

A Multiscale Hierarchy of $\times 10$ – $\times 3$ – $\times 12$ Scaling Recurrence Domains Across Earth History

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A Multiscale Hierarchy of $\times 10$ – $\times 3$ – $\times 12$ Scaling Recurrence Domains Across Earth History

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Abstract

Recurrent variability has been reported at many geological timescales, but whether these recurrences constitute a coherent cross-scale hierarchy remains unresolved. Here, this study developed a seven-domain hierarchy linked by the frozen adjacent multipliers $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10 \rightarrow \times 3 \rightarrow \times 12$, with empirical analyses focused on the P1–P5 domains. This study combined dense scans of continuous climate and geophysical records, blind phase-coherence searches of chronostratigraphic boundaries, chronology perturbation tests, nested global null models, multiplicity correction, frozen out-of-sample transfer tests, and theory-free inverse searches across independent Earth-history databases. Major recurrence centers were recovered near the defined domains, including approximately 0.098–0.120 Myr for P1, 1.136 Myr and neighboring continuous-record centers for P2, 3.507 Myr for P3, 47.09 and 57.85 Myr for P4, and 438.2 and 501.0 Myr for P5. Across adjacent scales, the GSSP-restricted peak-height hierarchy showed significant multiplicative alignment after Holm correction, and mass-weighted abstract centers retained significant integrated multi-domain support. P6 could not be tested independently, although propagation from the observed P5 peak-height center produced a diagnostic endpoint compatible with terrestrial and Solar-System age constraints. Frozen out-of-sample transfers were heterogeneous and predominantly non-significant, limiting claims of independent replication, whereas post-OOS and theory-free analyses generated additional candidate scales and ratio chains that remain exploratory. Together, these results support a statistically non-random organization of recurrence domains across geological scales, while not establishing a universal geological clock or a common causal mechanism.

Keywords: recurrence domains; multiscale hierarchy; Earth history; chronostratigraphy; phase coherence; out-of-sample validation

Introduction

Earth-system variability is expressed across a broad range of temporal scales. At Quaternary timescales, recurrent changes in Earth's orbital geometry provide a well-established framework for understanding climate variability associated with precession, obliquity, and eccentricity. The recognition of these astronomical signals in marine sedimentary records established the foundations of modern cyclostratigraphy and astrochronology, which are now widely used to refine geological timescales and investigate interactions among orbital forcing, climate, ice sheets, and sedimentary systems^{1,2}. A prominent example is the Mid-Pleistocene Transition, during which glacial variability shifted from a predominantly approximately 41-kyr pattern to higher-amplitude approximately 100-kyr cycles without a corresponding fundamental change in orbital forcing³. This transition demonstrates that recurrent climate variability can arise from state-dependent amplification and feedback within the coupled ice-sheet–ocean–atmosphere–carbon system rather than from external forcing alone. Long and highly resolved records, including the LR04 benthic $\delta^{18}\text{O}$ stack, provide a quantitative basis for examining these changes over several million years⁴.

Recurrent patterns have also been proposed at much longer geological timescales. Candidate periodicities have been reported in extinction, fossil diversity, tectonic and magmatic activity, sea-level change, and supercontinent assembly and breakup⁵⁻⁷. However, long-period claims are intrinsically difficult to evaluate. Geological records are finite, unevenly distributed, chronologically uncertain, and shaped by preservation and sampling biases. Events collected in a common database may also share age models, stratigraphic constraints, or geological causes and therefore cannot automatically be treated as statistically independent observations. Spectral leakage, selection of favorable analysis windows, examination of multiple candidate periods, and post hoc choice of the strongest peak can further increase the probability of apparently compelling but nonreproducible patterns^{8,9}. Consequently, a preferred peak must be distinguished from a recurrence domain that is robust to chronological uncertainty, and both must be distinguished from a signal that remains significant after the complete period-search procedure is reproduced under an appropriate null model.

The International Chronostratigraphic Chart provides a standardized hierarchical representation of Earth history through Eonothems, Erathems, Systems, Series, and Stages¹⁰. These units were established for global stratigraphic correlation rather than periodicity analysis, but their numerical boundaries form nested sets of discrete events at different temporal resolutions. The chart therefore provides a common chronological scaffold for comparing recurrence structures from the Myr scale to the scale of major divisions of Earth history. Continuous proxy records provide complementary evidence at shorter timescales. The Cenozoic Global Reference benthic foraminifer carbon and oxygen isotope Dataset (CENOGRID) provides an astronomically dated climate framework for the past 66 Myr¹¹, while orbital solutions and Quaternary records of benthic $\delta^{18}\text{O}$, temperature, sea level, ice volume, and atmospheric CO_2 characterize variability at kyr to lower-Myr scales^{4,12}. Together, these records make it possible to compare recurrence domains across datasets representing different components of the Earth system.

Most previous periodicity studies have focused on a particular record, geological interval, or

proposed cycle. Less attention has been given to whether preferred timescales at different levels of the geological record form a coherent multiplicative hierarchy. Addressing this question requires more than comparing individually favorable peaks. In Song dynasty China, Shao Yong developed a multilevel temporal system in which successive scales were constructed exclusively using $\times 12$ and $\times 30$ multipliers. But, this numerological-cosmological system was not formulated as a theory of geography, geology, or Earth-system dynamics.

Here, I developed a unified framework to evaluate recurrent timescale domains across Earth system history: P1 at 0.1296 Myr, P2 at 1.296 Myr, P3 at 3.888 Myr, P4 at 46.656 Myr, P5 at 466.56 Myr, P6 at 1,399.68 Myr, and P7 at 16,796.16 Myr with hierarchy $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10 \rightarrow \times 3 \rightarrow \times 12$. P1–P5 were evaluated using continuous climate and geochemical records, International Chronostratigraphic Chart boundaries, dense blind scans, timing-uncertainty propagation, and nested empirical null models. Cross-scale tests assessed whether independently identified recurrence centers retained the predefined multiplier structure, while competing rules and distribution-level abstract centers evaluated the dependence of the result on peak selection. The analytical hierarchy was then frozen before evaluation in external extinction, impact, large igneous province, and zircon databases. Finally, a strict theory-free inverse search was used to determine whether related periods or ratios could be rediscovered without supplying the predefined hierarchy. This design was intended to distinguish reproducible recurrence domains, cross-scale organization, partial external support, exploratory alternatives, and conceptual extrapolations within a single evidence architecture.

Results

A predefined multiscale framework guided a staged evaluation of recurrence domains across Earth history

I first established a hierarchical temporal framework spanning orbital-climate variability, stratigraphic organization, and deep-time Earth-system evolution (**Fig. 1**). The framework was constructed from nested scale relationships involving factors of 12 and 30. By resolving each 30-fold transition into successive 10- and 3-fold components, the resulting target sequence followed the ratio structure $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10 \rightarrow \times 3 \rightarrow \times 12$. Starting from 0.1296 Myr, this construction defined seven target domains: P1 = 0.1296 Myr, P2 = 1.296 Myr, P3 = 3.888 Myr, P4 = 46.656 Myr, P5 = 466.56 Myr, P6 = 1399.68 Myr, and P7 = 16796.16 Myr (**Fig. 1A,B**). These values were treated as predefined centers of temporal domains rather than assumed to be exact periodicities.

The data architecture was matched to the temporal resolution of each domain (**Fig. 1C,D**). P2–P6 were primarily interpreted through different hierarchical levels of the International Chronostratigraphic Chart (ICS). Stage boundaries were used for P2 and P3, System boundaries for P4, Erathem boundaries for P5, and Eonothem-scale divisions for the higher-order P6 interpretation. P3 was examined over both the complete 2.58–538 Ma interval and the better-resolved 2.58–66 Ma interval, whereas the ICS component of P2 was restricted to 2.58–66 Ma. P1 required continuous high-resolution records and was therefore evaluated using Quaternary climate and orbital datasets, including benthic $\delta^{18}\text{O}$, reconstructed temperature, sea level, ice volume, atmospheric CO_2 , and orbital eccentricity. Magnetochron boundaries, deep-time isotope records, and orbital data provided complementary evidence in the P2 domain. P7 was retained only as a conceptual upper-scale domain and was not subjected to empirical testing.

The study was organized into distinct evidential stages to prevent exploratory results from being interpreted as confirmatory evidence (**Fig. 1E**). Blind scans were used to discover candidate recurrence domains. The resulting candidate structure was then evaluated using a frozen joint test against an empirical null model. Generalization was assessed using frozen out-of-sample analyses without parameter retuning, while external databases were used for triangulation and evaluation of cross-record convergence and heterogeneity. Analyses conducted after the out-of-sample stage were classified as exploratory and were not promoted to confirmatory evidence.

Before implementing the full systematic scans, this study performed preliminary candidate-set comparisons to determine whether the proposed temporal domains showed any detectable alignment with stratigraphic boundaries and major Earth-history transitions (**Supplementary Figs. 1 and 2**). Each theoretical value was compared with a small set of neighboring rounded periods, while reported chronological uncertainties were propagated through repeated sampling. The normalized fit score quantified collective phase alignment after accounting for period length, whereas the probability of being selected as the best candidate quantified the stability of the

relative ranking under chronological perturbation.

These preliminary comparisons preferentially recovered periods located within or close to all four theory-defined P2–P5 domains. For P2, the 1.2-Myr candidate produced the highest median fit score from 1.0, 1.1, 1.2, 1.296, 1.3, 1.4, and 1.5 Myr ($\text{score_R} = 0.267$) and was selected in all uncertainty realizations, although its candidate-set global p value was 0.868. In the P3 domain, 3.5 Myr was preferred over the complete 2.58–538 Ma interval from 2.5, 3.0, 3.5, 3.888, 4.0, 4.5, and 5.0 Myr, with a median score_R of 0.174, a best-selection probability of 0.67, and a global p value of 0.327. Restriction to the better-resolved 2.58–66 Ma interval strengthened the relative preference for 3.5 Myr ($\text{score_R} = 0.478$; best-selection probability = 1.00; local $p \approx 0.016$), although the candidate-set global p value remained 0.093. Because the younger interval was a subset of the complete record, this result was treated as a temporal-window sensitivity analysis rather than an independent replication.

At longer timescales, the exact framework-derived P4 value of 46.656 Myr produced the highest fit score from 30, 40, 46.656, 50, and 60 Myr ($\text{score_R} = 0.513$) and was selected in 96% of the uncertainty realizations; the remaining 4% favored 60 Myr. Its local p value was approximately 0.054 and its candidate-set global p value was 0.233. Within the P5 domain, the neighboring 500-Myr candidate, rather than the exact 466.56-Myr value, produced the highest fit score from 300, 400, 466.56, 500, and 600 Myr ($\text{score_R} = 0.478$), was selected in all uncertainty realizations, and yielded a candidate-set global p value of 0.327.

Thus, the preliminary analyses did not uniformly recover the exact predefined values: the exact theoretical period was preferred for P4, whereas neighboring periods were preferred for P2, P3, and P5. Nevertheless, the preferred candidates consistently fell within the temporal neighborhoods specified by the framework. Their stable rankings provided an initial rationale for systematic investigation, but none reached candidate-set global significance at $p < 0.05$. These results were therefore treated as hypothesis-generating observations that motivated the subsequent dense period scans, frozen hierarchy test, out-of-sample evaluation, and external-data triangulation.

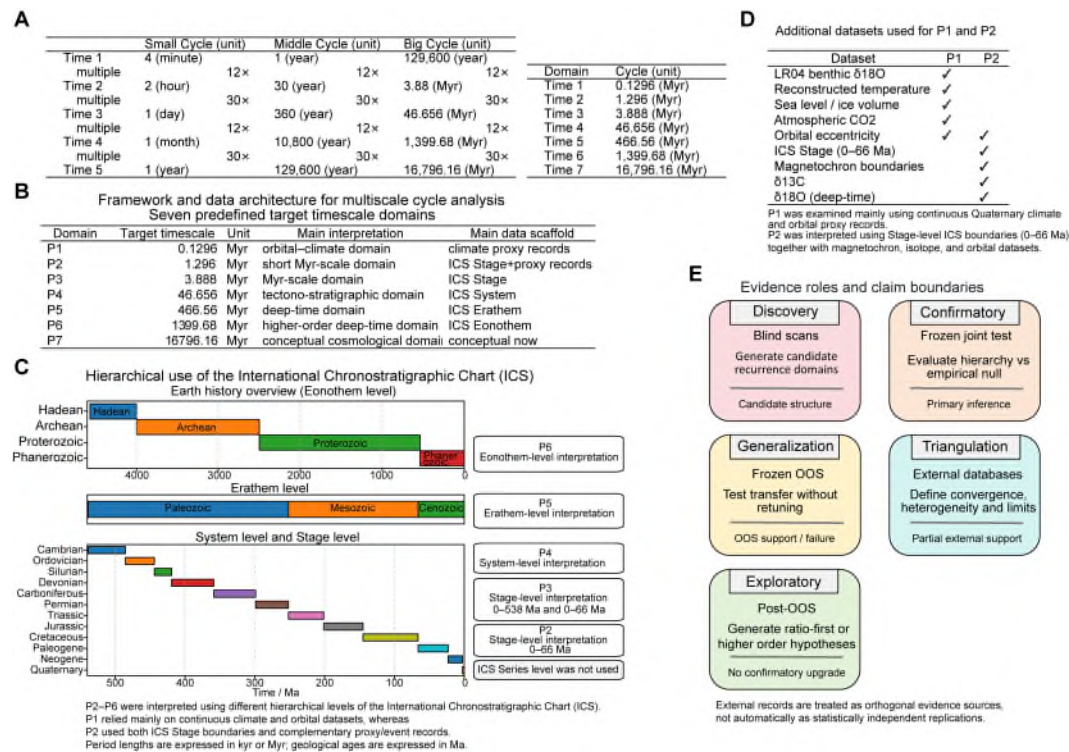


Figure 1 | Conceptual framework, data architecture, and evidential design for multiscale recurrence analysis across Earth history.

(A) Construction of the temporal framework from nested small-, intermediate-, and large-cycle scales. The underlying scale relationships involve alternating factors of 12 and 30. Resolving each 30-fold transition into successive factors of 10 and 3 generates the cross-domain sequence $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10 \rightarrow \times 3 \rightarrow \times 12$ and defines seven analytical domains from 0.1296 Myr to 16796.16 Myr.

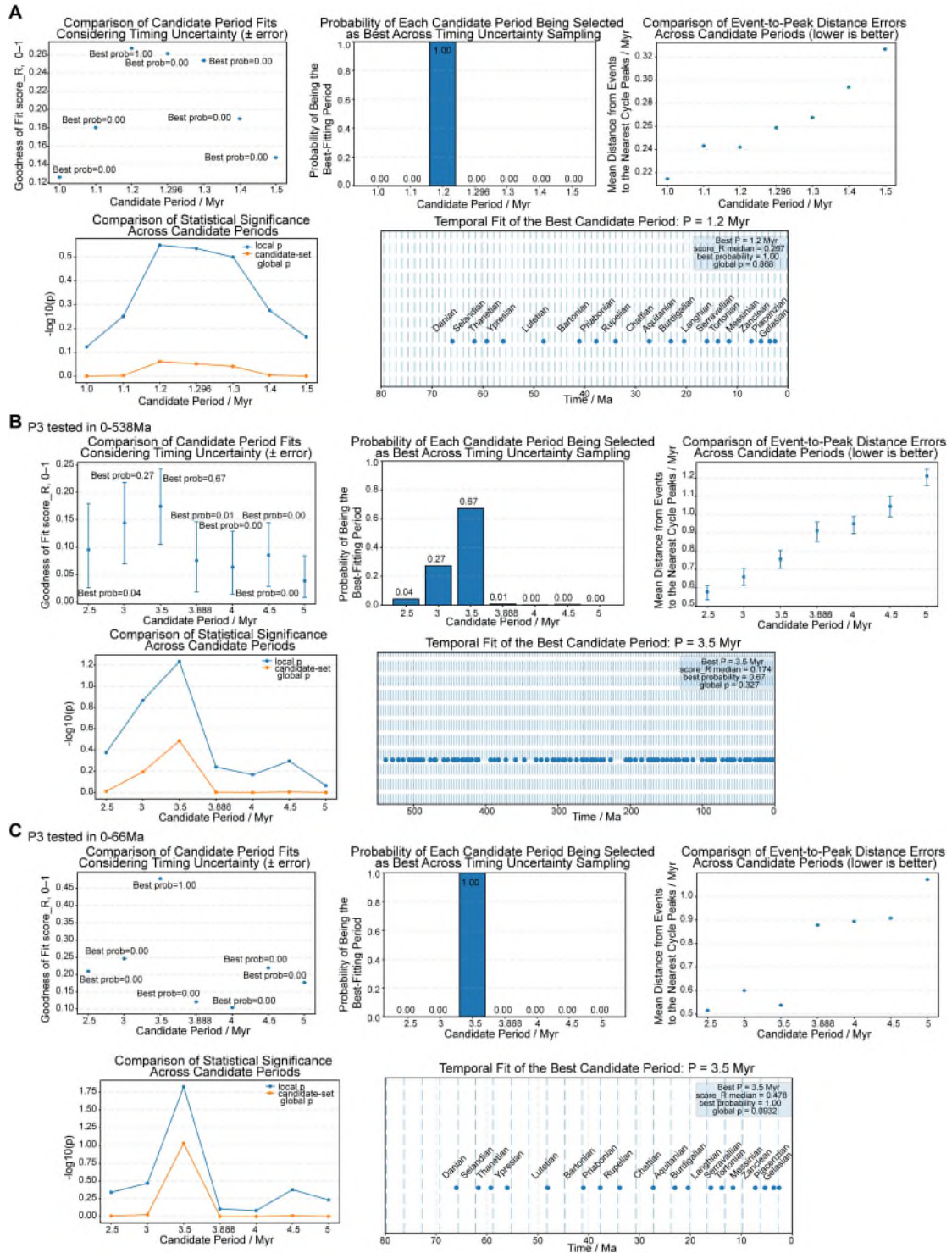
(B) Target timescales, interpretations, and principal data scaffolds for P1–P7. The predefined values represent centers of analytical domains and were not assumed to be exact final periodicities. P1 represents the orbital-climate domain; P2 and P3 represent lower- and intermediate-Myr domains; P4 and P5 represent tectono-stratigraphic and deep-time domains; and P6 and P7 represent higher-order conceptual domains.

(C) Hierarchical use of the International Chronostratigraphic Chart. Stage boundaries were used for P2 and P3, System boundaries for P4, Erathem boundaries for P5, and Eonothem-scale divisions for P6. P3 was examined over 0–538 Ma and 0–66 Ma, whereas the ICS analysis of P2 was restricted to 0–66 Ma. ICS Series boundaries were not used. Geological ages are expressed in Ma.

(D) Complementary datasets used for the P1 and P2 domains. P1 was evaluated mainly using continuous Quaternary climate and orbital records, including LR04 benthic $\delta^{18}\text{O}$, reconstructed temperature, sea level, ice volume, atmospheric CO_2 , and orbital eccentricity. P2 combined Cenozoic ICS Stage boundaries with magnetochron, deep-time isotope, and orbital records.

(E) Evidential roles and claim boundaries used throughout the study. Blind scans were used for discovery; the frozen joint hierarchy test provided the primary confirmatory inference; frozen out-of-sample analyses evaluated transfer without retuning; external databases provided triangulation; and post-OOS analyses were treated as exploratory. External records were considered orthogonal evidence sources but were not automatically assumed to constitute

statistically independent replications.



Supplementary Figure 1. Preliminary candidate-period comparisons in the P2 and P3 domains.

For each analysis, the five component plots show the distributions of normalized fit scores generated by timing-uncertainty sampling, the probability that each period was selected as the best-fitting candidate, the mean absolute distance from observed events to the nearest predicted

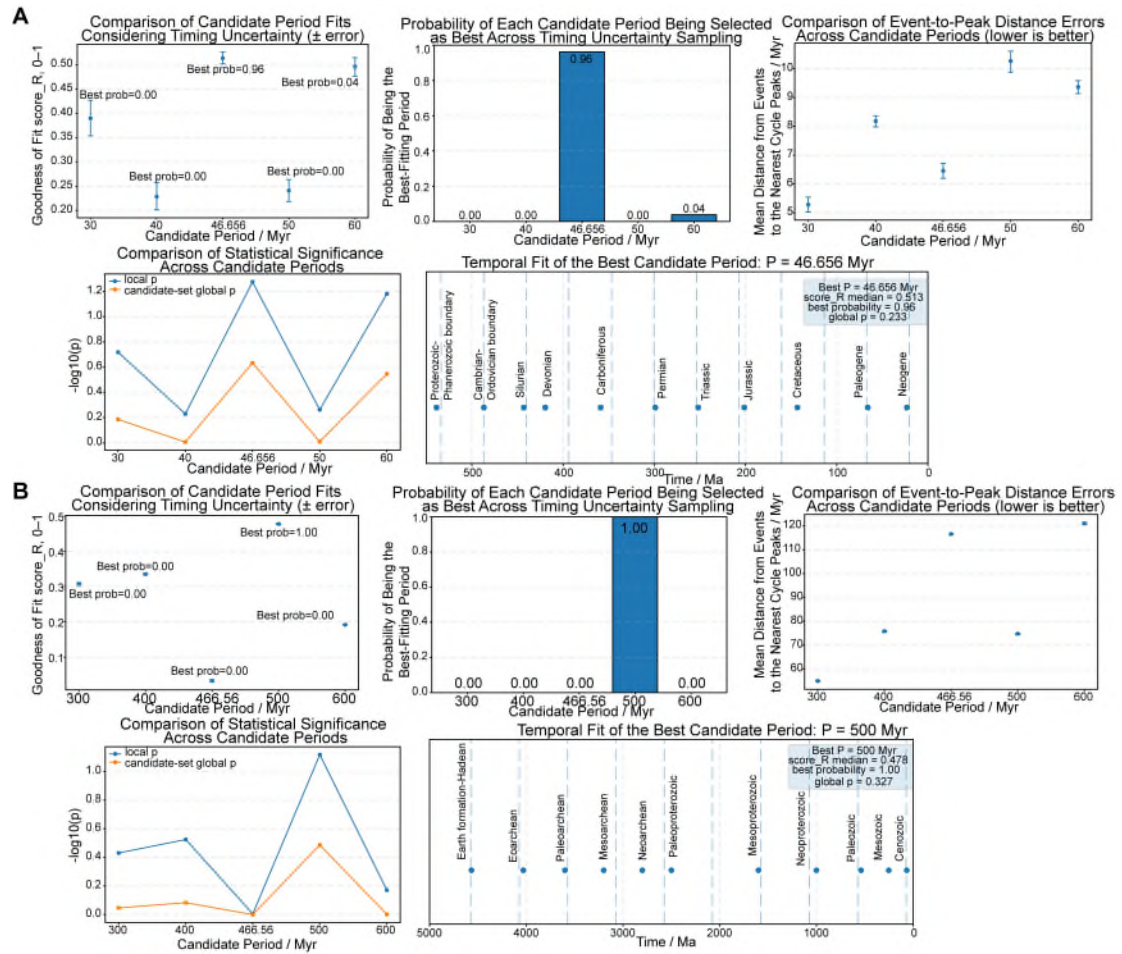
cycle peak, local and candidate-set global statistical significance expressed as $-\log_{10}(p)$, and the temporal projection of the preferred candidate.

(A) P2 comparison using Cenozoic ICS Stage boundaries from 66 Ma to 2.58 Ma. Candidate periods of 1.0, 1.1, 1.2, 1.296, 1.3, 1.4, and 1.5 Myr were evaluated. The 1.2-Myr candidate produced the highest median normalized fit score ($\text{score}_R = 0.267$) and was selected in all timing-uncertainty realizations. Its candidate-set global p value was 0.868. The temporal projection is displayed from 80 Ma to the present for visualization, although the analyzed boundaries were restricted to 2.58–66 Ma.

(B) P3 comparison over the complete 2.58–538 Ma interval. Candidate periods of 2.5, 3.0, 3.5, 3.888, 4.0, 4.5, and 5.0 Myr were tested. The 3.5-Myr candidate produced the highest median fit score ($\text{score}_R = 0.174$) and was selected in 67% of the timing-uncertainty realizations, followed by 3.0 Myr in 27%. The candidate-set global p value for 3.5 Myr was 0.327.

(C) Temporal-window sensitivity analysis of P3 using ICS Stage boundaries from 2.58–66 Ma. The 3.5-Myr candidate again produced the highest fit and was selected in all uncertainty realizations ($\text{score}_R = 0.478$). Its local p value was approximately 0.016 and its candidate-set global p value was 0.093. The axis extends to 80 Ma for visualization. Because the younger interval is contained within the full record, this analysis was treated as a temporal-window sensitivity analysis rather than an independent validation.

Points and error bars summarize fit-score or event-to-peak-distance distributions obtained by propagating chronological uncertainty. Absolute event-to-peak distance is shown descriptively because shorter candidate periods produce more closely spaced predicted peaks. Blue dashed lines in the temporal projections indicate fitted cycle peaks, and blue circles indicate the analyzed stratigraphic boundaries.



Supplementary Figure 2. Preliminary candidate-period comparisons in the P4 and P5 domains.

For each analysis, the five component plots show the distributions of normalized fit scores generated by timing-uncertainty sampling, the probability that each period was selected as the best-fitting candidate, the mean absolute distance from observed events to the nearest predicted cycle peak, local and candidate-set global statistical significance expressed as $-\log_{10}(p)$, and the temporal projection of the preferred candidate.

(A) P4 comparison using major System-level stratigraphic transitions. Candidate periods of 30, 40, 46.656, 50, and 60 Myr were tested. The framework-derived 46.656-Myr candidate produced the highest median normalized fit score (score_R = 0.513) and was selected as the best-fitting candidate in 96% of the timing-uncertainty realizations; the remaining 4% favored 60 Myr. The local p value for 46.656 Myr was approximately 0.054 and the candidate-set global p value was 0.233.

(B) P5 comparison using major deep-time Earth-history transitions. Candidate periods of 300, 400, 466.56, 500, and 600 Myr were evaluated. The 500-Myr candidate produced the highest median normalized fit score (score_R = 0.478) and was selected in all timing-uncertainty realizations. Its candidate-set global p value was 0.327. Thus, the preferred P5-scale candidate was close to, but did not exactly match, the predefined 466.56-Myr value.

Points and error bars summarize the distributions obtained after propagating the chronological uncertainties of the analyzed boundaries or events. Absolute event-to-peak distances are presented as descriptive measures because their expected magnitude depends on candidate-period length.

Blue dashed lines in the temporal projections indicate successive peaks of the fitted cycle, and blue circles denote the analyzed stratigraphic boundaries or major Earth-history transitions. Older ages are shown on the left and the present on the right.

Dense blind scans reveal a cross-record ~100-kyr P1 recurrence and a stable 1.136-Myr P2 stratigraphic optimum

This study first performed dense, theory-blind period scans of continuous records associated with the P1 and P2 domains. The theoretical values were displayed only as references and were not supplied to the peak-search procedure. This approach allowed the dominant recurrence domains to be identified directly from each record before cross-dataset aggregation. Within the P1 search range of 0.08–0.15 Myr, all six Quaternary climate records converged on a dominant peak between 0.0977 and 0.0982 Myr (**Fig. 2A**). The corresponding peak occurred at 0.09795 Myr, or approximately 98 kyr, in the evidence-only family-pooled consensus. A weaker secondary domain was consistently observed near 0.118–0.121 Myr and produced a pooled peak at 0.11985 Myr. Thus, the empirical P1 landscape contained a dominant ~98-kyr recurrence and a substantially weaker ~120-kyr secondary feature. The predefined 0.1296-Myr value did not constitute the principal empirical peak.

The ~100-kyr result was further supported by the expanded climate analyses (**Extended Data Fig. 1**). Band-limited reconstructions showed that the 80–150-kyr component increased markedly across the Mid-Pleistocene Transition and became prominent during the late Quaternary, whereas variability in the 35–50-kyr band remained present. Dense Lomb–Scargle spectra therefore retained a strong ~41-kyr component in several records while also recovering a distinct broad ~100-kyr signal. During the most recent 0.8 Myr, LR04 benthic $\delta^{18}\text{O}$, reconstructed global temperature, Spratt sea level, and atmospheric CO_2 yielded spectral peaks at approximately 98 kyr and independently selected 100 kyr as the best candidate, with family-wise error-rate-adjusted p values of 0.002–0.003. Across the broader candidate comparison, 100 kyr received the maximum relative score in all evaluated climate records. In contrast, the La2010 eccentricity series favored 129.6 kyr, while retaining nearly equivalent support at 100 and 130 kyr. These findings indicate that the P1 framework captured the relevant broad orbital-climate domain, but that its dominant empirical expression in late Quaternary climate was ~100 kyr rather than exactly 129.6 kyr.

The continuous-record P2 scans produced a multimodal but spatially concentrated recurrence landscape (**Fig. 2B**). Although the preferred peak positions varied among CENOGRID $\delta^{18}\text{O}$, CENOGRID $\delta^{13}\text{C}$, PINT palaeointensity, and the La2010 eccentricity positive control, the principal evidence-only consensus peaks were consistently concentrated around the predefined 1.296-Myr domain. The strongest pooled peaks occurred at 1.444 and 1.400 Myr, followed by peaks at 1.325, 1.363, and 1.188 Myr. Four of these five major peaks fell within $\pm 10\%$ of 1.296 Myr, while the highest peak at 1.444 Myr lay only slightly beyond this interval (+11.4%). Thus, despite differences in the precise optimum among records, both the dominant and subsidiary

consensus features were concentrated within an approximately $\pm 10\%$ neighborhood of the predefined P2 timescale, supporting a shared lower-Myr recurrence domain rather than an exact universal periodicity.

The expanded record-specific analyses supported this interpretation (**Extended Data Fig. 1E**). Magnetochron boundaries favored an approximately 1.30-Myr recurrence that remained significant after family-wise error-rate correction (adjusted $p = 0.022$). The $\delta^{13}\text{C}$ and $\delta^{18}\text{O}$ records instead favored approximately 1.40 Myr, with adjusted p values of 0.061 and 0.089, respectively, whereas orbital eccentricity provided no corresponding Myr-scale support. P2 was therefore characterized by record-dependent preferences distributed mainly across the ~ 1.3 – 1.4 -Myr range rather than by uniform recovery of a single exact period.

This study next conducted an independent dense blind scan of ICS Stage boundaries over 2.58–66 Ma. This analysis identified a sharp maximum at 1.136 Myr, with a median Rayleigh phase-coherence score of $R = 0.620$ (**Fig. 2C**). The 1.136-Myr peak was the highest pointwise peak in the complete 0.8–1.6-Myr search range. Other local maxima occurred near 0.892, 1.040, 1.110, 1.381, and 1.480 Myr. The 1.136-Myr optimum was 0.160 Myr, or approximately 12.3%, shorter than the predefined 1.296-Myr value. It therefore occupied the same lower-Myr recurrence domain but did not represent an exact recovery of the theoretical target. Peak height and domain width provided complementary assessments of this landscape. The 1.136-Myr feature had the greatest pointwise height, whereas the broader 1.040-Myr recurrence domain accumulated the largest integrated mass above its local baseline (**Fig. 2C and Supplementary Fig. 3A**). In a nested global null test accounting for the complete blind search, the observed maximum peak height yielded a global p value of 0.0666. This constituted suggestive evidence at the 0.10 level but did not reach the conventional $p < 0.05$ threshold. The high-and-wide domain-mass statistic was weaker, with a global p value of 0.2163. Thus, the P2 Stage-boundary result was driven primarily by a sharp 1.136-Myr optimum rather than by an unusually broad recurrence domain.

The tabulated ICS Stage boundaries did not provide event-specific chronological uncertainty intervals suitable for conventional uncertainty propagation. We therefore performed an artificial timing-jitter stress test across half-widths from 0 to 0.20 Ma (**Fig. 2D and Supplementary Fig. 3B**). The overall period landscape remained highly similar across the tested perturbation levels, and the maximum-median-score optimum remained at 1.136 Myr throughout the complete stress range. Even the conservative optimum based on the lower 95% score bound remained at 1.136 Myr through a jitter half-width of 0.10 Ma and shifted only marginally to 1.135 Myr at 0.20 Ma. The strength and localization of the peak nevertheless declined as the imposed perturbation became extreme. The median coherence of the tracked 1.136-Myr peak decreased from approximately 0.62 without perturbation to approximately 0.50 at a jitter half-width of 0.20 Ma, and the distribution of blind-search winners broadened at the highest stress level. Retention within the immediate 1.136-Myr region was nearly complete through moderate perturbations, whereas at ± 0.20 Ma approximately 48% and 57% of realizations remained within ± 0.02 and ± 0.05 Myr of 1.136 Myr, respectively. The stress test therefore demonstrated that the location of the primary P2 optimum was robust to moderate timing perturbation, while appropriately revealing reduced precision under deliberately severe chronological jitter.

Collectively, the P1 analyses showed strong cross-record convergence on an approximately 98–100-kyr climate recurrence, accompanied by a weaker ~120-kyr feature. P2 was less uniform: continuous and event-based records preferentially occupied a broader ~1.3–1.4-Myr range, whereas ICS Stage boundaries produced a sharp and perturbation-resistant 1.136-Myr optimum. These results support the existence of distinct P1 and lower-Myr P2 recurrence domains, but they also demonstrate that the empirical peak within a domain need not coincide exactly with its predefined theoretical center.

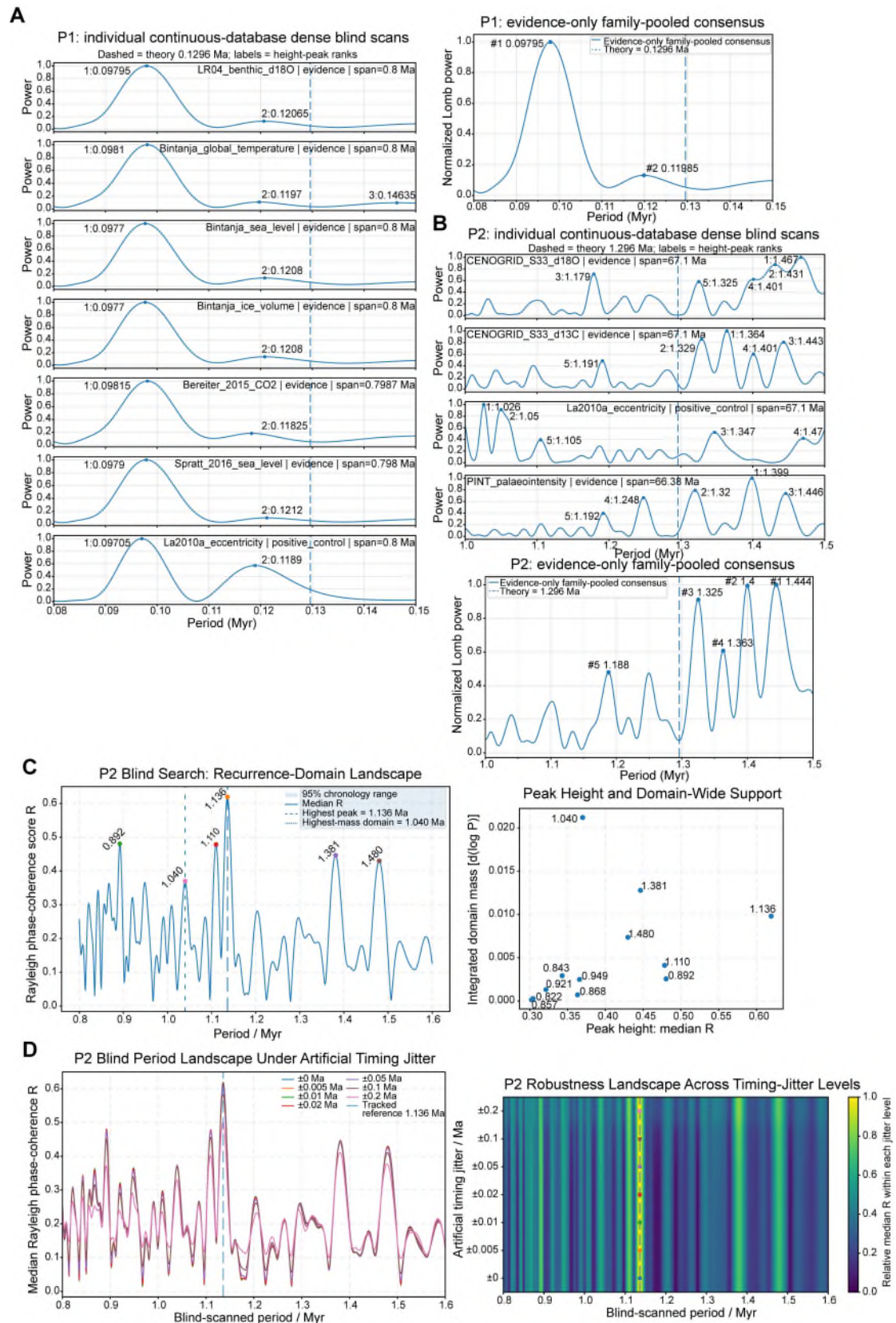


Figure 2 | Dense blind scans of the P1 and P2 recurrence domains.

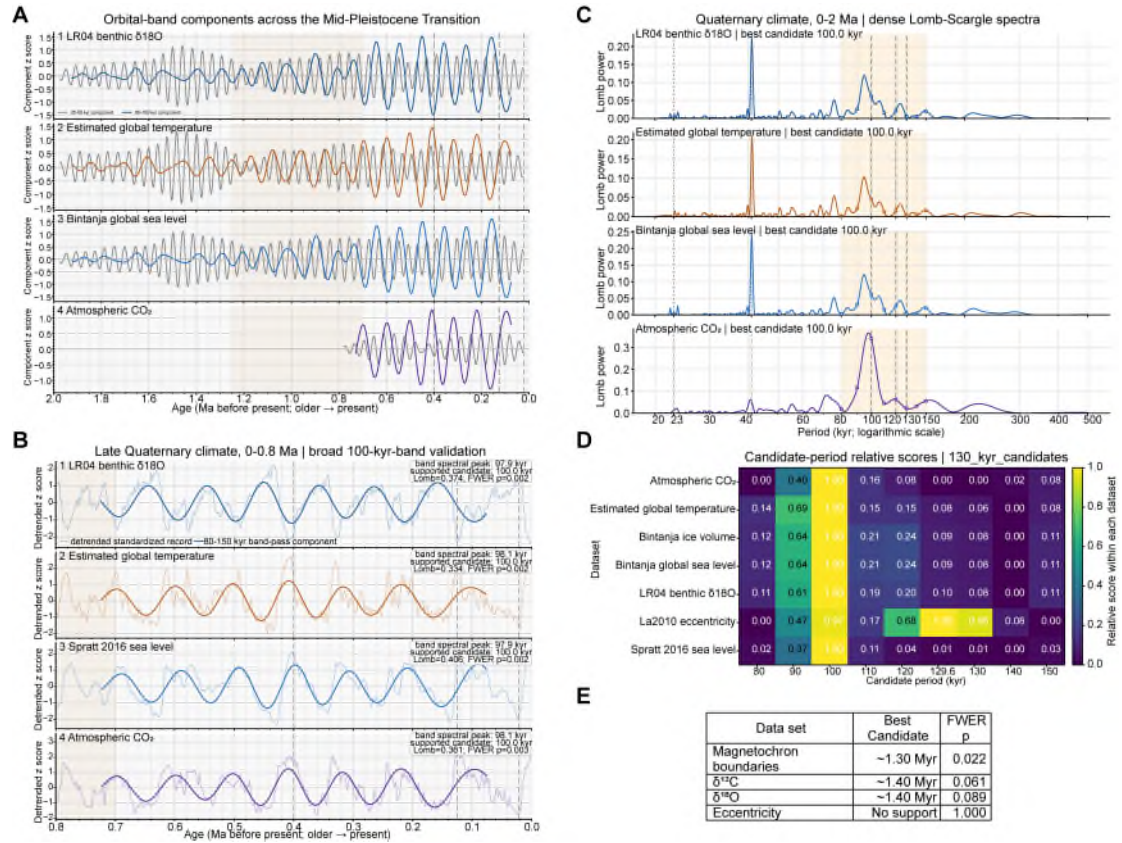
(A) P1 blind scans of continuous Quaternary records over 0.08–0.15 Myr. The left panels show normalized Lomb–Scargle power for LR04 benthic $\delta^{18}\text{O}$, Bintanja reconstructed global temperature, Bintanja global sea level, Bintanja ice volume, Bereiter atmospheric CO_2 , Spratt sea

level, and the La2010 eccentricity positive control. Labels identify local peaks and their height-based ranks. The vertical dashed line marks the predefined 0.1296-Myr value for reference and was not used to direct the blind search. All climate records produced a dominant peak near 0.098 Myr and a weaker secondary peak near 0.118–0.121 Myr. The evidence-only family-pooled consensus, excluding the positive control, identified a primary peak at 0.09795 Myr and a secondary peak at 0.11985 Myr.

(B) P2 blind scans of continuous records over 1.0–1.5 Myr. Individual panels show CENOGRID $\delta^{18}\text{O}$, CENOGRID $\delta^{13}\text{C}$, La2010 eccentricity as a positive control, and PINT palaeointensity. Peak labels indicate within-record height ranks. The lower panel shows the evidence-only family-pooled consensus, which identified major peaks at 1.444, 1.400, 1.325, 1.363, and 1.188 Myr. The dashed vertical line marks the predefined 1.296-Myr value. The multimodal pooled landscape indicates dataset-dependent P2 recurrence preferences rather than convergence on a single exact period.

(C) Blind search of ICS Stage boundaries over 2.58–66 Ma across candidate periods of 0.8–1.6 Myr. The left panel shows the median Rayleigh phase-coherence score, R , across the scanned period range. The highest pointwise peak occurred at 1.136 Myr, whereas the largest integrated high-and-wide domain mass was centered at 1.040 Myr. Other labeled local maxima occurred near 0.892, 1.110, 1.381, and 1.480 Myr. The right panel compares median peak height with integrated domain mass above the local baseline, illustrating the distinction between sharp, high peaks and broader recurrence domains.

(D) Artificial timing-jitter stress test of the P2 ICS blind scan. The left panel overlays median phase-coherence landscapes generated at jitter half-widths of 0, 0.005, 0.01, 0.02, 0.05, 0.10, and 0.20 Ma. The right panel shows the corresponding landscapes normalized within each jitter level. The dashed vertical line marks the unperturbed 1.136-Myr optimum. The dominant peak remained centered at approximately 1.136 Myr across the tested perturbation range, although peak contrast decreased under the strongest imposed jitter.



Extended Data Figure 1. Orbital-scale climate variability and complementary P1 and P2 recurrence evidence.

(A) Evolution of orbital-band components across the Mid-Pleistocene Transition. Standardized 35–50-kyr and 80–150-kyr band-limited components are shown for LR04 benthic $\delta^{18}\text{O}$, reconstructed global temperature, Bintanja global sea level, and atmospheric CO_2 . The longer-period component increased markedly toward the late Quaternary, whereas the 35–50-kyr component persisted, indicating redistribution of climate variability rather than complete replacement of the obliquity-scale signal.

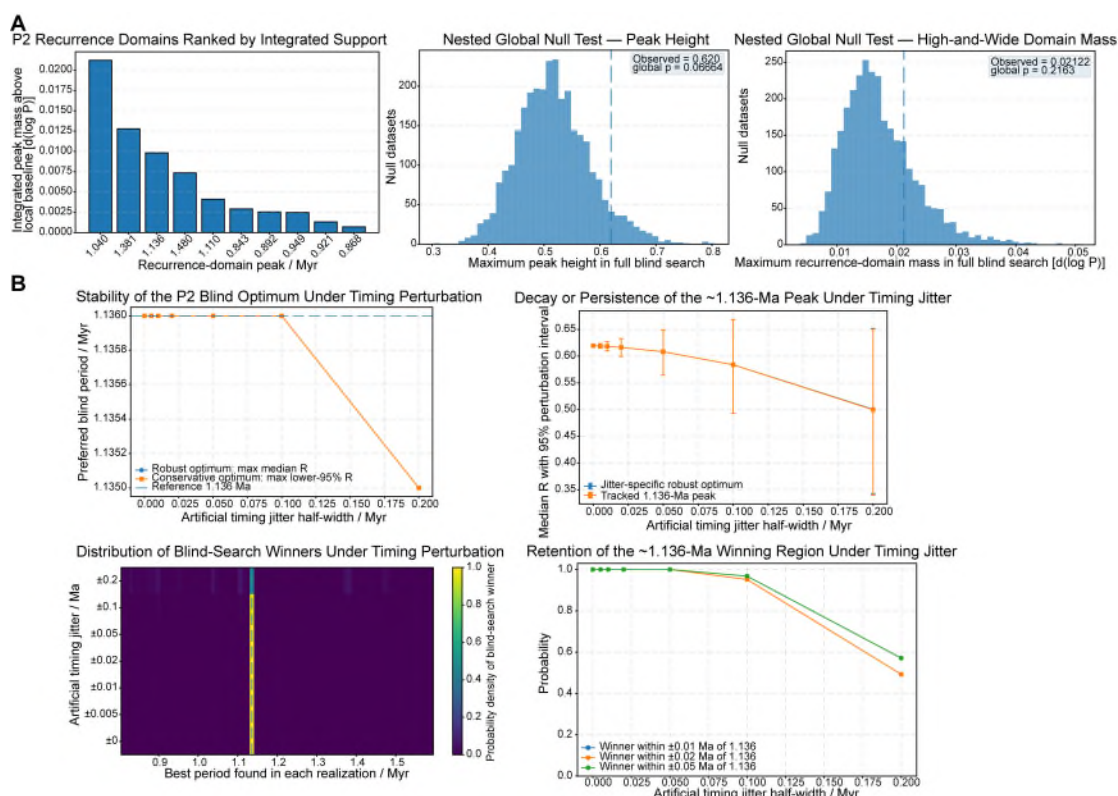
(B) Validation of the broad ~100-kyr recurrence during the most recent 0.8 Myr. Detrended standardized records and their 80–150-kyr band-pass components are shown for LR04 benthic $\delta^{18}\text{O}$, reconstructed global temperature, Spratt sea level, and atmospheric CO_2 . The corresponding band spectral peaks occurred at approximately 97.9–98.1 kyr, and 100 kyr was selected as the supported candidate in all four records. Family-wise error-rate-adjusted p values ranged from 0.002 to 0.003.

(C) Dense Lomb–Scargle spectra for Quaternary climate records over 0–2 Ma. Spectra are shown for LR04 benthic $\delta^{18}\text{O}$, reconstructed global temperature, Bintanja global sea level, and atmospheric CO_2 . The shaded region denotes the broad 80–150-kyr candidate band, and vertical reference lines indicate selected candidate periods within this interval. The ~41-kyr component remained prominent in several records, while each record also recovered 100 kyr as the best-supported candidate within the broad P1 band.

(D) Relative support for candidate periods from 80 to 150 kyr. Scores were normalized independently within each dataset so that its best-supported candidate equaled 1.00. The 100-kyr candidate received the maximum relative score in atmospheric CO_2 , reconstructed global

temperature, Bintanja ice volume, Bintanja global sea level, LR04 benthic $\delta^{18}\text{O}$, and Spratt sea level. La2010 eccentricity was maximized at 129.6 kyr but also showed high relative support at 100 and 130 kyr.

(E) Complementary P2 candidate-period tests. Magnetochron boundaries favored approximately 1.30 Myr and remained significant after family-wise error-rate correction (adjusted $p = 0.022$). The $\delta^{13}\text{C}$ and $\delta^{18}\text{O}$ records favored approximately 1.40 Myr but did not reach $p < 0.05$ after correction (adjusted $p = 0.061$ and 0.089 , respectively). Orbital eccentricity showed no corresponding Myr-scale support (adjusted $p = 1.000$). Together, these analyses indicate dataset-dependent expression within the broader P2 domain.



Supplementary Figure 3. Global null evaluation and timing-perturbation stability of the P2 ICS blind-search result.

(A) Statistical characterization of P2 recurrence domains identified from ICS Stage boundaries. The left panel ranks local domains according to their integrated peak mass above the local baseline. The broad 1.040-Myr domain produced the largest integrated mass, followed by the 1.381-, 1.136-, and 1.480-Myr domains. The center panel shows the nested global null distribution of the maximum peak height obtained anywhere in the complete blind search. The observed maximum at 1.136 Myr had a height of $R = 0.620$ and a global p value of 0.0666 . The right panel shows the corresponding nested null distribution for the maximum integrated high-and-wide domain mass. The observed mass was 0.02122 , with a global p value of 0.2163 . Dashed lines indicate the observed statistics.

(B) Stability of the 1.136-Myr optimum under artificial timing perturbation. The upper-left panel shows the period maximizing the median R and the period maximizing the lower 95% bound of R

at each jitter half-width. The maximum-median-score optimum remained at 1.136 Myr throughout the tested range, whereas the conservative optimum shifted only to 1.135 Myr at the maximum ± 0.20 -Ma perturbation. The upper-right panel shows the median R and its 95% perturbation interval for both the jitter-specific optimum and the tracked 1.136-Myr peak; coherence declined gradually as jitter increased. The lower-left heatmap shows the distribution of blind-search winners at each perturbation level, demonstrating strong concentration near 1.136 Myr under low and moderate jitter. The lower-right panel reports the probability that the winning period remained within ± 0.01 , ± 0.02 , or ± 0.05 Myr of 1.136 Myr. Retention was nearly complete through moderate perturbations but decreased at ± 0.20 Ma, when approximately 48% and 57% of realizations remained within ± 0.02 and ± 0.05 Myr, respectively.

Dense stratigraphic scans identify a stable ~3.5-Myr P3 recurrence and competing P4 and P5 domains

This study next extended the dense blind-search framework to the P3–P5 domains using hierarchical stratigraphic boundaries. Peak height and integrated domain mass were evaluated separately: peak height identified the strongest pointwise phase-coherence maximum, whereas domain mass quantified broader recurrence regions that combined peak strength and width.

The P3 analyses independently recovered an approximately 3.5-Myr optimum in both temporal windows (**Fig. 3A,B**). Over 2.58–66 Ma, the dominant peak occurred at 3.513 Myr and was the leader for both pointwise height and integrated domain mass. Its median Rayleigh phase-coherence score was $R = 0.481$, and its integrated mass was substantially greater than that of all other P3 domains. The same approximate period was recovered over the complete 2.58–538 Ma interval, where the highest pointwise peak occurred at 3.507 Myr. Thus, the preferred P3 position was effectively unchanged by expanding the analysis from the Cenozoic to the complete Phanerozoic record and differed from the predefined 3.888-Myr value by approximately 9.8%.

The relative dominance of the 3.5-Myr peak was stronger in the younger interval. In the 0–66 Ma analysis, the 3.513-Myr domain clearly separated from all competing peaks in both height and mass. Over 0–538 Ma, the landscape became noisier and more fragmented: 3.507 Myr remained the highest pointwise peak, whereas the broader 4.046-Myr region accumulated the largest integrated domain mass. The nested global p values were 0.352 for peak height and 0.127 for domain mass over 2.58–66 Ma, and 0.356 and 0.998, respectively, over 2.58–538 Ma (**Supplementary Fig. 4A,B**). The P3 result therefore showed strong within-landscape reproducibility of its peak position, particularly over 2.58–66 Ma, but was not individually significant under the complete search-wide null model.

The P4 scan using all System-level boundaries revealed two prominent recurrence domains (**Fig. 3C**). The highest pointwise peak occurred at 57.85 Myr, while a second strong peak occurred at 47.26 Myr. Both peak height and integrated domain mass initially favored the approximately

58-Myr region. The 47.26-Myr peak, however, differed from the predefined 46.656-Myr target by only approximately 1.3%, indicating that a prominent empirical recurrence feature was located very close to the theoretical P4 value. Under the nested global null, the all-boundary analysis yielded $p = 0.340$ for peak height and $p = 0.136$ for domain mass (**Supplementary Fig. 4C**).

Inspection of the P4 input revealed an important distinction in boundary-definition status. Ten of the 11 System-level boundaries were defined by ratified Global Boundary Stratotype Sections and Points, whereas the Jurassic–Cretaceous boundary lacked a ratified GSSP. When the analysis was restricted to GSSP-defined boundaries, the highest pointwise peak shifted from approximately 57.85 to 47.09 Myr (**Fig. 3D**). The broad approximately 58.17-Myr region retained the greatest integrated domain mass, showing that the longer-period feature remained wide, but the strongest sharp alignment was now located only 0.9% above the theoretical 46.656-Myr value. The GSSP-only nested global p values were 0.369 for peak height and 0.302 for domain mass (**Supplementary Fig. 4D**).

Leave-one-out analyses localized the source of the P4 peak competition (**Extended Data Fig. 2**). The Jurassic–Cretaceous boundary exerted one of the strongest positive influences on the 57.83-Myr peak relative to the 47.24-Myr peak, and its removal caused the leading period to switch to the approximately 47-Myr region. The Silurian–Devonian boundary produced a similarly strong preference for the 57.83-Myr peak, whereas several other boundaries favored the shorter candidate. Most single-boundary deletions retained the original approximately 58-Myr optimum, indicating that the landscape was not generally unstable; rather, the competition between 47 and 58 Myr could be traced to a small number of influential boundaries. Equal-size subset analyses did not identify the observed GSSP-defined subsets as statistically extreme, so this result was interpreted as a boundary-definition sensitivity analysis rather than an independent significance test.

This study additionally asked where the omitted Jurassic–Cretaceous boundary would be placed if its age were predicted solely from phase lattices fitted to the remaining training boundaries (**Fig. 3F** and **Supplementary Fig. 5**). The theoretical 46.656-Myr lattice, the empirical 47.09-Myr lattice, and a training-only blind-scan lattice near 47.07 Myr predicted nearest phase-aligned ages of approximately 161.23, 160.18, and 160.24 Ma, respectively, within the Oxfordian interval. A shorter 43.82-Myr lattice derived from one tenth of the 438.2-Myr P5 peak predicted approximately 153.19 Ma, within the Kimmeridgian interval. None of these predicted lattice points fell within the externally constrained Jurassic–Cretaceous boundary interval surrounding its current age. These values were therefore treated as phase-alignment diagnostics rather than proposed revisions of the geological timescale.

At the P5 scale, the highest pointwise peak occurred at 438.2 Myr, whereas the broader 501.0-Myr domain accumulated the greatest integrated mass (**Fig. 3E**). These two complementary statistics therefore identified competing short- and long-side expressions of the same broad P5 neighborhood. Importantly, both values were close to the predefined 466.56-Myr target: 438.2 Myr was 6.1% shorter and 501.0 Myr was 7.4% longer. The theoretical period was consequently bracketed by the strongest sharp peak and the strongest high-and-wide domain. The nested global

p values were 0.677 for peak height and 0.188 for domain mass (**Supplementary Fig. 4E**), indicating that neither feature was individually significant under the full blind-search null.

Because several P5 boundaries had broad or incompletely reported age uncertainties, this study tested the stability of the recurrence landscape by multiplying the reported errors from zero to three times their original values (**Fig. 3G**). Increasing uncertainty progressively broadened and reduced the contrast of the landscape but did not generate a new dominant recurrence region. The pre-existing domains near approximately 384, 438, and 501 Myr persisted across all uncertainty multipliers, and their relative positions remained stable. Because each heatmap row was normalized within its uncertainty level, this test evaluated preservation of landscape topology and peak location rather than absolute signal strength.

Temporal projections of the preferred periods showed their phase relationships with the underlying P2–P5 boundaries (**Extended Data Fig. 3**). Together, the analyses identify a reproducible ~3.5-Myr P3 peak across two temporal windows, a boundary-definition-dependent competition between approximately 47 and 58 Myr in P4, and complementary 438- and 501-Myr expressions bracketing the theoretical P5 value. Although none of these domains was individually significant under the complete search-wide nested null, their internally stable positions provided the multiscale peak structure subsequently evaluated by the joint hierarchy tests.

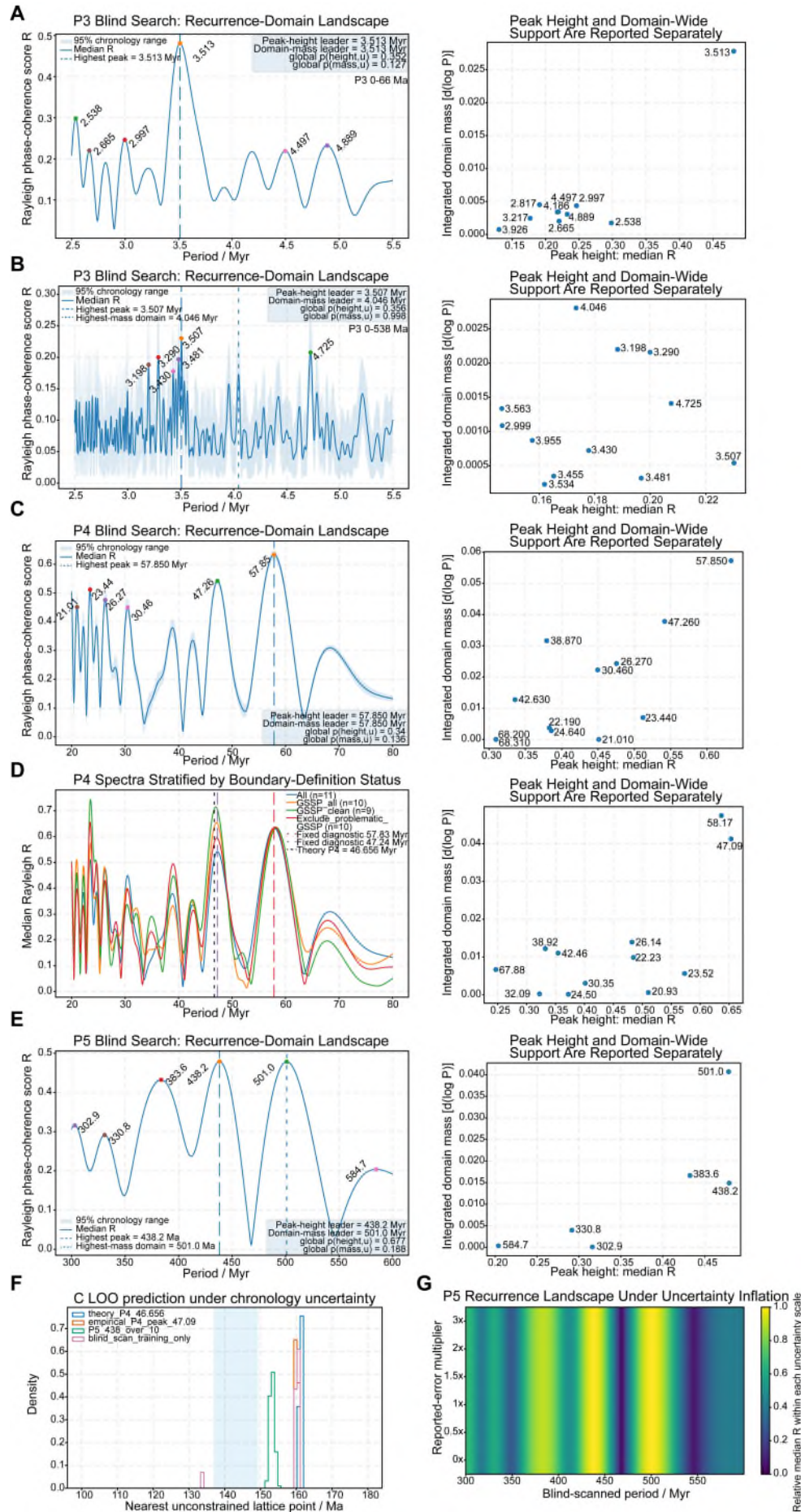


Figure 3 | Dense blind scans and sensitivity analyses of the P3–P5 recurrence domains.

(A) P3 blind search using ICS Stage boundaries over 2.58–66 Ma. The left panel shows the median Rayleigh phase-coherence score, R , across the scanned period range; shading represents the 95% chronology interval. The dominant peak occurred at 3.513 Myr and was the leader for both pointwise height and integrated domain mass. The right panel compares peak height with domain mass for the identified local recurrence regions.

(B) P3 blind search using ICS Stage boundaries over 2.58–538 Ma. The highest pointwise peak occurred at 3.507 Myr, whereas the broad 4.046-Myr domain accumulated the greatest integrated mass. Recovery of an approximately 3.5-Myr pointwise optimum in both P3 windows demonstrates stability of its location despite the greater fragmentation of the full-Phanerozoic landscape.

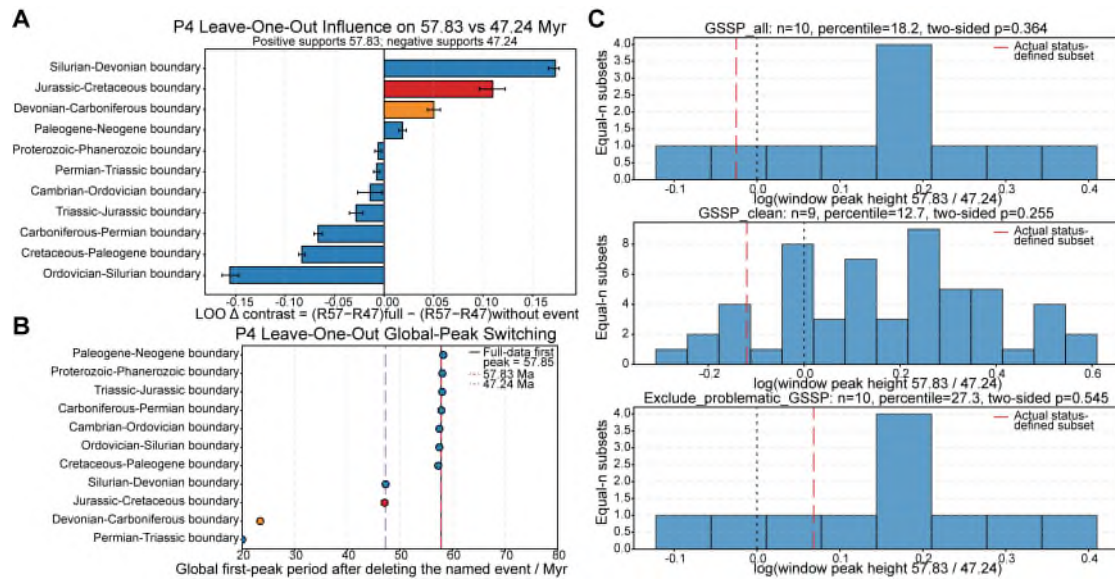
(C) P4 blind search using all System-level boundaries. The strongest peak and highest-mass domain occurred near 57.85 Myr, with a second prominent peak at 47.26 Myr. The latter lies close to the predefined 46.656-Myr target. The right panel reports pointwise height and integrated mass separately for each local recurrence domain.

(D) P4 spectra stratified by boundary-definition status. Curves show the landscapes obtained from all boundaries and from GSSP-based or diagnostic boundary subsets. Restriction to GSSP-defined boundaries shifted the highest pointwise peak to 47.09 Myr, whereas the broader 58.17-Myr region retained the largest integrated mass. Vertical reference lines mark the predefined 46.656-Myr value and the two empirical diagnostic regions. The right panel shows peak-height and domain-mass relationships for the GSSP-restricted landscape.

(E) P5 blind search using deep-time boundaries. The highest pointwise peak occurred at 438.2 Myr, whereas the broadest high-support domain was centered at 501.0 Myr. Both lie within $\pm 10\%$ of the predefined 466.56-Myr target. The right panel distinguishes their complementary peak-height and domain-mass support.

(F) Conditional leave-one-out prediction of the nearest phase-aligned age for the Jurassic–Cretaceous boundary. Distributions incorporate chronology uncertainty for lattices based on the theoretical P4 value, the empirical 47.09-Myr peak, one tenth of the 438.2-Myr P5 peak, and a training-only blind scan. The shaded region denotes the externally constrained Jurassic–Cretaceous interval.

(G) P5 landscape under inflation of reported chronological uncertainties from 0 to 3 times their original values. Colors represent median R normalized within each uncertainty level. Existing recurrence regions persisted without the emergence of a new dominant peak.

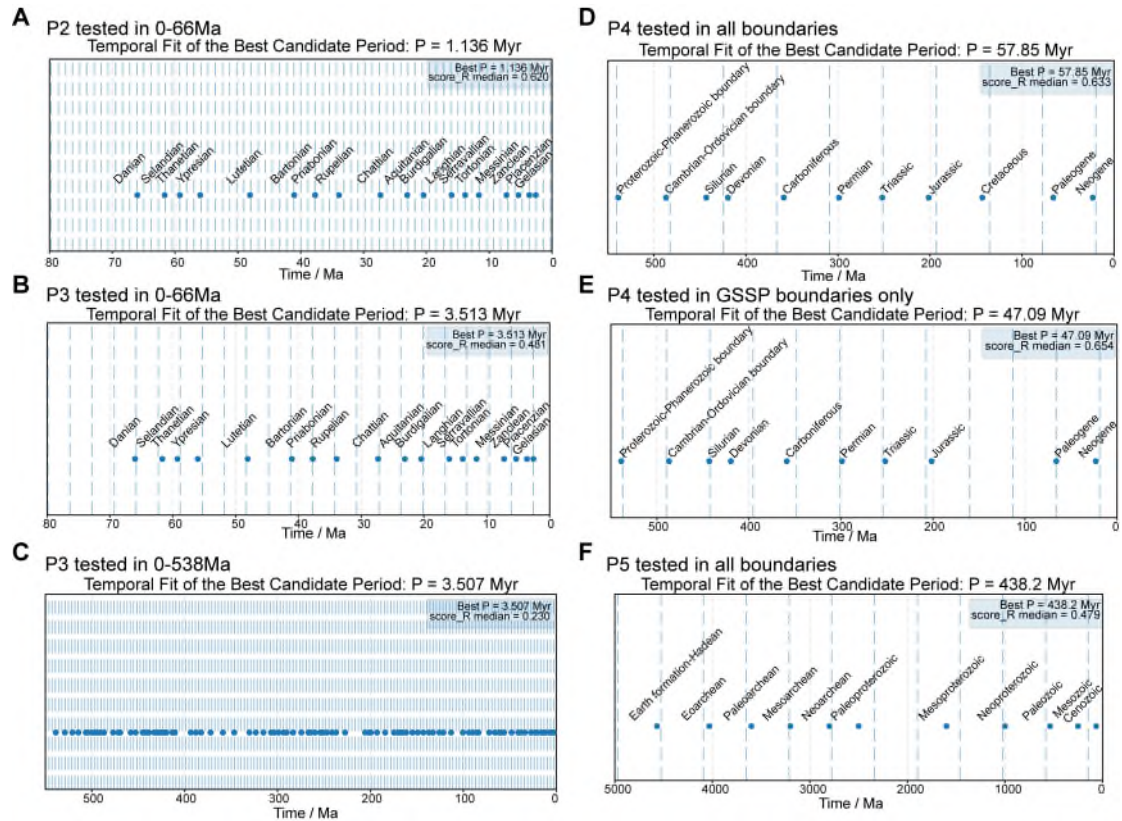


Extended Data Figure 2. Boundary-level influence and definition-status sensitivity of the competing P4 peaks.

(A) Leave-one-out influence of individual System boundaries on the competition between the 57.83- and 47.24-Myr regions. The contrast was calculated as the change in the difference between the two peak heights after removal of each boundary. Positive values indicate preferential support for 57.83 Myr, whereas negative values indicate preferential support for 47.24 Myr. The Silurian–Devonian and Jurassic–Cretaceous boundaries provided the strongest relative support for the approximately 58-Myr peak, while the Ordovician–Silurian boundary most strongly favored the approximately 47-Myr peak.

(B) Global leading period after deletion of each boundary. The full dataset was led by the 57.85-Myr peak. Most leave-one-out datasets retained this optimum, whereas removal of the Jurassic–Cretaceous or Silurian–Devonian boundary shifted the leader to approximately 47 Myr. Removal of a small number of other influential boundaries produced shorter-period optima.

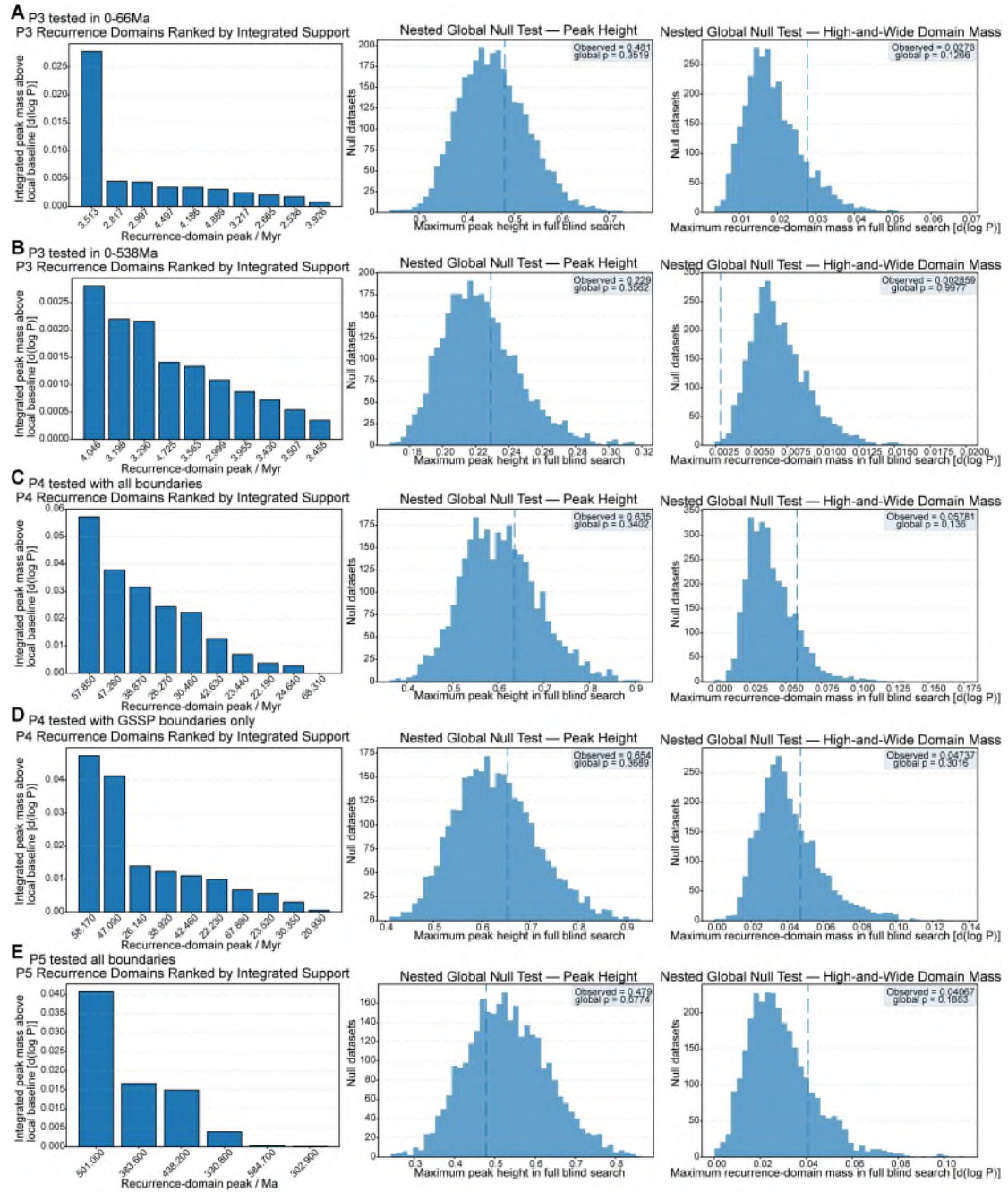
(C) Comparison of the observed boundary-definition subsets with equal-size resampled subsets. Histograms show the null distributions of the log peak-height ratio between the 57.83- and 47.24-Myr windows. Red dashed lines indicate the observed ratios for the GSSP-all, GSSP-clean, and diagnostic exclusion subsets, and black dashed lines mark equal support. The observed subset effects were directionally consistent with the approximately 47-Myr sensitivity but were not statistically extreme under equal-size resampling (two-sided $p = 0.364, 0.255, \text{ and } 0.545$, respectively).



Extended Data Figure 3. Temporal projections of the preferred P2–P5 recurrence periods.

- (A) Projection of the 1.136-Myr P2 lattice across ICS Stage boundaries from 2.58–66 Ma.
- (B) Projection of the 3.513-Myr P3 lattice across ICS Stage boundaries over 2.58–66 Ma.
- (C) Projection of the 3.507-Myr P3 lattice across the complete 2.58–538 Ma Stage-boundary record.
- (D) Projection of the 57.85-Myr P4 lattice across all System-level boundaries.
- (E) Projection of the 47.09-Myr P4 lattice after restriction to GSSP-defined System boundaries.
- (F) Projection of the 438.2-Myr P5 lattice across the deep-time boundary series.

In all panels, blue circles indicate the analyzed boundaries and dashed vertical lines represent successive peaks of the phase-aligned lattice. Time is expressed in Ma, with older ages on the left and the present on the right. Insets summarize the fitted period and median phase-coherence score.



Supplementary Figure 4. Integrated recurrence-domain support and nested global null tests for P3–P5.

For each analysis, the left panel ranks local recurrence domains by integrated mass above the local baseline. The center panel compares the observed maximum pointwise peak height with its nested full-search null distribution, and the right panel compares the observed maximum high-and-wide domain mass with its corresponding null distribution. Dashed lines indicate the observed statistics.

(A) P3 analysis over 2.58–66 Ma. The 3.513-Myr region produced the greatest integrated mass. The observed maximum height was $R = 0.481$ (global $p = 0.3519$), and the observed domain mass was 0.0278 (global $p = 0.1266$).

(B) P3 analysis over 2.58–538 Ma. The 4.046-Myr region produced the greatest integrated mass, whereas 3.507 Myr was the pointwise peak-height leader. The observed maximum height was $R = 0.229$ (global $p = 0.3562$), and the observed domain mass was 0.002859 (global $p = 0.9977$).

(C) P4 analysis using all System boundaries. The 57.85-Myr region ranked first by integrated mass, followed by 47.26 Myr. The observed maximum height was $R = 0.635$ (global $p = 0.3402$), and the observed domain mass was 0.05781 (global $p = 0.1360$).

(D) P4 analysis restricted to GSSP-defined boundaries. The broad 58.17-Myr region retained the largest integrated mass, followed by 47.09 Myr, whereas the highest pointwise alignment occurred in the approximately 47-Myr region. The observed maximum height was $R = 0.654$ (global $p = 0.3689$), and the observed domain mass was 0.04737 (global $p = 0.3016$).

(E) P5 analysis using deep-time boundaries. The 501.0-Myr region produced the greatest integrated mass, followed by the 383.6- and 438.2-Myr regions. The observed maximum pointwise height was $R = 0.479$ at 438.2 Myr (global $p = 0.6774$), and the maximum integrated domain mass was 0.04067 at 501.0 Myr (global $p = 0.1883$).



Supplementary Figure 5. Phase-lattice prediction of the omitted Jurassic–Cretaceous boundary.

(A) Training boundaries and fitted P4-scale phase lattices used to predict the nearest unconstrained lattice position to the Jurassic–Cretaceous interval. The four models correspond to the theoretical 46.656-Myr period, the empirical 47.09-Myr P4 peak, a 43.82-Myr period obtained as one tenth of the 438.2-Myr P5 peak, and a training-only blind-scan period of 47.068 Myr. Black points indicate the retained training boundaries, colored dashed lines show the phase-aligned lattice positions, and stars mark the nearest lattice point to the omitted boundary.

The theoretical, empirical-P4, and training-only lattices predicted nearest positions of 161.23, 160.18, and 160.24 Ma, respectively, whereas the P5-derived 43.82-Myr lattice predicted 153.19 Ma. The blue band denotes the externally constrained Jurassic–Cretaceous interval, and the dotted black line marks its current tabulated age of 143.1 Ma. None of the predicted lattice points fell within the constrained interval; the analysis was therefore used as a phase-alignment diagnostic rather than as a proposed re-dating of the boundary.

Peak-height recurrence centers support the predicted multiplicative hierarchy under the GSSP-defined P4 specification

Direct statistical estimation of P6 was not feasible because Earth history contains too few independent intervals at this scale. We therefore treated P6 exclusively as a diagnostic extrapolation from P5 and excluded it from all inferential scores and p values (**Fig. 4A,B**). Starting from the base of the Phanerozoic, endpoint closure was evaluated by propagating three P6 intervals toward the origin of the Earth–Solar System record. The frozen theoretical value $P6 = 1399.68$ Myr implied an endpoint age of 4737.8 Ma. This exceeded the reference ages of Earth formation and Solar System formation by 4.94% and 4.23%, respectively, although both residuals remained within the assumed $\pm 5\%$ tolerance. Propagation from the observed P5 peak-height center of 438.2 Myr yielded $P6 = 1314.6$ Myr and an implied endpoint of 4482.6 Ma. This estimate was substantially closer to both reference ages, with residuals of -1.43% relative to Earth formation and -2.10% relative to Solar System formation. In contrast, propagation from the broad P5 domain-mass center at 501.0 Myr yielded $P6 = 1503.0$ Myr and an endpoint of 5047.8 Ma, exceeding the two references by 12.69% and 11.93%. Thus, the P5 peak-height propagation provided the most plausible P6-scale closure, whereas the P5 mass-based extrapolation performed poorly. Because the tolerance bands were assumed diagnostic criteria rather than measured confidence intervals, this comparison was not treated as a statistical validation of P6.

This study next tested whether the empirically identified P1–P5 recurrence centers collectively followed the predefined cross-scale hierarchy (**Fig. 4C,D**). The theoretical sequence specified adjacent multipliers of $\times 10$, $\times 3$, $\times 12$, and $\times 10$. For the peak-height centers, the GSSP-defined sensitivity chain comprised approximately 0.09795, 1.136, 3.507, 47.09, and 438.2 Myr. The corresponding adjacent ratios were approximately 11.6, 3.09, 13.4, and 9.31, closely tracking the predefined multiplicative sequence despite modest deviations at individual transitions. Two measures of cross-scale agreement were evaluated. The absolute-alignment statistic quantified deviations between observed and theoretical centers in log-period space, whereas the ratio statistic tested the multiplicative organization of adjacent domains (**Fig. 4E,F**). Each statistic was calculated separately using pointwise peak-height centers and broader domain-mass centers, producing four primary tests. The strongest result was obtained for the peak-height ratio hierarchy. Its ratio-pattern deviation was $D = 0.1008$, with a raw Monte Carlo p value of 0.0044 and a Holm-adjusted p value of 0.018 (**Fig. 4F,G**). This result remained significant after correction across all four primary tests. Peak-height absolute alignment also produced nominal evidence ($D = 0.1486$, raw $p = 0.020$), but its Holm-adjusted p value was 0.060. The domain-mass ratio test yielded raw $p = 0.035$ and Holm-adjusted $p = 0.070$, while the domain-mass absolute test yielded raw $p = 0.061$ and Holm-adjusted $p = 0.070$. Therefore, only the peak-height ratio test remained significant after multiplicity correction.

The specification of the P4 center materially affected the hierarchy fit (**Fig. 4I,K**). When all System boundaries were used, the P4 peak near 57.85 Myr produced an observed fixed-target ratio log-RMSE of 0.2243. Restriction to GSSP-defined boundaries shifted the P4 peak-height center to 47.09 Myr and reduced the corresponding deviation to 0.1008. This improvement arose because

the $P3 \rightarrow P4$ and $P4 \rightarrow P5$ ratios moved substantially closer to the theoretical $\times 12$ and $\times 10$ transitions. Thus, the statistically significant result applies specifically to the objectively defined GSSP-only P4 sensitivity configuration and should not be interpreted as equivalent support under every possible P4 boundary specification. Leave-one-domain-out analyses further examined whether the ratio result was generated by a single recurrence domain (**Fig. 4H**). The full peak-height chain remained below $p = 0.05$, and the result was largely retained after removal of P1. In contrast, removing P2, P3, P4, or P5 attenuated the conditional evidence above the nominal threshold. Domain-mass chains remained non significant throughout these analyses. The result therefore depended primarily on the connected lower-Myr to deep-time sequence rather than on the shortest domain alone.

This study finally compared the predefined multiplier order with alternative cross-scale rules (**Fig. 4J**). The target sequence $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10$ ranked first among the 12 distinct permutations of the same multiplier multiset and second among all 81 four-transition rules constructed from $\{3, 10, 12\}$. These rankings were reproduced using both the all-System and GSSP-only P4 specifications. The predefined sequence therefore performed substantially better than most competing multiplier orders. These ranks were treated as descriptive model-comparison evidence rather than as an additional multiplicity-adjusted hypothesis test. Independent log-uniform and linear-uniform search-range nulls likewise placed the GSSP-only hierarchy closer to the target than the all-System configuration (**Fig. 4K**), while the formal inferential conclusion remained based on the four-test analysis and Holm correction.

Collectively, these analyses show that the empirically recovered peak-height centers exhibit significant multiplicative organization across P1–P5 under the GSSP-defined P4 specification. The primary ratio-based test survived correction for all four analytical variants, whereas absolute alignment and domain-mass tests provided weaker supporting evidence. P6 endpoint closure independently favored propagation from the 438.2-Myr P5 peak, but remained diagnostic because adequate P6-scale data were unavailable (**Fig. 4L**).

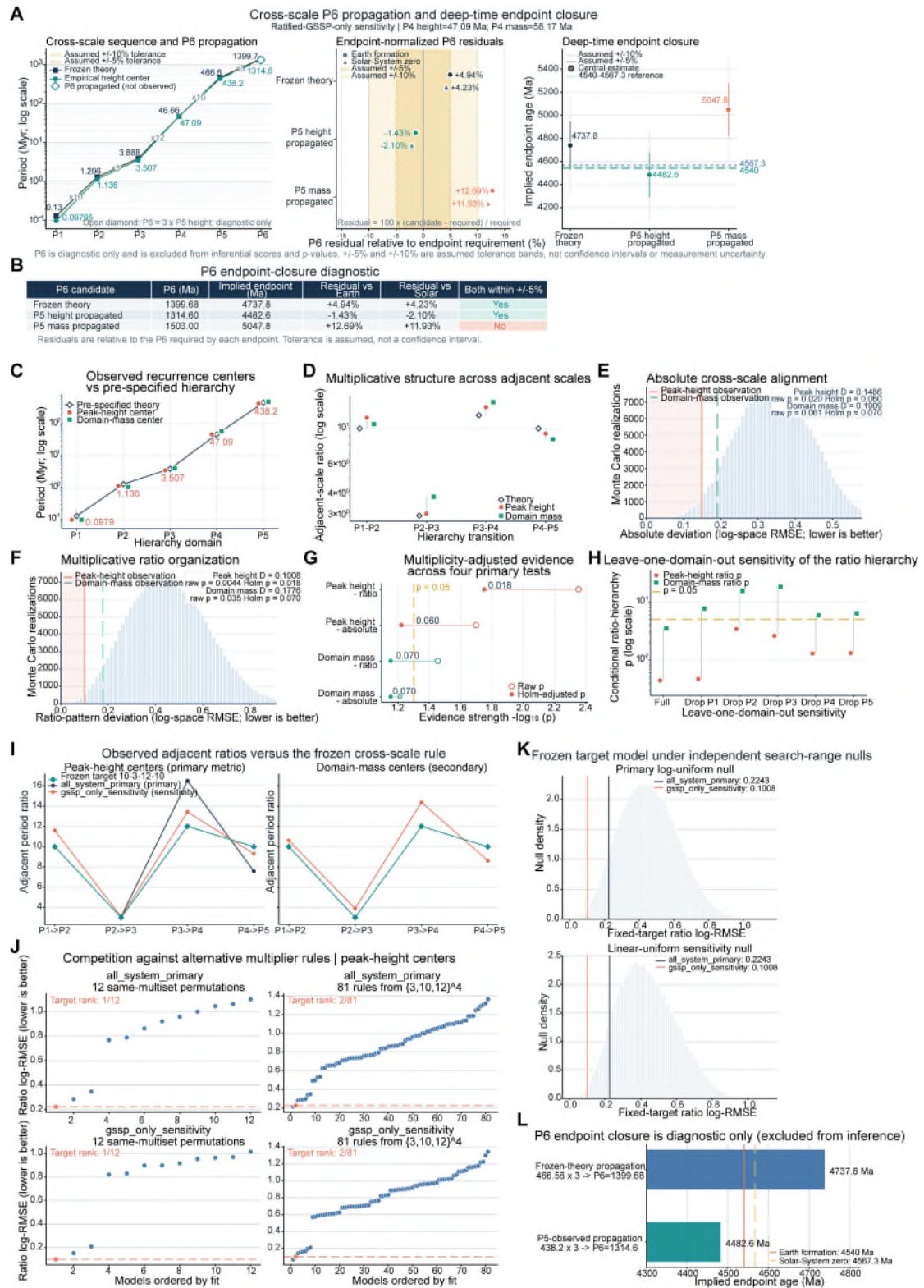


Figure 4 | Cross-scale hierarchy testing, competing multiplier rules, and diagnostic P6 endpoint closure.

(A) Cross-scale sequence and P6 propagation. The left panel compares the predefined P1–P6 hierarchy with empirically observed peak-height centers. The open P6 symbol indicates diagnostic propagation from three times the 438.2-Myr P5 peak rather than an independently observed period. The center panel shows residuals relative to the P6 values required to reach the Earth-formation

and Solar-System-formation endpoints. Shaded regions denote assumed $\pm 5\%$ and $\pm 10\%$ tolerance bands. The right panel shows the endpoint ages implied by the frozen theory, P5 peak-height propagation, and P5 domain-mass propagation.

(B) Numerical summary of the P6 endpoint-closure diagnostic. The frozen theoretical P6 value of 1399.68 Myr implied an endpoint of 4737.8 Ma. Propagation from the 438.2-Myr P5 peak yielded $P6 = 1314.6$ Myr and an endpoint of 4482.6 Ma, whereas propagation from the 501.0-Myr domain-mass center yielded $P6 = 1503.0$ Myr and an endpoint of 5047.8 Ma. Tolerance ranges are assumed diagnostic bands and not confidence intervals.

(C) Empirically observed P1–P5 recurrence centers compared with the predefined hierarchy on a logarithmic period scale. Diamonds denote theoretical values, circles denote peak-height centers, and squares denote domain-mass centers.

(D) Adjacent-scale ratios across the four P1→P2, P2→P3, P3→P4, and P4→P5 transitions. Empirical peak-height and domain-mass ratios are compared with the theoretical multiplier sequence $\times 10$, $\times 3$, $\times 12$, and $\times 10$.

(E) Absolute cross-scale alignment under the Monte Carlo null. Vertical lines indicate the observed log-space RMSE values for peak-height and domain-mass centers. Peak-height absolute alignment yielded raw $p = 0.020$ and Holm-adjusted $p = 0.060$; domain-mass absolute alignment yielded raw $p = 0.061$ and Holm-adjusted $p = 0.070$. Lower deviation indicates better agreement.

(F) Multiplicative ratio organization under the Monte Carlo null. The peak-height ratio statistic yielded $D = 0.1008$, raw $p = 0.0044$, and Holm-adjusted $p = 0.018$. The domain-mass ratio statistic yielded $D = 0.176$, raw $p = 0.035$, and Holm-adjusted $p = 0.070$.

(G) Multiplicity-adjusted evidence across the four primary hierarchy tests. Open symbols denote raw p values and filled symbols denote Holm-adjusted p values. The dashed line marks $p = 0.05$. Only the peak-height ratio test remained significant after correction.

(H) Leave-one-domain-out sensitivity of the ratio hierarchy. Conditional p values are shown after omitting each P1–P5 domain in turn. The peak-height result was largely retained after removal of P1 but weakened after removal of P2, P3, P4, or P5. Domain-mass tests remained above $p = 0.05$.

(I) Observed adjacent ratios under the all-System primary and GSSP-only P4 sensitivity specifications. The left panel shows peak-height centers and the right panel shows domain-mass centers. Restriction to GSSP-defined P4 boundaries moved the P3→P4 and P4→P5 ratios closer to the frozen $\times 12$ and $\times 10$ transitions.

(J) Competition between the predefined multiplier order and alternative rules using peak-height centers. The target sequence ranked first among 12 distinct permutations of the same $\{10, 3, 12, 10\}$ multiset and second among all 81 four-transition rules constructed from $\{3, 10, 12\}$. Results are shown for both the all-System and GSSP-only P4 specifications. Lower ratio log-RMSE indicates better fit.

(K) Frozen-target ratio log-RMSE under independent log-uniform and linear-uniform search-range nulls. Vertical lines show the observed deviations for the all-System configuration (0.2243) and the GSSP-only sensitivity configuration (0.1008). The GSSP-only chain showed closer correspondence to the frozen multiplier rule under both null constructions.

(L) Diagnostic comparison of the endpoint ages implied by the frozen theoretical P6 propagation and propagation from the observed 438.2-Myr P5 peak. Vertical reference lines indicate Earth formation at 4540 Ma and Solar System formation at 4567.3 Ma. P6 was excluded from all hierarchy scores and inferential p values.

Frozen out-of-sample testing constrains direct transfer while secondary analyses reveal conditional cross-scale structure

This study next evaluated whether the recurrence structure identified in the ICS training record transferred to independent Earth-history databases under a frozen out-of-sample protocol (**Fig. 5A**). Target periods, period windows, model definitions, and test statistics were fixed before evaluation of the held-out records. Hierarchy-fixed models tested direct transfer of the ICS-derived periods, whereas train-local-best models provided secondary tests using the locally preferred training peaks. The primary frozen OOS tests provided little evidence for direct period transfer. Hierarchy-fixed tests were nonsignificant for LIP P3 and P4, zircon P3–P5, and the corresponding impact analysis after multiplicity correction. Representative results included $p = 0.532$ for zircon P3, $p = 0.569$ for zircon P4, $p = 0.269$ for zircon P5, and Holm-adjusted $p = 1.000$ for the principal LIP fixed-period tests. A train-local-best LIP P3 model produced nominal evidence in the 0–66 Ma window (raw $p = 0.039$), and selected secondary sensitivity windows produced raw p values of 1×10^{-4} . However, these secondary models did not have a prespecified Holm family and were not promoted to confirmatory evidence (**Extended Data Fig. 4A**). The frozen OOS analysis therefore did not establish broad independent replication of the individual ICS periods.

After completion of the frozen analysis, this study conducted explicitly exploratory post-OOS analyses to examine whether partial ratio structure could nevertheless be detected (**Fig. 5B–E**). The LIP P3→P4 ratio was 11.73, close to the theoretical multiplier of 12, and produced an exploratory p value of 0.083. The extinction and zircon ratios were 15.45 and 14.83, respectively, and provided weaker alignment. When these ratios were compared with the frozen CORE4 reference ratio of 13.43, none survived correction: the robust LIP and extinction transfers yielded raw $p = 0.665$ and 0.854, respectively, with Holm-adjusted $p = 1.000$, while the zircon result remained diagnostic only. Direct transfer of the fixed 3.507-, 47.09-, and 438.2-Myr periods to extinction, impact, LIP, and zircon records was likewise nonsignificant (**Fig. 5D**). A separate post-OOS shared-scale analysis distinguished exact point alignment from broad recurrence-domain support (**Fig. 5C**). An exact diagnostic solution identified a common scale factor of $s = 1.287$ with $p = 9 \times 10^{-4}$, corresponding to periods near 5.00 and 60.04 Myr. However, this point solution was sensitive to precise peak locations. The prespecified broad-support statistic instead identified $s = 0.683$, corresponding to approximately 2.65 and 31.86 Myr, but was nonsignificant ($p = 0.785$). Dataset-level contributions suggested localized support from LIP P3 and weaker contributions from extinction and impact P4, but these component values were descriptive and were not treated as additional inferential tests.

The broader shared-absolute hierarchy analysis identified an exploratory $\times 3 \rightarrow \times 12$ structure with a fitted base near 1.64 Myr (**Fig. 5F**). The joint full-search p value was 0.0879. Support was heterogeneous across databases and levels, with the strongest positive cells contributed by LIP.

Complementary pair-level analyses produced nominal enrichment in some database combinations, but this enrichment did not extend to the complete three-level ladder, for which empirical partial-conjunction p values were 1.000 (**Extended Data Fig. 4B**). Collectively, the post-OOS analyses identified localized ratio-compatible features but did not upgrade the frozen OOS result to confirmatory replication.

This study next asked whether the complete collections of independently detected recurrence domains within P1–P5 were collectively centered near the theoretical hierarchy, without selecting only the single highest peak from each scan. All non-edge recurrence domains were retained and summarized by direct arithmetic weighted centers using integrated mass, peak height, peak prominence, or equal weighting (**Fig. 5F and Extended Data Fig. 5A,B**). Scan-edge maxima were excluded before abstraction. These abstract centers are distribution summaries and therefore need not coincide with individual spectral peaks. The primary integrated-mass weighting produced abstract centers of 0.09825, 1.151, 3.644, 45.58, and 454.6 Myr for P1–P5, respectively (**Fig. 5G**). Their adjacent ratios were 11.72, 3.17, 12.51, and 9.97, closely approximating the frozen $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10$ sequence. All five centers fell within the interior of their corresponding search ranges and near the core theoretical domains, indicating that the result was not generated by accumulation of scan-edge maxima. Under the primary empirical-width conditional null, the observed integrated multi-domain support was 0.269 (**Fig. 5H**). The mass-weighted hierarchy yielded a raw conditional p value of 0.0151 and remained significant after Holm correction across the four weighting schemes (Holm-adjusted p = 0.0453). Peak-prominence weighting also produced significant results in selected rigid-null configurations, whereas height and equal weighting were weaker (**Extended Data Fig. 5C**). This analysis therefore showed that the full distributions of detected domains were non-randomly centered around a coherent cross-scale hierarchy. Nevertheless, the test was conditional on the observed peak configuration and was conducted after the primary scans; it should be interpreted as a significant secondary abstraction analysis rather than as a raw-data nested global test.

Finally, this study tried abandon both the predefined periods and the multiplier sequence and performed strict theory-free inverse discovery across independent databases (**Fig. 5I,J**). The older 75% of each temporal record was used for discovery, the younger 25% for holdout evaluation, and the outer 10% of each period-search range was excluded to reduce edge selection. No theoretical target period or ratio was supplied to the search. In the primary CORE4 dataset, the unconstrained search identified a recurrent young-scale feature near 6.65 Myr. The free-period solution additionally selected approximately 4.92, 10.6, and 24.3 Myr (**Fig. 5I,J**). These candidates showed corrected discovery evidence but did not reproduce in the younger holdout. Addition of ICS or PINT generated further candidates near approximately 60–63, 200–216, 338, and 369–371 Myr, although their locations and holdout behavior depended on dataset composition (**Extended Data Fig. 6**). Among all tested configurations, the only model that satisfied corrected discovery and corrected holdout criteria was the unconstrained ALL6 free-period set comprising approximately 6.65, 63.5, 215, and 369 Myr (**Fig. 5J**). Importantly, these periods were selected freely and did not form the predefined hierarchy. The explicit ALL6 integer-ratio chain selected approximately 6.65, 66.5, 200, and 399 Myr, corresponding to $\times 10 \rightarrow \times 3 \rightarrow \times 2$, but did not pass the corrected holdout criterion. Other PINT-containing models achieved only partial support. Thus, theory-free analysis

recovered one reproducible multi-period set, but not a stable universal integer-ratio chain.

Together, these analyses define the boundary between validation and exploration. Frozen OOS testing did not confirm broad transfer of the individual ICS periods. Post-OOS analyses identified localized and database-dependent ratio-compatible signals, while conditional abstraction of all detected P1–P5 domains produced significant hierarchy-level organization after multiplicity correction. Strict theory-free discovery recovered a reproducible ALL6 multi-period set, but its periods and ratios differed from the predefined hierarchy. The combined results therefore support structured cross-scale organization while also demonstrating substantial heterogeneity across independent Earth-history records.

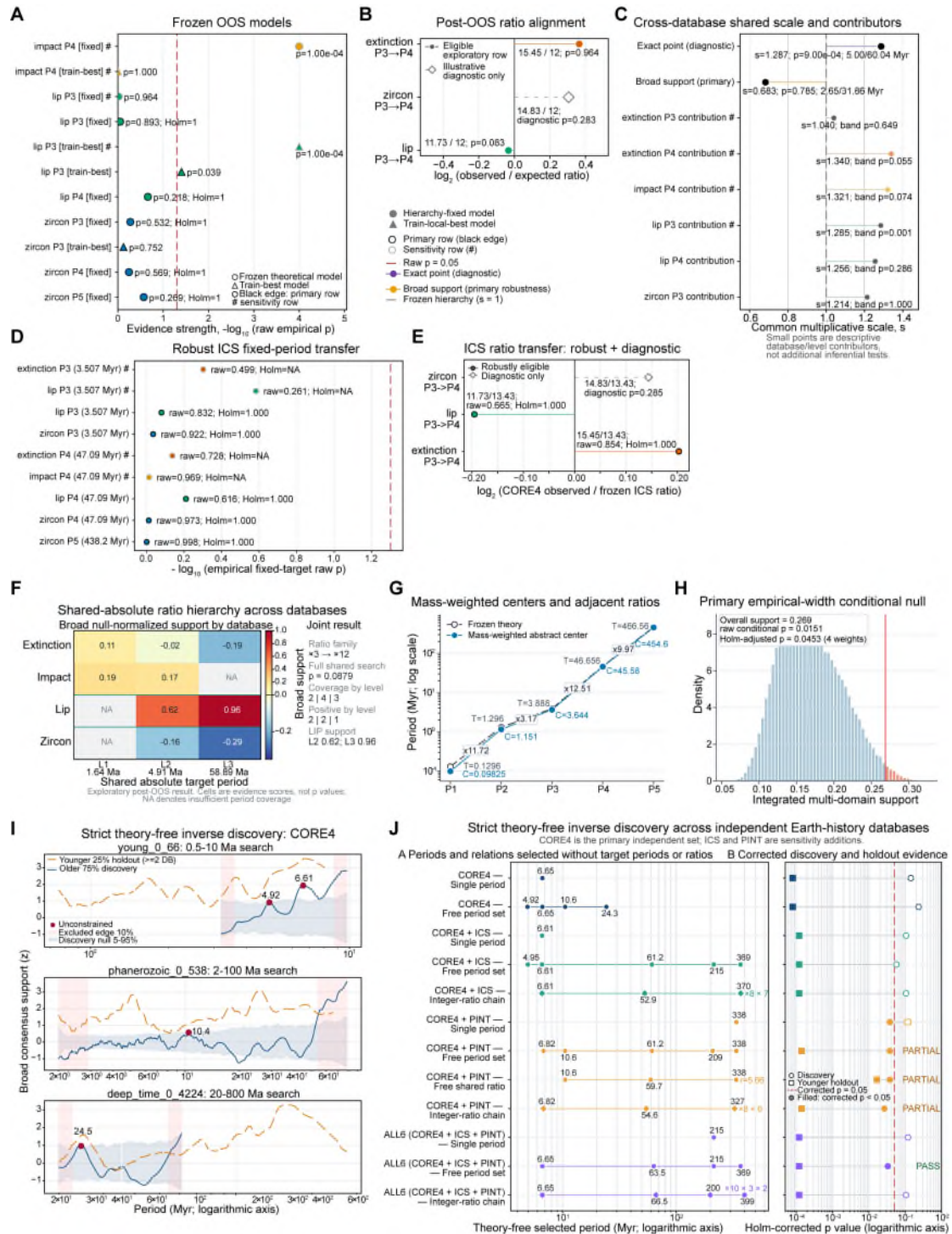


Figure 5 | Frozen out-of-sample evaluation, post-OOS exploration, abstract-domain hierarchy, and strict theory-free discovery.

(A) Frozen OOS tests in impact, LIP, and zircon records. Circles denote hierarchy-fixed theoretical models and triangles denote train-local-best models. Black-edged symbols identify primary rows, whereas rows marked with # are secondary sensitivity analyses. The red dashed line indicates raw $p = 0.05$. Holm-adjusted values are shown where prespecified.

(B) Exploratory post-OOS alignment of observed P3→P4 ratios with the theoretical multiplier of 12. LIP produced a ratio of 11.73 ($p = 0.083$), extinction produced 15.45 ($p = 0.964$), and zircon produced a diagnostic ratio of 14.83 ($p = 0.283$).

(C) Post-OOS cross-database shared-scale analysis. The exact-point diagnostic identified $s = 1.287$ and $p = 9 \times 10^{-4}$, whereas the primary broad-support statistic identified $s = 0.683$ and $p = 0.785$. Smaller points show descriptive database- and level-specific contributions and are not additional inferential tests.

(D) Transfer of fixed ICS periods to independent databases. Empirical p values are shown for 3.507-Myr P3, 47.09-Myr P4, and 438.2-Myr P5 targets. None of the robustly eligible tests survived Holm correction.

(E) Transfer of observed P3→P4 ratios relative to the frozen CORE4 reference ratio of 13.43. Robust LIP and extinction tests were nonsignificant after correction; zircon was retained as a diagnostic comparison only.

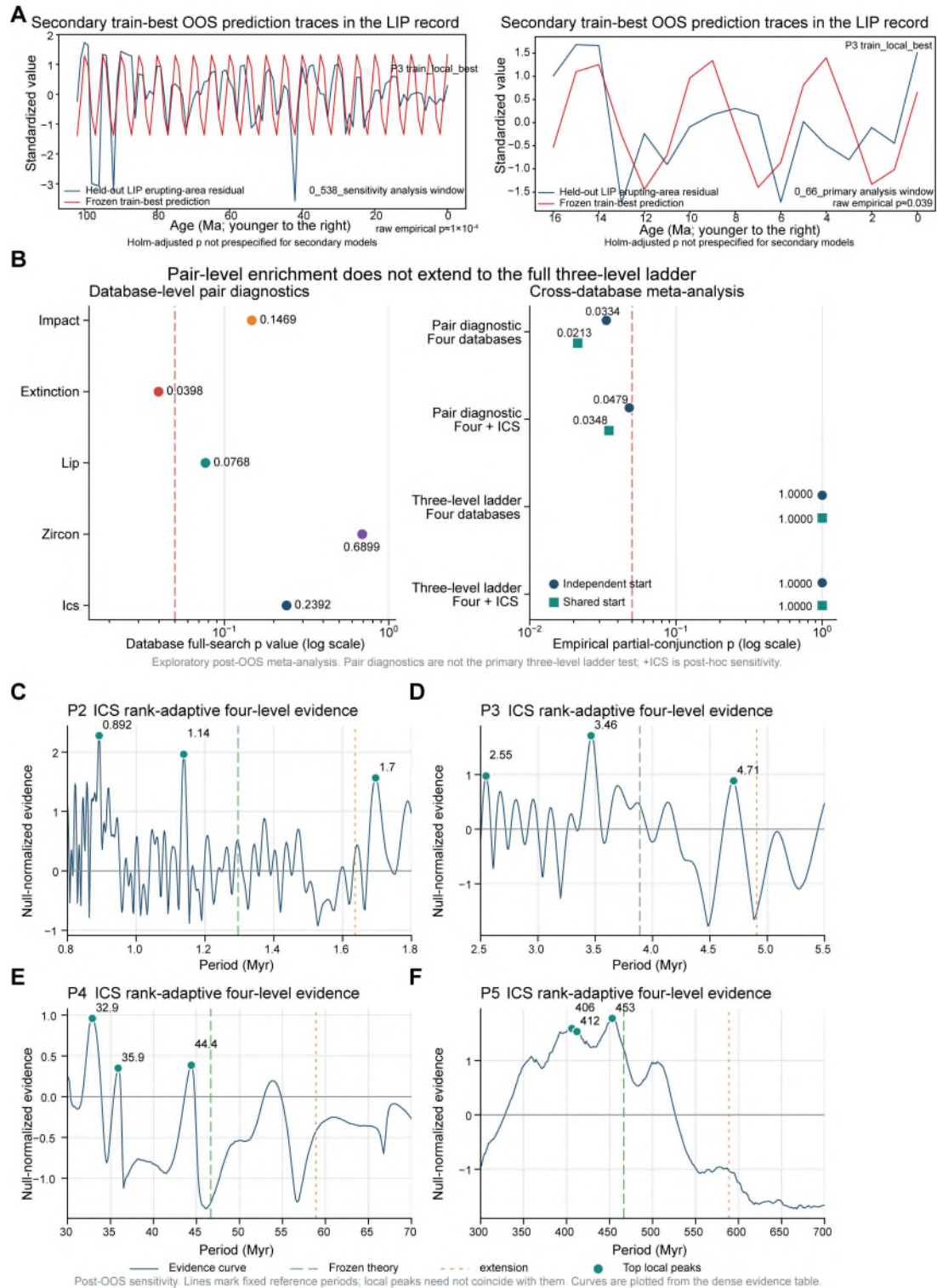
(F) Exploratory shared-absolute ratio hierarchy across databases. Cells show broad null-normalized evidence scores rather than p values. The fitted $\times 3 \rightarrow \times 12$ hierarchy yielded a full shared-search p value of 0.0879. NA indicates insufficient period coverage.

(G) Integrated-mass-weighted abstract centers across P1–P5. The resulting centers were 0.09825, 1.151, 3.644, 45.58, and 454.6 Myr, producing adjacent ratios of 11.72, 3.17, 12.51, and 9.97. Open symbols and the dashed line represent the frozen theoretical hierarchy.

(H) Primary empirical-width conditional null for integrated multi-domain support. The observed support was 0.269, with raw $p = 0.0151$ and Holm-adjusted $p = 0.0453$ across four weighting schemes. The red line marks the observed statistic; red histogram bars denote null realizations at least as extreme.

(I) Strict theory-free inverse discovery in the primary CORE4 databases. Curves show discovery support from the older 75% of each record and evidence in the younger 25% holdout. Shading denotes the 5th–95th percentile discovery-null envelope, and pink regions mark excluded edge intervals. Unconstrained candidates occurred near 4.92, 6.61, 10.4, and 24.5 Myr across the three search scales.

(J) Periods and relationships selected without predefined periods or ratios across CORE4 and sensitivity additions of ICS and PINT. The left panel shows selected single periods, free-period sets, free shared-ratio models, and integer-ratio chains. The right panel shows Holm-corrected discovery and younger-holdout p values. Filled symbols indicate corrected $p < 0.05$. Only the ALL6 free-period set passed both corrected discovery and holdout criteria; the explicit integer-ratio chains did not.

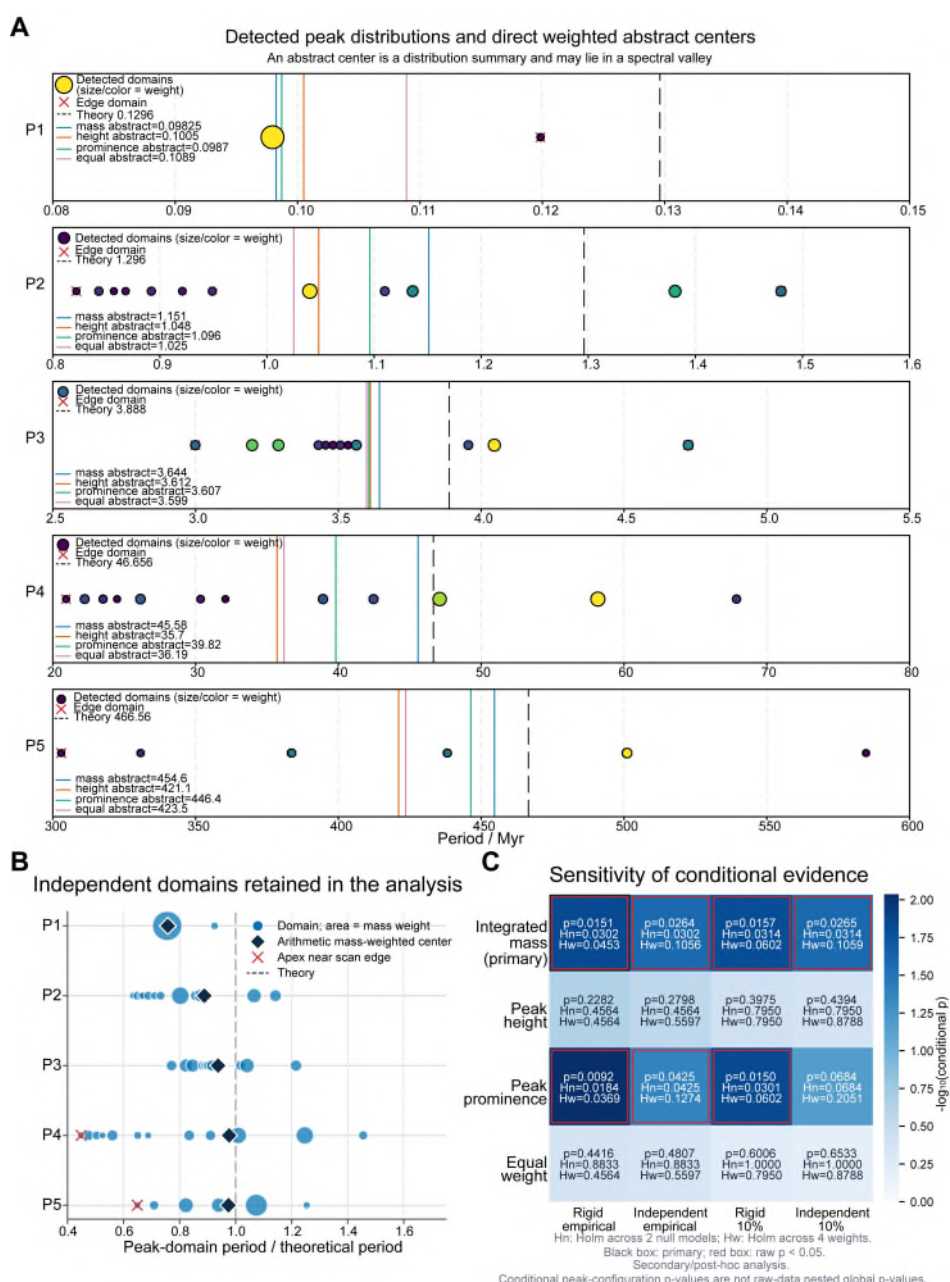


Extended Data Figure 4. Secondary OOS traces, pair-level enrichment, and post-OOS ICS rank-adaptive evidence.

(A) Secondary train-local-best OOS prediction traces in the LIP record over the 0–538 Ma sensitivity window and 0–66 Ma primary window. Blue curves show held-out standardized LIP erupting-area residuals and red curves show frozen training-derived predictions. Raw empirical p values were 1×10^{-4} and 0.039, respectively. Holm adjustment was not prespecified for these secondary models.

(B) Exploratory post-OOS pair-level diagnostics and cross-database meta-analysis. Extinction and LIP showed nominal database-level evidence, and selected pair-level meta-analyses yielded $p < 0.05$. However, the corresponding three-level ladder tests yielded $p = 1.000$, demonstrating that pair-level enrichment did not extend to a complete hierarchy.

(C–F) Post-OOS rank-adaptive evidence landscapes for P2, P3, P4, and P5 in the ICS record. Blue curves show null-normalized evidence, green dashed lines mark frozen theoretical periods, orange dotted lines mark propagated extensions, and labeled points identify the strongest local peaks. Local peaks were distributed near, but did not consistently coincide with, the frozen references. These panels are exploratory sensitivity analyses.



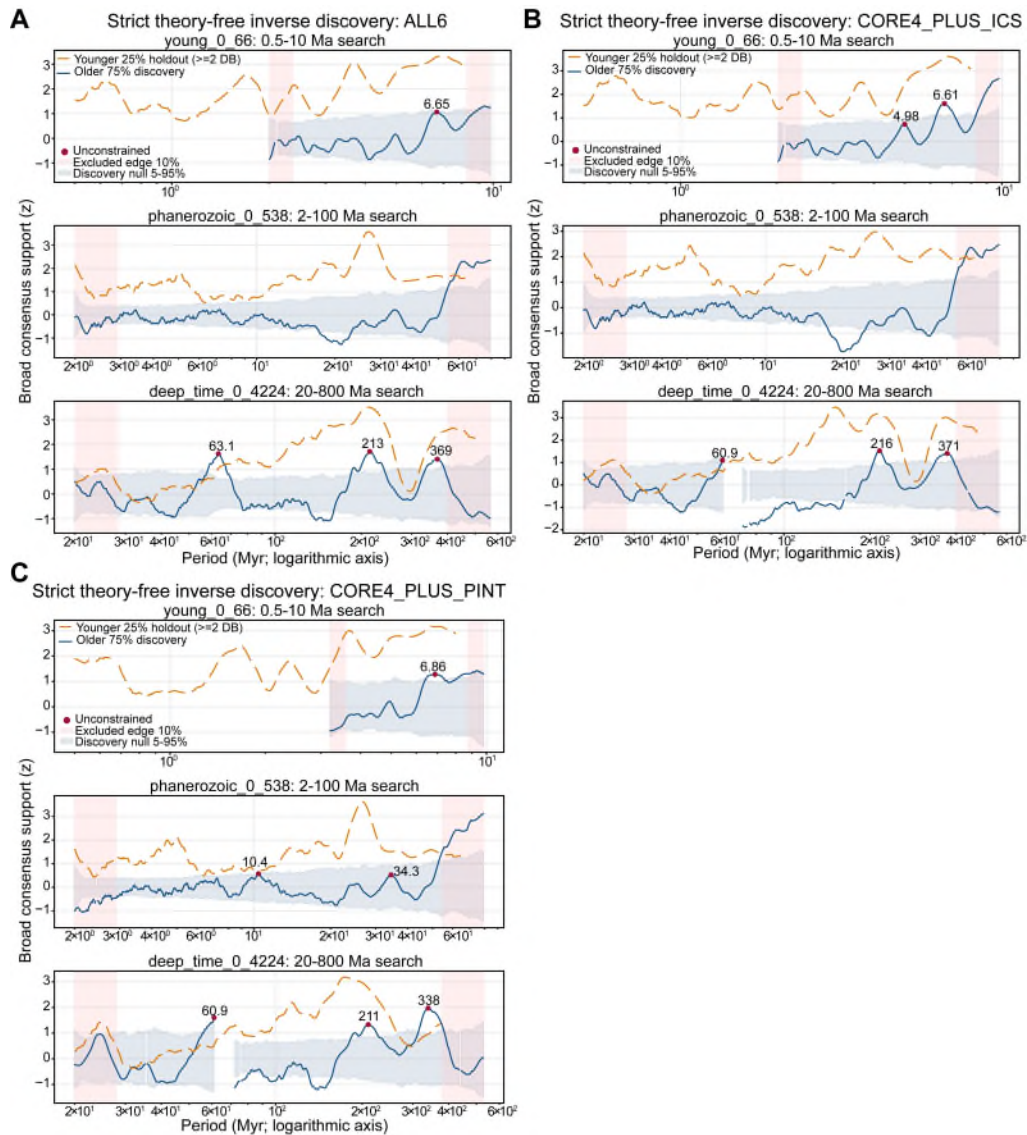
Extended Data Figure 5. Construction and sensitivity of abstract P1–P5 recurrence centers.

(A) Detected recurrence-domain distributions and direct weighted abstract centers for P1–P5.

Circle size and color represent domain weight. Vertical colored lines show arithmetic centers calculated using integrated mass, peak height, peak prominence, or equal weighting; dashed black lines mark theoretical periods. Red crosses identify scan-edge domains excluded from abstraction. An abstract center summarizes a distribution and may lie between individual spectral peaks.

(B) Independent domains retained after edge exclusion, plotted relative to each theoretical period. Circle area represents integrated mass and diamonds identify mass-weighted arithmetic centers. All mass-weighted centers were located near the interior theoretical domains rather than at scan edges.

(C) Sensitivity of conditional hierarchy evidence across four weighting schemes, rigid or independent empirical-width nulls, and fixed $\pm 10\%$ nulls. Values in each cell report the raw conditional p value, Holm adjustment across two null models (Hn), and Holm adjustment across four weighting schemes (Hw). Red outlines indicate raw $p < 0.05$ and black outlines identify the primary configurations. Integrated-mass weighting under the rigid empirical-width null remained significant after correction across all four weights (Hw = 0.0453). These are conditional peak-configuration tests rather than raw-data nested global tests.



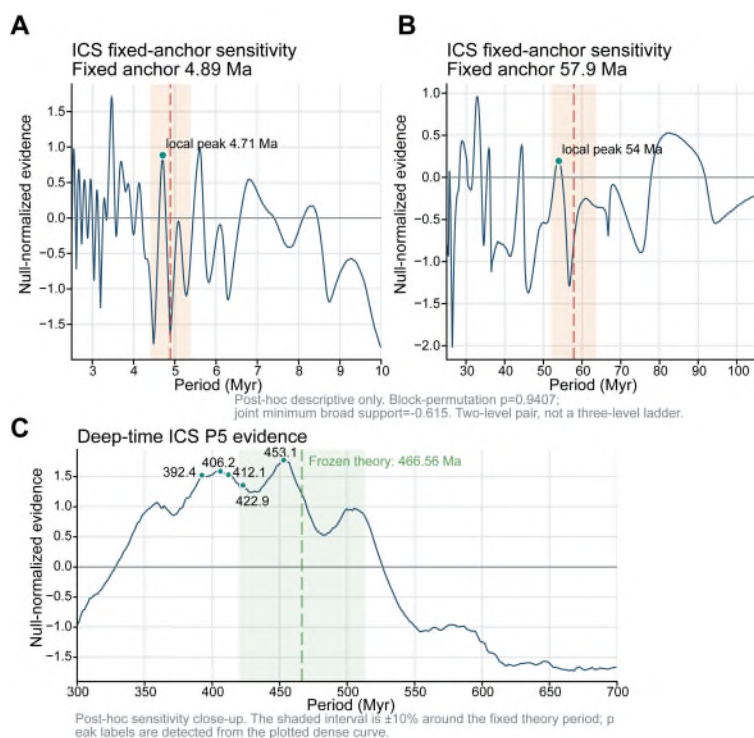
Extended Data Figure 6. Dataset-composition sensitivity of strict theory-free inverse discovery.

(A) Theory-free discovery using all six databases. Candidate features occurred near 6.65 Myr in the young search and near 63.1, 213, and 369 Myr in the deep-time search.

(B) CORE4 plus ICS sensitivity analysis. Candidate features occurred near 4.98 and 6.61 Myr in the young search and near 60.9, 216, and 371 Myr in the deep-time search.

(C) CORE4 plus PINT sensitivity analysis. Candidate features occurred near 6.86 Myr in the young search, 10.4 and 34.3 Myr in the Phanerozoic search, and 60.9, 211, and 338 Myr in the deep-time search.

In each panel, blue curves show consensus support in the older 75% discovery data and orange dashed curves show support in the younger 25% holdout. Blue shading represents the 5th–95th percentile discovery-null envelope, pink regions denote excluded edge intervals, and red points mark unconstrained candidate features. Changes among panels demonstrate sensitivity of theory-free candidates to database composition.



Supplementary Figure 6. Post-hoc ICS fixed-anchor and deep-time sensitivity analyses.

(A) ICS fixed-anchor sensitivity around 4.89 Myr. The strongest local feature within the highlighted region occurred at 4.71 Myr.

(B) ICS fixed-anchor sensitivity around 57.9 Myr. The corresponding local feature occurred near 54 Myr. Joint block-permutation analysis of the two anchors yielded $p = 0.9407$ and negative minimum broad support, indicating that the pair did not constitute a supported three-level ladder.

(C) Deep-time ICS P5 evidence over 300–700 Myr. Multiple local peaks occurred at approximately 392.4, 406.2, 412.1, 422.9, and 453.1 Myr. The shaded region represents $\pm 10\%$ around the frozen 466.56-Myr theory value. This panel is a post-hoc sensitivity close-up and does not constitute an independent confirmatory test.

Cross-scale synthesis reveals a coherent but non-exact recurrence hierarchy

This study finally integrated the recurrence centers identified across the theoretical, empirical, abstract, and theory-free analyses on a common logarithmic timescale (**Fig. 6A**). The frozen hierarchy spans more than five orders of magnitude, from $P1 = 0.1296$ Myr to the conceptual $P7 = 16,796.16$ Myr. The preferred record- or ICS-derived centers were 0.09795, 1.136, 3.507, 47.09, and 438.2 Myr for $P1$ – $P5$, respectively. Although these empirical centers did not coincide exactly with the predefined targets, each remained within its corresponding recurrence domain. Major competing centers were also retained in the synthesis, including the secondary $P1$ – $P3$ domains, the alternative $P4$ centers near 57.85–58.17 Myr, and the $P5$ competitor at 501 Myr.

The adjacent ratios of the preferred empirical centers were 11.60, 3.09, 13.43, and 9.31 for the $P1 \rightarrow P2$, $P2 \rightarrow P3$, $P3 \rightarrow P4$, and $P4 \rightarrow P5$ transitions, respectively (**Fig. 6B**). Thus, two transitions were within 10% of their frozen multipliers, whereas the remaining two were within the broader 10–25% interval. In contrast, substituting the all-System $P4$ center at 57.85 Myr generated larger departures in both transitions adjoining $P4$, and the chain constructed from competing mass-domain centers was less uniformly aligned with the frozen multiplier sequence. These comparisons show that the hierarchy depends not only on the presence of individually strong peaks but also on their joint organization across adjacent scales. Integration across all retained recurrence domains produced mass-weighted abstract centers of 0.09825, 1.151, 3.644, 45.58, and 454.6 Myr for $P1$ – $P5$. Their adjacent ratios were 11.72, 3.17, 12.51, and 9.97. Three of the four transitions were therefore within 10% of the frozen multipliers, with only $P1 \rightarrow P2$ falling in the 10–25% interval. The abstract hierarchy consequently showed closer overall ratio agreement than either the individual peak-height chain or the alternative competing chains. Importantly, these abstract centers summarize the complete retained peak distributions and need not correspond to the apex of any single spectral peak.

The strict theory-free CORE4 inverse search yielded an ordered candidate set at 4.92, 6.65, 10.6, and 24.3 Myr, with internal ratios of 1.35, 1.59, and 2.29. These periods were selected without supplying the predefined targets or multiplier sequence and cannot be assigned directly to $P1$ – $P5$. Moreover, the inverse search did not recover a multiplicity-significant cross-database hierarchy; only the approximately 6.65-Myr feature provided a single-period indication. The CORE4 result is therefore shown as an exploratory contrast rather than confirmatory support for the frozen hierarchy.

Mapping the preferred and abstract centers back onto the $P1$ – $P7$ framework illustrates the overall empirical realization of the proposed structure (**Fig. 6C**). The $P6$ value of 1,314.6 Myr was obtained by propagating the preferred $P5$ center using the frozen $\times 3$ transition and is diagnostic rather than independently observed. $P7$ remains a conceptual extrapolation without direct

empirical evaluation. Taken together, the synthesis indicates that the principal and distribution-level recurrence centers are compatible with a common cross-scale multiplicative organization, while preserving the distinctions between confirmatory evidence, exploratory alternatives, propagated diagnostics, and conceptual extensions.

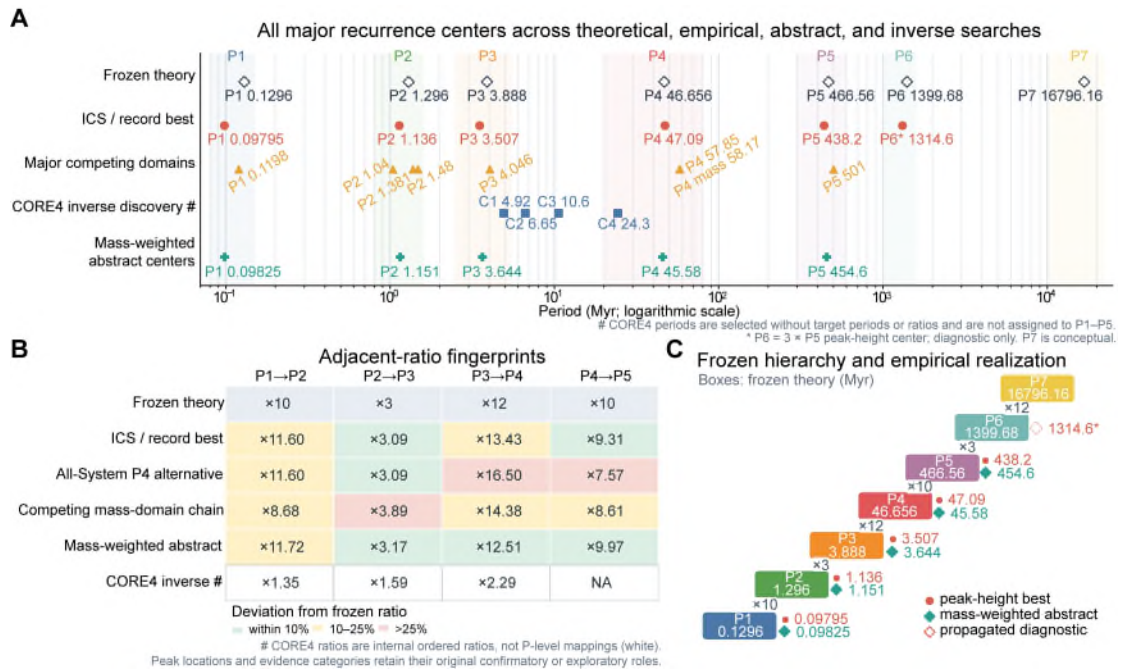


Figure 6 | Cross-scale synthesis of recurrence centers and multiplier structure.

(A) Major recurrence centers identified by the different analytical strategies, displayed on a common logarithmic period axis. Open diamonds indicate the frozen theoretical centers; red circles indicate the preferred record- or ICS-derived peak-height centers; orange triangles indicate major competing recurrence domains; blue squares indicate the ordered candidate periods selected by the theory-free CORE4 inverse search; and teal crosses indicate the mass-weighted abstract centers calculated across the retained peak distributions. Pale vertical bands indicate the predefined P1–P7 recurrence domains. The P6 value of 1,314.6 Myr was propagated as three times the preferred P5 peak-height center and is diagnostic only. P7 is conceptual and was not empirically tested. CORE4 periods were selected without supplying the target periods or multiplier sequence and are not assigned to P1–P5.

(B) Adjacent-period ratios for the frozen hierarchy and alternative empirical representations. The preferred record/ICS chain yielded ratios of 11.60, 3.09, 13.43, and 9.31. The all-System P4 alternative substitutes the 57.85-Myr P4 center while retaining the other preferred centers. The competing mass-domain chain was constructed from the retained competing centers at 0.1198, 1.040, 4.046, 58.17, and 501 Myr. The mass-weighted abstract centers yielded ratios of 11.72, 3.17, 12.51, and 9.97. Cell colors indicate multiplicative deviation from the corresponding frozen ratio: green, within 10%; yellow, 10–25%; and red, greater than 25%. The CORE4 row reports internal adjacent ratios only. Because the theory-free periods were not assigned to P1–P5, deviation-based cell colors were not applied. The fourth cell is marked NA because four CORE4 periods define only three adjacent ratios.

(C) Schematic realization of the frozen P1–P7 hierarchy. Colored boxes show the theoretical

periods and connecting labels show the frozen $\times 10$, $\times 3$, $\times 12$, $\times 10$, $\times 3$, and $\times 12$ transitions. Red circles show the preferred peak-height centers, teal diamonds show the mass-weighted abstract centers, and the open red diamond indicates the propagated P6 diagnostic. Periods are expressed in Myr. Peak positions and evidence categories retain their original confirmatory, exploratory, diagnostic, or conceptual roles.

Discussion

This study developed a predefined multiscale framework for testing whether recurrence structures across climate, chronostratigraphic, and deep-time records conform to a common multiplicative hierarchy. The principal empirical centers identified for P1–P5 did not reproduce the frozen target periods exactly, but they retained the predicted ordering and broadly preserved the $\times 10 \rightarrow \times 3 \rightarrow \times 12 \rightarrow \times 10$ transition structure. The preferred peak-height chain was statistically supported when the GSSP-defined P4 center near 47 Myr was used, including after correction for multiple testing. Moreover, integrating the complete retained peak distributions produced mass-weighted abstract centers with even closer ratio agreement, with three of four adjacent transitions falling within 10% of the frozen multipliers. These results suggest that the proposed organization is expressed more clearly at the level of recurrence domains than through exact individual spectral maxima.

The distinction between recurrence domains and exact periodicity is important. Geological and climatic records are affected by changing boundary definitions, chronological uncertainty, incomplete preservation, and scale-dependent physical processes. A hierarchical signal should therefore not be expected to behave as a perfectly stationary clock. The competing P4 centers illustrate this point: the all-System analysis favored a broad domain near 58 Myr, whereas restricting the analysis to GSSP-defined boundaries strengthened the approximately 47-Myr center and substantially improved its cross-scale compatibility. Similarly, uncertainty inflation broadened the P5 landscape but did not generate a fundamentally new optimum. These observations indicate that alternative local maxima can coexist within individual domains, while their cross-scale relationships provide an additional criterion for evaluating them.

The hierarchy was supported unevenly by external evidence. Frozen out-of-sample tests and post-OOS analyses identified several compatible database-level signals, but these did not constitute uniform independent replication across all scales or datasets. The target multiplier rule also ranked favorably relative to alternative multiplier sequences, although some model-competition and independent-null tests were not statistically significant. Conversely, the strict theory-free CORE4 search did not recover a stable multiplicity-significant hierarchy, with only an approximately 6.65-Myr single-period indication. This negative result limits any claim that the hierarchy can currently be rediscovered *de novo* from a small collection of heterogeneous external databases. At the same time, it provides a useful contrast with the stronger results obtained when the predefined structure was tested using the primary chronostratigraphic framework.

The P6 analysis should likewise be interpreted as a closure diagnostic rather than an independent discovery. Propagating the preferred P5 peak-height center produced a P6 estimate of 1,314.6 Myr, which yielded a deep-time endpoint compatible with estimates of Earth and Solar System formation. However, P6 was derived from the preceding hierarchy and was not measured from an independent P6-scale record. P7 remains entirely conceptual. These higher-order extensions therefore demonstrate internal consistency and generate testable future expectations, but they should not contribute to the inferential significance of the observed P1–P5 hierarchy.

Several limitations remain. The analyses combine continuous climate records, stratigraphic boundaries, and external event databases that differ substantially in resolution, uncertainty, completeness, and dependence structure. Chronostratigraphic boundaries are also historically and geologically related and cannot automatically be regarded as statistically independent observations. Although timing perturbations, nested null models, multiple-testing correction, competing rules, and out-of-sample analyses reduce the likelihood of simple overfitting, they cannot establish a common physical mechanism. The numerical multipliers should therefore be interpreted as an empirical organizational hypothesis rather than evidence for a universal deterministic clock.

Overall, the results identify a coherent but non-exact recurrence hierarchy spanning orbital–climate to deep-time scales. Its strongest support arises from the convergence of predefined testing, GSSP-based sensitivity analysis, cross-scale ratio organization, and distribution-level abstraction, rather than from any single peak or database. Future high-resolution chronologies and genuinely independent event records will be required to determine whether this hierarchy reflects a shared Earth-system process, interactions among distinct scale-dependent mechanisms, or a reproducible statistical organization without a single causal driver.

Methods

Data sources and preprocessing

Two principal data structures were analyzed: continuous Earth-system records and discrete geological boundary or event ages. Continuous P1 records included the LR04 benthic $\delta^{18}\text{O}$ stack⁴, reconstructed global temperature, global sea level¹³, ice volume, atmospheric CO_2 ¹⁴, and the La2010 orbital solution^{12,15}. Additional P2 records included CENOGRID benthic $\delta^{18}\text{O}$ and $\delta^{13}\text{C}$, orbital eccentricity, and palaeointensity data^{11,16–19}. Orbital solutions were treated as positive controls or forcing references rather than independent geological replications.

Chronostratigraphic names, ranks, numerical ages, and boundary-definition status were obtained from the International Chronostratigraphic Chart used in this study^{10,20}. Duplicate ages representing the same physical boundary were collapsed within each stratigraphic rank. Stage-level boundaries were used for P2 and P3, System-level boundaries for P4, and Erathem-level boundaries for P5. P2 and the younger P3 analysis used the 2.58–66 Ma Stage record, whereas P3 was also evaluated using the full approximately 2.58–538 Ma Phanerozoic

Stage record. The ICS Series level was not used.

External OOS and triangulation datasets comprised published compilations of extinction events^{5,21,22}, impact structures²³, large igneous provinces (LIP)²⁴, and zircon U–Pb ages²⁵. PINT Palaeointensity²⁶ and additional ICS-derived records were included in designated sensitivity analyses^{10,20}. These databases differ in temporal coverage, resolution, completeness, and event definition and were therefore treated as orthogonal evidence sources rather than automatically as statistically independent replications.

Rows with missing or nonfinite ages or measurements were removed. Continuous records were sorted by age, duplicate ages were averaged, and each record was restricted to its predefined analysis interval. Detrending and standardization were performed within each analysis window. Regular interpolation was applied only when required for filtering or time-domain visualization; Lomb–Scargle calculations used the original unevenly sampled ages.

Dense scans of continuous records

Continuous records were analyzed using dense Lomb–Scargle periodograms because geological time series were not uniformly sampled^{27,28}. P1 was scanned primarily over 0.08–0.15 Myr, and P2 continuous records were scanned over the corresponding lower-Myr search interval. No preference was imposed for the theoretical target during the dense scans.

Spectral power was normalized within each dataset to facilitate comparison among records with different units and variances. Local maxima were detected from the normalized spectra and ranked by peak height. For pooled summaries, records were first combined within related evidence families and then across families, preventing multiple closely related proxies from dominating the consensus. Positive controls were displayed separately and were not included in evidence-only pooled estimates.

Dense periodograms were also calculated over broader ranges to visualize orbital-band structure, including the approximately 41-kyr and broad approximately 100-kyr components. For late Quaternary time-domain displays, zero-phase Butterworth filters were used to isolate the 35–50-kyr and 80–150-kyr bands without introducing artificial phase shifts.

When statistical testing of continuous candidate periods was required, surrogate records preserving the approximate autocorrelation structure of the original series were generated. Observed spectral support was compared with the corresponding surrogate distribution. Family-wise correction within a candidate set was obtained from the maximum candidate support in each surrogate realization. Records sharing orbital tuning or closely related age models were not counted as fully independent confirmations.

For each candidate period, geological boundary ages were converted to positions within a repeating cycle. Their concentration around a common cycle phase was quantified using a circular

resultant-based fit score, denoted $score_R$ ²⁹. Larger values indicate stronger collective alignment of the observed boundaries with the fitted periodic model. The lag-one autocorrelation of each detrended record was estimated, and 2,000 AR(1) surrogate series of the same length were generated³⁰.

Blind recurrence-domain scans of discrete ages

Discrete boundary and event ages were scanned across predefined domain-specific period ranges: approximately 0.8–1.6 Myr for P2, 2.5–5.5 Myr for P3, 20–80 Myr for P4, and 300–600 Myr for P5. The grids were sufficiently dense to resolve narrow local maxima; P3 was evaluated at 0.001-Myr resolution in the final scans.

For a candidate period (P), each age (t_i) was mapped to a circular phase. Phase coherence was quantified using the Rayleigh resultant:

$$R(P) = \left| \frac{1}{n} \sum_{k=1}^n \exp \left(2\pi i, \frac{t_k}{P} \right) \right|.$$

Values of (R) range from 0 to 1, with larger values indicating stronger concentration around a common cycle phase. The phase of the circular mean defined the corresponding phase-aligned lattice. The distance from each event to its nearest lattice point was used only as a descriptive alignment measure because shorter periods inherently generate more closely spaced lattice points.

Peak height and domain-wide support were reported separately. Peak height was defined by the median or unperturbed (R) value at a local maximum. Integrated domain mass summarized the area of a high-and-wide recurrence domain above its local baseline. Peak prominence was retained as an additional descriptive property. This separation prevented a narrow high maximum and a broader moderate domain from being treated as equivalent forms of evidence.

Chronological uncertainty and stress tests

When published age uncertainties were available, ages were repeatedly sampled within their reported uncertainty intervals and the complete scan was recalculated. The median recurrence curve and its 95% chronology range were retained. The probability that a period or recurrence domain became the blind-search winner was calculated across perturbation realizations. This probability measures stability under chronological uncertainty and is not a statistical p value.

Because the 2.58–66 Ma P2 Stage boundaries did not have event-specific uncertainty estimates suitable for this procedure, robustness was examined using artificial timing jitter. Independent age perturbations were introduced over progressively larger half-widths from 0 to ± 0.2 Myr, and peak location, median coherence, uncertainty intervals, and the distribution of blind-search winners

were recorded. The persistence of a winner near 1.136 Myr was evaluated without using the theoretical target in peak selection.

For P5, reported uncertainties were inflated over a series of multipliers up to three times their original magnitude. The complete P5 landscape was rescanned at every uncertainty level to determine whether uncertainty inflation caused peak replacement or generated previously absent dominant domains.

Empirical global null tests

The statistical significance of blind recurrence scans was evaluated using nested empirical null models. Null event sets preserved the observed number of ages and analysis interval but removed their temporal organization. Each null dataset was passed through the same complete blind scan, peak detection, and domain-ranking procedure as the observed data.

Two complementary global statistics were evaluated. The peak-height test compared the largest observed recurrence maximum with the distribution of maximum peak heights obtained from the full null scans. The domain-mass test compared the largest observed integrated domain mass with the maximum mass obtained from each null scan. These procedures incorporate the look-elsewhere effect across the entire scanned range rather than testing only a period selected from the observed data.

Monte Carlo p values were calculated as

$$p_{\text{emp}} = \frac{b+1}{B+1}$$

where (b) is the number of null statistics at least as extreme as the observed value and (B) is the number of null realizations³¹. Peak-height and domain-mass p values were interpreted separately and were not combined into a single statistic.

P4 boundary-definition sensitivity

The P4 analysis was repeated using all System boundaries and a GSSP-defined subset. The Jurassic–Cretaceous boundary used in the chart lacked a ratified GSSP definition and was therefore examined as a predefined boundary-status sensitivity rather than being removed solely because of its effect on the result.

Leave-one-boundary-out analyses quantified the influence of every System boundary on the relative support for the approximately 58- and 47-Myr domains. The strongest blind-search period was recalculated after deleting each boundary in turn. Equal-size subset analyses were additionally

used to evaluate whether the shift observed for the GSSP-defined set exceeded that expected from deleting an arbitrary boundary. Phase-aligned lattices were examined to identify the nearest unconstrained lattice positions compatible with the theoretical P4 period, the empirical approximately 47-Myr center, and the propagated P5-based alternative. These lattice positions were descriptive and were not used to redefine the official ICS boundaries.

Cross-scale hierarchy tests

The primary empirical hierarchy used the preferred peak-height centers for P1–P5: 0.09795, 1.136, 3.507, 47.09, and 438.2 Myr. The P4 value from the GSSP-defined sensitivity analysis was selected before the final hierarchy test. Alternative analyses used the all-System P4 peak near 57.85 Myr and the corresponding domain-mass centers.

Agreement with the frozen hierarchy was quantified in log space. Ratio alignment was measured from the root-mean-square deviation between the observed adjacent ratios and the theoretical multiplier sequence. Absolute alignment measured the log-scale deviation between the observed centers and their corresponding frozen targets. Lower scores indicate closer alignment. Logarithmic scores were used because the tested periods span several orders of magnitude and because proportional deviations are more relevant than absolute differences.

Empirical significance was assessed against independent search-range nulls. Null centers were generated within the same P1–P5 search domains and passed through the identical peak-selection and hierarchy-scoring procedures. Separate tests were performed for peak-height and domain-mass centers and for ratio-based and absolute alignment. Holm correction was applied across the four primary hierarchy tests³².

The primary hierarchy used the highest-ranked peak in each domain. A rank-adaptive sensitivity analysis additionally allowed peaks up to rank five, subject to a predefined rank-cost budget. The same adaptive search was repeated for every null realization so that the additional selection flexibility was included in the null distribution. Leave-one-domain-out analyses were used to identify whether the hierarchy result depended disproportionately on a single scale.

The frozen multiplier sequence was also compared with competing models. Comparisons included all unique permutations of the same multiplier multiset and all 81 four-transition rules generated from $(\{3,10,12\}^4)$. Models were ranked by log-scale ratio error. Separate empirical null models tested whether the favorable rank of the frozen rule was stronger than expected when both recurrence centers and multiplier rules were searched.

Distribution-level abstract centers

To summarize the full recurrence landscapes rather than only their highest maxima, detected peak

domains were converted into abstract centers. Peak domains touching the edges of a search interval were flagged because their centers could lie outside the analyzed range. Overlapping or nonindependent detections were consolidated before the primary abstraction analysis.

The primary formulation, designated Method B, calculated an arithmetic center across independent retained domains using integrated domain mass as the weight. Secondary weighting schemes used peak height, peak prominence, or equal weights. A direct weighted summary of all detected peaks without independent-domain consolidation was retained as Method A and was treated as a secondary sensitivity analysis. Because an abstract center summarizes a distribution of peaks, it can fall between local maxima or within a spectral valley and should not be interpreted as a newly detected spectral peak.

Adjacent ratios were calculated from the P1–P5 abstract centers and compared with the frozen multipliers. Conditional empirical null models preserved the number, approximate widths, and weighting structure of the detected domains while disrupting their cross-domain alignment. Both rigid and empirically defined search windows were examined. The mass-weighted formulation was the primary analysis, and Holm adjustment was applied across the four weighting schemes.

Frozen out-of-sample and post-OOS analyses

Before evaluation of external databases, the target periods, hierarchy, search windows, primary statistics, and interpretation rules were frozen. Hierarchy-fixed models evaluated the predefined or preferred ICS-derived periods without retuning. Train-local-best models, in which a best period was selected from a training interval, were retained as secondary analyses and were not substituted for failed frozen predictions.

Within an external event database, significance was evaluated using null procedures that preserved major database characteristics, including temporal span, event number, and, where appropriate, internal clustering. Circular temporal shifts were used for ratio-transfer analyses when preservation of the internal event pattern was required. Fixed-period transfer and train-best transfer were distinguished explicitly.

Analyses designed after examination of the OOS results were labeled post-OOS. These included pair-level ratio alignment, cross-database shared-scale analyses, and rank-adaptive examination of local peaks near the frozen ICS periods. Post-OOS p values were reported descriptively and were not used to upgrade the primary confirmatory conclusion. External databases were treated as heterogeneous evidence sources rather than independent copies of the ICS analysis.

Theory-free inverse discovery

A strict theory-free inverse analysis was performed to determine whether related periods or

multiplier chains could be recovered without supplying the P1–P5 targets or the frozen ratios. The primary CORE4 set comprised extinction, impact, large igneous province, and zircon databases. ICS and palaeointensity records were added only in predefined sensitivity variants.

Three broad period ranges were searched: a young 0.5–10-Myr range, a Phanerozoic 2–100-Myr range, and a deep-time 20–800-Myr range. The oldest 75% of each eligible record was used for discovery and the younger 25% for temporal holdout evaluation. Candidate solutions near the outer 10% of a search interval were flagged as edge solutions. Support required contributions from at least two eligible databases where coverage permitted.

Single periods, unconstrained period sets, shared-ratio solutions, and integer-ratio chains were evaluated without reference to the frozen hierarchy. Dataset-specific null distributions were used to normalize evidence before cross-database combination. Model-search nulls repeated the complete period and ratio search, thereby accounting for selection among many possible models. Final theory-free analyses used 801 period-grid points, 5,000 dataset-level null realizations, 10,000 model-search realizations, and the fixed random seed 20260811. Holm correction was applied across the tested model families.

P6 endpoint-closure diagnostic

P6 was evaluated as a diagnostic propagation rather than a directly observed recurrence domain. Three candidates were compared: the frozen theoretical value of 1,399.68 Myr, three times the preferred P5 peak-height center, and three times the P5 domain-mass center. For a P6 candidate (P_6), the implied deep-time endpoint was calculated from the base of the Phanerozoic plus three P6 intervals and compared with reference ages for Earth and Solar System formation^{33,34}.

Residuals were expressed as percentages of the P6 value required to reach each endpoint. Fixed $\pm 5\%$ and $\pm 10\%$ bands were used as descriptive tolerance ranges and were not interpreted as confidence intervals or measurement uncertainties. P6 was excluded from hierarchy p values because the propagated candidates were mathematically derived from P5. P7 was conceptual and was not statistically tested.

Statistical reporting and reproducibility

All tests were two-sided or upper-tailed as appropriate to their predefined statistic, and exact empirical p values were reported whenever possible. Holm adjustment was used for prespecified families of related tests. Raw, adjusted, local, global, confirmatory, sensitivity, and exploratory results were labeled separately. A nonsignificant result was not reclassified on the basis of a favorable peak rank alone.

Final computationally intensive hierarchy null analyses used up to 200,000 realizations.

Theory-free dataset and model-search nulls used 5,000 and 10,000 realizations, respectively; other scan-level and sensitivity analyses used at least 2,000 realizations unless otherwise specified in the corresponding output. All random analyses used fixed seeds.

Analyses were conducted in Python using NumPy, pandas, SciPy, Astropy, Matplotlib, and related scientific-computing libraries. Geological ages are reported in Ma, whereas recurrence periods are expressed in kyr, Myr, or Gyr as appropriate. Analysis scripts, processed input tables, parameter records, and figure-generation code were retained to support reproducibility.

Declarations

Author contributions:

Y.J. conceived and designed the study, collected and curated the datasets, developed the computational methods, performed all analyses, interpreted the results, prepared the figures, and wrote the manuscript. The author approved the final version of the manuscript and accepts responsibility for the integrity of the work.

Data availability:

All datasets analyzed in this study were obtained from publicly accessible repositories and published data sources. The chronostratigraphic data were derived from the International Chronostratigraphic Chart. Climate, isotope, sea-level, ice-volume, atmospheric CO₂, magnetochron, orbital, extinction events, impact structures, large igneous provinces (LIP), zircon U-Pb ages⁵, and PINT Palaeointensity datasets were obtained from their respective public repositories and publications.

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