator functions make those regime boundaries explicit and testable.

Equation (7) - Who owns the market:

$$own = min\{0.95, hA/M\},\ \pi = min\{\pi_{cap}, \pi_{trend} + \beta_{\pi}\, own\},\ \beta_{\pi} = 0.60$$

Equation (7) defines modeled pension ownership as hA/M, capped at 95%, and maps it into the inelastic share π through the trend component and the ownership loading β_π, capped at 85%. The caps preserve share bounds rather than generate the result. With β_π = 0.60, ten percentage points of additional ownership add six percentage points to π before the cap. This closes the endogenous ownership–price-discovery feedback; S4 separately tests a design cap of π ≤ 0.60.

Equation (8) - Fundamentals: V grows at g, minus a misallocation drag:

$$(1/V)dV/dt = g - \varphi[ln(P/V) - \theta_{mis}]_{+}$$

Equation (8) subtracts a reduced-form misallocation term from fundamental growth only when overvaluation exceeds its threshold. The channel is one-sided because the model tests costs from persistently inflated signals and does not assume that undervaluation creates an offsetting productivity gain. Its magnitude is calibrated, not estimated. Setting φ = 0 removes the channel, which identifies how much of the adequacy result depends on this real-effects assumption.

Equation (9) - What savers earn, and the asset stock:

Equation (9a):

$$r_n^{P} = (P_{n+1}/P_n - 1)/\Delta t,\ r_n^{home} = r_n^{P} + d,\ r_n^{port} = h\, r_n^{home} + (1 - h)r_n^{w},\ r_n^{w} = r_{world} + \beta_w[r_n^{home} - (g + d)]$$

Equation (9b):

$$A_{n+1} = max\{0, A_n + [C_n{}^A - X_n - q_n A_n + r_n{}^{port} A_n]\Delta t\}$$

Equation (9a) constructs the portfolio return: the home share h earns the domestic price return plus dividends, and the remainder earns the world return. The β_w variant permits correlated stress and tests the diversification boundary. Equation (9b) is the asset-stock identity, adding contributions and investment income and subtracting payouts and redemptions. Quarterly ordering updates P first, converts that change into the realized return, and then updates A; the non-negativity floor only prevents infeasible asset balances.

Equation (10): The state put:

Equation (10a):

dep = A/(Y·1 yr), d*(dep) = max{0.10, 0.20 − 0.12·clip[(dep − 0.15)/0.50, 0, 1]}

Equation (10b):

- Trigger if put alive, inactive, dep > 0.15, dd > d*(dep), and B < B_max.
- While active, G = +0.03M yr⁻¹; exit if dd < 0.08. In S3b,
- if P/V > 1.15 and holdings remain, G = −min{0.02M yr⁻¹, SH·P/(1 yr)}.
- Between successful exits dT/dt = −δ_T𝟙[T > 0]; at a successful exit $T^+ = \min\{1, T^- + \gamma_T\}$.

Equation (10a) measures dependency as pension assets relative to one year's GDP and lets the support threshold decline from a 20% toward a 10% drawdown as dependency rises. Equation (10b) treats this as a calibrated political rule, not an estimated reaction function: when dependency exceeds 15%, the drawdown threshold is crossed, and fiscal room remains, the state buys 3% of the market per year until drawdown falls below 8%. Successful exits raise credibility T, which then affects panic and price correction. Positive G denotes purchases and negative G disposals throughout.

Equation (11) - The state's ledger, and the endgame:

Equation (11a):

Let D = B(Y·1 yr). Between failures: $dD/dt = G + r_D \cdot D$ and $dB/dt = G/(Y \cdot 1\,yr) + (r_D - g_Y)B$. State ledger: $dSH/dt = G/P, dDIV/dt = d \cdot SH \cdot P, NET = SH \cdot P + DIV - D$.

Equation (11b):

If $B \geq B_{max}$: jump resolution $P^+ = \min\{P^-, 0.72V\}$, $T^+ = 0$, $S^+ = S^- + 0.30$; close resolution $T^+ = 0$ only; unwind resolution = close plus gradual disposal of holdings.

Equation (11a) separates gross support financing D, its GDP ratio B, state holdings SH, dividends DIV, and the marked-to-model net position NET. This distinction is essential: a near-balanced net ledger can coexist with a binding gross financing constraint and with costs transmitted through prices, fundamentals, and pensions. In S3b, signed disposals reduce B, while cumulative purchases remain a separate diagnostic. Equation (11b) applies the finite-cap event and an imposed resolution map. Jump, closure, unwind, and restoration variants separate endogenous cap-hit incidence from assumed loss given failure. Because the cap is tested quarterly, the last active purchase can produce a quarter-end observation above 12%.

Equation (12) - The replacement rate:

$$RR(t) = payg(t) - \tau_B \cdot B(t) + [X(t)/R(t)]/[\Omega(t)/W(t)],\ \tau_B = 0.15$$

Equation (12) combines the PAYG component, a calibrated fiscal-consolidation drag, and funded payout per pensioner relative to the average wage. It is the common adequacy metric used to compare scenarios and policy designs. It is expressly a model replacement-rate index, not a household net-of-tax replacement rate.

X/R is the funded payout per pensioner and Ω/W is the average wage per contributor, so their ratio is dimensionless. The PAYG term adds statutory adequacy; τ_B·B subtracts the stylized consolidation incidence. RR is therefore a model comparison index, not a household net-of-tax replacement rate.

Equation and diagram conventions. A dot or d/dt denotes an annualized rate; $[z]_+ = \max(z,0)$; $\mathbb{1}[\text{condition}]$ equals one when the condition is true; and clip confines its argument to the stated interval. P and V are indices, M = M_0P is a monetary stock value, terms multiplied by M are monetary flows, and F/M is an annual rate. Figure 1 assigns a polarity to every causal link. Figure 2 instead distinguishes material flows, information links, and event logic; its algebraic signs appear in the labels rather than as edge polarities.

Discrete implementation. The code evaluates flows at the start of each quarter, updates P, computes the realized within-step price return, and then updates A as in equation (9b). The price update is bounded by an annualized rate clip [−0.9, 2.4]; widening these bounds does not change any deterministic benchmark outcome. Non-negativity floors apply to P, A, holdings and the buffer. B_max is a trigger tested on the quarterly grid rather than a prorated hard ceiling, so the final active trade can produce a reported value above 12%. Time-step refinement also moves event dates by roughly two years; dates are therefore reported as approximate scenario years rather than quarter-precise forecasts.

Put-failure event. The model separates incidence from severity. Whether the fiscal cap is reached by 2080 is generated endogenously by the path of drawdowns, dependency, rescues, and carrying costs. By contrast, the resolution map applied on cap exhaustion is a calibrated loss-given-failure assumption; Section 5.5 reports all three maps (jump, close, unwind) and shows that the scenario ranking does not depend on the choice. A smooth auction-based unwinding during the put's life is what S3b tests directly.

Design rationale and functional-form status. Each reduced form has one transparent task: logistic ramps avoid an instantaneous policy or maturity jump; exponential smoothing represents contractual and expectation inertia; positive-part terms encode mechanisms that activate only beyond a threshold; min/max operators implement institutional caps; and linear flow loadings preserve signs, units, and auditability. These choices are not claimed to be unique. Every behavioral block has a nested off-switch, and Table 9 replaces the main forms with a linear contribution ramp, level-ratio valuation gaps, and smooth thresholds. The coefficients β_π, φ, panic, credibility, fiscal drag, and failure maps remain calibrations rather than causal estimates.

4.5 Experimental design and calibration

Table 1 defines six experiments. Each adds or changes one important mechanism. S0 keeps funded saving small. S1 expands funded saving but assumes the market absorbs the flows without changing returns. S2 turns on the benchmark market feedback. S3 adds a limited public backstop, and S3b also allows public sales. S4 changes pension rules instead of adding a rescue: it uses a buffer, adjusts payouts, and keeps more price-sensitive investors. Population and deterministic shocks are the same in

every scenario, so differences come from the model's market and policy rules. S2 is not an estimate for Germany or Europe. S4 is a proof of concept because it uses unobserved V.

Scenario	Market structure	Policy regime	Purpose
S0 Baseline	μ = 5; passive trend	Funded saving ≈ 0.5%; no put	Status quo
S1 Elastic counterfactual	μ = 0.2; λ_0 = 0.6	Expansion to 4.5%; no put	Elastic benchmark
S2 Inelastic calibration	μ = 5; endogenous π; momentum, panic, and payout rigidity	Expansion to 4.5%; no put	Behavioral calibration
S3 State put	As S2	S2 + public purchases after the loss trigger; capacity 12% of GDP	Finite backstop
S3b Symmetric put	As S2	S3 + sales when P/V > 1.15	Backstop with disposals
S4 Stabilized design	As S2; π ≤ 0.60	Buffer + flexible payouts; no put	Preventive design

Table 1 Scenario definitions

The calibration separates observed anchors from assumptions (Table 3). Population, the PAYG path, the contribution ramp, and the passive-investing trend have data or policy support. The flow multiplier comes from wider research on inelastic markets. Ownership, misallocation, panic, credibility, public support, the fiscal limit, and exit rules are model choices rather than estimates. Three boundary choices are central. M_0 sets the size of the receiving market, h sets the share of pension flow entering it, and π_trend sets the background rise in price-insensitive ownership. Any German, European, or global application must choose M_0 and h for the same market. The smaller-market and lower-h cases therefore test different boundaries; they do not claim that one geography is correct.

4.6 Verification, validation, and robustness

The computational implementation is verified and validated in four stages. First, dimensional consistency and pension/state accounting identities are checked. Second, limiting cases confirm that price effects vanish as the flow multiplier approaches zero and that stronger price discovery closes valuation gaps faster. Third, time-step refinement evaluates the ordered quarterly update. Fourth, nested models, alternative equations, shared-shock simulations, Monte Carlo experiments, and broad sensitivity tests identify which results survive changes in mechanisms and functional form. These checks support internal validity and reproducibility; they do not establish empirical identification.

The numerical checks show that final outcomes are more stable than exact event dates. Halving the quarterly step changes the 2080 index by at most 0.11 percentage points and final assets by less than 1%; S3 still reaches the trigger in 500/500 runs. A smaller step moves the deterministic trigger by about two years. Trigger years are therefore approximate (about ±2 years), while trigger frequency and final comparisons are more stable.

Nested models show how the nonlinear result builds up. Adding flow-driven price effects, price-insensitive ownership, momentum and panic, and the real-economy channel raises S2's maximum drawdown from 1.0% to 47.2% and lowers the 2080 index from 58.4% to 50.2%. No single block creates the full result. A linear contribution ramp and smooth thresholds give almost the same benchmark. Using level ratios instead of log valuation gaps lowers the drawdown to 37.6% and delays the S3 trigger to about 2071, but the deterministic trigger still occurs (Table 9).

The broad sensitivity test separates the benchmark from a universal claim. All 500 benchmark S3 paths reach the trigger by 2080, usually between 2059 and 2073. Across the wider prespecified set of 400 parameter combinations, 57% reach it. The event is therefore produced by the benchmark assumptions, not inserted by hand. The 57% figure is not a real-world probability.

5 Computational experiments

5.1 Exogenous-return benchmarks (S0 and S1)

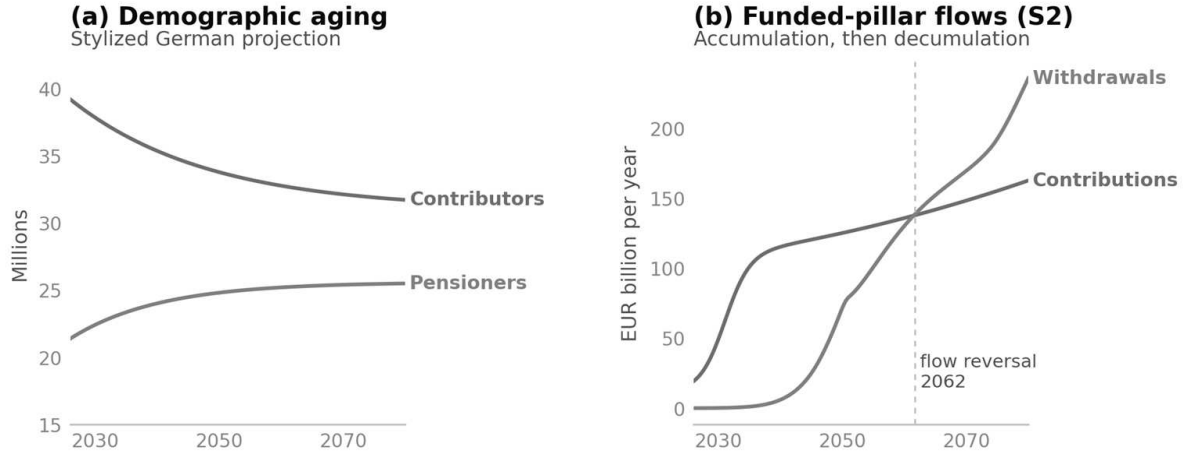


Fig. 3 Exogenous demographic drivers (a) and the funded pillar's flow reversal (b): contributions and withdrawals in S2

S0 and S1 provide the reference cases in which pension flows do not meaningfully change returns. S0 keeps funded saving small: assets reach €1.6 trillion and the 2080 index stays near 45%. S1 expands contributions but assumes the market absorbs them easily. Prices stay close to fundamental value, assets reach €8.5 trillion, the 2080 index reaches 58.3%, and the maximum drawdown is 3.6%. S1 therefore shows the model's gain from funded expansion when the reform does not change the returns it earns.

5.2 Endogenous-return dynamics and cohort incidence (S2)

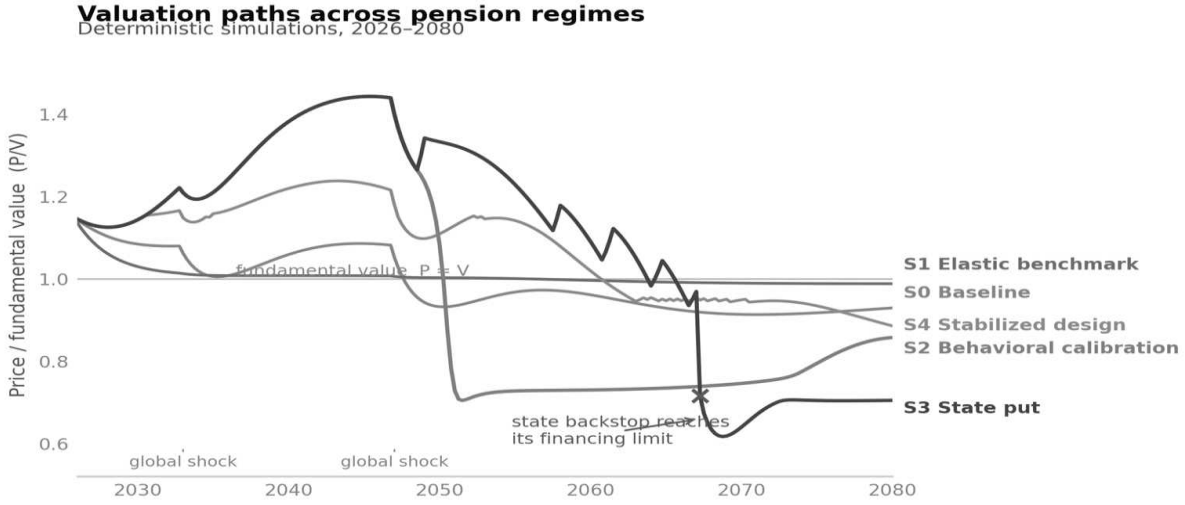


Fig. 4 Price/fundamental ratio across regimes, deterministic runs. Identical global shocks hit all scenarios in 2033 and 2047

S2 keeps the same contribution path but allows pension flows to affect returns. With a flow multiplier of five, P/V peaks at 1.44 around 2046 (quarterly maximum 2045.5). Higher measured returns increase participation and pension ownership. The price-insensitive share approaches its 85% cap, and prices

return to fundamental value much more slowly. The price-alignment index falls from 0.87 to 0.69. The loop is simple: policy flow raises price; higher price raises measured returns and ownership; more price-insensitive ownership slows correction.

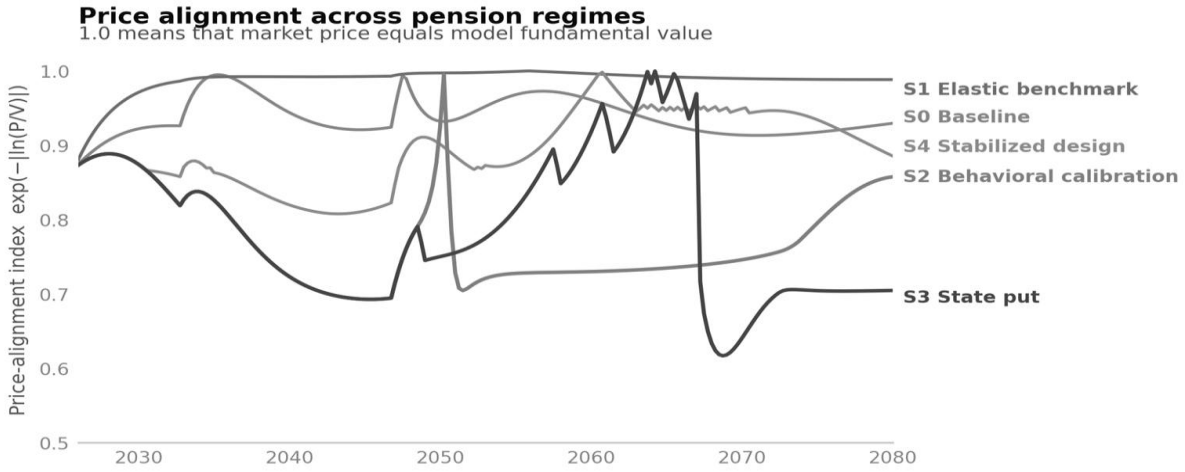


Fig. 5 Price-alignment index by scenario: exp(−|ln(P/V)|), where 1.0 means prices equal fundamental value. A transform of the valuation gap, not an informativeness measure in the sense of Bai et al. (2016)

The common shocks remain mild in S0, where the maximum drawdown is 8.9%. S2 also absorbs the 2033 shock because contributions are still strong. The 2047 shock arrives after the funded system has matured. Rigid payouts and panic selling then meet weak price-sensitive demand. The drawdown reaches 47% by 2051 and stays above 25% for 16.75 years. Withdrawals exceed contributions from 2062, which slows the recovery. Prices remain below fundamental value for a long period, so cohorts retiring during the downturn sell at low prices.

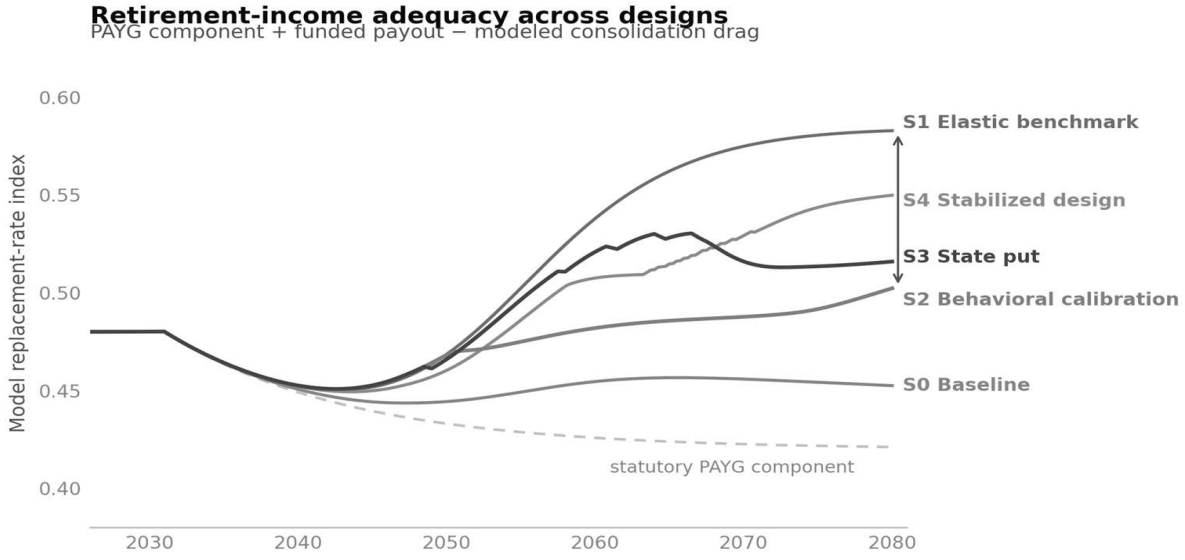


Fig. 6 Model replacement-rate index by scenario. The S1–S2 gap is the modeled market-structure cost; the S3 path shows short-run stabilization and its subsequent fiscal cost

The model shows the cost in retirement income. The 2080 index is 50.2% in S2 and 58.3% in S1. The 8.1-point gap comes from buying during the earlier overvaluation and selling during the later drawdown. This compares two model settings. It is not an estimated tax or welfare loss for a real pension reform.

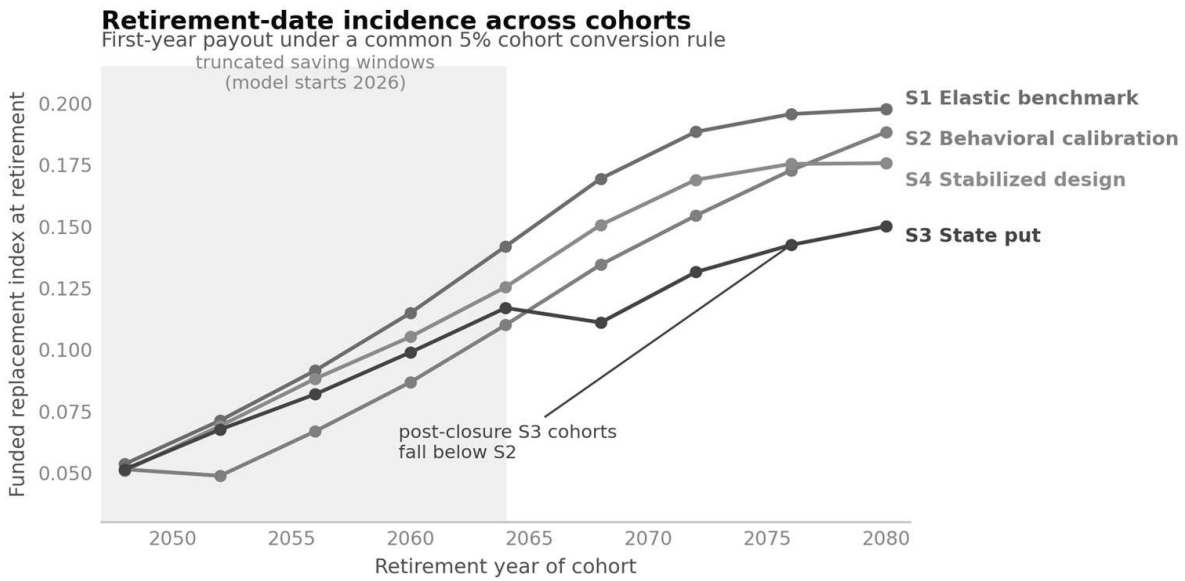


Fig. 7 Synthetic cohort outcomes: funded replacement index at retirement (first-year payout under a 5% rule on the cohort's own account, relative to the final wage), by retirement year. Cohorts retiring before ~2066 have truncated saving windows because the model starts in 2026

The cohort results show who bears the timing risk. The cohort retiring in 2052 receives funded income equal to 4.9% of final wage in S2 and 7.1% in S1, a gap of about 30%. By 2080, the gap narrows to 5% (18.8% versus 19.8%). The overall system can recover, but a cohort cannot move its retirement date. S4 stays close to S1 across cohorts. This shows that the loss comes from the modeled market structure, not from equity investment alone.

The two payout rates serve different purposes. The main model withdraws 5.5% from smoothed pooled assets once the system is mature. This represents benefit commitments and slow payout adjustment. The cohort comparison applies a 5% first-year rate only to each cohort's final account. That rate does not affect market prices or total pension assets. Changing it would rescale cohort levels but would not change the ranking of scenarios.

5.3 Finite-backstop experiments (S3 and S3b)

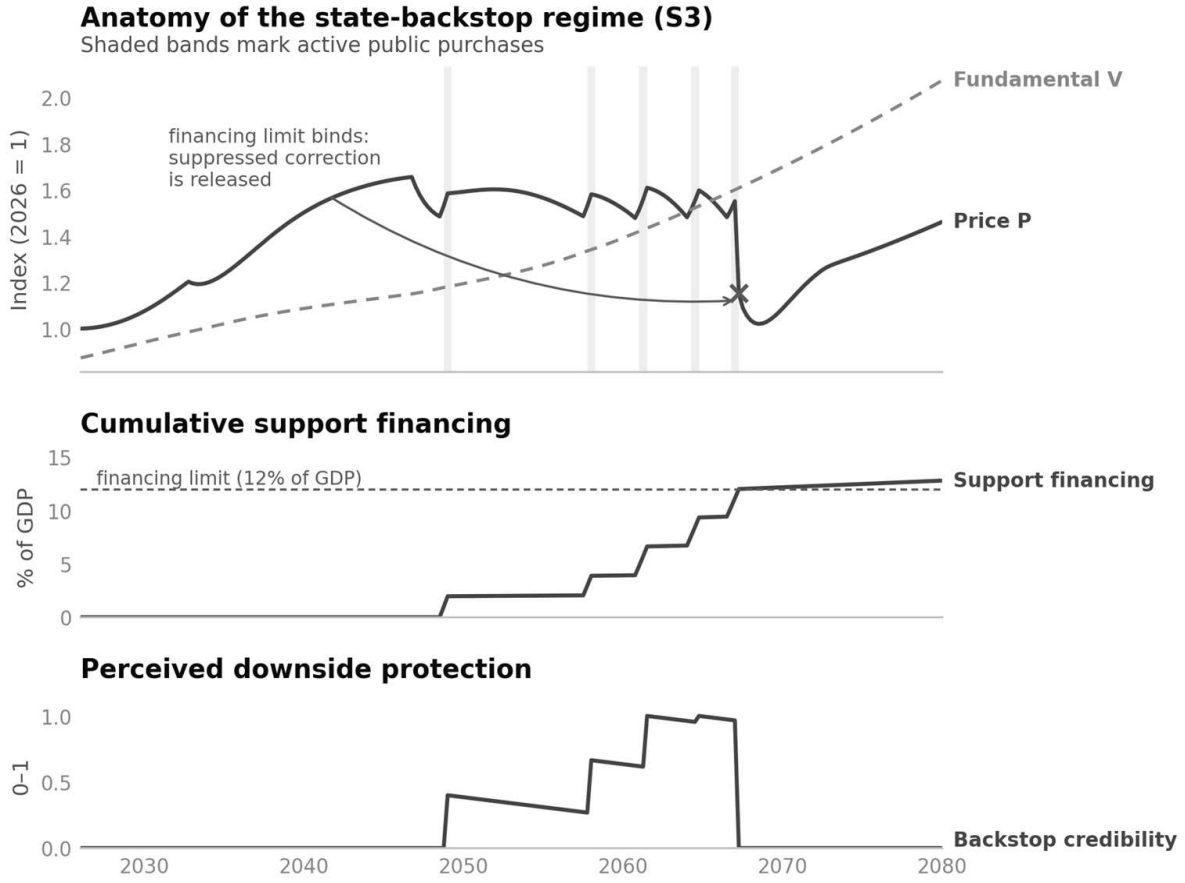


Fig. 8 Anatomy of the state-put regime: price versus fundamentals with rescue episodes (top), gross support financing against the fiscal cap (middle), and put credibility (bottom)

S3 follows S2 until the size of the pension system and the market drawdown activate the backstop. In the deterministic run, intervention starts in 2049 after the 2047 shock. Purchases equal to 3% of market value per year stop the fall within a few quarters. The first drawdown is about 10% rather than 47%, and pension assets and the index stay above S2 for roughly two decades. These visible short-run gains explain why continued support can become politically attractive.

The same rule creates later costs. Expected support slows price correction, existing holders sell into the public bid, and an aging population keeps payouts high. The financing ratio crosses the 12% trigger around 2067, with 12.8% recorded after the final active quarter. The later drawdown is 38%, and the 2080 index remains below S4. The exact year is uncertain, but the decision problem is clear: support helps current cohorts while using capacity that later cohorts may need.

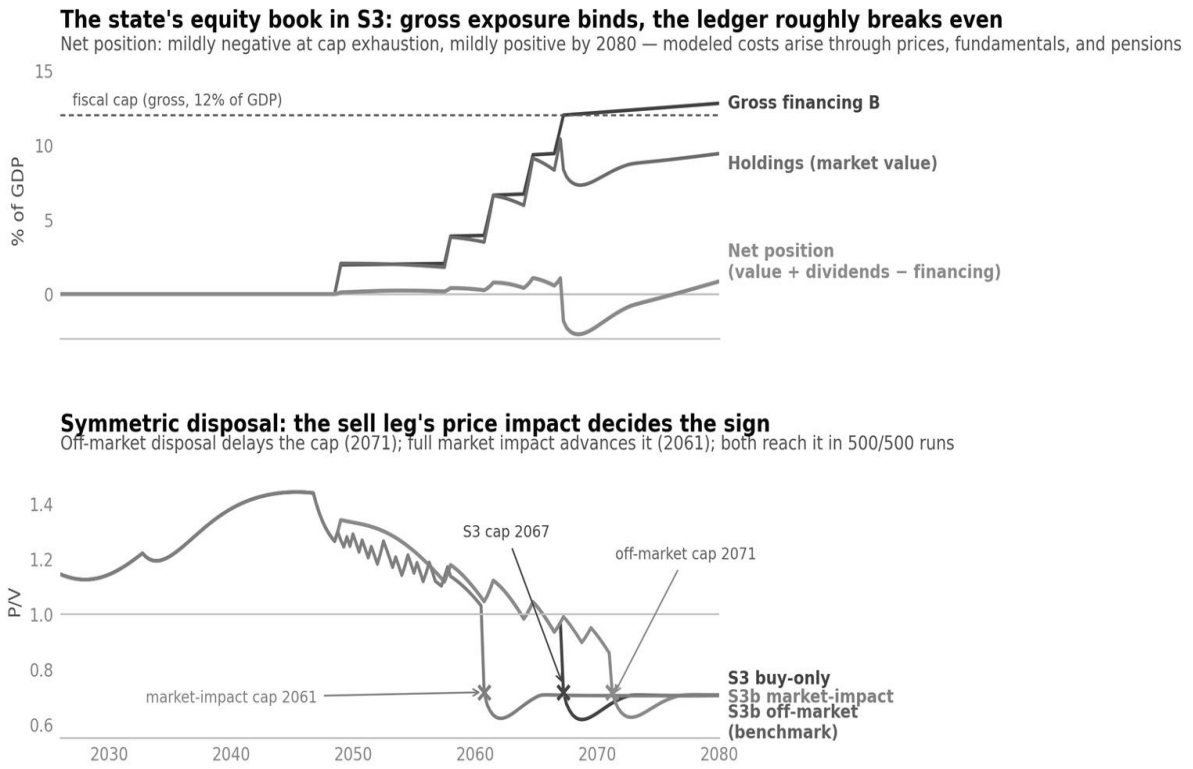


Fig. 9 Equation-consistent state ledger in S3 (top) and comparison of buy-only S3 with the two fully credited symmetric S3b variants, off-market disposal (benchmark) and full market impact (bottom). The corrected ledger uses $D = B \cdot Y$ and $NET = SH \cdot P + DIV - D$

The public account has two sides. At model prices, S3 ends with a net position of +0.8% of GDP because holdings and dividends slightly exceed recorded financing. This does not prove a fiscal profit or break-even. The account excludes liquidation costs, the price effect of selling, governance costs, and voting constraints. It also exists alongside lower fundamental value and lower payouts after the facility closes. A favorable account at model prices can therefore hide economic and distributional costs.

S3b sells assets during recoveries and subtracts the proceeds from financing. If these sales occur off-market, the trigger moves from about 2067 to 2071 and the maximum drawdown falls from 38.4% to 32.4%. The final index remains 51.6%, and all 500 stochastic paths still reach the trigger. Total purchases rise because sales free capacity for later intervention. The result depends on whether the legal limit covers net financing or total gross purchases.

The result also depends on how public sales affect prices. The benchmark assumes off-market sales that improve the public account without entering market flow. If sales move prices as strongly as purchases, the trigger comes about six years earlier, the drawdown deepens to 44.4%, and total purchases rise further. Public buying and selling are therefore not a costless exit. A credible plan must set the placement, pace, accounting rule, and market-impact safeguards before intervention begins. The Bank of Japan's slow sale plan shows why exit can take decades.

5.4 Preventive-design experiment (S4)

S4 changes pension rules instead of promising a rescue. It places some contributions in a buffer when valuation is high, uses the buffer when valuation is low, adjusts payouts within a corridor, and keeps more price-sensitive investors in the market. Because the model observes V perfectly, S4 improves price alignment and reaches a 55.0% final index. S4 is not an existing German fund and cannot be used

directly. A real version would need an observable signal, decision delays, error bands, independent governance, and protection against political use.

5.5 Uncertainty, sensitivity, and regime boundaries

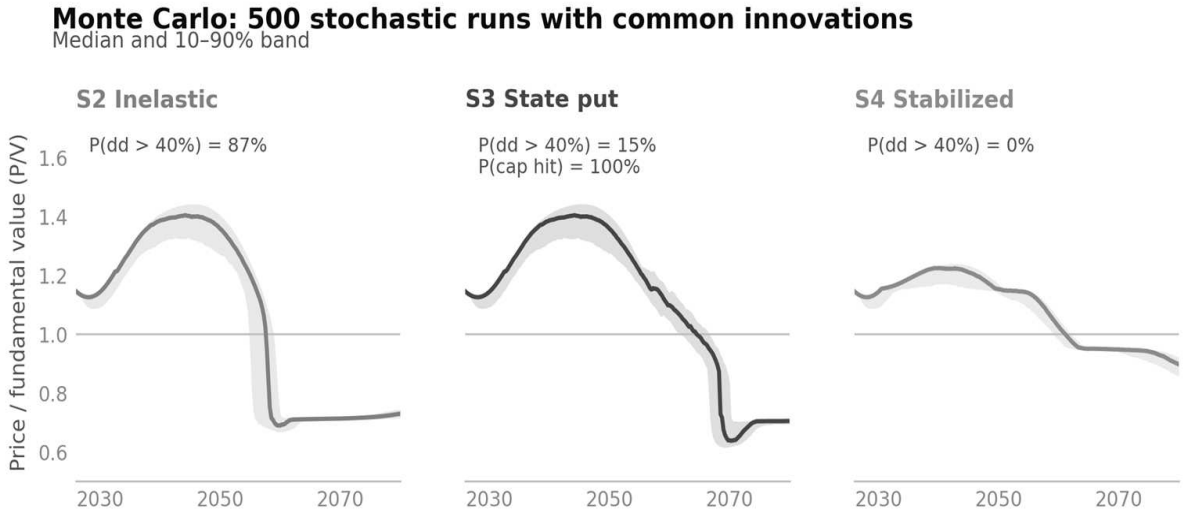


Fig. 10 Monte Carlo experiments: 500 runs per scenario using common random-number innovations; median and 10–90% bands of P/V

The stochastic experiment replaces the two fixed shocks with shocks that arrive at uncertain times and have uncertain sizes. In S2, 86.6% of runs have a drawdown above 40% (median 44.1%). In S3, 14.8% exceed 40%, but all 500 reach the fiscal trigger. Median financing after the trigger is 13.2% of GDP and the median net position is +0.9%. Off-market S3b also reaches the trigger in every run but lowers the median drawdown to 33.1%. If public sales move prices, 36.8% of S3bi runs exceed a 40% drawdown. S4 has no drawdown above 25% and ends near 54.6%. The backstop reduces the early market loss but does not remove the financing limit; preventive rules improve both.

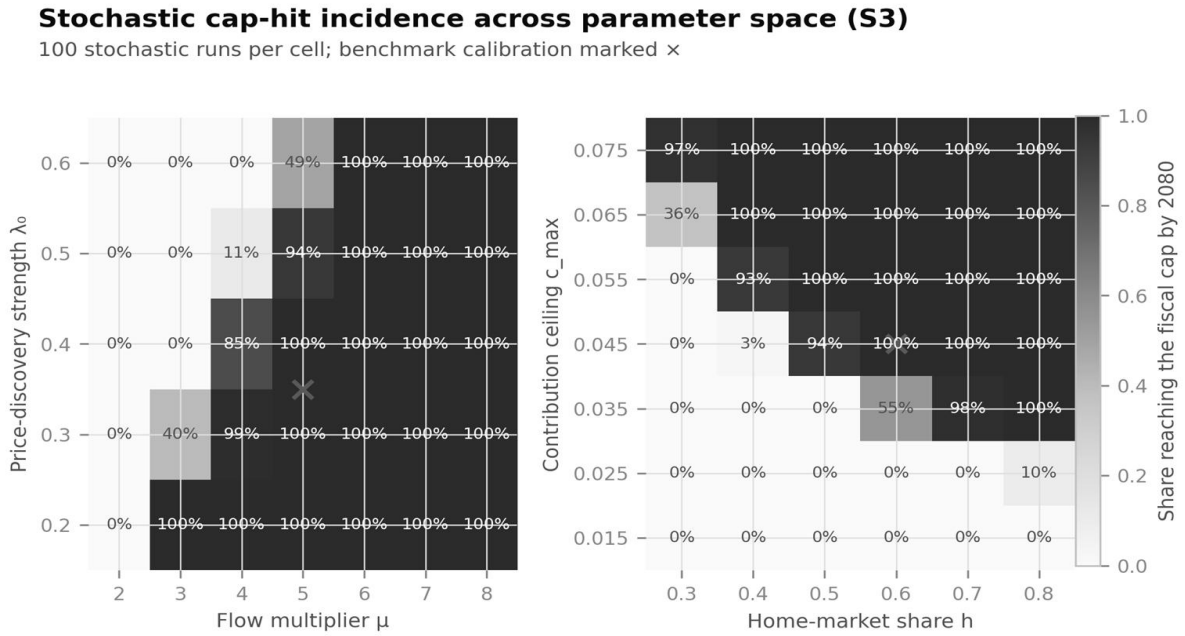


Fig. 11 Regime maps: share of 100 stochastic runs per cell in which S3 reaches the fiscal cap by 2080, over (μ, λ₀) and (h, c_max). The benchmark calibration is marked ×

Three checks show where the model moves from low to high trigger frequency. First, 57% of the 400 Latin-hypercube parameter sets reach the S3 trigger, and 67% produce an S2 boom and bust. Trigger frequency is most closely linked to the flow multiplier (+0.48), contribution scale (+0.46), home share (+0.31), and price-discovery strength (−0.29). Second, the regime maps show low frequency when μ is below about 3, the home share is below about 0.5, or contributions are below about 3%. Third, one-at-a-time tests tell the same story: a smaller receiving market worsens outcomes, stress in world markets weakens diversification, and a 2% contribution ceiling avoids the trigger but adds little retirement income. Larger flows create both the pension gain and the feedback risk.

Sampling design. The 400-draw box is $\mu \in [2, 8]$, $\lambda_0 \in [0.20, 0.60]$, $h \in [0.40, 0.80]$, $c_{max} \in [0.020, 0.070]$, $\varphi \in [0, 0.090]$, panic intensity $\rho \in [0.15, 0.45]$, ownership loading $\beta_\pi \in [0.30, 0.90]$, and credibility effect $\psi \in [0.50, 0.95]$. Marginals are uniform and Latin-hypercube stratified; independently permuted columns impose no cross-parameter dependence (seed 202609). Each vector uses the same scheduled shocks in paired S2/S3 runs, so this experiment samples parameters, not shock histories. The separate 500-run experiment reuses common quarterly innovations and lognormal sizes across scenarios. State-dependent hazards can still produce different realized event dates.

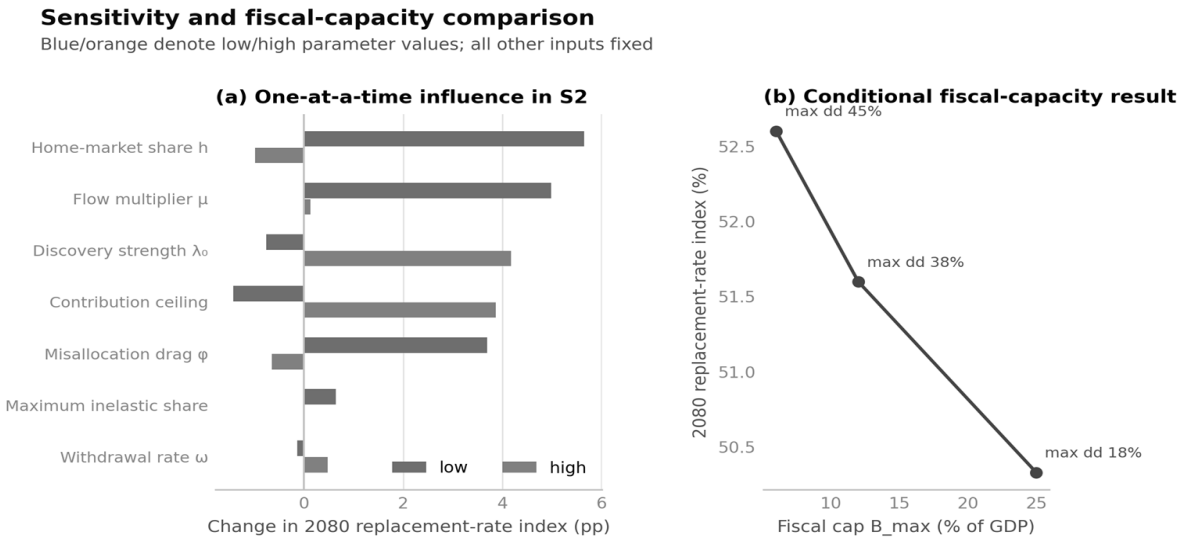


Fig. 12 Sensitivity: (a) one-at-a-time parameter sweeps around S2 (terminal replacement-rate index); (b) the fiscal-capacity comparison in S3

The sensitivity results show which assumptions drive the outcome. Lowering μ to 2 raises the S2 final index to 55.2%. Lowering h to 0.4 raises it by 5.7 points and cuts trigger frequency to 2%. Faster price correction raises the index by 4.2 points and cuts trigger frequency to 43%. Removing the real-economy channel raises the index by 3.7 points, showing that this channel changes the size of the result. A larger fiscal limit delays failure but may shift the adjustment to a later, more dependent population. This does not mean that smaller limits are always better. In every tested exit and fiscal-drag case, S4 ends above S3. Gradual sale is the least costly tested exit, but prevention still works better than repeated rescue within the model.

6 Economic interpretation and design implications

6.1 How pension flows create cohort risk

The experiments show that pension flows affect returns only under certain conditions. What matters is not funded saving alone, but how large and persistent the flows are relative to the market receiving them. During the saving phase, regular demand can raise prices and reported pension assets. If high

measured returns attract more participants or more price-insensitive ownership, prices return to fundamental value more slowly. During the payout phase, the process can reverse: withdrawals and rigid benefits meet fewer price-sensitive buyers. This creates timing risk. The overall system may recover later, but a cohort retiring during the fall cannot recover income already lost.

Two channels carry the effect. The accounting channel raises reported assets while prices are high and reverses when later sales meet weak demand. The real-economy channel in equation (8) lets long-lasting price distortions reduce fundamental growth. This second channel is assumed, not estimated. Turning it off changes how severe the outcome is, but the market-and-accounting feedback remains. The direction is therefore more reliable than the exact size.

The backstop shifts risk across time, cohorts, and balance sheets. Public purchases hide part of the first adjustment, but they also delay price correction, let private holders sell to the state, and create financing and exit duties. The state can appear to end in a good position when its holdings are valued at model prices, even though sale costs, market impact, governance costs, and weaker fundamentals are not included. Support buys time and changes who bears the loss; it does not remove the exposure.

6.2 Five choices for pension design

The model points to five design choices. Each weakens a specific part of the feedback. None is always best for every country.

1. **Match flow scale to market capacity.** Price effects depend on pension flow relative to the market receiving it. Wider global diversification and a gradual rise in contributions reduce concentration. A smaller receiving market makes every feedback stronger. M_0 and h must therefore refer to the same geographic market. The Frühstart-Rente's global mandate points in the stabilizing direction, while a domestic-investment rule would concentrate risk.
2. **Use a rules-based buffer with observable signals.** A collective fund can save part of contributions when markets are expensive and use the buffer when markets are weak. The rule cannot rely on unobserved V. It needs a published valuation measure, decision delays, error bands, independent governance, and protection against political use. This separates preventive stabilization from discretionary rescue.
3. **Adjust payouts gradually.** Payouts tied rigidly to past asset values can force sales after a crash. Corridors, phased annuitization, or withdrawals based on smoothed assets can spread the adjustment over time. Clear automatic rules are better than sudden cuts because retirees and fund managers can plan for them.
4. **Keep enough price-sensitive investors.** Low-cost default funds can remain passive, but some investors must be able to buy when assets are cheap and sell when they are expensive. Rebalancing rules, active allocation, and market-making provide that function. The sensitivity tests show that less price-insensitive ownership and faster price correction reduce both mispricing and pressure on the backstop. Price discovery is a resource the pension system uses; it is not free (Grossman and Stiglitz, 1980).
5. **Set the backstop's limit and exit before it starts.** The law must say whether the limit covers net financing or total purchases, whether sale proceeds restore capacity, how assets will be sold, and what happens when the limit is reached. Off-market sales can delay the constraint, while sales that move prices can make it worse. The tests favor gradual exit over sudden liquidation and prevention over repeated rescue. The model does not find one best limit. It shows that the limit, payout, accounting, governance, and exit rules must be chosen together before political pressure becomes too strong.

The German debate shows how to use these comparisons without treating the model as a national forecast. Gradual rollout and global diversification weaken the feedback in the experiments. Payout adjustment, enough price-sensitive investors, and advance rules for any future support remain separate policy choices. The model makes these links visible before funded assets become large. It does not claim that Germany already follows S3 or S4.

7 Limitations and external validity

Scope and distribution. This is an aggregate scenario model, not a forecast. It contains one receiving market and a world return set outside the model. Its results cannot be assigned to Germany, Europe, or a global portfolio until M_0 and h are set for the same market. The retirement cohorts are synthetic. They do not differ by earnings, taxes, means-tested benefits, life expectancy, sex, or gender. RR is a model index, not a household replacement rate after taxes.

Evidence and calibration. Price correction, momentum, panic, credibility, political triggers, misallocation, and fiscal drag are simplified model rules. Several values are chosen by order of magnitude. The broad sensitivity test explores the model's own parameter box; it does not estimate parameters from history or test them on new data. Nested and alternative versions show that the main mechanism does not depend on one ramp or threshold, but the timing and depth of a crisis still depend on the model form. Historical estimation or indirect inference would be needed before using the model for policy probabilities.

Market and public-finance boundary. The model leaves out bonds, interest-rate feedback, changing tax revenue, sovereign risk, new share issuance, dilution, and company balance sheets. The state account values holdings at model prices and excludes sale costs, market impact, and governance costs. S3b depends on whether sale proceeds restore legal capacity and whether sales can occur off-market. S4 can observe unobserved V and assumes that politics cannot misuse the buffer. These assumptions make S4 a test of a mechanism, not a ready policy rule.

Failure and numerical rules. Whether the financing limit is reached follows from the model, but the rule applied afterward is chosen in advance. Sudden loss, closure, gradual sale, and restoration keep the main scenario ranking, but none has independent empirical support. Exact dates and crisis depths are more sensitive to the time step than final outcomes and carry about ±2 years of numerical uncertainty. The 500/500 benchmark result, the 57% parameter-box share, and the regime boundary describe this calibrated model, not the real world.

8 Conclusion

This paper presents a reproducible computational framework in which funded-pension returns depend on the scale and destination of pension flows. The model integrates policy contributions, endogenous market prices, price-insensitive ownership, payout inertia, population aging, cohort accounts, and finite public support in one nonlinear dynamic system. Its purpose is to identify mechanisms and regime boundaries rather than to forecast a national pension path.

The central result is a market-capacity condition. When the receiving market absorbs flows elastically, funded expansion improves the model replacement-rate index without materially changing the returns it earns. When flows are large relative to market capacity and price discovery is weak, accumulation-phase buying can raise prices and reported assets, while later payouts reverse the flow and concentrate losses on cohorts retiring during the adjustment. Monte Carlo and parameter-space experiments show that the direction is robust, but the incidence and severity are calibration dependent.

A finite state backstop changes the timing and allocation of risk rather than eliminating it. Public purchases can limit the initial drawdown, but they create gross financing exposure, state holdings, credibility effects, and an exit problem. In the model, preventive rules—diversification, gradual rollout, flexible payouts, buffers, and sufficient price-sensitive ownership—reduce both market instability and reliance on rescue.

The methodological implication is that pension design, market structure, and public support should be simulated jointly when reform flows are potentially large. The model provides a transparent platform for empirical calibration, alternative market boundaries, richer household heterogeneity, and estimation or indirect-inference work. The useful output is not one crisis date or probability, but a computational map of the conditions under which pension reform can endogenize its own returns.

References

Abel, A. B. (2001). Will bequests attenuate the predicted meltdown in stock prices when baby boomers retire? Review of Economics and Statistics, 83(4), 589–595.

Alterssicherungskommission (2026). Empfehlungen der Alterssicherungskommission. Report submitted to the Federal Government, 23 June 2026. Berlin: Federal Ministry of Labour and Social Affairs. https://www.bmas.de/DE/Soziales/Rente-und-Altersvorsorge/Rentenreform-2025/Rentenkommission-2026/rentenkommission-2026.html (accessed 28 July 2026).

Anadu, K., Kruttli, M., McCabe, P., & Osambela, E. (2020). The shift from active to passive investing: Risks to financial stability? Financial Analysts Journal, 76(4), 23–39.

Australian Prudential Regulation Authority (2025). Quarterly superannuation performance statistics, June 2025. Sydney: APRA. https://www.apra.gov.au/quarterly-superannuation-statistics (accessed 28 July 2026).

Bai, J., Philippon, T., & Savov, A. (2016). Have financial markets become more informative? Journal of Financial Economics, 122(3), 625–654.

Baker, M., & Wurgler, J. (2002). Market timing and capital structure. *Journal of Finance, 57(1)*, 1–32.

Bank of Japan (2025). Guidelines for Sales of Exchange-Traded Funds and Japan Real Estate Investment Trusts. 19 September 2025. https://www.boj.or.jp/en/mopo/measures/term_cond/etf_s1.htm (accessed 28 July 2026).

Barberis, N., Greenwood, R., Jin, L., & Shleifer, A. (2015). X-CAPM: An extrapolative capital asset pricing model. *Journal of Financial Economics, 115(1)*, 1–24.

Barlas, Y. (1996). Formal aspects of model validity and validation in system dynamics. *System Dynamics Review, 12(3)*, 183–210.

Bengen, W. P. (1994). Determining withdrawal rates using historical data. *Journal of Financial Planning, 7(4)*, 171–180.

Board of Governors of the Federal Reserve System (2026). Financial Accounts of the United States and Distributional Financial Accounts. Washington, DC. https://www.federalreserve.gov/releases/z1/current/recent_developments.htm (accessed 28 July 2026).

Bond, P., Edmans, A., & Goldstein, I. (2012). The real effects of financial markets. *Annual Review of Financial Economics, 4*, 339–360.

Bundesministerium der Finanzen (2026). Entwurf eines Gesetzes zur Einführung einer Frühstartrente (FruehStRG). Referentenentwurf, 21 July 2026. https://www.bundesfinanzministerium.de/Content/DE/Gesetzestexte/Gesetze_Gesetzesvorhaben/Abteilungen/Abteilung_IV/21_Legislaturperiode/2026-07-21-FruehStRG/0-Gesetz.html (accessed 28 July 2026).

Börsch-Supan, A., & Wilke, C. B. (2004). The German public pension system: How it was, how it will be. NBER Working Paper 10525.

Börsch-Supan, A., Ludwig, A., & Winter, J. (2006). Ageing, pension reform and capital flows: A multi-country simulation model. *Economica, 73(292)*, 625–658.

Castellacci, F. (2018). Co-evolutionary growth: A system dynamics model. *Economic Modelling, 70*, 272–287. https://doi.org/10.1016/j.econmod.2017.11.010.

Charoenwong, B., Morck, R., & Wiwattanakantang, Y. (2021). Bank of Japan equity purchases: The (non-)effects of extreme quantitative easing. *Review of Finance, 25(3)*, 713–743.

Cieslak, A., & Vissing-Jorgensen, A. (2021). The economics of the Fed put. *Review of Financial Studies, 34(9)*, 4045–4089.

Cui, J., de Jong, F., & Ponds, E. (2011). Intergenerational risk sharing within funded pension schemes. *Journal of Pension Economics and Finance, 10(1)*, 1–29.

De Long, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1990). Noise trader risk in financial markets. *Journal of Political Economy, 98(4)*, 703–738.

Deutscher Bundestag (2025). Bundestag beschließt das Rentenpaket. Textarchiv, December 2025. https://www.bundestag.de (accessed 28 July 2026).

Deutscher Bundestag (2026a). Gesetz zur Reform der geförderten privaten Altersvorsorge (Altersvorsorgereformgesetz). Berlin, March 2026.

Deutscher Bundestag (2026b). Abschließende Beratungen zur Altersvorsorge and status of the 2024 Generationenkapital proposal. Textarchiv, March 2026. https://www.bundestag.de/dokumente/textarchiv/2026/kw13-de-abschliessende-beratungen-1143114 (accessed 28 July 2026).

Deutsches Aktieninstitut (2026). Aktionärszahlen 2025/2026: 14,1 Millionen Aktiensparerinnen und Aktiensparer. Frankfurt am Main. https://www.dai.de (accessed 28 July 2026).

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance, 25(2)*, 383–417.

Forrester, J. W. (1961). *Industrial Dynamics*. Cambridge, MA: MIT Press.

Gabaix, X., & Koijen, R. S. J. (2022). In search of the origins of financial fluctuations: The inelastic markets hypothesis. NBER Working Paper 28967 (revised version).

Gallup (2025). What percentage of Americans own stock? Gallup Economy and Personal Finance survey. https://news.gallup.com/poll/266807 (accessed 28 July 2026).

GAO (2006). Baby boom generation: Retirement of baby boomers is unlikely to precipitate dramatic decline in market returns, but broader risks threaten retirement security. GAO-06-718. Washington, DC: US Government Accountability Office.

Gollier, C. (2008). Intergenerational risk-sharing and risk-taking of a pension fund. *Journal of Public Economics, 92(5–6)*, 1463–1485.

Greenwood, R., & Shleifer, A. (2014). Expectations of returns and expected returns. *Review of Financial Studies, 27(3)*, 714–746.

Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *American Economic Review, 70(3)*, 393–408.

Haddad, V., Huebner, P., & Loualiche, E. (2025). How competitive is the stock market? Theory, evidence from portfolios, and implications for the rise of passive investing. *American Economic Review, 115(3)*, 975–1018.

Investment Company Institute (2026). The US Retirement Market, First Quarter 2026; 2026 Investment Company Fact Book. Washington, DC. https://www.ici.org/statistical-report/ret_26_q1 (accessed 28 July 2026).

Miller, M., Weller, P., & Zhang, L. (2002). Moral hazard and the US stock market: Analysing the 'Greenspan put'. *Economic Journal, 112(478)*, C171–C186.

Poterba, J. M. (2001). Demographic structure and asset returns. *Review of Economics and Statistics, 83(4)*, 565–584.

Poterba, J. M. (2004). The impact of population aging on financial markets. NBER Working Paper 10851.

S&P Dow Jones Indices (2026). S&P 500 index data and constituent weights, 30 June 2026. https://www.spglobal.com/spdji/en/indices/equity/sp-500/ (accessed 28 July 2026).

Sammon, M. (2024). Passive ownership and price informativeness. *Management Science*, https://doi.org/10.1287/mnsc.2023.00836.

Sterman, J. D. (2000). *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Boston: Irwin/McGraw-Hill.

Swedish Pensions Agency (2026). The Swedish pension system and the premium pension. https://www.pensionsmyndigheten.se/other-languages/english-engelska/english-engelska/the-swedish-pension-system (accessed 28 July 2026).

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Author contributions

Martin Prause: Conceptualization, Methodology, Software, Formal analysis, Investigation, Validation, Visualization, and Writing.

Data and code availability

The Python source code, figure-generation scripts, parameter values, fixed seeds, and tidy scenario-path files needed to reproduce the deterministic results, Monte Carlo summaries, global parameter sample, and regime maps are provided as Supplementary Information with the submission. Figures were generated with the accompanying Python scripts.

Use of generative AI

During manuscript preparation, the author used OpenAI ChatGPT for language editing, structural reframing, consistency checking, and document formatting. The author reviewed and edited all outputs and takes responsibility for the final manuscript.

Appendix: Numerical specification, notation, and calibration

The implementation advances an ordered quarterly Euler scheme from 2026 to 2080 ($\Delta t = 0.25$ years). The PAYG component is held at 48% of wages through 2031 and then declines smoothly toward 42%. This stylized demographic path creates the retirement-income gap that the funded pillar is intended to offset. The funded pillar reaches maturity around 2052, when participants with full contribution histories begin to retire; the passive-ownership trend has its midpoint in 2036.

Mom follows $dMom/dt = [\ln 2/(1\ yr)](r^p - Mom)$. Between impulses, S follows $dS/dt = -0.9S$; deterministic impulses arrive in 2033 and 2047 with sizes 0.28 and 0.45. Stochastic arrivals use hazard $0.06\{1 + 4[\ln(P/V)]_+\}\ yr^{-1}$ and lognormal sizes with $\ln z \sim N(\ln 0.15, 0.4^2)$. Endogenous bursts occur if $P/V > 1.45 + 0.35T$, subject to a six-year refractory period. Panic redemptions use $q = \rho(dd - 0.10)/0.2$ above a 10% drawdown, multiplied by the credibility shield and a deep-value fade. At a successful put exit T rises by $\gamma_T = 0.40$; otherwise it decays linearly at $\delta_T = 0.015$ per year until it reaches zero.

Mom summarizes recent price returns with a one-year half-life and therefore represents momentum demand. S is a transitory risk-off shock stock that decays within roughly one year. In the deterministic comparisons, identical shocks occur in 2033 and 2047, so differences across scenarios reflect model structure rather than shock realizations. The Monte Carlo experiment instead draws stochastic arrival times and sizes.

Endogenous corrections occur when P/V exceeds $1.45 + 0.35T$. Higher backstop credibility T therefore delays correction and permits a larger valuation gap before the threshold binds. Panic redemptions begin after a 10% drawdown, increase with drawdown depth, and are moderated by credibility and deep undervaluation. A successful intervention raises T by 0.40; otherwise T decays by 0.015 per year. This

mechanism captures the possibility that effective short-run support weakens subsequent market discipline.

D records support financing in currency units; B = D/(Y × 1 year) expresses the same ledger relative to annual GDP. GDP normalization prevents a nominal cap from tightening mechanically as the economy grows, and each quarterly update enforces the identity between D and B. The (r_D − g_Y) term is the carrying cost relative to GDP growth. State purchases acquire G/P market units and receive dividends. Consequently, the marked-to-model net position can remain near zero even when gross financing reaches the cap. This is a distinction between valuation and balance-sheet exposure, not evidence of costless intervention.

Baseline S3 permits purchases but not disposals, so B rises monotonically toward the cap. S3b adds sales of up to 2% of the market per year when P/V exceeds 1.15 and credits the proceeds against B. The central uncertainty is whether a large public disposal affects market prices. With off-market placement, the cap is reached around 2071 rather than 2067. With full market impact (S3bi), public selling depresses prices, reactivates support, and advances the cap to 2061. The two variants bracket this implementation assumption. Cumulative gross purchases are reported separately so that sale proceeds do not obscure intervention scale.

S4 combines countercyclical contributions with flexible payouts. When valuations are elevated, half of contributions enter a cash buffer; when valuations are low, the buffer is deployed subject to annual limits. Payout adjustment is bounded between −35% and +10%, reducing forced sales during downturns. S4 observes latent fundamental value V, which is unavailable to a real policymaker. It is therefore a proof of mechanism rather than a directly implementable rule; an operational version would require observable valuation signals, decision lags, and robustness bands. Bounds on returns, prices, and nonnegative stocks are numerical safeguards for extreme paths rather than economic assumptions.

Symbol	Plain-language meaning and role in the formulas	Unit
$t, \Delta t, 1\ yr$	Calendar time, numerical time step, and the one-year factor used to convert annual GDP (a flow) into a stock-sized denominator.	years
$A, \bar{A}, A_{buf}$	Pension assets; smoothed asset reference used to set payouts; countercyclical buffer assets in S4.	EUR trn
P, V	Market-price index and fundamental-value index. P/V > 1 means the market price is above fundamentals.	index; dimensionless
$M_0, M = M_0 P$	Initial domestic market value and its current market value. Multiplying the fixed scale M_0 by P converts the price index into EUR trillion.	EUR trn
C, C^A, b	Total policy-set contributions; contributions entering pension assets; buffer diversion, with C^A=C−b.	EUR trn/yr
$X, q, qA, X/A$	Scheduled payout flow; panic-withdrawal rate; panic withdrawals; and annual payout pressure relative to pension assets.	EUR trn/yr; q: 1/yr; X/A: 1/yr
F, G	Net domestic-market flow (buying minus selling) and signed state trade. G>0 is a purchase; G<0 is a disposal.	EUR trn/yr
$h, own = hA/M$	Home-market portfolio share and the share of the domestic market owned by pension assets.	shares, 0–1
$Y, dep = A/(Y \cdot 1\ yr)$	Annual real GDP and pension-system dependency: pension assets relative to one year of GDP.	Y: EUR trn/yr; dep: share
Mom, χ	Smoothed recent price return and the coefficient translating momentum into the flow χ·Mom·M.	Mom: 1/yr; χ: dimensionless
S, κ	Risk-off shock state and its loading in the selling flow κSM.	S: dimensionless; κ: 1/yr

Symbol	Plain-language meaning and role in the formulas	Unit
μ	Flow multiplier: how strongly net flow relative to market size, F/M, changes the annual price-growth rate.	dimensionless
π	Price-inelastic/passive ownership share. A higher π weakens valuation-based price correction.	share, 0–1
λ_0, λ_{eff}	Baseline and effective speed at which prices move back toward fundamentals; λ_eff is reduced by passive ownership and put credibility.	1/yr
$\lambda_{deep}, \delta_{deep}$	Additional deep-undervaluation correction and the log-gap threshold at which it activates.	1/yr; log units
ν_{iss}, ν_{eff}	Baseline and effective response of valuation-sensitive secondary-market selling; a live put can raise the effective response.	1/yr
$\theta_{iss}, \theta_{mis}$	Log-valuation thresholds that activate, respectively, extra market supply and the misallocation drag on fundamentals.	dimensionless log gaps
g, d, r_{world}	Baseline fundamental-growth rate, dividend yield, and exogenous world-market return.	1/yr
$r^{\mathrm{P}}, r^{\mathrm{V}}, r^{port}$	Annual market-price growth, annual fundamental-value growth, and the pension portfolio's annual return.	1/yr
g_Y, r_D	Annual GDP growth and the annual financing/carrying rate on state support.	1/yr
φ	Strength of the one-sided reduction in fundamental growth when overvaluation exceeds θ_mis.	1/yr
$dd, d * (dep)$	Drawdown from the previous price peak and the dependency-dependent drawdown threshold that activates support.	shares, 0–1
$D, B = D/(Y \cdot 1\ yr), B_max$	Support-financing amount; support financing relative to one year of GDP; and the fiscal capacity limit.	D: EUR trn; B and B_max: shares
SH	State holdings measured in market units. Their current value is SH·P.	market units
DIV, NET	Cumulative dividends on state holdings and the state's net position NET=SH·P+DIV−D.	EUR trn
T, γ_T, δ_T	Put credibility (0 to 1), its upward jump after a successful exit, and its gradual decay rate.	T, γ_T: dimensionless; δ_T: 1/yr
$[z]_+$	Positive-part operator max(z,0): the enclosed channel is active only when z is positive.	same unit as z
$ln(P/V), ln(V/P)$	Log valuation gaps. They are zero at fair value, positive in the direction indicated by the numerator, and negatives of each other.	dimensionless
$P^+/P^-, T^+/T^-, S^+/S^-$	Values immediately after (+) and immediately before (−) a discrete cap or resolution event.	same unit as the state
d/dt	Time derivative: the instantaneous annual rate of change of a stock or index.	variable per year
RR	Model replacement-rate index used later in the paper; used in the results sections (Section 5).	share of wage
W, R, Ω	Contributors, pensioners, and the annual wage bill used in contribution, payout, and adequacy calculations.	W, R: million people; Ω: EUR trn/yr
c, m	Policy contribution rate and funded-pillar maturity ramp.	shares, 0–1
$\omega, \omega_max, x_cap$	Current payout rate, mature-system withdrawal rate, and institutional payout ceiling in equation (2).	1/yr
$payg, \tau_B$	PAYG benchmark component and consolidation-drag coefficient in the replacement-rate index.	dimensionless shares

Table 2 Principal notation: plain-language meaning, role in the formulas, and units for the main symbols in equations (1)-(12) and Appendix

Parameter	Value	Meaning	Anchor
μ	5 (0.2 in S1; 2–8 in SA)	Flow multiplier dM/M per F/M	Gabaix & Koijen (2022)
λ₀	0.35/yr (0.6 in S1)	Arbitrage / price-discovery strength	Mispricing half-life 2–6 yrs
λ_deep, δ_deep	0.70/yr for ln(V/P) > 0.28	Deep-value correction independent of π	Floor calibration; not a direct estimate

Parameter	Value	Meaning	Anchor
π(t)	0.35→0.55 trend + 0.60·own; cap 0.85	Inelastic (passive or mandated) share	Trend anchored; ownership loading judgmental
χ	0.10 (1-yr EMA)	Momentum flow coefficient	De Long et al. (1990); Greenwood & Shleifer (2014)
ν	0.010/yr per unit log overvaluation; ×(1+2T) under live put	Valuation-sensitive secondary-market supply rate	Reduced-form calibration; primary issuance is not modeled
φ	0.045/yr (0–0.09 in SA)	Misallocation drag on fundamental growth	Real-effects literature; magnitude judgmental
g, d, r_world	2.0%, 2.5%, 5.0% real	Fundamental growth, dividend yield, world return	Long-run equity aggregates
β_w	0 (0.5 stress variant)	World-return correlation with domestic excess	Generosity check on diversification
c(t)	0.5%→4.5% of wage bill by 2035 (2–7% SA)	Funded-pillar contribution ramp	Commission 2% pillar + depots + bAV
ω_max	5.5%/yr on smoothed assets	Mature withdrawal rate	Bengen (1994)-style drawdown
h	0.6 (0.4–0.8 SA)	Home-market share of flows	Receiving-market concentration parameter
M_0	1.0×GDP (0.6 variant)	Modeled receiving-market scale	Benchmark 1×GDP; 0.6×GDP is a smaller-market stress test
ρ, dd*	0.30/yr above 10% drawdown	Panic redemption intensity	Crisis redemption episodes
κ	0.06/yr per unit shock stock	Risk-off outflow loading κSM	Calibrated so S0 shocks stay ordinary
Shocks	2033 (0.28), 2047 (0.45); stochastic in MC	Global confidence shocks	2000/2008-style spacing
Put trigger	dep>0.15; d* declines 20%→10%	State-dependent rescue threshold	Reduced-form political rule
Put scale	3% of M per yr while active	Government purchase intensity	Facility-scale order (TARP ≈5% GDP authorized)
S3b disposal	≤2% of M per yr when P/V>1.15	Symmetric sell rule (benchmark: off-market disposal; market-impact variant in §5.3)	BoJ disposal logic
B_max	12% of one year's GDP (6–25% SA)	Quarterly event trigger for support-financing ratio; the final active step may overshoot	BoJ order of magnitude
Resolution	jump (0.72V + shock) | close | unwind 10%/yr	Loss-given-failure maps	Imposed stress maps; varied in Table 6
ψ, shield	0.85, 0.75	Credibility effects on correction and panic	Judgmental behavioral calibration
τ_B	0.15 (0–0.30 SA)	Consolidation drag per unit of B	Stylized incidence
PAYG path	48% through 2031, then toward 42%	PAYG benchmark component of RR	Rentenpaket 2025; stylization
Demography	W: 39.2→31m; R: 21.4→25.5m	Contributors, pensioners	Destatis-style stylization

Table 3 Parameters, values, and calibration anchors

Outcome (2026–2080)	S0	S1	S2	S3	S3b	S4
Peak P/V (year)	1.15 (2026)	1.15 (2026)	1.44 (2046)	1.44 (2046)	1.44 (2046)	1.24 (2043)
Trough P/V	0.91	0.99	0.70	0.62	0.62	0.89
Maximum drawdown	8.9%	3.6%	47.2%	38.4%	32.4%	6.7%
Years in crisis (dd>25%)	0.0	0.0	16.8	5.3	3.3	0.0
Replacement-rate index 2050	44.4%	46.9%	46.8%	46.6%	46.8%	46.0%
Replacement-rate index 2065	45.7%	56.2%	48.6%	52.8%	53.3%	51.5%
Replacement-rate index 2080	45.3%	58.3%	50.2%	51.6%	51.6%	55.0%
Pension assets 2080 (€ trn)	1.58	8.48	4.74	6.11	6.05	8.38
Support-financing ratio B, 2080 (% GDP; signed in S3b)	0.0%	0.0%	0.0%	12.8%	12.6%	0.0%
State net position 2080 (% GDP)	0.0%	0.0%	0.0%	0.8%	0.9%	0.0%
Mean price alignment	0.94	0.99	0.77	0.79	0.81	0.89
Cap reached (year)	n.a.	n.a.	n.a.	yes (~2067)	yes (~2071)	n.a.

Table 4 Key outcomes of the deterministic scenario runs, including equation-consistent state positions and fully credited S3b disposals

Monte Carlo statistic	S0	S1	S2	S3	S4
Median RR index 2080 (10–90%)	45.2% (45.1%–45.2%)	58.3% (58.3%–58.3%)	51.8% (51.2%–52.7%)	51.5% (51.3%–51.8%)	54.6% (54.0%–55.0%)
Median max drawdown	3.2%	3.6%	44.1%	36.4%	3.8%
P(drawdown > 25%)	0.0%	0.0%	100.0%	97.2%	0.0%
P(drawdown > 40%)	0.0%	0.0%	86.6%	14.8%	0.0%
Observed cap-hit share by 2080	n.a. (no put)	n.a. (no put)	n.a. (no put)	100.0% (500/500)	n.a. (no put)
Median gross financing 2080	0.0%	0.0%	0.0%	13.2%	0.0%
Median price alignment	0.93	0.99	0.76	0.79	0.89

Table 5 Monte Carlo summary (500 runs per scenario; common random-number innovations, state-dependent realized arrivals)

Resolution variant	RR index 2080	Max drawdown	B at 2080	Net position 2080	Cap year
jump, $\tau_B = 0.0$	53.5%	38.4%	12.8%	0.8%	2067
jump, $\tau_B = 0.15$	51.6%	38.4%	12.8%	0.8%	2067
jump, $\tau_B = 0.3$	49.7%	38.4%	12.8%	0.8%	2067
close, $\tau_B = 0.0$	53.1%	29.8%	12.8%	1.1%	2067
close, $\tau_B = 0.15$	51.2%	29.8%	12.8%	1.1%	2067
close, $\tau_B = 0.3$	49.3%	29.8%	12.8%	1.1%	2067
unwind, $\tau_B = 0.0$	53.1%	34.6%	6.1%	−0.5%	2067
unwind, $\tau_B = 0.15$	52.2%	34.6%	6.1%	−0.5%	2067

Resolution variant	RR index 2080	Max drawdown	B at 2080	Net position 2080	Cap year
unwind, $\tau_B = 0.3$	51.2%	34.6%	6.1%	−0.5%	2067
restore (rebuilt put), $\tau_B = 0.15$	52.3%	38.4%	9.6%	n.m.	2067
S4 (comparison, no put)	55.0%	6.7%	0.0%	0.0%	n.a.

Table 6 Resolution-map and fiscal-drag sensitivity (S3), using the equation-consistent state ledger: the S3 < S4 terminal ranking survives every variant. In the restore variant, post-failure amortization retires financing without appearing as a cost in the ledger; its net position is therefore not meaningful (n.m.)

S3 variant	Cap-hit share	Median max drawdown	Median RR index 2080	Median net position
benchmark	100%	36.4%	51.5%	0.9%
mu=2	0%	4.2%	55.1%	0.0%
mu=8	100%	61.0%	51.2%	−0.9%
h=0.4	2%	10.6%	55.0%	1.8%
h=0.8	100%	46.6%	49.5%	−0.1%
lam0=0.6	43%	11.0%	52.7%	2.7%
c_max=0.02	0%	9.8%	48.6%	0.0%
c_max=0.07	100%	57.0%	56.4%	−1.8%
B_max=0.25	54%	11.9%	50.4%	1.8%
M0=0.6	100%	62.2%	50.5%	−2.7%
beta_w=0.5	99%	32.8%	50.3%	1.2%
S3b (off-market benchmark)	100%	33.1%	51.5%	1.0%

Table 7 Stochastic cap-hit incidence and outcomes across parameter variants (500 runs each), with equation-consistent net positions

Retirement year	S1 funded RI	S2 funded RI	S3 funded RI	S4 funded RI
2048	5.3%	5.1%	5.1%	5.1%
2056	9.1%	6.7%	8.2%	8.8%
2064	14.2%	11.0%	11.7%	12.5%
2072	18.8%	15.4%	13.1%	16.9%
2080	19.8%	18.8%	15.0%	17.6%

Table 8 Synthetic cohort outcomes: funded replacement index at retirement (5% payout rule), by retirement year

Table 9 Nested-model and functional-form tests, deterministic paths

Specification	Peak P/V	Max. drawdown	RR index, 2080	S3 cap year
N0 Accounting only; feedbacks off	1.14	1.0%	58.4%	n.a.
N1 + flow-price channel	1.25	10.2%	55.3%	n.a.
N2 + endogenous inelastic ownership	1.29	18.7%	54.8%	n.a.
N3 + momentum and panic	1.38	30.5%	53.9%	n.a.
N4 Full S2 (+ real-effects drag)	1.44	47.2%	50.2%	n.a.
N5 Full S3 (+ state put)	1.44	38.4%	51.6%	~2067

Specification	Peak P/V	Max. drawdown	RR index, 2080	S3 cap year
Linear contribution ramp, 2027–2035	1.44	47.3%	50.2%	~2067
Level-ratio rather than log valuation gap	1.46	37.6%	51.8%	~2071
Soft rather than hard valuation thresholds	1.45	47.5%	50.2%	~2068

Notes: Nested rows add mechanisms cumulatively to S2; N5 reports S3. Functional-form rows report S2 outcomes and the corresponding S3 trigger year. All deterministic comparisons reuse the same 2033 and 2047 shocks. The endogenous-burst switch is active in the full model but does not bind on the benchmark deterministic path because peak P/V remains below its 1.45 trigger.

Reproducibility. A self-contained Python implementation with fixed seeds reproduces the deterministic tables, 500-run summaries, Latin-hypercube sample, and regime maps. Common innovations are reused across scenarios, while state-dependent hazards can shift realized arrival dates. Section 4.6 documents high stability of terminal levels and failure incidence, with lower stability of exact failure dates and maximum drawdowns. Code and data files accompany the submission.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- simcore.zip