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Coordinating Mathematical Structure and Pedagogical Appropriateness: Preschool Teachers' Problem Posing Across Differently Structured Tasks

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Abstract

Problem posing is widely recognised as a central mathematical activity, yet little is known about how preschool teachers coordinate the pedagogical and mathematical demands of posing problem situations for young children. This qualitative study examined 15 preschool teachers' conceptions, reported classroom practices, professional learning needs, and performance across structured, semi-structured, and visually supported open problem-posing tasks. Teachers responded to two visual contexts construction materials and grocery shopping and produced 90 problem situations. Interview data were analysed through descriptive and pattern coding, while task responses were classified as not a problem, non-mathematical, impossible, insufficient, or sufficient. Teachers consistently valued play, concrete materials, everyday-life contexts, and child participation. However, these pedagogical orientations did not always become mathematically explicit and coherent problem situations. Of the 90 responses, 68 were sufficient, 16 insufficient, and 6 were not problems; no response was classified as non-mathematical or impossible. Performance was strongest in structured tasks and became more variable as task structure decreased. All responses classified as not a problem occurred in the open tasks. The two visual contexts yielded the same overall number of sufficient responses but foregrounded different mathematical possibilities. The findings suggest that preschool teachers' problem-posing competence is best understood as the coordination of pedagogical appropriateness, an explicit mathematical relationship, and a meaningful requirement for children's action. Open prompts did not automatically produce productively open mathematical problems; instead, they transferred greater responsibility for selecting the mathematical relationship, coordinating information with the requirement, and specifying a meaningful goal. Open task formats may therefore reveal, rather than automatically promote, this coordination. Implications are proposed for practice-based teacher education centred on comparing, revising, and discussing problem situations.

1 Introduction

Mathematical problem posing—the generation of new problems and the reformulation of existing ones has long been treated as both a companion to problem solving and a mathematical activity in its own right (Silver, 1994, 1997). Recent scholarship has renewed attention to problem posing as an instructional tool, a learning goal, and an object of research (Cai & Leikin, 2020; Cai et al., 2022; Liljedahl & Cai, 2021). This renewed interest reflects a broader view of mathematics learning in which reasoning, representation, connection, explanation, and generalisation depend not only on solving given tasks but also on recognising and constructing situations that are mathematically problematic.

The issue is particularly consequential in early childhood education. Young children encounter quantity, space, pattern, measure, comparison, classification, and data in play and everyday routines. Yet the presence of a familiar context or manipulatives does not, by itself, make an activity mathematically problematic. Asking a child to count objects may elicit a known procedure; asking how three plates could be shared among five children so that everyone has one may invite the child to identify a quantitative discrepancy, represent what is missing, and consider multiple actions. The latter situation coordinates a

meaningful context, a mathematical relationship, information, and a requirement that makes action or explanation necessary (Freudenthal, 1968, 1981; Lesh, 1981; Van den Heuvel-Panhuizen & Drijvers, 2014).

Research on problem posing has concentrated primarily on school-aged students, prospective teachers, and teachers of later grades (English, 1998; Guo et al., 2021; Leikin & Elgrably, 2020; Xie & Masingila, 2017). Studies with young children show that representations, real-world interests, and task variables can shape the problems children pose (Kwon & Capraro, 2021; Carmona-Medeiro et al., 2024). Teacher-focused studies indicate that posing mathematically productive tasks draws on subject-matter knowledge, pedagogical content knowledge, sensitivity to learners, and the capacity to design or transform tasks (Crespo, 2003; Crespo & Sinclair, 2008; Lee et al., 2018; Klein & Leikin, 2020; Marco & Palatnik, 2024; Martín-Díaz & Montes, 2022). Klein and Leikin (2020), for example, showed that teachers' ways of opening tasks were related to their experience and that perceived difficulty in posing open tasks was connected to confidence and willingness to use such tasks. Marco and Palatnik (2024) further demonstrated that context-based tasks can be examined through dimensions such as correctness, authenticity, task assortment, task flow, and student involvement. In early childhood education, however, the teacher's challenge is distinctive: a problem situation must remain developmentally accessible and connected to play or experience while also preserving an identifiable mathematical relationship and a genuine need to reason.

This challenge creates a tension between pedagogical attractiveness and mathematical structure. A teacher may design an engaging story, game, or hands-on activity that is appropriate for young children but does not contain a mathematical requirement. Conversely, a formally correct numerical question may be mathematically explicit but poorly connected to children's experience or possibilities for action. Treating these dimensions separately obscures the competence required to coordinate them. Problem-posing competence in preschool settings therefore cannot be inferred only from teachers' stated commitment to play, concrete materials, or child-centred learning, nor only from the grammatical presence of a question. It must be examined through the problem situations teachers actually produce.

A second unresolved issue concerns task structure and the meaning of openness. Problem-posing tasks are often described as structured, semi-structured, or free according to the amount of information and constraint provided to the poser (Stoyanova & Ellerton, 1996). Yet recent scholarship cautions that openness should not be treated as a single undifferentiated task property. Völlinger et al. (2026) distinguish openness in the initial state (missing or unspecified information), the intermediate state (more than one possible model, procedure, or solution path), and the goal state (a target that requires interpretation or specification). This distinction is important because productive openness differs from ambiguity, under-specification, or the absence of a mathematical problem. The present study therefore differentiates the openness of the prompt presented to the teacher from the openness and quality of the problem subsequently posed for children.

The present study addresses these gaps by examining the same preschool teachers' conceptions and reported practices together with their direct performance across differently structured tasks and two visual contexts. Rather than treating the teachers' products as isolated question sentences, the study analyses problem situations as coordinated configurations of context, mathematical relationship, information, and requirement. The study asks what teachers say they value, what they find difficult, and what becomes visible when they are required to pose problems under decreasing levels of task structure.

2 Theoretical framing

2.1 Problem posing as the construction of a problem situation

Problem posing can involve producing a new problem under specified conditions or transforming an existing problem into another one (Leung, 1993; Silver, 1994). In the present study, the unit of analysis is not merely an interrogative sentence. It is a problem situation in which a context, a mathematical relationship, available information, and a requirement are sufficiently coordinated to invite a response, action, solution, or explanation.

Drawing on Malaspina's work and the model proposed by Burgos et al. (2025), four components are especially relevant. Context refers to the real or imagined situation in which the problem is located. Environment refers to the mathematical domain or general structure. Information consists of the data and conditions available to the solver. Requirement specifies what must be found, explained, decided, or constructed. A prompt may sound child-friendly and still fail as a mathematical problem if it lacks a mathematical relationship or requirement. Conversely, a mathematically determinate question may offer little opportunity for meaningful participation if the context and expected action are not accessible to young children.

For preschool mathematics, this coordination includes a developmental and pedagogical dimension. Play, stories, visual representations, manipulatives, and routine events can provide powerful settings, but they become problem situations only when they orient children toward relationships such as quantity, comparison, measure, pattern, classification, spatial organisation, data, or reasoning. The teacher therefore needs to mathematise a situation without stripping it of experiential meaning (Björklund & Palmér, 2024; Gravemeijer, 2016).

2.2 Task structure and responsibility for mathematical specification

Structured, semi-structured, and visually supported open prompts differ in how much of the posing situation is specified for the teacher (Stoyanova & Ellerton, 1996). In structured prompts, the mathematical relationship and expected form of the problem are comparatively constrained. Semi-

structured prompts specify a context or broad mathematical purpose while leaving some decisions to the poser. In visually supported open prompts, the context is provided but the mathematical focus, information, requirement, and degree of openness must largely be selected by the teacher. This study uses these terms to characterise the problem-posing prompt, not automatically the product created by the teacher. A response to an open prompt may still be a closed problem, and a response with missing information may be merely ambiguous rather than productively open.

Task variables including context, numerical information, representation, prompt wording, and level of constraint shape both posing processes and products (English, 1998; Leung & Silver, 1997; Zhang et al., 2022; Cai et al., 2022). To analyse the products more precisely, the present study also draws on the distinction among initial-, intermediate-, and goal-state openness (Völlinger et al., 2026). Initial-state openness concerns whether children must identify or assume information; intermediate-state openness concerns whether several models, strategies, or representations are possible; and goal-state openness concerns whether the problem objective itself requires interpretation or decision. These forms of openness are analytically distinct from insufficient information or unclear wording.

2.3 Coordinating mathematical and pedagogical demands

Teacher problem posing is a didactic-mathematical practice. It can be directed toward particular concepts, strategies, anticipated difficulties, or forms of reasoning (Burgos et al., 2025; Malaspina et al., 2019). The mathematical dimension concerns whether a posed situation contains a discernible relationship and can be acted on or reasoned about. The pedagogical dimension concerns developmental appropriateness, meaningfulness, connection to the context, and the opportunity afforded for children's participation. In early childhood education these dimensions are inseparable: a problem must be mathematically consequential without becoming detached from children's ways of acting, representing, and communicating.

The analytical stance of this study is consequently relational. Sufficient problem posing is not defined by mathematical correctness alone, by openness alone, or by the use of an attractive context. It is defined by coordination among context, mathematical focus, information, requirement, task demand, and child action. This stance informed the integrated analytical framework shown in Fig. 1. The framework is not proposed as a validated general theory; it is a study-specific synthesis used to examine how teachers' products satisfy or fail to satisfy— he coordinated demands of early mathematics problem posing.

A further distinction concerns reported orientation and demonstrated performance. Problem posing has been described as a low-floor, high-ceiling activity: participation may be accessible, whereas posing mathematically demanding, unambiguous, and non-routine problems requires substantially more knowledge and judgement (Cai, 2022; Cai et al., 2026). Cai et al. (2026) also showed that task-specific self-evaluation in problem posing does not align with performance in a simple linear manner. Although the present study did not measure teacher self-efficacy, this work supports examining teachers' reported classroom orientations separately from the mathematical quality visible in their task products.

3 Research questions

The study addressed four questions:

1. How do preschool teachers conceptualise early mathematics problem posing?

2. What approaches and criteria do they report using when creating early mathematics problem situations?

3. What difficulties and professional learning needs do they identify?

4. How does the quality of their posed problem situations vary across task structure and visual context?

4 Methods

4.1 Design

The study used a qualitative descriptive design (Sandelowski, 2000, 2010) combining semi-structured interviews with task-based elicitation. Qualitative description was selected because the study sought a detailed, data-near account of how teachers conceptualised and reported using early mathematics problem situations, while also documenting the products they generated under shared task conditions. The design did not aim to derive a grounded theory or to reconstruct the essence of experience. Instead, it supported systematic comparison of teachers' professional accounts, the recurring patterns within those accounts, and the mathematical-pedagogical characteristics of their posed products.

The two data sources served complementary but analytically distinct purposes. Interviews elicited teachers' definitions, perceived contributions of problem situations, reported classroom practices, selection criteria, experienced difficulties, and professional learning needs. The task component provided direct evidence of what teachers produced when required to coordinate a visual context, mathematical focus, information, and a prompt at different levels of structure. Bringing these sources together made it possible to examine convergence and tension between espoused pedagogical orientations and observable task-design performance. This product-oriented perspective is consistent with research that treats teachers as designers of mathematical opportunities and examines posed tasks not only for formal correctness but also for authenticity, diversity, flow, and student involvement (Marco & Palatnik, 2024).

4.2 Participants and context

Participants were 15 practising preschool teachers working in different early childhood institutions in Türkiye. Purposeful sampling was used to include teachers who were actively responsible for preschool classrooms and had experience implementing early mathematics activities. Eligibility criteria were active employment as a preschool teacher, prior experience conducting early mathematics activities, and

voluntary agreement to participate. The group included 11 women and four men, with reported professional experience ranging from 3 to 22 years. Fourteen participants held a bachelor's degree and one held a master's degree. Teachers worked with four-year-old, five-year-old, or mixed-age groups; reported class sizes ranged from 4 to 25 children. Most reported conducting mathematics activities several times per week or integrating them into daily routines. This variation was relevant because it allowed the study to examine problem-posing products across differing levels of professional experience and classroom conditions, rather than restricting the sample to novice teachers. Participants were assigned codes T1-T15 and identifying institutional information was not reported.

Table 1
Participant characteristics

Teacher	Gender	Age	Degree	Experience	Age group	Class size	Math frequency
T1	Woman	48	Bachelor's	18	Age 5	9	Daily
T2	Woman	44	Bachelor's	19	Age 5	17	3–4 days/week
T3	Woman	38	Bachelor's	16	Age 4	14	3–4 days/week
T4	Woman	38	Bachelor's	15	Mixed	21	3–4 days/week
T5	Woman	44	Master's	15	Age 5	20	Daily
T6	Woman	34	Bachelor's	10	Age 5	15	Almost daily
T7	Woman	31	Bachelor's	7	Mixed	4	Daily
T8	Woman	28	Bachelor's	5	Not reported	13	Not reported
T9	Woman	39	Bachelor's	15	Age 4	19	3–4 days/week
T10	Woman	25	Bachelor's	3	Age 5	10	2–3 days/week
T11	Woman	NR	NR	NR	Mixed, ages 4–5	25	Daily
T12	Man	32	Bachelor's	9	Not reported	5 currently	About 3 days/week
T13	Man	34	Bachelor's	10	Ages 4–6	16	About 3 days/week
T14	Man	33	Bachelor's	8	Age 5	17	2–3 days/week
T15	Man	45	Bachelor's	22	Age 5	18	3 days/week

4.3 Data collection instruments

The data collection form had four parts: (a) demographic and professional information; (b) semi-structured questions about knowledge and experience of mathematical problem posing; (c) visual problem-posing tasks; and (d) questions about experienced difficulties and professional development. The interview section asked how teachers defined problem posing, how problem situations might contribute to children's mathematical thinking, where such situations were used in classroom activity, who usually initiated them, whether children were given opportunities to pose problems, what criteria guided problem selection or creation, and which forms of support teachers considered necessary. These questions were deliberately broad so that participants could illustrate their responses through classroom routines, play, stories, drama, materials, and child-generated examples rather than respond only to abstract definitions.

The task section used two visual contexts chosen to provide contrasting but familiar starting points for mathematization. The first showed coloured construction blocks, a common manipulative that could support relationships involving number, colour, length, grouping, classification, pattern, equalising, comparison, and spatial organisation (Fig. 2). The second showed a family shopping in a grocery store, providing possibilities involving counting, classification, comparison, completing missing quantities, sharing, budgeting, money, and everyday quantitative reasoning (Fig. 3). The contrast was purposeful: the construction-material image foregrounded perceptual and spatial attributes, whereas the shopping scene foregrounded everyday quantitative relations. In line with research showing that context and representation shape what problem posers notice and produce (Cai et al., 2022; Carmona-Medeiro et al., 2024), the two images were used to examine whether task quality and mathematical focus varied across representational settings.

Both images were generated with ChatGPT according to researcher-defined design criteria: developmental appropriateness for preschool children, recognisable objects and actions, limited distracting detail, and the presence of several potential mathematical relationships rather than one predetermined operation. The research team reviewed successive versions for alignment with the study purpose and retained the final images because each offered multiple plausible routes for posing early mathematics problems. The images functioned as elicitation materials, not as instructional interventions; they were not presented as validated assessment items or as equivalent measures of a single latent construct.

For each image, teachers were asked to create three kinds of problem situation: structured, semi-structured, and visually supported open. Structured posing referred to situations in which the intended mathematical relationship, relevant information, and expected response were comparatively explicit (e.g., determining how many blocks remain after some are removed). Semi-structured posing specified a broad context or mathematical purpose while leaving the strategy, representation, or some decisions to the child (e.g., deciding how two towers could be made equal). Visually supported open posing retained the image as a common starting point but left the mathematical focus, information to be selected, and

solution pathway largely to the teacher. Thus, open did not mean posing from an entirely blank situation. Each teacher produced three responses for each of the two images, yielding six products per participant and 90 task products in total. The three task types were included because they place progressively greater responsibility on the poser for specifying the mathematical object, conditions, and requirement (Stoyanova & Ellerton, 1996; Klein & Leikin, 2020).

4.4 Procedure

Before data collection, participants received information about the purpose and scope of the study, the voluntary nature of participation, confidentiality, the intended use of the data, and their right to withdraw without penalty. Written informed consent was obtained before participation. Data collection proceeded in two stages. First, semi-structured interviews were conducted using open-ended prompts that encouraged teachers to draw on examples from their classrooms. Second, teachers completed the visual problem-posing tasks. The construction-material image was presented first and the shopping image second; for each image, responses were elicited in the order structured, semi-structured, and open. This fixed sequence should be considered when interpreting the findings because task order was not counterbalanced.

No correctness feedback, model answers, hints, or evaluative comments were provided while teachers completed the tasks. The purpose was to document teachers' current ways of noticing and coordinating context, mathematical relationship, information, and requirement, rather than to measure learning after instruction. The resulting products therefore represent initial responses under the stated elicitation conditions. This distinction is important because iterative feedback and sustained professional development can substantially alter teachers' task-design products (Marco & Palatnik, 2024), whereas the present study examined performance before any such design cycle.

Ethical approval

was granted by the **** University Rectorate, Human Research Ethics Committee (protocol no. 2026 – 171; decision letter no. E-34183927-020-00001188635, dated 23 March 2026). The committee unanimously determined that the study was ethically appropriate. Participation was voluntary, written informed consent was obtained, and participants were informed about confidentiality, data use, and withdrawal rights. Teacher identities were replaced with codes (T1-T15), and the data were used only for research purposes.

4.5 Data analysis

Analysis proceeded in three stages. First, participant characteristics were summarised descriptively to contextualise the sample. Second, interview transcripts were analysed through descriptive coding followed by pattern coding (Saldaña, 2025). All transcripts were read holistically to establish familiarity

and to preserve the relation between individual statements and the broader interview account. Descriptive codes remained close to the manifest content of teachers' statements for example, everyday-life connection, noticing mathematical relationships, generating a question or problem, producing a solution path, play-based practice, use of concrete materials, developmental appropriateness, difficulty with mathematical language, distinguishing task types, workshop need, example banks, mentoring, and technology integration. Codes were then compared across participants and grouped into broader patterns and themes. Frequencies in the interview findings indicate the number of teachers who contributed at least one statement to a code, not the number of utterances; repeated statements by the same participant were counted once for that code.

Third, the 90 task products were analysed using an adaptation of Leung's (2013) evaluative categories: not a problem, non-mathematical, impossible, insufficient, and sufficient. A sufficient response contained a coherent mathematical relationship, enough information or a defensible invitation to make assumptions, a meaningful requirement, and developmental appropriateness. An insufficient response had mathematical potential but lacked coordination among context, information, mathematical focus, and requirement. In this coding, missing information was not automatically treated as a flaw: it was considered productive initial-state openness when the task clearly invited children to make reasonable assumptions. By contrast, information gaps that made the requirement indeterminate without such an invitation were coded as insufficient. Similar distinctions were made between multiple solution paths and vague wording, and between an interpretable open goal and the absence of a discernible goal (Völlinger et al., 2026).

After quality classification, responses were compared across structured, semi-structured, and open task formats and across the construction-material and shopping contexts. The analysis also considered the mathematical relationships made salient in the responses, including counting and operations, comparison and equalising, classification, pattern, measurement, sharing, spatial relations, and broader reasoning. Frequencies and percentages were used only as descriptive summaries. No inferential claims were made from the 90 products because responses were nested within 15 teachers and the study was not designed for statistical generalisation.

Table 2
Classification criteria for task-based responses

Category	Definition	Operational interpretation
Not a problem	No question or problem situation	A statement such as “I could not think of one,” “let the child find one,” or a general activity description without a requirement.
Non-mathematical	A problem-like statement without a mathematical relationship	A context is present, but no relationship involving number, measure, geometry, pattern, classification, comparison, data, or reasoning is identifiable.
Impossible	Contradictory or unsolvable with the information provided	Conditions are logically incompatible or the requested result cannot be determined or discussed.
Insufficient	Mathematical potential but incomplete coordination	The activity is potentially mathematical, but information, context, requirement, or mathematical focus is unclear.
Sufficient	Mathematically coherent, developmentally appropriate, and actionable	The problem largely fits the requested task type; the mathematical relationship and expected child action are identifiable.

Coding was conducted through a collaborative, consensus-based analytic process involving three researchers. The researchers reviewed the interview transcripts and task products, discussed the operational meanings and boundaries of the codes and categories, and revisited the original **** data when interpretations differed. Category definitions were refined iteratively during analysis, and final coding decisions were reached through discussion and agreement among the three researchers. Because the data were not coded independently, an intercoder agreement coefficient was not calculated; the procedure is therefore reported as collaborative coding rather than independent reliability testing.

4.6 Research quality and reflexivity

Research quality was supported through data-source triangulation, explicit decision rules, and attention to disconfirming cases. Teachers’ interview accounts were examined alongside the products they posed, allowing claims about competence to be checked against direct task evidence rather than inferred solely from self-report. Category definitions and inclusion boundaries were documented, and representative quotations and task examples were retained to make the analytical pathway visible. The analysis did not assume that pedagogically attractive contexts were necessarily mathematical problems; responses that challenged this assumption were examined closely. Similarly, the absence of non-mathematical and impossible responses was retained rather than forcing all theoretical categories to appear in the dataset.

The three-researcher team brought complementary backgrounds in early childhood education and mathematics education. These backgrounds supported attention to both developmental appropriateness and mathematical structure, while also creating a risk of interpreting teachers’ responses through researchers’ preferred pedagogical or mathematical norms. To address this risk,

interpretations were repeatedly checked against the full *** transcripts and the teachers' complete task responses rather than isolated quotations. Participants were not ranked or evaluated professionally, and the task was presented as a research activity rather than an appraisal of teaching performance. **** quotations were selected after the thematic analysis, translated into English for reporting, and checked against the original wording to preserve meaning, emphasis, and the level of certainty expressed by participants.

5 Findings

5.1 Teachers' conceptions: everyday experience, noticing, and solution activity

Teachers most often conceptualised early mathematics problem posing through everyday experience. Fourteen of the 15 teachers connected posing to situations children encounter in daily life. T1 described the central aim as "relating mathematics to life," whereas T2 defined posing as children "finding a solution path and creating a question about what they see in daily life." T3 offered a more explicit mathematical account: children notice mathematical situations in everyday life, express them, and produce solution paths. Across the transcripts, however, teachers more readily named contexts and child actions than the mathematical relationship, information structure, or requirement that would make the situation a coherent mathematical problem.

Teachers attributed both cognitive and socio-emotional value to problem situations. Reasoning, questioning, and analysis were mentioned by 13 teachers, and ten emphasised concretisation and durable learning. T3 stated that problem situations "make mathematics concrete and support more lasting learning." Creativity was explicitly mentioned by only one teacher. Socio-emotional benefits were less frequent: four teachers associated problem situations with reducing anxiety or increasing liking for mathematics, two with confidence and a sense of success, and one with collaboration. Thus, teachers' discourse foregrounded meaningful experience and reasoning, but did not consistently articulate the mathematical architecture of the problem situation itself.

Table 3
Summary of interview themes and frequencies

Theme	Code	n	Illustrative quotation
Conceptions	Everyday-life connection	14	“Children notice mathematical situations from everyday life.” (T3)
	Solution path/problem solving	9	“A process that enables children to produce solution paths.” (T3)
Perceived contribution	Reasoning, questioning, analysis	13	“Problems that push children to think and question.” (T14)
	Concretisation and durable learning	10	“Problem situations become concrete and meaningful.” (T11)
Classroom approach	Play-based practice	15	“It is generally play based.” (T5)
	Concrete materials	15	“Blocks, beads, magnets, and construction materials.” (T9)
	Everyday routines	14	“Attendance, weather, and counting routines.” (T9)
Design criterion	Real-life connection	14	“My basic starting point is life.” (T1)
	Appropriate difficulty	12	“Neither too difficult nor too easy.” (T4)
Difficulty	Developmentally appropriate posing	8	“It is difficult to pose a problem suitable for every child.” (T3)
	Generating a suitable problem/question	6	“Finding the right question forms.” (T1)
	Distinguishing task types	5	“In the open task, it turns into structured or semi-structured.” (T12)
Professional learning	Practice-based workshops	9	“Workshops would be more effective.” (T6)
	In-service and sustained support	8	“We most need in-service education.” (T12)

5.2 Reported classroom practices: strong pedagogical orientation

All teachers reported using play and concrete materials when creating mathematical situations. Fourteen referred to routines such as distributing materials or food, taking attendance, checking the weather, and counting classroom objects. Eight mentioned stories or drama. T10 described interrupting a story and asking, “What would you do to solve it?” These accounts show a repertoire of contexts that are developmentally plausible and potentially mathematisable.

All teachers also stated that children were given opportunities to pose problems. Yet the transcripts showed that ‘giving opportunity’ covered different practices: asking children to suggest a question, inviting them to choose a solution, allowing spontaneous problems to emerge in play, or jointly constructing a problem after teacher prompting. Eight teachers explicitly said that they themselves commonly initiated the problem, and nine described joint posing. The contrast is analytically important: reported child-centredness often concerned participation in a teacher-framed situation rather than children’s responsibility for specifying the mathematical problem.

The criteria teachers reported using were predominantly pedagogical. Real-life connection was mentioned by 14 teachers, appropriate difficulty by 12, and curiosity or interest by nine. These criteria are indispensable in preschool education. At the same time, explicit reference to the internal mathematical relationship, sufficiency of information, or coherence of the requirement was less visible in interview discourse.

5.3 Difficulties and professional learning needs

Teachers’ difficulties clustered around four demands. Eight reported difficulty adapting problems to children’s developmental, linguistic, or readiness differences. Six described difficulty producing an appropriate problem or question. Five found it difficult to distinguish structured, semi-structured, and open tasks. Four mentioned mathematical language and expression. T1 summarised the coordination challenge as “mathematising it, making it appropriate for preschool level, and being sure that the mathematics is correct.”

Professional learning preferences were strongly practice oriented. Nine teachers requested workshops, and eight requested in-service education or longer-term support. Five wanted an activity bank or examples, two requested mentoring, and six referred to technology or artificial intelligence. Their comments suggested that they wanted more than explanations of task categories: they wanted opportunities to generate examples, compare alternatives, receive feedback, and judge whether a problem was mathematically and developmentally appropriate.

5.4 Task performance: openness revealed increasing coordination difficulty

Across the 90 task products, 68 responses (75.6%) were classified as sufficient, 16 (17.8%) as insufficient, and six (6.7%) as not a problem. No response was classified as non-mathematical or impossible. The absence of non-mathematical and impossible products indicates that teachers generally remained within recognisable mathematical contexts and did not formulate contradictory situations. The main variation concerned whether the mathematical focus, information, requirement, and requested task structure were sufficiently coordinated.

Table 4
Classification of responses by visual context and task structure

Visual context	Task	Sufficient	Insufficient	Not a problem	Total
Construction materials	Structured	15	0	0	15
	Semi-structured	12	3	0	15
	Open	7	6	2	15
	Total	**34**	**9**	**2**	**45**
Grocery shopping	Structured	14	1	0	15
	Semi-structured	11	4	0	15
	Open	9	2	4	15
	Total	**34**	**7**	**4**	**45**
All responses	**Total**	**68**	**16**	**6**	**90**

Note. "Open" refers to the visually supported open prompt given to the teacher; it does not imply that the resulting problem product was open or of high quality. Productive openness was distinguished from ambiguity and insufficient specification.

Task structure produced the clearest pattern. In the construction-material context, all 15 structured responses were sufficient; this fell to 12 in the semi-structured condition and seven in the open condition. In the shopping context, 14 structured responses were sufficient, compared with 11 semi-structured and nine open responses. All six products classified as not a problem occurred in open tasks. The pattern therefore did not indicate a general inability to pose problems. Rather, it indicated increasing variability when teachers were required to select the mathematical focus and formulate the problem with less support.

The visual contexts produced the same total number of sufficient products (34 of 45 in each context), suggesting that neither scene was globally easier. They nevertheless afforded different mathematical relationships. The construction-material scene foregrounded comparison, equalising, pattern, colour, length, and spatial organisation; the shopping scene foregrounded quantity, sharing, missing amounts, and everyday numerical reasoning. Context shaped what teachers noticed, whereas task structure shaped whether the noticed possibility was transformed into a sufficiently specified problem situation.

The convergence between interview and task evidence is important. Teachers reported difficulty distinguishing task types and generating appropriate open problems, and this difficulty became visible in the response distribution. Their strong pedagogical commitments remained evident responses were generally meaningful and developmentally plausible but under open conditions some products remained activities, invitations, or broad prompts without a sufficiently explicit mathematical requirement.

6 Discussion

6.1 Pedagogical richness is necessary but not sufficient

The study's central contribution is to distinguish pedagogical richness from the coordination required for a mathematical problem situation. Teachers valued precisely the features expected in strong early childhood pedagogy: play, manipulatives, routines, familiar contexts, curiosity, and child participation. These orientations are not deficits. They are essential resources for engaging young children and for grounding mathematics in meaningful activity (Pramling et al., 2019; Gravemeijer, 2016). Yet the task products showed that these resources do not automatically produce a mathematical problem. A context becomes mathematically productive when a relationship is made salient and a requirement creates a reason for children to act, represent, compare, justify, or decide.

This finding extends work on teachers' problem-posing knowledge by showing why early childhood settings require a double coordination. Teachers must preserve developmental accessibility while specifying mathematical structure. The challenge is not simply to know mathematics or to know children; it is to make one visible through the other. This resonates with research on the challenges of mathematizing in early childhood education (Björklund & Palmér, 2024) and with accounts of specialised knowledge becoming visible through problem posing (Martín-Díaz & Montes, 2022).

Teachers' interview language also illuminates this coordination problem. They readily described contexts and child actions but less often named the mathematical relationship or the adequacy of information and requirement. This does not mean that they lacked mathematical awareness: most task products were sufficient and none was impossible or wholly non-mathematical. Rather, mathematical structure appeared more often embedded in practice than explicitly articulated as a design criterion. Professional learning may therefore need to help teachers make this tacit dimension discussable.

6.2 Open tasks increase responsibility; they do not guarantee richer posing

A second contribution concerns task openness. The decline in sufficient responses from structured to semi-structured and visually supported open prompts suggests that reduced constraint increases the poser's responsibility for selecting the mathematical relationship, deciding what information is relevant, formulating a requirement, and calibrating the task for young children. This pattern should not be interpreted as evidence that open problems are educationally inferior. Rather, it shows that prompt openness and product quality are different constructs. Consistent with Völlinger et al. (2026), openness may occur in the initial, intermediate, or goal state, and each requires deliberate design. When these design decisions were not coordinated, openness deteriorated into ambiguity, under-specification, or no problem at all.

This interpretation cautions against treating open-endedness as an intrinsic marker of quality. Open tasks can support creativity, flexibility, multiple strategies, and learner agency, but only when the

mathematical object of attention remains discernible. Cai et al. (2026) characterise problem posing as accessible to participation yet capable of very high mathematical demand. The present teacher data show the same distinction at the level of task design: producing a question or inviting choice was relatively accessible, whereas constructing an unambiguous, developmentally appropriate, and productively open problem required a more complex coordination of mathematical and pedagogical judgement.

The findings also suggest that structured and open tasks should not be arranged on a simple developmental hierarchy in which more open is necessarily more advanced. They place different demands on the poser. A teacher may produce a high-quality structured problem by recognising and expressing an established relationship, yet struggle to originate the relationship and requirement in an open context. Assessment and professional development should therefore sample multiple task structures rather than infer competence from a single format.

6.3 Context affords mathematics; the prompt organises attention

The identical number of sufficient responses across the two visual contexts is analytically useful. It indicates that overall quality was not determined by whether the image represented manipulatives or everyday life. Instead, the scenes afforded different mathematical possibilities. The construction materials made perceptual attributes and spatial arrangements salient; shopping made quantity and sharing relations salient. The context provided a field of possibilities, but the task prompt and teacher's selection organised attention within that field.

This distinction helps refine claims about the value of real-life contexts. Familiarity can make a situation meaningful, but familiarity alone does not define the mathematics. A grocery scene may invite social conversation, naming objects, or storytelling unless the poser identifies a quantitative, comparative, classificatory, or spatial relation. Similarly, manipulatives do not guarantee conceptual engagement if the requirement reduces activity to following a direction. Context should therefore be analysed in relation to the mathematical relationship and requirement it supports.

6.4 Implications for teacher education and professional learning

Teachers' requests for workshops, examples, mentoring, and sustained support align closely with the performance pattern. A productive professional learning design would not begin with a list of definitions alone. It would place contrasting problem situations side by side and ask teachers to identify the context, mathematical relationship, information, requirement, and expected child action. Teachers could then revise an insufficient activity prompt into a problem, vary the degree of structure while preserving the mathematics, and anticipate multiple child strategies.

A possible learning cycle would include five moves: noticing the mathematics afforded by a context; specifying a meaningful requirement; analysing developmental accessibility and possible representations; anticipating children's choices and solution strategies; and revising the task after peer or expert feedback. Such cycles are consistent with practice-based approaches to developing didactic analysis through problem posing (Malaspina et al., 2019), with work inviting prospective teachers to judge what makes a problem mathematically interesting (Crespo & Sinclair, 2008), and with evidence that teachers' products evolve through iterative design, feedback, classroom testing, and revision (Marco & Palatnik, 2024). A single workshop may raise awareness, but sustained opportunities are more likely to support movement beyond familiar task forms.

Example banks may be useful, but they should not become repositories for imitation. Their value lies in supporting comparison: Why is one prompt a problem and another merely an instruction? How does changing one piece of information alter the requirement? What mathematical ideas become visible in a block scene, a snack routine, or a shopping context? How might the same relationship be posed in structured, semi-structured, and open forms? Technology and generative AI may support variation, but teachers still require criteria for judging mathematical coherence, developmental appropriateness, and bias in generated content.

7 Limitations and future research

The study involved 15 teachers and two researcher-selected visual contexts. Its purpose was analytic description rather than population estimation; the frequencies should not be interpreted as prevalence estimates. Coding was collaborative and consensus based rather than independent, so no intercoder reliability statistic is available. Future studies should retain the collaborative interpretive work while also independently coding a substantial proportion—or all—of a similarly sized task dataset and reporting an appropriate agreement coefficient.

The ordering of the task types and images was fixed. Performance may therefore have been influenced by practice, fatigue, or order effects. Future studies should counterbalance context and task order, and should examine how alternative prompt wordings redistribute responsibility between the task and the teacher.

The visual stimuli were reviewed by the researchers but the source manuscript did not document an external expert-validation procedure. Future work should involve early mathematics and early childhood experts, cognitive interviewing, or pilot testing to examine what mathematical affordances participants perceive in each image.

The study analysed posed products but did not observe how teachers enacted them in classrooms. A written or spoken problem may be transformed through questioning, gesture, materials, children's responses, and contingent teacher decisions. Video-based studies could investigate the relationship between posing a problem and sustaining its mathematical potential in interaction.

Finally, the study captured current performance without feedback or revision. Intervention research could examine how comparing examples, peer discussion, expert feedback, and repeated revision change teachers' ability to coordinate mathematical and pedagogical demands. Longitudinal designs would be especially valuable for studying whether gains transfer from workshop tasks to spontaneous classroom problem posing.

8 Conclusion

Preschool teachers in this study brought strong pedagogical resources to early mathematics problem posing. They valued play, concrete materials, everyday routines, meaningful contexts, appropriate difficulty, and child participation. Most of their task products were sufficient. Nevertheless, performance became more variable as task structure decreased, and all products classified as not a problem appeared in open tasks.

These findings support a relational account of problem-posing competence. Competence lies not merely in producing an engaging context or a question, but in coordinating pedagogical appropriateness with an explicit mathematical relationship, sufficient information, and a meaningful requirement for children's action. Open tasks can make this competence visible because they transfer more responsibility for mathematical specification to the teacher. Teacher education should therefore provide repeated opportunities to analyse, compare, revise, and enact problem situations across different levels of structure.

Declarations

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Competing interests

The authors have no relevant financial or non-financial interests to disclose.

Ethics approval

Ethical approval for this study was granted by the Human Research Ethics Committee of **** University Rectorate under protocol number **2026-171**. The committee unanimously determined that the study was ethically appropriate. The approval was issued on **23 March 2026**, with decision letter number **E-34183927-020-00001188635**.

Consent to participate

Before data collection, all participants were informed about the purpose and scope of the study, the data collection procedures, the voluntary nature of participation, the confidentiality of their information, and their right to withdraw from the study at any time without providing a reason. Informed consent was obtained from all participants prior to their participation.

Consent for publication

The participants were informed that anonymised excerpts from their interview responses and problem-posing products could be used in scientific publications. No personally identifiable information is included in the manuscript.

Data availability

The qualitative data generated and analysed during the current study are not publicly available because the interview transcripts and task-based responses contain detailed contextual information that could compromise participant confidentiality. Anonymised excerpts supporting the findings are included in the manuscript. Further information may be obtained from the corresponding author upon reasonable request and subject to ethical approval and confidentiality restrictions.

Authors' contributions

All authors contributed to the conceptualisation and design of the study. Data collection, coding, and analysis were conducted collaboratively by the research team. The authors jointly interpreted the findings, reviewed the manuscript critically for important intellectual content, and approved the final version for submission.

Use of artificial intelligence

Artificial intelligence tools were used to generate the two visual contexts employed in the problem-posing tasks. The pedagogical and mathematical content, suitability, and final selection of the visuals were reviewed and determined by the researchers. Artificial intelligence was not used to generate, code, or interpret the research data.

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Figures

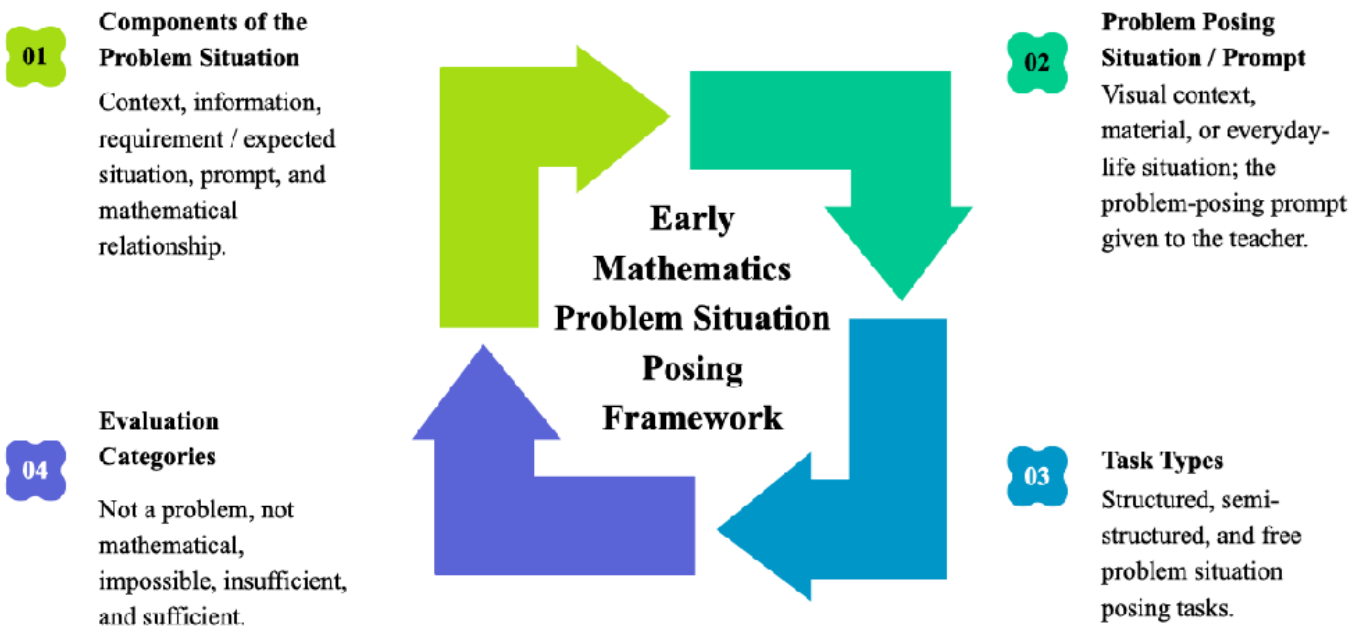


Figure 1

Integrated analytical framework for early mathematics problem-situation posing

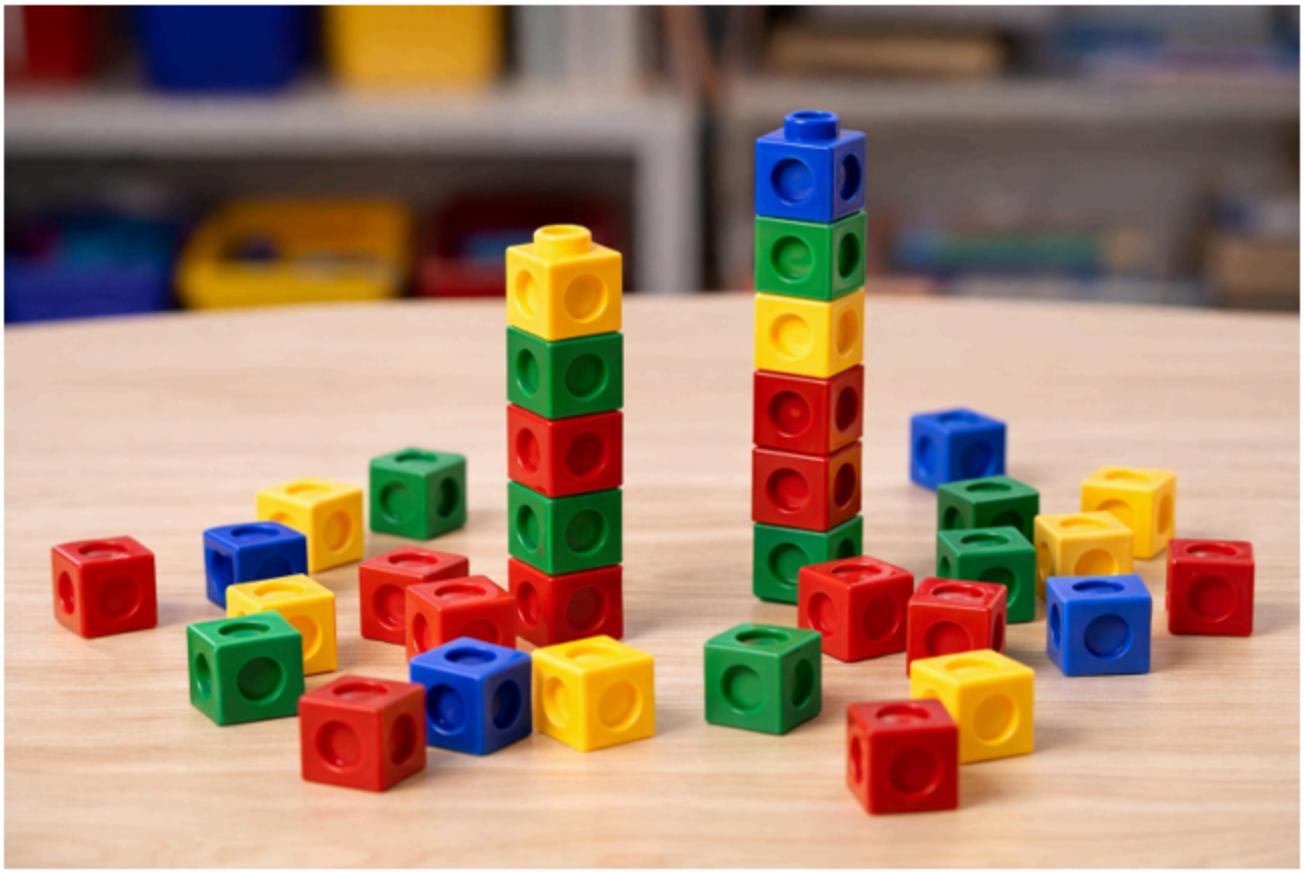


Figure 2

Construction-material context used in the problem-posing tasks



Figure 3

Grocery-shopping context used in the problem-posing tasks